Wrinkling of Sandwich Panels for Marine Applications

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Preface

The work presented in this doctoral thesis has been carried out at the Department of Aeronautical and Vehicle Engineering at the Royal Institute of Technology, between February 1999 and August 2003. The Swedish Defence Material Administration, FMV, has provided the financial support through the contract 63823-LB126424. The research was initiated through the design of the Visby class corvette and a close cooperation with Kockums AB Karlskronavaret has been held through the more than four years of research.

I would like to express my gratitude to the department and all the staff that made me feel welcome and appreciated. During my research I have always been able to exchange ideas with fellow colleagues, both in their offices and also of course in the coffee room. Special thanks to my supervisor Professor Dan Zenkert who has always been there for me when I have needed support and new fresh ideas. My contact with FMV, and especially Mr. Anders Lönno, has also provided valuable insights that has helped me throughout the work.

The diversity of the tasks I have been doing during the time spent at the department has been very valuable to me personally. I have chosen to investigate wrinkling of sandwich plates, a problem some people would argue is narrow but the more time I spent with it, the more vast and intriguing it seems.

My deepest gratitude to my family and friends for being there and making my life fun to live. I would not be the person I am if not my parents Mats and Malin had raised me and my brothers Magnus and Jonas like they did. Letting us learn by our mistakes and encouraging us to go through life with a curious mind.

Finally, all my love to Kristina for choosing to share her life with me. Not everybody would appreciate my sometimes all too present absent-mindedness towards life and responsibilities, when my mind was occupied with new interesting formulas coming to life.

Stockholm, August 2003

Linus Fagerberg
Abstract

The recent development in the marine industry with larger ships built in sandwich construction and also the use of more advanced materials has enforced improvements of design criteria regarding wrinkling. The commonly used Hoff’s formula is not suited for the highly anisotropic fibre reinforced sandwich face sheets of today.

The work presented herein investigates the wrinkling phenomenon. A solution to wrinkling of anisotropic sandwich plates subjected to multi-axial loading is presented. The solution includes the possibility of skew wrinkling where the wrinkling waves are not perpendicular to the principal load direction. The wrinkling angle is obtained from the solution together with the maximum wrinkling load. This method has been supported with tests of anisotropic plates subjected to uni-axial and bi-axial loading.

The effect of the face sheet local bending stiffness shows the importance of including the face sheet stacking sequence in the wrinkling analysis. The work points out the influence of the face sheet local bending stiffness on wrinkling. Three different means of improving the wrinkling load except changing core material is evaluated. The effect of the different approaches is evaluated theoretically and also through comparative testing.

The transition between wrinkling and pure face sheet compression failure is investigated. Theoretical discussions are compared with compressive test results of two different face sheet types on seven different core densities. The failure modes are investigated using fractography. The results clearly show how the actual sandwich compression failure mode is influenced by the choice of core material, changing from wrinkling failure to face sheet micro buckling failure as the modulus density increases.

Finally, a new approach is presented where the wrinkling problem is transferred from a pure stability problem to a material strength criterion. The developed theory provides means on how to decide which sandwich constituent will fail first and at which load it will fail. The method give insight to and develop the overall understanding of the wrinkling phenomenon. A very good correlation is found when the developed theory is compared with both finite element calculations and to experimental tests.

Keywords: wrinkling, local buckling, imperfection, stability, anisotropy, sandwich
Aims and scope

Throughout the more than four years of research the goal has been to investigate and further increase the understanding of the wrinkling phenomenon. This has been done with the simple theories and without complicating things more then necessary. The research should be applicable and made known to the sandwich community. All findings should be presented so that the largest possible audience can understand them.

In this thesis sandwich wrinkling is investigated. This is done in the context of linear-elastic material constituents and isotropic core materials. Issues like dimpling, where the face sheet buckles into the honeycomb cells, are not included, neither is the local buckling or local indentation that can occur close to load introductions. The thesis specifically deals with the effects of anisotropy in the face sheets typically derived from the use of aligned fibres and different stacking sequences. It is investigated and exemplified how the wrinkling load can be increased through altering the composition of the face sheet as well as changing core material. The transition from wrinkling to the pure compression failure is also examined. Finally, wrinkling is transferred from a traditional stability criterion into a material failure criterion in order to study the effect of initial imperfections. All theories developed within the thesis are thoroughly compared to experimental tests and numerical finite element models. Thus, the contents of this thesis provide an extension of the classical work on wrinkling.
Dissertation

The thesis includes a short introduction to the common wrinkling theories where the most well known and most often referred to formulae are derived and discussed. Some special cases of wrinkling and their proposed solutions are presented. The introduction is followed by four appended papers:

**Paper A**

Fagerberg L. and Zenkert D., “Effects of Anisotropy and Multi-Axial Loading on the Wrinkling of Sandwich Panels”, *Submitted for publication*.

**Paper B**


**Paper C**

Fagerberg L., “Wrinkling and Compression Failure Transition in Sandwich Panels”, Accepted for publication in *Journal of Sandwich Structures and Materials*.

**Paper D**

Fagerberg L. and Zenkert D., “Imperfection Induced Wrinkling Material Failure in Sandwich Panels, *Submitted for publication*.
Division of work between authors

Paper A
Fagerberg performed the analytical calculations, tests and finite element calculations and wrote most of the paper. Zenkert initiated and outlined the work and assisted in all analytical calculations and helped writing the paper.

