Traffic Load Effects on Bridges

Statistical Analysis of Collected and Monte Carlo Simulated Vehicle Data

by

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Preface

The research work presented in this thesis was carried out at the Department of Civil and Architectural Engineering at the Royal Institute of Technology (KTH), at the division of Structural Design and Bridges between April 1999 and February 2003. The project was financed by the Swedish National Road Administration (Vägverket) and the Royal Institute of Technology (KTH).

First, I thank my supervisor Professor Håkan Sundquist whose belief in me from our first meeting has given me the lift I needed to ensure that the work was completed. Preface sections are often full of phrases such as "without whom" but in this case it is true, without Professor Håkan Sundquist, this work would not have been done. Similar thanks must also go to my co-supervisor Dr. Raid Karoumi for his guidance and support.

In particular I would like to acknowledge the discussions with Dr. Christian Cremona from LCPC (Laboratoire Central des Ponts et Chaussées) in France and for his unbelievably quick replies of my questions via e-mail.

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A warm thank you to all the staff at the Department of Civil and Architectural Engineering especially the staff from the former Department of Structural Engineering for creating a stimulating environment.

Finally, thanks must go to my gorgeous Tsegereda Derar who has encouraged me throughout the years in my higher education and for the love that she has brought into my life.

I would like to dedicate this thesis to my mother Tsedale Negery and to the memory of my father Getachew Wolde-Tsadik.

Stockholm, February 2003,
Abraham Getachew.
Abstract

Research in the area of bridge design has been and still is concentrated on the study of the strength of materials and relatively few studies have been performed on traffic loads and their effects. Traffic loads have usually been assumed to be given in codes. This is mainly because it is very difficult to model traffic loads in an accurate manner because of their randomness.

In this work, statistical evaluations of traffic load effects, obtained from real as well as Monte Carlo (MC) simulated vehicle data, are presented. As the dynamic contribution of the vehicle load was filtered by the system used for measuring vehicle weight, no attention was paid in the present study to the dynamic effects or the impact factor. The dynamic contribution of the traffic load models from codes was deducted wherever they were compared with the result from the evaluation of the real data. First, the accuracy of the collected data was investigated. This was done to examine the influence of what was most probably unreasonable data on the final evaluated results. Subsequently, the MC simulation technique, using a limited amount of the collected data, was used to generate fictitious vehicle data that could represent results from field measurements which would otherwise have to be recorded under a long period. Afterwards, the characteristic total traffic loads for bridges with large spans were determined by probabilistic analysis. This was done using real as well as simulated data and the two were compared. These results were also compared with the corresponding values calculated using the traffic load model from the Swedish bridge design code.

Furthermore, using traffic data, different load effects on bridges (girder distribution factor of slab-on-girder bridges and the mid-span deflection as well as the longitudinal stress at critical locations on box-girder bridges) were investigated. The main task was to obtain a more accurate knowledge of traffic load distributions on bridges as well as their effects for infrastructure design. The results showed that the traffic load models from codes gave considerably higher load effects compared to the current actual traffic load effects. These investigations were based on the available data for the actual position of the vehicles on a single bridge and might not cover all possible traffic scenarios. The results showed only how the real traffic loads, under "normal" conditions and their transverse positions relate to the load model according to the codes.

**KEYWORDS:** bridge, traffic load, load effect, transverse distribution, characteristic value, weigh in motion, Monte Carlo simulation, Rice’s formula, level crossing histogram, vehicle queue.
Sammanfattning (Summary in Swedish)


Forskningen inom brokonstruktionstekniken koncentreras och har koncentrerats i stor utsträckning på bärförmågesidan. Till följd därav har förhållandevis få undersökningar kring laster och lasteffekter utförts. De sistnämnda brukar antas vara givna i normer. För att studera bärförmågan hos olika ingående delar av en bro, kan relativt enkla modeller av dessa göras och provas i laboratorier. Däremot kan det vara ganska svårt att utforska den verkliga lasteffekten då det behövs mycket information i form av data från fältmätningar. Dessutom är måtdata från fältmätningar relativt sett mer behäftade med fel jämfört med data från laboratoriemätningar, vilket gör att mätvärdenas noggrannhet bör ifrågasättas och undersökas noggrant.

De trafiklastmodeller som är angivna i många normer anses vara konservativa. Detta är bland annat p.g.a. att de är baserade på gamla insamlade trafikdata. Detta gör att dessa laster inte motsvarar de laster som genereras av dagens fordon då fordonens utformning och dämpningsmekanismer har förändrats markant under den senaste tiden. Därför är det mycket viktigt att kontinuerligt uppdatera de i normer angivna trafiklastfallen. Kostnadsökningen för byggandet av en ny bro som är dimensionerad med ett konservativt trafiklastvärde är obetydlig. Denna kostnadsökning är en följd av osäkerheten i trafiklastvärdena samt för att förenkla brodimensioneringsförfarandet. Efter att bron har tagits i drift är dock kostnaden för uppklassning av den mycket högre. Det mest noggranna sättet att bestämma dimensionerande trafiklastfall för en bro är troligen att utföra sannolikhetsanalyser med hjälp av insamlade trafikdata, simulerade trafikdata eller en kombination av dessa.

Trafikens sammansättning är naturligtvis olika på olika ställen. Detta medför att den verkliga trafiklasten, särskilt på broar med långa spänningar, varierar beroende på var bron befinner sig. För en befintlig eller en framtida bro bör då speciella undersökningar av den lokala trafiksituationen genomföras för att kunna fastställa det trafiklastvärde som gäller just för den betraktade bron. Den senaste tiden har olika system utvecklats för trafikdatainsamling med syfte att bland annat kalibrera trafiklastmodeller givna i olika normer. Ett av dessa system använder sig av den så kallade ”Weight In Motion” (WIM) tekniken. Detta är ett måtsystem för vagnning av fordon i rörelse. Normalt är utförandet av WIM-mätningar både kostsamt och

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Alla typer av mätningar innehåller självklart fel. Därför har insamlade data från mätserie E undersöks i första delen av detta arbete för att avgöra vilka olika typer av felaktiga data som mätresultaten innehåller. Analysen har visat att bland måtdata finns fordon som är registrerade med orimliga längder och/eller vikter. Undersökningen har också visat att cirka 10 % av registrerade fordondata är felaktiga och bör exkluderas före vidare bearbetning av måtresultaten. Bland de vidare bearbetade måtdata är cirka 10 % fordon vilka är registrerade som enaxliga. Vad dessa fordon kan ha varit kunde inte avgöras. Vidare har måtdata utvärderats om enligt den metod som är beskriven i [50]. Syftet med denna metod är att bestämma ett karakteristisk trafiklastvärde, med utgångspunkt från vissa grunddata, som gäller för broar med stora spännvidder, d.v.s. spännvidder större än 200 meter. Enligt denna metod har karakteristiska trafiklastvärden för olika kölängder beräknats, från data både före och efter filtryring av felaktiga data. Det har visat sig att filtyringen av felaktiga mätvärden inte har påverkat resultaten i så hög grad som förväntats. Skillnaderna i de karakteristiska lastvärdena beräknade, för olika kölängder, före och efter filtryringen var som högst 3,5 %. Detta beror förmodligen på att felen ”liten vikt på stor längd” samt ”stor vikt på liten längd” har jämnats ut för det icke filtrerade fallet vilket gör att köväktarna blir nästan detsamma som efter filtryringen.

Resultat av jämförelser mellan de karakteristiska trafiklastvärden, bestämda från WIM-data, och motsvarande värden från normlasten visar att normvärdena är betydligt högre i samtliga fall. Här ska det påpekas att de karakteristiska trafik-
lastvärdena bestämda av data från mätserien E är lägre än motsvarande värden från de andra mätserierna (d.v.s. mätserierna A, B, C, D och F), se [51, 58]. Detta är förmodligen en följd av den lokala trafiksituati
gen.

Monte Carlo simuleringsstekniken har efteråt använts för generering av fordonssdata. Målet var att simuleran

fikta fordonssdata, genom att använda resultat från kort-

periods WIM-mätning, som kan representera data från fältmätningar. Vidare har färskningsfunktioner från såväl simulerade som insamlade fordonssdata beräknats och jämförts med vanan. Resultaten visar att färskningsfunktionerna, särskilt för höga kövikter, stämmer väl överens med vanan. Som ett resultat från denna

analys föreslås att insamling av nya fordonssdata utförs på ett systematiskt sätt under relativt korta perioder. Sedan kan resultat från dessa användas som grunddata för att generera önskat antal fiktiva fordonssdata genom simulering. Detta leder till om-

fattande minskning av både tid och kostnader som läggs ut på trafikdatainsamlingar vilka normalt utförs kontinuerligt för långa perioder.


Den andra studerade brotypen är en lådbro. Två lasteffekter har valts för denna studie. Dessa är nedbörningar och längsgående spänningar i bromitssnitt som har beräknats från mätdata för olika spännvidder. Dessa lasteffekter har bestämts med

finit elementanalys. De ovannämnda lasteffekter som har beräknats från mätdata har norr som mätdata och normalsten beräknad från trafiklastfallet enligt Bro94. Vidare har värden beräknade för olika fraktorer hos färskningsägarna av ovannämnda

nedbörnings- och spänningskvoter bestämts genom att använda analysen enligt Rices formel. Resultaten antyder att normalstenen är från tre till fyra gånger större än motsvarande värden beräknade från mätdata.
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List of Symbols and Abbreviations

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\( A \) Concentrated axle load according to Bro94, p. 2

\( D \) The statistics of the variation between two probability functions, p. 25

\( \hat{F}_n(x) \) The empirical distribution function for a discrete random variable \( X = x_1, x_2, \ldots, x_n \), p. 19

\( F_X(x) \) Probability distribution function or cumulative distribution function of the stochastic variable \( X \), p. 9

\( L \) Bridge span, p. 3

\( L_0 \) An apriori chosen vehicle queue length, p. 57

\( N \) The average number of vehicle queues that is assumed to occur per year, p. 63

\( N \) The number of class intervals of level crossing histogram, p. 25

\( P(X \leq x) \) The probability that \( X \leq x \), p. 9

\( Q \) The vehicle queue weight, p. 79

\( Q_{KS} \) The value of K-S statistics, p. 25

\( R_A \) The reaction force, from a given vehicle queue weight on a bridge, that acts on Beam A, p. 79

\( R_T \) The return period, p. 15

\( T \) The reference time, p. 15

\( T_{rec} \) The record period, p. 24

\( T_{ref} \) The reference period, p. 23

\( W \) The total weight of all vehicles in a queue, p. 57

\( \dot{X} \) The derivative of the stochastic process \( X \), p. 24

\( X \) Stochastic variable, p. 8
Roman Lower Case

$a_i$ The required space of vehicle number $i$, $a_i = d_i + 2$ meters, p. 57

$d_i$ The length of vehicle number $i$, p. 57

$f_X(x)$ Probability density function of the stochastic variable $X$, p. 10

$\ell$ The vehicle queue length, p. 57

$m$ The mean value of $X$, p. 24

$m_{opt}$ The mean value that corresponds the optimal fitting, p. 27

$p$ Uniformly distributed traffic loads according to Bro94, p. 2

$p_i$ The proportion of population $i$, p. 14

$q_i$ Uniformly distributed load from vehicle number $i$, p. 57

$u$ An outcome from $\Omega$, p. 8

$v_0$ $v_0 = \dot{\sigma}/2\pi\sigma$, p. 24

$w$ The characteristic load value according to Bro94, p. 63

$w_k$ The characteristic load value of the queue weights from the measurement, p. 63

$x_0$ A threshold value, p. 24

$x_i$ An outcome from stochastic variable $X$, p. 9

$x_k$ The characteristic value of the stochastic variable $X$, p. 16

$x_{opt}$ The threshold value for optimal fitting, p. 26

$\hat{x}_{opt}$ The threshold value for absolute fitting, p. 26

Greek Upper Case

$\Phi(\cdot)$ The standard normal distribution function, p. 12

$\varphi(\cdot)$ The standard normal density function, p. 12

$\Omega$ Sample space, p. 8

Greek Lower Case

$\beta_0$ Confidence level, p. 26

$\chi^2$ The chi-square merit function, p. 28

$\delta$ The vertical deflection calculated from traffic data, at node 13, p. 112
$\delta_{\text{Bro94}}$ The vertical deflection calculated using the load model from Bro94, at node 13, p. 112

$\varepsilon$ The dynamic contribution of point traffic load according to [66], in $\%$, p. 3

$\kappa$ A factor for the consideration of the type of influence function, [50], p. 61

$\mu$ The mean value of $X$, p. 11

$\mu_i$ The mean value of population $i$, p. 14

$\sigma_{\text{opt}}$ The standard deviation that corresponds the optimal fitting, p. 27

$\mu'$ The mean value of the queue weight for the first subpopulation, p. 60

$\mu''$ The mean value of the queue weight for the second subpopulation, p. 60

$\mu Q$ The average vehicle weight, p. 58

$\mu_q$ The average load intensity, $\mu_q = W/\ell$, p. 58

$\rho$ The correlation coefficient, p. 22

$\dot{\sigma}$ The standard deviation of $\dot{X}$, p. 24

$\sigma$ The standard deviation of $X$, p. 12

$\sigma_i$ The standard deviation of population $i$, p. 14

$\sigma$ The longitudinal stress calculated from traffic data, 250 mm from node 13 into the bottom slab of the box 13, p. 112

$\sigma_{\text{Bro94}}$ The longitudinal stress calculated using the load model from Bro94, 250 mm from node 13 into the bottom slab of the box 13, p. 112

$\sigma'$ The standard deviation of the queue weight for the first subpopulation, p. 60

$\sigma''$ The standard deviation of the queue weight for the second subpopulation, p. 60

$\sigma Q$ The standard deviation for $Q_i$, p. 58

$\sigma_q$ The standard deviation for $q_i$, p. 58

**Mathematical Symbols**

$\wp$ The girder distribution factor, p. 79

$\wp_k$ The characteristic value of the girder distribution factor, p. 106

$\wp_{\text{opt}}$ The threshold value for optimal fitting, p. 103
\( \hat{\varphi}_{\text{opt}} \) The threshold value for absolute fitting, p. 103

## Abbreviations

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<td>The Swedish bridge design code</td>
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<td>CEN</td>
<td>The European Committee for Standardization</td>
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Chapter 1

Introduction

1.1 Background and Motivation

Design and assessment of highway bridge structures requires accurate prediction of the maximum load effects which may be expected during the lifetime of the structures. Traffic loads represent the largest part of the total value of the external action to be considered in the design of a bridge. However, the actual traffic load on bridges is very difficult to model in an accurate way because of its high degree of randomness. The design traffic load models, which are given in different codes, are believed to have a conservative nature. These loads are closely related to the largest loads acting on a bridge during its lifetime. Obviously, underestimated design loads may lead to the collapse of the structure with many induced damages as a consequence. On the contrary, overestimated design loads lead to uneconomical structures and large waste of money. Nevertheless, different codes give conservative design traffic load models because of the uncertainty in traffic loads at the design stage and because the models must be valid for all types and sizes of bridges. The increased cost of construction of a new bridge due to the use of an overestimated design load model is small and necessary to allow for uncertainty and to simplify the design process. However, once a bridge is in service, the cost of an over-conservative evaluation could be much greater. Upgrading of bridges to a new standard is potentially an expensive task. One obvious method of upgrading is to physically increase the strength of a bridge by various strengthening methods. However, a less expensive method is to recalculate the strength of the bridge using better knowledge of the actual bridge in question and especially the actual traffic loading. This justifies the use of an approach which considers the actual traffic and the induced traffic load effects on bridges. A correct design is possible only if the statistical properties of the largest loads are well known.

Generally, the maximum allowable vehicle gross weight is much greater in Sweden and other Nordic countries compared to other European countries. Moreover, Sweden also allows the maximum lengths for the road trains. Figure 1.1 illustrates the maximum allowable vehicle weights and total lengths in different European countries. As seen in the figure the highest values are allowed in Sweden. This is mainly
due to the need for timber transport, [57]. Sweden is a very long country with relatively low population density where the distance between cities are usually very large. The distance between the components suppliers and industries as well as between industries and the customers are normally very great. This consequently leads to long and expensive goods transport. Therefore, the Swedish industry is much in favour of a road infrastructure on which high vehicle weight is been allowed.

It is mostly the bridge bearing capacity that is decisive in deciding how heavy vehicles are allowed to be on the road infrastructures. Today, there are approximately 14600 bridges in the Swedish road network, approximately 86 % of which have a span less than 40 meters. Up to 1938, bridges in Sweden, had been designed for real truckloads. This traffic load model that is recognizable from realtrucks was abandoned and the concept of the so-called equivalent load models was introduced in association with the nationalization of the Swedish National Road Administration (Vägverket). These load models that incorporate many different traffic loading scenarios are given in the Swedish bridge design code Bro94 [65]. According to this code, the vertical characteristic traffic load, acting both on the transversal and longitudinal direction of a bridge deck is illustrated in Figure 1.2. The model is valid for bridges with spans less than 200 meters. This loading system consists of three axles, which produces concentrated loads. Each axle has a weight of $A$. The magnitude of this load, $A$, equals 250 kN and 170 kN for the first and second lane respectively. The model also consists of uniformly distributed loads having a weight density per square meter $p$. The magnitude of $p$ is equal to 4 kN/m$^2$, 3 kN/m$^2$ and 2 kN/m$^2$ for the first, second and third lane, respectively. The distance between the axle loads in the length direction is greater than or equal to 1.5 and 6.0 meters,
1.2 Aims and Scope

The traffic load models given in many codes are based on old collected traffic data. This implies that the models do not represent the traffic loads induced by today’s vehicles, since vehicle formations and properties have changed a great deal in recent years. Consequently, using these load models, especially with the intention of repairing or reconstructing existing bridges to meet current design traffic loads could result in a great waste of money. Therefore, it is very important to continuously update the design traffic load models given in codes. A new era has now begun.
where simulations and extrapolations are used to statistically analyze recorded vehicle data to study different load effects on bridges with the intention of calibrating the traffic load models given in different codes.

The primary aim of this work is to show how different statistical tools can be implemented, using a limited amount of field data, to investigate different traffic load effects on bridges. This hopefully helps future studies intended to calibrate traffic load models that are given in different codes. For this purpose different traffic load effects on bridges are investigated—girder distribution factor of slab-on-girder bridges and the mid-span deflection as well as the longitudinal stress at critical locations on box-girder bridges. These load effects are evaluated for bridges with medium and short spans. The main task is to obtain a more accurate knowledge of the traffic load distributions on bridges as well as their effects on infrastructure design.

Because of the variation of traffic flow with respect to time, traffic data collection is usually performed continuously for long periods of time in order to predict the actual traffic loads and traffic compositions. Consequently, performing this kind of measurement is not only time-consuming but also very expensive. Another aim of this work is therefore to find and test a method for the generation of fictitious vehicle data, using a limited amount of collected data, which can represent the actual site-specific vehicle data. This requires a statistical evaluation of the collected as well as the simulated vehicle data and a comparison of the results with each other.

All measured data contains, of course, errors. Therefore, it is also intended to develop a simple method for the investigation of the accuracy of measured data for each vehicle from a database. Another ambition of this work is to study the influence of the measurement errors on the final results of traffic load effect evaluations.

1.3 General Structure of the Thesis

The following outline gives an overview of the general structure of this thesis.

In Chapter 2, the fundamental concepts used in this research are discussed.

In Chapter 3, previous works that adopt a similar approach to this research are presented. Extensive literature searches for this work have been made. However, few previous works that have near relation to the presented work could be found. One of which is the determination of the traffic load models that are given in the Eurocode, which is briefly presented in this chapter. Two other works by O’Connor and O’Brien [40] and Cremona [13] that are very close to the present research are also briefly discussed in this chapter.

In Chapter 4, the part of this research project that was presented as a licentiate work, which was carried out by the author of this thesis, is summarized and reviewed. The work contains a re-evaluation of the results of existing traffic load measurements that were performed by the Swedish National Road Administration (Vägverket). First,
the accuracy of the collected data is investigated. Then, the data both before and after filtration of unreasonable data are evaluated according to the method discussed in [50]. This is done in order to investigate the influence of measurement errors on the final results of data evaluation. Afterwards, the Monte Carlo simulation technique is used to generate fictitious vehicle data. Finally, the results from the evaluation of measured and simulated vehicle data are compared. These results are also compared with the corresponding values calculated using the traffic load model from the Swedish bridge design code.

In Chapter 5, the procedure adopted for the collection of data measuring the transverse position of vehicles on bridges is described. The data accumulation is performed on the highway E4 south of Stockholm 400 meters after the turn-off for Järna. A detailed description of the measurement results is also presented in the chapter. Afterwards, a method for the investigation of girder distribution factor, using the collected data, for medium and short span slab-on-girder bridges is presented.

In Chapter 6, two statistical tools for the analysis of traffic load effects are introduced. The first one uses the Monte Carlo simulation technique, where fictitious vehicle data is simulated and evaluated. The second one utilizes Rice’s formula. The last mentioned analysis is performed under the assumption of normality to drive the theoretical upcrossing distribution that is asymptotically normal for large values, i.e. above a given threshold. For this matter, the level upcrossing distribution, given by the Rice’s formula [46] is used. Also in this chapter, a comparison of results obtained using these two approaches are made. These results are also compared with the corresponding values calculated using the traffic load models of the Swedish bridge design code, as well as the Eurocode.

In Chapter 7, numerical calculations of traffic load effects on box-girder bridges are performed. For this purpose, finite element models of box-girder bridges with the same cross-sections and different lengths have been developed. These are performed using the commercial finite element software SOLVIA [49]. The loadings are modelled using the collected data. The calculated load effects are normalized by the corresponding values calculated using the traffic load model from the Swedish bridge design code. Finally, the results from the numerical calculations are analyzed using Rice’s formula.

In Chapter 8, general conclusions of this study are presented and proposals for further research are stated.
Chapter 2

Fundamental Concepts

2.1 General

In this chapter the fundamental concepts used in the research are presented. Most of the theories described in the section 2.2-2.4 are taken from [2,10,24]. The concepts in section 2.5-2.8 are gathered, among other reports and litterateurs, from [7,15,48]. Further, most of section 2.9 is taken from [13,14,27,28,46]. Finally the theories in section 2.10 and section 2.11 are taken from [43,45].

Some of the concepts, especially in the first few sections of the chapter, might seem elementary mathematics for a few readers. However, this thesis is written primarily for civil engineers and the author believes that the mathematical statistic knowledge of most of civil engineers is quite limited. It is therefore judged to be most important that almost all of the mathematical statistical concepts dealt with in this work should be briefly discussed in order to fully understand the thesis.

2.2 Probability Concepts

A mathematical description of the term probability is discussed in [2]. The classical interpretation of the word probability can be explained as follow:

If there is a total of $n$ possible outcomes, i.e. a result of a random test, and if there is not any reason to suspect that any outcome is more probable than an other, then the probability for each outcome is $1/n$. If the event consist of $m$ outcomes, then the probability becomes $m/n$.

As discussed in [62], some events, which civil engineers mostly deal with, have very low probabilities. One has the desire to ensure that bridges and dams should not collapse, but there is always a slight possibility that this could happen. A tower is maybe built to withstand a wind velocity up to $50 \text{ m/s}$. Other rare events of the same type are extremely large masses of snow, flooding, earthquakes, etc. Often, it is very difficult to say anything about unlikely events. For example, if one wants to
determine the wind-force which is allowed to be exceeded with the probability 0.001 under the next year, it is preferable to observe the weather for several hundred years to be sure of predicting the 1000 years return load with a high degree of accuracy.

2.3 Statistic

2.3.1 Stochastic Variable

The term stochastic in statistics refers to random or chance variables, or that which involves chance or probability. A stochastic variable is neither completely determinable nor completely random—in other words, it contains an element of probability. A system containing one or more stochastic variables is probabilistically determined. A stochastic variable is often defined as a function on sample space, Ω, i.e. a number of possible outcomes.

In [2], it is described that the use of the term stochastic variable is misleading and it would be better to say stochastic function or random function but the linguistic usage is unfortunately decided. To explain that a stochastic variable $X$ actually is a function from $\Omega$ to $R^1$, it can explicitly be written as $X(u)$, where $u$ is an outcome from $\Omega$, see Figure 2.1. This is often expressed as $X: \Omega \rightarrow R^1$; which implies that a stochastic variable is a function that maps events in the sample space $\Omega$ into the real line $R^1$.

Some simple examples of one-dimensional stochastic variables are the number of heads or tails that fall during a series of flips of a coin, a gambler’s winnings in one play-round of roulette in Monte Carlo, the number of children in one randomly selected Swedish family and the length of life of a randomly selected Swedish citizen. A stochastic variable dose not always has to be one-dimensional. Sometimes, a random experiment can give many results at the same time. In that case, we get multi-dimensional stochastic variable.

