Calculations of Wind Turbine Flow in Yaw using the BEM Technique

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ABSTRACT

The earlier EU-sponsored project MEXICO (model experiments in controlled conditions) provided a huge database for flows past an experimental rotor in standard and yaw conditions. This study aims to determine the eligibility of different models under various conditions by using the MEXICO data. The main purpose of this project is to improve the BEM technique for yawed flows by using the new yaw model. Additionally, the BEM technique with new yaw model is compared with the CFD and measurement results. The Glauert's yaw model is also applied in BEM model to compare the effectiveness of the new yaw model. It is proved that the CFD technique is still better than the BEM technique except at the high yaw and wind conditions. Furthermore, new yaw model is favored against Glauert’s yaw model. This project also aims to implement the new tip loss correction model in the BEM code and the results are validated with the CFD results.

Key words: New Yaw Model, Glauert's Yaw Model, AL/NS Model, BEM technique, CFD technique, axial and tangential loads, axial and tangential induction factors, new tip loss correction model.
NOMENCLATURE

\( a, a' \) Axial and tangential induction factors
\( \bar{a}, \bar{a}' \) Average axial and tangential induction factors
\( a_c \) Critical axial induction factor
\( A \) Area of the rotor plane
\( A_0 \) Area of the disc at the upstream
\( A_1 \) Area of the disc at the downstream
\( A_{CV} \) Area of the disc in the control volume
\( B \) Number of the Blades
\( c \) Chord length
\( C_a \) Coefficient of new inner and new yaw model
\( C_b, C_d \) Lift and drag force coefficients
\( C_{l,inv} \) Lift coefficient of the fully attached flow
\( C_{l,fs} \) Lift coefficient of the fully separated flow
\( C_n, C_t \) Normal and tangential dimensionless force coefficients
\( C_p \) Power coefficient
\( C_T \) Thrust coefficient
\( dM \) Torque
\( dr \) Thickness of the blade element
\( D \) Drag Force
\( f_s \) Separation function
\( F \) Prandtl’s tip loss factor
\( F1 \) New tip loss correction coefficient
\( F_{air} \) Force on the air
\( g \) Coefficient for the new tip loss correction model
\( K \) Experimental coefficient of the new inner and new yaw model
\( L \) Lift force
\( m \) Overall air mass in the control volume
\( m_{side} \) Air mass leaving the control volume
\( M \) Angular momentum
\( p_0 \) Pressure at the far upstream and downstream
\( p^+ \) Pressure at the upstream
\( p^- \) Pressure at the downstream
\( P_D \) Power potential of the undisturbed wind
\( P_{actual} \) Actual power
\( P_{nu}, P_T \) Normal and tangential force loads
\( P_{shaft} \) Power output of the rotor shaft
\( r \) Local radius
\( R \) Radius of the rotor
\( T \) Thrust force
\( u \) Wind velocity on the rotor plane
\( u_z \) Wind velocity in the wake
\( V_0 \) Undisturbed wind velocity
\( V_{rel,y}, V_{rel,z} \) Relative wind velocities in \( y \) and \( z \) directions
\( V_{rot} \) Rotational speed
\( V_y, V_z \) Velocity components of the undisturbed wind in \( y \) and \( z \) directions
\( V_{wake} \) Induced tangential velocity in the wake
\( W \) Induced velocity
\( W_0 \) Mean induced velocity
\( W_y, W_z \) Induced velocities in \( y \) and \( z \) directions
\( \alpha \) Angle of attack
\( \beta \) Twist angle
\( \gamma \) Yaw angle
\( \Delta p \) Pressure difference on the rotor
\( \theta \) Local pitch angle
\( \theta_0 \) Measured maximum angle for the blade in the wake
\( \theta_{cone} \) Cone angle
\( \theta_{yaw} \) Yaw angle
\( \theta_{wing} \) Wing angle
\( \lambda \) Tip speed ratio
\( \rho \) Air density
\( \sigma \) Coefficient of solidity
\( \sigma_1 \) Coefficient of solidity in the new yaw model calculations
\( \phi \) Flow angle
\( \chi \) Wake skew angle
\( \psi \) Azimuth angle
\( \omega \) Angular velocity
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1. INTRODUCTION

The increasing energy demand of the world and the environmental concerns gave rise to focus on alternative power sources. As a result of this, the concept of using clean energy became more common subject to argue. Since the renewable resources are environmentally friendly and replenished naturally, they are regarded as clean energy sources. Wind is assumed as one of these renewable resources and it can be said that the wind turbine technology is the most reliable option for sustainable energy production at the present time.

The working principal of the wind turbine is simply based on extracting the mechanical energy of the wind and convert it into the electrical energy. This can be achieved by the rotation of the turbine blades and for that reason the determination of the forces acting on the blade are quite necessary. However, the determination of the loads is not so simply and there are many parameters that have to be considered for the calculations. Therefore, the need for a wind turbine model becomes more apparent.

Blade element momentum (BEM) method provides a very good insight into the wind turbine modeling to estimate the performance at the possible conditions. For instance, the simple wind turbine modeling is generally carried out according to the standard flow conditions. However, due to the nature of the wind, the flow direction changes in time and it might not be normal to the turbine rotor. This type of flow is called yawed flow and it is important to determine the load distribution in yawed situation.

The main interest in this project is to improve the BEM technique for the yawed flows by including the New Yaw Model into the BEM code. It is also aimed to compare the results of the BEM technique with New Yaw and Glauert’s Yaw Model with the CFD results. For this purpose, the same properties of the experimental rotor which was used in previously performed EU-sponsored project MEXICO are taken into consideration. Since the MEXICO project also provided a huge database for the flows in standard and yaw conditions, the measurement results can be used as references for the eligibility of the computational techniques in different conditions.

This project is also interested in the application of a new tip loss correction model. Since the estimation of the loads around the blade tip does not match with the measured values, the various tip loss correction models were developed to fit the curve to experimental data. The new tip loss correction model for the Nordtank 500 wind turbine is discussed and implemented in the BEM code. The force distribution along the blade for different wind speeds are determined and validated with CFD results.
2. METHODOLOGY

- Literature survey for the subject.

- Derivation of the averaged axial and tangential induction factor formulas according to new yaw and new inner models by using the thrust and torque relationships on the rotor.

- Derivation of the modified axial induction factor formula at higher axial induction factors.

- Investigation of the previously developed BEM code for the MEXICO rotor at standard flows.

- Determination of the sample yaw conditions for investigations.

- Development of the computer code for the yawed flows according to BEM technique.

- Extraction of the loading data for the last rotation at each sample condition.

- Determination of the loading data by using Glauert’s Yaw Model in the BEM code.

- Comparison of the results obtained by BEM technique with New Yaw Model and Glauert’s Yaw Model.

- Comparison of the results obtained by BEM technique with the CFD and measurement results.

- Implementation of the new tip loss correction model in the BEM code for Nordtank 500 wind turbine.

- Extraction of the loading data for certain wind speed conditions.

- Comparison of the force distribution on Nordtank 500 wind turbine blade with the CFD results.
3. STATE OF ART
Basic information about the wind turbine aerodynamics and the related theories are provided in this chapter.

3.1 Simple Momentum Theory

A wind turbine is simply a machine that converts the kinetic energy of the wind into mechanical energy to produce electricity. The extraction of the mechanical energy is achieved on the rotor plane and it is transferred to the generator to produce electrical energy. For the simplicity of the 1-D momentum theory, the rotor disc is assumed to be in the ideal conditions in which there is not any frictional force acting on it and the rotational velocity in the wake is neglected (Hansen, 2008). The flow is also assumed to be incompressible, stationary, frictionless and there are infinite number of blades on the rotor plane (Burton et al., 2001).

Since the mechanical energy is extracted when the wind is passing through the rotor disc, it is expected that the undisturbed wind speed $V_0$ in the far upstream is going to be decreased and become $u$ at the rotor plane and $u_1$ in the wake. The decrease in the wind speed causes an expansion of the disc area behind the rotor due to the conservation of mass theorem. Because, the air mass flow rate must be same in everywhere inside the defined control volume. Thus, the area of the rotor disc is larger than the area of the disc on the upstream and smaller than the disc on the downstream as it is displayed in Figure 1 (Hansen, 2008).

![Figure 1. Velocity and pressure distribution on the wind turbine system (modified from Burton et al., 2001)](image)

The overall air mass flow along the wind direction is defined as;

$$\rho A_0 V_0 = \rho A u = \rho A_1 u_1$$

Eq. 1

Where,

The area of the discs on the upstream and downstream denoted by $A_0$ and $A_1$ respectively and the area of the rotor plane is shown with the capital $A$ in Figure 1. The reduction of the wind speed on the rotor is easily defined by the parameter called induction factor. The induction factor in the axial direction is defined as “a” and for that reason the magnitude of the induced flow is expressed as $aV_0$. Hence, the definition of the wind speed on the rotor disc become,

$$u = (1 - a)V_0$$

Eq. 2
The velocity difference between the upstream and downstream causes a momentum difference in the system. Since the momentum change directly related with this velocity change and the air mass flow, the resultant rate of a momentum change equals to:

\[ dM = (V_0 - u_1)\rho Au \]  
\[ \text{Eq. 3} \]

It is obvious that the pressure difference on the rotor plane is the result of this momentum change and it is formulized in Eq. 4.

\[ (p_d^+ - p_d^-)A = (V_0 - u_1)\rho A(1 - a)V_0 \]  
\[ \text{Eq. 4} \]

The pressure difference on the disc can be found by applying Bernoulli equation between two points. There are going to be two Bernoulli equations for both upstream and downstream. As it is predicted by Bernoulli, the overall energy of the flow is constant when the conditions are steady (Burton et al., 2001). The Eq. 5 shows the application of the first Bernoulli equation between two points which are assumed to be located in upstream and just in front of the rotor disc.

\[ \frac{1}{2}\rho V_0^2 + p_0 + \rho_0 g h_0 = \frac{1}{2}\rho_d u^2 + p_d^+ + \rho_d g h_d \]  
\[ \text{Eq. 5} \]

Based on the previous assumptions about the incompressibility of the flow, the Eq. 5 is simplified to Eq. 6.

\[ \frac{1}{2}\rho V_0^2 + p_0 = \frac{1}{2}\rho u^2 + p_d^+ \]  
\[ \text{Eq. 6} \]

The second Bernoulli equation is applied between the points which are assumed to be located in downstream and just behind the disc. The simplified version of the equation is displayed in Eq. 7.

\[ \frac{1}{2}\rho u_1^2 + p_o = \frac{1}{2}\rho u^2 + p_d^- \]  
\[ \text{Eq. 7} \]

It is clear that subtracting Eq. 7 from Eq. 6 gives the pressure difference along the rotor disc.

\[ p_d^+ - p_d^- = \frac{1}{2}\rho(V_0^2 - u_1^2) \]  
\[ \text{Eq. 8} \]

By putting the definition of pressure difference in terms of velocities into the Eq. 4 and make the appropriate simplifications, the velocity of the wind in the wake is found in terms undisturbed wind velocity \( V_0 \) in Eq. 9.

\[ u_1 = (1 - 2a)V_0 \]  
\[ \text{Eq. 9} \]

The definition given in Eq. 4 is modified by putting the outcome of the Eq. 9 into it. Finally, the force on the air is described as,

\[ F_{air} = (p_d^+ - p_d^-)A = 2\rho Aa(1 - a)V_0^2 \]  
\[ \text{Eq. 10} \]

The extracted power of the wind is directly related with this force and the wind velocity on the rotor plane. Thus, the actual power obtained from the wind can be specified as it is given in Eq. 11.

\[ P_{actual} = F_{air}u = 2\rho Aa(1 - a)V_0^2(1 - a)V_0 \]  
\[ \text{Eq. 11} \]

The power potential of the undisturbed wind is defined in Eq. 12. The ratio between the extracted power on the disc and the available power of the wind in the upstream is called power coefficient and it is denoted as \( C_p \) in Eq. 13 (Burton et al., 2001).

\[ P_o = \frac{1}{2}\rho V_0^3A \]  
\[ \text{Eq. 12} \]
\[
C_p = \frac{P_{\text{actual}}}{P_0} = \frac{2\rho Aa(1-a)^2v_0^3}{\frac{1}{2} \rho v_0^2 A} = 4a(1-a)^2 \quad \text{Eq. 13}
\]

The definition of power coefficient implies that there is a limit for the extraction of power from the wind (Burton et al., 2001). This upper limit can be estimated by taking the derivative of the power coefficient according to the axial induction factor “a”.

\[
\frac{dc_p}{da} = 4(1-a)(1-3a) = 0 \quad \text{Eq. 14}
\]

According to the Eq. 14, it is seen that at the point where the axial induction factor equals \(\frac{1}{3}\), the power coefficient \(C_p\) reaches its maximum value 0.593. This limit is simply known as Betz limit because it is firstly mentioned in his study (Betz, 1919). Nevertheless, for any wind turbine machine, it is impossible to reach the Betz limit. Because, all the calculations are based on the ideal turbine concept and there are always other external and internal factors that need to be considered during the design process.

It is mentioned previously that the pressure difference along the disc yields force on the blade. This force in other words is called thrust force and it is used to define the dimensionless thrust coefficient which is shown in Eq. 15.

\[
C_T = \frac{2\rho Aa(1-a)^2v_0^2}{\frac{1}{2} \rho v_0^2 A} = 4a(1-a) \quad \text{Eq. 15}
\]

However, it is proved by the experiments that this definition is only valid for a limited interval of the axial induction factor which is up to 0.4. After that level the wind velocity in the wake firstly becomes zero and then goes for a negative value which is actually not a realistic case. This can also be explained by the theoretical expressions. According to the momentum theory, the velocity of the wind in the wake \(u_1\) = \((1-2a)\)\(V_0\). When the axial induction factor equals to 0.5, the wake velocity becomes zero (Burton et al., 2001). The upper limit of the axial induction factor for the applicability of the momentum theory is determined more precisely with the experimental methods (Hansen, 2008).

### 3.1.1 Rotational Effects of the Rotor Disc

The blades on the rotor plane are one of the most important components of the wind turbine system. The pressure difference occurred on the disc is mainly the outcome of the rotating blades which are designed for that purpose. When the blades start to rotate, the torque is produced on the rotor. As a result of this produced torque, the generator inside the nacelle provides a reaction torque which is equal to the magnitude of the torque produced by the rotor but opposite to the direction of the flow. Since the blades rotate with a certain rotational speed, the torque produced by the generator balances the torque of the rotor shaft in order to hold the rotational speed constant. The electricity production in the generator also depends on the work done by this torque. The airflow which is passing through the rotor plane is also affected by the reactive torque. As a result of this reactive torque on the air particles, they start to rotate in the opposite direction of the rotor blades. Hence, there appears to be an additional velocity component of the air flow behind the rotor which is tangential to the rotation (Burton et al., 2001).

The new term for the change of the tangential velocity is emerged as \(a'\) which can also be named as tangential induction factor in other words. Since the tangential velocity is a function of rotational speed and the radial distance, the induced tangential velocity on the disc equals to \(\dot{a \omega}\). Similarly, the induced tangential velocity in the wake equals to \(2\dot{a \omega}\). Due to the lack of tangential velocity component in the upstream, the induced
tangential velocity is zero. The tangential and the axial velocity components on the rotor are shown below (Burton et al., 2001).

![Figure 2. Velocity components of the airflow on the rotor plane (Hansen, 2008)](image)

### 3.1.2 Angular Momentum Theory

As it is stated previously, the tangential velocity depends on the radial position. Therefore, the contribution of momentum from each section on the rotor disc is going to be different. If the rotor disc is hypothetically divided into several annular rings, the rotor torque on these annular rings will give rise to the generation of the tangential velocity component. On the other hand, the decrease in the axial velocity is expected as a result of the axial force on each ring (Burton et al., 2001).

The torque on each annular ring is related with the rate of change of the angular momentum. The angular momentum is itself a function of the radius of the ring, the air mass flow rate and the variation of the tangential velocity. It can be defined as in Eq. 16 (Burton et al., 2001).

\[
M = r \rho AV_0 (1 - a) 2 a' \omega r
\]  
Eq. 16

In order to find the power output of the rotor shaft, the angular velocity “\( \omega \)” of the rotor must be multiplied with the produced torque. The estimated power output must be same as the actual power output that is obtained from the wind in Eq. 11.

\[
P_{\text{shaft}} = M \omega = \rho AV_0 (1 - a) 2 a' \omega^2 r^2 = P_{\text{actual}} = 2 \rho A a (1 - a) V_0^2 (1 - a) V_0
\]  
Eq. 17

Finally the Eq. 18 is obtained after the simplifications are carried out and the term of local speed ratio \( \lambda = \frac{\omega r}{V_0} \) is added into the equation (Burton et al., 2001).

\[
a (1 - a) = \lambda^2 a'
\]  
Eq. 18
Assuming that the area of each annular ring is $2\pi r dr$, the derivative of the power coefficient in terms of radius of each ring gives the maximum power coefficient in Eq. 19 (Burton et al., 2001).

\[
\frac{dC_p}{dr} = \frac{4npV_0^2(1-a)\omega^2 r^2}{2\rho V_0^2 R^2} = 0 \quad \text{Eq. 19}
\]

### 3.1.3 The Blade Element Theory

The blade element theory is based on an approach that the blades consist of series of strips along the span direction. It is assumed that the thickness of each strip is infinitesimal and there is not any interaction between them. The contribution of the lift and drag forces from each strip are determined and integrated along the span to calculate the power and torque of each blade. Then of course the number of blade on the rotor plane is taken into consideration to find the overall rotor power and torque (Mathew, 2006).

The velocity and the force distribution on the blade element are displayed in Figure 3. Axial and tangential components of the velocity and the resultant wind seen by the blade section are provided. The angle of attack is denoted as $\alpha$ and $\beta$ is the twist angle of the blade. The angle $\Phi$ refers to the flow angle which is quite important to determine the tangential and axial velocity components.