Paper D
Fagerberg initiated the work, developed the analytical model, performed the tests and calculations. Zenkert supported throughout the work and the paper was written jointly by the authors.
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Introduction

A structural sandwich comprises of two stiff face sheets on a thick lightweight core. One of the benefits is that the sandwich has a very high bending stiffness compared to its weight and it is therefore often used within the aeronautical, vehicle and marine industry where low weight and high load carrying capabilities immediately allow for higher payload. The high bending stiffness comes from that the stiff face sheets are separated (in space) and the core bonds them together, making them work in concert much more effectively then if they would have been alone. However, the sandwich is not that good at carrying in-plane compressive loads and it has a unique stability failure mode of its own – wrinkling. In this thesis this failure mode is investigated. The thesis starts with some notes on sandwich within the marine industry and why wrinkling is nowadays getting more attention. The background to the problem is discussed and thereafter follows an introduction to the classical wrinkling theories. A discussion on wrinkling related problems is given, including finite element analysis methods. The summary of the thesis providing information on how and where this thesis makes a contribution to the subject is then given, followed by a short discussion on possible future work.

Figure 1. The first Visby class corvette at the Karlskrona shipyard the 5 of March 2002.

Background

The sandwich concept, using two thin stiff face sheets in combination with a thick lightweight core, has been utilised in structures for some time. During the last decade the development within the Swedish marine industry has been expanding the boundaries of what previously was believed possible. Larger vessels then ever before are now being built in sandwich construction and this brings some new considerations to the attention of the engineer. Kockums Karlskronavarvet, who is part of the HDW group, builds the Visby class corvettes for the Swedish Navy, see Figure 1. Visby is a surface combatant, designed for a wide range
of roles: anti-surface warfare, anti-submarine warfare, mine countermeasures, patrol and much more.

A small boat is usually designed to be tough enough to handle, i.e. it should feel comfortably solid and it should not break when drawn up on a beach. A high-speed powerboat has to be able to sustain high slamming loads in addition. All these loads are localised and mainly perpendicular to the boat hull. Therefore sandwich boats of small to medium dimensions often have a rather high-density core to prevent local face indentations and proportionally thick face sheets to avoid face failure.

A larger boat hull is often more slender, since the drag through water is less for a slender hull than for a bulky one. For longer hulls other types of forces have to be included in the design of the ship.

A boat travelling through heavy seas is subjected to wave induced loads. Figure 2 shows what the shipwright refers to as “hoggimg” and “sagging” motion. When the ship is subjected to hogging, the midsection is lifted on a wave while the bow and aft is not supported. This causes tensile stresses in the deck and compressive stresses in the bottom. In sagging it is the other way around, the bow and aft is lifted, while the midsection is not supported. Sagging therefore creates compressive stresses in the deck and tensile in the bottom. If the ship also travels through the waves misaligned with the wave front the ship will be subjected to torsion as well.

![Figure 2. Hogging and sagging of ship hull.](image)

The two types of loading are usually not of the same importance for different ship types. The weight distribution compared to the volume distribution of the ship decides which of the cases is worst. For the type of ship that Visby is (see Figure 1), sagging is more severe.

This wave induced hogging and sagging leads to high in-plane stresses in the deck and bottom of the ship. These stresses in long and slender boats are much higher than almost any localised load can give rise to. Therefore this kind of ship has to have deck and bottom panels designed to sustain high in-plane loads, both compressive and tensile.

It is often more difficult to design a panel for compressive forces than for tensile. In compression there are several different possible failure modes that have to be considered. One of these is the compressive failure of the face sheet, which often for fibre composites takes the form of microbuckling. In addition to compressive failure, a structure loaded in compression can sometimes collapse through loss of stability. There are several classic examples of this
and the most well known is possibly the buckling of a column (Figure 3). Columns are however rather rare in ships but plates can of course buckle as well. The critical number of waves is not necessarily the same for a sandwich plate as for a single skin composite or isotropic metallic plate since the sandwich have much lower shear stiffness. A sandwich can in addition to the previously described global form of buckling also lose stability through local buckling, where the instability is within the sandwich, i.e. the sandwich face sheet buckles but the symmetry plane of the sandwich remains more or less undeformed.

![Figure 3. The first three buckling modes of a simply supported a thin column and the first buckling mode of a simply supported long plate.](image)

The local buckling can in turn be divided into several subgroups; Localised buckling, which occur close to load introductions or loads applied perpendicular to the sandwich face sheets, dimpling, where the face sheet buckles into the cavities of honeycomb cores, and finally wrinkling, which in theory can occur simultaneously on the entire sandwich plate. Wrinkling is sometimes referred to as natural wavelength buckling since it is associated with periodic waves of a wavelength depending on the material properties, core- and face thickness, and not on the in-plane dimensions of the sandwich. Wrinkling will be more thoroughly described below.

The local types of buckling have not been much of a concern for shipwrights building in sandwich using glass fibre composites and high density core materials. But the development in the material field has led to a decrease in cost for materials which formerly where used only in aerospace structures. The high performing materials allow for a more full-blown utilisation of the sandwich concept with thinner face sheets and more lightweight cores. This development has led to that wrinkling can be the active design constraint. The theories now in use when designing against wrinkling is often rather old and they were derived when most face sheets used were metallic and they do therefore badly suit the fibre composite face sheets of today.

**Introduction to wrinkling theory**

In this section a short introduction to some of the most commonly referred theories on wrinkling are presented together with the derivation of a few frequently used and refereed to formulae.

The first reported work on sandwich face wrinkling was published in early 1940’s by Gough et al. [1] and Williams et al. [2]. The results from these investigations are in essence covered
by the following. Local instability of the sandwich face is of course possible in pure in-plane compression of the sandwich, but also when it is subjected to ordinary bending, since one of the faces will be in membrane compression.

The wrinkling wave is a periodic wave with the (half) wavelength \( l \), see Figure 4.

\[
D_f \frac{d^4w}{dx^4} + P \frac{d^2w}{dx^2} - \sigma_z = 0
\]

where \( D_f \) is the bending stiffness of the face sheet alone, \( P \) is the compressive load acting on the face sheet and \( \sigma_z \) the support pressure from the core, the elastic foundation. There are several different cases of wrinkling, depicted in Figure 5, and ways that this problem has been solved.