![Figure 2.1: Description of the stochastic variable, $X$, as a function of $u$, where $u$ is an outcome from $\Omega$.](image)
2.3.2 Probability Distribution Function

Let $x_1, x_2, \ldots, x_n$ where $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ be independent observations which can be seen as outcomes of $X_i = (X_{i1}, X_{i2}, \ldots, X_{id})$ which is a $d$-dimensional stochastic variable with the distribution function, denoted $F_X(x)$, expressed as

\[ F_X(x) = P(X_1 \leq x_1, X_2 \leq x_2, \ldots, X_n \leq x_n) \]
\[ = P(X_{11} \leq x_{11}, \ldots, X_{id} \leq x_{id}, X_{21} \leq x_{21}, \ldots, X_{nd} \leq x_{nd}). \]  

$F_X(x)$ is called a probability distribution function or a cumulative distribution function (cdf) for the stochastic variable $X$. Again, to elucidate that $X$ actually is a function from $\Omega$ to, in this case $\mathbb{R}^d$, $P(X \leq x)$ should be understood as $P(\{u : X(u) \leq x\})$, cf. section 2.3.1. Hopefully the following example would clarify the meaning of the probability distribution function.

Suppose that $X$ is one-dimensional stochastic variable, which is for example a result of a random experiment, as a rule it is impossible to theoretically determine the appearance of the probability distribution function. However, something about the appearance of the distribution can be stated. Assume that we know that the value of the measurement result lies between two numbers $a$ and $b$ and it can take any value in-between them. Consequently, $F_X(x)$ must be 0 for the $x$-values that are less that $a$ and it must be 1 for the $x$-values that are greater that $b$. Moreover, the distribution function must be monotonic increasing in the interval $(a, b)$, because the probability that $X \leq x$, i.e. $P(X \leq x)$, must of course increase as $x$ increases. Therefore, the probability distribution function, $F_X(x)$, has the general appearance as illustrated in Figure 2.2.

\[ F_X(x) \rightarrow \begin{cases} 
0 & \text{when } x \rightarrow -\infty \\
1 & \text{when } x \rightarrow \infty 
\end{cases} \]  

\[ (2.2) \]

$F_X(x)$ is an increasing function of $x$ and is continuous to the right of each $x$. 

Figure 2.2: The probability distribution function for the stochastic variable $X$. 

Thus, for the probability distribution function, $F_X(x)$, for the stochastic variable $X$ the following is valid.
It is often appropriate to use the derivative of the probability distribution function. This function is called the probability density function (pdf) and is defined, assuming of course that the derivative exists, as

\[ f_X(x) = \frac{dF_X(x)}{dx}. \]  

(2.3)

## 2.4 Probability Distributions

### 2.4.1 Uniform Distribution

The uniform distribution (also called rectangular distribution) has a constant pdf between its two parameters \(a\), the minimum, and \(b\), the maximum. The stochastic variable \(X\) is said to be uniformly distributed if it has the probability density function, \(f_X(x)\), according to

\[
f_X(x) = \begin{cases} 
\frac{1}{b-a} & \text{if } a < x < b \\
0 & \text{otherwise}. 
\end{cases}
\]  

(2.4)

The probability distribution function, \(F_X(x)\), of uniformly distributed stochastic variable is obtained through an integration of (2.4) giving

\[
F_X(x) = \begin{cases} 
0 & \text{if } x < a \\
\frac{x-a}{b-a} & \text{if } a \leq x \leq b \\
1 & \text{if } x > b. 
\end{cases}
\]  

(2.5)

**Code notation**: \(X \sim R(a,b)\)

Figure 2.3 illustrates the probability density and distribution functions, \(f_X(x)\) respectively \(F_X(x)\) of uniformly distributed stochastic variable \(X\).

![Figure 2.3: The probability density and distribution functions for stochastic variable having uniform distribution.](image)
2.4.2 Exponential Distribution

The stochastic variable $X$ is said to be *exponentially distributed* if its density function, $f_X(x)$, is

$$f_X(x) = \begin{cases} \frac{1}{\mu} \exp(-x/\mu) & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(2.6)

where $\mu > 0$ is the mean value of the stochastic variable $X$.

The probability distribution function for the exponential distribution is obtained through an integration of (2.6) giving

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - \exp(-x/\mu) & \text{otherwise.} \end{cases}$$

(2.7)

*Code notation:* $X \sim \text{Exp}(\mu)$

An example of the density and the distribution functions for an exponentially distributed stochastic variable with $\mu = 5$ is illustrated in Figure 2.4. For this distribution, the bigger the value of $\mu$, the more stretched the probability mass is in the interval $(0, \infty)$.

![Figure 2.4: The probability density and distribution functions for a stochastic variable having an exponential distribution.](image)

2.4.3 Normal and Log-Normal Distributions

The *normal distribution* was first studied in the eighteenth century when scientists observed an astonishing degree of regularity in errors of measurement. They found that the patterns (distributions) they observed were closely approximated by a continuous distribution which they referred to as the ”normal curve of errors” and attributed to the laws of chance [24].
The normal distribution is often used to describe the variation of different phenomena. That is why a vast part of statistical theory is based on this distribution. However, it should be noted that the normal distribution is not the only distribution, and divergence from it does not mean anything abnormal. There exist unlimited possibilities to find theoretical density functions that can fit the observed data very well. The reason why the normal distribution should be chosen primarily is because it has many good mathematical properties which make it very easy to use.

The stochastic variable $X$ is said to be normally distributed if its density function, $f_X(x)$, is according to (2.8).

$$f_X(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( \frac{-(x - \mu)^2}{2\sigma^2} \right), \quad (-\infty < x < \infty) \quad (2.8)$$

where $\mu$ and $\sigma$ are respectively the mean value and the standard deviation of the stochastic variable $X$.

The probability distribution function for the normal distribution is obtained through an integration of (2.8) giving

$$F_X(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( \frac{-(t - \mu)^2}{2\sigma^2} \right) dt. \quad (2.9)$$

**Code notation:** $X \sim N(\mu, \sigma)$

The **standard normal distribution** is the special case of (2.8) and (2.9) where $\mu = 0$ and $\sigma = 1$ and is denoted $X \sim N(0, 1)$. Its density and distribution functions are denoted by $\varphi(\cdot)$ and $\Phi(\cdot)$ respectively, and are given by (2.10) and (2.11).

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{x^2}{2} \right), \quad (-\infty < x < \infty) \quad (2.10)$$

$$\Phi(x) = \int_{-\infty}^{x} \varphi(t) dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{t^2}{2} \right) dt \quad (2.11)$$

If $x$ is standard normal, then $x\sigma + \mu$ is also normal with mean $\mu$ and standard deviation $\sigma$. This implies that any normally distributed stochastic variable $Y$ with mean $\mu$ and standard deviation $\sigma$ can be transformed into standard normal $X$ by

$$x = \frac{y - \mu}{\sigma}. \quad (2.12)$$

Figure 2.5 illustrates the density and distribution functions for standard normal distribution with mean zero and different standard deviations.

The **log-normal distribution** occurs in practice whenever we encounter a stochastic variable which is such that its natural logarithm has a normal distribution. The density and distribution functions for log-normal distribution for the stochastic variable $X$ are shown in (2.13) and (2.14) respectively.

$$f_X(x) = \frac{1}{x\sigma \sqrt{2\pi}} \exp \left( -\frac{(\ln x - \mu)^2}{2\sigma^2} \right) \quad (2.13)$$
2.4. PROBABILITY DISTRIBUTIONS

\[ F_X(x) = \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{x} \frac{1}{t} \exp \left( \frac{-(\ln t - \mu)^2}{2\sigma^2} \right) dt \]  

(2.14)

where \( \mu \) and \( \sigma \) are the mean value and the standard deviation of the stochastic variable \( X \).

Figure 2.6 illustrates the density and distribution functions for log-normal distribution with mean 1 and standard deviation 0.5.

Figure 2.5: The probability density and distribution functions for stochastic variable having normal distribution with \( \mu = 0 \) and different standard deviations.

Figure 2.6: The probability density and distribution functions for stochastic variable having log-normal distribution.
2.4.4 Multimodal Distribution

As described previously, because of the good mathematical properties of normal distribution function, it is often the preferred choice for use in different kinds of probabilistic applications. For example, many results from traffic data measurement have shown that different vehicle’s gross weight and total length have a multimodal distribution, see [10]. Numerous studies have shown that even the traffic load effects can, with sufficient accuracy, be modelled by the sum of several normal distributions. Often multimodal distributions obtained are a result of different populations. They can, for example, be written as the sum of several normal distributions as shown in (2.15).

\[
F_X(x) = \sum_{i=1}^{n} p_i \Phi \left( \frac{x - \mu_i}{\sigma_i} \right) \tag{2.15}
\]

where \( p_i, \mu_i \) and \( \sigma_i \) are the proportion, the mean value and the standard deviation for mode \( i \), respectively.

For the entire population (2.16), (2.17) and (2.18) are then valid.

\[
\sum_{i=1}^{n} p_i = 1 \tag{2.16}
\]

\[
\mu = \sum_{i=1}^{n} p_i \mu_i \tag{2.17}
\]

![Figure 2.7: An example of a multimodal probability density function having two populations. The distribution is constituted from two normal distributions having different mean values as well as standard deviations.](image)

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\[
\sigma = \sqrt{\sum_{i=1}^{n} p_i \sigma_i^2 + \sum_{i=1}^{n} p_i (\mu - \mu_i)^2}
\] (2.18)

The probability density functions corresponding to \( N(\mu_i, \sigma_i) \) and the multimodal distribution function, \( F_X(x) \), with \( n = 2 \), are shown as an example in Figure 2.7. The solid curve in the figure shows a multimodal distribution which is the sum of two normal distribution having different mean values and standard deviations, shown by the dashed curves.

### 2.5 Return Period

Let \( A \) be an event e.g. the exceedance of a value \( x \), and \( T \) the random time between consecutive occurrences of events \( A \). The mean value, \( \tau \), of the random variable \( T \) is called the return period, denoted \( R_T \), of the event \( A \). In other words the return period of any value of \( x \) is the mean time interval between two exceedances of the value \( x \) by the stationery time series \( X_i, i = 1, \ldots, n \), or rather the mean time elapsed before the first exceedance of \( x \). Therefore, if \( x_\alpha \) is the \((1 - \alpha)\) quantile of the stochastic variable for the load or the load effect, then the return period \( R_T \) can be expressed as

\[
R_T \approx -\frac{T}{\ln(1 - \alpha)} \approx \frac{T}{\alpha} \text{ if } 0 < \alpha < < 1
\] (2.19)

where \( T \) is called the reference time [20].

Note that if \( F_X(x) \) is the probability distribution function of the yearly maximum of a random variable, the return period of that random variable to be exceeded the value of \( x \) is \( 1/[1 - F_X(x)] \) years. Similarly, if \( F_X(x) \) is the probability distribution function of the yearly minimum of a random variable, the return period for the variable to fall below the value \( x \) is \( 1/F_X(x) \) years.

Also note that if a given engineering work fails when, and only when, the event \( A \) occurs, its mean lifetime coincides with the return period of \( A \). The importance of return periods in engineering is due to the fact that many design criteria are defined in terms of return periods.

### 2.6 Extreme Value Distribution

Here follows a short motivation for the selection of the extreme value distributions to describe model loads. Assume that the maximum loads during individual days are almost independent and equally distributed. Let us now choose the larger time period of one month, say (Observe that we will choose \( dt = 1 \) month). Then obviously the maximums of a load during successive months are still independent and identically distributed, but in addition, each of them will also be a maximum of
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30 daily values. Classical extreme value theory deals principally with the distribution of maximum of \( n \) independent and identically distributed random variables \( X_1, X_2, \ldots, X_n \), i.e.

\[
M_n = \max(X_1, X_2, \ldots, X_n).
\] (2.20)

The distribution of \( M_n \) is easily written down as (2.21) because of the independent and identically distributed assumption for the \( X \).

\[
P(M_n \leq x) = P(\max(X_1, X_2, \ldots, X_n) \leq x)
\]
\[= P(X_1 \leq x) \cdot P(X_2 \leq x) \cdots P(X_n \leq x)\] (2.21)
\[= F_{X_1}(x) \cdot F_{X_2}(x) \cdot \cdots F_{X_n}(x)
\]
\[= (F_X(x))^n\]

As \( n \to \infty \), the above equation tends to the so-called asymptotic extreme value distribution which has three types. These three types are the Gumbel-type, the Fréchet-type and the Weibull-type distribution, respectively see [7].

2.7 Characteristic Load Value

Characteristic load value corresponds to the loads that are certainly rare but yet, with a small probability, can be expected to occur some time during the constructions normal design working life. The characteristic values of load parameters are chosen to be high but measurable quantiles. The characteristic value of an action is defined in [8] as its principal representative value. The representative value of an action is a value used for the verification of a limit state, where the constructions are at such a limit that they no longer fulfil their given design demands. A mathematical definition of this value, described in [7], is as follow. A certain value \( x \) of a random variable \( X \) is said to be the characteristic value, denoted \( x_k \), for a period of duration of \( n \) units, if the mean value of the number of exceedances of that value is such a period is unity. That is

\[
n \left[ 1 - F_X(x_k) \right] = 1 \quad \Rightarrow \quad F_X(x_k) = 1 - \frac{1}{n}\] (2.22)

The probability of exceeding the characteristic value in the period is

\[
1 - [F_X(x_k)]^n = 1 - \left( 1 - \frac{1}{n} \right)^n
\] (2.23)

which for large \( n \) tends to \( 1 - e^{-1} = 0.6321 \).

According to [3, 8, 31, 38] the characteristic value of a load is defined as the 98th percentile of the annual maximum load distribution. This means that this value exceeded with the probability of 0.02 under one year or alternatively it is exceeded on average once every fifty year. This value is normally used for loads that are caused by nature such as wind and snow. However, this definition is used even for
traffic loads in the Swedish bridge design codes. In Eurocode, the characteristic value for the traffic loads has been defined for a return period of 1000 years, i.e. the value with a probability of exceedance of 5% in 50 years. Hopefully, the following illustrative example will clarify the computation of characteristic load value from a yearly maximum load distribution.

Suppose that we want to determine the characteristic traffic load value for a bridge with span of 30 meters. Assume that this case corresponds to the gross weight of two trucks that happen to be on the bridge simultaneously. Say that we have gathered data for the gross weights of all vehicles that have passed over the bridge under one entire year. Let \( \hat{F}_X(x) \) be the empirical distribution function, see section 2.8.1, where \( x_i \) is gross weights of two trucks which follow each other successively and can be assumed to be on the bridge simultaneously. \( x_i \) can therefore be assumed to be independent outcome of the stochastic variable \( X \). The characteristic load value, \( x_k \), can then be calculated as

\[
x_k = \hat{F}_X^{-1}(0.98)
\]  

(2.24)

where \( \hat{F}_X^{-1}(\cdot) \) is the inverse function of \( \hat{F}_X(x) \). This means that it is assumed that only once during a period of one year are there two trucks present on the bridge simultaneously. However, if it is assumed that this event happens \( N \)-times in one year, then the observed yearly maximum loading \( y \) is the maximum value of sets of \( N \)-values of \( x \) with distribution function \( \hat{F}_Y(y) \). That means \( y_i \) can be seen as independent outcome of the stochastic variable \( Y \). Equation (2.25), cf. (2.21) on page 16, shows that relationship between \( \hat{F}_X(x) \) and \( \hat{F}_Y(y) \) which can easily be shown to be

\[
\hat{F}_Y(y) = [\hat{F}_X(x)]^N.
\]  

(2.25)

The characteristic load value, in this case, can be calculated as

\[
y_k = \hat{F}_Y^{-1}(0.98) = \hat{F}_X^{-1}(0.98^{1/N}).
\]  

(2.26)

Here follows another illustrative example. The purpose of the example is only to verify the validity of (2.25) and has no practical meaning. Assume that we have gathered data during one entire year and have obtained the observations \( X = x_1, x_2, \ldots, x_n \), where \( x_i \)'s are gross weights of two successively following trucks assumed to be on the bridge simultaneously. For the sake of simplicity, \( X \) is assumed to have been generated from \( N(15 \text{kN}, 9 \text{kN}) \) and we simulate the \( x_i \)'s from this distribution. Say that we want to determine the characteristic load value for different \( N \)-values from the empirical distribution for \( X \). As before, this value can be calculated for \( N = 1 \) as 0.98 percentile of the distribution \( \hat{F}_X(x) \). For \( N = 2 \) the characteristic value can be calculated as (2.26) either as \( \hat{F}_X^{-1}(0.98^{1/2}) \) or as \( \hat{F}_Y^{-1}(0.98) \), where \( \hat{F}_Y(y) \) is obtained according (2.25). The calculated values for different levels, i.e. for \( p = (0.98, 0.99, 0.996, 0.9996) \), and for both assumptions, i.e. \( N = 1 \) and \( N = 2 \) are shown in Figure 2.8. As clearly seen, this figure proves the validity of (2.25).
CHAPTER 2. FUNDAMENTAL CONCEPTS

2.8 Monte Carlo Simulation Technique

The Monte Carlo simulation (or MC simulation) technique was first used during World War II by scientists named Fermi, von Neumann, Ulam, Metropolis, and Richtmeyer who developed it for the solution of problems related to neutron transport during the development of the atomic bomb [29, 37]. This was performed at the Los Alamos National Laboratory in New Mexico. The name Monte Carlo is used since the method is based on the selection of random numbers. In this sense it is related to the gambling casinos at the city Monte Carlo in Monaco. The Monte Carlo method can be considered as a very general mathematical method to solve a great variety of problems.

During the last two decades, the rapid increase of computer power has facilitated the development of the Monte Carlo techniques in statistics. More complicated multidimensional models can now be handled using computer intensive statistical algorithms. Often, the use of the Monte Carlo simulation eliminates the cost of building and operating expensive equipment for performing different types of experiments. The Monte Carlo methods are also useful in situations where direct experimentation is impossible—say, in studies of the spread of cholera epidemics, which of course, are not induced experimentally on human populations.

The theory regarding the Monte Carlo simulations has been used to describe many problems in scientific literature including applications. A classical example of the use of the Monte Carlo methods in the solution of a problem of pure mathematics is the determination of $\pi$ (the ratio of the circumference of a circle to its diameter) by probabilistic means. Early in the eighteenth century George de Buffon, a
French naturalist, proved that if a very fine needle of length $a$ is thrown at random onto a board ruled with equidistant parallel lines, the probability that the needle will intersect one of the lines is $2a/\pi b$, where $b$ is the distance between the parallel lines. What is remarkable about this fact is that it involves the constant $\pi = 3.1415926535\ldots$, which in elementary geometry is approximated by the circumferences of regular polygon enclosed in a circle of radius $1/2$. Buffon’s result implies that if such a needle is actually tossed a great many times, the proportion of the time it crosses one of the lines give an estimate of $2a/\pi b$ and, hence, an estimate of $\pi$ since $a$ and $b$ are known. Early experiment of this kind yielded an estimate of 3.1596 (based on 5000 trials) and an estimate of 3.155 (based on 3204 trials) in the middle of nineteenth century.

Another important use of the Monte Carlo simulation technique is the so-called resampling. The basic idea is to construct artificial new data sets (from the original one) by means of simulation.

### 2.8.1 The Empirical Distribution Function

All the simulations in this work are done using the empirical distribution functions, denoted $\hat{F}_n(x)$, and no assumptions of parametric distributions are made. The empirical distribution function for a discrete random variable $X = x_1, x_2, \ldots, x_n$ is discontinuous. It makes jumps at the points that are possible values $x_i$ of this random variable, and the sizes of the jumps are equal to $1/n$. Between the jumps the function is constant. In other words, if the $n$ events are located at values $x_i$, $i = 1, 2, \ldots, n$, then $\hat{F}_n(x)$ is the function giving the fraction of data points to the left of a given value $x$. As said before, this function is obviously constant between consecutive (i.e. sorted into ascending order) $x_i$’s, and jump by the same constant $1/n$ at each $x_i$. The empirical distribution function can be written as

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq x) \quad (2.27)$$

where

$$I(A) = \begin{cases} 
1 & \text{if } A \\
0 & \text{otherwise.}
\end{cases}$$

Clearly, the sum in (2.27) is equal to zero for $x$ that are smaller than all values $x_i$ of the random variable $X$, and equal to 1 to the right of all these $x_i$; within one interval between two adjacent values $x_i$ no new value is added to the sum or deleted from it, so the function remains constant.

### 2.8.2 The Inverse Method

Inversion is a general method for simulating random variables. It makes use of the fact that the transformation $X = F_X^{-1}(U)$, where $U$ is uniformly distributed
stochastic variable between 0 and 1 (i.e. \( U \sim R(0, 1) \)), yields a random variable \( X \) with distribution function \( F_X(x) \) provided the inverse function \( F_X^{-1}(x) \) exists. This is a simple consequence of the change of variables formula \( g_X(U) = F_X^{-1}(U) \).

Since \( g_X^{-1}(x) = F_X(x) \), the density of \( X \) becomes \( \frac{d}{dx} F_X(x) = f_X(x) \), which is the probability density corresponding to the distribution function \( F_X(x) \).

Let \( F_X(x) \) be the probability distribution function for the stochastic variable \( X \). Since \( F_X(x) \) is a strictly increasing function, the inverse \( F_X^{-1}(u) \) exists for all values of \( u \) between 0 and 1, see Figure 2.9. \( F_X^{-1}(u) \) is the smallest value of \( x \) which satisfies \( F_X(x) \geq u \) i.e.

\[
F_X^{-1}(u) = \inf \{ x : F_X(x) \geq u \}, \quad 0 \leq u \leq 1.
\]

As \( U \sim R(0, 1) \) and \( X = F_X^{-1}(U) \), the inverse of \( F_X(x) \) can be constructed in such a way that \( F_X^{-1}(u) > x \) if and only if \( u > F(x) \). Thus

\[
P(X \leq x) = 1 - P(X > x)
= 1 - P(F^{-1}(U) > x)
= 1 - P(U > F(X))
= 1 - (1 - F(x))
= F(x).
\]

**2.8.3 Data Generation**

In this section, procedures for fictitious data simulation using samples are discussed. The procedures are very simple but very powerful.
2.8. MONTE CARLO SIMULATION TECHNIQUE

One-dimensional Data Generation

First, let us generate 200 data \( x_1, x_2, \ldots, x_{200} \) from \( \text{Exp}(10) \) which can be seen as outcomes of the stochastic variable \( X \). This means that data is generated from \( F_X(x) \) which in this case is \( \text{Exp}(10) \). Assume that the data is a result of a random experiment, i.e. observations noted. The generation of data is complicated if the \( x_i \)'s comes from another arbitrary distribution \( G_X(x) \). In real situations the distribution from which data is generated is not known. All information comes from the samples, and the empirical distribution, see section 2.8.1 on page 19, contains all the information. As explained before, in this work, all the generations of fictitious data are done using the empirical distribution functions, \( \hat{F}_n(x) \), and no assumptions of parametric distributions are made. The empirical distribution, as explained before, can be obtained by estimating the unknown distribution \( G_X(x) \) from which data is generated by \( \hat{F}_n(x) \), from the observations \( x_1, x_2, \ldots, x_n \). The following illustrative example shows the procedure of the data generation.

New data is drawn from the original samples with the help of \( \hat{F}_n(x) \), which implies that picking the values \( x_1, x_2, \ldots, x_{200} \) each with the probability of 1/200. This assumes, of course, that all possible outcomes are equally likely. Drawing a data repeatedly is the same thing as drawing data with replacement. Figure 2.10 illustrates a comparison between the empirical distributions \( \hat{F}_{10000}(x) \) and \( \hat{F}_{200}(x) \) for 10000 newly generated and the original 200 samples respectively. Also shown in

Figure 2.10: A comparison of the empirical distribution functions calculated using 1000 simulated data and the original 200 samples. Also plotted is the distribution function \( \text{Exp}(10) \) from which the original data is drawn from. For one-dimensional data generation.
the figure is the distribution function from which the original data is generated, i.e. \( \text{Exp}(10) \). As seen in the figure, the two empirical distributions not only agree well with each other but also reflect very well the “true” distribution function, i.e. \( \text{Exp}(10) \).

Two-dimensional Data Generation

The method used in this work to generate two-dimensional dependent data is described here. Assume that we have the observations \((x_1, y_1), (x_2, y_2), \ldots, (x_{200}, y_{200})\) which can be seen as dependent outcomes of the stochastic variables \((X, Y)\). We assume that the \((x, y)\) are a result of a random test that are normally distributed according to (2.28) and generate 200 data.

\[
\begin{bmatrix} X \\ Y \end{bmatrix} \sim N \left( \begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix}, \begin{bmatrix} \sigma_X & \rho \\ \rho & \sigma_Y \end{bmatrix} \right) = N \left( \begin{bmatrix} 5.3 \\ 5.6 \end{bmatrix}, \begin{bmatrix} 2.0 & 0.5 \\ 0.5 & 2.0 \end{bmatrix} \right) \tag{2.28}
\]

where \( \rho \) is the correlation coefficient of \( X \) and \( Y \).