The resultant velocity $V_{rel}$, which is also the relative velocity seen by the blade, can be expressed in terms of velocity components. It is easy to formulize the resultant velocity from the Figure 3 and it is shown in Eq. 20.

\[
V_{rel} = \sqrt{V_0^2(1-a)^2 + \omega^2 r^2(1+a')^2} \quad \text{Eq. 20}
\]

The lift “$L$” and the drag “$D$” forces are also shown in Figure 3. These forces on the blade section with a thickness of $dr$ are expressed in Eq. 21 and Eq. 22.

\[
dL = \frac{1}{2} \rho V_{rel}^2 C_l dr \quad \text{Eq. 21}
\]

\[
dD = \frac{1}{2} \rho V_{rel}^2 C_d dr \quad \text{Eq. 22}
\]

$C_l$ and $C_d$ are the lift and drag coefficients which are obtained from the airfoil data. The chord length is denoted by “$c$” in these equations.
According to the defined lift and drag forces, the thrust and the torque calculations on the blade section are
carried out and represented in Eq. 23 and Eq. 24.

\[ dT = dL\cos\phi + dD\sin\phi \quad \text{Eq. 23} \]
\[ dM = r(dL\sin\phi - dD\cos\phi) \quad \text{Eq. 24} \]

3.2 The Blade Element Momentum (BEM) Theory

3.2.1 Steady BEM model

The behavioral concept of the wind turbine and the related force and thrust distributions along the rotor are
specified in terms of simple physical equations by using various theories in the previous chapters. What makes
the difference between the blade element momentum theory and the other theories is that the addition of
other factors to the BEM model such as twist and chord distribution, airfoil structure and the number of
blades. The blade element momentum theory, shortly BEM method, enables to estimate the loads on the
blades. Then these loads are used to calculate the thrust and power for the various conditions such as varying
wind speed, rotational speed and pitch angle (Hansen, 2008). The blade element momentum theory that is
explained here is modeled by Glauert (1935).

In this method, it is assumed that the rate of momentum change of the air particles passing through the
elemental annulus, results only from the force of each blade element (Burton et al., 2001). Thus, it is assumed
that each flow on the radial blade element is totally independent than the others and each radial blade
element section is evaluated separately. This assumption could only be realistic if the axial induction factor
would not change radially (Burton et al., 2001). However, the axial induction factor varies radially and the
examination of this variation along radial direction is one of the objectives of this study.

There is another crucial assumption for the application of BEM method and it is about the forces on the
annular elements. The forces from the blades on the annular elements are considered to be constant. This
assumption can only be valid in case of having infinite number of blades on the rotor plane (Hansen, 2008).
However, it is impossible to have this kind of situation in reality. Since most of the wind turbines have two or
three bladed rotor systems, a need for a correction factor is emerged to make the model more realistic.

In order to calculate the loads on each blade element section, the axial and tangential induction factors must
be calculated with an iterative procedure. For that reason, it is useful to remember the definition of thrust and
torque again. The trust force occurs due to the rate of momentum change in the axial direction. Thus the
definition of the trust force in Eq. 3 is combined with Eq. 9 by replacing the area notation \( A \) with \( 2\pi r dr \) to get
the Eq. 25.

\[ dT = 4\rho rV_o^2a(1 - a)dr \quad \text{Eq. 25} \]

Similarly, the previously defined torque expression in Eq. 16 according to the rate of change of angular
momentum is modified by putting \( 2\pi r dr \) instead of the area notation \( A \) in Eq. 26.

\[ dM = 4\rho r^3\omega V_o(a'(1 - a)dr \quad \text{Eq. 26} \]

After that, the normal and the tangential forces on the rotor plane have to be discussed. In order to estimate
these forces, it is important and required to define the angles on the blade element.
As it is mentioned before briefly, $\alpha$, $\beta$ and $\Phi$ refers to the angle of attack, twist angle and the flow angle respectively. As it is seen, the new term $\theta$ is used instead of $\beta$ in Figure 4. It refers to the local pitch angle of the blade. In other words, it is the angle between the chord line and the rotation plane. Since the pitch angle of the blade might be different than zero in most of the time, the local pitch angle $\theta$, is the sum of this pitch angle and the twist angle $\beta$. The pitch and the twist angles are always given for the blade element (Hansen, 2008).

The flow angle $\Phi$ is the angle between the relative wind velocity seen by blade and the rotation plane (Hansen, 2008). As it can be clearly seen from the Figure 4, the flow angle can be estimated easily by applying the simple geometrical calculations as it is given in Eq. 27.

$$\tan\phi = \frac{(1-a)V_0}{(1+a')\omega r}$$

Eq. 27

When the local pitch and flow angles are known, the local angle of attack can be calculated by subtracting the local pitch angle from the flow angle.

$$\alpha = \phi - \theta$$

Eq. 28

The local angle of attack is used to determine the lift and drag coefficients $C_l$ and $C_d$ from the airfoil data. Then these coefficients are used to calculate the lift “$L$” and drag “$D$” forces per length as it is shown in Eq. 29 and Eq. 30 (Hansen, 2008).

$$L = \frac{1}{2} \rho c V_{rel}^2 C_l$$

Eq. 29

$$D = \frac{1}{2} \rho c V_{rel}^2 C_d$$

Eq. 30

The normal “$P_n$” and the tangential “$P_t$” forces acting on the blade element can be determined by using the relationship between the lift and drag forces and the flow angle (Hansen, 2008). The Eq. 31 and Eq. 32 provide the required equations to calculate these forces.
\[ P_N = L\cos \phi + D\sin \phi \quad \text{Eq. 31} \]
\[ P_T = L\sin \phi - D\cos \phi \quad \text{Eq. 32} \]

The relationship between these definitions and the thrust and torque was provided in Eq. 23 & Eq. 24. In order to calculate the overall thrust and torque, the Eq. 33 and Eq. 34 can be used. In these equations, the normal and the tangential forces in each section with a thickness of \( dr \) are considered and the results are multiplied with the number of blades (Hansen, 2008).

\[ dT = B(L\cos \phi + D\sin \phi) \, dr \quad \text{Eq. 33} \]
\[ dM = Br(L\sin \phi - D\cos \phi) \, dr \quad \text{Eq. 34} \]

Since the aim is to find the axial and tangential induction factors, this can be achieved by equating the Eq. 25 to Eq. 33 and Eq. 26 to Eq. 34. Therefore, the Eq. 33 & Eq. 34 have to be modified to put \( \alpha \) and \( \alpha' \) into the equations.

If the Figure 4 is examined again, the relative velocity can be expressed in terms of its components as it is given in Eq. 35.

\[ V_{rel} = \frac{V_0(1-a)}{\sin \phi} = \frac{\omega r (1+a')}{\cos \phi} \quad \text{Eq. 35} \]

Additionally, the normal and the tangential dimensionless force coefficients are defined like in Eq. 36 and Eq. 37 (Hansen, 2008).

\[ C_n = \frac{P_N}{\frac{1}{2} \rho V_0^2 a} \quad \text{Eq. 36} \]
\[ C_t = \frac{P_T}{\frac{1}{2} \rho V_0^2 a} \quad \text{Eq. 37} \]

By putting the definition of \( V_{rel} \) into the Eq. 36 & Eq. 37, the normal and the tangential forces can be obtained in terms of axial and tangential induction factors. Then these forces are used in Eq. 33 & Eq. 34 to determine the overall thrust and torque (Hansen, 2008). The final equations are given in Eq. 38 and Eq. 39.

\[ dT = \frac{1}{2} B p c \frac{V_0^2}{(\sin \phi)^2} C_n \, dr \quad \text{Eq. 38} \]
\[ \frac{1}{2} \quad dM = \frac{1}{2} B p c \omega r V_0 \frac{(1-a)(1+a')}{(\sin \phi \cos \phi)} C_t \, dr \quad \text{Eq. 39} \]

The normal and the tangential coefficients used in Eq. 38 & Eq. 39 can be expressed in terms of lift and drag coefficients as they are given in Eq. 40 and Eq. 41 (Hansen, 2008).

\[ C_n = C_c \cos \phi + C_a \sin \phi \quad \text{Eq. 40} \]
\[ C_t = C_c \sin \phi - C_a \cos \phi \quad \text{Eq. 41} \]

The Eq. 38 and Eq. 25 are used to define the axial induction factor \( \alpha \) (Hansen, 2008). After applying the proper simplifications in those equations the axial induction factor is found to be,

\[ \alpha = \frac{1}{\frac{4 \sin^2 \phi}{\pi e_n} + 1} \quad \text{Eq. 42} \]

Similarly the Eq. 39 & Eq. 26 are equated to determine the tangential induction factor \( \alpha' \) (Hansen, 2008).
\[a' = \frac{1}{4 \sin \phi \cos \phi} \frac{1}{\sigma C_t} \quad \text{Eq. 43}\]

Where, the constant \(\sigma\) stands for the solidity of the section which is expressed as \(\frac{cR}{2 \pi \alpha} \).

Eventually all of the required terms are defined for the iteration process of BEM method. As it is mentioned in the beginning of this chapter, the precise values of axial and tangential induction factors can be found by applying iterative solution and this iterative solution consists of several steps which are summarized below (Hansen, 2008).

I. Set the values of the axial and tangential induction factors \(a\) and \(a'\) into zero.
II. Determine the flow angle \(\Phi\).
III. According to the given pitch and twist angles, calculate the local angle of attack \(\alpha\).
IV. By using the angle of attack, find the corresponding \(C_l\) and \(C_d\) values from the airfoil data.
V. Calculate the normal and tangential force coefficients \(C_n\) and \(C_t\) in terms of \(C_l\) and \(C_d\).
VI. Calculate the axial and tangential induction factors \(a\) and \(a'\) to check the difference between the newly calculated values and the previous values.
VII. Keep running the iteration by going back to second step until \(a\) and \(a'\) converge.
VIII. For the final values of \(a\) and \(a'\), calculate the local axial and tangential loads on each segment of the blade.

Since the primary assumption of the BEM theory is the independency of the blade element sections, this iterative solution must be applied for each section separately.

3.2.2 Unsteady BEM Model

The steady BEM approach is quite useful to predict the loads on the blades and the total energy production roughly. However, the assumption of having steady wind is hypothetical because the wind profile is generally unsteady. In reality, there are various factors that make the profile unsteady (Hansen, 2008). For instance, the atmospheric turbulence caused by the heat convection from the ground to the air during the day is one of the main reasons for unsteady wind. This is also named as thermal turbulence. The other factor is called wind shear which relates the wind velocity with the height from the ground. The wind speed on the ground is assumed to be zero due to the surface friction and it increases with the increasing height. Hence, this effect can also be considered as mechanical turbulence. The tower of the wind turbine also affects the steadiness of wind profile and this tower effect has to be considered in unsteady BEM model to come up with more precise results.

Unlike the steady model, it is required to determine the position of the wind on the blade element sections in each time step. Since the wind profile has three dimensions in space, it is necessary to define a rigid coordinate system and this coordinate system can be considered at the bottom of the tower. It is also possible to define more coordinate systems for the different parts of wind turbine such as blades, nacelle or shaft like it is shown in Figure 5. In this situation, the coordinates of the points in each coordinate system has to be transformed to a main coordinate system with the help of transformation matrices (Hansen, 2008).
It is important to support the unsteady BEM model with the structural model of the turbine to compute the loads on the blade. According to the stiffness ratio of the blades and the tower, they all vibrate with certain velocities. Therefore, these vibrational velocities have to be considered for the calculation of the local relative wind speed seen by the blade (Hansen, 2008). However, the complete structural analysis of the wind turbine is beyond the scope of this study and it is not included for the simplicity of the calculations. More information about the transformation matrices and the structural modeling of the wind turbine can be found in Hansen (2008).

The undisturbed wind speed is generally defined according to main coordinate system and for that reason it has to be transformed into the blade coordinate system to calculate the wind speed seen by sections on the blade in all directions. Apart from the undisturbed wind, the rotational \( V_{rot} \) and the induced velocities \( W \) have to be included in the general relative wind velocity equation as it is shown in Eq. 44 (Hansen, 2008).

\[
V_{rel} = V_o + V_{rot} + W
\]  

**Eq. 44**

The velocity components on the airfoil cross section are demonstrated in Figure 6. Since only the velocity components in the y and z directions have an influence on the airfoil cross-section, the velocity component in the x direction is neglected in the calculation.
The relative wind speed equations in Eq. 45 and Eq. 46 can be derived by using the scheme in Figure 6. The cone angle in Eq. 46 refers to an angle between the rotor plane and the blade axis.

\[ V_{rel,x} = V_z + 0 + W_z \]  
Eq. 45
\[ V_{rel,y} = V_y + (-\omega r \cos \theta_{cone}) + W_y \]  
Eq. 46

The axial and tangential induced velocities can be simply defined by the induction factors as they are seen in Eq. 47 and Eq. 48. Since the local induction factor varies along the blade, the induced velocities for each blade segment have to be calculated separately. Additionally, it is necessary to use the averaged value of the local wind velocity on each blade in order to calculate the axial induced velocity. Because the wind velocities seen by blade are not identical due to the previously stated reasons that make the wind unsteady throughout the time.

\[ W_z = a(V_{z,1} + V_{z,2} + V_{z,3})/3 \]  
Eq. 47
\[ W_y = a' r \omega \]  
Eq. 48

The determination of the induced velocities enables to calculate the local flow and attack angles (Hansen, 2008). The flow angle is defined previously as the ratio of the relative wind velocities seen by the blade and it is shown in Eq. 49. However, the local attack angle is determined by using the Eq. 28.

\[ \tan \phi = \frac{V_{rel,x}}{V_{rel,y}} \]  
Eq. 49

The iterative solution algorithm of the unsteady BEM model is almost same as the steady BEM algorithm with some additional modifications. Unlike the steady BEM model, the induced velocity components are the main parameters which have to be converged in the iterative procedure instead of induction factors. The loads on the blade sections are determined in each iteration step according to the calculated induced velocities after series of calculation steps. However, there is always a possibility of a gust or a sudden pitch angle variation. In case of these situations thrust force changes and the loads on the blade change according to that variation. However, loads couldn’t able to respond to that sudden thrust change instantaneously and there is always a time lag for the loads to come to equilibrium with the induced velocities. Therefore, it is required to use a dynamic wake model which is simply based on modifying the calculation of the induced velocities in order to take this time lag into consideration (Hansen, 2008). More information about the outcomes of the various dynamic models can be found in two project studies which are carried out by Snel and Schepers (1995) and Schepers and Snel (1995).

As like as the pitch angle, it is a well-known fact that the angle of attack varies due to the atmospheric turbulence, wind shear, presence of tower and yaw-tilt orientation in operation time. Similarly, the response of the loads for this change can be observed with time lag and it is related with the condition of the boundary layer (Hansen, 2008). For instance, Theodersen (1935) proposed a theory to estimate the time lag for unsteady lift in a condition of attached flow. On the other hand, if the flow is separated at the trailing edge and increases with the increasing attack angles, dynamic stall model has to be used. According to Øye (1991), separation function \( f_s \) can be used to modify the lift in terms of dynamic stall model.

\[ C_l = f_s C_{l,inv} + (1 - f_s) C_{l,fs} \]  
Eq. 50

The lift coefficients on the right hand side of the Eq. 50 refer to fully attached “\( C_{l,inv} \)” and fully separated “\( C_{l,fs} \)” flows and they can be obtained from the airfoil data.
The unsteady BEM model can also be used for the yawed rotors. However, in this case the induced velocity is going to vary according to the position of the blade in each revolution. When the blade is at the downward position, the induced velocity on the blade is always larger in comparison to one at the upward position (Hansen, 2008). The reason of this situation can be clearly seen from the Figure 7. When the blade is at the downward position it is closer to the wake and therefore the blade experiences higher induced velocity. As a result of this, the loads are higher when the same blade is at the upward position (Hansen, 2008).

![Figure 7. Yawed rotor plane (Hansen, 2008)](image)

The Eq. 51 was derived by Glauert to observe the distribution of the induced velocity.

\[
W = W_0(1 + \frac{r}{R} \tan(\frac{\chi}{2}) \cos(\theta_{\text{wing}} - \theta_0))
\]

Eq. 51

In this equation, \(W_0\) is the previously calculated value of the mean induced velocity. The magnitude of the induced velocity for the yawed rotor is related with the angle between the rotational axis and the wind velocity in the wake. This angle is named as wake skew angle and denoted by \(\chi\). \(\theta_{\text{wing}}\) refers to an azimuth position and \(\theta_0\) is the measured maximum angle for the blade in the wake.

The iterative solution algorithm of the unsteady BEM model consists of several steps which are explained below (Hansen, 2008).

1. Read the geometry and the parameters of the wind turbine.
2. Set the blade positions and velocities.
3. Set the induced velocities \(W_z\) and \(W_y\) into zero for each time step, blade and blade sections.
4. By using the initialized values of the induced velocities, calculate the relative velocity for the blade elements.
5. Determine the flow and the attack angles.
6. Read the static lift \(C_l\) and drag \(C_d\) coefficients from the airfoil data.
7. By using the dynamic stall model, read the dynamic airfoil data and calculate lift.
8. Calculate the loads in axial and tangential directions.
9. Calculate the induced velocities \(W_z\) and \(W_y\).
10. By using the dynamic wake model, compute the unsteady induced velocities \(W_z\) and \(W_y\).
11. Calculate the induced velocities for each blade if the rotor is yawed according to the yaw model.
The blade element momentum theory is generally accepted as a very suitable way of determining the loads on the blade segments. However, based on the assumption of having the infinite number of blades on the rotor is still needed to be corrected. Therefore, the suitable tip loss correction model has to be used. Apart from that, the correction for the high values of axial induction factor has to be included in the BEM method.