The occurrence of the different wrinkling modes depends on the geometry of the sandwich (thickness of components) and the material properties. In the first case (Case I) “rigid base” or single-sided wrinkling, only one face sheet of the sandwich wrinkles. This mode can appear when the sandwich is under bending and only one face sheet under compression, or if the sandwich has un-symmetric face sheets, one with lower buckling load than the other.

The second mode (Case II) is the anti-symmetrical, in-phase or “snake” mode where the two face sheets wrinkle in phase. In this case the two face sheet deformations share the same phase. This mode is common for anisotropic core materials where the shear stiffness is considerably lower than the Young’s modulus (perpendicular to the face sheet), for example honeycombs, and for sandwich configurations with a thin core. A sandwich configuration prone to snake mode wrinkling must carefully be checked against all global buckling modes. It is likely that a global buckling analysis will predict even lower buckling loads than Case II wrinkling formulae.

The third case (Case III) is defined as symmetrical wrinkling, out-of-phase or “hourglass” wrinkling. Hourglass wrinkling is a special case of rigid base wrinkling with a smaller
distance to the rigid plane. A smaller distance to the rigid base provides a firmer support of the face sheet and therefore makes the wavelength shorter and critical load higher. Hence Case III wrinkling formulae always predict a higher wrinkling load than Case I formulae. This mode is accordingly a bit theoretical and of less importance to the engineer.

In wrinkling the displacements of the faces are transmitted to the core in which they damp out rapidly in the thickness direction. If the core thickness exceeds a certain value the mid-plane of the core remains flat for all three wrinkling mode cases. In the real world foreign to symmetry and perfect materials it is of course possible to have wrinkling where the waves are not exactly in phase or anti-symmetrical and where the wrinkling amplitude differs between the face sheets. Both face sheets of the sandwich will most probably have some sort of imperfection or not being entirely symmetric and this will in most cases lower the wrinkling load that we are about to derive.

The wrinkling load can be derived in several different ways and the following sections describe some of them. The first and fourth methods described are based on solving the governing differential equations, while the second and third are based on energy methods. Each approach has a different assumption for the core stress. Although the approaches are different they yield approximately the same result, apart from the first one, and the differences are mainly due to different assumptions on the decay of the transverse stress.

**No decay: Winkler foundation**

When attempting to solve a buckling problem some assumptions must be made regarding the displacement field. This is also done in ordinary beam theory where for example the beam section is supposed to remain flat and perpendicular to the neutral axis of the beam. A simple assumption regarding the core as a medium supporting the sandwich face is the “Winkler foundation” model. This assumes that the support material consists of an array of continuously distributed linear springs, as shown in Figure 6.

In the snake mode case it is seen that the springs remain unloaded even after wrinkling. It is further seen that the mode of deformation in the core is shear rather than tension/compression, which the set of springs are unable to model, and hence no solution can be derived in this case. In the symmetrical case, on the other hand, the model becomes more realistic since the mode of deformation in the core is both tension/compression and shear. In this case it assumes a core with some modulus perpendicular to the faces ($E_{zz}$) but with no out-of-plane shear stiffness, i.e., $G_{czz} = 0$. The Winkler model then suggests that the core stress due to the wrinkling of the face is proportional to the face sheet deformation, and may be written as

$$\sigma_{czz} (x, 0) = -K_z w(x, 0) = -K_z w_f (x)$$

(2)

where $K_z$ is the so called “foundation modulus”. Since the stress is constant through the core thickness and core shear is neglected the wrinkling wave ($w(x, z)$) damps out linearly through the core. It further has to be equal to zero at the symmetry plane (Case III wrinkling, see
Figure 4 and 6). Therefore the wrinkling wave can be written as \( w(x,z) = w_f (1 - 2z/t_c) \).
Assuming further that one may treat the set of springs as a continuous media so that \( \varepsilon_z = dw/dz \) and \( \sigma_{cz} = E_{cz} \varepsilon_{cz} \), we may then for \( z = 0 \) write

\[
\sigma_{cz}(x,0) = -K_z w_f(x) = -E_{cz} \frac{2}{t_c} w_f(x) \quad \text{so that} \quad K_z = \frac{2E_{cz}}{t_c}
\]

To derive the governing equation following this approach, a small section of the sandwich face sheet can be examined, see Figure 7.

![Figure 7. Forces, bending moments and stresses acting on the face sheet.](image)

By treating the face sheet as an ordinary beam or strut and neglecting transverse shear deformations (a most reasonable assumption for a thin composite or metal face), the governing equation for the beam is derived from the two equilibrium equations for the face

\[
\frac{dM_x}{dx} - T_x = 0 \quad \text{and} \quad \frac{dT_x}{dx} + \sigma_{cz} + N_x \frac{d^2w}{dx^2} = 0
\]

leading to the differential equation (also using that \( d^2w/dx^2 = -M/D_j \) and taking the in-plane load as compressive, i.e., \( N_x = -P \))

\[
\frac{d^4w}{dx^4} + \frac{P}{D_j} \frac{d^2w}{dx^2} + \frac{K_z}{D_j} = 0
\]

This is solved by assuming some appropriate function for the deflection field \( w \) that fulfils Equation (5). One such is simply the usual wrinkling wave equation

\[
w = W \sin \frac{\pi x}{l}
\]

Inserting this into Equation (5) yields an expression for \( P \) as

\[
P = D_j \left( \frac{\pi}{l} \right)^2 + K_z \left( \frac{1}{\pi} \right)^2
\]

To find the critical load one must further minimise \( P \) with respect to the unknown wavelength \( l \), which gives

\[
\frac{dP}{dl} = -\frac{2D_j \pi^2}{l^2} + \frac{2K_z l}{\pi^2} = 0 \quad \Rightarrow \quad l^* = \frac{D_j \pi}{K_z}
\]

with the critical load as

\[
P_{cr} = 2D_j K_z = 2 \sqrt{\frac{E_{ji} l^4 E_{cz}}{6t_c}} \quad \text{or} \quad \sigma_{w} = 0.8165 \sqrt{\frac{E_{ji} l^4 E_{cz}}{t_c}}
\]
To derive a Winkler based formula for rigid base (Case I) wrinkling it simply to assume that \( w \) must be zero for \( z = t_c \) and to redo the derivation (the foundation modulus is in that case equal to \( E_c/t_c \) providing only half the previous support).

