Arching Stability in Shallow Tunnels

A comparison between analytical and numerical solutions

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Acknowledgments

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Abstract

Depending on the budget of a project and the level of complexity in case of ground conditions and tunnel geometry a choice has to be made if a numerical solution should be used or if analytical methods will be enough when arching stability of shallow tunnels are analyzed. Analytical solutions are easier to use and gives a faster and cheaper solution, and can often be used as a pre-study to get a first insight into the problems that can arise. However, since the analytical solutions are based on simplified models of a more complex reality, they are usually associated with a significant model uncertainty. This thesis will investigate how analytical methods stands in comparison with numerical models when arching stability is analyzed, since the numerical models are built with a higher level of complexity and can explain the reality better. This will be done by creating four different models in the numerical program UDEC, where the results from the numerical models are compared against the results from the analytical models. The results shows that the difference between the analytical and the numerical solution does not differ more than 10% for the chosen geometries. These results indicate that the analytical solution can constitute acceptable tools for shallow tunnels in blocky rock masses. Further research are recommended to study the applicability for different dip angles and also investigate the effect from rock bolts.

Key words: UDEC, numerical modelling, arching stability, beam theory.
## Contents

1 Introduction 4  
1.1 Objective ........................................ 4  
1.2 Outline of thesis ............................... 4  
1.3 Limitations .................................... 4  

2 Theoretical background 6  
2.1 Introduction to beam theory and arching stability .... 6  
2.2 Analytical model .................................. 7  
2.3 Voussoir beam theory ........................... 8  
2.4 Analytical model with inclined joints .......... 10  
2.5 Analytical model with rock support .......... 11  

3 Methodology 16  
3.1 UDEC ........................................... 16  
3.2 Geometry ...................................... 17  
3.3 Joint model area .................................. 18  
3.4 Row of blocks .................................. 19  
3.5 Arch-tunnel .................................... 20  
3.6 Circular-tunnel .................................. 22  
3.7 Circular tunnel with support .................. 24  

4 Calculations and Results 25  
4.1 Row of blocks .................................. 25  
4.2 Arch-tunnel .................................... 28  
4.3 Circular-tunnel .................................. 30  
4.4 Circular-tunnel with support ................. 31  

5 Discussion 31  
5.1 Row of blocks .................................. 31  
5.2 Arch-tunnel .................................... 32  
5.3 Circular-tunnel .................................. 32  
5.4 General discussion of the results ............. 33  

6 Conclusions and further research 33  

7 References 34  

8 Appendix 1 - Row of blocks 35  

9 Appendix 2 - Arch-tunnel 37  

10 Appendix 3 - Circular-tunnel 39  

11 Appendix 4 - Circular tunnel with support 41
1 Introduction

Rock is a complex material that can behave both anisotropic and heterogeneous. In addition, the limited numbers of field investigations makes it very challenging to work with, due to the large uncertainties. For a shallow tunnel to be stable, three different failure criterion’s has to be fulfilled. The crushing criterion makes sure that the rock is strong enough, and not to break from the stresses that will arise. Another is the rotation criteria; if this is fulfilled the blocks will not rotate from the differences between the horizontal and vertical stresses. The last criteria that has to be fulfilled is the sliding criteria which takes the inclination and friction angle of the joints into account and tells us if the blocks will start to slide along the joints. These criteria are mainly derived from beam-theory. Based on these criteria, simple analytical models have been created to analyze the arching stability and design the rock support. Common for these analytical models are that they all contains model uncertainties that at the present time is unknown, hence some cases are modelled with numerical methods instead though they are more time consuming and thus more expensive. It would be desired to quantify the uncertainties of the analytical methods and how they stand in comparison with a numerical method.

1.1 Objective

The objective with this master-thesis is to try and understand how the arching effect works for shallow tunnels and to quantify the analytical model uncertainty compared against a numerical model created in UDEC.

1.2 Outline of thesis

Following this introduction a literature study is presented in chapter 2. This will serve as a basis for the following chapters. In chapter 3 a glimpse into the theories and parameters needed when building a model with the numerical software UDEC is presented and also how the different models have been built up in UDEC. Four different models were created in UDEC, starting with a simple geometry in form of a row of blocks and two model geometries with the shape of a tunnel. The influence of rock-support in form of shotcrete was also tested for the last model. The results from these models are presented in chapter 4 and all models have been compared with an analytical solution to see how the results differs between a numerical and a analytical solution. The results are discussed in chapter 5 and conclusion and suggestions for further research are presented in chapter 6.

1.3 Limitations

The subject that this thesis is dealing with is broad and complex, there exists many different rock types around the world but the rock parameters in this thesis has been chosen to mainly represent a hard crystalline rock. Since the analytical
model does not have as much input parameters as in UDEC, some assumptions and simplifications have been done when setting up the UDEC-model, which are described in chapter 3.
2 Theoretical background

2.1 Introduction to beam theory and arching stability

Beam theory takes the basics of mechanics into account when deriving expressions for deformations and stresses in a beam with known material properties, load and boundary conditions. One of the most simple cases is for a simply supported beam exposed to a vertical distributed load and with horizontal loads acting on the abutments, see figure 1.

An arch is a construction which mainly transfer load in compression. The classical arch is a number of blocks arranged in such a way that the joints transfer only compressive forces (Stille et al., 2004). This arch can fail in three different ways either by crushing the joint surfaces, by sliding or by rotational failure (Töyrä J., 2006), all of these three criteria has to be fulfilled to keep the system stable. In shallow tunnels, the two most common ways of failure are the two last ones, at least in hard crystalline rock. Those are also the ones that are most difficult to calculate accurately and this thesis will therefore focus on them.

![Figure 1: Arching effect in a row of blocks (Stille et al., 2004).](image)

The inclination of the pressure line for an arch can be expressed as follows (Stille et al., 2005):

\[ y'' = \frac{q(x)}{Hq} \]

(1)

By integrating the equation above the level of the pressure line in the y-direction becomes:

\[ y = \frac{qL^2}{8Hq} \left[ 1 - \left( \frac{2x}{L} \right)^2 \right] \]

(2)
The maximum height of the arching line \( f \) will occur at the middle of the beam with \( x = 0 \)

\[
f = \frac{qL^2}{8Hq}
\]  

(3)

If the ground would contain intact rock it would behave as any of the usual construction materials that are common today, and accordingly traditional mechanics would apply here as well.

With this in mind the arching effect that is taking place in a beam that is put under compression would also work for rock. By simplifying a shallow tunnel one could say that the roof acts like a beam. And with this simplification a analytical method has been developed by using the beam theory and applying it to rock masses.