We use these data as samples we have for the simulation of new fictitious data and see them as data pairs, for example data for vehicles’ gross weights and total lengths. The distribution function in (2.28) is estimated, as usual, by the empirical distribution function \( \hat{F}_{200}(x, y) \). In this case the empirical distribution function is a function that gives the fraction of 1/200 to each 200 data pairs. Likewise, as for the one-dimensional case, new data, in this case data pairs, is drawn from the original samples with the help of \( \hat{F}_{200}(x, y) \), which implies that picking the data pairs \((x_1, y_1), (x_2, y_2), \ldots, (x_{200}, y_{200})\) each with the probability of 1/200. Again, this assumes, of course, that all possible outcomes are equally likely. Figure 2.11 illustrates a comparison of the empirical distribution functions for newly simulated 10000 data pairs and for the original 200 samples. From the figure, it can be concluded that \( \hat{F}_{10000}(x, y) \) and \( \hat{F}_{200}(x, y) \) agree very well.

The mean values, standard deviations and the correlation coefficients for the simulated and the original data are,

for the original 200 data (samples)

\[
N \left( \begin{bmatrix} 5.285 \\ 5.542 \end{bmatrix}, \begin{bmatrix} 2.032 & 0.564 \\ 0.564 & 2.103 \end{bmatrix} \right)
\]

for the simulated 10000 data

\[
N \left( \begin{bmatrix} 5.254 \\ 5.477 \end{bmatrix}, \begin{bmatrix} 2.026 & 0.576 \\ 0.576 & 2.095 \end{bmatrix} \right).
\]
2.9 Level Crossings and Rice’s Formula

In this section, the methodology for extrapolating minimal and maximal load effects presented by Cremona and Cremona and Carracilla in [13, 14], is discussed. In these articles the potential usage of the method is illustrated, where Cremona and Carracilla have implemented it to the study of different traffic load effects on several bridges. This study is summarized and presented in section 3.3 on page 36.

In many situations, the engineers or scientists, instead of dealing with maxima and minima, are interested in the events associated with the exceedances of a certain value of the random variable under study. Exceedance can be defined as follow. Let $X$ be a random variable and $\nu$ a real number, we say that the event $X = x$ is an exceedance of the level $\nu$ if $x > \nu$ [7].

There exists a very simple formula for the mean number of occasions per unit time that a stationary Gaussian process crosses a fixed constant level. This formula is known as Rice’s formula and is named after S.O. Rice, who in two pioneering papers developed the theory of Gaussian process in an electrical engineering context, see [46]. Rice’s formula, (2.29), expresses the mean rate $v_X(x)$ of upcrossings for a level $x > 0$ respectively down-crossings $x < 0$ during a reference period $T_{\text{ref}}$.

$$v_X(x) = \frac{1}{2\pi} \frac{\sigma}{\sigma} e^{-\frac{(x - m)^2}{2\sigma^2}}$$  \hspace{1cm} (2.29)
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Figure 2.12: Level crossings. $x_1$ is increasingly crossed 9 times and $x_2$ is decreasingly crossed 5 times. Redrawn from [13].

where $\sigma$, $\dot{\sigma}$ and $m$ are the standard deviation of $X$, the standard deviation of the stochastic process derivative $\dot{X}$, and the process mean, respectively.

It is possible to obtain level crossing histograms for different load effects, i.e. bending moments, shear forces etc., for example using recorded traffic load data. These histograms of level crossings represents the number of times at which positive values are increasingly crossed and where negative values are decreasingly crossed, see Figure 2.12. These histograms are particularly interesting for extrapolation of extreme values for any return period. It should be noted that the hypothesis of a stationary process is necessary as the extrapolation may not take account of any further changes in traffic patterns in the future.

If the level crossing histogram is normalized with respect to the record period $T_{rec}$, each normalized class interval gives an approximation of the mean rate $v_X(x)$. Therefore, it is intuitive to fit Rice’s formula on the large values, i.e. above a given threshold, of that normalized histogram. This consequently leads to the need to identify the three parameters $\sigma$, $m$ and $v_0 = \dot{\sigma}/2\pi\sigma$ for each tail. This estimation requires a choice of a value $x_0$ representing the starting class interval, i.e. the threshold value, from which the fitting is made, see Figure 2.13.

It is obvious that, if the starting interval is chosen very close to the tail, the fitting is expected to be a good approximation of the very far tail. However, this will provide few data points on which to estimate the parameters and to extrapolate the load effects. Contrarily, if the starting interval is far from the end of the tail, the fitting can be expected to be more representative for extrapolating load effects, but would provide a poor approximation of the tail. Therefore, the starting interval must be chosen in an effective manner. In the preparatory studies of Eurocodes, the choice of the optimal value of $x_0$ was performed by successive tests. An automatic and rigorous method needed to be sought in order to optimize the choice of $x_0$.

By taking the logarithm of (2.29) on page 23, the fitting is brought back to the
2.9. LEVEL CROSSINGS AND RICE’S FORMULA

identification of the parameters of a second order polynomial function as

\[ y = \ln(v_X(x)) = a_0 + a_1 x + a_2 x^2 \quad (2.30) \]

\[ a_0 = \ln \left( v_0 - \frac{m^2}{2\sigma^2} \right); \quad a_1 = \frac{m^2}{\sigma^2}; \quad a_2 = -\frac{1}{2\sigma^2}. \]

The determination of the polynomial coefficients can be carried out by the least squares method. Parameter estimation is therefore straightforward. For the parameter estimation, the linear least square method described in section 2.10 on page 27 is used in this work.

2.9.1 Confidence Level

As mentioned earlier, the crucial point when performing the fitting is related to the choice of an appropriate number of class intervals. The Kolmogorov-Smirnov (or K-S) test, described in section 2.11 on page 29, is applied for the fitting of data and Rice’s formula. This statistical test compares two probability functions corresponding respectively to the reference and the tested distributions. If \( \hat{F}_X(x) \) and \( S_X(x) \) are the empirical respectively the probability distribution functions, the K-S test studies the statistics of the variation \( D \) as seen in (2.54) on page 30. For a value \( d \), the probability \( P(d > D) \) is approximated by the Kolmogorov-Smirnov function \( Q_{KS} \) as, cf. (2.56) on page 30,

\[ P(d > D) = Q_{KS}(\sqrt{N}d) = 2\sum_{j=1}^{\infty} (-1)^{j-1}e^{-2j^2(\sqrt{N}d)^2} \quad (2.31) \]

where \( N \) is the number of class intervals.

In our problem, we should like to assess the pertinence of Rice’s formula and to perform an automatic determination of the number of intervals to use for the fitting.

Figure 2.13: Principle for optimal fitting. i) Rice’s formula fitting on a tail of the upcrossing rates histogram. ii) Fitted and normalized histograms versus the remaining part. Redrawn from [13].
Let us first fix the interval $x_0$ corresponding to the right tail of the level crossing rate histogram, see Figure 2.13. The histogram is truncated in order to take into account only $N(x_0)$ intervals. If $v_X(x)$ and $f_X(x)$ denote the fitted function, i.e. Rice’s formula, and the level crossing rate distribution, the objective is to compare the truncated part of $v_X(x)$ and $f_X(x)$ for $x > x_0$, see Figure 2.13. Let us therefore define the renormalized tail of $v_X(x)$ and normalized tail of $f_X(x)$ for $x > x_0$ as

$$v'_X(x) = \begin{cases} 0 & \text{if } x < x_0 \\ \frac{v_X(x)}{\int_{x_0}^{+\infty} v_X(x) dx} & \text{otherwise} \end{cases} \quad (2.32)$$

$$f'_X(x) = \begin{cases} 0 & \text{if } x < x_0 \\ \frac{f_X(x)}{\int_{x_0}^{+\infty} f_X(x) dx} & \text{otherwise.} \end{cases} \quad (2.33)$$

Since $\int_{x_0}^{+\infty} v_X(x) dx = \int_{x_0}^{+\infty} f_X(x) dx = 1$, the above two functions are density functions. Once this procedure is applied, it is then possible to determine the probability functions $S'_X(x)$ of $v'_X(x)$ and $\hat{F}'_X(x)$ of $f'_X(x)$.

The optimal class interval can be different if an absolute optimal fitting or relative optimal fitting is applied. In the first case a solution can be obtained. Whereas, in the second case, if a too high KS value is fixed, no relative optimal fitting can be obtained. This can happen when all the KS test values are lower than the target one. What will be good is a mixture of the two approaches—when the target KS value is not obtained, no optimal fitting can be provided. If this is the case, then one must keep $x_0$ provided the largest KS value is above the target one. In the relative fitting, one must always keep the smallest $x_0$ which corresponds to the best KS test value. The optimal fitting is obtained for $x_{\text{opt}}$ such that

$$x_{\text{opt}} = \min_{\beta(x_0) \geq \beta_0} (x_0) \quad (2.34)$$

where $\beta_0$ is a conventional confidence level for the Kolmogorov-Smirnov test which can be chosen between 0.9 and 1. The optimal interval number is therefore $N(x_{\text{opt}})$. Let us note that the value $\beta(x_{\text{opt}})$ is the confidence level of the optimal fitting with regard to Rice’s formula. Of course, this confidence level depends on the threshold level $\beta_0$. This is why the fitting is only a relative fitting in the sense that it depends on $\beta_0$. An absolute fitting can nevertheless be defined by

$$\hat{x}_{\text{opt}} = \min_{\max(\beta(x_0))} (x_0). \quad (2.35)$$

In other terms, the absolute fitting is issued from the smallest $x_0$ corresponding to the highest Kolmogorov-Smirnov value obtained from all the successive fittings.
optimal fittings present the advantage to provide a sensitivity analysis versus the threshold level. If the fitting (number of optimal class intervals and Kolmogorov-Smirnov test) is only slightly sensitive to $\beta_0$, the fitting can be considered as very robust. In which case Rice’s formula can be considered as a good representation of the level crossing rate histogram of the tail. If this is not the case, fitting must be carefully employed for extrapolating load effects.

### 2.9.2 Extrapolation of Load and Load Effects

When the optimal fittings are obtained for each tail, the extrapolation of maximum and minimum effects, for any return period $R_T$ can be assessed. Indeed, as the return period $R_T$ for $x$ is defined as the mean period between two occurrences of the value $x$ see section 2.5 on page 15, then it follows that

$$v_X(x)R_T = 1. \quad (2.36)$$

Equation (2.36) permits expressing the $x$ value in terms of the return period as

$$v_0 e^{-\frac{(x - m)^2}{2\sigma^2}} = \frac{1}{R_T}. \quad (2.37)$$

Hence finding the value of $x$ with a return period $R_T$ means solving (2.37) as

$$x = m \pm \sigma \sqrt{2 \ln(v_0 R_T)}. \quad (2.38)$$

Consequently, the extrapolated maximum and minimum effects, $x_{\text{max}}(R_T)$ and $x_{\text{min}}(R_T)$, related to a return period $R_T$ can be obtained from (2.39) and (2.40), respectively.

$$x_{\text{max}}(R_T) = m_{\text{opt}}^r + \sigma_{\text{opt}}^r \sqrt{2 \ln(v_{0,\text{opt}}^r R_T)} \quad (2.39)$$

$$x_{\text{min}}(R_T) = m_{\text{opt}}^l - \sigma_{\text{opt}}^l \sqrt{2 \ln(v_{0,\text{opt}}^l R_T)} \quad (2.40)$$

The indexes $r$ and $l$ in (2.39) and (2.40) indicate respectively fitting to the right and left tail of a level crossing histogram of load effects.

### 2.10 General Linear Least Square

The general linear least square can be used to fit a set of data $(x_i, y_i)$ to a model which is a linear combination of any $M$ specified functions of $x$. For example, the functions could be $1, x, x^2, \ldots, x^{M-1}$, in which case their general combination

$$y(x) = a_1 + a_2 x + a_3 x^2 + \cdots + a_M x^{M-1} \quad (2.41)$$
is a polynomial of degree $M - 1$. Or, the function could be sines and cosines, in which case their general linear combination can be a harmonic series. A general form of this kind of model is seen in (2.42).

$$y(x) = \sum_{k=1}^{M} a_k X_k(x)$$  \hspace{1cm} (2.42)

where $X_1(x), X_2(x), \ldots, X_M(x)$ are arbitrary fixed function of $x$, called the basis function.

To measure how well the model agrees with the data, we use the chi-square merit function, which in this case is

$$\chi^2 = \sum_{i=1}^{N} \left[ y_i - \frac{\sum_{k=1}^{M} a_k X_k(x_i)}{\sigma_i} \right]^2.$$  \hspace{1cm} (2.43)

Thus to determine $a_k$ we have to minimize (2.43), i.e. we have to pick the best parameters those that minimize $\chi^2$. We get this when the derivative of $\chi^2$ with respect to each $a_k$ vanish. Before doing this let us define the following.

Let $A$ be a matrix whose $N \times M$ components are constructed from the $M$ basis functions evaluated at the $N$ abscissas $x_i$ and from $N$ measurement errors $\sigma_i$, by the prescription

$$A_{ij} = \frac{X_j(x_i)}{\sigma_i}.$$  \hspace{1cm} (2.44)

The matrix $A$ is called the design matrix of the fitting problem. Notice that in general $A$ has more rows than columns, $N \geq M$, since there must be more data points than model parameters to be solved for. Also define a vector $b$ of length $N$ by

$$b_i = \frac{y_i}{\sigma_i}$$  \hspace{1cm} (2.45)

and denote the $M$ vector, $a_1, a_2, \ldots, a_M$, whose components are the parameters to be fitted by $a$.

### 2.10.1 Solution by use of the Normal Equations

The minimum of (2.43) occurs when the derivative of $\chi^2$ with respect to all $M$ parameters $a_k$ vanishes.

$$0 = \sum_{i=1}^{N} \frac{1}{\sigma_i} \left[ y_i - \sum_{j=1}^{M} a_j X_j(x_i) \right] X_k(x_i); \quad k = 1, 2, \ldots, M$$  \hspace{1cm} (2.46)

Interchanging the order of summation, we can write (2.46) as the matrix equation

$$\sum_{i=1}^{M} \alpha_{kj} a_j = \beta_k$$  \hspace{1cm} (2.47)
2.11. Kolmogorov-Smirnov Goodness of Fit Test

In this section, a test is discussed that considers the goodness of fit between a hypothesized distribution and an empirical distribution function. This test is called the *Kolmogorov-Smirnov* (or K-S) test. This goodness of fit test is applicable to unbinned distributions that are functions of a single independent variable, that is, to data sets where each data point can be associated with a single number (life time of each lightbulb when it burns out, or declination of each star). In such cases, the list of data points can be easily converted to an unbiased estimator $\hat{S}_N(x)$ of the cumulative distribution function of the probability distribution from which it was drawn. If the $N$ events are located at values $x_i$, $i = 1, 2, \ldots, N$, then $\hat{S}_N(x)$ is the function giving the fraction of data points to the left of a given value $x$. This function is obviously constant between consecutive (i.e. sorted into ascending order) $x_i$'s, and jumps by the same constant $1/N$ at each $x_i$, see Figure 2.14, cf. section 2.8.1 on page 19.

Different distribution functions, or sets of data, give different cumulative distribution function estimates by the above procedure. However, all cumulative distribution functions agree at the smallest allowable value of $x$ (where they are zero), and at the largest allowable value of $x$ (where they are unity). The smallest and largest values might be $\pm \infty$. So it is the behaviour between the largest and smallest value that distinguishes distributions.

One can think of a number of statistics to measure the overall difference between two cumulative distribution functions. The absolute value of the area between them,
Figure 2.14: Kolmogorov-Smirnov statistic $D$. A measured distribution of values in $x$ (shown as $N$ dots on the lower abscissa) is to be compared with a theoretical distribution whose cumulative probability distribution is plotted as $F_X(x)$. $D$ is the greatest distance between the two cumulative distributions.

For example. Or their integrated mean square difference. The Kolmogorov-Smirnov statistic $D$ is a particularly simple measure. It is defined as the maximum value of the absolute difference between two cumulative distribution functions. Thus, for comparing one data set’s $\hat{S}_N(x)$, i.e. the empirical distribution function, to a known cumulative distribution function $F_X(x)$, the K-S statistic is

$$D = \max_{-\infty<x<\infty} |\hat{S}_N(x) - F_X(x)|$$

while when comparing two different empirical distribution functions, $\hat{S}_{N1}(x)$ and $\hat{S}_{N2}(x)$, the K-S statistic is

$$D = \max_{-\infty<x<\infty} |\hat{S}_{N1}(x) - \hat{S}_{N2}(x)|.$$

What makes the K-S statistic useful is that its distribution in the case of the null hypothesis, data set drawn from the same distribution can be calculated, at least to useful approximation, thus giving the significance of any observed nonzero value of $D$ as

$$Q_{KS}(\lambda) = 2 \sum_{j=1}^{\infty} (-1)^j e^{-2j^2\lambda^2}$$
which is a monotonic function with limiting values

\[ Q_{KS}(0) = 1 \quad \text{and} \quad Q_{KS}(\infty) = 0. \]  (2.57)

In terms of this function, the significance level of an observed value of \( D \), as a disproof of the null hypothesis that the distributions are the same, is approximately given by the formulas

\[ P(D > \text{observed}) = Q_{KS}(\sqrt{N} D) \]  (2.58)

for the case (2.54) of one distribution, where \( N \) is the number of data points, and

\[ P(D > \text{observed}) = Q_{KS}(\sqrt{\frac{N_1 N_2}{N_1 + N_2}} D) \]  (2.59)

for the case (2.55) of two distributions, where \( N_1 \) is the number of data points in the first distribution and \( N_2 \) the number in the second.

The nature of the approximation involved in (2.58) and (2.59) is that it becomes asymptotically accurate as the \( N \)’s become large. In practice, \( N = 20 \) is large enough, especially if one is conservative and require a strong significance level of 0.01 or smaller [43].
Chapter 3

Related Works

3.1 General

Research in the area of bridge design has been and still is concentrated, to a great extent, on the study of the strength of materials and relatively few studies have been preformed on traffic loads and their effects. Traffic loads have usually been assumed to be given in codes. This is mainly because it is very difficult to model the actual value of the traffic load in an accurate way as much information, in the form of field data, is needed. For the investigation of the bearing capacity of different parts of a bridge, relatively simple models can be made and tested in laboratories. In other words, we have a much better knowledge of the strength of our bridges than the loads to which they are subjected during their lifetime.

In 1992, COST 323, one of the actions supported by the COST (CoOperation in Science and Technology), DG VII [12], was initiated and the main objectives were:

1. Inventory of Weigh-In-Motion (WIM) requirements in Europe.
2. Collection and evaluation of existing WIM information.
3. Preliminary work on the development of a European technical specification on WIM.
4. Agreement of mechanisms and protocols for a pan-European database of WIM sites and data.
5. Collection and dissemination of scientific and technical information.
6. Exchange of experiences and conclusions from other international projects.

Since 1993, COST 323 has been run by the Management Committee, a group of scientific and technical experts, to promote the development and implementation of weigh-in-motion (WIM) techniques and their applications, and to facilitate an exchange of experiences between different European countries. In 1998, 19 countries were taking part in the action: Austria, Belgium, Switzerland, Czech republic,
CHAPTER 3. RELATED WORKS

Denmark, Germany, Finland, France, Hungary, Ireland, Iceland, Italy, Netherlands, Portugal, Slovak Republic, Slovenia, Spain, Sweden and United Kingdom. Since then many have been working on the development of the WIM techniques. Consequently, not only the quality of the WIM data has improved but also the performance of the measuring devices have been made better and easier. Currently, a two-dimensional bridge-WIM algorithm is been developed at the Royal Institute of Technology (KTH), at the division of structural design and bridges, see [44]. The system uses the bridge as a weighing scale and measure the axle weights of many vehicles simultaneously. For interested reader the recent development of the performance of WIM measurement can be found in [33].

Unfortunately, it is the author’s opinion that the development of the analysis methods of WIM data has not been as effective. Consequently, only a few previous works that are closely related to the presented work could be found. In the next section the background and the methods used for the determination of the load models in Eurocode 1 is briefly presented. Thereafter, two other works by Cremona [13] as well as O’Connor and O’Brien [40], that are closely related to the present research, are briefly discussed.

3.2 Eurocode 1

The structural Eurocodes which have ten parts are a unified set of international codes of practice which are intended to harmonize technical rules for all the European countries. They provide the basis for the limit state design of wide range of building and civil engineering structures. The part that deals with action on structures is Eurocode 1. This document, which had, in the beginning, been given the number ENV (European pre-standards) 1991, has many parts. The one that is relevant here, which defines models of traffic loads for the design of road bridges, footbridges and railway bridges, is ENV 1991-2 (1995) Part 3 Traffic Loads on Bridges, see [8]. This part was renamed in 2002 as EN (European Standard) 1991-2 Traffic Loads on Bridges, see [9]. In the document, actions for the design of road bridges with individual spans less than 200 meters are defined. Road traffic actions are represented by a series of load models which represent different traffic and different components, e.g. horizontal force, of traffic actions. The work for the definition of these traffic loads started in 1987. A working group was appointed by EEC. At the end of 1991, the work for the development of what was then included in ENV 1991 Part 3 was transferred to the CEN (European Committee for Standardization) and allocated to a project-team, see [4].

The load models, presented in Eurocode 1, representing road traffic loads have been calibrated on traffic recorded in Europe. The traffic data collection was performed in the eighties. Using the collected data, different realistic traffic scenarios, such as free flow and traffic jams, regarding various influence lines and bridge spans, were extrapolated. To evaluate the extreme values of axle and lorry loads, three different extrapolation methods have been adopted using the half-normal, the Gumbel distribution as well as Monte Carlo simulation. These methods are described briefly
All the studies showed that the various methods lead to practically equivalent results, see [17,30]. The first idea was to mix all the traffic records in order to get a European sample, however some of the extrapolations methods based on simulations needed a sample of homogenous traffic. For this reason, it was decided by the investigating groups that all statistical analysis would be done on the recorded traffic on the A6 motorway (Paris-Lyon) near Auxerre year 1986 in France. Because the period of traffic measurement is limited, usually a maximum of one week, it was necessary to predict, from the measurements of the traffic level $R$, the traffic in the future as a function of return period and overtaking probabilities. Extrapolation methods were used to obtain a corrected traffic level $R_{y\alpha}$. The extrapolation was extended to a return period of 1000 years. This value is determined from (3.1) cf. (2.19) on page 15, where the design life $T$ is 100 years, assuming 50 working weeks per year and the quantile $\alpha$ equal to 10%.

$$R_{y\alpha} \approx \frac{-T}{\ln(1 - \alpha)} \approx \frac{T}{\alpha}$$  \hspace{1cm} (3.1)

A reassessment of the main load model, i.e. Load Model 1 (LM1), of the Eurocode with modern traffic data has been performed [39,41]. New target values were specified. Significant advances have been made in improving the accuracy of weigh-in-motion (WIM) systems in recent years, and it was decided to assess the effect that these improvements might have on the target values. In addition, it was desirable to assess if changes in traffic patterns might have been experienced in the 10 years since the original calibration studies, and if so, to quantify if such changes might influence the original target values. These modern European WIM data were collected from a number of representative sites and used in simulations in exactly the same way as those employed in the original calibration studies. In addition, an attempt was made to replicate the results of the original studies, using the Auxerre data. The results of these simulations were compared with the results from the original studies, thereby assessing the adequacy of LM1, [42].

Figure 3.1 illustrates the main load model, LM1, which mixes axles and uniformly distributed loads. This main model covers most of the effects of lorry traffic for both general and local conditions. LM1 consists of two partial systems per notional lane. It consists of double-axle concentrated loads each axle having the weight of $\alpha Q_k$ and uniformly distributed loads having the weight per square meter of $\alpha q_k$. The values of $\alpha$ factors correspond to the classes of traffic. When they are taken equal to 1, they correspond to a traffic for which a heavy industrial international traffic is expected, representing a large part of the total traffic of heavy vehicles. For more common traffic compositions (highways or motorways), a moderate reduction of $\alpha$ factors applied to tandems systems and the uniformly distributed loads on Lane 1 may be recommended (10 to 20%). The characteristic values of $Q_k$ and $q_k$, dynamic amplification included, should be taken as illustrated in Figure 3.1.
3.3 Optimal Extrapolation of Traffic Load Effects

This section summarizes an article [13] by C. Cremona which illustrates the practical applications of Rice’s formula.