The winkler approach suffers from the drawback that the shear in the core is neglected. This is only reasonable either if the core truly has a very low shear modulus, or if the wavelength of the wrinkling is sufficiently long. In all other cases, there will be some shear lag effect smoothing out the stress \( \sigma_{cz} \) so that it decreases with increasing \( z \). The use of Equation (9) may therefore be somewhat unreliable.

**Linear decay: Hoff and Mautner’s method**

The next method to assess the wrinkling problem is that of Hoff and Mautner [3]. In this case the shear stress in the core is accounted for and the shear lag is modelled by a linear decay function. The centre line of the core is in the first case assumed to be undeformed and this model thus simulates Cases I and III.

The deformation of the face induces tensile and compressive stresses in the core perpendicular to the face. If the wavelength, \( l \), is short it would hardly effect the material in the middle of the core and hence it is assumed that the core is only affected in a small zone with depth \( h \). Since the buckling is symmetrical the core deforms only in the \( z \)-direction. Assuming the face to undergo a sinusoidal displacement (Equation (7)), and that the wave damps out linearly (linear decay) with coordinate \( z \), and we can then write

\[
w = \frac{W(h-z)}{h} \sin \frac{n \pi x}{L} = \frac{W(h-z)}{h} \sin \frac{\pi x}{l},
\]

where \( L \) is the length of the beam or strut, \( l \) the wavelength of the wrinkles (see Figure 4) and that \( n \) is allowed to be a large number. Assuming the core to have low (or zero) in-plane modulus the tensile/compressive stress in the core can be written \( \sigma_{cz} = E_c \partial w/\partial z \) and if the deflections are in the \( z \)-direction only the core shear stress is \( \tau_{cz} = G_c \partial w/\partial x \). It may be worth noticing that the assumed stress field in the core not satisfies the 2-D equilibrium condition \( \sigma_{ij} = 0 \). Anyhow, using this assumption the strain energy stored in the core over the length \( l \) will become

\[
U_c = \frac{1}{2} \int_0^l \int_0^h \sigma_{cz}^2 dxdz + \frac{1}{2} \int_0^h \tau_{cz}^2 dxdz = \frac{E_c W^2 l}{4h} + \frac{G_c \pi^2 W^2 h}{12l}
\]

The strain energy stored in the face due to bending is

\[
U_f = \frac{D_f}{2} \int_0^l \left( \frac{d^2w}{dx^2} \right)^2 dx = \frac{W^2 \pi^2 l E_f}{48l^3}
\]

The work done by the applied load is

\[
U_p = -\frac{1}{2} \int_0^l P \left( \frac{dw}{dx} \right)^2 dx = -\frac{W^2 \pi^2 l P}{4l} = -\frac{W^2 \pi^2 l P}{4l} \sigma_f
\]

The energy equation \( U = U_f + U_c + U_p \) can now be solved for \( \sigma_f \) which after some rearrangement results in
The critical stress in this equation depends on the parameters \( l \) and \( h \). The values of these are those that minimise the critical stress. Therefore, they are found by

\[
\frac{d\sigma_f}{dh} = -\frac{E_i}{\pi^2 t_f h^2} + \frac{G_e}{h^3} = 0 \quad \text{and} \quad \frac{d\sigma_f}{dl} = \frac{2E_i}{\pi^2 t_f h^2} - \frac{\pi^2 E_i t_f^2}{6l^3} = 0
\]  

Solution of the two equations are

\[
\frac{h}{t_f} = 0.911 \sqrt{\frac{E_f E_i}{G_e^2}} \quad \text{and} \quad \frac{l}{t_f} = 1.656 \sqrt{\frac{E_f^2}{E_i G_e}}
\]

Substitution into Equation (14) yields the critical face stress as

\[
\sigma_{Ho} = 0.911 \sqrt{\frac{E_f E_i G_e}{E_i}}
\]

Now, this formula is only correct when the zone \( h \), as given in Equation (16), is smaller than half the core thickness \( \frac{t_c}{2} \) (or \( t_c \) if Case I wrinkling is treated). If not, the assumed displacement field in Equation (10) must be changed so that \( \frac{t_c}{2} \) is substituted for \( h \) and the same derivation as outlined above is again performed. This will result in the following relations [3].

\[
\frac{l}{t_f} = 1.421 \sqrt{\frac{E_f E_i}{E_t f}} \quad \text{and} \quad \sigma_{Holl} = 0.817 \sqrt{\frac{E_f E_i t_f}{E_t}} + 0.166 G_i \left( \frac{t_c}{t_f} \right)
\]

As seen, Equation (18) gives the same result as the Winkler model in the previous section but with an extra term for the shear part. This is not surprising since even the Winkler model assumes some linear decay of the deformation and that \( w = 0 \) for \( z = \frac{t_c}{2} \). Hence, Equation (17) should be used for symmetrical (Case III) wrinkling with thick cores and Equation (18) for Case III wrinkling on thin cores.