The maximum height of the arching line will occur at the middle of the beam/roof, according to equation 3, and to keep the blocks from rotating the height \( f \) can not be bigger than the height of the blocks which gives a criterion for the maximum vertical load to keep the system from rotating.

\[
q_{\text{max}} = \frac{8HqB}{L^2}
\]  

(4)

The other failure mode accounted for in this thesis is sliding. It has the biggest risk of occurring at the abutments where the compression line has the highest inclination against a vertical joint calculated according to equation 5.

\[
y'(L/2) = \frac{q}{Hq} \frac{L}{2} = \frac{4f}{L} = \tan \alpha
\]  

(5)

To keep the blocks from sliding the friction angle has to be greater than the angle \( \alpha \), this gives a maximum height of the compression line as in equation 6 and hence the maximum vertical load to keep the joints from sliding are according to equation 7.

\[
\frac{4f}{L} \leq \tan \phi
\]  

(6)

\[
q_{\text{max}} = \frac{2Hq}{L} \cdot \tan \phi
\]  

(7)

2.2 Analytical model

Assuming that the roof in a shallow tunnel behaves somewhat like a beam with vertical joints unable to take tensile forces the equations from Section 2.1 can be used to create two different failure criteria for the tunnel, rotation and sliding failure which are expressed as:
\[ q_{max} = \min \left\{ \frac{2Hq}{L^2} \cdot \tan \phi, \frac{8HqB}{L^2} \right\} \]

Where:
- \( q_{max} \): Vertical load \([\text{N/m}^2]\)
- \( H_q \): Horizontal force \([\text{N/m}]\)
- \( L \): Total length \([\text{m}]\)
- \( B \): Height of the blocks \([\text{m}]\)
- \( \phi \): Friction angle \([\circ]\)

### 2.3 Voussoir beam theory

The Voussoir beam theory deals with the development of a compression arch in a laminated rock mass. The model considers deflection due to self weight, external loads, water pressure, support, and the deformability of the beam (Töyrä J., 2006).

Rock mass behaviour dominated by parallel laminations is often encountered in underground excavations. The laminations can be a result of sedimentary layering, extensile jointing, fabric created through metamorphic or igneous flow processes or through excavation-parallel stress fracturing of massive ground. This lamination can be the main factor controlling the stability of roofs in large excavations.

If the lamination partings is the only joint set presented in the rock mass, the roof stability and deflections can be calculated using conventional beam deflection and lateral stress calculations. However, it is more common to encounter other joint sets cutting through the laminations. These joints reduce the ability of the rock mass to sustain boundary parallel stresses such as those assumed in conventional beam theory. However, if these joints cut through the laminations at steep angles, or if reinforcement has been installed, it is possible to assume that a compression arch can be generated within the beam which will transmit the beam loads to the abutments (Diederichs and Kaiser, 1999), see figure 2.
The primary modes of failure assumed in the Voussoir beam theory are buckling or snap-through failure, lateral compressive failure (crushing) at the midspan and abutments, abutment slip and diagonal fracturing, see Figure 3. Snap-through failure (Fig. 3a) and crushing (Fig. 3b) is observed at high span to thickness ratios (thin beams) while shear failures (Fig. 3c) is observed in thicker beams. Ran et al. (1994) showed that if the angle between the cross cutting joints and the normal to the lamination plane is less than one third to one half of the effective friction angle of these joints the Voussoir beam theory can be applied. If this is not the case, and the angle between the cross cutting joints and the normal to the lamination plane is larger, then sliding along the cross cutting joints occur and premature shear failure of the beam is likely to be initiated.

Stimpson and Ahmed (1992) also showed that for thick beams, external loading can produce diagonal tensile ruptures (Fig. 3d).
2.4 Analytical model with inclined joints

The method for calculating the safety of a tunnel is quite straightforward if the model is simplified with vertical joints as in section 2.2, but this is seldom the case in reality. The joint pattern is often more complex than simplified vertical joints. The analytical model handles the problem with inclined joints by decreasing the friction angle of the rock with an angle $\beta$ to an equivalent friction angle, where $\beta$ is the angle between the vertical plane and the joint (Stille et al. 2005).

$$\phi' = \phi - \beta$$  \hspace{1cm} (8)
Figure 4: Relationship between the dip of the joint and the pressure line

$\beta$: angle between the vertical plane and the joint

$\phi'$: equivalent friction angle

2.5 Analytical model with rock support

This section will go through the theories and equations when calculating the required thickness for rock support in shallow tunnels and why it is used.

Two of the most commonly used rock supports are shotcrete and/or rock bolts.
This is used in order to increase the confining stress of the rock mass and hence increase the arching effect. If adequate support is not provided, joints may form loosened rock blocks that tend to fall out and change the geometry of the roof arch which will decrease the stability (Huang et al 2002). Shotcrete is used in the way that it is building up an arch that helps with carrying the load.

By assuming a thickness of the arch, \( t \), equal to the bolt length minus the anchor length of the bolt, a distance \( L_{dis} \) can be calculated with trigonometry, where \( L_{dis} \) is the length of each element effected by a persistent joint going through the arch. This is done for each joint starting from the center of the tunnel and out to the boundary. For each element is the resulting arch pressure divided into a shear load parallel with the joint (\( T_R \)) and a normal load perpendicular to the joint (\( N_R \)).
Figure 5: Illustration of angles and acting forces
\[ T_R = R \cdot \cos((90 - \alpha_{dis}) ) \]  
\[ N_R = R \cdot \sin((90 - \alpha_{dis}) ) \]

\( R \) is the resultant force of the arching pressure.

\[ R = \sqrt{V^2 + H_q^2} \]

Where:

- \( H_q \) : Horizontal force \([\text{kN/m tunnel}]\)
- \( V \) : Resulting support force, \( \frac{q \cdot x}{2} \) \([\text{kN/m tunnel}]\)

The shotcrete applies a counterpressure

\[ \sigma_{3,SB} = \frac{f_{cc} \cdot t}{r} \]

Where:
- \( f_{cc} \) : design compression strength of the shotcrete \([\text{MPa}]\)
- \( t \) : thickness of the shotcrete \([\text{m}]\)
- \( r \) : radius of the tunnel \([\text{m}]\)

The total counteracting force of the shotcrete becomes:

\[ S_{SB} = \sigma_{3,SB} \cdot L_{dis} \]

\( S_{SB} \) can be divided into a shear component parallel with the joint, \( T_{SB} \), and a normal component perpendicular to the joint, \( N_{SB} \):

\[ T_{SB} = S_{SB} \cdot \sin(\alpha_{dis}) \]
\[ N_{SB} = S_{SB} \cdot \cos(\alpha_{dis}) \]

With the forces acting on each element divided into a shear and a normal direction an equilibrium equation can be put up which gives the needed thickness of the shotcrete to keep the system stable.