3.3.1 The Burgundy Bridge

The first implementation of the method described in the article is done for the study of the traffic-induced added tensions in stay cables of a cable-stayed bridge, called the Burgundy bridge. The bridge supports two traffic lanes and is 351.5 meters long. In order to assess traffic load effects, WIM data has been collected during a three week period. The collected data together with the influence surface, for added tensions in a cable, are used to determine the added-tension history during the measurement period. Figure 3.2 illustrates the level crossings histogram of the added tensions in one of the longest cables (marked H4C10). It is explained in the paper that the histogram does not resemble the bell curve of Rice’s formula. However, referring to [32] (in French), it is discussed that previous studies have
3.3. **OPTIMAL EXTRAPOLATION OF TRAFFIC LOAD EFFECTS**

![Figure 3.2](image)

Figure 3.2: Level crossings of the added tensions in cable H4C10. Reproduced from [13].

Shown that the load effects can be sometimes approximated as the sum of two or three Gaussian processes.

Figure 3.3 illustrates the extrapolated values of minimal and maximal added tensions versus the return period, $R_T$, and the test threshold value, $\beta_0$. For threshold values $\beta_0 \geq 0.95$, the extrapolation is stable and robust. Furthermore, it is described that the negative added tensions are tensions which are subtracted from the static tension. Figure 3.3 shows that all the negative extrapolated added tensions are intermingled, i.e. they give the same results independent of the chosen $\beta_0$ value. Figure 3.4 provides the optimal number of intervals and the Kolmogorov test value for the set of thresholds $\beta_0$. The figure demonstrates that the Rice’s formula gives a rather good approximation of the outcrossing rates histogram since $\beta$ values are very close to 1. For the maximal effects, threshold values greater than 0.95 give very close results.

Before putting the bridge into service, load tests were performed in order to assess load effects under extreme truck load configurations, which is a usual method to perform a load test value on a bridges. The negative and positive added tensions were measured. The load effects obtained from the load tests were compared to the extrapolated values induced by the “real” traffic conditions. Table 3.1 shows that the extreme load effects obtained from the load test. The extrapolated negative values are larger than those obtained from the load test (in absolute value), see Table 3.1 and Figure 3.4. It can nevertheless be argued that only positive added tensions are of interest. The article states that the results from the load test gave higher positive values in the cables than the extrapolated load effects with a return period of 1000 years obtained using the WIM data.

**Table 3.1: Extreme load effects during load tests [13].**

<table>
<thead>
<tr>
<th>Negative added tensions</th>
<th>Positive added tensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>-26.65 kN</td>
<td>165.04 kN</td>
</tr>
</tbody>
</table>
CHAPTER 3. RELATED WORKS

Figure 3.3: Extrapolation of minimum and maximum added tensions versus $R_T$ and $\beta_0$. Reproduced from [13].

Figure 3.4: Effect of the threshold value $\beta_0$ on the optimal fitting. Reproduced from [13].
3.3. OPTIMAL EXTRAPOLATION OF TRAFFIC LOAD EFFECTS

Figure 3.5: Level crossings of the total supported load (main span). Reproduced from [13].

Figure 3.6: Extrapolated total supported load (main span). Reproduced from [13].

3.3.2 The Tancarville Bridge

The Tancarville bridge which is 960 meters long and supports four lanes of traffic, is the first rehabilitation of a large suspension bridge involving the replacement of the supporting cables. The rehabilitation has been preceded by numerous studies such as the assessment of the total load supported by each cable. A study of the bridge behaviour under traffic has been commissioned to the LCPC, Laboratoire Central des Ponts et Chaussées. Figure 3.5 shows the histogram of level crossings for the total traffic load supported by the main span. To determine this total traffic load, a unit constant influence surface has been chosen. Figure 3.6 provides the extrapolated values with a threshold value $\beta_0 = 0.95$. It was decided not to close the bridge to traffic (including trucks) during the replacement. The article states that the total load from which the bridge is designed, is found to be four times greater that the extrapolated value corresponding to the 1000 year of return load. For this reason, data acquisition has been stopped during all the rehabilitation period. But further measures are planned after repair in order to assess the load development. Similar computations using WIM records have been made for assessing the total loads supported by each suspension cable.
3.3.3 Series of Multi-Span Bridges

Here, the mid-span normal stresses due to bending for six composite bridge spans are analyzed. The bridges are multi-spans except one (called the Joigny bridge) and are composed of only two girders. All bridges carry national traffic. Their total length varies from 75 (one span) to 450 meters (multi-spans). It is mentioned in the paper that the Gaussianity has been verified in previous studies, referring to [30], especially for the short bridge lengths. Further, it is stated, as for the Burgundy bridge, that Gaussianity is different for the upper and lower tails. The computations consist in letting a traffic flow cross the bridge and in calculating the corresponding load effects. The traffic flows are full WIM records, performed during a 3 week recording period, from the national road networks. Figure 3.7 provides an example of a level crossings histogram for the mid-span normal stress for the second span of one of the bridges (called the Beaucaire bridge) which is 441 meters long.

Extrapolated values for different $\beta_0$ are shown in Figure 3.8. As seen in the figure the computations have shown a good stability of the extrapolations versus the confidence threshold $\beta_0$. The article stats that this demonstrates the particular interest of such an approach for calibrating loads and load effects. The results from the extrapolations calculated for $\beta_0 = 0.95$, are illustrated in Figure 3.8.
3.3. OPTIMAL EXTRAPOLATION OF TRAFFIC LOAD EFFECTS

Figure 3.8: Sensitivity of the extrapolations versus the confidence threshold, $\beta_0$ (Beaucaire bridge, normal stress at mid-span for the second span). Reproduced from [13].

Figure 3.9: Extrapolation of traffic effects on some parts of bridges. Reproduced from [13].
3.4 Characteristic Load Effect Prediction

3.4.1 General

There are different methods of determining the characteristic traffic load and load effect values. In 1999, O’Connor and O’Brien, [40], used three different methods to determine the characteristic values of the total load and mid-span moment of simply supported as well as double fixed spans, for four lanes of traffic flow. The paper describes the use of WIM statistics in the prediction of characteristic load effects. The results of simulations performed using WIM data for both frequent and infrequent traffic flow conditions are presented and compared with those obtained through Monte Carlo simulation. The implications of the accuracy of the recorded WIM data on the predicted extreme load effect are assessed along with the sensitivity of the extreme to the method of prediction.

In the first method, simulations are performed using WIM data for both frequent and infrequent traffic flow conditions. In the second method, Monte Carlo simulation has been employed to generate vehicle flow patterns from assumed values of the statistical parameters describing the traffic flow. For the last method, extrapolations of the extreme load effects are performed, under the assumption that the load effect behaves as a stationary Gaussian process. The last mentioned is performed by fitting Rice’s formula to the level crossing distribution of the result from the simulations done using real WIM data.

The characteristic values for total load and mid-span moment of simply supported and double fixed spans with nine different span lengths (i.e. 5, 10, 20, 30, 50, 75, 100, 150 and 200 meters) are determined. The longitudinal influence lines that are considered for the simulations are illustrated in Table 3.2.

<table>
<thead>
<tr>
<th>Influence line number</th>
<th>Representation</th>
<th>Description of the influence line</th>
</tr>
</thead>
<tbody>
<tr>
<td>I0</td>
<td></td>
<td>Total load.</td>
</tr>
<tr>
<td>I1, I2</td>
<td></td>
<td>Maximum bending moment of a simply supported and double fixed(^1) span, respectively.</td>
</tr>
</tbody>
</table>

\(^1\) with an inertia strongly varying between mid-span and the ends

3.4.2 Simulation from WIM Data

Today’s WIM systems provide a much more accurate picture of the random variables governing traffic flow, i.e. vehicle gross weights, axle loads, spacings, speeds, headway etc. As such, it may be expected that characteristic values determined from scenarios generated with actual WIM data will be more representative than those determined from artificially generated traffic data.
3.4.3 Monte Carlo Simulation

For the Monte Carlo simulations, a total number of thirteen vehicle classes are adopted, demonstrating the varying vehicle forms for a given number of axles. When generating axle weights and spacing, correlations between the vehicle gross weight and the governing axle or axle group is identified. Subsequent correlation between axles is employed to determine individual axle weights. It is observed in the article that no statistically significant correlation exists between gross or axle weights and axle spacing, i.e. axle spacing within a given class are relatively constant independent of vehicle or axle weight. By referring to [21], the gamma distribution, which has the probability density function according to (3.2), is used to simulate the inter-vehicle distances. Vehicle speed is modelled as normally distributed.

\[
f_X(x) = \frac{\lambda(\lambda x)^{k-1}e^{-\lambda x}}{\Gamma(k)} \quad x \geq 0 \quad \lambda > 0 \quad k > 0
\]

(3.2)

3.4.4 Simulation Results

The results of simulations from measured and statistically generated traffic files show that there exist differences between the calculated load effects. These differences are illustrated in Table 3.3. It is apparent that the largest differences exist for short span lengths (< 30 meters) when a small number of vehicles or axles are on the influence surface. With increasing span length the differences attenuate. The article states that this is to be expected, as artificial regeneration of traffic records cannot reproduce the subtle correlations which exist between vehicle in adjacent lanes in real traffic flow. Herein lies the huge advantage of WIM data.

<table>
<thead>
<tr>
<th>Span [m]</th>
<th>UDL</th>
<th>SSM</th>
<th>FFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-11.7</td>
<td>-5.5</td>
<td>1.2</td>
</tr>
<tr>
<td>10</td>
<td>-12.2</td>
<td>-18.0</td>
<td>-14.5</td>
</tr>
<tr>
<td>20</td>
<td>-10.9</td>
<td>-15.7</td>
<td>-10.7</td>
</tr>
<tr>
<td>30</td>
<td>-1.3</td>
<td>-8.8</td>
<td>-12.4</td>
</tr>
<tr>
<td>50</td>
<td>-2.5</td>
<td>-6.4</td>
<td>-8.0</td>
</tr>
<tr>
<td>75</td>
<td>2.8</td>
<td>-0.4</td>
<td>-9.4</td>
</tr>
<tr>
<td>100</td>
<td>1.6</td>
<td>2.6</td>
<td>-5.2</td>
</tr>
<tr>
<td>150</td>
<td>2.3</td>
<td>5.8</td>
<td>3.4</td>
</tr>
<tr>
<td>200</td>
<td>1.3</td>
<td>3.3</td>
<td>4.7</td>
</tr>
</tbody>
</table>

UDL : Uniformly distributed load.
SSM : Mid-span moment of simply supported span.
FFM : Mid-span moment of double fixed span.
3.4.5 Prediction of Extremes

Rice’s Extrapolation

Rice’s formula, see (2.29) on page 23, is used for the extrapolation of the extreme values, thus one only has to verify that the distribution of the effects has an asymptotically Gaussian decreasing behaviour. In order to do so the program CASTOR-LCPC [16] adjusts the normal density proposed in the right hand side of (2.29) on the upper part of the level crossing histogram computed during the simulation. A typical form being illustrated in Figure 3.10.

Gumbel Extreme Value Distributions

The extreme values of the load effects may be modelled using a Gumbel type extreme value distribution. A Weibull (i.e. Gumbel type III) distribution results when the maximum values are sampled from a parent frequency distribution having a finite upper bound. An alternate distribution is the Gumbel I distribution. In this case, the maximum values are sampled from a parent distribution with no upper bound.

The principle of tail equivalence is employed in determining an appropriate extreme value distribution. The extreme value and parent distributions, $G_X(x)$ and $F_X(x)$ respectively, are considered tail equivalent if

$$\lim_{x \to \infty} \frac{1 - G_X(x)}{1 - F_X(x)} = 1$$

(3.3)
3.4. CHARACTERISTIC LOAD EFFECT PREDICTION

where the extreme value distribution is modelled by either the Gumbel or the Weibull distribution, given by (3.4) and (3.5), respectively.

\[
G_X(x) = \exp \left[-\exp \left(-\frac{(x - \lambda)}{\delta}\right)\right] \quad -\infty < x < \infty \quad \delta > 0 \quad (3.4)
\]

\[
G_X(x) = \exp \left[-\left(\frac{\lambda - x}{\delta}\right)^\beta\right] \quad -\infty < x < \lambda \quad (3.5)
\]

The parameters of the Gumbel law \(G_X(x)\) are estimated by the maximum likelihood approach.

The suitability of either distribution is assessed by plotting the extreme data on probability paper, where linearity indicates the appropriateness of the mathematical model. Figure 3.11 illustrates this technique. In Figure 3.11(a) the extreme values for simply supported moment in a 200 meters span are plotted on Gumbel I probability paper, along with the Gumbel I approximation. A linear trend is clearly apparent indicating the suitability of this distribution in the extreme. Figure 3.11(b) illustrates a convex trend in the right hand tail. As the tail region is of prime importance in extrapolation, clearly the Weibull distribution is unsuited in this case.

3.4.6 Comparison of Method of Prediction of Characteristic Extremes

Figure 3.12 illustrates, in ascending order, the predicted extremes of simply supported moment for span lengths 5, 10, 20, 50, 100 and 200 meters. It should be noted that the extremes predicted by the Rice extrapolation are performed from the simulations using real WIM data. Whilst those calculated using the Gumbel I and Weibull extreme value distributions are from MC simulations. The relative errors, with respect to the Rice extrapolations, are listed in Table 3.4. It is apparent, that the characteristic extremes predicted by all methods are in reasonable agreement, when the differences already noted between direct simulation and MC, listed in Table 3.4 under the heading SSM, are taken into account.
Figure 3.12: Comparison of characteristic extreme prediction methods. Reproduced from [40].

Table 3.4: Extrapolation results, [40].

<table>
<thead>
<tr>
<th>Return Period</th>
<th>Rice</th>
<th>Gumbel</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1 year</td>
<td>20 years</td>
<td>100 years</td>
</tr>
<tr>
<td>5</td>
<td>-11.1 (-14.6)</td>
<td>-11.0 (-12.6)</td>
<td>-11.1 (-10.7)</td>
</tr>
<tr>
<td>10</td>
<td><strong>10.1</strong> (-1.1)</td>
<td><strong>12.4</strong> (1.1)</td>
<td><strong>14.1</strong> (2.9)</td>
</tr>
<tr>
<td>20</td>
<td>-9.1 (-5.9)</td>
<td>-9.1 (-3.7)</td>
<td>-9.1 (-1.7)</td>
</tr>
<tr>
<td>50</td>
<td>3.5 (6.9)</td>
<td>4.1 (9.1)</td>
<td>4.5 (10.9)</td>
</tr>
<tr>
<td>100</td>
<td><strong>10.1</strong> (13.7)</td>
<td><strong>10.8</strong> (15.9)</td>
<td><strong>11.3</strong> (17.7)</td>
</tr>
<tr>
<td>200</td>
<td><strong>4.4</strong> (10.4)</td>
<td><strong>4.2</strong> (12.2)</td>
<td><strong>4.0</strong> (13.7)</td>
</tr>
</tbody>
</table>

**Bold**: % Difference between Rice’s & Gumbel.

( ) : % Difference between Rice’s & Weibull.

SSM : Mid-span moment of simply supported span.
Chapter 4

Traffic Load Models for Long-Span Bridges

4.1 General

This chapter provides a summary of part of the research project that was presented by the author as a licentiate work [18] in January 2000. The work contains a re-evaluation of existing traffic load measurement results. First, a simple method for the investigation of the accuracy of the collected vehicle data from one measurement series is introduced. Results of the filtration of the data are presented, the filtration removes data which is judged with a high degree of probability to be erroneous. Then, the data both before and after filtrations are evaluated according to the method discussed in [50]. This is performed in order to investigate the influence of measurement errors on the final results of data evaluation. Subsequently, the Monte Carlo simulation technique, using a limited amount of the collected data, is implemented. The main objective is to generate fictitious vehicle data that can represent results from field measurement which would otherwise have to be recorded under a long period. Finally, the results from the evaluations of measured and simulated vehicle data are compared. These results are also compared with the corresponding values calculated using the traffic load model from the Swedish bridge design code.

4.2 Background

Traffic load models given in different codes cannot be used directly for the design of long-span bridges, i.e. bridges with spans larger than 200 meters. In order to determine the characteristic loads for such bridges, the Swedish National Road Administration (Vägverket) conducted Weigh-In-Motion (WIM) measurements. The measurements were performed at five different test sites in four different regions—in the north, east, west and south of Sweden. For each vehicle, among other things, data on serial number, direction, velocity, number of axles, axle weight, length and
point-in-time for passage of the measurement station were registered and saved on
a PC as a binary file. The test locations and the measurement periods are shown
in Figure 4.1. The locations chosen were close to some with large span planned
bridges on the main roads E4 and E6. As an example, part of one measurement file,
after unpacking of raw-data and converting it into EXCEL format, is shown in Ta-
ble 4.1. The results present the static traffic loads where the dynamic contribution
of the vehicles’ load is filtered by the measurement system. The WIM data from
all measurement series were analyzed and presented in [51,53–56]. These were done
by using a method developed by L. Östlund, see [50]. The data set from two of the
measurement series were recently used in [6] for the reliability based assessment of
short span bridges.

Of course, all measurement data are disturbed by errors. Therefore, in this work,
the collected data from one of the above mentioned measurement series was care-
fully investigated and re-evaluated. The analyzed data was the one from the E6 at
Torp collected during July 1993 to August 1994, see Figure 4.1. In that measure-
ment series vehicle data was collected once a month under 14 measurement periods.
These 14 different measurement periods were denoted by E1, E2, E3, . . . , and E14,
respectively. A primary investigation of the WIM data showed that among the col-
lected data there were vehicles that were registered with unreasonable lengths and/or
weights. A closer study revealed that a total of approximately 10 % of the entire
collected data was unreasonable and should be excluded before further analysis of
the data. About 10 %, of the entire collected data came from vehicles registered
with one axle. Furthermore, the characteristic traffic load values for different loaded

<table>
<thead>
<tr>
<th>Road</th>
<th>Test site (Series)</th>
<th>Measurement period</th>
</tr>
</thead>
<tbody>
<tr>
<td>E4</td>
<td>Sprängviken (B1-B6,C)</td>
<td>Nov. 1991-April 1992</td>
</tr>
<tr>
<td>E6</td>
<td>Torp (E1-E14)</td>
<td>July 1993-Aug. 1994</td>
</tr>
</tbody>
</table>

Figure 4.1: The measurement locations and periods.
### Table 4.1: Data registered for each vehicle during the WIM measurements. After unpacking of raw-data and converting it into EXCEL format.

<table>
<thead>
<tr>
<th>Veh. No.</th>
<th>Date</th>
<th>Time</th>
<th>L</th>
<th>D</th>
<th>Gap</th>
<th>Hdw</th>
<th>Ax</th>
<th>Spd</th>
<th>Ax wts 1</th>
<th>Ax wts 2</th>
<th>Ax wts 3</th>
<th>Ax wts 4</th>
<th>Ax wts 5</th>
<th>Ax wts 6</th>
<th>Weight (kg)</th>
<th>Leng. (cm)</th>
<th>Wbase (cm)</th>
<th>Cl</th>
<th>R.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8-Aug-94</td>
<td>6:25:23</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>2</td>
<td>82</td>
<td>1070</td>
<td>880</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1950</td>
<td>386</td>
<td>263</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>8-Aug-94</td>
<td>6:25:25</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1.4</td>
<td>2</td>
<td>80</td>
<td>1280</td>
<td>930</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2200</td>
<td>433</td>
<td>268</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>8-Aug-94</td>
<td>6:25:38</td>
<td>1</td>
<td>0</td>
<td>12.7</td>
<td>13.1</td>
<td>2</td>
<td>74</td>
<td>1000</td>
<td>880</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1880</td>
<td>428</td>
<td>289</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>8-Aug-94</td>
<td>6:25:42</td>
<td>1</td>
<td>0</td>
<td>3.6</td>
<td>4.1</td>
<td>2</td>
<td>91</td>
<td>1010</td>
<td>1080</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>620</td>
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<td>1290</td>
<td>367</td>
<td>238</td>
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</tbody>
</table>
lengths were calculated using vehicle data both before and after filtration of unreasonable data and compared with one another. The loaded lengths were coupled to the loaded lengths for two large bridges—the Höga Kusten Bridge and the Uddevalla Bridge that were to be built. Summary and results of the licentiate work will be presented in the following sections.

### 4.3 Filtration of Unreasonable Data

#### 4.3.1 General

In the work, the accuracy of data collected on the highway E6 at Torp was investigated. The result showed that the WIM data contained many unreliable values. Some of the most obvious errors among the registered data were the following.

- Vehicles that were registered with "0" axle and "0" weight having a length up to approx. 30 meters.
- Vehicles that were registered with only one axle. In most cases these vehicles weighed up to 11 tonnes and were up to 25 meters long.
- Vehicles that were registered with unreasonable length and weight combinations, i.e. vehicles with long length having low weight and vice versa. To illustrate this, the smallest length in combination with the heaviest weight obtained in measurement periods E1 and E3 are shown in Table 4.2.

<table>
<thead>
<tr>
<th>E1</th>
<th>E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length [m]</td>
<td>Weight [ton]</td>
</tr>
<tr>
<td>3.5</td>
<td>67.5</td>
</tr>
<tr>
<td>4.6</td>
<td>81.3</td>
</tr>
</tbody>
</table>

Of course, the above mentioned measurement errors influence the results that were obtained by forming vehicle queues from the collected data, see [54]. Because there were many vehicles that were registered with zero-weights and long lengths, as well as vehicles with unreasonably long lengths, the queue weights in [54] become lower (unfiltered data) than those of the actual vehicle queues (filtered data). This will be discussed further at a later stage of this chapter.

An algorithm was developed, using the Matlab language [34–36], for the filtration of the data, which was considered to be highly unreasonable. First, the registered vehicles were grouped in to different vehicle types with respect to their number of axles and their lengths. Then, each vehicle group was studied separately. The filtration were done by comparing data for the vehicle gross weights and total lengths as well as the weight of each axle with the corresponding maximum allowable values.
according to the regulations of the Swedish National Road Administration, see [64, 67]. The result from such an investigation of data from the measurement period E1 is given hereafter.

### 4.3.2 Vehicles with One Axle

During the measurement period E1, there were 2495 vehicles (9 % of the total) registered with only one axle. The heaviest registered axle weight was 10.3 tonnes, see Figure 4.2. According to the Swedish National Road Administration [64, 67], the maximum allowable axle weight is 10 tonnes for driving axle and 11.5 tonnes for non-driving axle. If we accept all these one axle vehicles without attempting to explain them, Figure 4.2 shows that all the data was acceptable with respect to the allowable axle weights. However, if we look at the vehicles length there were vehicles that had a length which was even greater than the maximum allowable vehicle total length. It is reasonable to believe that these axles could have been a part of, for example, articulated lorries. It is also possible, because of road surface irregularities, that some axles could have jumped up just at the position of the sensors and hence were not registered. Although it is not believable that there exist vehicles that are 12 meters long and have only one axle, all recorded vehicle data with lengths that did not exceed 12 meters were accepted. According to this assumption, from this group, only 43 vehicle data (i.e. 1.7 %) were excluded from further evaluation.

![Figure 4.2: A) Weight and B) length for vehicles registered with one axle sorted in ascending weight respectively length.](image)

### 4.3.3 Vehicles with Two Axles

In total, 21817 vehicles (78 % of the total) registered with two axles with gross weight between 0.65 tonnes and 25.3 tonnes, see Figure 4.3. This weight variation was due to the fact that this vehicle category includes both cars (or other similar...
light vehicles) and heavy trucks. In this case, the vehicles were re-grouped into two groups, namely cars and trucks including buses, according to their gross weights. Further, the weight and length of each vehicle was investigated and compared with the corresponding maximum allowable values according to the Swedish National Road Administration. There were 17873 vehicles (81.9 % of the vehicles registered with two axles) that weighed less than 2.0 tonnes of which 16985 (95.0 %) had lengths that were less than 5.0 meters, see Figure 4.4A. These vehicles were assumed to be cars. 3933 vehicles (18.0 %) weighed in the range 2.0–18.0 tonnes and were assumed to be trucks or buses. The smallest observed vehicle length in the last mentioned group was 2.72 meters and weighed 2.25 tonnes, see Figure 4.4B.

The investigation of the above data resulted in that approximately 95 % of the entire vehicle data from this group were acceptable and kept for further evaluations.
4.3. FILTERATION OF UNREASONABLE DATA

4.3.4 Vehicles with Three Axles

1273 vehicles (4.6 % of the whole) were registered with three axles with weights between 1.73 tonnes (7.86 meters) and 26.95 tonnes (17.9 meters) and with the lengths between 2.03 meters (3.1 tonnes) and 25.54 meters (5.13 tonnes). Vehicles with three axles could of course include cars with trailers, trucks, buses and articulated bus. In this case it was the trucks and articulated buses which were of interest in determining the upper limits of the weights respectively the length of this vehicle group. Therefore, the maximum allowable vehicle gross weights and total lengths for the last mentioned vehicles were used to investigate the accuracy of the collected data, see Figure 4.5. In this case, the data investigation resulted in approximately 99 % of the entire number of vehicles registered with three axles were accepted and kept for further evaluations.