3.2.3 The Tip Loss Correction Models

The rate of change of momentum of the air particles passing the rotor is the driving force of the pressure difference on the rotor disc and the rotation of the blades. According to the BEM method, it is assumed that there are infinite blades on the rotor so that all the air particles passing through the rotor can experience the momentum change. However, this theoretical assumption is not valid in reality. Because, most of the wind turbines have two or three blades on their rotor and the momentum of the particles passing through the space between these blades will remain unchanged. In fact, the momentum change can only be observed for the particles which are in the vicinity of the blades. From that, it can be predicted that the axial induction factor may not be uniform on the rotor disc. Therefore, the azimuthally averaged axial induction factor is used to compute the axial momentum change (Burton et al., 2001).

The term “loss” undoubtedly refers to a reduction in the power production of the system or the unconverted energy potential. First of all, it is crucial to investigate the reason for that and determine how it occurs. At this point, it is beneficial to remind Figure 3 to examine the velocity and force distribution on the blade section. When the axial induction factor gets larger, the velocity component on the axial direction becomes very small and this situation results in the decrease of the flow angle \( \Phi \). For the smaller values of flow angle, the axial component of the lift force increases and the tangential component become very small. The tangential component of the lift force is quite important due to its contribution to maintain torque. As a result of this, the reduction of the torque will give rise to a reduction in the power production. The amount of the reduction simply designated as tip loss because very high induction factor is only observed on the sections very close to the tip of the blade (Burton et al., 2001). The reason for having high values of axial induction factor is the existing vortices at the tip. These tip vortices are illustrated in Figure 8.

![Figure 8. The tip vortices in the wake (Burton et al., 2001)](image)
Prandtl (1927) proposed a tip-loss function to correct the assumption of having infinite number of blades. This tip-loss function is generally entitled as Prandtl's tip loss factor and it was formulated by Glauert (1935) to improve the BEM method.

\[ F = \frac{2}{\pi} \cos^{-1} \left( e^{-\frac{B}{2\sqrt{r}} \left( r - \frac{B}{2}\right)} \right) \tag{Eq. 52} \]

In Eq. 52, B refers to the number of blade; r and R are the local and the total radius values respectively.

If the Glauert’s tip loss correction model is included into the BEM code to calculate the axial and tangential induction factors, the Eq. 42 and Eq. 43 have to be modified to get Eq. 53 and Eq. 54 for the induction factors.

\[ a = \frac{1}{\sqrt{\sin^2 \lambda - 1}} \tag{Glauert, 1935} \text{ Eq. 53} \]

\[ a' = \frac{1}{\sqrt{\sin^2 \lambda - 1}} \tag{Glauert, 1935} \text{ Eq. 54} \]

As like as Glauert’s contributions on tip loss correction model, Wilson and Lissaman (1974) and De Viries (1979) studied on this subject to develop better models. Apart from these models, there is another technique called New Tip Loss Correction model and the new tip loss function which was derived in Shen et al. (2005) is used in this model. The new tip loss function is almost same as the formulated version of Prandtl’s tip loss function with an addition of a coefficient \( g \). The formula of the new tip loss function and the \( g \) coefficient are given in Eq. 55 and Eq. 56.

\[ F_1 = \frac{2}{\pi} \cos^{-1} \left( e^{-\frac{B}{2\sqrt{r}} \left( r - \frac{B}{2}\right)} \right) \tag{Shen et al., 2005} \text{ Eq. 55} \]

\[ g = \exp(-0.125(B\lambda - 21)) + 0.1 \tag{Shen et al., 2005} \text{ Eq. 56} \]

As it was always before, B is the number of blades, \( \Phi \) is the flow angle, r and R are the local and the total radius values and \( \lambda \) refers to tip speed ratio. The determination of the axial and tangential induction factors according to the new tip loss correction model are given in Eq. 57 and Eq. 58.

\[ a = \frac{2 + Y_1 - \sqrt{4F_1(1-F_1) + Y_1^2}}{2(1+F_1)} \tag{Shen et al., 2005} \text{ Eq. 57} \]

\[ a' = \frac{1}{\sqrt{\sin^2 \lambda - 1}} \tag{Shen et al., 2005} \text{ Eq. 58} \]

Where, the constants \( Y_1 \) and \( Y_2 \) are shown in Eq. 59 and Eq. 60.

\[ Y_1 = \frac{4F\sin^2 \Phi}{\sigma_c F_1} \tag{Eq. 59} \]

\[ Y_2 = \frac{4F\sin \phi \cos \phi}{\sigma_c F_1} \tag{Eq. 60} \]

The comparison of the Glauert’s and new tip loss correction models against the CFD results is carried out for the non-yawed rotor in the results part. However, the new tip loss correction model is used for the new yaw model in this study and it is going to be shown in the following chapters.
3.2.4 Breakdown of the Momentum Theory

The simple momentum theory is only valid for the values of axial induction factor below 0.4. After that level, the flow in the wake becomes turbulent (Hansen, 2008). Therefore, the expression of the thrust coefficient $C_T$ has to be modified to predict the best fit for the experimental results. The sample empirical fit is displayed in Figure 9.

There are various empirical expressions which are used for this modification. Among all other empirical expressions, the expression derived by Spera (1994) is used in this study.

$$C_T = \begin{cases} 
4a(1-a)F & a \leq a_c \\
4(a_c^2 + (1-2a_c)a)F & a > a_c 
\end{cases}$$

Eq. 61

The Prandtl's tip loss factor is included in the formula as well as the critical value of axial induction factor $a_c$ which is set to 0.2 in this expression. The use of the correction in the new yaw model and the related derivations are explained in detail in the next chapter.

![Figure 9. Theoretical and Experimental values of $C_T$ corresponding to axial induction factor (Burton et al., 2001)](image)

The dashed line represents the calculated theoretical values of $C_T$ according to the momentum theory. On the other hand, the values marked with small boxes refer to a kind of sample experimental values. As it is seen from the figure, there is a large difference between the experimental and the theoretical values after a certain point of axial induction factor. The empirical fit is shown with the straight line.

Both corrections for the high values of induction factor and the tip losses have to be considered in the iterative BEM solution. Then the loads on each blade segment are used to compute the power curve to estimate the annual energy production.
4. YAW MODELS

4.1 The Glauert`s Yaw Model

It is well known fact that the direction of the wind affects the distribution of the loads on the blade and the annual energy production of the wind turbine system. Due to the various reasons mentioned in the previous sections, the character of the wind is unsteady in general and its direction is not stable throughout the operation time. Hence, it is very crucial to measure the wind direction precisely and align the rotor axis according to this direction. Even if, the direction of the wind is measured correctly, it is not possible for the rotor to align itself rapidly. As a reason of this situation, there appears to be an angle between the axial direction of the rotor plane and the wind direction. This angle is called yaw angle and it is showed in Figure 10. The presence of the yaw angle results in power output losses by decreasing the efficiency of the system.

Therefore the control of the yaw angle is very important in the turbine system. There are different types of control mechanisms which are used to regulate the power production such as pitch, stall, active stall and yaw controls. The control of the yaw angle is carried out by the yaw control mechanism. For instance, minimizing the yaw angle could be beneficial to avoid power losses but responding for very small yaw changes might harm the components. Hence, it is important to optimize the magnitude of the yaw angle (Hau, 2006). On the other hand, in case of wind condition which is higher than the specific rated wind speed for the turbine, yaw control can be used to increase the yaw angle to keep the power production at the same level (Mathew, 2006).

Glauert (1926) proposed a yaw model by using the simple momentum calculations. He examined the aerodynamic behavior of the auto gyro which is a simple aircraft model with a rotor and a propeller to yield lift force and forward thrust. Since the alignment of the auto gyro rotor is same as the 90 degrees yawed rotor of the wind turbine, the obtained results should be valid for the wind turbine system. Based on this study, Glauert came to a conclusion that the equation for calculating the axial induced velocity for the auto gyro can.
be used for any yaw angle. Another important outcome of Glauert’s auto gyro theory is about the uniformity of the induced velocity through the rotor.

The axial component of the relative wind velocity and the axial average induced velocity are uniform as they are shown in Figure 11. On the other hand, it is known that the magnitude of the induced velocity at the leading edge is less than the induced velocity at the trailing edge. Therefore, the pattern of the flow reveals the fact that there must be another non-uniform induced velocity component acting on the rotor disc (Burton et al., 2001). The non-uniform induced velocity component is related with the radius and the azimuth angle in the rotational direction. According to Glauert, total induced velocity can be calculated with the Eq. 62 given below.

\[ W = W_0(1 + \frac{r}{R} \sin \psi) \]  

Eq. 62

![Figure 11. The induced velocity components for the yawed rotor (Burton et al., 2011)](image)

4.2 New Yaw Model

The distribution of the velocity vectors in case of having yawed flow is displayed in Figure 12. It can be clearly seen from the figure that the axial wind velocity acting on the rotor is directly related with the cosine of the yaw angle. Similarly, the tangential component of the wind velocity is appeared at the yaw conditions and it is related with the sine of the yaw angle.

The derivation steps of the induction factors according to the new yaw model are provided in this section. It is important to mention that all axial and tangential load calculations are based on the first blade of the wind turbine. Additionally, the azimuth angle is assumed as zero when the blade is at the upward position and it increases in time until the blade completes a rotation.
4.2.1 Derivation of Axial (Normal) Induction Factor

Figure 13 represents the control volume around a wind turbine and the thrust force for the yawed rotor which is represented by $T$ can be calculated as follows.

$$-T = p u_1^2 A_1 + p v_0^2 (c o s y)^2 (A_{CV} - A_1) + m_{side} v_0 c o s y - p v_0^2 (c o s y)^2 A_{CV}$$  

Eq. 63
Where,

The undisturbed wind velocity and the wind velocities at the rotor plane and wake are denoted by \( V_0 \), \( u \) and \( u_1 \) respectively. The area of the rotor plane is shown by capital \( A \) and the area of the same amount of air mass at the wake is denoted by \( A_1 \). The area of the control volume is symbolized by \( A_{CV} \) and \( \rho \) refers to the air density.

The mass of the air particles leaving the control volume can be found by using the conservation of mass theory.

\[
p u_1 A_1 + \rho V_0 \cos \gamma (A_{CV} - A_1) + m_{\text{side}} = \rho V_0 \cos \gamma A_{CV}
\]

Eq. 64

In which,

\[
m_{\text{side}} = \rho A_1 (V_0 \cos \gamma - u_1)
\]

Eq. 65

The overall air mass passing through the turbine equals to the mass on the area \( A_1 \) behind the turbine. Thus, the relationship between \( A \) and \( A_1 \) can also be found by using the mass conservation theory.

\[
m = \rho u A = \rho u_1 A_1
\]

Eq. 66

Then the thrust force can be determined by combining the Eq. 64 and Eq. 66.

\[
T = \rho u_1 A_1 (V_0 \cos \gamma - u_1) = \rho u A (V_0 \cos \gamma - u_1)
\]

Eq. 67

Thrust force is the result of the pressure drop on the rotor plane. Since it is in the opposite direction to the wind direction, it reduces the wind speed \( V_0 \) to \( u_1 \). If \( A \) is the area of the rotor and \( \Delta p \) is the pressure difference, thrust force can be defined as \( T = \Delta p A \). In order to estimate the pressure drop on the rotor plane, Bernoulli equations are used between points 1-2 & 2-3 in Figure 13.

\[
p_0 + \frac{1}{2} \rho V_0^2 (\cos \gamma)^2 = p + \frac{1}{2} \rho u^2 \quad \text{&} \quad p - \Delta p + \frac{1}{2} \rho u_1^2 = p_0 + \frac{1}{2} \rho u_1^2
\]

Eq. 68

By using the equations given above, the pressure drop over the rotor plane is determined.

\[
\Delta p = \frac{1}{2} \rho (V_0^2 (\cos \gamma)^2 - u_1^2)
\]

Eq. 69

Then the thrust definitions in Eq. 67&Eq. 70 are combined to find the velocity on the rotor plane.

\[
T = \Delta p A = \frac{1}{2} \rho (V_0^2 (\cos \gamma)^2 - u_1^2) A
\]

Eq. 70

It is important to see that the velocity on the rotor plane is the average of the wind speed values \( V_0 \) in the upstream and \( u_1 \) in the far wake.

\[
u = \frac{1}{2} (V_0 \cos \gamma + u_1)
\]

Eq. 71

Since the wind speed on the rotor plane is dependent on the axial induction factor and defined as in Eq. 72 for the yawed rotor,

\[
u = (1 - a) V_0 \cos \gamma
\]

Eq. 72

The wind speed in the wake can be found by combining Eq. 71& Eq. 72.

\[
u_1 = (1 - 2a) V_0 \cos \gamma
\]

Eq. 73
The derived $u$ and $u_1$ are used in the thrust equation which is found in Eq. 67. It is also crucial to put the tip loss correction coefficients for the induction factors.

$$T = 2\rho V_0^2 (\cos \psi)^2 a F (1 - a F) A ; A = r dr d\psi \quad \text{Eq. 74}$$

The area of the rotor is a function of radius and the azimuth position of the blade. The azimuth angle of the blade ($\psi$) is shown in Figure 14. In order to find the rotor area, the equation has to be integrated for a complete rotation. This integration along the full circle is also required to find the average induction factor “$a$” for each section.

$$\frac{d\tau}{dr} = \int_0^{2\pi} 2\rho V_0^2 (\cos \psi)^2 a F (1 - a F) \, d\psi \quad \text{Eq. 75}$$

The axial loading can also be defined like the formula in Eq. 76.

$$\frac{d\tau}{dr} = B P_N = \frac{1}{2} \rho c B V_{rel}^2 C_n \quad \text{Eq. 76}$$

In which,

$$C_n = C_c \cos \phi + C_d \sin \phi \quad \text{Eq. 77}$$

Where, $B$ denotes the number of blades, $\rho$ denotes air density, $c$ denotes chord length, $V_{rel}$ denotes relative wind velocity seen by the blade, $C_n$ denotes normal components of lift and drag forces.

The Figure 15 represents the velocity and force distribution on the blade in the yaw condition. When the rotor plane is yawed against the flow, it becomes normal to the cosine component of $V_0$. The sine component of $V_0$ contributes to the rotational speed in the tangential direction and it is shown in Figure 16.
The relationship between the flow angle ($\Phi$) and the relative wind velocity can be derived from Figure 15. The relative wind velocity $V_{rel}$ according to the yaw condition is formulized as:

$$V_{rel} = \frac{V_0(1-a)\cos \phi}{\sin \phi}$$  \hspace{1cm} \text{Eq. 78}

By using the Eq. 78 in Eq. 76 and including the new tip loss correction coefficient, the axial load is defined as below.

$$\frac{dT}{dr} = \frac{1}{2} B p c \left( \frac{V_0^2 \cos \phi}{\sin \phi} \right)^2 \frac{(1-a)^2}{(1-F)^2} C_\alpha F_1$$  \hspace{1cm} \text{Eq. 79}

The definition has to be integrated along the full circle according to the azimuth angle ($\psi$) and divided into $2\pi$ to estimate the average induction factors for each section. Then the Eq. 75 & Eq. 79 become identical and they are used to calculate the axial induction factor "$a$".

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2} B p c \left( \frac{V_0^2 \cos \phi}{\sin \phi} \right)^2 \frac{(1-a)^2}{(1-F)^2} C_\alpha F_1 \, d\psi = \int_0^{2\pi} 2 \rho V_0^2 (\cos \psi)^2 a F (1 - a F) \, d\psi$$  \hspace{1cm} \text{Eq. 80}

The induction factor "$a$" in the formula is replaced by the average induction factor "$\bar{a}$" and the corresponding coefficients for the sections on the blade. According to the new yaw model, two different coefficients are derived and used to estimate the local induction factors (Shen, 2010). The Eq. 81 (the new inner model) is valid for the sections up to %35 of the overall blade length and the Eq. 82 (the new yaw model) can be used to estimate the induction values at the sections beyond the %60 of the blade. For the sections between %35 - %60, the interpolation between these equations is carried out. The coefficients are simply denoted as $C_\alpha$ in Eq. 83.

$$a = a_{\text{average}} (1 - K \cos \psi)$$  \hspace{1cm} \text{Eq. 81}

$$a = a_{\text{average}} \left[ 1 + K \left( \frac{\rho}{\rho_R} \right)^2 \sin \left[ \psi + \frac{\pi}{2} \left( \frac{\rho}{\rho_R} - 1 \right) \right] \right]$$  \hspace{1cm} \text{Eq. 82}

In which, the experimental K values are shown in Table 1.
Table 1. K values at different yaw angles

<table>
<thead>
<tr>
<th>Yaw angle (degrees)</th>
<th>New Yaw Model</th>
<th>New Inner Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.27</td>
<td>0.12</td>
</tr>
<tr>
<td>30</td>
<td>0.52</td>
<td>0.22</td>
</tr>
<tr>
<td>45</td>
<td>0.80</td>
<td>0.25</td>
</tr>
</tbody>
</table>

The final equation is derived as;

\[
\sigma_1 \int_0^{2\pi} \frac{C_a^2}{(\sin \phi)^2} C_n F_1 d\phi + \int_0^{2\pi} C_a^2 F^2 d\phi \cdot a^2 = [2 \sigma_1 \int_0^{2\pi} \frac{C_a}{(\sin \phi)^2} C_n F_1 d\phi + \int_0^{2\pi} C_a F d\phi] a + \sigma_1 \int_0^{2\pi} \frac{1}{(\sin \phi)^2} C_n F_1 d\phi = 0
\]

Eq. 83

In which,

\[
\sigma_1 = \frac{c B}{8\pi r}
\]

Eq. 84

The contribution from each blade is taken into account by simply including the number of blades “B” into the equation. However, it is not totally true in a real case. Since the blades are oriented as to be 120 degrees far away from each other, the azimuth position of each blade is going to be different in the very beginning of the rotation. Thus, the contribution of each blade throughout the time step is calculated separately and summed up before the integration is carried out in the BEM code.