If rock-bolts is needed similar calculations can be done. The equation for transferring shear stress for bolts in a joint is (without taking any consideration to possible reduction in tensile capacity due to shear stresses):

\[ f_f = (\rho \cdot f_{st} + \sigma_{fc}) \cdot \mu \]

- \( f_f \) : Shear strength of the joint \([\text{MPa}]\)
- \( \mu \) : Friction coefficient of the joints \([-]\)
- \( \rho \) : Ratio of bolt area to shotcrete area \([-]\)
- \( f_{st} \) : Yield stress of the bolt \([\text{MPa}]\)
- \( \sigma_{fc} \) : Compressive stress of the joint \([\text{MPa}]\)
By multiplying $\rho$ with $\sin \beta + \frac{\cos \beta}{\mu}$ gives:

$$f_f = \mu \left( \rho (\sin \beta + \frac{\cos \beta}{\mu}) f_{st} + \sigma f_c \right)$$

(17)

To obtain forces acting in the element:

$$T_B = f_f \cdot A$$

(18)

$$N_B = \sigma f_c \cdot A$$

(19)

$$A_s = \rho \cdot A \cdot \eta$$

(20)

$$\mu = \tan \phi_{joint}$$

(21)

Where:

- $\phi_{joint}$: friction angle of the joint
- $A$: Area of joint [m$^2$]
- $A_s$: Area of bolt [m$^2$]

By using equation 18 - 21 with equation 17 gives:

$$T_B = (N_B + (\sin \beta + \frac{\cos \beta}{\tan \phi_{joint}}) \eta \cdot A_s f_{st}) \cdot \tan \phi_{joint}$$

(22)

Where $\eta = \frac{L_{dis}}{s}$ and $\eta =$ number of bolts/m tunnel

The condition that has to be fulfilled throughout every point of the tunnel is:

$$T_B \geq T_{rot}$$

(23)
3 Methodology

The main purpose of this thesis is to see how well analytical solutions stand in comparison with a discrete numerical solution for shallow tunnels. This has been done by creating four different models in the numerical program UDEC. The four different models are used to see how different geometries and supports effect the results.

The analysis starts with a model containing a simple row of block with vertical joints. The second model is a tunnel with an arch-shaped roof and the joints will be modelled with a dip. The two last models will have an circular-shaped roof and inclined joints where the last model also will have support in form of shotcrete. The impact of rockbolts will not be tested in this thesis.

In this chapter, an insight into the numerical program UDEC will be given, including geometries and parameters that have been chosen for each model. The overall methodology of the work can be seen in Figure 6.

![Figure 6: Scheme of work-order.](image)

3.1 UDEC

This section is based on the UDEC manual made by ITASCA Consulting Group (UDEC Distinct-Element Modeling of Jointed and Blocky Material in 2D., Version 6.0). UDEC stands for Universal Distinct Element Code and is a two-dimensional numerical program based on the distinct element method for discontinuum modeling. It simulates the dynamic response of discontinuous media subjected to loading. The discontinuous media is represented as an assemblage of discrete blocks. The blocks can behave either as rigid or deformable material. The deformable blocks are subdivided into a mesh of finite-difference elements and each element responds according to a prescribed linear or nonlinear stress-strain.
law. UDEC has several built in material models both for the intact blocks and for the discontinuous, which permit the simulation of response representative of discontinuous geological material. UDEC is based on a Lagrangian calculation scheme that is well suited to model the large movements and deformations for a blocky system.

### 3.2 Geometry

The size of the models in section 3.5 and 3.6 are built up with a width equal to 3 times the width of the tunnel on each side of the tunnel and two times the width of the tunnel in the vertical direction.

![Figure 7: Figure showing the size of model geometry](image)

**Rigid/Deformable-blocks:**

Early tests on the row of blocks model (section 3.4) showed that rigid blocks would not rotate and thus the system would not fail when exposed to relatively low stresses, hence deformable blocks will be used in all of the models.

**Mesh:**

The smaller each elements get the more accurate the solution becomes though this is at the cost of time since smaller elements means a larger amount of calculations. For each created model a sensitivity analysis has been done for the mesh size. By starting the calculations with a large mesh and from there decrease the element size until the result starts to converge to a constant solution which means a coarser mesh will not give a significantly better result. To reduce the simulation time even further the mesh size is different depending how far the element is from the tunnel face.
Rounding length:
Each corner is rounded with a circle that is tangential to the two corresponding edges at a given distance from the corner. The analytical method assumes perfectly squared blocks; hence the starting value for the rounding length will be put to a small value, though a sensitivity analysis will be done to see how it affects the results.

3.3 Joint model area

A rock joint is represented numerically as a contact surface formed between two block edges. For deformable blocks, point contacts are created at all gridpoints located on the block edge in contact; thus the number of contacts can be increased with a refined mesh. The point of contact between a corner and an edge is located at the intersection between the edge and the normal taken from the center of the radius of the circular arc at the corner with the edge.

In the normal direction, the stress-displacement relation is assumed to be linear and governed by the stiffness $k_n$:

$$\Delta \sigma_n = -k_n \Delta u_n$$ (24)

Where:
$\Delta \sigma_n$ : effective normal stress increment
$\Delta u_n$ : normal displacement increment

The shear-displacement relation is controlled by a constant shear stiffness, $k_s$ and the maximum shear strength follows the Mohr-Coulomb criteria:

$$\tau_{max} = C + \sigma_n \tan \phi$$ (25)

and

$$\Delta \tau_s = -k_s \Delta u^e_s \leq \tau_{max}$$ (26)

$\Delta u^e_s$ : is the elastic component of the incremental shear displacement.

The joints may also dilate if the normal stress is not to high or the accumulated shear displacement exceeds a limiting value. This limitations corresponds to the observation that crushing of asperities at high normal stress or large shearing would eventually prevent the joint from dilating.

The contacts will have the following properties assigned:

$k_n$ : normal stiffness of the joint
$k_s$ : shear stiffness of the joint
$\phi$ : friction angle of the joint
$\Psi$ : dilation angle of the joint
$c$ : cohesion of the joint ($= 0$)
$\sigma_t$ : tensile strength of the joint ($= 0$)
3.4 Row of blocks

In order to analyze how well a numerical model in UDEC can represent the analytical models in section 2.2, a simple numerical model that consists of a row of blocks has been built. The model consists of 10 blocks with the dimensions 1x1 meters and are separated with vertical joints. The boundary conditions will be the same as for the analytical model, with both the outermost blocks fixed in the y-direction and with one of those blocks also fixed in the x-direction.