4.3.5 Vehicle with Four Axles

In total, 382 vehicles (1.4 % of the total) were registered with four axles. One vehicle was registered with 0 weight and 0 length. The rest were registered with the weights between 2.1 tonnes (4.13 meters) and 42.54 tonnes (15 meters). The largest observed length was 24.74 meters (weighed 24.54 tonnes), see Figure 4.6. According to [67] the maximum allowable weight for a lorry and a trailer with four axles are 32 tonnes respectively 36 tonnes. Since vehicles with four axles could be trucks with trailers having two axles each, the last mentioned limits could not be used to filter unreasonable vehicle data. Therefore, the filtration was performed by investigating each vehicle length and axle weight. The axle weights were obtained by distributing the gross weight of the vehicle to respective axles with the proportion according to the standard in [67]. Then, if the distance between the first and last axle is less than...
The shortest dist. between the first and the last axle

Figure 4.6: A) Weight and B) length for vehicles registered with four axles sorted in ascending weight respectively length.

3 meters or if the axle weights do not meet the allowable limits, the vehicle data was considered to be unreasonable and was excluded.

As a result of the investigation, approximately 96% of the entire vehicle data from this group was acceptable and kept for further evaluations.

4.3.6 Vehicles with Five or more Axles

Since vehicles of this type could have been both individual lorries and lorries with a trailer, all vehicles that were registered with weights less than 60 tonnes were kept. This weight limit had been increased somewhat with regard to some illegal overloading of trucks or vehicles with special permission. The maximum and minimum weights and lengths of vehicles registered with five to nine axles are shown in Table 4.3. For the filtration of unreasonable data of vehicles in this group, the weights of each axle of each vehicle and the distance between them was investigated. As in the last case, the axle weights were obtained by distributing the gross weight of the vehicle to respective axles with the proportion according to the standard in [67]. Among the 331 vehicles that were registered with five axles, 4 vehicles had total lengths which were less than 4 meters (4×1 meter, i.e. 1.0 meter between each axles) and therefore were excluded. All 71 vehicles registered with seven axles had lengths greater than 6 meters (6×1 meter) and were kept. The smallest vehicle were 9.5 meters long and weighed 6.84 tonnes. Among these vehicles, two weighed greater than 60 tonnes, one 62.03 tonnes and were 22.62 meters long and another weighed 63.38 tonnes and were 22.53 meters long. There was one vehicle which was registered with eight axles and one with nine axles with reasonable weights and lengths, see Table 4.3.
Table 4.3: The maximum and minimum weights and lengths of vehicles registered with five to nine axles.

<table>
<thead>
<tr>
<th>Number of axles</th>
<th>Number of vehicles</th>
<th>Gross weight [ton]</th>
<th>Total length [m]</th>
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<td>Max</td>
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<td>29.4</td>
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4.3.7 Vehicles Registered with "0" Axle

Totally, 1306 vehicles (approx. 5% of the total) were registered with "zero" axle and with lengths between 0 and 22.84 meters, see Figure 4.7A. These vehicle data were obviously unreasonable and excluded from the data. The distribution of vehicle lengths registered with "zero" axle during the entire measurement series is illustrated in Figure 4.7B. These vehicles were registered with "zero" weight.

Figure 4.7: A) Length for vehicles registered with "zero" axle sorted in ascending length. B) Histogram showing the distribution of vehicle lengths registered with "zero" axle taken from the entire measurement series.

4.3.8 Result of Filtration of Unreasonable Data for the Entire Measurement Series

About 91.5% of the total vehicle data collected under the measurement period E1 were assumed to be reasonable and accepted for further evaluation. In a similar manner as for E1, the collected data during the rest of the measurement periods, i.e. E2 to E14, were investigated. Table 4.4 shows the result of the filtration of
Table 4.4: The results of filtration of unreasonable data for all measurement periods. Et indicate the values for the entire measurement series.

<table>
<thead>
<tr>
<th>Measurement period</th>
<th>No. of registered veh. data</th>
<th>No. of veh. data after filtration</th>
<th>Percentage of unreasonable veh. data, [%]</th>
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<td>51299</td>
<td>7.9</td>
</tr>
<tr>
<td>E13</td>
<td>69620</td>
<td>65565</td>
<td>5.8</td>
</tr>
<tr>
<td>E14</td>
<td>69505</td>
<td>64143</td>
<td>7.7</td>
</tr>
<tr>
<td>Et</td>
<td>760727</td>
<td>688813</td>
<td>9.6</td>
</tr>
</tbody>
</table>

unreasonable data for all measurement periods. During the filtration, the vehicles or axles with maximum weights that slightly exceed a given upper limits were closely investigated. Vehicles or axles which were assumed to be registered with reasonable weights were kept in order to take into account some over-loaded vehicles. This was done automatically in the developed Matlab algorithm.

4.4 Data Analysis

4.4.1 General

As explained earlier, the results of the traffic load measurements are intended to be used for the design of bridges when the loaded length is large, i.e. above 200 meters. Therefore, the characteristic load value was assumed to be determined by the occurrence of a closed-packed queue of stationery vehicles, see [50]. Thus, queues with a length of 250 and 450 meters and with a free distance of 2 meters between each vehicle were formed from the collected data, both before and after filtration of unreasonable data. It should be noted here that, in real vehicle queue formations, the distance between different types of vehicles varies. Nevertheless, a careful investigation discussed in [1] shows that it is reasonable to assume that this distance on average is 2 meters. In this way a number of queues were formed which successively follow each other, as shown in Figure 4.8. Furthermore, the weight per unit length for each queue, for different queue lengths, was calculated, from which the
4.4. DATA ANALYSIS

Queue 1, $t_1$

Queue 2, $t_2$

etc.

Figure 4.8: The formation of traffic queues with 2 meters free distances between vehicles.

Probability distribution functions for the queue weights were obtained. A Matlab algorithm was developed to determine the probability distribution functions for the queue weights. The computation procedure for this algorithm was according to the following.

For each vehicle the following parameters were calculated [50]:

- the required space
  \[ a_i = d_i + 2 \text{ meters} \]  (4.1)
  where $d_i$ was the total length of vehicle number $i$

- uniformly distributed load
  \[ q_i = Q_i / a_i \]  (4.2)
  where $Q_i$ was the total weight of vehicle number $i$.

For each vehicle queue the following were calculated:

- the number of vehicles, $n$, so that
  \[ \sum_{i=1}^{n-1} a_i \leq L_0 \leq \sum_{i=1}^{n} a_i \]  (4.3)
  where $L_0$ was a priori chosen queue length.

- the actual queue length
  \[ \ell = \sum_{i=1}^{n} a_i \]  (4.4)

- the total weight of all vehicles in a queue
  \[ W = \sum_{i=1}^{n} Q_i \]  (4.5)

- the average load intensity
  \[ \mu_q = \frac{1}{\ell} \sum_{i=1}^{n} q_i a_i = \frac{W}{\ell} \]  (4.6)
- the average vehicle weight
\[ \mu_Q = \frac{1}{n} \sum_{i=1}^{n} Q_i \]  
(4.7)

- standard deviation, \( \sigma_q \), for \( q_i \)
\[ \sigma^2_q = \frac{1}{\ell} \sum_{i=1}^{n} (q_i - \mu_q)^2 a_i \]  
(4.8)

- standard deviation, \( \sigma_Q \), for \( Q_i \)
\[ \sigma^2_Q = \frac{1}{n} \sum_{i=1}^{n} (Q_i - \mu_Q)^2 \]  
(4.9)

- the correlation \( \rho \)
\[ \rho(\nu) = \frac{1}{\sigma^2_Q} \frac{1}{n - \nu} \sum_{i=1}^{n-\nu} (Q_i - \mu_Q)(Q_{i+\nu} - \mu_Q) \quad \nu = 1, 2, 3, \ldots \]  
(4.10)

4.4.2 Effect of Filtration of Unreasonable Data

The number of vehicle queues, the mean values and the standard deviations for queue weights for different values of \( \ell \) were calculated according to the procedure described in the previous section. It was obvious that the number of vehicle queues would decrease as the consequence of the reduction of data, which was the result of the filtration of unreasonable values. Apparently, the queue weights became greater after the filtration because of most of the excluded data were either vehicles that were registered with "zero" weights or vehicles with unreasonably large lengths. Consequently, the mean values for the queue weights became greater after the filtration of unreasonable data (approximate increase of 5 to 10 %). The values for the standard deviations after the filtration were insignificantly smaller that the corresponding values before the filtration. As an example, Figure 4.9 illustrates a comparison of the mean values for queue weights, for \( \ell = 250 \) meters.

4.4.3 Distribution of Queue Weights

The queue weight distributions for different queue lengths were calculated according to the procedure described in section 4.4.1 on page 56. For each value of \( \ell \), fourteen histograms were obtained, from the fourteen measurement periods. The class interval for the histograms was chosen to be 200 kN. Through an addition of the frequency values of those fourteen histograms, the distribution for the whole measurement series could be obtained. An example of such a distribution, for \( \ell = 250 \) meters is illustrated in Figure 4.10. By normalizing the frequency values, i.e. by adjusting them so that their sum becomes one, the frequency functions could be obtained which of course could give the probability distribution function for queue weights. Since the queue weights were obtained by the summation of each vehicle’s weight in a queue, one would expect that the queue weights should be normally distributed.
4.4. DATA ANALYSIS

Figure 4.9: The influence of unreasonable data filtration on the mean value of the queue weights, $\ell = 250$ meters.

Figure 4.10: Histogram for the queue weights calculated using data from the whole measurement series. Queue length 250 meters.
4.4.4 Periodical Variation of the Queue Weights

The primary evaluation of the measured data showed that the population of the vehicles, which pass the test site, was not homogeneous. The proportion of heavy vehicles were much greater at night than during the day. Consequently, the queue weights calculated using data collected during the night were generally much greater than the corresponding values calculated using data collected during the day. This implied that a simple normal distribution could not be assumed to describe the distribution of the queue weights. Therefore, the variations of the traffic flow with respect to time is considered by dividing the population of the queue weights into two populations—the normal (daytime) and the heavy (nighttime) queue weights, see Figure 4.11.

For these two populations the parameters shown in Table 4.5 could then be calculated.

Table 4.5: The proportion, the mean value and the standard deviation for each population.

<table>
<thead>
<tr>
<th></th>
<th>Population one</th>
<th>Population two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>1-p</td>
<td>p</td>
</tr>
<tr>
<td>Mean value</td>
<td>μ′</td>
<td>μ″</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>σ′</td>
<td>σ″</td>
</tr>
</tbody>
</table>

4.4.5 Probability Distribution Functions of the Queue Weight

From the mean values and standard deviations of the two populations the mean value and the standard deviation for the total population could be obtained as in [50]

\[
\mu = (1-p)\mu' + p\mu''
\]

(4.11)
\[ \sigma^2 = (1 - p)\sigma'^2 + p\sigma'^2 + (1 - p)(\mu - \mu')^2 + p(\mu - \mu'')^2 \] (4.12)

\[ \sigma^2 = (1 - p)\sigma'^2 + p\sigma'^2 + \frac{p}{1 - p}(\mu - \mu'')^2 \] (4.13)

\(\sigma'\) and \(\sigma''\) were assumed to be proportional to \(1/\sqrt{\ell}\), see [50]. Equation (4.13) could then be written as

\[ \sigma^2 = A + \frac{B}{\ell} \] (4.14)

where

\[ A = \frac{p}{1 - p}(\mu - \mu'')^2 \] (4.15)

\[ \frac{B}{\ell} = (1 - p)\sigma'^2 + p\sigma'^2 \] (4.16)

From (4.11) and (4.15), the following could be obtained

\[ \mu' = \mu - \sqrt{\frac{p}{1 - p}A} \] (4.17)

\[ \mu'' = \mu + \sqrt{\frac{p}{1 - p}A} \] (4.18)

With the values \(\mu', \mu'', \sigma', \sigma''\) and \(p\) calculated and with the assumption of normal distributed for the two populations, the probability distribution for the queue weights for the entire population could be written as

\[ F_W(w) = p\Phi\left(\frac{w - \mu'}{\sigma'}\right) + (1 - p)\Phi\left(\frac{w - \mu''}{\sigma''}\right) \] (4.19)

where \(\Phi(\cdot)\) is the normal probability distribution function. From now on, the indexes of the probability density as well as distribution functions will be excluded and will be written as \(f(w)\) and \(F(w)\) rather than \(f_W(w)\) and \(F_W(w)\).

Further, to consider the type of structural system of a bridge, the \(\sigma'\) and \(\sigma''\) values were multiplied by the factor \(\kappa\) from the three different types of influence functions according to [50]. These three types of influence functions with the corresponding values of \(\kappa\) are shown in Figure 4.12.

![Figure 4.12: Different types of influence functions, [50].](image)

As an example, Figure 4.13A illustrates the probability density function according to (4.19) fitted to the histogram for queue weight with \(\ell = 400\) meters. The distribution
was determined from vehicle data after filtration of unreasonable values. These two distributions are also plotted in normal probability paper and are shown in Figure 4.13B. As can be seen in this figure, the pdf agrees well with the distribution of the queue weight especially for higher values, i.e. on the right tail of the histogram.

Also seen in Figure 4.13A is the log-normal pdf fitted to the histogram of the queue weights. The log-normal pdf agrees very well with the distribution of the queue weight. However, since the queue weights are the sum of weights of individual vehicles, and these weights can be assumed to be independent, the distribution for the queue weight should be close to a normal distribution, according to the Central Limit Theory. The assumption for this is that the statistical variables do not vary with time. This however is not fulfilled for the data used to determine the distribution of the queue weights. The proportion of heavy vehicles varies with the time of day. However, if the WIM data collected during day and during night would be analyzed separately, the queue weights in both cases would be normally distributed. Therefore, the queue weights should be modeled by a normal distribution [52].

4.4.6 Results from the Analysis of Collected Data

The load values obtained from the measurement were compared with the specified traffic load according to the Swedish bridge design code, Bro94 [65]. This code specifies a loading type 5 which is intended to be used for bridges with spans larger than 200 meters. The load system for the lane which is assumed to be heavily loaded is shown in Figure 4.14. It is considered as a characteristic load.
Figure 4.14: The load system according to Bro94, [65], for bridges with spans larger than 200 meters.

According to different codes the characteristic value for a load is determined as the 98th percentile of the probability distribution function for the annual maximum value of the load, see section 2.7 on page 16. Therefore, the characteristics value for the queue weights for different values of \( \ell \), before and after filtration of unreasonable data was calculated as (4.20), cf. (2.25) on page 17. These values were then compared with the corresponding characteristic load values \( w_e \) according to the Swedish code.

\[
  w_k = F^{-1}(0.98^{1/N})
\]

where \( F^{-1} \) is the inverse function of (4.19) on page 61 and \( N \) is the average number of vehicle queues that is assumed to occur per year.

Thus, for \( N = 1 \) and \( N = 5 \), the characteristic load values were calculated as \( F^{-1}(0.98) \) respectively \( F^{-1}(0.996) \). The results for different values of \( \ell \) and for the three different influence lines are presented in Figure 4.15–4.17. In all cases the values of \( w_k \) after the filtration of unreasonable data were approximately 3.5\% higher than the corresponding values before the filtration. There is a reason to believe that during the filtration mostly light vehicles with long lengths as well as heavy vehicles with short lengths were excluded, whereby their effects on the vehicle queue weights cancelled each other out.

The quota between of \( w_e \) and \( w_k \), after the filtration, is also illustrated in the figures. The results showed that, for \( N = 1 \) and \( \ell = 100 \) meters the values from the code were twice as high as the value from the measurements. The corresponding value for \( N = 5 \) and \( \ell = 100 \) meters was approximately 1.7. The value of \( w_e/w_k \) decreased with increasing \( \ell \) and became 1.5 for \( N = 1 \) and 1.3 for \( N = 5 \), for \( \ell = 1000 \) meters. These relations were approximately the same for all three influence line types.
CHAPTER 4. TRAFFIC LOAD MODELS FOR LONG-SPAN BRIDGES

Figure 4.15: A comparison of $w_k$ before and after filtration of unreasonable data as well as the values $w_e$ according to the code. Influence line type A. In the indexes the letters a and b indicate values after and before filtration, respectively.

Figure 4.16: A comparison of $w_k$ before and after filtration of unreasonable data as well as the values $w_e$ according to the code. Influence line type B. In the indexes the letters a and b indicate values after and before filtration, respectively.
4.5 Monte Carlo Simulations

4.5.1 General

In recent years, advanced Weigh-In-Motion (WIM) technology has been used in many countries to collect traffic load data. These measurements are conducted in order to get a reliable background for the development of traffic load models for bridge design codes. Because of the variation of traffic with respect to time, these measurements have usually been performed continuously for long periods of time in order to predict the actual traffic loads and traffic composition. Therefore, performing this kind of measurement is not only time-consuming but also very expensive. Today’s computers provide us the opportunity of making millions of simulations within just a few minutes. Therefore, it was believed that vehicle data, based on shorter WIM measurement series, could easily be generated that provide good agreement with results from long term of WIM measurements. Thus, one of the main objectives of this work was to generate site-specific vehicle data using the Monte Carlo simulation technique.
4.5.2 Variation of Traffic Flow

As explained earlier, the population of vehicles that pass the test location, which is on E6 at Torp, was not homogeneous. This phenomenon can be seen in Figure 4.18 where the mean value of the number of axles of vehicles that pass the test location during the whole measurement period is plotted versus the time of passage. The continuous line, in the figure, shows the variation of the mean value of the number of axles for 528522 vehicles that were registered during weekdays. Whereas, the dashed line shows similar mean value variation as above for 137833 vehicles which were registered during holidays. It was concluded that the proportion of heavy vehicles was usually much greater during the night than during the day and greater during the week than at weekends and during holidays.

![Figure 4.18: The variation of mean value of the number of axles of vehicles that pass the test location versus the time of passage. The circles represent the calculated points.](image)

4.5.3 Vehicle Weight Distributions during Different Measurement Periods

For the MC simulations, it was appropriate to classify the vehicles into three different groups. This classification was done in order to simplify the generation of vehicle data, as will be discussed in detail later. The first group included passenger cars including vans and was denoted by Gr1. The second group included rigid lorries with two to four axles including buses and was denoted by Gr2, and the last group included articulated lorries and lorries with trailer and was denoted by Gr3, see Table 6.1 on page 86. As will be explained in detail later, the collected data during one of the measurement periods was chosen as being representative for all the data and was thereafter used for the MC simulation. Therefore, it was necessary to study the gross vehicle weight distributions of especially Gr2 and Gr3 in different measurement periods in order to investigate how these distributions vary in different periods. The multi-modal histograms in Figure 4.19 show the gross vehicle weight distributions of Gr2 and Gr3 for the measurement period E3, which was randomly chosen for this illustration. Because of the limited quality of the WIM data, it
was not possible to classify the observed vehicles into more than three classes. As a consequence, according to the vehicle classification described above, some of the observed vehicles might have been grouped into a wrong class. For example, because private cars with caravans were registered as vehicles with three axles, they were grouped into Gr2. With newer WIM data, however, it may be possible to make a more accurate vehicle classifications that may lead to better simulation results.

The weight distributions for the other measurement periods were very similar to those shown in Figure 4.19 with insignificant variation in mean values and standard deviations for vehicle gross weight. These differences in mean values and standard deviations, for all periods, are shown in Figure 4.20. The values for the measurement period E1 were usually small compared to the others. The reason for this is most probably because during this period only a small amount of vehicle data had been collected compared with the other periods. As mentioned above, with the exception of E1, the difference in mean value and standard deviation were insignificant.

The author believes that it was the same vehicles that used this part of road month after month, most probably with a different composition of vehicles. Thus, the weight distributions of different types of vehicles in different periods should be equivalent and the results shown above are not so surprising. As a result, the vehicle data that was collected during period E6 in combination with the actual traffic flow were chosen to be used for the MC simulation. The reason why this period had been chosen had to do with the measurement errors that the WIM data contained, see Table 4.4 on page 56. The percentage of unreasonable data in the measurement period E6 was approximately the average of all the series combined.
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4.5.4 Generation of Vehicle Data

For the determination of the characteristic traffic load values for long-span bridges, the influence of the weight of private cars and vans are negligible. Therefore the weights of all vehicles in Gr1 were simply put equal to 1 ton and only the total lengths of these vehicles were simulated. For Gr2 and Gr3, both vehicle gross weights and total lengths were simulated and the correlation between vehicle weight and length was compared with the corresponding values calculated from the WIM data.

As mentioned in the previous section, WIM data from period E6 in combination with the actual traffic flow were used for the MC simulation. Figure 4.21–4.22 illustrate the probability distribution functions of the WIM and simulated vehicle gross weight and total length for Gr2 and Gr3, respectively. There were 51257 vehicles of Gr2 type and 40299 of Gr3 type observed and the same number of vehicles for each class were simulated. As seen in the figure, in all cases the MC simulated vehicle data agree very well with the WIM data. In addition, the correlation coefficient, between vehicle gross weight and total length, for both collected and simulated vehicle data were compared. For Gr2 the correlation coefficients were calculated to 0.52 for the WIM data and 0.58 for simulated vehicles. In the case of Gr3 these values were 0.22 and 0.20, respectively. For a new simulation, with the same amount of vehicles as above, the correlation coefficients were 0.54 for Gr2 and 0.21 for Gr3. These results indicated that it was reasonable to assume that the MC simulation generates vehicle data with the same correlation as the WIM data.
4.5. MONTE CARLO SIMULATIONS

Figure 4.21: Probability distribution function comparison for measured and simulated vehicle data for Gr2. The left figure shows vehicle gross weight cdf and the right one vehicle total length cdf.

Figure 4.22: Probability distribution functions comparison for measured and simulated vehicle data for Gr3. The left figure shows vehicle gross weight cdf and the right one vehicle total length cdf.

4.5.5 Results from the Analysis of MC Simulated Data

According to the traffic variation illustrated in Figure 4.18 on page 66, 3.5 million vehicle data were simulated using the WIM data collected during period E6. Afterwards, and in the same way as described in section 4.4 on page 56, vehicle queues with a length of 200 to 1800 meters and with the free distance of 2 meters between each vehicle were formed, from both the recorded WIM data and the generated vehicle data. Furthermore, the weight per unit length for each queue was calculated, from which the probability distribution function for the queue weight was obtained. As an example, Figure 4.23 illustrates in the form of histograms and probability
Figure 4.23: An example of a comparison between queue weight distributions calculated using WIM data and MC generated vehicle data. For 400 meters queue length. A) Histogram B) Probability distribution function.

distribution functions, the comparison between queue weight distributions obtained by using the recorded and simulated vehicle data. The figure shows the results calculated for queue lengths of 400 meters.

In order to compare the results in the form of histogram the number of queue weights obtained using the MC simulated vehicle data was reduced so that it had the same number of queue weights calculated using the WIM data. The reduction was done randomly. The queue weight distributions calculated using the MC simulated and the collected vehicle data had very good agreement especially for the larger queue weight values. As is illustrated in the histogram, for very small queue weights, the MC simulation gave higher frequency values than the WIM data. This was the result of the assumption that the weight of vehicles in Gr1 was set equal to 1 ton.

High queue weight values are the most important in the determination of the characteristic traffic load values. Therefore, the 98th and 99.96th percentile from the distribution functions of the queue weight, calculated using the simulated vehicle data and WIM data, were determined in order to study the agreement of the queue weight values at the higher levels. It should be noted that there was no probabilistic model applied to the queue weight distributions in the determination of the above values. The obtained queue weights were simply sorted into ascending order and the values that correspond to the desired percentiles were picked. The resulting values are shown in Figure 4.24. As illustrated in the figure, both the MC generated vehicle data and WIM data gave nearly equal queue weight values at the investigated levels. One should bear in mind that it is possible that in some cases an individual value may have diverged from the queue weight distribution and could give either higher or smaller value than expected. This implies that the values shown in the figure do not represent the whole queue weight distributions, as a consequence of no probabilistic model being applied to the distributions. However, using these values for the comparison discussed above was believed to be reasonable.
4.6 Results and Discussion

The investigation conducted in this chapter showed that among the collected data there were vehicles that were registered with unreasonable lengths and/or weights. Totally about 10% of the total collected data were considered to be unreasonable and were excluded before further analysis. About 10%, of the total collected data, were vehicle data registered with one axle. Because of the limited information, these vehicles could not be identified and therefore were not excluded.