4.2.2 Glauert Correction for High Values of Axial Induction Factor

The simple momentum theory is valid when the induction value is not higher than 0.4. Thus, the correction model has to be used when the induction factor is above that level. In this study, the Glauert’s correction model is used for the induction values which are above 0.3. The aim of this correction is to modify thrust coefficient \( C_T \) according to the induction factor “a” in order to get closer results with the experimentally measured ones. The thrust coefficient expression (Spera, 1994) given below is used in this model and the value of \( a_c \) is defined approximately as 0.2.

\[
C_T = \frac{4a(1-a)F}{4(a_c^2 + (1-2a_c)a)F} \quad a \leq a_c
\]

Eq. 85

Since the correction model is used when the axial induction factor is higher than 0.3, there is no need to consider the condition in which \( a \leq a_c \). The definition of the thrust coefficient is shown in Eq. 86 for the yawed rotor.

\[
C_T = \frac{T}{\frac{1}{2} \rho V^2 \cos \gamma A} = 4(a_c^2 + (1-2a_c)a)F
\]

Eq. 86

Then the thrust and the axial loading are expressed as in Eq. 87 & Eq. 88 respectively.

\[
T = \frac{1}{2} \rho V^2 \cos \gamma A \times 4(a_c^2 + (1-2a_c)a)F \quad ; \quad A = \pi dr d\phi
\]

Eq. 87
The local induction factor “a” is again replaced by the average induction factor “\( \bar{a} \)” multiplied with the corresponding section coefficients \( C_n \) in Eq. 89.

\[
\left[ \int_{0}^{2\pi} \frac{\sigma_1}{(\sin\phi)^2} C_n F_1 d\phi \right] a^2 = \left[ 2\sigma_1 \int_{0}^{2\pi} \frac{C_n F_1}{(\sin\phi)^2} d\phi + \int_{0}^{2\pi} (1 - 2\bar{a}\alpha) \sigma_1 F d\psi \right] a + \left[ \int_{0}^{2\pi} \frac{1}{(\sin\phi)^2} C_n F_1 d\phi - \int_{0}^{2\pi} a^2 F d\psi \right]
\]

Eq. 90

4.2.3 Derivation of the Tangential Induction Factor

When the wind turbine blades start to rotate, there appears to be an additional velocity component on the system which has to be considered for the calculations. Since the rotational direction of the blades is tangential to the rotor plane, the force resulted from this velocity component is called tangential force \( F_T \). The tangential force calculations are very important for the design of the wind turbine due to the produced torque of the system. The tangential velocity component is a function of angular velocity of the rotor \( \omega \) and the radial distance \( r \) from the rotational axis. There is also another component of the induction factor which is in the tangential direction and denoted as \( \alpha \). Thus, the tangential induced velocity equals to \( \alpha \omega r \). The value of tangential induction factor is generally between 0-0.1 and it is smaller than the axial induction factor. However, it should be added into the velocity profile of the system to observe its trend along the blade. The axial and the tangential velocity components and the forces on the blade in the yawed rotor are shown in Figure 16.

![Figure 16. Velocity and force profile of the yawed rotor (modified from Hansen, 2008)](image-url)
The tangential induction factor $a'$ can be calculated by using the torque equation on annular elements according to the BEM theory. The definition of the torque that is created by the tangential force is given in Eq. 91.

$$\frac{dM}{dr} = BrP_T = \frac{1}{2} \rho c B r^2 v_{rel} C_t$$  \hspace{2cm} \text{Eq. 91}$$

The relative wind speed seen by the blade is defined previously in Eq. 78. It can also be defined by means of tangential velocities on the system. As it is seen from the Eq. 92, the tangential component of $V_0$ is also multiplied with the additional term $\cos \psi$. It is crucial to add this term because the magnitude of the component directly related with the azimuth angle which changes throughout the rotation period.

$$V_{rel} = \frac{w r (1+a')-V_0 \sin \gamma \cos \psi}{\cos \phi}$$  \hspace{2cm} \text{Eq. 92}$$

By combining the Eq. 79, Eq. 81, Eq. 92 and including the tip loss correction, torque equation becomes;

$$\frac{dM}{dr} = \frac{1}{2} \rho c B r \left( \frac{V_0 \cos \gamma (1-a)}{\sin \phi} \right) \left( \frac{w r (1+a')-V_0 \sin \gamma \cos \psi}{\cos \phi} \right) C_t F_1$$  \hspace{2cm} \text{Eq. 93}$$

The definition of $C_t$, which refers to the tangential components of lift and drag forces, can be derived from the geometry shown in Figure 16.

$$C_t = C_{l \sin \phi} - C_{\alpha \cos \phi}$$  \hspace{2cm} \text{Eq. 94}$$

There is another definition of torque which is derived from the moment of momentum theorem. In this definition, the torque on the annular element is formulized by means of induced tangential velocity in the wake “$V_{\text{wake}}$” and the axial velocity “$u$” on the rotor plane.

$$\frac{dM}{dr} = \int_0^{2\pi} \rho V_{\text{wake}} u r^2 d\psi$$  \hspace{2cm} \text{Eq. 95}$$

The magnitude of the tangential induced velocity in the wake is two times larger than the magnitude of the tangential induced velocity on the rotor which is $a \omega r$. The definition of axial velocity “$u$” is also provided previously in Eq. 72. By using all of these definitions and including the tip loss correction, the torque equation becomes,

$$\frac{dM}{dr} = \int_0^{2\pi} 2\rho r^3 \omega a' F_1 (1-aF) V_0 \cos \gamma d\psi$$  \hspace{2cm} \text{Eq. 96}$$

Since tangential induction factor $a'$ varies with the azimuth angle, the Eq. 93 is also needed to be integrated along the full circle. Then the Eq. 93 and Eq. 96 become identical with each other.

$$\frac{1}{2\pi} \int_0^{2\pi} \rho c B r \left( \frac{V_0 \cos \gamma (1-a)}{\sin \phi} \right) \left( \frac{w r (1+a')-V_0 \sin \gamma \cos \psi}{\cos \phi} \right) C_t F_1 d\psi = \int_0^{2\pi} 2\rho r^3 \omega a' F_1 (1-aF) V_0 \cos \gamma d\psi$$  \hspace{2cm} \text{Eq. 97}$$

In order to calculate average $a'$, the axial and tangential induction factors in the equations are replaced by the average induction factors multiplied by corresponding coefficients which are given in Eq. 81 & Eq. 82. Then the tangential induction factor is taken to the left side of the equation.

$$\bar{a'} = \frac{\int_0^{2\pi} c B (1-aC_{\alpha \cos \phi}) (w r - V_0 \sin \gamma \cos \psi) C_t F_1 d\psi}{\int_0^{2\pi} C_{\alpha F} (1-aC_{\alpha F}) - \frac{c B}{2\pi r} (1-aC_{\alpha \cos \phi}) C_{\alpha F} F_1 d\psi}$$  \hspace{2cm} \text{Eq. 98}$$
As a result of this derivation, it is seen that the tangential induction factor is dependent on the axial induction factor. Thus, it is required to calculate the axial induction factor before carrying out the calculations for the tangential one.

5. RESULTS

5.1. The Comparison of the Different Yaw Models at various Yaw and Wind Conditions

The derived formulas for the yawed rotor are used in the BEM code to determine the induction factors and the related induced velocities acting on the rotor. However, these calculated induced velocity components are basically the averaged values on the blade. Hence, the local values have to be computed in each iteration step. The contributions of the local induced velocity components are quite necessary for the estimation of the local relative wind velocity. Because, the local lift, drag forces and the local flow angle which are used to calculate the local loads are influenced by the changes in relative wind velocity.

As it is mentioned previously, one of the main aims of this study is to improve the BEM model for the yawed flows and compare the results with the other models. Actually the results of three different computational models are compared with the measurement results. The results according to new yaw and Glauert’s yaw models are determined thanks to the improved BEM technique which is developed in this study. The third computational technique is called computational fluid dynamics (CFD) with Actuator Line-Navier Stokes (AL-NS) yaw model. This technique includes the Navier-Stokes solver and the actuator line model. The Navier-Stokes solver is basically the EllipSys3D code which is developed at DTU (Michelsen, 1992) and Risø National Laboratory (Sørensen, 1995). The working principle of the EllipSys3D code is based on a multi block/cell-centered finite volume discretization of the steady/unsteady incompressible Navier-Stokes equations in velocity-pressure variables (Shen, Zhu and Sørensen, 2011).

The flow past the wind turbine is generally turbulent so that the different turbulence models are taken into consideration in the AL/NS model. Since the turbulence is produced from the wall boundary layer and maintaining this boundary layer is so expensive at high Reynolds numbers, the importance of using the actuator line technique is become evident. Because, actuator line technique enables to determine the forces on the blade according to blade element method by using airfoil data. Hence, it is unnecessary to calculate the blade boundary layer (Shen, Zhu and Sørensen, 2011).

The results of the CFD computations are obtained from Shen, Sørensen and Yang (2011). At the same time, the experimental results are acquired from the EU-sponsored MEXICO (model experiments in controlled conditions) project (Schepers and Snel, 2007). The rotor and the tower specifications of the turbine which was used in MEXICO project are given on Table 2. The geometrical and structural properties as well as the airfoil data have to be specified for the BEM code.
Table 2. MEXICO Rotor and Tower Specifications

<table>
<thead>
<tr>
<th>Rotor and Tower Specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>3</td>
</tr>
<tr>
<td>Angular velocity (rad/s)</td>
<td>44.4</td>
</tr>
<tr>
<td>Cone angle (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Tilt angle (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Total pitch (deg)</td>
<td>-2.3</td>
</tr>
<tr>
<td>Tower height (m)</td>
<td>6.5</td>
</tr>
<tr>
<td>Shaft length (m)</td>
<td>2.13</td>
</tr>
<tr>
<td>Approx. total mass of tower (kg)</td>
<td>8419</td>
</tr>
<tr>
<td>Tower ground radius (m)</td>
<td>0.3</td>
</tr>
<tr>
<td>Tower top radius (m)</td>
<td>0.2</td>
</tr>
<tr>
<td>Rotor radius (m)</td>
<td>2.25</td>
</tr>
</tbody>
</table>

In addition to these specifications, wind velocity and the yaw angle have to be specified for each conditional analysis to carry on the calculations. Therefore, three different yaw angles 15°, 30°, 45° and wind velocities 10 m/s, 15 m/s, 24 m/s were chosen to investigate the aerodynamic behavior of the wind turbine system. The effects of the varying yaw angle and wind velocity on the induction factors and load distributions are evaluated according to new yaw model in each condition.

The comparison of the models is carried out in terms of the load distribution on the blade. Since the load distribution is not uniform along the blade, five different blade sections %25, %35, %60, %82, %92 were chosen for the examination and each section is represented with the different color on the load distribution figures. For the simplicity of the load analysis, the results of the BEM technique with new yaw and Glauert’s yaw models are denoted as Model 1 and Model 2 respectively. Similarly, the CFD results with AL-NS technique are expressed as Model 3 in the following case analyses.

5.1.1 CASE I. Yaw 15° & Wind Speed 10 m/s

The yaw angle and the wind velocity are set to the smallest values for the first case. The axial and the tangential induction factors are represented with the blue and red colors respectively in the figure below. The blade is divided into 30 sections and each small square refers to a certain section on the blade. The first three points are very close to the hub and the last point is at the tip. Hence, the axial induction values at these sections are very small as it can be seen from Figure 17. The axial induction factor at the inner sections is about 0.3 and it keeps rising up to 0.45 along the blade. Then it follows a reverse pattern to decrease slowly until it gets close to the tip. The axial induction factor decreases dramatically at the last section because it is exactly the tip of the blade and the axial induction factor at the blade tip is almost below 0.1. On the other hand, the tangential induction factor is generally very small along the blade when it is compared with the axial one and it could not even reach to 0.1 at its highest value.
The observation of the load distribution pattern for a complete rotation of specific blade sections is also a part of this project. The normal forces, in other words, the axial loads on the specified sections are displayed in Figure 18 for the varying azimuth angle. The five blade sections are specified with five different colors and there are four different lines to indicate the three different computational yaw model and the experimental results. For instance, the solid line stands for the measurement results. The dashed line, the bold points and the dotted line represent the results of the BEM technique with new yaw model, BEM technique with Glauert’s yaw model and CFD technique with the Actuator Line-Navier Stokes(AL-NS) yaw model respectively.

It is easily observed from the figure that all the models for the inner sections (%25-%35) provided fairly well results which are very close to the experimental results. The magnitude of the normal loads at the inner sections at 15° yaw and 10 m/s wind speed does not vary so much throughout the rotation. For the outer sections such as %60, %82, %92, there appears to be small differences between the estimated axial loads and the measurement results. It is seen that the results predicted by Model 3 at the sections %60 and % 82 are better than the other models. On the other hand, Model 1 and Model 2 follow almost the same pattern at the section %92 and they provide closer results than the results of Model 3 at the last quarter of the rotation.
5.1.2 CASE II. Yaw 15° & Wind Speed 15 m/s

In the second case, the yaw angle is kept constant but the wind velocity is increased up to 15 m/s. The effect of the increasing wind velocity on the axial and tangential induction factors can be seen below. The axial induction factor starts to increase rapidly at the inner sections and it keeps increasing until it reaches the value of around 0.3. Then the axial induction factor is kept almost at that level along the blade. The value of the induction factor starts to decrease at sections close to the tip. The axial induction factor is smaller than 0.1 and almost close to zero at the tip. On the other hand, the tangential induction factors are very slightly larger at the inner sections when they are compared with the previous case. Then the tangential induction factor decreases smoothly along the blade.
Figure 19. Axial and Tangential Induction Factors at 15° yaw and 15 m/s wind speed

The effect of increasing the wind velocity on the normal load distribution is observed in Figure 20. At this condition, the magnitude of the normal forces acting on the sections %25, %35, %60 are doubled and they get 1.5 times larger at sections %82, %92. The measurement results show that the loads at the sections %25, %35, %60 are almost constant during the rotation. All three yaw models work properly at the inner sections %25, %35. At the outer sections %60, %82, %92, Model 1 and Model 2 provide quite similar results and it can be concluded that Model 3 is more suitable to use. The Model 3 is especially beneficial to estimate the axial load on the blade sections very close to the tip.
5.1.3 CASE III. Yaw 15° & Wind Speed 24 m/s

The yaw angle is still kept constant at 15 degrees in the third case to investigate the effect of the extreme wind condition. As it can be noticed from Figure 21, the magnitude of the axial induction factor decreased dramatically at every section on the blade. The value of the axial induction factor seems like almost constant along the blade and it is around 0.15. However, it is reduced down to a value which is very close to zero at the blade tip. On the other side, the effect of the extreme wind on the tangential induction factor is not obvious as it can be observed from the figure below. As a result of the first, second and the third cases, it can be said that the tangential induction factor does not respond so much to the changes in wind velocity.

![Figure 21. Axial and Tangential Induction Factors at 15° yaw and 24 m/s wind speed](image)

The normal load distributions at the extreme wind condition are shown in Figure 22. The normal load at each defined section is larger than the first and second cases due to the higher wind condition. By examining the outcomes of the third case, it is verified that the magnitude of the normal load acting on the rotor is directly related with the wind speed independent of its yaw angle.

The experimental results at this condition are quite irregular and do not follow a smooth pattern in a rotation period. The instant fluctuations on the normal force line are observed throughout the time at various azimuth angles. Therefore, it is not easy to estimate the loads on the sections. For instance, at the inner sections especially at the section %25, the results obtained by all three models are far beyond the correct estimation of the normal force. The same comment is also valid for the outer sections %82, %92. Even if the estimated normal force patterns are similar with the measurement results, they are overestimated by all three models. It seems like the best results are obtained for the section %60. The examination of the third case demonstrated that the results of Model 1, Model 2 and Model 3 are similar at the inner and outer sections.
5.1.4 CASE IV. Yaw 30° & Wind Speed 10 m/s

The yaw angle is chosen as 30 degrees and the wind speed is set to 10 m/s in the fourth case. The results can be compared with the results of the first case to observe the influence of the yaw. The variation of the axial and tangential induction factors along the blade are shown in Figure 23. The axial induction is around 0.3 at the inner sections and it keeps increasing along the blade up to 0.5. From that level, the induction factor starts to decrease as it gets close to the blade tip. The axial induction factor at the blade tip is estimated at around 0.1. The examination of the fourth case proved that the magnitude of the axial induction factor directly related with the yaw of the rotor and it increases with an increasing yaw angle. However, it is not possible to come up with the same consequence for the tangential induction factor. It is very small and does not change too much along the blade as in the first case. Therefore, it can be stated that the tangential induction factor is not affected by the changes in yaw angle or the effect of the yaw is negligible.
The axial load distribution curves for five reference sections at 30 degrees of yaw and 10 m/s wind velocity are shown below. When the Figure 24 is compared with the Figure 18 in the first case, it is seen that the magnitude of the normal loads slightly decreased in all sections. As a consequence of this observation, it can be claimed that the magnitude of the normal load acting on the blade section is indirectly related with the yaw condition of the rotor. If the normal load distribution results are taken into consideration in terms of the previously used yaw models, it is seen that the Model 3 provide better results at all sections. Model 1 and Model 2 can also be regarded as useful at the inner sections. At the outer sections %60, %82, %92 Model 1 is favored against Model 2.
5.1.5 CASE V. Yaw 30° & Wind Speed 15 m/s

The conditional property of the fifth case is chosen to determine the influence of the increasing wind speed on the induction factors and normal loads at 30 degrees of yaw angle. When the wind speed becomes 15 m/s, the calculated axial induction factor decreased at all sections. Unlike the fourth case, the axial induction factor does not increase so much after it reaches 0.3 at the inner sections and its magnitude reaches at most to 0.35 along the blade. Furthermore, the value of the axial induction factor at the blade tip decreases sharply to approximate value of 0.05. The tangential induction factor slightly increased and gets closer to 0.1 at inner sections when it is compared with the value at the fourth case. It is also important to compare the results of Figure 19 and Figure 25 in order to see the impact of a yaw angle on the axial induction factors at 15 m/s wind speed. As a result of this comparison, it can be stated that the increase in the yaw degree give rise to an increase in the axial induction factor.