![Figure 8: Geometry of the row of block-model](image)

The blocks will be assigned an elastic and isotropic model and are characterized by reversible deformations upon unloading, with a linear and path-independent stress-strain law. The relation of stress to strain in incremental form is expressed by Hooke’s law. The properties for the blocks will have the values as follows, and will not be changed during the course of this thesis:

- Density = 2650 kg/m$^3$
- Young’s modulus = 39 GPa
- Bulk modulus = 26 GPa
- Shear modulus = 15.6 GPa
- Poisson’s ratio = 0.25

Seven different values of the vertical load have been tested, going from only gravitation, 0.0265 MPa up to 1.5 MPa.

The model are solved to equilibrium with only a horizontal stress applied and after that a distributed vertical load is applied on top of the beam. In the next step a fish function is called with the purpose to decrease the horizontal stress gradually when the y-velocity in the middle of the model is less than $1\times10^{-5}$ m/s. The decrease in horizontal stress will stop when the displacement is larger than a predefined value of 0.5 meters. The displacements and the velocities are plotted against the horizontal and Figure 12 show the threshold for how much horizontal force that is needed before the model goes to failure.

The parameters and their different values that have been tested for their impact are:
• Vertical load, $\sigma_v = 0.0265, 0.1, 0.3, 0.5, 0.7, 0.9, 1.5$ MPa
• Dilation angle, $\Psi = 0, 2.5, 5, 7.5, 10, 12.5, 15$ degrees
• Joint shear stiffness, $k_n = 1, 10, 100$ GPa
• Joint normal stiffness, $k_s = 0.1, 1, 10$ GPa
• Rounding lengths

Only one parameter have been changed at a time, for example when evaluating the impact from different vertical loads, the other parameters have been kept constant. The starting values for the parameters are as in Table 1.

<table>
<thead>
<tr>
<th>$\sigma_v$ [MPa]</th>
<th>$\Psi$ [$^\circ$]</th>
<th>$k_n$ [GPa]</th>
<th>$k_s$ [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0265</td>
<td>0</td>
<td>100</td>
<td>10</td>
</tr>
</tbody>
</table>

Limitations
When using the analytical method the vertical load is often the self weight of the rock above including the beam and if an external load exists for example a building of top of the tunnel, it is just added. The weight of the blocks will therefore be assumed to act on top of the blocks, and not distributed inside. The blocks will not have rounded corners since the analytical method does not take that into account, though it will be tested which impact the rounding length have. The ratio between normal- and shear-stiffness of the joint has been assumed to have a ratio of 10. Though this model is created to mimic the analytical model as much as possible it is still far away from the reality.

3.5 Arch-tunnel
In Sweden we often don’t build tunnels with a perfect circular roof, the usual way is to have an arch-shaped roof with a radius of the tunnel roof that is less than the width of the tunnel, see Figure 9. This could raise a problem when we are using the analytical model since the arching effect could be minimized when the roof does not have the shape of a circle. To analyze this, the model is done with a flatter roof to see how it behaves compared to the analytical model. In the next section, a model with a circular roof with a radius equal to half of the width of the tunnel will be shown and how it stands in comparison with the model in this section.

Previous models have been done in a simplified manner with vertical joints and rectangular geometry. To make it more realistic this model was done by using inclined joints instead of vertical and a tunnel shaped geometry instead of a beam.
The rock and joints parameters has been chosen to represent a hard crystalline rock. The tunnel geometry is built up with a width, $L$, of 25 meters and a height, $H$, of 22 meters, but the roof has a radius, $r$, of 5 meters. The joints will have an inclination of 75 degrees and a spacing of 2 meters. The rock cover, $B$, is set to 7 meters.

![Figure 9: Model geometry](image)

The values that has been used for the rock mass are as follows:

- Density = 2650 $kg/m^3$
- Young’s modulus = 39 GPa
- Bulk modulus = 26 GPa
- Shear modulus = 15.6 GPa
- Poisson’s ratio = 0.25
- Joint normal stiffness = 10 GPa
- Joint shear stiffness = 1 GPa

<table>
<thead>
<tr>
<th>L [m]</th>
<th>H [m]</th>
<th>r [m]</th>
<th>B [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>17</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>dip direction [°]</th>
<th>$\phi$ [°]</th>
<th>$q_v$ [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>30</td>
<td>185.5</td>
</tr>
</tbody>
</table>
The solving process is a bit different for this model compared to the model in section 3.4. Instead of applying a constant vertical load and finding the break point by calling a fish function that decreases the horizontal force over time the solving process was done by decreasing the horizontal in situ stress manually.

The rock mass includes continuously jointing with spacing of 2 meter and with a dip direction of 75 degrees and input parameters as in Table 3 and with a tunnel geometry as in Table 2. The vertical load that is acting on the system will only be due to self-weight.

**Limitations**
Since the model has gotten a bit more complex some assumptions has to be made. In the analytical model a horizontal force is used to calculate the maximum vertical load that the roof can carry and the first problem is to evaluate how this horizontal force should be derived. The way this has been handled is by printing the stresses acting in the middle of the model and integrating them from the roof boundary up to the surface for the overlying rock and this value has later been used as the horizontal force in the analytical calculations. Since the analytical method is derived for shallow tunnels the models in UDEC are built up with an overburden less than half of the tunnel width.

### 3.6 Circular-tunnel
This model is somewhat similar to the previous one, the only difference is that a circular roof is used, see Figure 10. Otherwise the setup is the same, with near vertical joints with a dip of 75 degrees as illustrated in Figure 11. Rock parameters are the same as for the arch-shaped model, and input values as in Table 4 and 5.

<table>
<thead>
<tr>
<th>Table 4: Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>L [m]</td>
</tr>
<tr>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5: Input values</th>
</tr>
</thead>
<tbody>
<tr>
<td>dip direction [°]</td>
</tr>
<tr>
<td>75</td>
</tr>
</tbody>
</table>
The parameters for the rockmass that were used in section 3.5 has also been used for this model:

- Density: 2650 kg/m³
- Young’s modulus: 39 GPa
- Bulk modulus: 26 GPa
- Shear modulus: 15.6 GPa
- Poisson’s ratio: 0.25
- Joint normal stiffness: 10 GPa
- Joint shear stiffness: 1 GPa
Limitations
Since the model is more complex compared to the row of blocks simplifying assumptions has to be made. In the analytical model a horizontal force is used to calculate the maximum vertical load that the roof can carry and the first problem is to evaluate how this horizontal force should be derived. The way this has been handled is by printing the stresses acting in the middle of the model and integrating them from the roof boundary up to the surface for the overlaying rock and this value has later been used as the horizontal force in the analytical calculations. Since the analytical method is derived for shallow tunnels the models in UDEC are built up with a overburden less than half of the tunnel width. It is therefore believed that this approximation is acceptable.