By making a similar analysis of the anti-symmetrical case (Case II) the following results are obtained [3]. For \( h < \frac{t_c}{2} \) (thick cores)

\[
\frac{h}{t_f} = 1.516 \sqrt{\frac{E_f E_i}{G_e^2}} \quad \text{and} \quad \frac{l}{t_f} = 2.156 \sqrt{\frac{E_f^2}{E_i G_e}}
\]

\[
\sigma_{Holl} = 0.513 \sqrt{\frac{E_f E_i G_e}{E_i}} + 0.33 G_i \left( \frac{t_c}{t_f} \right)
\]

and for \( h = \frac{t_c}{2} \) (for thin cores)

\[
\frac{l}{t_f} = 1.671 \sqrt{\frac{E_f E_i}{E_t f}} \quad \text{and} \quad \sigma_{Holl} = 0.59 \sqrt{\frac{E_f E_i t_f}{E_t}} + 0.387 G_i \left( \frac{t_c}{t_f} \right)
\]

Hoff and Mautner [3] concluded from their analysis that the very simple formula, Equation (17), is a conservative estimate of the buckling load for all cases except for thin cores where Equation (21) can predict a lower load. Hoff also found that the prediction of the buckling
load trends agreed very well with tests but suggested that in practical cases the load should be predicted using the conservative formula

$$\sigma_{ho} = 0.5 \sqrt{E_E E_c G_c} \quad (22)$$

This is the formula a large part of the industry uses today and is often referred to as the Hoff’s formula. According to this formula, the wrinkling stress is independent of the sandwich geometry and a function only of face and core properties. It also follows that the core will have the most influence on the wrinkling load with improving load bearing capacity for cores with high elastic properties.

**Exponential decay: Plantema’s approach**

Shear lag problems are usually described by some exponential functions in hyperbolic sines or cosines. Therefore, another deflection function was used by Plantema [4] for the stress decay assuming the waves to damp out exponentially (exponential decay) as

$$w = We^{-k_c} \sin \frac{n \pi x}{L} = We^{-k_c} \sin \frac{n \pi x}{l} \quad (23)$$

We further assume that there are no displacements in the x-direction, i.e. $$u = 0$$. The following derivation based on this assumption also assumes basic wrinkling, or rather that the buckling of one face has no effect on the face sheet on the other side. Using this function in the same way as outlined above, one can write

$$\sigma_{zz} = E_e E_{zz} = E_e \frac{\partial w}{\partial z}, \quad \sigma_{zz} = 0, \quad \tau_{zz} = G_c \gamma_{zz} = G_c \left( \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z} \right) = G_c \frac{\partial w}{\partial x} \quad (24)$$

Again, this assumption violates the 2-D equilibrium condition $$\sigma_{ij,j} = 0$$ in the core. The bending strain energy of the face sheet can from this assumption be written

$$U_f = \frac{D_f}{2} \int_0^l \left( \frac{d^2 w_f}{dx^2} \right)^2 dx = \frac{W^2 D_f \pi^4}{4l^3} \quad (25)$$

and the strain energy in the core as

$$U_c = \frac{1}{2E_c} \int_0^l \sigma_{zz}^2 dx dz + \frac{1}{2G_c} \int_0^l \tau_{zz}^2 dx dz = \frac{E_c W^2 k l}{8} + \frac{G_c \pi^2 W^2}{8 k l} \quad (26)$$

and finally

$$U_p = -\frac{1}{2} \int_0^l \rho \left( \frac{dw_f}{dx} \right)^2 dx = -\frac{PW^2 \pi^2}{4l} \quad (27)$$

which results in the total strain energy as

$$U = \frac{\pi^2 W^2 D_f}{4l^3} + \frac{E_c W^2 k l}{8} + \frac{G_c \pi^2 W^2}{8 k l} - \frac{PW^2 \pi^2}{4l} \quad (28)$$

Minimising with respect to $$W$$ leads to an expression for the load as

$$\frac{\partial U}{\partial W} = 0 \rightarrow W = 0 \text{ (trivial solution)} \quad (29)$$
\[ P = \frac{\pi^2 D_f}{l^2} + \frac{E_v k l^2}{2\pi^2} + \frac{G_{cv}}{2k} \]  
(non-trivial solution)  

(30)

and by then letting \( \partial P/\partial l = \partial P/\partial k = 0 \) one arrives at

\[ P_{nl} = \frac{3}{2} \sqrt{2D_f E_v G_{cv}} \]  

(31)

Which is often referred to as the Plantema formula. If the face sheet is isotropic Equation (31) can be rewritten to

\[ \sigma_{nl} = 0.825\sqrt{E_f E_v G_{cv}} \]  

(32)

As seen, this is very close to the result obtained by Hoff and Mautner for the symmetrical case and thick core (see Equation (17)). In fact, the only difference is that another decay function has been used, otherwise the assumptions are the same. It must be understood that Equation (31) is much more powerful than Equation (10) or (32). The Plantema formula incorporates the effect of the local bending stiffness and not only the in-plane stiffness of the sandwich face sheet. This is of the greatest importance when dealing with anisotropic and layered composite face sheets commonly used today.

**Differential equation method: Allen’s approach**

The following derivation assumes a thin beam (the face) under compressive end loads, continuously supported by an elastic medium, which extends infinitely on one side of the beam. Hence, the face is unaffected by the opposite face. The differential equation for a homogeneous beam or plate in cylindrical bending (the face) with \( N_i = -P \) is

\[ D_f \frac{d^4 w}{dx^4} + P \frac{d^2 w}{dx^2} = q \]  

(33)

Now, the transverse load \( q \) equals \( \sigma_c \). Assume that the core is isotropic and the same shape of the face as above, i.e., \( w = W \sin(\pi x/l) \). By assuming an Airy's stress function the stress, \( \sigma_{cz} \), necessary to deform the core in this manner may as Allen [5] showed be written

\[ \sigma_{cz} = -\frac{a}{l} W_m \sin\left(\frac{\pi x}{l}\right) \text{ with } a = \frac{2\pi E_v}{(3-v_c)(1+v_c)} \]  

(34)

This leads to expressions for the natural wavelength and instability loads reading [5]

\[ l = \sqrt{\frac{2\pi^4 D_f}{a}} \quad \text{and} \quad P_{Allen} = \left( \frac{1}{2} + \frac{1}{2} \right)^{\frac{1}{2}} \frac{1}{\pi^2} \sqrt{D_f a^2} \approx 0.88 \sqrt{D_f a^2} \]  

(35)