Figure 26 can be used to observe the change of the normal loads in a rotation time when the wind velocity increases up to 15 m/s at 30° yaw. As it can be expected, the comparison of the loading curves in Figure 26 with the Figure 24 proves that the normal forces are larger at each section due to the higher wind velocity. However, the examination of the Figure 20shows that the normal loads are reduced when the yaw angle becomes 30 degrees at the same wind velocity. According to Figure 26, the estimated load distributions obtained by using all models are quite acceptable at the inner sections %25 and %35. At the outer sections %60, %82, %92, Model 1provides closer results than the other models until the one-third of the rotation is completed. For the rest of the rotation time, it is observed that the curve estimated by Model 3 is closer to the experimental results. In general, the Model 3 seems to be the best model for the normal load estimation and Model 1 can be preferred instead of Model 2. Because Model 2 estimates very high normal forces especially at the outer sections when the azimuth angle is between 200°-250°.
5.1.6 CASE VI. Yaw 30° & Wind Speed 24 m/s

The effect of the extreme wind condition is examined in the sixth case by keeping the yaw angle at 30 degrees. Figure 27 shows the pattern of the axial and tangential induction factors along the blade. It can be seen that the axial induction factor decreases sharply at all sections. The axial induction factor reaches to a value very close to 0.2 at the inner sections. Then it almost stays at that level along the blade until it comes to the blade tip. At the blade tip, the axial induction factor is very small and close to the tangential induction factor in terms of its magnitude. On the other hand, the tangential induction factor seems like increased very slightly at the inner sections and then it keeps decreasing at the outer sections. The comparison of Figure 27 with Figure 25 and Figure 21 proves one more time that the axial induction factor increases with the increasing yaw angle of the rotor and decreases with the increasing wind velocity. Additionally, the tangential induction factor is not affected so much by the changes in yaw angle and wind velocity.
The load distributions at the extreme wind condition according to the various models are shown in Figure 28. As it can be seen from the figure, normal loads increased with the increasing wind velocity acting on the rotor. Moreover, the measurement results in Figure 22 demonstrated that the normal loads acting on all blade sections get smaller in the beginning when the yaw increases to 30°. However, the load distribution curves obtained by all three models are getting more bell shaped curves. As a result of this, the estimation of the normal force at the 180 degrees azimuth angle, which means that the blade is at the downward position and parallel to the tower, is way higher than the measured value at the outer sections %60, %82, %92. In contrast to this situation, the results provided by all three models fit properly with the experimental results at section %60 at the first and the last quarter of the rotation period which refer to the azimuth angles between 0°-90° and 270°-360°. The same argument is valid for the sections %82, %92 because the results are closer to the experimental values in these intervals.

On the other hand, for the inner sections %25 and %35 all three techniques work perfectly only for a short period of time. For instance, at section %25, all three techniques provide better approximations when the blade azimuth angle is between 150°-220°. Similarly, the best results can be obtained for the section %35 in the azimuth range of 100°-200°. None of the techniques for the inner sections is useful except in these mentioned intervals due to the some irregularities and the fluctuations in the measurement results during the first and the last quarter of the rotation period.

As a conclusion for this case, it can be said that the results acquired from all three techniques are close to each other in both inner and outer sections. However, Model 1, Model 2 and Model 3 are only eligible to use for certain azimuth ranges.
5.1.7 CASE VII. Yaw 45° & Wind Speed 10 m/s

The influence of the high yaw angle on the induction factors and the load distribution is examined in this case by keeping the wind velocity at 10 m/s. The distribution of the axial and the tangential induction factors are displayed in Figure 29. It can be clearly seen that the axial induction factor rapidly increases up to 0.3 at the inner sections. After it reaches to that point, it decreases slightly at the next section and then it keeps increasing at the consecutive sections until it reaches around 0.6. The magnitude of the axial induction factor starts to decrease when it gets closer to the blade tip and eventually it becomes 0.1 at the blade tip. On the other side, there is a huge difference between the axial and the tangential induction factor because the tangential induction factor is so small along the blade.

The effect of the yaw on the induction factors can be determined by taking the Figure 17 and Figure 23 into consideration. The wind velocity is same in these cases and it is set to 10 m/s. According to the comparison, it is confirmed that there is a direct relationship between the yaw angle and the axial induction factor whereas the impact of the yaw angle on the tangential induction factor is not obvious.
The load distributions at the high yaw condition according to the various models are shown in Figure 30. The evaluation of the measurement results in Figure 18, Figure 24 and Figure 30 verify that the normal force acting on all blade sections are getting smaller when the yaw angle increases. However, the shape of the load curves determined by Model 1 and Model 2 are getting more sinusoidal at the outer sections.

According to Figure 30, the normal load distribution curves determined by Model 3 for the sections %25 and %35 follow quite similar path with the experimental curve. Hence, it can be assumed that Model 3 is better than the other models. Apart from the Model 3, the results maintained by Model 2 is closer to the measurement results and that’s why it is favored against Model 1 at the inner sections.

Figure 30 demonstrates that at outer sections %60, %82, %92, Model 3 is the best method which can be used to estimate the normal loading on the blade. Since the curves representing the results of the BEM techniques follow the sinusoidal pattern, the normal force estimations especially at the sections %60, %82 are underestimated between the azimuth ranges of 0°-150°. On the contrary, they are overestimated in the azimuth interval of 200°-300°.

As a consequence, Model 1 is favored against Model 2 at the outer sections %60, %82, %92 whereas Model 2 works slightly better at the inner sections %25, %35. However, Model 3 is the best method to be used at this condition and it provides fairly well results in all sections.
5.1.8 CASE VIII. Yaw 45° & Wind Speed 15 m/s

The conditional property of this case is chosen to determine the influence of the increasing wind speed on the induction factors and loads at 45 degrees of yaw angle. The axial and the tangential induction factors along the blade are shown in Figure 31. It is observed that the axial induction factor at the inner sections is around 0.3 and it increases slowly along the blade. At the outer sections, the axial induction factor reaches up to a point close to 0.45. Then it decreases sharply below 0.1 at the blade tip. The tangential induction factor is very small as it was in every case and it is decreasing along the blade after it reaches to a value around 0.05. The effect of the yaw on the induction factors is again investigated by comparing Figure 31 with the Figure 19 and Figure 25 for the same wind velocity conditions. It is seen that the axial induction factors at all sections are larger when the yaw angle is 45 degrees.

The influence of the increasing wind velocity at 45° yaw can also be evaluated by using the results displayed in Figure 29. As it is previously proved, there is an indirect relationship with the magnitude of the wind velocity and the axial induction factor. On the other hand, the effects of the yaw and the wind velocity are not so obvious for the tangential induction factor.
The normal load distributions according to the various techniques at 45 degrees yaw and 15 m/s wind velocity are shown in Figure 32. The measurement results show that the normal forces acting on the rotor blade increase at every section when the wind speed increases to 15 m/s and the yaw angle is kept constant. The effect of the yaw at the constant wind speed 15 m/s can be studied by comparing Figure 20, Figure 26 and Figure 32. It is observed that the normal loads at the inner and outer sections decrease with the increasing yaw angle. However, the load curves obtained by the BEM techniques become more sinusoidal at the outer sections.

If the normal load distribution curves at the inner sections %25, %35 are taken into consideration, it is seen that the points on the CFD curve follow almost similar path with the experimental curve. Although the curve obtained from Model 2 provides slightly closer results at the azimuth range of 200°-300°, the pattern of the CFD curve is proper than the other curves in general. The results of the Model 1 are close to Model 2 at inner sections except when the azimuth angle is between 150°-200°.

Model 1 works better than the Model 2 at the outer sections %60, %82, %92. However, as a result of having more sinusoidal curves for both BEM techniques, the magnitude of the normal forces are underestimated in an azimuth interval of 0°-150°. On the contrary, they are overestimated at the azimuth range of around 200°-300°.

In conclusion, the estimated normal load distribution curves by using Model 3 at all sections fit more properly with the experimental curves. Model 1 works better than the Model 2 at the outer sections whereas the results of the two models are quite similar at the inner sections.
5.1.9 CASE IX. Yaw 45° & Wind Speed 24 m/s

The effects of the high yaw and the extreme wind on the induction factors and the normal load distribution are studied in this case. The axial and the tangential induction factors at these conditions are shown in Figure 33. It is observed that the axial induction factor at the inner sections increases rapidly to a value 0.2 and then it increases slowly up to a value around 0.3 along the blade. The axial induction factor gets very close to zero at the blade tip. On the other side, the tangential induction factor increases up to very close to 0.1 in the beginning and then it decreases smoothly along the blade. The comparison of the Figure 29, Figure 31 and Figure 33 verifies that for the same yaw angle of 45 degrees, the magnitude of the axial induction factor decreases with the increasing wind velocity. Additionally, the comparison of the Figure 21, Figure 27 and Figure 33 shows that for the same wind velocity the axial induction factor increases with the increasing yaw angle. However, the effects of the yaw and the wind velocity on the tangential induction factor are not so obvious.
The normal load distributions according to the various techniques at 45 degrees yaw and extreme wind condition are shown in Figure 34. The effect of the high yaw angle can be studied by comparing Figure 34 with the Figure 22 and Figure 28. As a result of this comparison, it is observed that the magnitude of the normal loads at 45° yaw are less than the values obtained at 30° and 15° yaw especially in the beginning of the rotation. The load distribution curves are again bell shaped and reach to their maximum value at all sections when the blade azimuth angle is around 200°.

It is apparent that there are some irregularities in the measurement results in the first and the last quarter of the rotation period. These instant fluctuations are more common at the inner sections and they get smaller at the outer sections. Due to these intense fluctuations at the inner sections, none of the techniques work properly between the azimuth intervals of 0°-120° and 260°-360°. However, when the azimuth range is between 120°-260°, the curves of all three models follow the same pattern with the experimental curve. At this condition, the normal load curve obtained by Model 1 is closer to the experimental curve and therefore it is favored against the other techniques at sections %25 and %35.

At the section %60, it is seen that all the models follow the same pattern with the experimental curve but the best fit can be achieved by using the Model 1. At the outer sections %82 and %92, estimations of BEM techniques are close the experimental values until the blade azimuth angle reaches up to 120°. After that level, the results of the BEM techniques increase dramatically and the normal loads are overestimated at the azimuth interval of 120°-260°. However, in general the pattern of the curves obtained by Model 3 is closer to the experimental curves throughout the rotation at the outer sections.

In conclusion, Model 1 is the best method for the sections %25, %35 and %60 whereas the Model 3 is favored at the outer sections %82 and %92.
5.1.10 The Distribution of the Tangential Force at All Cases

The distributions of the tangential loads in all cases are represented in the following figures. It is observed that the magnitude of the tangential force acting on the blade is always smaller than the axial force at the same conditions. This situation is quite understandable because the wind turbine is always installed against the wind flow in order to take it from the axial direction. The figures also demonstrate that the tangential force is higher at the inner sections and it decreases along the blade.

According to the figures, it is seen that the tangential forces are so small especially when the wind velocity is 10 m/s. There seems to be a very small difference in the results obtained by different models. Therefore, it makes sense to determine the proper model for each condition according to the evaluation of normal force figures. Nevertheless, it is still beneficial to analyze the distribution of the tangential loads to comprehend the general trend.

The Figure 35 shows that Model 3 is slightly favored at both inner and outer sections in all wind velocity conditions. However, it can be also said that the Model 1 and Model 2 provide fairly well results at the outer sections.

Figure 34. Axial load distributions for various yaw models at 45° yaw and 24m/s wind speed
The relationship between the yaw angle and the tangential force is studied according to the following figures. Since the magnitudes of the tangential forces are small, the effect of an increasing yaw degree seems so small. The Figure 36 proves that the tangential forces slightly decreased when the yaw angle becomes 30 degrees.

The same conclusion can be drawn for the comparison of the different models because the patterns of the distribution curves are almost similar to the previous yaw condition.
The Figure 37 represents the change of a tangential force at the high yaw condition. The tangential force on each section is again slightly decreased due to the increasing yaw. The distribution curves for the outer sections become more sinusoidal. The irregularities and instant fluctuations are observed at high wind velocities. Therefore, it is not so easy to fit the model curves with the measurement results. However, in all cases the patterns of the curves obtained by different models are quite similar.

The same conclusions about the different models are still valid in this case. Model 3 is slightly favored especially at the inner sections. However, Model 1 provides also good estimations at the outer sections in extreme wind condition.
5.2 Implementation of the New Tip Loss Correction Model

Since the implementation of the New Tip Loss Correction model is another objective of this study, the BEM model is modified according to the formulas given above. However, at this time the different wind turbine (NordTank 500) model is chosen for the examination. The Nordtank turbine is larger in terms of size and the corresponding rotor and tower specifications are provided on Table 3.

The normal and the tangential forces along the blade at wind velocities 5 m/s, 7 m/s, 10 m/s, 12 m/s are determined and validated with the CFD results which are obtained from Shen, Zhu and Sørensen (2011). The Glauert’s tip loss correction model is also implemented to compare the results of various models on the same
blade (Glauert, 1935). However, Nordtank turbine has LM 19.1 blades with sizes almost ten times larger than the former blade types. Hence, the new airfoil data as well as the new geometry and structure data according to LM 19.1 blades are used as input files in the BEM code.

Table 3. Rotor and Tower Specifications of Nordtank 500 Turbine

<table>
<thead>
<tr>
<th>Rotor and Tower Specifications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of blades</td>
<td>3</td>
</tr>
<tr>
<td>Angular velocity (rad/s)</td>
<td>2.96</td>
</tr>
<tr>
<td>Cone angle (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Tilt angle (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Total pitch (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Yaw angle (deg)</td>
<td>0.0</td>
</tr>
<tr>
<td>Tower height (m)</td>
<td>36</td>
</tr>
<tr>
<td>Shaft length (m)</td>
<td>20</td>
</tr>
<tr>
<td>Approx. total mass of tower (kg)</td>
<td>46930</td>
</tr>
<tr>
<td>Tower ground radius (m)</td>
<td>3</td>
</tr>
<tr>
<td>Tower top radius (m)</td>
<td>2</td>
</tr>
<tr>
<td>Rotor radius (m)</td>
<td>20.5</td>
</tr>
</tbody>
</table>

The normal and the tangential loads at various wind velocities are shown in Figure 38, Figure 39, Figure 40 and Figure 41. The examination of these figures demonstrated that both the normal and the tangential forces along the blade increase when the wind velocity increases. However, at the same wind velocity condition, they keep on rising up to certain point on the blade and then start to decrease as it gets close to the blade tip.

For instance, the normal force reaches to a maximum value at the radial position of 17 meters on the blade according to all models. After this point, it decreases sharply until it becomes almost zero at the blade tip. The same conclusion can be drawn for the tangential force acting on the blade. The magnitude of the tangential force along the blade is directly related with the wind velocity. However, at the certain wind velocity, the tangential force reaches its maximum value around the middle of the blade. Then it decreases along the blade until it reaches to the tip.

The comparison of the different tip loss correction models showed that the results obtained from each model are similar in general. However, the estimated force trends at the radial positions close to the blade tip vary slightly. Therefore, the difference of the various tip loss correction models becomes perceptible around the blade tip.

Thanks to the implementation of the tip loss correction models in this chapter, the resulting figures can be used to determine the effectiveness of the new tip loss correction model against the other models if the measurement data is known.
Figure 38. Normal and Tangential Force distribution for various tip loss correction models at 5 m/s wind velocity

Figure 39. Normal and Tangential Force distribution for various tip loss correction models at 7 m/s wind velocity
Figure 40. Normal and Tangential Force distribution for various tip loss correction models at 10 m/s wind velocity

Figure 41. Normal and Tangential force distribution for various tip loss correction models at 12 m/s wind velocity
6. DISCUSSION

This project aims to improve the BEM technique for the yawed flows passing through the wind turbine rotor by using the new yaw model. For this purpose, the experimental MEXICO rotor is used for the modeling computations. As it is mentioned before, the MEXICO project provided a database for the various yaw conditions and this database is used to estimate the effectiveness of the BEM technique with new yaw model. In fact, the comparison of the BEM technique is not only carried out against the measurement results but it is also validated against the CFD technique in this project. Additionally, the implementation of Glauert’s yaw model in BEM technique provided interesting results for comparison.

The conditional case study for various yaw and wind velocity conditions provided crucial information about the influence of those parameters on the load distribution and the induction factors. One of the important outcomes of these conditional analyses is that, regardless of the experienced wind velocity, the axial induction factor along the blade increases with the increasing yaw angle. The direct relationship between the yaw angle and the axial induction factor can be simply explained by analyzing the velocity components on the rotor plane. In case of having yawed flows, the axial velocity on the rotor plane becomes the cosine of the yaw angle times the undisturbed wind velocity. Hence, the larger yaw angle means the smaller axial wind velocity seen by blade. The change in the relative wind velocity affects the series of calculation steps to give larger axial induction factor. The conditional analyses also demonstrated that there is a relationship between the wind velocity and the magnitude of the axial induction factor. Unlike the situation which is experienced in varying yaw condition, the axial induction factor decreases when the wind velocity increases at constant yaw angle. The explanation of this indirect relationship between the wind velocity and the axial induction factor is related again with the magnitude of the relative wind velocity. However, the axial component of the relative wind velocity increases at this time. As a result of this, several parameters that are used in the calculation steps such as flow angle and lift coefficient change to provide smaller axial induction factor. On the other hand, the effect of the yaw and wind velocity on the tangential induction factor couldn’t be determined precisely. Because, the calculated values in each case are so small along the blade and do not change so much along the blade.