3.7 Circular tunnel with support
To see how the analytical method with support works in comparison with UDEC a model was built with the same input parameters as in Table 5 but also with a support lining installed in the model. The properties that has been used for the shotcrete are as in Table 6.

Table 6: Input parameters for shotcrete.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density [kg/m$^3$]</td>
<td>2300</td>
</tr>
<tr>
<td>Elastic modulus [GPa]</td>
<td>16</td>
</tr>
<tr>
<td>Poisson’s ratio [-]</td>
<td>0.25</td>
</tr>
<tr>
<td>Compressive yield strength [MPa]</td>
<td>30.5</td>
</tr>
<tr>
<td>Tensile yield strength [MPa]</td>
<td>4</td>
</tr>
<tr>
<td>Residual tensile yield strength [MPa]</td>
<td>3</td>
</tr>
</tbody>
</table>

To evaluate how the support lining affects the UDEC calculations a horizontal in situ stress of 0.45 MPa were applied which is a value that would make the model unstable without support. The thickness of the lining where changed manually in the model to find the minimum thickness to keep the tunnel stable.

Limitations
The same assumptions as for the previous models have been made. Only shotcrete as support will be taken into account, hence the impact of rockbolts will not be investigated.
4 Calculations and Results

In this section an overview of the results are presented, and a close up on some chosen values. For all results see Appendices 1-4.

4.1 Row of blocks

Below are the results from the model with input data as follows:

- Vertical load, $\sigma_v = 0.5$ MPa
- Dilation angle, $\Psi = 0$
- Joint normal stiffness, $k_n = 100$ GPa
- Joint shear stiffness, $k_s = 10$ GPa

With these input values for the Udec model a plot was done with horizontal force vs y-displacement (see Fig 12) and as it shows the model breaks when the horizontal force is smaller than 4.7 MN.

Figure 12: Plot showing horizontal stress against deformation. (Horizontal stress on y-axis and vertical displacement on x-axis.)
The major principal stresses are shown in Figure 13 and it can be seen how the pressure line has formed a arch like shape as expected. Different values for the vertical load has been tested to see how it affects the result. Table 7 shows the results from the different values of the vertical load and how the horizontal force needed to keep the analytical solution stable stands in comparison with the horizontal force needed to keep the UDEC-model stable.
Table 7: Results of horizontal force needed to keep the model stable with a given vertical load.

<table>
<thead>
<tr>
<th>$q$ [MN/m$^2$]</th>
<th>$Hq_{\text{Analytical, Eq. 7}}$ [MN/m]</th>
<th>$Hq_{\text{UDEC}}$ [MN/m]</th>
<th>$Hq_{\text{Analytical}} / Hq_{\text{UDEC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>12.9</td>
<td>14.6</td>
<td>0.88</td>
</tr>
<tr>
<td>0.9</td>
<td>7.7</td>
<td>8.5</td>
<td>0.91</td>
</tr>
<tr>
<td>0.7</td>
<td>6.0</td>
<td>6.6</td>
<td>0.91</td>
</tr>
<tr>
<td>0.5</td>
<td>4.3</td>
<td>4.7</td>
<td>0.92</td>
</tr>
<tr>
<td>0.3</td>
<td>2.6</td>
<td>2.9</td>
<td>0.90</td>
</tr>
<tr>
<td>0.1</td>
<td>0.86</td>
<td>1.1</td>
<td>0.78</td>
</tr>
<tr>
<td>0.0265</td>
<td>0.23</td>
<td>0.5</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Dilation
To see how the dilation angle would affect the results gotten from UDEC the same model were tested with a constant vertical load of 0.5 MN/m$^2$ applied and by decreasing the horizontal force until failure. The results of the minimum horizontal force to keep the model stable can be seen in Table 8. As Table 8 shows the dilation angle does not have a significant impact on the results, for this particular model.

Table 8: Influence by changing the dilation angle.

<table>
<thead>
<tr>
<th>Dilation angle [-]</th>
<th>$Hq_{\text{UDEC}}$ [MN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.7</td>
</tr>
<tr>
<td>2.5</td>
<td>4.8</td>
</tr>
<tr>
<td>5</td>
<td>4.8</td>
</tr>
<tr>
<td>7.5</td>
<td>4.8</td>
</tr>
<tr>
<td>9</td>
<td>4.8</td>
</tr>
<tr>
<td>10.5</td>
<td>4.8</td>
</tr>
<tr>
<td>12</td>
<td>4.8</td>
</tr>
<tr>
<td>15</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Joint shear and normal stiffness
Different values of the joint shear and normal stiffness were also tested in UDEC. Those parameters have no impact on the analytical solution but it is interesting to see how they change the result in UDEC. It is assumed that the values of these parameters are in the range of 0.1 to 100 GPa and with a ratio between them of 10. The results shows that increasing the stiffness’s will increase the overall stability of the tunnel, see Table 9.
Table 9: Influence by changing the joint shear and normal stiffness.

<table>
<thead>
<tr>
<th>Jkn/Jks [GPa/m]</th>
<th>Hq (UDEC) [MN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000/100</td>
<td>5.0</td>
</tr>
<tr>
<td>100/10</td>
<td>4.8</td>
</tr>
<tr>
<td>10/0.1</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Rounding length
Changes of the rounding length of the blocks has the biggest impact of all tested parameters. By rounding the corners the row tends to fall apart easier and sharper edges made the block-row more stable. This could be explained with each blocks ability to rotate against each other, with more rounded corners the blocks have a more free space to rotate and hence tends to fall easier. Since the analytical model does not take the rounding length of the blocks into account a rounding length equal to 0.05 has been used for the rest of the models.

4.2 Arch-tunnel
The results shows that with only self-weight applied the in situ stress needed to keep the tunnel stable has to be larger than 0.55 MPa, and the integrated horizontal force acting above the tunnel roof equals 7.8 MN. A tunnel failure occuring with horizontal stress equal to 0.55 MPa is shown in Figure 14.