If the face sheet is isotropic the bending stiffness can be substituted and the resulting formulæ for the wrinkling wavelength and critical stress are obtained as

\[ l = \pi t_f \sqrt{\frac{(3-v_c)(1+v_c) E_f}{12(1-v_c^2) E_v}} \quad \text{and} \quad \sigma_{Allen} = \frac{3}{\sqrt{12(3-v_c^2)(1+v_c)}} \sqrt{E_f E_v^2} \]  

(36)
This gives for $\nu_c = 0.3$

$$\sigma_{AI} = 0.58\sqrt{\frac{E_f E^2}{G_c}}$$

(37)

and with $G_c = E_c / 2(1 + \nu_c)$

$$\sigma_{AI} = 0.78\sqrt{\frac{E_f E G_c}{(1 - \nu^2_c)}}$$

(38)

which is very close to the result in Equation (17) and (32) and also the proposed formula in Equation (22). Again the formula for anisotropic face sheets, Equation (35), is more powerful than Equation (37) or (38), since it incorporates the effect of the local bending stiffness of the face sheet.

Several papers have been written on this topic since these more classical works were published. In general, the approaches have been to find a more unified approach to solve the different wrinkling modes. One such approach was published recently by Niu and Talreja [6]. Others attempt to combine global buckling of sandwich columns and wrinkling, e.g. [11]. In essence, there is little new to these approaches, they come to basically the same conclusion that the "old" formulae in principle agrees well. Surprisingly, almost all the recent work on sandwich panel wrinkling have in common that they treat uni-axial loading of sandwich beams/columns, without initial imperfections and there is very, very seldom any experimental verification.

**Special situations**

**Wrinkling of sandwich panels**

The above analyses have been derived for a state of plane stress, which is legitimate provided that the column is narrow. When it is wide, the column must be treated as a panel and a state of plane strain predominates. In an analysis similar to that of the column but for panels, Norris et al. [7] derived the expression for the wrinkling stress as

$$\sigma_{cr} = \Phi \sqrt{\frac{E_f E_c}{(1 - \nu^2_f)(1 - \nu^2_c)}} G_c$$

(39)

using differential equations where the constant $\Phi$ equals 0.72 for $\nu_f = 0.3$ and $\nu_c = 0.2$. This indicates that the formula experimentally verified by Hoff and Mautner, Equation (22), can be used even for panels, but with the $E$ substituted by $E/\nu (1 - \nu^2)$. In fact, the tests verifying this formula were performed on panels under uniaxial compressive loads [3].

**Wrinkling under multi-axial load**

The results from previous sections are only valid for beams or plates under uni-axial loading. Plantema [4] has presented results indicating that the bi-axial loading case for a plate can be treated by simply comparing the lowest wrinkling load in any of the two main directions. For multi-axial loading, i.e., including all edge forces $N_x$, $N_y$ and $N_{xy}$, and generally anisotropic sandwich plates, no solution or solution procedure is known except the suggested solution presented in Paper A in this thesis.
In Paper A the problem is addressed using a different approach where the ratio between applied load and sustainable wrinkling load is accessed for all directions within the plate. This method also include the effect of skew wrinkling where the wrinkling wave is not perpendicular to the highest compressive principal stress within the sandwich plate.

Sullins et al. [8] presented some recommendations regarding wrinkling under multi-axial load. They may help the designer to pursue this task and the recommendations are stated as follows:

1. Calculate the principal stresses $\sigma_1$ and $\sigma_2$, either by means of analytical or numerical methods.

2. If only one of the principal stresses is in compression, then neglect the tension stress and treat the problem as one-dimensional.

3. If both principal stresses are in compression then use the interaction formula

$$\frac{\sigma_1}{\sigma_{1cr}} + \left(\frac{\sigma_2}{\sigma_{2cr}}\right)^3 \leq 1 \quad \text{with} \quad \sigma_1 > \sigma_2$$

(40)

to determine the wrinkling strength. The critical stresses $\sigma_{1cr}$ and $\sigma_{2cr}$ are the one-dimensional wrinkling stresses calculated in the directions of the principal stresses.

The interaction formula of Equation (40) is taken from analyses and tests of global (Euler-type) buckling of sandwich shells and is just, without physical justification, translated to be used for wrinkling as well. One can argue over the validity of this kind of approach but it does anyway provide some means of determining the wrinkling strength for a more general case. A scheme like the above is also very well suited for implementation in finite element codes or used as a constraint in optimisation programs. The most serious criticism against this scheme is that for highly anisotropic faces the directions for the principal stresses do not coincide with directions for the principal strains. Furthermore, the anisotropy also implies that the wrinkling strengths, here denoted $\sigma_{1cr}$ and $\sigma_{2cr}$, may vary strongly with the in-plane direction. However, for isotropic and moderately anisotropic faces, the approach should be more accurate.

Vonach and Rammerstorfer [9] has also addressed the problem of wrinkling of orthotropic sandwich plates under general loading. They tackle the problem by assuming the core to be infinitely thick and the wrinkling wave at the interface between the face sheet and core to be sinusoidal. Thereafter they are able to solve the governing differential equation describing the face sheets deformation.

**Face stress exceeding elastic limit**

The described methods all predict the wrinkling load and not the failure load. In practice, the actual wrinkling failure will occur as one buckle becomes unstable. Whether it buckles into or away from the core depends on the strength of the core and/or the adhesive joint. Buckling into the core occurs if the compressive strength of the core is lower than the tensile strength of the core or the adhesive joint and buckling outwards breaking the core or the adhesive joint if the other way. If the wrinkling stress exceeds the yield strength of the face it is customary to replace $E_f$ by a reduced modulus just as in the overall buckling. One such reduced modulus proposed by Norris et al. [7] is
where $E_t$ is the tangent modulus. The application of Equation (22) then requires a trial-and-error operation to find the correct wrinkling stress. A more conservative approach would be to use $E_r = E_t$.

