Frequency Response Analysis of IEMI in Power Line Network by Using Monte Carlo Approach

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***Abstract*—In this paper, we investigate the targeted load frequency responses of Intentional Electromagnetic Interference (IEMI) in low voltage power line network, which consists of multiple junctions and branches. A disturbance that is injected at a random position in the network is considered in our work, and we study the impact of the position of the injection point in the sense of probability distribution, through the Monte Carlo method. To increase the precision, in the Monte Carlo simulation model, we introduce three variance reduction techniques, namely, complementary random numbers, correlated sampling and stratified sampling, and we use them in combination. Results show that they can significantly reduce the variance and increase the simulation precision. More importantly, simulations quantitatively show that, controlling the probability of criminal accessing the targeted load can effectively reduce the influence level, which is crucial for ensuring the security and robustness of whole networks.**

***Index Terms*—Monte Carlo, variance reduction, IEMI, frequency response**

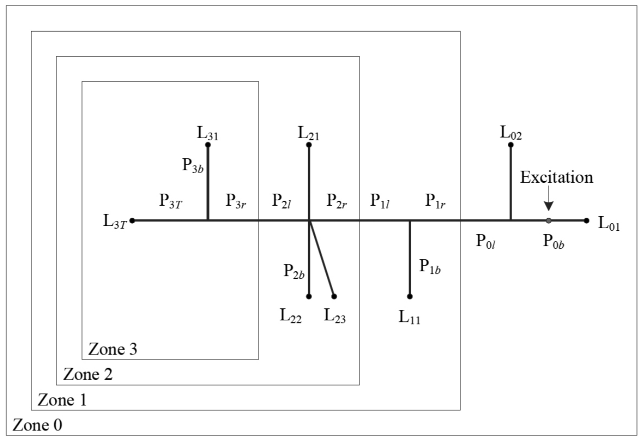

Fig. 1. A low voltage power line network installed in the building divided into four zones.

I. Introduction

In this work, we study the propagation of Intentional Electromagnetic Interference (IEMI) [1], [2] in a low voltage power line network with multiple junctions and branches. The point of injection of the disturbance can be randomly distributed in the network. The probabilities of the criminal gaining access to different regions of the network are different. To evaluate the potential risks of the target being damaged by the disturbance, due to the uncertainty of the point of injection of the disturbance, the Monte Carlo method is used to analyze the expectation value of the targeted load frequency response.

The remainder of the paper is organized as follows. In Sec. II, we describe the scenario and formulate our problem by showing a mathematical model. To improve the simulation precision, which is mainly decided by the variance of the estimated expectation value, we introduced three typical variance reduction techniques used in the Monte Carlo simulation in Sec. III. Subsequently, a practical example with specific simulation parameters is studied in Sec. IV. Finally, the conclusions are drawn in Sec. V.

II. Mathematical Model

Among various approaches for solving the IEMI problem, electromagnetic topology [3] is an effective and popular tool that is based on the concept of zones.

We consider a low voltage power line network with four junctions and eight branches, which can be divided into four zones, namely, Zone 0, 1, 2 and 3, as shown in Fig. 1. From Zone i to Zone j, $i, j = 0, 1, 2, 3$, there are totally $|i-j|$ zone boundaries required to cross. Also, all the loads are general systems, except for the system labeled as Z_{3T}, which performs critical functions.

Assume that an EMI source appears in Zone i with probability P_i, carried by a criminal with the aim of destroying the system Z_{3T}. In each zone, EMI injection may happen at different positions (lines) of the power line network. We denote by P_{mb} the probability of injection point occurring on the branch; otherwise, the probability is represented by P_{md}, i.e., the injection point is between junctions. Here, m is the index of zone, b is the index of branch, $d = l$ or $d = r$ is the position of the injection point that is on the left-hand side or right-hand side of the junction in Zone m.

Preliminarily, the input, output, model parameters, and the mathematical model are formulated as follows.

- Input:

x_{s} denotes the distance between the interference source and the junction which located in the same zone, and the probability functions are given by

$$f_1(x_{\mathrm{s}}) = \{P_i : \text{in Zone } i, i = 0, 1, 2, 3\},$$

$$f_2(x_{\mathrm{s}}) = \begin{cases} P_{mb}, & \text{injected on branches,} \\ P_{md}, & \text{injected between junctions,} \\ P_{3T}, & \text{injected on the same branch as target.} \end{cases}$$