Apart from the evaluation of the induction factors, the effects of the yaw and the wind velocity on the normal and tangential forces are observed. According to the measurement results it is seen that the magnitude of the forces on the blade decreases at higher yaw angles and increases with an increasing wind velocity. The reason of this situation is exactly the same reason which is indicated for the induction factors. The axial component of the relative wind velocity decreases when the yaw angle increases and this give rise to have smaller lift and drag forces on the blade. On the other side, the relative wind velocity is larger at the high wind velocity condition. Hence, the lift force increases to provide higher loads on the blade.

The observation of the load distribution in various conditions enabled to compare the different yaw models. For instance, the determination of the effectiveness of new yaw model at different yaw and wind velocity conditions can be carried out in two ways. Firstly, three different conditions at the same wind velocity are chosen to investigate the situation at different yaw angles. Secondly, three different conditions at the certain yaw angle are chosen to observe the outcomes at different wind velocities. According to the first comparison method, the figure sets such as Figure 18, Figure 24, Figure 30; Figure 20, Figure 26, Figure 32; Figure 22, Figure 28, Figure 34 can be chosen for the evaluation. The comparison of these figures showed that the CFD technique is favored at every section in general. At the same time, it is seen that BEM technique with the new yaw model is more successful than Glauert’s yaw model at the outer sections %60, %82, %92. The new yaw
model is only favored against the CFD model at the section %60 in Figure 34. The situation is more or less the same at the inner sections %25 and %35. The CFD technique is proved to be the best method in all conditions except in Figure 34 where the new yaw model provides better results at sections %25, %35. At the inner sections, the new yaw model is favored against Glauert’s yaw model at higher wind velocities such as 15 m/s and 24 m/s in all the yaw conditions. However, the first set of figures show that, at 10 m/s wind velocity, Glauert’s yaw model slightly better than the new yaw model at the sections %25, %35 at the higher yaw angles.

The effectiveness of the models according to the different wind velocities can be determined by examining the figure sets such as Figure 18, Figure 20, Figure 22; Figure 24, Figure 26, Figure 28; Figure 30, Figure 32, Figure 34. The comparison of these figures demonstrated that the CFD technique is the best method almost in all conditions. It is also observed that the new yaw model is slightly better than Glauert’s yaw model at 10 m/s and 15 m/s wind velocities in all the yaw conditions. However, in case of having extreme wind velocity 24 m/s, all three models provide the same results especially at the yaw angles 15 and 30 degrees. The new yaw model is only more successful than any other models especially at sections %25, %35 and % 60 when the yaw angle is at its highest value of 45 degrees and the wind velocity is 24 m/s.

The tangential load distributions according to the different models are also evaluated in this study. Since the tangential force provides torque for the rotation, its magnitude is larger at the inner sections. The tangential forces at all sections increase with the increasing wind velocity just as same as the axial forces. The effect of the yaw is also very small but it appears to be that the tangential forces decrease slightly when the yaw angle increases. The conditional analyses of the tangential loads proved that the CFD technique is the most successful method. Nevertheless, the new yaw model is favored against the Glauert’s yaw model in general.

The application of the new tip loss correction model is another objective of this project. For this purpose, the new tip loss correction model is tested on the Nordtank 500 wind turbine. The distribution of the forces at various wind velocities is validated with the CFD results of the previous study. The new tip loss correction model is simply the modification of the Prandtl’s correction function in which the coefficient $g$ is included in the formula. The coefficients used in the $g$ formulation are determined from experimental data. Therefore, the new tip loss correction provide better curve fitting with the experimental results than other correction models.
7. CONCLUSION

As an aim of this project, the BEM technique for the yawed flows is improved. This is carried out by including the derived formulas for the new yaw model into the BEM code. In order to determine the effectiveness of the BEM technique with new yaw model, the CFD and the measurement results obtained from the MEXICO project are taken into consideration. Moreover, the Glauert’s yaw model is employed in the BEM code for the comparison.

One of the important outcomes of this study is that the CFD technique still seems to be better than any other techniques in general. However, the BEM technique with new yaw model is only favored against the CFD technique at the high yaw and wind conditions. Nevertheless, this project proved that the new yaw model is favored against the Glauert’s yaw model almost in all conditions.

The second part of this project is about the implementation of the new tip loss correction model in the BEM code. The calculated force distribution on Nordtank 500 wind turbine blade is validated with the CFD results. The results from the Glauert’s tip loss correction model are also obtained from the BEM code and put into the same figures for comparison. The force distribution of each model differs than the others around the blade tip region. Therefore, this study can be used as a good source for comparisons against the measurement results.

As a recommendation for future studies on this thesis subject, the dynamic wake and stall models can be included into the BEM code to estimate the induced velocities and angle of attack more precisely in each time step.
8. REFERENCES


APPENDIX

Matlab Codes

1. Input

```matlab
clear
load geometry.dat
load Ngeometry.dat
r = geometry(:,1);
r1 = Ngeometry(:,1) + 0.21;
c1 = Ngeometry(:,2);
tw1 = Ngeometry(:,3);
th1 = Ngeometry(:,4);

c = interp1(r1, c1, r);
tw = interp1(r1, tw1, r);
 th = interp1(r1, th1, r);

[r, c, tw, th]
```

2. Functions

```matlab
function [Ca] = func1(radius, Rb, Psi, theta_yaw)
K = 0;

if (radius/Rb) <= 0.35
    if theta_yaw == 15*pi/180
        K = 0.45837*theta_yaw;
    elseif theta_yaw == 30*pi/180
        K = 0.45837*theta_yaw;
    elseif theta_yaw == 45*pi/180
        K = 0.45837*theta_yaw;
    end

    Cnst1 = 1 - K*cos(Psi);
    Ca = Cnst1;
end

if (radius/Rb) >= 0.6
    if theta_yaw == 15*pi/180
        K = 1.03*theta_yaw;
    elseif theta_yaw == 30*pi/180
        K = 1.03*theta_yaw;
    elseif theta_yaw == 45*pi/180
        K = 1.03*theta_yaw;
    end

    Cnst2 = 1 + K*(radius/Rb)^2*sin(Psi+pi/2*(radius/Rb-1));
    Ca = Cnst2;
end

if (radius/Rb) > 0.35 && (radius/Rb) < 0.6
    if theta_yaw == 15*pi/180
        K = 0.45837*theta_yaw;
    elseif theta_yaw == 30*pi/180
        K = 0.45837*theta_yaw;
    end
```

elseif theta_yaw==45*pi/180
    K=0.45837*theta_yaw;
end
Cnst1=1-K*cos(Psi);

if theta_yaw==15*pi/180
    K=1.03*theta_yaw;
elseif theta_yaw==30*pi/180
    K=1.03*theta_yaw;
elseif theta_yaw==45*pi/180
    K=1.03*theta_yaw;
end
Cnst2=1+K*(radius/Rb)^2*sin(Psi+pi/2*(radius/Rb-1));

Ca=(Cnst1*(0.6-(radius/Rb))+(Cnst2*((radius/Rb)-0.35)))/0.25;
end

function [Ca]=func2(radius,Rb,Psi,theta_yaw)
K=0;

if (radius/Rb)<=0.35
    if theta_yaw==15*pi/180
        K=0.45837*theta_yaw;
    elseif theta_yaw==30*pi/180
        K=0.45837*theta_yaw;
    elseif theta_yaw==45*pi/180
        K=0.45837*theta_yaw;
    end
    Cnst1=1+(radius/Rb)*K*sin(Psi);
    Ca=Cnst1;
end

if (radius/Rb)>=0.6
    if theta_yaw==15*pi/180
        K=1.03*theta_yaw;
    elseif theta_yaw==30*pi/180
        K=1.03*theta_yaw;
    elseif theta_yaw==45*pi/180
        K=1.03*theta_yaw;
    end
    Cnst2=1+(radius/Rb)*K*sin(Psi);
    Ca=Cnst2;
end

if (radius/Rb)>0.35 && (radius/Rb)<0.6
    if theta_yaw==15*pi/180
        K=0.45837*theta_yaw;
    elseif theta_yaw==30*pi/180
        K=0.45837*theta_yaw;
    elseif theta_yaw==45*pi/180
        K=0.45837*theta_yaw;
    end
    Cnst1=1+(radius/Rb)*K*sin(Psi);
end
if theta_yaw==15*pi/180
    K=1.03*theta_yaw;
elseif theta_yaw==30*pi/180
    K=1.03*theta_yaw;
elseif theta_yaw==45*pi/180
    K=1.03*theta_yaw;
end
Cnst2=1+(radius/Rb)*K*sin(Psi);
Ca=(Cnst1*(0.6-(radius/Rb))+(Cnst2*((radius/Rb)-0.35)))/0.25;
end

3. Dynamic Structural Model of Wind Turbine (BEM Code)

function DSMWT
    clear
    clc
    close all
    tic
    % warning off
    % read profile and geometry data
    [r_,dr_,c_,theta_twist_,EI1_,EI2_,M_,alpha_,Cl_,Cd_,Cm_]=readprofile;
    % read configuration data
    [theta_yaw,theta_cone,pitch,theta_tilt,Omega,gravity,...
     density,B,gear_ratio,k,shaftlength,H,m_tower,a_ground,a_top,gamma,...
     stall_trigger,Vo,iter,dt,nbs,Ge_trigger,Gf,n0,...
     stall_trigger,Tp, Tf, b1, b2, A1, A2] = readconfig;
    % start the rotor at zero speed (takes longer time)
    % if(Ge_trigger==1);
    %     Omega=0;
    % end
    if(Wd_trigger==1);
        winddata = importdata('../input/winddata.dat');
        if(length(winddata)<iter)
            fprintf(1,'increase the wind data or decrease iteration number to %g
            
            ',iter)
        end
        Vomean = mean(winddata((1:iter),1));
        Vo (1:iter) = winddata((1:iter),1);
        Voy(1:iter) = winddata((1:iter),2);
        Vox(1:iter) = winddata((1:iter),3);
    else
        Vomean = Vo;
        Vox(1:iter)=0;Voy(1:iter)=0;Vo (1:iter)=Vo;
    end
    % interplate profile data according to the number of blade elements
    [r,dr,c,theta_twist,EI1,EI2,M,alpha,Cl,Cd,Cm]= ... 
    interprofile(r_,c_,theta_twist_,EI1_,EI2_ ,M_,alpha_,Cl_,Cd_,Cm_,nbs);
    % save twist theta_twist
% initialization
[X,X1,X2,GF,uy,uz,ut] = initialize(dofs,r,Omega);

% call mass and stiffness matrices
[mass,K,u]=MKmatrix(dofs,M,k,r,dr,c,m_tower,theta_twist,theta_cone,pitch,B,gravity,
    density,EI1,EI2,alpha,Cl,Cd,Cm,Omega);

m_hat=M;
N=length(r);
Rb=max(r);

% give initial values
Wy(1:N)=0;
Wz(1:N)=0;
an(1:N)=0;at(1:N)=0;

An(1:N,1:B)=0;At(1:N,1:B)=0;

deltaZ(1,1) = 0;
deltaZ(2,1) = 0;
deltaZ(3,1) = 0;
FcZ(1:B,1:N)=0;FcY(1:B,1:N)=0;
Uold(1:N,1:B)=0;Aold(1:N,1:B)=0;
x1(1:N,1:B)=0;x2(1:N,1:B)=0;x3(1:N,1:B)=0;x4(1:N,1:B)=0;
gravZ = m_hat.'*gravity*dr*sin(theta_cone);
mgconst = gravity*m_hat*dr;

pi23=2*pi/3;
pi43=4*pi/3;

sum1(1:N)=0; sum2(1:N)=0; sum3(1:N)=0; sum4(1:N)=0; sum5(1:N)=0;
sum6(1:N)=0; sum7(1:N)=0; sum8(1:N)=0; sum9(1:N)=0;
sum10(1:N)=0; sum11(1:N)=0; sum12(1:N)=0; sum13(1:N)=0;

for n=1:iter

[Vx,Vy,Vz]=fun_Vwind(n,N,r,X(2),dr,Vox(n),Voy(n),Vo(n),theta_yaw,theta_tilt,theta_cone,Omega,pitch,dt,B,...
    shaftlength,H,gamma,a_ground,a_top);

% plot cldy(:,:,n),clst(:,:,n) as function of time, otherwise put
% cldy,clsy to reduce dimension
W1=Wy;W2=Wz;
Ann=an;Att=at;

Psii=Omega*n*dt;

if((gear_ratio*30*X1(2)/pi-n0)>0 && Ge_trigger==1)
MG(n)=Gf*(gear_ratio*30*X1(2)/pi-n0)*gear_ratio;  % generator mode
else
MG(n)=0;  % motor mode
end

MR(n)=sum(((Py(:,1)+Py(:,2)+Py(:,3))*dr).*r')*cos(theta_cone);

if ((gear_ratio*30*X1(2)/pi-n0)>0 && Ge_trigger==1)
Pmech(n)=MR(n)*X1(2);  % power output
else
Pmech(n)=MR(n)*Omega;  % wjz
end
\[ GF(1) = \text{sum}((Pz(:,1) + Pz(:,2) + Pz(:,3) \cdot dr) \cdot \cos(\theta_{\text{cone}})) \quad \% \text{total rotor thrust} \]

\[
\text{if}(Ge\_\text{trigger}==1);
\quad GF(2) = MR(n) - MG(n);
\quad GF(2) = MR(n);
\text{end}
\]

\[ \text{gravY}(1,:) = mgconst \cdot \sin(X(2)); \]
\[ \text{gravY}(2,:) = mgconst \cdot \sin(X(2) + \pi/3); \]
\[ \text{gravY}(3,:) = mgconst \cdot \sin(X(2) + \pi/4); \]

\[
\text{Fc1f} = \text{sum}(\text{FcY}(1,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{FcZ}(1,:) \cdot \text{'.} \cdot u.z1f');
\]
\[
\text{Fc1e} = \text{sum}(\text{FcY}(1,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{FcZ}(1,:) \cdot \text{'.} \cdot u.z1e');
\]
\[
\text{Fc2f} = \text{sum}(\text{FcY}(1,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{FcZ}(1,:) \cdot \text{'.} \cdot u.z2f');
\]
\[
\text{G1f} = \text{sum}(\text{gravY}(1,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{gravZ} \cdot u.z1f');
\]
\[
\text{G1e} = \text{sum}(\text{gravY}(1,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{gravZ} \cdot u.z1e');
\]
\[
\text{G2f} = \text{sum}(\text{gravY}(1,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{gravZ} \cdot u.z2f');
\]

\[
\text{GF}(3) = \text{sum}(\text{Py}(1,:) \cdot u.y1f' \cdot dr) + \text{sum}(\text{Pz}(1,:) \cdot u.z1f' \cdot dr) - \text{Fc1f} + \text{G1f};
\]
\[
\text{GF}(4) = \text{sum}(\text{Py}(1,:) \cdot u.y1e' \cdot dr) + \text{sum}(\text{Pz}(1,:) \cdot u.z1e' \cdot dr) - \text{Fc1e} + \text{G1e};
\]
\[
\text{GF}(5) = \text{sum}(\text{Py}(1,:) \cdot u.y2f' \cdot dr) + \text{sum}(\text{Pz}(1,:) \cdot u.z2f' \cdot dr) - \text{Fc2f} + \text{G2f};
\]

\[
\text{Fc1f} = \text{sum}(\text{FcY}(2,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{FcZ}(2,:) \cdot \text{'.} \cdot u.z1f');
\]
\[
\text{Fc1e} = \text{sum}(\text{FcY}(2,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{FcZ}(2,:) \cdot \text{'.} \cdot u.z1e');
\]
\[
\text{Fc2f} = \text{sum}(\text{FcY}(2,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{FcZ}(2,:) \cdot \text{'.} \cdot u.z2f');
\]
\[
\text{G1f} = \text{sum}(\text{gravY}(2,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{gravZ} \cdot u.z1f');
\]
\[
\text{G1e} = \text{sum}(\text{gravY}(2,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{gravZ} \cdot u.z1e');
\]
\[
\text{G2f} = \text{sum}(\text{gravY}(2,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{gravZ} \cdot u.z2f');
\]

\[
\text{GF}(6) = \text{sum}(\text{Py}(2,:) \cdot u.y1f' \cdot dr) + \text{sum}(\text{Pz}(2,:) \cdot u.z1f' \cdot dr) - \text{Fc1f} + \text{G1f};
\]
\[
\text{GF}(7) = \text{sum}(\text{Py}(2,:) \cdot u.y1e' \cdot dr) + \text{sum}(\text{Pz}(2,:) \cdot u.z1e' \cdot dr) - \text{Fc1e} + \text{G1e};
\]
\[
\text{GF}(8) = \text{sum}(\text{Py}(2,:) \cdot u.y2f' \cdot dr) + \text{sum}(\text{Pz}(2,:) \cdot u.z2f' \cdot dr) - \text{Fc2f} + \text{G2f};
\]

\[
\text{Fc1f} = \text{sum}(\text{FcY}(3,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{FcZ}(3,:) \cdot \text{'.} \cdot u.z1f');
\]
\[
\text{Fc1e} = \text{sum}(\text{FcY}(3,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{FcZ}(3,:) \cdot \text{'.} \cdot u.z1e');
\]
\[
\text{Fc2f} = \text{sum}(\text{FcY}(3,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{FcZ}(3,:) \cdot \text{'.} \cdot u.z2f');
\]
\[
\text{G1f} = \text{sum}(\text{gravY}(3,:) \cdot \text{'.} \cdot u.y1f') + \text{sum}(\text{gravZ} \cdot u.z1f');
\]
\[
\text{G1e} = \text{sum}(\text{gravY}(3,:) \cdot \text{'.} \cdot u.y1e') + \text{sum}(\text{gravZ} \cdot u.z1e');
\]
\[
\text{G2f} = \text{sum}(\text{gravY}(3,:) \cdot \text{'.} \cdot u.y2f') + \text{sum}(\text{gravZ} \cdot u.z2f');
\]

\[
\text{GF}(9) = \text{sum}(\text{Py}(3,:) \cdot u.y1f' \cdot dr) + \text{sum}(\text{Pz}(3,:) \cdot u.z1f' \cdot dr) - \text{Fc1f} + \text{G1f};
\]
\[
\text{GF}(10) = \text{sum}(\text{Py}(3,:) \cdot u.y1e' \cdot dr) + \text{sum}(\text{Pz}(3,:) \cdot u.z1e' \cdot dr) - \text{Fc1e} + \text{G1e};
\]
\[
\text{GF}(11) = \text{sum}(\text{Py}(3,:) \cdot u.y2f' \cdot dr) + \text{sum}(\text{Pz}(3,:) \cdot u.z2f' \cdot dr) - \text{Fc2f} + \text{G2f};
\]