### Wrinkling and initial imperfections

The actual wrinkling load is very much affected by any initial imperfections or waviness of the face. Some design guidelines to account for such effects are given in [7] and [8] where it is found that the factor preceding the cube root expression in Equations (17), (22), (32) and (39) decreases from between 0.5 - 0.9 down to such low numbers as 0.05 to 0.1 for very high waviness. However, waviness in the faces is usually carefully avoided, not only because of a reduction in the wrinkling stress, but mainly because a smooth surface is a sought after characteristic when using a sandwich design.

The reason for using the conservative formula in Equation (22) is mainly due to the effect of initial irregularities. Derivations of this effect are performed in [7] and [8] concluding that the effect has a maximum when the irregularities have a wavelength equal to $2l$. In practical cases, initial irregularities are likely to reduce the wrinkling strength to about 80% of the theoretical [4].

Within the higher order sandwich theory, first published by Benson and Mayers [11], the effect of imperfections has been investigated by for example Frostig [12], Sokolinsky et. al. [13] and Wadee [14]. Their work is interesting but sometimes difficult to understand and implement.

The pure geometrical nonlinearity of wrinkling, which can be predicted both by traditional analytical methods and with FEM, does not necessarily mean that the sandwich structure fails at that point. It just predicts the load where the deformation starts to increase more rapidly. The structure is not damaged until the material fails. If for example a sandwich panel in a boat hull starts to wrinkle this panel deforms. If the surrounding panels still have not reach their wrinkling load, the total load will be redistributed and the wrinkled panel may not break. The wrinkling induced material failure is more difficult to predict than pure wrinkling. In non-linear wrinkling analyses it is possible to monitor both strains and stresses in the sandwich through the loading process. Therefore it is possible to predict when the material will fail and also which constituent will fail first, the core or the face sheet. This information could be utilised in the selection of material combinations. In Paper D herein a method is suggested able to find the appropriate knockdown factor and corresponding failure load.

### Wrinkling and finite element modelling

With the development of computer power during the last 20 years the use of numerical methods provide new means for structural analysis. Finite element (FE) software is one such tool nowadays available both to academic researchers and to the industry. Several articles has been published in the wrinkling field where FE plays an important role. Vonach [15] has published work on for example the supporting effects from the opposing face sheet where FE
was a key instrument. Several authors, e.g. Hadi [16], has also published work based on the comparison between different wrinkling theories and FE calculations.

Finite elements is a useful tool when investigating the wrinkling phenomenon but a few issues are good to bear in mind. The following ideas are based on the experiences from the FE calculations performed in this thesis. If the sandwich face sheet is much thinner than the core, different elements should be used to model the constituents. In a 2D model the face sheet should be modelled with beam elements and the core with membrane elements. In a 3D model the face should be modelled with shell elements and the core with solid (brick) elements. If two different element types is not used a poor mesh is almost always obtained unless a very fine mesh is used. Using mixed models, where elements with different number of nodal degrees of freedoms are used, is sometimes questioned. However, the mathematical error in using mixed models is small compared to modelling the thin face sheets and relatively thick core using only one type of element, where a high number of elements is required to avoid too high element aspect-ratio and distortion.

When modelling wrinkling, elements with quadratic shape (displacement) functions are preferred over elements with linear shape functions. The quadratic displacement functions model the buckling mode much better with fewer elements. To achieve a decent load estimate the element length should be shorter than half the wrinkling wavelength. If also the shape of the wrinkling wave is important and a better load estimate is wanted, at least four elements per wave length should be used. The core should at least be modelled with four elements trough the thickness. The sides of the core elements should be of equal lengths, forming a square in 2D or cube in 3D. If fewer elements are used through the core thickness, or in the length direction of the model, buckling modes might be missed and the predicted loads erroneous.

The model needs to be long enough in the length direction to incorporate four wrinkling waves or the length of the model must be equal to the natural wrinkling wavelength, otherwise a too high buckling load will be obtained.

The boundary conditions of the model can be carefully modelled using periodic boundary conditions as in [9] and [10]. But still the length of the model is important. The length must be an even multiple of the natural wavelength or a too high buckling load is obtained. The model boundaries can also be modelled with simpler non-periodic boundary conditions as in, Paper A and D in this thesis. If a unit cell is used it should be starting and ending at the peaks and bottoms of the wrinkling wave.

Figure 8 shows a few typical finite element models used to model wrinkling. The first model one (Figure 8a) is a 2D model with beam and membrane elements. It has a fine mesh but relatively few degrees of freedom making it fast to run. This model was used in nonlinear calculations investigating the post-buckling behaviour of wrinkling (see Paper D for details). The second model (Figure 8b) is similar to the first but it is a 3D model with solid elements in the core and layered shell elements to model the face sheet. It has the same function as the previously described model, is more computationally intense, but can model anisotropic face sheets easily. Using coupled degrees of freedom can make the model less computationally intense. The stiffness matrix will however still be larger then for the previous model.
Figure 8. Different finite element models used in wrinkling analysis.
(a) 2D model of a unit cell. (b) 3D model of a unit cell with layered shell element
modelling the face sheet. (c) 3D model with periodic boundary conditions.
(d) 3D model of a small sandwich plate used for tests.

The third model (Figure 8c) is a plate model with periodic boundary conditions modelling a
sandwich face sheet on the sandwich core. This model can be used if the load is multi-axial
and it can be used to decide for example at which load and angle the face sheet wrinkles. The
last and fourth model (Figure 8d) is a FE model used to model an actual test situation. The
boundaries for this last model are modelled as rigid plates.

When using FE calculations always remember that the result is never better than the
underlying code and always compare the results obtained from a new FE model with some
known data from tests or analysis.

Nonlinear response of compressively loaded sandwich structures

The nonlinear response of compressively loaded sandwich structures can be studied either by
means of higher order theory, e.g. [12-14] or with nonlinear finite element calculations, e.g.
Paper D herein. Both of these methods can give insight into the post-buckling response of the
structure. If the structure is “beam type” a limit load can be expected. If the wrinkling load is
below the global buckling load the onset of wrinkling will trigger global buckling.