Figure 14: Tunnel failure when horizontal stress falls below 0.55 MPa
To compare this result with the analytical model the equivalent friction angle has to be calculated according to equation 8.

\[ \phi' = \phi - \beta \rightarrow \phi' = 30^\circ - (90^\circ - 75^\circ) = 15^\circ \]

With input values according to Table 3 a minimum horizontal force can be calculated with the analytical equations for the rotation and sliding criterions.

\[
H_q \geq \frac{\sigma_v L^2}{8B} = \frac{188.5 \cdot 10^3 \cdot 25^2}{8 \cdot 7} = 2.1 \text{ MN/m (Rotation)}
\]

\[
H_q \geq \frac{\sigma_v L}{2 \tan \phi} = \frac{185.5 \cdot 10^3 \cdot 25}{2 \tan 15} = 8.65 \text{ MN/m (Sliding)}
\]

The failure criterion that should be used is the sliding criteria since that is the one needing the highest horizontal force. As for the first model in section 3.4 the two different solutions only differs with 10 \%.

Figure 15: Contour plot of major principal stresses.
4.3 Circular-tunnel

The results show that with only self-weight applied the in situ stress needed to keep the tunnel stable has to be larger than 0.6 MPa. In the same manner as in section 3.5 a horizontal force were integrated in the tunnel to a value of 8.76 MN. Failure of the tunnel with a horizontal stress of 0.6 MPa is shown in Figure 16.

![Figure 16: Tunnel failure when horizontal stress falls below 0.6 MPa](image)

To compare this result with the analytical model the equivalent friction angle has to be calculated according to equation 8.

\[
\phi' = \phi - \beta \rightarrow \phi' = 30 - (90 - 75) = 15^\circ
\]

With input values according to Table 5 a minimum horizontal force can be calculated with the analytical equations for the rotation and sliding failure.

\[
H_q \geq \frac{\sigma_v L^2}{8B} = \frac{188.5 \cdot 10^3 \cdot 25^2}{8 \cdot 7} = 2.1 \text{ MN/m} \quad (\text{Rotation})
\]

\[
H_q \geq \frac{\sigma_v L}{2 \tan \phi} = \frac{185.5 \cdot 10^3 \cdot 25}{2 \tan 15} = 8.65 \text{ MN/m} \quad (\text{Sliding})
\]
The result from UDEC with a horizontal force of 8.76 MN/m integrated above the tunnel is in close agreement with the value of 8.65 MN/m calculated with the analytical model.

![UDEC Contour Plot](image)

**Figure 17**: Contour plot of major principal stresses.

### 4.4 Circular-tunnel with support

The results showed that a shotcrete thickness of 0.15 meters where needed to keep the tunnel stable (horizontal stress equal to 0.45 MPa). To see how this result stands in comparison with the analytical method, calculations where done according to section 2.5. The results from the analytical model shows that a thickness of 0.16 meters for the shotcrete is needed to keep the tunnel stable, see Appendix 4 for calculations. The value of the horizontal force used in the calculations were evaluated by integrating the stresses in the middle of the tunnel, to the tunnel surface.

### 5 Discussion

#### 5.1 Row of blocks

As Table 7 shows the difference between the analytical model and UDEC tends to differ with 10 % which has to be seen as a good result. To keep the beam from failure the horizontal force that has to be applied in UDEC needs to be bigger than what the analytical solution gives, which means that the numerical model will fail earlier, assuming that both are put under the same vertical stress.
It could be argued that this model is dangerous since one of the results seems to show a 50% difference but it has to be said that this is for very small forces, which will never occur in reality, the vertical load of 0.0265 MN/m is only the self weight of a one meter high row of blocks and as the results shows it only needs 0.5 MN in horizontal force which could be translated into a horizontal stress of 0.5 MPa, which is quite low. The in situ stresses in typical Swedish rock is usually higher than this without the arching effect and hence this result should not have any impact. Because of this, the conclusion could be that the two models differs with approximately 10 percent.

It is important to remember that those results are with sharp edged blocks and that the difference would be bigger if the corners would be more rounded, though first and foremost this thesis is about how well the analytical model is compared to UDEC, and the analytical model does not take the rounding length of the blocks into account since it uses perfectly squared blocks. It can also be assumed that the hard crystalline rock behaves more like sharp edged blocks rather than rounded blocks. Therefore, no rounding length has been included in the other models.

5.2 Arch-tunnel

For this model, as well as the previous one in section 3.4 the results from the numerical method differs with approximately 10% compared with the analytical results. Though for this case the numerical model is more safe than the analytical, since this one needs a smaller value on the horizontal force to keep it stable. However, it is still only 10% and that should not be considered a significant model error, compared to the errors that the model has when describing the reality, such as the geometry, both in regard of joint patterns and shape of the tunnel boundary.

5.3 Circular-tunnel

Both results seems to coincide with each other quite well both without and with support. This model gives a lower stability compared with the analytical result, though it is still in the same range, 8.76 MN against 8.65 MN which has to be seen as a very good result.

The shotcrete thickness differs only with 0.01 meters when comparing the analytical results with the numerical results from UDEC. The small difference should not be seen as they work perfectly. The small amount of test for this model implies that it might be a coincidence that they give approximately the same thickness.
5.4 General discussion of the results

All three models in UDEC seems to give a satisfying result with a difference of approximately 10% compared to the results calculated with the analytical methods, except for the last model which is even closer. All models shows that the analytical model does work and gives satisfying results. The question is if the few analysis performed in this thesis is enough to draw any firm conclusions. As stated in this thesis a lot of assumptions have been made and it should not be considered to describe the reality in a perfect way. One parameter that has been significantly simplified is the joint pattern; in reality the joints are often more winding and thus it could be assumed that they interlock each other which might give a higher stability.

In Sweden we often build tunnels with a flatter roof, like the model in section 3.5. The results from that section shows that the analytical model underestimates the stability of the tunnel, but this is just a comparison with that specific model. Though it is built in an advanced program like UDEC a lot of simplifications has still been made. We will never get a case were the rock parameters, joint pattern and tunnel geometry in this model describes a project in reality. With this said; the analytical model should be used carefully. Especially for tunnels with a flatter roof, which the model with support is not developed for.

6 Conclusions and further research

The conclusion that can be drawn from this work is that the analytical methods used today seems to show a difference of approximately 10%. Though it should still be investigated how good it stands against a real project and a numerical model in UDEC that is describing that project as close as possible.

For further research it would be interesting to see how it would work if there was a model with a joint set of higher complexity and how this model would work compared to the analytical results. The analytical methods can not handle interlocking joints, though this can be modelled in UDEC. This will probably show a bigger difference in the results between an analytical and a numerical solution and hence probably the limit of what is achievable with the analytical methods.