\% ver.1 first order
X2 = gfunc(X, X1, GF, mass, K);

X1 = X1 + X2*dt;
X = X + X1*dt;

\text{if}(K(1,1)>=1e9);X(1)=0;X1(1)=0;X2(1)=0;\text{end}

\text{if}(Ge\_\text{trigger}==1);
\quad \Omega = X(1);\quad \Omega = X(2);
\text{else}
\quad X2(2)=0;
\quad X(1)=0;
\quad X1(1)=0;
\quad \Omega = X(2);
\text{end}
end

x(:,n)=X;        % x: displacement;   x1: velocity;   x2: acceleration
x1(:,n)=X1;
x2(:,n)=X2;

% % % ver.2 higher order
% % % X2=gfunc(X,X1,GF,mass,K);
% % % x(:,n)=X;        % x is displacement;   x1 is differential of x;
x2 is differential of x1.
% % x1(:,n)=X1;
% % x2(:,n)=X2;
% % A=dt/2*x2(:,n);
% % b=dt/2*(x(:,n)+1/2*A);
% % Bx=dt/2*gfunc(X+b,X1+A,GF,mass,K);
% % C=dt/2*gfunc(X+b,X1+Bx,GF,mass,K);
% % d=dt*(x(:,n)+C);
% % D=dt/2*gfunc(X+d,X1+2*C,GF,mass,K);
% %
% % x(:,n+1)=x(:,n)+dt*(x1(:,n)+1/3*(A+Bx+C));
% % x1(:,n+1)=x1(:,n)+1/3*(A+2*Bx+2*C+D);
% % X=x(:,n+1);
% % X1=x1(:,n+1);
% % X2=gfunc(X,X1,GF,mass,K);

% X is the column of displacement ,e.g. X(2) is the angular displacement.
% X1 is the column of speed ,e.g. X1(2) is the rotation speed of rotor.
% X2 is the column of acceleration ,e.g. X2(2) is the acceleration of
% shaft.

% vibration speeds for 3 blades respectively.
uy.blade1=x(3,n)*u.y1f + x(4,n)*u.y1e + x(5,n)*u.y2f;
uz.blade1=x(3,n)*u.z1f + x(4,n)*u.z1e + x(5,n)*u.z2f;
uy.blade2=x(6,n)*u.y1f + x(7,n)*u.y1e + x(8,n)*u.y2f;
uz.blade2=x(6,n)*u.z1f + x(7,n)*u.z1e + x(8,n)*u.z2f;
uy.blade3=x(9,n)*u.y1f + x(10,n)*u.y1e + x(11,n)*u.y2f;
uz.blade3=x(9,n)*u.z1f + x(10,n)*u.z1e + x(11,n)*u.z2f;

ut=X1(1);  % this is the vibration speed of the tower(or cart).

% and this speed should be transformed into local blade coordinate system.

% flapwise blade deflections at each step
deflectY(1,:,n)=x(3,n)*u.y1f + x(4,n)*u.y1e + x(5,n)*u.y2f;
deflectZ(1,:,n)=x(3,n)*u.z1f + x(4,n)*u.z1e + x(5,n)*u.z2f;
deflectY(2,:,n)=x(6,n)*u.y1f + x(7,n)*u.y1e + x(8,n)*u.y2f;
deflectZ(2,:,n)=x(6,n)*u.z1f + x(7,n)*u.z1e + x(8,n)*u.z2f;
deflectY(3,:,n)=x(9,n)*u.y1f + x(10,n)*u.y1e + x(11,n)*u.y2f;
deflectZ(3,:,n)=x(9,n)*u.z1f + x(10,n)*u.z1e + x(11,n)*u.z2f;

deflectZ(1,2:N) = diff(deflectZ(1,:,n));
deflectZ(2,2:N) = diff(deflectZ(2,:,n));
deflectZ(3,2:N) = diff(deflectZ(3,:,n));
deflectY(1,2:N) = diff(deflectY(1,:,n));
deflectY(2,2:N) = diff(deflectY(2,:,n));
deflectY(3,2:N) = diff(deflectY(3,:,n));

% sin(teta), teta is deformation
sintetaZ(1,:) = deflectZ(1,:)./sqrt(deflectZ(1,:).^2+dr^2);
sintetaZ(2,:) = deflectZ(2,:)./sqrt(deflectZ(2,:).^2+dr^2);
sintetaZ(3,:) = deflectZ(3,:)./sqrt(deflectZ(3,:).^2+dr^2);
sintetaY(1,:) = deltaY(1,:)./sqrt(deltaY(1,:).^2+dr^2);
sintetaY(2,:) = deltaY(2,:)./sqrt(deltaY(2,:).^2+dr^2);
sintetaY(3,:) = deltaY(3,:)./sqrt(deltaY(3,:).^2+dr^2);

% call centrifugal forces
Fr = centrifugal(Omega,r,dr,M);

% projection of centrifugal forces normal to blades.
FcZ(1,:) = Fr.*(sintetaZ(1,:)+sin(theta_cone));
FcZ(2,:) = Fr.*(sintetaZ(2,:)+sin(theta_cone));
FcZ(3,:) = Fr.*(sintetaZ(3,:)+sin(theta_cone));

FcY(1,:) = Fr.*sintetaY(1,:);
FcY(2,:) = Fr.*sintetaY(2,:);
FcY(3,:) = Fr.*sintetaY(3,:);

% interpolation to find the Pz & Py values at sections r/R=35,60,82,92
Pz25=Pz1;
Pz35(n,1)=Pz2(n,1)+(Pz3(n,1)-Pz2(n,1))*(0.35-r(9)/Rb)/(r(10)/Rb-r(9)/Rb);
Pz60(n,1)=Pz4(n,1)+(Pz5(n,1)-Pz4(n,1))*(0.60-r(17)/Rb)/(r(18)/Rb-r(17)/Rb);
Pz82(n,1)=Pz6(n,1)+(Pz7(n,1)-Pz6(n,1))*(0.82-r(24)/Rb)/(r(25)/Rb-r(24)/Rb);
Pz92(n,1)=Pz8(n,1)+(Pz9(n,1)-Pz8(n,1))*(0.92-r(27)/Rb)/(r(28)/Rb-r(27)/Rb);
Py1=AoA1;
Py25=Py1;

% interpolation to find the AoA values at sections r/R=35,60,82,92
AoA25=AoA1;
AoA35=AoA2;

% interpolation to find the Ft & Fn values at sections r/R=35,60,82,92
Ft(n,1:N,1:B,n) = Py; %*dr;
Fn(n,1:N,1:B,n) = Pz; %*dr;

% % interpolation to find the Pz & Py values at sections r/R=35,60,82,92
Pz25=Pz1;
Pz35(n,1)=Pz2(n,1)+(Pz3(n,1)-Pz2(n,1))*(0.35-r(9)/Rb)/(r(10)/Rb-r(9)/Rb);
Pz60(n,1)=Pz4(n,1)+(Pz5(n,1)-Pz4(n,1))*(0.60-r(17)/Rb)/(r(18)/Rb-r(17)/Rb);
Pz82(n,1)=Pz6(n,1)+(Pz7(n,1)-Pz6(n,1))*(0.82-r(24)/Rb)/(r(25)/Rb-r(24)/Rb);
Pz92(n,1)=Pz8(n,1)+(Pz9(n,1)-Pz8(n,1))*(0.92-r(27)/Rb)/(r(28)/Rb-r(27)/Rb);
Py1=AoA1;
Py25=Py1;

% interpolation to find the AoA values at sections r/R=35,60,82,92
AoA25=AoA1;
AoA35=AoA2;

% interpolation to find the Ft & Fn values at sections r/R=35,60,82,92
Ft(n,1:N,1:B,n) = Py; %*dr;
Fn(n,1:N,1:B,n) = Pz; %*dr;

torq(1,n)=sum((Py(:,1)*dr).*r');
torq(2,n)=sum((Py(:,2)*dr).*r');
torq(3,n)=sum((Py(:,3)*dr).*r');
MR(n) = sum(((Py(:,1)+Py(:,2)+Py(:,3))*dr).*r')*cos(theta_cone);

% blade deflection at final step
% deflectY(1,:) = x(3,iter) * u.y1f + x(4,iter) * u.y1e + x(5,iter) * u.y2f;
% deflectZ(1,:) = x(3,iter) * u.z1f + x(4,iter) * u.z1e + x(5,iter) * u.z2f;
% deflectY(2,:) = x(6,iter) * u.y1f + x(7,iter) * u.y1e + x(8,iter) * u.y2f;
% deflectZ(2,:) = x(6,iter) * u.z1f + x(7,iter) * u.z1e + x(8,iter) * u.z2f;
% deflectY(3,:) = x(9,iter) * u.y1f + x(10,iter) * u.y1e + x(11,iter) * u.y2f;
% deflectZ(3,:) = x(9,iter) * u.z1f + x(10,iter) * u.z1e + x(11,iter) * u.z2f;

fprintf(1, 'total time: %g second \n', tt(n))
fprintf(1, 'number of rotations 1 (structure ): %g \n', max(x(2,1:iter))/(2*pi))
fprintf(1, 'number of rotations 2 (no struct.): %g \n', tt(n)*Omega/(2*pi))
fprintf(1, 'time averaged rotor power: %g MW \n', mean(Pmech(floor(iter/4):iter))*1e-6)
toc

resplots = showplots(r,x,x1,x2,Pmech,tt,iter,deflectY,deflectZ,Ft,Fn,an,at,AoA,cldy,clst);

if n<72
figure(21)
pplot(mod(Psii1,2*pi)*180/pi,Pz25,mod(Psii1,2*pi)*180/pi,Pz35,mod(Psii1,2*pi)*180/pi,Pz60,mod(Psii1,2*pi)*180/pi,Pz82,mod(Psii1,2*pi)*180/pi,Pz92)
xlabel('azimuth angle (degree)')
ylabel('P\ z (Normal Force)')
legend('%25','%35','%60','%82','92')
figure(22)
pplot(mod(Psii1,2*pi)*180/pi,Py25,mod(Psii1,2*pi)*180/pi,Py35,mod(Psii1,2*pi)*180/pi,Py60,mod(Psii1,2*pi)*180/pi,Py82,mod(Psii1,2*pi)*180/pi,Py92)
xlabel('azimuth angle (degree)')
ylabel('P\ y (Tangential Force)')
legend('%25','%35','%60','%82','92')
else
  j=1;
  for i=(n-70):n
    Pzz25(j,1)=Pz25(i,1);
    Pzz35(j,1)=Pz35(i,1);
    Pzz60(j,1)=Pz60(i,1);
    Pzz82(j,1)=Pz82(i,1);
    Pyy25(j,1)=Py25(i,1);
    Pyy35(j,1)=Py35(i,1);
    Pyy60(j,1)=Py60(i,1);
    Pyy82(j,1)=Py82(i,1);
    Pyy92(j,1)=Py92(i,1);
    Psii11(j,1)=Psii1(i,1);
    AoAa25(j,1)=AoA25(i,1);
    AoAa35(j,1)=AoA35(i,1);
    AoAa60(j,1)=AoA60(i,1);
    AoAa82(j,1)=AoA82(i,1);
    AoAa92(j,1)=AoA92(i,1);
  end
end
tableZ(j,1)=mod(Psii11(j,1),2*pi);tableY(j,1)=mod(Psii11(j,1),2*pi);
tableZ(j,2)=Pzz25(j,1);
tableZ(j,3)=Pzz35(j,1);tableZ(j,4)=Pzz60(j,1);tableZ(j,5)=Pzz82(j,1);tableZ(j,6)=Pzz92(j,1);
tableY(j,3)=Pyy35(j,1);tableY(j,4)=Pyy60(j,1);tableY(j,5)=Pyy82(j,1);tableY(j,6)=Pyy92(j,1);

j=j+1;
end
tableZ;
tableY;

mod(Psii11,2*pi);

figure(20)

plot(mod(Psii11+pi,2*pi)*180/pi,AoAa35,mod(Psii11+pi,2*pi)*180/pi,AoAa60,mod(Psii11+pi,2*pi)*180/pi,AoAa82,mod(Psii11+pi,2*pi)*180/pi,AoAa92)

xlabel('azimuth angle(degree)')
ylabel('AoA (rad)')
legend('%35','%60','%82','92')

figure(21)

plot(mod(Psii11,2*pi)*180/pi,Pzz25,mod(Psii11,2*pi)*180/pi,Pzz35,mod(Psii11,2*pi)*180/pi,Pzz60,mod(Psii11,2*pi)*180/pi,Pzz82,mod(Psii11,2*pi)*180/pi,Pzz92)

xlabel('azimuth angle(degree)')
ylabel('Pz (Normal Force)')
legend('%25','%35','%60','%82','92')

figure(22)

plot(mod(Psii11,2*pi)*180/pi,Pyy25,mod(Psii11,2*pi)*180/pi,Pyy35,mod(Psii11,2*pi)*180/pi,Pyy60,mod(Psii11,2*pi)*180/pi,Pyy82,mod(Psii11,2*pi)*180/pi,Pyy92)

xlabel('azimuth angle(degree)')
ylabel('Py (Tangential Force)')
legend('%25','%35','%60','%82','92')

end

% Find force on rotor

function [Py,Pz, Wy_mean, Wz_mean, cldy, clst, An, At, AoA, Aold, U, Uold, xs1, xs2, xs3, xs4] = fun_Vrel(n,r,N,Rb,xwing,dr,Vomean,Vx,Vy,Vz,Wy_mean,Wz_mean,Omega,uy,uz,ut,dt,c,theta_yaw,theta_cone,theta_tilt,theta_twist,alpha,pitch,Cl,Cd,B,shaftlength,H,gamma,density,a_ground,a_top,Uold,Aold,xx1,xx2,xx3,xx4,stall_trigger,Tp,Tf,b1,b2,A1,A2)

for k=1:B

for mm=1:N

Vw=[Vx(mm,k);Vy(mm,k);Vz(mm,k)]; % wind velocity seen by blade
Vrote=[0;-r(mm)*Omega*cos(theta_cone);0];

W=[0;Wy_mean(mm);Wz_mean(mm)]; % induced velocity

if k==1

vb=[0;-uy.blade1(mm);-uz.blade1(mm)];% -vb is vibration speed of blade
elseif k==2
vb=[0;-uy.blade2(mm);-uz.blade2(mm)];
else
vb=[0;-uy.blade3(mm);-uz.blade3(mm)];
end

vt=[0;0;-ut*cos(theta_cone)];
Vrel = Vw + Vrote + W;

Vrelx=Vrel(1); % in x,y,z direction respectively
Vrely=Vrel(2);
Vrelz=Vrel(3);

phi=abs(atan(Vrelz/Vrely)); % flow angle
Alpha=phi-theta_twist(mm)-pitch;
Aoa(mm,k)=Alpha;

% 1. steady (simple) case
% cl = interp1(alpha,Cl(:,mm),Alpha);
% cd = interp1(alpha,Cd(:,mm),Alpha);

% 2. dynamic stall model

U(mm,k) = norm(Vrel);

if(n==1)
    Uold(mm,k) = U(mm,k); Aold(mm,k) = AoA(mm,k);
end

testdU (mm,k) = (U(mm,k)-Uold(mm,k))/dt;
testAoa(mm,k) = (AoA(mm,k)-Aold(mm,k))/dt;

[cl,cd,CLst,CDst,xs1(mm,k),xs2(mm,k),xs3(mm,k),xs4(mm,k)] = dynamicstall(n,U(mm,k),Uold(mm,k),c(mm),AoA(mm,k),Aold(mm,k),alpha,Cl(:,mm),Cd(:,mm),dt,stall_trigger,xx1(mm,k),xx2(mm,k),xx3(mm,k),xx4(mm,k),Tp,Tf,b1,b2,A1,A2);

Uold(mm,k) = U(mm,k); Aold(mm,k) = AoA(mm,k);

% find the lift and drag force
L=1/2*density*(Vrelx^2+Vrely^2+Vrelz^2)*cl*c(mm);
D=1/2*density*(Vrelx^2+Vrely^2+Vrelz^2)*cd*c(mm);

% find induced velocity
Cn = (cl*cos(phi)+cd*sin(phi));
Ct = (cl*sin(phi)-cd*cos(phi));