The characteristic traffic load value for long-span bridges was assumed to be determined by the occurrence of a closed-packed queue of stationery vehicles. Therefore, vehicle queues with different lengths were formed from the collected data, both before and after filtration, from which the characteristic load value could be obtained. The result showed that the influence of unreasonable data was insignificant. The largest difference between the characteristic load values calculated before and after filtration of unreasonable data was 3.5%. Comparing the characteristic traffic load values with corresponding values according to Bro94 showed that the code values were considerably higher than the actual values.

Also the Monte Carlo simulation technique was used to generate fictitious vehicle data. It has been demonstrated that by using the MC simulation, with a limited amount of WIM data, it was possible to generate vehicle data that could represent the actual site-specific vehicle data. Because of the limited quality of the WIM data used in this work, the vehicle classification made for the MC simulation was rather crude. Despite this, the obtained results from the vehicle data generation were very satisfactory. It is believed that by using newer more accurate WIM data, better simulation results can be obtained. Therefore, it is suggested that future vehicle data collection should be done systematically under relatively shorter periods. Then, the measurement results can be used to generate fictitious data using MC simulation technique.
Furthermore, a comparison of the probability distribution functions calculated using the simulated and the collected vehicle data were done. The queue weight distribution obtained using the MC simulated and the WIM data agreed very well especially for the large queue weight values. Obviously, it is the larger queue weights that are of most important for the determination of the characteristic traffic load values.
Chapter 5

Field Measurements of the Transverse Distributions of Vehicles on Bridges

5.1 General

The purpose of this part of the thesis was to study the transverse distribution of traffic loads and their effects on bridges. In order to achieve this, data was collected that measures the transverse distribution of vehicles, on two bridges in the Stockholm area. The first and the main data accumulation was performed on the highway E4 south-west of Stockholm, 400 meters after the turn-off for Järna. This highway has two lanes in each direction, separated by a central median. The second set of data collection was performed on a road just before Muskö, south of Stockholm. This road has two lanes of traffic, one in each direction. Both measurements were performed using the same procedure. The data collection at the later test site was performed over three days, the 4th, the 11th and 12th of December 2001. The temperature during these days was between -15 and -20 Celsius, which of course influence the traffic flow condition. Because this measurement was performed under a relatively short period and done on a single month, it is planned to be complemented at a later date. The evaluation of data from this measurement is intended to be presented separately in a report. The procedure of the data recording, as well as the results of the first measurement, are presented in this chapter. For the second measurement series, an overview of the test site, as well as the results from the measurement are shown respectively in Figure B.1 and Figure B.2 on page 138. Finally in this chapter, a method for the investigation of the girder distribution factor, using the collected data, for medium and short span slab-on-girder bridges is presented.
5.2 Vehicle Data Collection

As mentioned previously, data recording of the transverse distribution of vehicles, on the highway E4 south-west of Stockholm, was performed. Figure 5.1 gives an overview of this test site. The highway is a dual carriageway with two traffic lanes in each direction. These lanes are each 3.6 meters wide. The highway also contains two verges, with a width of 2.73 meters on the outer, and 1.5 meters on the inner side of the freeway respectively, see Figure 5.2. Two independent bridges carry the traffic on this site, one for each traffic direction. The measurements were performed for the south-going traffic, i.e. the traffic that leaves Stockholm.

A system called Metor was used for the data collection. This system is used by the Swedish National Road Administration (Vägverket) to collect traffic data at a number of locations in Sweden. This apparatus is developed by Allogg [61,63]. The procedure of the data collection, which is simple but very effective, is now described. Three rubber tubes were installed on the road which send signals to Metor. Two of the rubber tubes were installed parallel to each other but perpendicular to the traffic direction, and were 3.30 meters apart. The third tube was placed diagonally according to Figure 5.2. As shown in the figure Tube A, Tube B and Tube C were installed respectively along the traffic direction. In order to make the measurement procedure easier, the tubes were placed just before the bridge. The shortest distance from Tube C to the bridge was 4.6 meters. The width of both the lanes and the verges of the bridge are equal to the respective widths at the test site. Therefore, it is reasonable to assume that the transverse distribution of the traffic on the bridge is equivalent to that of the test position. Finally, when the data accumulation started,
5.2. VEHICLE DATA COLLECTION

Figure 5.2: Arrangement of tubes for data collection. The dimensions are given in meter.

the points-in-time when each wheel of every vehicle crossed each tube were registered and saved in Metor. Then, using this data, the lateral position of each vehicle on the road was calculated. Moreover, the accuracy of the registered data was investigated by checking, for each collected vehicle data, the validity of

\[
\begin{align*}
& t_{C,\text{front}} - t_{A,\text{front}} \approx t_{C,\text{rear}} - t_{A,\text{rear}} \\
& t_{C,\text{front}} - t_{B,\text{front}} \approx t_{C,\text{rear}} - t_{B,\text{rear}}
\end{align*}
\]

where \( t_{A,\text{front}} , t_{B,\text{front}} \) and \( t_{C,\text{front}} \) are the points-in-time when the front wheels of a vehicle cross Tube A, B and C respectively. Similarly, \( t_{A,\text{rear}} , t_{B,\text{rear}} \) and \( t_{C,\text{rear}} \) are points-in-time when the rear wheels of a vehicle cross Tube A, B and C respectively. In addition, before the first measurement period, see Table 5.1, data for the transverse position of a small number of vehicles was noted using simultaneously a video camera and the tubes. The analysis of these two recordings resulted in good agreement between the transverse positions calculated by the tubes and by the camera.

For each vehicle, using the points-in-time data from Tube A and B, the velocity, number of axles and axle spacing can be calculated. Hence the time between Tube B and C can be transferred to distance, the transverse position of the vehicle can be calculated using the points-in-time data from Tube B and C. Metor sorted the vehicle data into 15 different vehicle classes, shown in Table A.1 on page 134, according to the numbers and spacing of their axles. For the determination of the lateral position of vehicles from the collected Metor data, a program developed, using a Matlab language, by Karoumi [25] is used.

The measurement periods, the total number of registered vehicles and the proportion of heavy vehicles are given in Table 5.1. As seen in the table, in order to cover the actual traffic composition, the measurements were performed systematically during
Table 5.1: The measurement periods, the total number of registered vehicles and the proportion of heavy vehicles.

<table>
<thead>
<tr>
<th>Measurement period</th>
<th>Total registered</th>
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<th>Trucks</th>
<th>Percentage of trucks [%]</th>
</tr>
</thead>
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</tr>
<tr>
<td>Fri. 21-09-01 14-16</td>
<td>1942</td>
<td>1803</td>
<td>139</td>
<td>7.2</td>
</tr>
<tr>
<td>Mon. 24-09-01 17-21</td>
<td>1841</td>
<td>1631</td>
<td>210</td>
<td>11.4</td>
</tr>
<tr>
<td>Wed. 26-09-01 11-15</td>
<td>1819</td>
<td>1500</td>
<td>319</td>
<td>17.5</td>
</tr>
<tr>
<td>Fri. 28-09-01 19-01</td>
<td>1821</td>
<td>1635</td>
<td>186</td>
<td>10.2</td>
</tr>
<tr>
<td>Sun. 30-09-01 18-05</td>
<td>1743</td>
<td>1530</td>
<td>213</td>
<td>12.2</td>
</tr>
<tr>
<td>Mon. 01-10-01 23-10</td>
<td>1602</td>
<td>1308</td>
<td>294</td>
<td>18.4</td>
</tr>
<tr>
<td>Total</td>
<td>26633</td>
<td>23622</td>
<td>3011</td>
<td>11.3</td>
</tr>
</tbody>
</table>

different times of the day and on different weekdays. The first six measurements were performed in 1999 by Karoumi [25], where under each period, data for an average of 8318 wheels was registered. The rest of the measurements were conducted by the author in 2001 and during each period, data for an average of 6290 wheels was registered. This difference in the number of the registered wheels was due to the fact that in the last mentioned case more information about the collected vehicles in the form of, among other things, velocity was processed. Totally, 26633 vehicles were registered, 3011 (11.3 %) of which were heavy vehicles.

5.3 Primary Analysis of the Collected Data

In total, 23622 (89 %) of the registered vehicle data were passenger cars including vans. The corresponding figures for rigid lorries with two to four axles including buses, and for articulated lorries including lorries with trailer were 1346 (5 %) and 1665 (6 %), respectively. Figure 5.3 illustrates the probability density function for the lateral distributions of the right front wheel of each vehicle from the collected data. The upper figure shows the distribution of cars including vans and the lower figure shows the distribution of lorries. Obviously, as can be seen in the figure, a large number of vehicles primarily use the right lane. Of the number of cars and vans that were registered, 71 % drove in the right lane whereas 29 % drove in the left lane. In the case of lorries 93 % drove in the right lane and the rest drove in the left lane.
Figure 5.3: Probability density function for transverse position of right front wheel from the collected data. The upper pdf shows the distribution for cars including vans and the lower pdf shows the distribution for lorries.

Figure 5.4: Distribution of the transverse position of right front wheel for different vehicle types from the collected data.
Table 5.2: The portion of cars and trucks in different lanes under different measurement periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>Percentage of cars and vans [%]</th>
<th>Percentage of trucks [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both lanes</td>
<td>Right lane</td>
</tr>
<tr>
<td>1</td>
<td>80.0</td>
<td>62.2</td>
</tr>
<tr>
<td>2</td>
<td>91.3</td>
<td>62.7</td>
</tr>
<tr>
<td>3</td>
<td>95.5</td>
<td>67.8</td>
</tr>
<tr>
<td>4</td>
<td>85.8</td>
<td>61.8</td>
</tr>
<tr>
<td>5</td>
<td>89.6</td>
<td>63.4</td>
</tr>
<tr>
<td>6</td>
<td>88.8</td>
<td>64.9</td>
</tr>
<tr>
<td>7</td>
<td>86.4</td>
<td>58.9</td>
</tr>
<tr>
<td>8</td>
<td>92.8</td>
<td>47.3</td>
</tr>
<tr>
<td>9</td>
<td>88.6</td>
<td>60.9</td>
</tr>
<tr>
<td>10</td>
<td>82.5</td>
<td>60.7</td>
</tr>
<tr>
<td>11</td>
<td>89.8</td>
<td>52.0</td>
</tr>
<tr>
<td>12</td>
<td>87.8</td>
<td>55.7</td>
</tr>
<tr>
<td>13</td>
<td>81.6</td>
<td>53.1</td>
</tr>
</tbody>
</table>

Also shown in Figure 5.4 are the lateral distributions of the right front wheel of different types of vehicles in the form of histograms. The upper histogram shows the lateral distribution of the right front wheel of passenger cars including vans. Similar distribution for rigid lorries with two to four axles including buses and for articulated lorries including lorries with trailer are shown in the middle and lower histogram respectively. An analogous conclusion as before can be made here i.e. a large amount of vehicles primarily used the right lane. Comparison of the first peaks of the histograms in the figure shows that it is the very heavy vehicles, i.e. articulated lorries and lorries with trailer, that drove closer to the verge. This result is of course not surprising.

Table 5.2 shows the percentage of cars including vans, as well as trucks, that drove on the respective lanes during the measurement.

### 5.4 Determination of Girder Distribution Factor from the Collected Data

In order to determine a characteristic value of the girder distribution factor value, it was assumed, as a basic principle, that the occurrence of a close-packed queue of stationary vehicles was the determining design case. In other word, it was assumed that the design case was determined by the occurrence of vehicles queue on a bridge. Thus, the primary evaluation of the collected data was made in the following way: The lateral positions of each vehicle were calculated from the collected data, as discussed in section 5.2 on page 74. The vehicles were moved together so that they...
Queue 1, $\ell$

Queue 2, $\ell$

Queue 3, $\ell$

etc.

Figure 5.5: The formation of vehicle queues. The distance between each vehicle is assumed to be one meter.

The mutual positions of the vehicles in the queue were the same as in the free traffic. In this way a number of vehicle queues were formed which successively follow each other and the position of each vehicle’s wheels on the bridge was determined.

The above results were used to calculate the girder distribution factor distributions for a typical slab-on-girder bridge with a cross-section as seen in Figure 5.6. The girder distribution factor, denoted here as $\wp$, is defined in this work as the proportion of the total traffic load on the bridge, with cross-sections as seen in Figure 5.6, that acts on the respective beams. The determination of the girder distribution factor was done by using the simplified model described in [59, 60]. This model implies that, for bridges having cross-sections as seen in Figure 5.6, the deck acts as a simply supported beam between the two girders, for the determination of girder distribution factors. Therefore, the part of the traffic load, from a given vehicle queue on a bridge, that acts on Beam A, see Figure 5.6, could be obtained simply by calculating the reaction force $R_A$ and by dividing this value by the total queue weight, $Q$. $R_A$ can for example be calculated as

$$R_A = \frac{1}{2aL_0} \sum_{j=1}^{m} \left[ \sum_{i=1}^{n} w_{ij} (a + B - 2x_{ij}) \right]$$  (5.2)

where $a$ is the spacing of the girders, see Figure 5.6, $\ell$ is the length of a queue, $B$ is the total width of the carriageway and $w_{ij}$ and $x_{ij}$ are the weight and the corresponding position of each wheel.

The determination of $\wp$ was done as follows: The first processing of the collected data for transverse distribution of vehicles on the bridge provided, for each vehicle, the position of the right front wheel on the road as well as the type of the vehicle according to the classification shown in Table A.1 on page 134. From these two pieces of information, the number and position of the rest of the wheels of each vehicle were simulated. For the simulation of the position of the wheels in the longitudinal direction, the values in the last mentioned tables were used. In the transverse direction, the spacing of the wheels was set to 1 meter for cars, vans as well as other similar light vehicles and 2 meters for the rest of the vehicles. Using this data, vehicle queues as described above were formed. The vehicle wheels from the queues were assumed to act as point loads on the bridge. Furthermore, it was assumed that each vehicle was fully loaded. Therefore, fully loaded cars and vans were assumed to weigh 2.0 tonnes. Hence, a weight 0.5 ton was assigned for
each wheels of cars and vans. For heavier vehicles, the maximum allowable weights for individual axles, double bogie and triple bogie axles were assigned according to the limits given by Swedish National Road Administration (Vägverket) [64,67]. These values are 10 tonnes for an individual driving axle and 11.5 tonnes for an individual non-driving axle. For the double bogie and triple bogie axles, the values are 18 tonnes and 24 tonnes respectively. These axle weights were then divided by two and assigned to the corresponding wheels.

To simplify the calculations, the determination of $R_A$, for a given vehicle queue, was done by using an influence surface. That is, an expression for the influence surface for the reaction force along Beam A, see Figure 5.6, as a function of the wheel positions of the vehicles, on the the bridge deck, was determined. This function gave the influence coefficient for any wheel load located at any position on the bridge deck. Clearly, the summation of the products of the influence coefficients and the corresponding vehicles’ wheel loads gave the reaction force, $R_A$. The determination of the distribution of $\varphi$ for different queue lengths was done by forming a number of vehicle queues from the collected data, as illustrated in Figure 5.5, for different values of $\ell$, i.e. for different desired queue lengths. Moreover, $R_A$ was calculated for each queue and divided by the corresponding value of the total queue weight, $Q$. This ratios, $R_A/Q$ and $1 - R_A/Q$ are the desired factors $\varphi$ and $1 - \varphi$, i.e. the ratio of the total vehicle queue weight acting on the respective beams. Adopting this method calculated distributions of $\varphi$ for queue lengths 15 and 20 meters are illustrated in Figure 5.7 in the form of histograms.

As illustrated in the figure, the histograms have two main peaks. Very likely, the first part of the histograms represent the lighter queue weights, which most probably are formed by cars and vans in both lanes. The second part of the histogram most probably represents heavier queue weights that are mainly formed by heavy vehicles in the right lane and light vehicles in the left lane. Figure 5.8 shows the distribution for the ratio of the reaction force $R_A$ obtained from the collected data and the corresponding value calculated using the traffic load model according Bro94 [65].
According to this result, as can clearly be seen in the figure, even though maximum allowable load was applied to each wheel, the $R_A$ values determined from the collected data were considerably smaller than the corresponding value calculated using the load model from the code. The maximum value of this quota was 0.37. This result however was not so surprising since for the calculations, the traffic load from the code had been placed right out on the side of the bridge deck (the dimensioning position), while the vehicle loads from the collected data had mostly been measured in the middle of the roadway. This matter will be discussed further at a later stage. It should also be mentioned here that the object of this work is not to compare the $R_A$ values from the collected data with the value from the code, but to study the distributions of $R_A/Q$, i.e. $\varphi$, from the measurement and compare them with the corresponding values calculated using the code load model.

![Histograms](image.png)

Figure 5.7: Distributions for the part of total queue weight that load Beam A, denoted $\varphi$, from the collected data. The upper histogram shows the result for queue length of 15 meters and the lower histogram shows the result for queue length of 25 meters.
Figure 5.8: A histogram for $R_A$ from the collected data divided by the corresponding value from the code. For a queue length of 20 meter.
Chapter 6

Lateral Traffic Load Distribution in Slab-on-Girder Bridges

6.1 General

In this chapter, two statistical methods are implemented to determine the characteristic values of the girder distribution factor, $\varphi$, for different queue lengths. The first method uses the Monte Carlo (MC) simulation technique, where fictitious vehicle data is simulated and evaluated. This method is discussed in section 2.8 on page 18. For the determination of the characteristic value of $\varphi$ in the second method, Rice’s formula discussed in section 2.9 on page 23 is applied. The implementation of the last mentioned method and the corresponding results were discussed in detail with Dr. Christian Cremona from LCPC, who is one of the leading experts in this area. The results from these two data evaluation methods are compared with each other. Finally, the results from the evaluations are compared with the corresponding values calculated using the load model from the Swedish bridge design code as well as the Eurocode.

6.2 Evaluation using the Monte Carlo Simulation Technique

It is shown in [18, 19] that fictitious data, that can represent data from field measurements, can be generated using the Monte Carlo simulation technique with a limited amount of collected vehicle data. Therefore, in this work, data for both the lateral positions of vehicles on bridges and their weights were generated using the MC simulation technique. Likewise as explained earlier, the characteristic value of $\varphi$ was assumed to be determined by the occurrence of queues of stationary vehicles. Thus, vehicle queues in a similar manner as described in section 5.4 on page 78, with different lengths, were formed using, in this case, the simulated data to determine the girder distribution factor distributions for different values of $t$. The
major purpose of the implementation of the MC simulation was to take as many as possible combinations of different vehicles into consideration in the formation of vehicle queues. Thus, the vehicle queues from the simulated data are assumed to reasonably correspond to many possible real queues that could occur during a longer period of time than the period during which data recording had been performed. This implies that the larger the amount of vehicle data generated, the better the simulated data represents an actual traffic situation. Moreover, the stability of the MC simulation was checked by comparing the distributions of $\varphi$ calculated using a different amount of simulated vehicle data. The result showed no significant difference for the distributions of $\varphi$ calculated using 1, 3.5 and 5 million simulated vehicle data. A similar investigation is discussed in [18] and hence is not presented here.

6.2.1 Data used for the Simulation of Wheel Load

As distinguished from section 5.4 on page 78, where the maximum allowable axle weights were used to assign the gross weights for the recorded vehicles, the weights for the simulated vehicles were generated using the result from a WIM measurement. This WIM measurement was performed during year 1994 on E4 at Salem, which is reasonably near the site where data for the lateral location of vehicles had been collected. This data was collected once a month for 7 to 8 days successively, see [56]. The collection of this WIM data was performed by the Swedish National Road Administration (Vägverket). This data was collected with the aim of determining the design traffic load value for bridges with large loaded lengths, see Chapter 4 on page 47. The evaluation of this data is presented in [56].

The measurement results in the form of gross weight distributions of rigid lorries with two to four axles including buses and for articulated lorries including lorries with trailer are illustrated in Figure 6.1 and Figure 6.2, respectively. These two density distribution functions were used to simulate the gross weight for different vehicles.

6.2.2 Data Generation

As mentioned previously, data measuring the transverse distribution of vehicles, see Chapter 5 on page 73, and the WIM data, see section 6.2.1, were used for the MC simulation. Vehicle data simulations were performed for three main classes of vehicles as seen in Table 6.1. The first group includes passenger cars including vans and is denoted Gr1. The second group includes rigid lorries with two to four axles including buses and is denoted Gr2, and the last group includes articulated lorries and lorries with trailer and is denoted Gr3. The number and weights of axles including the positions of each wheel of the vehicles on the bridge were generated for different amount of vehicle simulations. This means, each time, using the inverse method one vehicle was simulated with number and position of wheels on a bridge from the empirical distribution obtained from the collected vehicle data for the lateral position measurement. The inverse method and the empirical distribution are discussed in
6.2. EVALUATION USING THE MONTE CARLO SIMULATION TECHNIQUE

**Gr2**

- No. of vehicles = 66562
- Mean value = 94.2 kN
- Std. deviation = 60.3 kN
- Max weight = 280.0 kN

![Figure 6.1: Probability density function for the gross weight of lorries with two to four axles including buses, from WIM data.](image)

**Gr3**

- No. of vehicles = 51299
- Mean value = 327.8 kN
- Std. deviation = 126.9 kN
- Max weight = 649.6 kN

![Figure 6.2: Probability density function for the gross weight of articulated lorries including lorries with trailer, from WIM data.](image)
section 2.8.2 and section 2.8.1 on page 19, respectively. Furthermore, each vehicle was examined to establish which vehicle group, according to Table 6.1, it belonged. If the vehicle was of type Gr1, i.e. passenger cars, then 2 tonnes was assigned as a gross weight. If the vehicle was of either type Gr2 or Gr3 then the gross weight of the vehicle was simulated according to the probability density function shown in Figure 6.1 and Figure 6.2, respectively. Moreover, the gross weights of the vehicles in Gr2 and Gr3 were distributed to their wheels using the same proportioning as the axle weight limits given in [67]. The determination of the positions of the vehicle wheels on the bridge was done in a similar manner to that discussed in section 5.4 on page 78.

### 6.2.3 Analysis of the Simulated Data

As explained earlier, the characteristic girder distribution factor value was assumed to be determined by the occurrence of a closed-packed queue of stationery vehicles, see section 5.4 on page 78. Thus, from simulated vehicle data, vehicle queues with lengths from 10 to 45 meters and with the free distance of one meter between each vehicle were formed on both lanes. Furthermore, for each vehicle queue, the reaction force $R_A$ for the bridge cross-section as shown in Figure 5.6 on page 80 was calculated. Finally, the $R_A$ values were divided by the respective queue weight to get a factor which gives the proportion of each queue weight acting on the respective beams. For example, the part of the total weight of each vehicle queue that acts on Beam A, denoted $\varphi_i$, is calculated as $R_{Ai}/Q_i$ where $R_{Ai}$ is the reaction force from the vehicle queue $i$ along Beam A and $Q_i$ the respective total queue weight. In this
way, a number of vehicle queues were formed, for different values of $\ell$, for 3.5 million simulated vehicle data and for each queue the $\varphi$-value was calculated from which the distribution for $\varphi$ was obtained.

Figure 6.3 and Figure 6.4 illustrate the distributions of $\varphi$, calculated using the MC simulated vehicle data, for different queue lengths in the form of histograms. As illustrated in the figures, in all cases the histograms have two peaks. The frequency values of the first peak is greater than the second one. Also shown in the figures are the 98th percentile values from the respective distributions and the corresponding values calculated using the traffic load model according to Bro94. At this stage, these values are shown just to see how they relate to each other and a careful comparison between them is made later.

A similar explanation, as discussed in section 5.4 on page 78, can be made here, that is the first part of the histograms represent the lighter queue weights, which most probably are formed from cars and vans in both lanes. In all cases, the $\varphi$-values for the first peak of the histograms are somewhat less than 0.5, i.e. 50%. This indicates that on average, for queues with the same weights in both lanes, a larger portion of the total queue weight is transmitted to Beam B. This is a consequence of the two beams not being placed symmetrically about the traffic lanes which is because of the different width of the right and left verges. The second part of the histogram represents heavier queue weights that are most likely formed by heavy vehicles in the right lane and light vehicles in the left lane. However, for instance all $\varphi$-values that constitute the right tails of the histograms do not necessary have to have heavy queue weights. In other words there is a possibility of getting large $\varphi$-values from light queue weights than from heavy ones. This can be the case if we have relatively heavier queues, which are not necessary constructed of very heavy vehicles, in the right lane than in the left lane. When later, one vehicle queue is chosen to represent a characteristic value of $\varphi$, i.e. a given percentile from the distributions for $\varphi$, it will be pointless if this queue has a very light weight. Therefore, it is necessary to investigate the relationship between the ratio $R_A/Q$, i.e. $\varphi$, and the corresponding $R_A$-value. As an example, the relation between $\varphi$ and $R_A$ are illustrated in the form of scatter diagrams in Figure 6.5 and Figure 6.6. The figures are plotted for the queue lengths of 20 meters and 40 meters, respectively. Also given in the figures are the correlation coefficient, $\rho$, between $\varphi$ and $R_A$. Table 6.2 shows the $\rho$-values for $\varphi$ and $R_A$ calculated for different queue lengths. As can be seen from both the scatter plots and from the $\rho$-values shown in Table 6.2, $\varphi$ and $R_A$ are strongly correlated. This implies that the queue weights increase from the left to the right tails in the histograms shown in Figure 6.3 and Figure 6.4. Therefore it is reasonable to assume that the values that are from the right tail of the histograms have, with a very high probability, heavy queue weights.