For the shotcrete calculations more runs should be done with different joint inclinations. The same model could also be used but with a change in the horizontal in situ stress.
7 References


8 Appendix 1 - Row of blocks

With the equations for sliding and rotation criteria and with known vertical load the minimum horizontal load needed to keep the model stable can be calculated.

\[ q_{\text{max}} = \frac{2Hq}{L} \cdot \tan \phi \]

\[ \text{max} = \frac{8Hq \cdot B}{L^2} \]

\( q=1.5 \text{ MN/m} \)

\[ Hq = \frac{1.5 \cdot 8}{2 \tan 25} = 12.9 \text{MN} \]

\[ Hq = \frac{1.5 \cdot 8^2}{8 \cdot 1} = 12 \text{MN} \]

\( q=0.9 \text{ MN/m} \)

\[ Hq = \frac{0.9 \cdot 8}{2 \tan 25} = 7.7 \text{MN} \]

\[ Hq = \frac{0.9 \cdot 8^2}{8 \cdot 1} = 7.2 \text{MN} \]

\( q=0.7 \text{ MN/m} \)

\[ Hq = \frac{0.7 \cdot 8}{2 \tan 25} = 6.0 \text{MN} \]

\[ Hq = \frac{0.7 \cdot 8^2}{8 \cdot 1} = 5.6 \text{MN} \]

\( q=0.5 \text{ MN/m} \)

\[ Hq = \frac{0.5 \cdot 8}{2 \tan 25} = 4.3 \text{MN} \]

\[ Hq = \frac{0.5 \cdot 8^2}{8 \cdot 1} = 4.0 \text{MN} \]
q=0.3 MN/m

\[ H_q = \frac{0.3 \cdot 8}{2 \tan 25} = 2.6\text{MN} \]

\[ H_q = \frac{0.3 \cdot 8^2}{8 \cdot 1} = 2.4\text{MN} \]

q=0.1 MN/m

\[ H_q = \frac{0.1 \cdot 8}{2 \tan 25} = 0.86\text{MN} \]

\[ H_q = \frac{0.1 \cdot 8^2}{8 \cdot 1} = 0.80\text{MN} \]

q=0.0265 MN/m

\[ H_q = \frac{0.0265 \cdot 8}{2 \tan 25} = 0.22\text{MN} \]

\[ H_q = \frac{0.0265 \cdot 8^2}{8 \cdot 1} = 0.21\text{MN} \]

36
9 Appendix 2 - Arch-tunnel

To keep the tunnel stable in UDEC an applied horizontal insitu stress of 0.55 MPa were needed. In the middle of the tunnel, from the roof boundary up to the model boundary, \((x_1 = 0, y_1 = 22; x_2 = 0, y_2 = 29)\) the vertical distance were divided into smaller elements and for each element with a known distance the horizontal stress were printed (\((\sigma_1)\) and with the known distance a horizontal force acting on that length could be derived.

<table>
<thead>
<tr>
<th>(y)</th>
<th>(\sigma_1) [MN]</th>
<th>delta (Y)</th>
<th>(F_x) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>1.197</td>
<td>0.175</td>
<td>209</td>
</tr>
<tr>
<td>22.4</td>
<td>1.197</td>
<td>0.35</td>
<td>419</td>
</tr>
<tr>
<td>22.7</td>
<td>1.158</td>
<td>0.35</td>
<td>405</td>
</tr>
<tr>
<td>23.1</td>
<td>1.165</td>
<td>0.35</td>
<td>408</td>
</tr>
<tr>
<td>23.4</td>
<td>1.165</td>
<td>0.35</td>
<td>408</td>
</tr>
<tr>
<td>23.8</td>
<td>1.137</td>
<td>0.35</td>
<td>398</td>
</tr>
<tr>
<td>24.1</td>
<td>1.125</td>
<td>0.35</td>
<td>394</td>
</tr>
<tr>
<td>24.5</td>
<td>1.121</td>
<td>0.35</td>
<td>392</td>
</tr>
<tr>
<td>24.8</td>
<td>1.112</td>
<td>0.35</td>
<td>389</td>
</tr>
<tr>
<td>25.2</td>
<td>1.108</td>
<td>0.35</td>
<td>388</td>
</tr>
<tr>
<td>25.5</td>
<td>1.105</td>
<td>0.35</td>
<td>387</td>
</tr>
<tr>
<td>25.9</td>
<td>1.098</td>
<td>0.35</td>
<td>384</td>
</tr>
<tr>
<td>26.2</td>
<td>1.107</td>
<td>0.35</td>
<td>387</td>
</tr>
<tr>
<td>26.6</td>
<td>1.115</td>
<td>0.35</td>
<td>390</td>
</tr>
<tr>
<td>26.9</td>
<td>1.124</td>
<td>0.35</td>
<td>393</td>
</tr>
<tr>
<td>27.3</td>
<td>1.127</td>
<td>0.35</td>
<td>394</td>
</tr>
<tr>
<td>27.6</td>
<td>1.146</td>
<td>0.35</td>
<td>401</td>
</tr>
<tr>
<td>28.0</td>
<td>1.149</td>
<td>0.35</td>
<td>402</td>
</tr>
<tr>
<td>28.3</td>
<td>1.163</td>
<td>0.35</td>
<td>407</td>
</tr>
<tr>
<td>28.7</td>
<td>1.176</td>
<td>0.35</td>
<td>412</td>
</tr>
<tr>
<td>29.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(F_{x_{tot}}\) 7770

Table 10: Horizontal forces acting in the tunnel roof

The total horizontal force needed to keep the tunnel stable according to UDEC is as can be seen in Table 10 is 7.77 MN, and to see how this solution stands in comparison with an analytical solution following calculations have been done:

*Equivalent friction angle:*

\[
\phi' = \phi - \beta \rightarrow \phi' = 30 - (90 - 75) = 15^\circ
\]

With input values according to table 3 a minimum horizontal force can be calculated with the analytical equations for the rotation and sliding criterions.

\[
H_q \geq \frac{\sigma_v L^2}{8B} = \frac{188.5 \cdot 10^3 \cdot 25}{8 \cdot 7} = 2.1 \text{ MN } (\text{Rotation})
\]

37
\[ H_q \geq \frac{\sigma_v L}{2 \tan \phi} = \frac{185.5 \cdot 10^3 \cdot 25}{2 \tan 15} = 8.65 \text{ MN} \quad \text{(Sliding)} \]

The deciding horizontal force from the analytical solution differs with 10% compared with the horizontal force needed in UDEC:

\[ F_{x_{\text{tot}}} = 7.77 \text{ MN} \]

\[ H_{q,\text{analytical}} = 8.65 \text{ MN} \]

\[ \frac{F_{x_{\text{tot}}}}{H_{q,\text{analytical}}} = \frac{7.77}{8.65} = 0.9 \]
10 Appendix 3 - Circular-tunnel