%% ver 1: model with new tip correction

tipratio=Omega*Rb/Vomean;
sigma=B*c(mm)/(2*pi*r(mm));
if(B*tipratio>30)
g0=exp(-1.125)+0.1;
else
g0=exp(-0.125*(B*tipratio-21))+0.1;
end
f = abs(B*(Rb-r(mm))/(2*r(mm)*sin(phi)));
f1=abs(g0*B*(Rb-r(mm))/(2*r(mm)*sin(phi)));
F =2/pi*acos(exp(-f)) +1e-12;
F1=2/pi*acos(exp(-f1))+1e-12;
Y = (4*sin(phi)*sin(phi))*F/(sigma*Cn*F1);
if(Y<0)
an(mm)=(2+Y+sqrt(abs(4*Y*(1-F)+Y^2)))/(2*(Y*F+1));
else
an(mm)=(2+Y-sqrt(4*Y*(1-F)+Y^2))/(2*(Y*F+1));
end
ac = 0.3;
if(an(mm)>ac)
an(mm)=0.5d0*(2.+Y*(1.d0-2*ac*F)...)
\[-\sqrt{\text{abs}}\left(\text{Y}*(1-2*\text{ac}^2*\text{F})+2+4*(\text{Y}^2+\text{ac}^2*\text{F}-1)\right)\];

\[
\text{Y1} = \frac{4\sin(\phi)\cos(\phi)}{\sigma\text{Ct}\text{F1}};
\]

\[
\text{at}(\text{mm}) = \frac{1}{(1-\text{an}(\text{mm})*\text{F})*\text{Y1}/(1-\text{an}(\text{mm}))-1};
\]

\[
\text{Wz}(\text{mm},k) = -\text{an}(\text{mm})*\text{Vz}(\text{mm},k);
\]

\[
\text{Wy}(\text{mm},k) = \text{at}(\text{mm})*\text{r}(\text{mm})*\Omega;
\]

\% for the new tip model
\[
\text{Pz}(\text{mm},k) = (L\cos(\phi) + D\sin(\phi))*\text{F1};
\]

\[
\text{Py}(\text{mm},k) = (L\sin(\phi) - D\cos(\phi))*\text{F1};
\]

\[
\text{cldy}(\text{mm},k) = \text{cl};
\]

\[
\text{clst}(\text{mm},k) = \text{CLst};
\]

\[
\text{Wz}_\text{mean} = (\text{Wz}(:,1) + \text{Wz}(:,2) + \text{Wz}(:,3))/3;
\]

\[
\text{Wy}_\text{mean} = (\text{Wy}(:,1) + \text{Wy}(:,2) + \text{Wy}(:,3))/3;
\]

\begin{verbatim}
function[Pz6,Pz9,Pz10,Pz17,Pz18,Pz24,Pz25,Pz27,Pz28,Py6,Py9,Py10,Py17,Py18,Py24,Py25,Py27,Py28,AoA6,AoA9,AoA10,AoA17,AoA18,AoA24,AoA25,AoA27,AoA28,...
\end{verbatim}

\[
\text{Py},\text{Pz},\text{Wz},\text{cldy},\text{clst},\text{an},\text{at},\text{AOA},\text{AoA},U,U\text{old},x1,x2,x3,x4,sum1,sum2,sum3,sum4,sum5,sum6,sum7,sum8]=\text{fun_VrelYaw}(n,r,N,Rb,...
\]

\[
xwing,dr,\text{Vmean},Vx,Vy,Vz,\Omega,\text{u},\text{u},\text{u},\text{d},\text{c},\theta_yaw,\theta_cone,\theta_tilt,\theta_twist,\alpha,\text{pitch},\text{CL},\text{Cd},\text{B},\text{shaftleng},\text{H},...
\]

\[
\text{gamma},\text{density},\text{a ground},\text{a top},\text{Uold},\text{AoA},\text{aa},x1,x2,x3,xx4,sum\text{trigger},\text{Tp},\text{Tf},b1,b2,A1,A2,W1,W2,\text{Ann},\text{Att},\text{psi},\text{sum1},\text{sum2},\text{sum3},\text{sum4},\text{sum5},\text{sum6},\text{sum7},\text{sum8},\text{sum9}
\]

\[
\text{ac}=0.2;
\]

\[
\text{an} = \text{Ann};
\]

\[
\text{at} = \text{Att};
\]

\[
\text{Wz} = \text{Wz};
\]

\[
\text{sum1} = \text{sum1};\text{sum2} = \text{sum2};\text{sum3} = \text{sum3};\text{sum4} = \text{sum4};\text{sum5} = \text{sum5};\text{sum6} = \text{sum6};\text{sum7} = \text{sum7};\text{sum8} = \text{sum8};\text{sum9} = \text{sum9};
\]

\begin{verbatim}
for mm=1:N
\end{verbatim}

\begin{verbatim}
for k=1:B
\end{verbatim}

\% calculate induced velocity of three blades
\[
\text{theta}_\text{wing} = xwing + (k-1)*2*\text{pi}/3;
\]

\[
\text{theta}_\text{wing} = \text{mod}(\text{theta}_\text{wing},2*\text{pi});
\]

\[
\text{Psi} = \text{Psi}+(k-1)*2*\text{pi}/3;
\]

\[
\text{Vw} = \text{[Vx(mm,k);Vy(mm,k);Vz(mm,k)];} \quad \% \text{wind velocity seen by blade}
\]

\[
\text{Vr} = \text{[0;-r(mm)*Omega*cos(\text{theta}_\text{cone});0]};
\]

\[
\text{radius} = \text{r(mm)};
\]

\[
\text{[Ca(mm,k)] = func1(radius,Rb,Psi,theta_yaw);}
\]

\[
\text{Wy}_\text{local}(\text{mm}) = \text{Wy}(\text{mm})*\text{Ca}(\text{mm},k);
\]

\[
\text{Wz}_\text{local}(\text{mm}) = \text{Wz}(\text{mm})*\text{Ca}(\text{mm},k);
\]

\[
\text{W} = \text{real([0;Wy}_\text{local}(\text{mm});\text{Wz}_\text{local}(\text{mm}))}; \quad \% \text{induced velocity}
\]

\begin{verbatim}
if k==1
\end{verbatim}

\[
\text{vb} = \text{[0;-u}y.bladel1(mm);-u}z.bladel1(mm)]; \quad \% -vb is vibration speed of blade
\begin{verbatim}
\end{verbatim}

\begin{verbatim}
else if k==2
\end{verbatim}

\[
\text{vb} = \text{[0;-u}y.blade2(mm);-u}z.blade2(mm)];
\end{verbatim}
else
    vb=[0;-uy.blade3(mm);-uz.blade3(mm)];
end

vt=[0;0;-ut*cos(theta_cone)];
Vrel = Vw + Vrote + W;
% + vb + vt; % wjz

Vrelx=Vrel(1);                        % in x, y, z direction respectively
Vrely=Vrel(2);
Vrelz=Vrel(3);
phi(mm,k)=atan2(Vrelz,-Vrely);% flow angle

Alpha=phi(mm,k)-theta_twist(mm)-pitch;
if(Alpha> pi);Alpha=Alpha-2*pi;end
if(Alpha<-pi);Alpha=2*pi+Alpha;end
AoA(mm,k)=Alpha ;

% % 1. steady (simple) case
% cl = interp1(alpha,Cl(:,mm),Alpha);
% cd = interp1(alpha,Cd(:,mm),Alpha);

% 2. dynamic stall model

U(mm,k) = norm(Vrel);
if(n==1)
    Uold(mm,k) = U(mm,k); Aold(mm,k) = AoA(mm,k);
end

testdU (mm,k) = (U(mm,k)-Uold(mm,k))/dt;
testAoA(mm,k) = (AoA(mm,k)-Aold(mm,k))/dt;

[cl,cd,CLst,CDst,xs1(mm,k),xs2(mm,k),xs3(mm,k),xs4(mm,k)] = dynamicstall(n,U(mm,k),Ucomponents(mm,k),c(mm),AoA(mm,k),alpha,cl(:,mm),cd(:,mm),dt,stall_trigger,xx1(mm,k),xx2(mm,k),xx3(mm,k),xx4(mm,k),Tp,Tf,b1,b2,A1,A2);

Uold(mm,k) = U(mm,k); Aold(mm,k) = AoA(mm,k);

% find the lift and drag force
L=1/2*density*(Vrely^2+Vrelz^2)*cl*c(mm);
D=1/2*density*(Vrely^2+Vrelz^2)*cd*c(mm);

Cn(mm,k) = (cl*cos(phi(mm,k))+cd*sin(phi(mm,k)));
Ct(mm,k) = (cl*sin(phi(mm,k))-cd*cos(phi(mm,k)));

%% ver 1: model with new tip correction

tipratio=Omega*Rb/Vomean;
sigma=B*c(mm)/(2*pi*r(mm));
if(B*tipratio>30)
g0=exp(-1.125)+0.1;
else
    g0=exp(-0.125*(B*tipratio-21))+0.1;
end
f = abs(B*(Rb-r(mm))/(2*r(mm)*sin(phi(mm,k))));
f1=abs(g0*B*(Rb-r(mm))/(2*r(mm)*sin(phi(mm,k))));
F(mm,k) =2/pi*acos(exp(-f)) +1e-12;
F1(mm,k)=2/pi*acos(exp(-f1))+1e-12;
\[ P_z(mm,k) = (L \cdot \cos(\phi(mm,k)) + D \cdot \sin(\phi(mm,k))) \cdot F_1(mm,k) \]
\[ P_y(mm,k) = (L \cdot \sin(\phi(mm,k)) - D \cdot \cos(\phi(mm,k))) \cdot F_1(mm,k) \]

\[ \text{clyy}(mm,k) = \text{cl} \]
\[ \text{clst}(mm,k) = \text{CLst} \]

\[ \text{cnst1}(mm,k) = \text{Ca}(mm,k)^2 \cdot \text{Cn}(mm,k) \cdot F_1(mm,k) / \sin(\phi(mm,k))^2 \]
\[ \text{cnst2}(mm,k) = \text{Ca}(mm,k)^2 \cdot F(mm,k)^2 \]
\[ \text{cnst3}(mm,k) = \text{Ca}(mm,k) \cdot \text{Cn}(mm,k) \cdot F_1(mm,k) / \sin(\phi(mm,k))^2 \]
\[ \text{cnst4}(mm,k) = \text{Ca}(mm,k) \cdot F(mm,k) \]
\[ \text{cnst5}(mm,k) = \text{Cn}(mm,k) \cdot F_1(mm,k) / \sin(\phi(mm,k))^2 \]
\[ \text{cnst6}(mm,k) = F(mm,k) \]

\[ \text{C1} = \text{c}(mm) / (8 \cdot \pi \cdot r(mm)) \]
\[ \text{sum1}(mm) = \text{sum1}(mm) + (\text{cnst1}(mm,1) + \text{cnst1}(mm,2) + \text{cnst1}(mm,3)) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum2}(mm) = \text{sum2}(mm) + \text{cnst2}(mm,1) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum3}(mm) = \text{sum3}(mm) + (\text{cnst3}(mm,1) + \text{cnst3}(mm,2) + \text{cnst3}(mm,3)) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum4}(mm) = \text{sum4}(mm) + \text{cnst4}(mm,1) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum5}(mm) = \text{sum5}(mm) + (\text{cnst5}(mm,1) + \text{cnst5}(mm,2) + \text{cnst5}(mm,3)) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum6}(mm) = \text{sum6}(mm) + \text{cnst6}(mm,1) \cdot (\Omega \cdot \text{dt}) \]

\[ p = [\text{C1} \cdot \text{sum1}(mm) + \text{sum2}(mm) - (2 \cdot \text{C1} \cdot \text{sum3}(mm) + \text{sum4}(mm)) \cdot \text{C1} \cdot \text{sum5}(mm)] \]
\[ a = \text{roots}(p) \]

\[ \text{if} \ a(1,1) < 1 \land a(1,1) < a(2,1) \]
\[ \quad \text{an(mm)} = \text{real}(a(1,1)) \]
\[ \text{else} \]
\[ \quad \text{an(mm)} = \text{real}(a(2,1)) \]
\[ \text{end} \]

\[ \text{if} \ \text{an(mm)} > 0.3 \]

\[ p = [\text{C1} \cdot \text{sum1}(mm) - (2 \cdot \text{C1} \cdot \text{sum3}(mm) + (1 - 2 \cdot \text{ac}) \cdot \text{sum4}(mm)) \cdot (\text{C1} \cdot \text{sum5}(mm) - \text{ac}^2 \cdot \text{sum6}(mm))] \]
\[ a = \text{roots}(p) \]

\[ \text{if} \ a(1,1) < 1 \land a(1,1) < a(2,1) \]
\[ \quad \text{an(mm)} = \text{real}(a(1,1)) \]
\[ \text{else} \]
\[ \quad \text{an(mm)} = \text{real}(a(2,1)) \]
\[ \text{end} \]

\[ \text{C2} = \text{c}(mm) \cdot B / (8 \cdot \pi \cdot \Omega \cdot \text{r}(mm))^2 \]
\[ \text{C3} = \text{c}(mm) \cdot B / (8 \cdot \pi \cdot \text{r}(mm)) \]

\[ \text{sum7}(mm) = \text{sum7}(mm) + (1 - \text{an(mm)} \cdot \text{Ca}(mm,1)) / (\sin(\text{phi}(mm,1)) \cdot \cos(\text{phi}(mm,1))) \cdot (\Omega \cdot \text{r}(mm) - \text{Vomega} \cdot \sin(\text{theta}_\text{yaw}) \cdot \cos(\text{Psii})) \cdot \text{Ct}(mm,1) \cdot F_1(mm,1) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum8}(mm) = \text{sum8}(mm) + \text{Ca}(mm,1) \cdot F(mm,1) \cdot (1 - \text{an(mm)} \cdot \text{Ca}(mm,1) \cdot F(mm,1)) \cdot (\Omega \cdot \text{dt}) \]
\[ \text{sum9}(mm) = \text{sum9}(mm) + (1 - \text{an(mm)} \cdot \text{Ca}(mm,1)) \cdot \text{Ca}(mm,1) \cdot \text{Ct}(mm,1) \cdot F_1(mm,1) / (\sin(\text{phi}(mm,1)) \cdot \cos(\text{phi}(mm,1))) \cdot (\Omega \cdot \text{dt}) \]
at(mm)=C2*sum7(mm)/(sum8(mm)-C3*sum9(mm));

Wz(mm)=-an(mm)*(Vz(mm,1)+Vz(mm,2)+Vz(mm,3))/3;
Wy(mm)=at(mm)*r(mm)*Omega;
end

Pz6=Pz(6,1); Pz9=Pz(9,1); Pz10=Pz(10,1); Pz17=Pz(17,1);
Pz18=Pz(18,1);Pz24=Pz(24,1);Pz25=Pz(25,1);Pz27=Pz(27,1);Pz28=Pz(28,1);
Py6=Py(6,1); Py9=Py(9,1); Py10=Py(10,1); Py17=Py(17,1);
Py18=Py(18,1);Py24=Py(24,1);Py25=Py(25,1);Py27=Py(27,1);Py28=Py(28,1);
AoA6=AoA(6,1);AoA9=AoA(9,1);
AoA10=AoA(10,1);AoA17=AoA(17,1);AoA18=AoA(18,1);AoA24=AoA(24,1);AoA25=AoA(25,1);AoA27=AoA(27,1);AoA28=AoA(28,1);

4. Tip Loss Graphs

clc
clear
close all

load force05.txt;
load force07.txt;
load force10.txt;
load force12.txt;
load forceNewtiploss.txt;
load forceGlauerttiploss.txt;

% normal and tangential forces at 5 m/s

hold on
grid on
plot(force05(:,1),force05(:,4),forceNewtiploss(:,1),forceNewtiploss(:,2),forceGlauerttiploss(:,1),forceGlauerttiploss(:,2))
xlabel('radial position(m)')
ylabel('Pz (Normal Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure
hold on
grid on
plot(force05(:,1),force05(:,3),forceNewtiploss(:,1),forceNewtiploss(:,3),forceGlauerttiploss(:,1),forceGlauerttiploss(:,3))
xlabel('radial position(m)')
ylabel('Py (Tangential Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure

% normal and tangential forces at 7 m/s

hold on
grid on
plot(force07(:,1),force07(:,4),forceNewtiploss(:,1),forceNewtiploss(:,4),forceGlauerttiploss(:,1),forceGlauerttiploss(:,4))
xlabel('radial position(m)')
ylabel('Pz (Normal Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure
hold on
grid on
plot(force07(:,1),force07(:,3),forceNewtiploss(:,1),forceNewtiploss(:,5),forceGlauerttiploss(:,1),forceGlauerttiploss(:,5))
xlabel('radial position(m)')
ylabel('Py (Tangential Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure

% normal and tangential forces at 10 m/s
hold on
grid on
plot(force10(:,1),force10(:,4),forceNewtiploss(:,1),forceNewtiploss(:,6),forceGlauerttiploss(:,1),forceGlauerttiploss(:,6))
xlabel('radial position(m)')
ylabel('Pz (Normal Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure
hold on
grid on
plot(force10(:,1),force10(:,3),forceNewtiploss(:,1),forceNewtiploss(:,7),forceGlauerttiploss(:,1),forceGlauerttiploss(:,7))
xlabel('radial position(m)')
ylabel('Py (Tangential Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure

% normal and tangential forces at 12 m/s
hold on
grid on
plot(force12(:,1),force12(:,4),forceNewtiploss(:,1),forceNewtiploss(:,8),forceGlauerttiploss(:,1),forceGlauerttiploss(:,8))
xlabel('radial position(m)')
ylabel('Pz (Normal Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off
figure
hold on
grid on
plot(force12(:,1),force12(:,3),forceNewtiploss(:,1),forceNewtiploss(:,9),forceGlauerttiploss(:,1),forceGlauerttiploss(:,9))
xlabel('radial position(m)')
ylabel('Py (Tangential Force)(N/m)')
legend('CFD','Newtiploss','Glauert')
hold off