Table 6.2: The correlation coefficient, $\rho$, between $R_A$ and $\varphi$ for different queue lengths calculated using the MC simulated vehicle data.

<table>
<thead>
<tr>
<th>$\ell$ [m]</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Figure 6.3: A distribution function for $\rho$ calculated using the MC simulated vehicle data. For queue lengths of 10, 15 and 20 meter.

Figure 6.4: A distribution function for $\rho$ calculated using the MC simulated vehicle data. For queue lengths of 25, 30 and 35 meters.
6.2. EVALUATION USING THE MONTE CARLO SIMULATION TECHNIQUE

Figure 6.5: The scatter plot of $R_A$ versus $\wp$, for queue length of 20 meters. The $\rho$ values indicate the correlation coefficient between $R_A$ and $\wp$.

Figure 6.6: The scatter plot of $R_A$ versus $\wp$, for queue length of 40 meters. The $\rho$ values indicate the correlation coefficient between $R_A$ and $\wp$. 
6.2.4 Probabilistic Model for the Distributions of Girder Distribution Factor

A first observation of the histograms, shown in Figure 6.3 and Figure 6.4 on page 88, indicates that the data set for \( \varphi \) in each distribution most likely appears to be generated by a sum of two normal distributed random variables, as shown in (6.1), cf. section 4.4.4–4.4.5 on page 60.

\[
F(\varphi) = p \Phi \left( \frac{\varphi - \mu'}{\sigma'} \right) + (1 - p) \Phi \left( \frac{\varphi - \mu''}{\sigma''} \right) \tag{6.1}
\]

where \( p, \mu', \mu'', \sigma' \) and \( \sigma'' \) are parameters as shown in Table 6.3 and \( \Phi(\cdot) \) is the standard normal probability distribution function, cf. (4.19) on page 61.

To verify the above assumption, the distributions for \( \varphi \) obtained from the simulations were divided into two groups where each groups was associated with a certain population of vehicle queues. The portion of the first population was chosen to be \( p \) and the portion of the second population then was \( 1 - p \). Furthermore, for each group there is a defined probability distribution function, which is assumed to be normal with mean values and standard deviations as shown in Table 6.3.

Table 6.3: The proportion, mean value and standard deviation for each population.

<table>
<thead>
<tr>
<th>Population one</th>
<th>Population two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>1-( p )</td>
</tr>
<tr>
<td>Mean value</td>
<td>( \mu' )</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>( \sigma' )</td>
</tr>
</tbody>
</table>

From the mean value and standard deviation of the two groups the mean value and the standard deviation for the total population can be obtained as

\[
\mu = (1 - p)\mu' + p\mu'' \tag{6.2}
\]

\[
\sigma^2 = (1 - p)\sigma'^2 + p\sigma''^2 + (1 - p)(\mu - \mu')^2 + p(\mu - \mu'')^2 \tag{6.3}
\]

The heavier queue weight values, i.e. values on the right tail of the histograms shown in Figure 6.3 and Figure 6.4 on page 88, are the most important in the determination of the characteristic values of \( \varphi \). Therefore, the value for \( p \) was determined in such a manner that a high percentile from the distribution of \( \varphi \) was best fitted to the corresponding value from the mathematical model of (6.1). The percentile that was chosen to determine the \( p \) value for different queue lengths was 0.9999. In this way calculated values for \( p, \mu', \mu'', \sigma' \) and \( \sigma'' \) for different queue lengths are shown Table 6.4. Using these values, probability distribution functions, that most especially, agree at the higher \( \varphi \)–values from the simulations were obtained. Figure 6.7 shows an example of the histogram for \( \varphi \) together with the probability density function drawn using the parameters according to Table 6.4 for the queue length of 25 meters. As can be seen in the figure, the probability density function agrees very well with the histogram especially for the right tail of the distribution.
Table 6.4: A table showing values for $p$, $\mu'$, $\mu''$, $\sigma'$ and $\sigma''$, for different queue lengths calculated using vehicle data from the Monte Carlo simulations.

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>$p$</th>
<th>$\mu'$</th>
<th>$\sigma'$</th>
<th>$\mu''$</th>
<th>$\sigma''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.492</td>
<td>0.106</td>
<td>0.20</td>
<td>0.451</td>
<td>0.074</td>
<td>0.655</td>
<td>0.031</td>
</tr>
<tr>
<td>15</td>
<td>0.494</td>
<td>0.103</td>
<td>0.21</td>
<td>0.454</td>
<td>0.073</td>
<td>0.648</td>
<td>0.030</td>
</tr>
<tr>
<td>20</td>
<td>0.498</td>
<td>0.101</td>
<td>0.23</td>
<td>0.455</td>
<td>0.071</td>
<td>0.641</td>
<td>0.031</td>
</tr>
<tr>
<td>25</td>
<td>0.500</td>
<td>0.099</td>
<td>0.25</td>
<td>0.456</td>
<td>0.070</td>
<td>0.634</td>
<td>0.030</td>
</tr>
<tr>
<td>30</td>
<td>0.503</td>
<td>0.097</td>
<td>0.27</td>
<td>0.456</td>
<td>0.068</td>
<td>0.627</td>
<td>0.030</td>
</tr>
<tr>
<td>35</td>
<td>0.505</td>
<td>0.095</td>
<td>0.30</td>
<td>0.456</td>
<td>0.066</td>
<td>0.619</td>
<td>0.031</td>
</tr>
<tr>
<td>40</td>
<td>0.506</td>
<td>0.092</td>
<td>0.32</td>
<td>0.456</td>
<td>0.065</td>
<td>0.614</td>
<td>0.030</td>
</tr>
<tr>
<td>45</td>
<td>0.508</td>
<td>0.090</td>
<td>0.33</td>
<td>0.457</td>
<td>0.064</td>
<td>0.609</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Figure 6.7: Model according to (6.1) fitted to the distribution for $\rho$ calculated using vehicle data from the Monte Carlo simulations. $\ell = 25$ m and $p = 0.25$.

Also compared were the probability distribution functions for $\rho$ obtained from the simulated vehicle data and from (6.1) calculated using the values according to Table 6.4. This was done for several queue lengths and are illustrated in Figure 6.9–6.15, where the distributions are plotted on a normal distribution paper. As can be seen in the figures, in all cases the two probability distribution functions for every queue lengths agree well especially for higher $\rho$-values.
Figure 6.8: A comparison of the values of \( \varphi \) calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 10 meters.

Figure 6.9: A comparison of the values of \( \varphi \) calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 15 meters.
6.2. EVALUATION USING THE MONTE CARLO SIMULATION TECHNIQUE

Figure 6.10: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 20 meters.

Figure 6.11: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 25 meters.
Figure 6.12: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 30 meters.

Figure 6.13: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 35 meters.
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Figure 6.14: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 40 meters.

Figure 6.15: A comparison of the values of $\varphi$ calculated using the MC simulated vehicle data and the distribution according to (6.1) plotted on a normal probability paper. For queue length of 45 meters.
6.2.5 Results and Discussion

The results and discussions made in this section are based on the evaluations of the available data for transverse distribution of vehicle wheels on the bridge. The data however might not cover all possible "abnormal" traffic scenarios that are possible in real life. A bridge is usually designed for a working life of about 100 years and might need to be repaired every 30–50 years. Normally, during repair work, traffic is allowed, according to vehicle dimensions and weights, on different temporary lanes. These temporary lanes may often use the outer limits of the bridge deck while the other part of the bridge deck is being repaired. This would have given different lateral distribution of the traffic on the bridge than the one used in this work. It is also possible that a bridge with a roadway width like the one where the present data was collected, might in the future be rearranged to give three traffic lanes. Cases like these were not considered in this work. The results show only how the current "normal" real traffic loads and their positions correspond to the load model according to the codes. Keeping this in mind the following discussions are made.

As it has been explained earlier, according to different codes, the characteristic traffic load value is often determined as the 98th percentile of the probability distribution function for the annual maximum value of the load. This means that \( F(\wp) = 0.98 \) gives the characteristic value for \( \wp \) if, for a given bridge, a vehicle queue is assumed to occur on average once a year, see section 2.7 on page 16. \( F(\wp) \) is the probability distribution function for \( \wp \) which is illustrated in Figure 6.9–6.15 for different queue lengths. Further, as it has been explained in the last mentioned section, with the assumption of the occurrence of on average \( N \) vehicle queues per year that the characteristic value for \( \wp \), for different queue lengths, can be obtained as \( F^{-1}(0.98^{1/N}) \). The assumption of the number of occurrences of vehicle queues per year can, for example, be coupled with the traffic flow for a desired road and at a desired location. The traffic flow of course is different from place to place. Therefore, in order to be able to determine the value of \( N \), the traffic situation in a given location should be studied. In this work, the characteristic values of \( \wp \) for different values of \( N \), i.e. in a range of one queue per year (\( N = 1 \)) to one queue per day (\( N = 365 \)), were calculated. Hence, the influence of the value of \( N \) on the characteristic values of \( \wp \) could be assessed. The results calculated for different values of \( N \) and for different queue lengths, are presented in Table 6.5 together with the corresponding values calculated using the load model according to Bro94. These results are also illustrated in Figure 6.16. The dashed curves, in the figure, show the values from the simulations whereas the solid curve shows the values determined according to the load model in Bro94. The results showed that the values calculated using the load model from Bro94 were considerably higher than the corresponding values obtained from the simulated vehicle data. For \( N = 1 \), the values from the simulated data for all queue lengths were approximately 81 % of the corresponding values determined according to the load model in Bro94. This value was 85, 86 and 88 % for the \( N \) values of 12, 52 and 365 respectively.

The values calculated using the traffic load model according to Bro94, for all queue lengths, corresponded to values determined from the simulated data with approxi-
6.2. **EVALUATION USING THE MONTE CARLO SIMULATION TECHNIQUE**

approximately \( N = 950000 \), i.e. the values which could be obtained with the assumption of the occurrence of approximately 2603 vehicle queues per day or 2 vehicle queues per minute.

Table 6.5: Characteristic values of \( \wp \) for different \( N \)-values, i.e. with the assumption of the occurrences of 1, 12, 52 and 365 vehicle queues per year. The values are calculated, for different queue lengths, using data from the MC simulations. In the last row, the corresponding values calculated from the load model according to Bro94 are given.

<table>
<thead>
<tr>
<th>( N )</th>
<th>Percentile</th>
<th>Queue length [m]</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
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<td>0.661</td>
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<tr>
<td>12</td>
<td>0.99832</td>
<td>0.729</td>
<td>0.722</td>
<td>0.716</td>
<td>0.709</td>
<td>0.703</td>
<td>0.698</td>
<td>0.692</td>
<td>0.688</td>
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<tr>
<td>52</td>
<td>0.99961</td>
<td>0.745</td>
<td>0.738</td>
<td>0.731</td>
<td>0.724</td>
<td>0.718</td>
<td>0.713</td>
<td>0.707</td>
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</tr>
<tr>
<td>365</td>
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<td>0.763</td>
<td>0.756</td>
<td>0.750</td>
<td>0.742</td>
<td>0.736</td>
<td>0.730</td>
<td>0.724</td>
<td>0.720</td>
<td></td>
</tr>
<tr>
<td>Bro94</td>
<td></td>
<td></td>
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<td>0.848</td>
<td>0.838</td>
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<td>0.820</td>
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</table>

Figure 6.16: Characteristic values for \( \wp \) obtained for different assumptions of the occurrence of queues per year. The values are calculated, for different queue lengths, using data from the MC simulations. The solid curve shows the value calculated using the traffic load model according to Bro94.
CHAPTER 6. LATERAL TRAFFIC LOAD DISTRIBUTION IN SLAB-ON-GIRDER BRIDGES

6.3 Evaluation using Rice’s Formula

6.3.1 General

The primary aim of this section is to compare the results evaluated using Rice’s formula with the corresponding results from the Monte Carlo simulations, which are discussed in the previous section. This is done in order to confirm the last mentioned results. This will therefore provide another method for the assessment of the girder distribution factors. The theory concerning the level upcrossing intensities and Rice’s formula are described in section 2.9 on page 23.

6.3.2 Level Upcrossing Intensity

The girder distribution factors for different queue lengths calculated in the same manner as discussed in section 5.4 on page 78, i.e. directly using the collected data, were used to determine of the characteristics values of \( \varphi \) according to the analysis using Rice’s formula. When determining the level upcrossing intensity histograms of \( \varphi \), for different values of \( \ell \), the sequence of local extremes called the sequence of turning points were first calculated. The turning points are a vector with local extremes of \( \varphi \). This means the length of this vector is equal to the number of local maxima and minima. Figure 6.17 illustrates an example of the turning points for queue length of 25 meters.

Histogram for level upcrossing intensities was then obtained from the turning points. That was done by successively increasing the level, i.e. the value of \( y \)-axis in Figure 6.17, and calculate the number of times at which each level has been crossed with an increasing vale of \( \varphi \). Figure 6.18 illustrates an example of histogram for level upcrossing intensity for queue length of 25 meters. The bar height gives the number of upcrossings for a given level, i.e. for a given value of \( \varphi \). The width of the bar in the histogram can affect the final results—the more class intervals, i.e. smaller bar widths, are used the better will be the tail description. Nevertheless, a very small bar width can lead to a larger fluctuations in the bar height especially in the tails where there are usually few data. Therefore, the data must be analyzed in terms of sensitivity or the precision that is desired in the data description. Since, this is an obvious and simple matter, it is not presented here.

For the queue length of 25 meters, there were 3805 values of \( \varphi \) obtained. Since each value of \( \varphi \) is assumed to represent a yearly maxima, the turning points in Figure 6.17 illustrate 3805 years signal at one year intervals. Therefore, in order to get the level upcrossing intensity for one year, the histogram in Figure 6.18 had to be normalized by 3805.
6.3. EVALUATION USING RICE’S FORMULA

Figure 6.17: Turning points. For queue length of 25 meters. Each value is a yearly maximum value, therefore the figure shows values that represent 3805 years’ signal with one year interval.

Figure 6.18: Level upcrossing intensity for queue length of 25 meters. Since each value of \( \varphi \) is a yearly maximum value and there are 3805 \( \varphi \)-values obtained, the level upcrossing intensity histogram represent a distribution for 3805 years.
6.3.3 Fitting to Rice’s Formula

Once the level upcrossing intensities were obtained, the fitting of Rice’s formula could be carried out, using the method described in section 2.9 on page 23. The fittings were performed for different confidence levels, i.e. for different values of \( \beta_0 \), see section 2.9.1 on page 25.

As described previously, the optimal fitting corresponds to the fitting providing the highest value for the Kolmogorov-Smirnov test from successive fittings. Consequently, the procedure starts from the last three class intervals and then goes up the histogram tail, i.e. the number of class intervals increases. For a given confidence level \( \beta_0 \), the smallest \( x_0 \) for which the highest KS test value is obtain, is defined as yielding the optimal fitting class interval. Successive fitting gives, for each \( x_0 \),

\[
Q_{KS}(\sqrt{N(x_0)D(x_0)}) = \beta(x_0),
\]

where \( N(x_0) \) and \( D(x_0) \) are respectively the number of class intervals used in the fitting and the value of the Kolmogorov-Smirnov statistics. Figure 6.19–6.21 illustrate the result from the successive fittings calculated for queue lengths of 10, 25 and 45 meters respectively. As seen in the figures, in all cases, the \( \beta_0 \) values decrease with decreasing \( x_0 \) values. Obviously, as seen in the figures, for a wide interval of the threshold value, very good fitting, i.e. \( \beta_0 \approx 1 \), could be obtained. Table 6.6 shows the steps, number of class intervals, threshold values, KS statistic \( D \) and \( \beta_0 \) values obtained from the successive fittings for \( \ell = 25 \) meters.

As can be seen in the table, the \( Q_{KS} \), i.e. \( \beta_0 \), values are 1 for the first 7 steps and very close to 1 for many steps after that. The consequence of this is that if for instance the confidence level is chosen to be as low as 0.9, the optimal fitting could be first obtained at step 42 with the corresponding \( x_0 \) value of 0.491. This \( x_0 \) value is, as can be seen in Figure 6.18, very far from the tail of the histogram. This implies that, for the present case, Rice’s formula fits well with the level upcrossing intensity histograms even for low threshold values, i.e. for values very far from the histograms’ right tail. Therefore, an absolute fitting or a relative fitting with a very high target value, say at least \( \beta_0 = 0.999 \), should be chosen for the fittings here.

It should however be pointed out that according to the discussion made with Cremona, his experiences have shown that, in some cases good fittings as for the present case could not be obtained. Therefore, in such cases a very careful investigation should be carried out in order to determine a threshold value corresponding to a desired confidence level.
6.3. EVALUATION USING RICE’S FORMULA

Figure 6.19: The relationship between $x_0$, $\sqrt{ND}$ and $\beta_0$ obtained from the successive fitting of Rice’s formula to the histogram of level upcrossing intensity. For a queue length of 10 meters.

Figure 6.20: The relationship between $x_0$, $\sqrt{ND}$ and $\beta_0$ obtained from the successive fitting of Rice’s formula to the histogram of level upcrossing intensity. For a queue length of 25 meters.
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Figure 6.21: The relationship between $x_0$, $\sqrt{N}D$ and $\beta_0$ obtained from the successive fitting of Rice's formula to the histogram of level upcrossing intensity. For a queue length of 45 meters.

Table 6.6: Values obtained from the successive fitting of Rice’s formula to the histogram of level upcrossing intensity. For a queue length of 25 meters.

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<th>$D_{\text{max}}$</th>
<th>$Q_{KS} = \beta_0$</th>
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Absolute Fitting

Figure 6.23–6.25 illustrate the absolute fittings of Rice’s formula to the histogram of level upcrossing intensities as well as comparisons of the probability distribution functions of these two distributions. The figures show the results calculated for queue lengths of 10, 25 and 45 meters. Note that the histograms are renormalized so that they fit with the probability density functions according to Rice’s formula, which of course have the area under the function equal to 1. The values of the threshold, \( \hat{\wp}_{\text{opt}} \), and the number of class intervals, \( N \), for different queue lengths are illustrated in Figure 6.22. From this figure it can be concluded that both the threshold values, \( \hat{\wp}_{\text{opt}} \), and the number of class intervals, \( N \), show no special relationship with the queue length, i.e. with the \( \ell \)-values.

Relative Fitting, \( \beta_0 = 0.999 \)

Figure 6.26–6.28 illustrate relative fittings, with \( \beta_0 = 0.999 \), of Rice’s formula to the histogram of level upcrossing intensities and comparison of the probability distribution functions of these two distributions. The figures show the results calculated for queue lengths of 15, 25 and 30 meters respectively.

![Figure 6.22: The threshold values, \( \hat{\wp}_{\text{opt}} \), and the number of class intervals, \( N \), for different queue lengths. \( \beta_0 = 1 \).](image-url)
Figure 6.23: Absolute fitting of Rice’s formula to the distribution of level upcrossing intensity of $\varphi$, $\ell = 10$ m.

Figure 6.24: Absolute fitting of Rice’s formula to the distribution of level upcrossing intensity of $\varphi$, $\ell = 25$ m.

Figure 6.25: Absolute fitting of Rice’s formula to the distribution of level upcrossing intensity of $\varphi$, $\ell = 45$ m.
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Figure 6.26: Relative fitting of Rice’s formula to the distribution of level upcrossing intensity of $\varphi$, $\ell = 15\text{m}$.

Figure 6.27: Relative fitting of Rice’s formula to the distribution of level upcrossing intensity of $\varphi$, $\ell = 25\text{m}$.

Figure 6.28: Relative fitting of Rice’s formula to the distribution of the level upcrossing intensity of $\varphi$, $\ell = 30\text{m}$.
6.4 Comparison of the Results Calculated using Rice’s Formula and the MC Simulations

As described in the previous section, Rice’s formula fits well with the histograms of level upcrossing intensities for the present study. Therefore, it was possible to compute the characteristic girder distribution factor values for absolute fittings as well as for fittings with high confidence levels. This was performed for different return periods and different values of queue lengths. The computations was done according to (6.4), cf. (2.39) on page 27.

\[ \phi_k(R_T) = m_{opt} + \sigma_{opt} \sqrt{2 \ln(v_{0, opt} R_T)} \]  

(6.4)

In the case of the evaluation using the MC simulated vehicle data, the characteristic values of \( \phi \) was determined as the 98th percentile of the probability distribution function for the annual maximum value of \( \phi \). Equivalent value from the later analysis could easily be met by calculating the return period which corresponds to \( \alpha = 1 - 0.98 = 0.02 \) from (2.19), on page 15, with \( T_{ref} = 1 \) year and inserting the value into (6.4). This \( \alpha \) value corresponds to, \( R_T = 1/\ln(1 - 0.02) \), 50 years of return period. In this way the calculated characteristic values for \( \phi \) verses queue lengths, for different confidence level, are illustrated in Figure 6.29. As seen in the figure, the chosen values for the confidence levels were between 0.9 and 1. Even the corresponding values that were obtained from the MC simulated vehicle data evaluations are shown in the figure with dashed curve. It could be concluded from the figure that the results from the two methods were in good agreement, for all confidence levels that were chosen for the fittings of Rice’s formula to the level upcrossing intensity histograms. Note the much-zoomed scale of the vertical axis in Figure 6.29. The best agreement was obtained for \( \beta = 0.999 \).

Figure 6.30 gives a comparison of the characteristic values of \( \phi \) evaluated using vehicle data from the MC simulation, similar values evaluated using the Rice’s analysis with \( \beta = 0.999 \) as well as \( \beta = 1 \), and the corresponding values calculated using the load models according to Bro94 as well as Eurocode. The load models from the codes are shown respectively in Figure 1.2 on page 3 and Figure 3.1 on page 36. As mentioned in section 2.7 on page 16, according to Eurocode, the characteristic value for traffic load is determined for a return period of 1000 years. Thus, the characteristic values of \( \phi \) that corresponds to demand of the Eurocode was calculated by setting \( R_T = 1000 \) in (6.4), which correspond to the 99.9th percentile of the distributions for \( \phi \). The confidence level that was chosen for the last mentioned calculation was 0.999.

Again, as can be seen from the figure, the values obtained by the evaluation using the vehicle data from the MC simulations were in a good agreement with the corresponding values from upcrossing intensity analysis, especially for \( \beta = 0.999 \). The relationship of these values with the corresponding values calculated using the load model according to Bro94 was the same as discussed in section 6.2.5. The figure also illustrates that the values calculated using the upcrossing intensity analysis with \( R_T = 50 \) and \( \beta = 0.999 \) were, for all queue lengths, approximately 86 % of the
6.4. COMPARISON OF THE RESULTS CALCULATED USING RICE’S FORMULA AND THE MC SIMULATIONS

Figure 6.29: A comparison of the characteristic values for $\varphi$, for different queue lengths, obtained from the analysis according to Rice’s formula, for different confidence level, and the values evaluated using the MC simulated vehicle data.