To keep the tunnel stable in UDEC an applied horizontal in situ stress of 0.55 MPa were needed. In the middle of the tunnel, from the roof boundary up to the model boundary, \((x_1 = 0, y_1 = 22; x_2 = 0, y_2 = 29)\) the vertical distance were divided into smaller elements and for each element with a known distance the horizontal stress were printed \((\sigma_1)\) and with the known distance a horizontal force acting on that length could be derived.

\[
\begin{array}{|c|c|c|c|}
\hline
y & \sigma_1 [MN] & \text{delta Y} & Fx [kN] \\
\hline
22 & 1.704 & 0.175 & 298 \\
22.4 & 1.704 & 0.35 & 596 \\
22.7 & 1.573 & 0.35 & 551 \\
23.1 & 1.593 & 0.35 & 558 \\
23.4 & 1.593 & 0.35 & 558 \\
23.8 & 1.437 & 0.35 & 503 \\
24.1 & 1.365 & 0.35 & 478 \\
24.5 & 1.330 & 0.35 & 466 \\
24.8 & 1.284 & 0.35 & 449 \\
25.2 & 1.254 & 0.35 & 439 \\
25.5 & 1.218 & 0.35 & 426 \\
25.9 & 1.169 & 0.35 & 409 \\
26.2 & 1.155 & 0.35 & 404 \\
26.6 & 1.133 & 0.35 & 397 \\
26.9 & 1.110 & 0.35 & 389 \\
27.3 & 1.108 & 0.35 & 388 \\
27.6 & 1.054 & 0.35 & 369 \\
28.0 & 1.054 & 0.35 & 369 \\
28.3 & 1.032 & 0.35 & 361 \\
28.7 & 1.002 & 0.35 & 351 \\
29.0 & - & - & - \\
\hline
\text{Fx}_{\text{tot}} & 8760 \\
\hline
\end{array}
\]

Table 11: Horizontal forces acting in the tunnel roof

The total horizontal force needed to keep the tunnel stable according to UDEC is as can be seen in Table 11 is 78.76 MN, and to see how this solution stands in comparison with an analytical solution following calculations have been done:

**Equivalent friction angle:**

\[\phi' = \phi - \beta \rightarrow \phi' = 30 - (90 - 75) = 15^\circ\]

With input values according to table 3 a minimum horizontal force can be calculated with the analytical equations for the rotation and sliding criterions.

\[
Hq \geq \frac{\sigma_u \cdot L^2}{8B} = \frac{188.5 \cdot 10^3 \cdot 25^2}{8 \cdot 7} = 2.1 \text{ MN} \quad (\text{Rotation})
\]
\[ H_q \geq \frac{\sigma_v L}{2 \tan \phi} = \frac{185.5 \cdot 10^3 \cdot 25}{2 \tan 15} = 8.65 \text{ MN} \quad (Sliding) \]

The deciding horizontal force from the analytical solution differs with 1% compared with the horizontal force needed in UDEC:

\[ F_{x_{\text{tot}}} = 8.76 \text{ MN} \]

\[ H_{q,\text{analytical}} = 8.65 \text{ MN} \]

\[ \frac{F_{x_{\text{tot}}}}{H_{q,\text{analytical}}} = \frac{8.76}{8.65} = 0.99 \]
11 Appendix 4 - Circular tunnel with support

Figure 18 shows the results for the shotcrete calculations for each element, (x=0 to x=12.5). To get the factor of safety equal to 1 or more for each element a thickness of 0.16 meters was needed for the shotcrete.

Closer look on how the calculations have been done for x = 0.

**Input values:**

\[ x = 0 \]

\[ \alpha = 75^\circ \] (Joint dip direction)

\[ \phi = 30^\circ \] (Friction angle)

\[ q = 185.5 \text{ kPa} \] (Vertical load)

\[ H_q = 5640 \text{ kN/m} \] (Horizontal force, integrated from UDEC)

\[ V = \frac{q \cdot x}{2} = \frac{185.5 \cdot 0}{2} = 0 \] (Resulting support force)

\[ f = 5 \text{m} \] (Assumed thickness of arch)

\[ f_{cc} = 30.5 \text{ MPa} \] (Compressive yield strength for shotcrete)

\[ t = 0.16 \text{m} \] (Shotcrete thickness)

\[ r = 12.5 \text{m} \] (Radius of tunnel)
Calculations:

$\alpha_{comp} = \arctan \frac{8 \cdot x}{B} = 0$ (Inclination of pressure line)

$\alpha_{Dis} = \alpha - \alpha_{comp} = 75 - 0 = 75^\circ$

$R = \sqrt{V^2 + H^2} = \sqrt{0^2 + 5640^2} = 5640 \text{kN/m}$ (Resultant force of the arching pressure)

$T_R = R \cdot \sin(90 - \alpha_{Dis}) = 5640 \cdot \sin(90 - 75) = 1460 \text{ kN}$ (Shear component of R)

$N_R = R \cdot \cos(90 - \alpha_{Dis}) = 5640 \cdot \cos(90 - 75) = 5448 \text{ kN}$ (Normal component of R)

$L_{Dis} = \frac{f}{\tan \alpha_{Dis}} = \frac{5}{\tan 75} = 1.3 \text{m}$

$\sigma_{3,SB} = \frac{L_{Ext}}{r} = \frac{30.5 \cdot 10^3 \cdot 0.16}{12.5} = 0.390 \text{ MPa}$ (Counterpressure from shotcrete)

$S_{SB} = \sigma_{3,SB} \cdot L_{Dis} = 0.390 \cdot 10^3 \cdot 1.3 = 523 \text{ kN}$ (Total counteracting force from shotcrete)

$N_{SB} = S_{SB} \cdot \cos \alpha_{Dis} = 523 \cdot \cos 75 = 135 \text{ kN}$ (Normal component of $S_{SB}$)

$T_{SB} = S_{SB} \cdot \sin \alpha_{Dis} = 523 \cdot \sin 75 = 505 \text{ kN}$ (Shear component of $S_{SB}$)

$T_{Tot} = T_R - T_{SB} = 1460 - 505 = 955 \text{ kN}$

$N_{Tot} = N_R + N_{SB} = 5448 + 135 = 5583 \text{ kN}$

$T_{BF} = \frac{T_{Tot}}{\tan \phi} = \frac{5583}{0.57} = 3.88$ (Factor of safety)

For $x=0$ gives a factor of safety above 1 which means that the shotcrete thickness is too thick, but if the same calculations are carried out for $x_1 = 0 \text{ m to } x_n = 12.5 \text{ m}$ a table as in Figure 18 can be created and as can be seen in the figure the safety factor for $x=12.5$ is 1 and hence the thickness of the shotcrete is correct.