Figure 9 shows examples of nonlinear response of sandwich structures. The solid lines show
the equilibrium paths for a unit cell (basic wrinkling) in compression. The three lines arise
from three different amplitudes of initial imperfections in the wrinkling wave (0.25, 0.1 and
0.01 mm). This is a local form of small scale imperfection further investigated in Paper D in
this thesis. It can further be seen from the figure that even if the basic wrinkling case shows a
nonlinear behaviour and a stiffness change in the vicinity of the predicted buckling load it still
cares an increased load. This is explained with the fact that even if the face sheet buckles
and carries no increased load the core is continuously compressed and carries a larger portion
of the load. To avoid confusion it should be said that all equilibrium paths presented in Paper D are for the face sheet alone without including the load carried by the core, as all paths in Figure 9 does.

![Figure 9](image)

**Figure 9.** Nonlinear paths for two different FE models and a few different degrees of initial imperfection. The solid lines show the nonlinear response for the basic wrinkling case and The dashed lines show the equilibrium paths for a pre-bent sandwich beam.

The dashed lines in Figure 9 show the equilibrium paths for a sandwich beam that, in addition to wrinkling, can buckle in a global mode. The beam model has a global form of imperfection and is not entirely straight prior to loading but has a pre-bent shape, or in other words, a radius of curvature. The different degrees of global imperfection (and the three dashed lines) is from three different beams with increasing radius of curvature. The lowest dashed line is for a highly bent beam where the radius of curvature was five times the length of the beam. The middle line corresponds to a radius of curvature twenty times the length of the beam and the upper line is for a beam with a radius of curvature eighty times its length, being almost perfectly straight. All beam models also has a very small local imperfection in the upper face sheet that triggers wrinkling and mathematically makes it easier to find the onset of wrinkling in the nonlinear analysis. The beams were modelled with an initial curvature because the nonlinear response is easier to find with FE analysis when the buckling direction is known. It can be seen from the equilibrium paths that the initial curvature of the beam models both affect the initial stiffness of the beam and at which load wrinkling will occur. At the onset of wrinkling a limit load is reached and the load drops for an increased strain. If this would happen to a real structure it would collapse. If the structure is plate type the post buckling behaviour (equilibrium path) is much more smooth and without a pronounced limit load. This is also the case if wrinkling is the dominant stability failure mode and no large interaction between global and local modes exists.
Summary of thesis

In Paper A both the effect of through-the-thickness anisotropy and the effect of multi-axially loaded plates are investigated. It is shown how the anisotropy of the face sheets should be tackled and the wrinkling load decided. The developed method further provides means to decide at which angle in the plane wrinkling occurs and that this angle does not have to be perpendicular to the highest principal compression stress as been assumed in earlier work, e.g. [7]. The proposed analytical method is verified with both comparison with detailed FE calculations and experimental tests.

In the second paper, Paper B, three possible methods to increase the load bearing capacity of wrinkling sensitive sandwich panels are developed and exemplified. This is done using simple mathematics and several examples. All methods described are focused on the face sheet. It is common to always tackle the wrinkling problem by choosing a core material with higher elastic modulus. It is shown that this is often unnecessary and that by altering the face sheet lay-up the load bearing capacity is increased with virtually no additional weight. It is also shown with extreme clarity that the commonly used Hoff’s formula (Equation (22)) is a blunt instrument that should not be used unless dealing with isotropic (through-the-thickness) face sheets.

In Paper C the transition from wrinkling failure to compression failure of the face sheet is investigated. Theoretical discussions are compared with test results for sandwich panels with two different face sheet types and seven different core densities. For the first time in the open literature this transition from wrinkling to compression failure is shown through actual tests. The failure modes are investigated using fractography. The results clearly show how the actual sandwich compression failure mode is influenced by the choice of core material, changing from wrinkling failure to face sheet micro buckling failure as the core density increases.

Finally, in the last paper, Paper D, a new approach is presented where the wrinkling problem is transferred from a pure stability problem to a material strength criterion. A theory is presented providing means on how to decide which sandwich constituent will fail first and at which load it will fail. This is accomplished using the effect of initial imperfections in the shape of the wrinkling wave and the effect this imperfection has on the strains within the compressively loaded sandwich panel. It makes it possible to estimate the required knockdown factor and failure load for each initial imperfection amplitude and material combinations. A very good correlation is found when the developed theory is compared with both finite element calculations and to experimental tests. The additional information possible to attain from this derived method give insight to and develop the overall understanding of the wrinkling phenomenon.
Future work

The formulae now used for multi-axial loaded panels are only explicit fittings to the results from tests that often were difficult to conduct. Some are even used with the only argument that they predict the buckling load of single skin plates. One can thus seriously doubt the validity of these approaches for practical purposes. Instead, most engineers found themselves having to use one of the formulae derived for uni-axial loading, simply checking one or a few critical directions. The recent findings regarding wrinkling behaviour in anisotropic multi-axially loaded sandwich plates, e.g. [10] and Paper A herein, are currently being implemented as a failure criterion in FE codes and will be making the life easier for the engineer designing large sandwich structures. The post-buckling behaviour and wrinkling-global buckling interaction is still difficult to fully predict and has to be investigated further.

There is still some lack of experimental data available and the researchers and engineers publishing work on wrinkling and stability should try to make their test results more available to others. A lot more experimental work is still to be done in the field of sandwich stability and wrinkling. High-speed photography and digital speckle photography are new tools currently at disposal that should more often be used in combination with the traditional compression testing.

Wrinkling is a sandwich specific and sometimes unknown failure mode to people starting out designing sandwich structures. It is therefore important to the sandwich community, as a whole, to understand the wrinkling phenomenon and to make simple-to-use guidelines available to the engineers. If this fails, poor structures and catastrophic failures is the possible result. This might give the sandwich concept a poor reputation and not the proper appreciation it deserves.

References