Figure 6.30: The characteristic values for $\varphi$ versus queue lengths obtained from the analysis according to Rice’s formula, for $\beta = 0.999$ and $\beta = 1$, from the values evaluated using the MC simulated vehicle as well as the corresponding values according to Bro94 and Eurocode.
corresponding values calculated using the load model from Eurocode. However, the values from Eurocode were in relatively good agreement with the values calculated using the uppcrossing intensity analysis with $R_T = 1000$ and $\beta = 0.999$. 
Chapter 7

Load Effects on Box-Girder Bridges

7.1 General

This chapter presents the investigation of different load effects on box-girder bridges calculated using a combination of the collected data for vehicle transverse distribution and the WIM data. For this purpose, Finite Element (FE) models of box-girder bridges with the same cross-sections but different lengths have been developed. This is done using the commercial finite element software SOLVIA [49]. The files for pre- and post-processing used in SOLVIA were automatically generated using a code developed in Matlab. In addition, all the output from SOLVIA are interpreted using a Matlab developed code. To verify the accuracy of the results from the FE model, midspan deflections calculated using the standard handbook formulas, when both the bridge was subjected to two concentrated symmetrical point loads as well as when it was subjected to uniformly distributed load over the whole bridge deck are compared with the corresponding results obtained from the FE model analysis. For the interested reader, comparison of results from FE models using SOLVIA and theoretical models described in [59] can be found in [26], for similar bridge types as those studied in this work. The loadings of each bridge are performed using the data collected for the transverse distributions of vehicles on the bridge in combination with the data from the WIM measurements. The first data is used to determine the locations of the vehicles’ wheels on the bridge and the second is used to determine the magnitude of their weights. The calculated load effects are normalized by the corresponding values calculated using the traffic load model from the Swedish bridge design code. Further the load effects are evaluated using the method described in section 2.9 on page 23, i.e. analysis using Rice’s formula. Finally, the results are presented in the form of the load effects as a function of the return period.
7.2 FE Model

All the generation of coordinates, the meshings and the loadings of the bridge models are performed automatically using a code developed in Matlab for a desired, among other things, bridge length and width. The output data from SOLVIA are processed in Matlab. An example of one file for pre-processing is shown next:

```
SET MODE=BATCH
HEADING 'Box-girder bridge, L=20m'
DATABASE CREATE
KINEMATICS DISPLACEMENT=SMALL
*
SYSTEM 1 CARTESIAN
COORDINATES
ENTRIES NODE X Y Z
READ 'COORDINATESdataL20.dat'
*
MATERIAL 1 ELASTIC E=32,E9 NU=0.3
*
READ 'STRAIGHTLINEdataL20.dat'
READ 'GSURFACEdataL20.dat'
READ 'FIXBOUNDARIESdataL20.dat'
READ 'LOADdataL20.dat'
SET SMOOTHNESS=YES NSYMBOLS=NO
SUBFRAME 21
MESH NNUMBERS=MYNODES VECTOR=LOAD
MESH ENUMBER=NO BCODE=ALL NNUMBER=NO
SHELLCONT
EDGECONT
SOLVIA
END
```

where all the input files with the extension .dat (i.e. the files that are in front of the command READ above), containing coordinates, meshings, loadings, etc. These are generated in Matlab for different bridge spans. The node numbering as well as the cross-section and material data are illustrated in Figure 7.1. A linear 4-node shell element, see [11], was chosen for the analysis.

![Figure 7.1: Bridge model.](image)

As in Chapter 6 on page 83, the design case was assumed to be determined by the occurrence of vehicle queues. Thus, queues from the collected vehicle data
for the lateral position of vehicles, see Chapter 5 on page 73, were formed with different lengths, each of which constitutes one loading case, cf. section 5.4 on page 78. This means that the number and position of wheels of the vehicles in each queue, on a bridge of given length, could be obtained from the observations. In the similar manner as in section 6.2.2 on page 84, for the determination of the gross vehicle weights, each vehicle in a queue was examined to which group of vehicle class (according to Table 6.1 on page 86) it belonged. If the vehicle was of type Gr1 then 2 tonnes was assigned as a gross weight. Whereas, if the vehicle was of either type Gr2 or Gr3 then the gross weight of the vehicle was simulated according to the probability density function shown in Figure 6.1 and Figure 6.2 on page 85 respectively. Moreover, as been done before, the gross weights of the vehicles in Gr2 and Gr3 were distributed to their wheels with the proportion according to the standard in [67]. Furthermore, the vehicle wheels were assumed to act on the bridge as point loads with a pressure area of 0.1 m$^2$. An example of a complete finite element model, for a bridge with a span length of 25 meters, obtained from SOLVIA, is shown in Figure 7.2. Also illustrated in the figure are two load cases—the load according to Bro94, see Figure 1.2 on page 3, and one randomly selected load case from the vehicle queues.

**Figure 7.2:** The finite element model, $L = 25$ m. The upper figure shows the bridge loaded by the load model according to Bro94, see Figure 1.2 on page 3, and the lower figure shows the bridge loaded by one randomly selected vehicle queue.
7.3 Analysis of Outputs from SOLVIA

7.3.1 General

The results shown in this section were from the output of SOLVIA processed in Matlab. Data for two different load effects, calculated using the collected vehicle data, were chosen to be analyzed—the vertical deflection at node 13, denoted \( \delta \), see Figure 7.1 on page 110, and the longitudinal stress 250 mm from node 13 into the bottom slab of the box, denoted \( \sigma \). The vertical deflection at node 13 was chosen because it gives an integrated effect of the whole deformation of the bridge. A choice of any node on the top slab of the bridge would be misleading because of possible local deformations, such as bending of the flanges. Also calculated was the deflection at node 13 from the load model according to Bro94 when being applied at the most unfavorable position, see Figure 7.2. This load position also gives the maximum longitudinal moment at node 13, see [47]. This was the reason why the longitudinal stress at the above mentioned point was also chosen to be investigated. These load effects were calculated for different bridge spans—from 10 to 35 meters. For each bridge span, among other things, deflections and stresses for the FE model are obtained from SOLVIA for a number of load cases, i.e. for a number of vehicle queues, from which the distribution for \( \delta \) and \( \sigma \) were obtained.

7.3.2 The Output Data Evaluation using Rice’s Formula

The results discussed in the previous section were analyzed in a similar manner to that performed earlier in section 6.3 on page 98, i.e. using Rice’s formula. The obtained results of \( \delta \) and \( \sigma \), for different bridge spans were first normalized by the corresponding values calculated using the load model according to Bro94, denoted \( \delta_{\text{Bro94}} \) and \( \sigma_{\text{Bro94}} \), respectively. Furthermore, the turning points for \( \delta/\delta_{\text{Bro94}} \) and \( \sigma/\sigma_{\text{Bro94}} \), for different values of \( \ell \) were determined and from that histograms for level upcrossings were obtained. As described earlier, each values of \( \delta/\delta_{\text{Bro94}} \) and \( \sigma/\sigma_{\text{Bro94}} \), are assumed to represent yearly maxima. Therefore, when these histograms are normalized by the respective number of the values of \( \delta/\delta_{\text{Bro94}} \) and \( \sigma/\sigma_{\text{Bro94}} \) for different values of \( \ell \), they give histograms of level upcrossing intensity representing one year, cf. section 6.3.2 on page 98.

Using the same procedure as described in section 6.3.3 on page 100, once the level upcrossing intensities were obtained, fitting of Rice’s formula to the level upcrossing intensity histograms could be carried out. The fittings were performed for different confidence levels, i.e. for different values of \( \beta_0 \), see section 2.9.1 on page 25. As described before, the optimal fitting corresponds to the fitting providing the highest value for the Kolmogorov-Smirnov test from successive fittings. Therefore, the procedure starts from the last three class intervals and then goes up the tail of the histogram, i.e. the number of class intervals increases. For a given confidence level \( \beta_0 \), the smallest \( x_0 \) for which the highest KS test value is obtain, yields the optimal fitting class interval. Successive fittings give, for each \( x_0 \), the value
7.3. ANALYSIS OF OUTPUTS FROM SOLVIA

\[ Q_{KS}(\sqrt{N(x_0)D(x_0)}) = \beta(x_0). \]

Where \( N(x_0) \) and \( D(x_0) \) are respectively the number of class intervals used in the fitting and the value of the Kolmogorov-Smirnov statistics.

The investigation showed that the level upcrossing intensity histograms in all cases are well described by Rice’s formula. Analogous results as in section 6.3.3 on page 100 were obtained even here and hence are not discussed again. Figure 7.3–7.8 illustrate the absolute fittings of Rice’s formula to the histograms of level upcrossing intensity. The first three figures show the fittings of Rice’s formula to the distributions of \( \delta/\delta_{\text{Bro94}} \) obtained for bridge span of 10, 20 and 30 meters, respectively. The last three figures show the fittings of Rice’s formula to the distributions of \( \sigma/\sigma_{\text{Bro94}} \) obtained for bridge span of 15, 25 and 35 meters, respectively.

![Figure 7.3: Relative fitting of Rice’s formula to the histogram of level upcrossing intensity of \( \delta/\delta_{\text{Bro94}}, \ell = 10 \text{ m}. \)](image)

Figure 7.3: Relative fitting of Rice’s formula to the histogram of level upcrossing intensity of \( \delta/\delta_{\text{Bro94}}, \ell = 10 \text{ m}. \)
Figure 7.4: Relative fitting of Rice’s formula to the histogram of level uppcrossing intensity of $\delta/\delta_{Br94}$, $\ell = 20$ m.

Figure 7.5: Relative fitting of Rice’s formula to the histogram of level uppcrossing intensity of $\delta/\delta_{Br94}$, $\ell = 30$ m.
7.3. ANALYSIS OF OUTPUTS FROM SOLVIA

Figure 7.6: Relative fitting of Rice’s formula to the histogram of level uppcrossing intensity of $\sigma/\sigma_{\text{Bro94}}$, $\ell$ = 15 m.

Figure 7.7: Relative fitting of Rice’s formula to the histogram of level uppcrossing intensity of $\sigma/\sigma_{\text{Bro94}}$, $\ell$ = 25 m.
7.4 Results and Discussion

As been described in the previous section, Rice’s formula fits well to the histograms of the level upcrossing intensities for all cases. One observation that could be made from Figure 7.3–7.8 was that the maximum value of both $\delta/\delta_{\text{Bro94}}$ and $\sigma/\sigma_{\text{Bro94}}$, in all cases, were less than approximately 0.46. Figure 7.9 illustrates the maximum values of $\delta/\delta_{\text{Bro94}}$ and $\sigma/\sigma_{\text{Bro94}}$ obtained for different values of $\ell$. According to this investigation, the load model from Bro94 gave a deflection at the studied node that was between, slightly more than, three times, for $\ell = 10$ meters, and two times, for $\ell = 35$ meters, higher than the corresponding maximum values obtained from the real traffic loadings. The corresponding figures for the longitudinal stress at the studied node were also approximately three and two respectively. This result however was not so surprising since for the calculations, the traffic load from the code had been placed at the outer edge of the bridge’s roadway, the dimensioning position, while the vehicle loads from the collected data mostly occurred in the middle of the roadway, see Figure 7.2 on page 111, cf. section 6.2.5 on page 96. Since this investigation was performed using the available data for the actual positions of vehicles on the bridges it might not cover all possible real traffic loadings scenarios. As discussed in the last mentioned section, during a possible repair work of a bridge, traffic is allowed, according to vehicle dimensions and weights, on different temporary lanes which may often use the outer limits of the bridge deck while the other part of the bridge deck is being repaired. This of course would have given different
7.4. RESULTS AND DISCUSSION

transverse distribution of the traffic on the bridge than the one used in this work. Also, as previously discussed, there is a possibility of, in the future, increasing the lanes into three of a bridge with a roadway width similar to the one on which data was collected for this work. Again, cases like these were not considered in this work. The results showed only how the current ”normal” real traffic loads correspond to the load model according to Bro94. Keeping that in mind, it is now possible to estimate the values of $\delta/\delta_{\text{Bro94}}$ and $\sigma/\sigma_{\text{Bro94}}$ for any return period according to (2.39) on page 27.

For the estimation of the values of deflection and stress ratios for different return periods, relative fittings with the confidence level $\beta_0 = 0.999$ were used. The result are illustrated in Figure 7.10 and Figure 7.11. The first figure illustrates the deflection ratios at the studied node for different return periods whereas the second figure shows similar relationship for the stress ratios. The values that correspond to the return period of 50 years (i.e. $\approx 98$th percentile) as well as the values that correspond to the return period of 1000 years from the respective distributions are also shown in the figures.

As seen in both figures, according to the present study the values for the deflection and stress ratios were considerably small, suggesting that the traffic load value from the code is possibly very high. The values from the code, in both cases, were approximately three to four times greater than the actual traffic load for $R_T = 50$ years. The maximum value obtained for $R_{A,\text{measured}}/R_{A,\text{Bro94}}$ was also of the same magnitude, see Figure 5.8 on page 82. The last mentioned figure as well as Figure 7.10 and Figure 7.11 show how large the load value from the code is compared

<table>
<thead>
<tr>
<th>$\ell$ [m]</th>
<th>$\sigma$/$\sigma_{\text{Bro94}}$</th>
<th>$\delta$/$\delta_{\text{Bro94}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.3134</td>
<td>0.3469</td>
</tr>
<tr>
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<tr>
<td>30</td>
<td>0.4193</td>
<td>0.4529</td>
</tr>
<tr>
<td>35</td>
<td>0.4199</td>
<td>0.4597</td>
</tr>
</tbody>
</table>

Figure 7.9: The maximum values of $\delta/\delta_{\text{Bro94}}$ and $\sigma/\sigma_{\text{Bro94}}$ obtained for different values of $\ell$. 
Figure 7.10: Values of $\delta/\delta_{\text{Bro94}}$ for different bridge spans verses return period.

Figure 7.11: Values of $\sigma/\sigma_{\text{Bro94}}$ for different bridge spans verses return period.
to the current actual traffic loads. It should be reminded that the formations of the vehicle queues were done on the safe side, where only one meter free distance between the vehicles was allowed. Nevertheless, in all cases the ratio of the actual traffic load to the design traffic load is very low. Therefore, according to this investigation, strengthening or reconstructing an existing bridge to meet the load model from the code could be a very uneconomical and unnecessary task.
Chapter 8

Conclusions and Discussions

8.1 General

In this chapter, the conclusions of this research are presented in two main sections. The first section states the conclusions drawn from the part of this project that was presented as a licentiate work, see Chapter 4 on page 47. Conclusions from the rest of the work are presented in the second section of this chapter. Finally, proposals for further research are stated in the last section.

Most certainly, this study has not provided a comprehensive answer to the question concerning the actual traffic loads and their effects on bridges. However, the author hopes that the results of this study will be of assistance to bridge designers, bridge code authors and researchers and provide a basis for future studies.

8.2 Traffic Load Models for Long-Span Bridges

8.2.1 Analysis of the Collected Data

A careful investigation of the accuracy of the collected WIM data reveals that the data for the vehicle lengths contain more errors compared with the other recorded parameters. This can clearly be seen in Figure 4.9 on page 59 where the mean value of the queue weights, in all cases, became greater when calculated using data after the filtration than before the filtration of unreasonable data. However, data for the axle weights as well as the vehicle gross weights seem to be relatively accurate. The excluded unreasonable data shows no significance affect on the final results, i.e. the characteristic traffic load values for long-span bridges calculated in the different ways. This is believed to be a consequence of the fact that during the filtration mostly light vehicles with long lengths as well as heavy vehicles with short lengths are excluded, whereby their effects on the vehicle queue weights cancelled each other out. It should however be pointed out that the filtration is performed in a conservative manner where for example vehicles with single axles have been included in the
further evaluation. Vehicles with four axles could be individual vehicles as well as articulated vehicles. Because of the limited quality of the WIM data, vehicle of this type could not be identified. Therefore, all vehicle data that have lengths less than the maximum allowable length for articulated vehicles are accepted and are analyzed further. Consequently, among these vehicles individual vehicles with unreasonable lengths might be found. Moreover, because there is not enough information about the vehicles registered and judged as unreasonable data, they are simply excluded without replacing them by ”corrected” vehicle data.

The most critical situations for long span bridges appears when the traffic is disturbed while for short span bridges it is the heaviest individual axles, double bogie axles, triple bogie axles or the single vehicle’s gross weight that determine the characteristic load effects value. Therefore, it should be borne in mind that any measurement errors will, most probably, have considerable influence, when using the WIM data to investigate load effects for short span bridges.

### 8.2.2 Analysis of the Simulated Data

As expected, the evaluation of the WIM data has shown that the distributions of different vehicle gross weights in different measurement periods are equivalent. Furthermore, it has been shown that by utilizing the MC simulation technique, using a limited amount of WIM data, it is possible to generate fictitious vehicle data that can well represent the actual site-specific vehicle data. Because of the limited quality of the WIM data used in this work, the vehicle classification made for the MC simulation is rather crude. Despite this, the obtained results from the vehicle data generation are very satisfactory. It is, therefore, believed that by using newer more accurate WIM data better simulation results can be obtained. The queue weight distributions obtained using the MC simulated and the WIM data are in good agreement especially for the large queue weight values. Obviously, it is the larger queue weights that are most important for the determination of the characteristic traffic load values.

As a result of this investigation, it is suggested that future traffic data recording should be done systematically under relatively short periods. This collected data together with the information of the traffic composition can then be used to generate fictitious vehicle data using the MC simulation technique. This leads to an extensive minimization of both the time and money that will be spent on the recording of traffic loads, which usually is performed continuously during long periods.

It should be mentioned here that the characteristic traffic load values calculated from the data recorded at Torp (i.e. measurement series E, see Figure 4.1 on page 48) are lower than the corresponding values calculated from the other measurement series (i.e. A, B, C, D and F). This is believed to be a consequence of the local traffic composition.
8.3 Traffic Load Effects on Medium and Short Span Bridges

8.3.1 General

The primary investigation of the collected data for transverse distribution of vehicle wheels on the bridge illustrated in Figure 5.2 on page 75, has unsurprisingly shown that a large number of vehicles primarily use the right lane. It is also seen that very heavy vehicles drive closest to the verge. Of the 23622 recorded vehicle data for cars and vans, 199 drove with at least the wheels on one side of the vehicle outside the right lane, i.e. on the right verge. The corresponding figure for the 3011 heavy vehicles recorded with two to four axles including buses as well as for articulated lorries including lorries with trailer was 162. Most probably, these vehicles result in higher values of the investigated load effects and are assumed to cover some abnormal traffic scenarios. However, since the data evaluations, performed in this work, are based on the available data for the actual positions of vehicles on the bridges, the results might not cover all possible real traffic scenarios. They show only how the current real traffic loads and their transverse distributions relate to the load model according to the codes. Keeping this in mind the following conclusions can be made.

8.3.2 Slab-on-Girder Bridges

The histogram, illustrated in Figure 5.8 on page 82, for the ratio of $R_A$ obtained from recorded data and the corresponding value calculated using the traffic load model according Bro94 shows that the $R_A$ values determined from data are considerably smaller than the corresponding value calculated using the load model from the code. The maximum value of this quota is 0.37. This first investigation clearly reveals the high safety margin with regard to the traffic design load compared with the actual traffic loading.

Using the recorded data for transverse distribution of vehicles on the bridge as well as the results from the WIM measurement, performed on highway E4 at Salem, the characteristics values of the girder distribution factors for different queue lengths are determined. These values for different queue lengths are obtained using two different statistical evaluation methods—the Monte Carlo simulation technique and the evaluation using Rice’s formula. The investigation has shown that the two approaches give very similar results. The characteristics values of the girder distribution factor for different queue lengths are also compared with the corresponding values calculated using the traffic load models according to the Swedish bridge design code, i.e. Bro94, as well as the Eurocode. The results show that the values obtained from the codes’ load models are higher than the corresponding values calculated using the collected data. This indicates, according to this study, the conservative nature of the placement of the traffic load models from the codes for the determination of the girder distribution factors.
8.3.3 Box-Girder Bridges

Under the same circumstance as above, i.e. the relationship between the current "normal" real traffic loads as well as their transverse placements and the load model according to the codes, load effects on box-girder bridges are evaluated using the recorded data. The traffic load effect values, i.e. deflections and longitudinal stresses at critical locations, calculated for different bridge spans and different return periods show that the corresponding values calculated using the load model according to Bro94 are considerably higher. It should be remembered that during the formation of the vehicle queues only a free distance of one meter between the vehicles is allowed, which is on the safe side. However, the ratio of the actual traffic load to the design traffic load is very low. Therefore, according to this investigation, using the load models from the codes, especially with the intention of repairing or reconstructing an existing bridge to meet current design traffic load models can be a very uneconomical and unnecessary task.

8.4 Suggestions for Further Research

As discussed previously, statistical analysis of recorded vehicle data to study different load effects on bridges, with the intention of calibrating the traffic load models given in different codes, is a relatively new area of study. More research studies are required to get a better knowledge of the actual traffic load effects on bridges. It is often observed during traffic load measurements that the design load capacity of a bridge is much higher than the actual traffic loads to which it is subjected. This extra safety reserve can be utilized to avoid rehabilitation thus justifying the need for a reliability-based bridge design specifications. The term reliability-based bridge design refers to procedures in which specification bodies consider the statistical distributions of loadings and the statistical distribution of component strengths. The reliability is calculated from these load and resistance distributions.

There has been considerable research and data gathering in recent years on highway bridge loadings and component resistances. Referring to Chapter 4, there is a huge amount of traffic load data that has been recorded at different locations in Sweden. These measurement results should be used to build a database for future researchers. A complete evaluation of the condition of a bridge also requires the knowledge of actual traffic load and how they are distributed to different bridge components. The first step therefore should be using this data, to perform reliability based modelling of existing bridges. The aim should be to provide a comprehensive description of the material and loading properties of the structure, which can then be applied in a reliability based bridge evaluation. There are many textbooks and articles devoted to structural reliability. The excellent work [22, 23] conducted parallel to the present research, which deals with the raising of the allowable axle loads on existing railway bridges is also a very good reference. The WIM data set from two of the measurement series were recently used in [6] for the reliability based assessment of short span bridges. This work is also very interesting.
Also given information in the results from the WIM measurements are the speeds of the vehicles. Therefore, it could be interesting to perform reliability based assessment of fatigue as well as dynamic traffic loadings.

The present work does not cover all possible traffic scenarios. Therefore, it is of great importance to study how the consideration of other traffic situation can influence the result of this work.

To complete the data set recorded at Muskö, see section 5.1 on page 73, further testing should be performed and the traffic load distribution amongst the girders of the bridge should be evaluated. This bridge can for example be used to perform a case study of reliability-based assessment of an existing bridge structure.
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Appendix A

Vehicle Classification used by Metor

For the recording of traffic data for the transverse distribution of vehicles on the bridge, a system called Metor was used, see Chapter 5. This system processes and analyzes the pulse received through the tubes, see Figure 5.2 on page 75, and provides different vehicle data sorted in 15 different vehicle classes. This vehicle classification is shown in Table A.1. The classification of the collected data is done with regard to the axle spacing and number of axles of the traction vehicle and the trailer. There are four different vehicle groups denoted MC, P, L and XXX. The first group, i.e. MC includes light traction vehicles with axle spacings between 80 cm and 180 cm, such as moped, motorcycle. The second group, i.e. P includes traction vehicles with axle spacings between 180 cm and 330 cm, such as cars, vans, light trucks, etc. The group denoted by L includes traction vehicles with axles spacing between 330 cm and 1050 cm or 80 cm and 1050 cm if the vehicle has more than two axles. The last vehicle group denoted XXX includes vehicles that do not belong to the groups MC, P or L. In some cases, the vehicles group are divided into different vehicle classes, see Table A.1. These vehicle classes are denoted by the group name followed by two numbers. The first number indicates the number of axles the traction vehicle has and the second number indicates the number of axles the trailer (if any) has.
Table A.1: The vehicle classification used by the measurement system Metor.

### Vehicle Group MC

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### Vehicle Group L

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<td>330 cm &lt; D &lt; 600 cm</td>
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<td>600 cm &lt; E &lt; 1050 cm</td>
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<th>Vehicle Class L21</th>
<th>Allowable axle spacing</th>
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<td>180 cm &lt; C &lt; 330 cm</td>
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<th>Vehicle Class L22</th>
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<td>Vehicle Class</td>
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| L23           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L24           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L30           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L31           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L32           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L33           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
| L34           | $80 \text{ cm} < B < 180 \text{ cm}$  
                | $180 \text{ cm} < C < 330 \text{ cm}$  
                | $330 \text{ cm} < D < 600 \text{ cm}$  
                | $600 \text{ cm} < E < 1050 \text{ cm}$  |
Appendix B

Measurement in Muskö

In this appendix, the result of the second data measuring the transverse position of vehicles on a bridge is presented. This data was collected on the road just before Muskö, south of Stockholm. The measurement was performed under a relatively short period. In total, data for 5985 vehicles were recorded during three days. 5261 vehicle data were for cars including vans and 724 data were heavy vehicles. As explained earlier, this measurement was performed over only three days in December 2001 when the temperature was between -15 and -20 Celsius. Histograms for load effects calculated using this data showed high fluctuation of the frequency values especially in the tails. This is believed to be a result of the insufficient amount of recorded data. Therefore, this data is planned to be complemented at a later date and the evaluation of data from this measurement is intended to be presented separately in another report. An overview of the test site is illustrated in Figure B.1.

Figure B.2 shows the measurement results in the form of histograms for the transverse distributions of the right front wheel of different types of vehicles. The upper histogram shows the distribution for cars including vans and the lower histogram shows the distribution for lorries.
Figure B.1: An overview of the test site.

Figure B.2: Distribution of the transverse position of right front wheel for different vehicle types from the collected data. The upper histogram shows the distribution for cars including vans (5261 vehicles) and the lower histogram shows the distribution for lorries (724 vehicles).
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