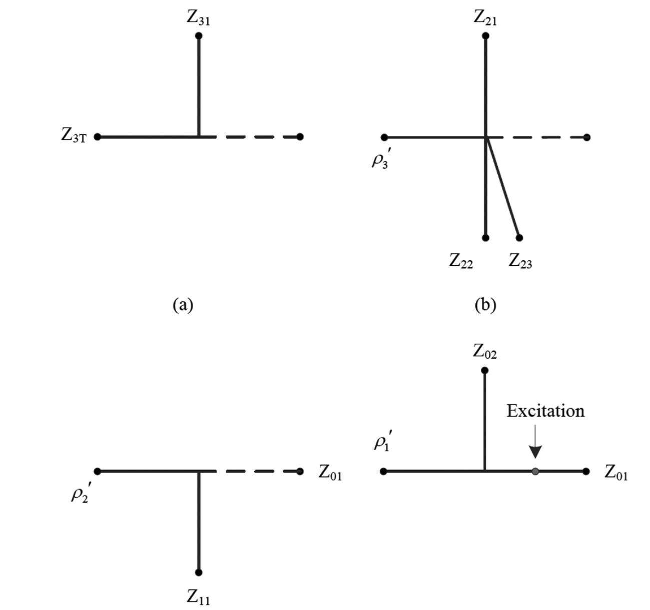

Fig. 2. Decomposition schematic of the network with multiple junctions.

- Output:

V denotes frequency response of the targeted load.

- Model parameters:

f_{int} : frequency of the interference source,
V_{s} : voltage amplitude of the interference source,
I_{s} : current amplitude of the interference source,
Z_{c} : characteristic impedance of power line,
γ : propagation constant,
L_{mb} : length of branch line,
L_{md} : length of line between junctions,
Z_{mb} : load impedance.

Here, m, b and d similarly define the index of zone, index of branch and the position relative to the junction, respectively.

To simplify the problem, we assume that all the power lines have the same characteristic parameters, which means that the value of Z_{c} is the same for each line, as well as γ. This can be changed without causing large problems for the method itself.

- Mathematical model:

The mathematical model is established based on the modified BLT equation. To calculate the frequency responses of the loads in a low voltage power line network, we used the method proposed in [4]. The basic idea of the method is dividing the network into several single junction networks and introducing a virtual branch with zero length to preserve the characteristics of single junction networks. The step-by-step decomposition of the network studied in this paper, iteratively from Zone 3 to Zone 0, is shown in Fig. 2.

As a specific example, for the one-junction network shown in Fig. 2d), the modified BLT equation is

$$\mathbf{R} = (\mathbf{I} + \mathbf{P})\,(\mathbf{I} - \mathbf{\Gamma}\mathbf{P})^{-1}\,\mathbf{S}, \tag{1}$$

where $\mathbf{R} = [V_1, V_2, V_3]^T$ is the voltage vector showing the responses of the three loads, $\mathbf{I}$ is the identity matrix of size 3, $\mathbf{P} = \operatorname{diag}(\rho_{01}, \rho_{02}, \rho_1')$ is the reflection matrix with $\rho_{0i} = (Z_{0i} - Z_{\text{c}})\,/\,(Z_{0i} + Z_{\text{c}})$, $i = 1, 2$, denoting the reflection coefficients at the two loads, ρ_1' is the corrected reflection coefficient at the first junction, which is calculated by (2). Besides, the transmission matrix $\mathbf{\Gamma}$ and excitation source vector $\mathbf{S}$ are respectively given by

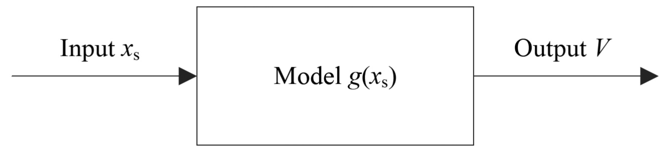

Fig. 3. Diagram of the mathematical model.

$$\mathbf{\Gamma} = \begin{bmatrix} \rho^{(01)} e^{-2\gamma L_{01}} & T^{(02)} e^{-\gamma(L_{01}+L_{02})} & T^{(0l)} e^{-\gamma(L_{01}+L_{0l})} \\ T^{(01)} e^{-\gamma(L_{01}+L_{02})} & \rho^{(02)} e^{-2\gamma L_{02}} & T^{(0l)} e^{-\gamma(L_{02}+L_{0l})} \\ T^{(01)} e^{-\gamma(L_{01}+L_{0l})} & T^{(02)} e^{-\gamma(L_{02}+L_{0l})} & \rho^{(0l)} e^{-2\gamma L_{0l}} \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} -\frac{V_{\text{s}} - Z_{\text{c}} I_{\text{s}}}{2} e^{-\gamma x_{\text{s}}} + \rho^{(01)} \frac{V_{\text{s}} + Z_{\text{c}} I_{\text{s}}}{2} e^{-\gamma(2L_{01} - x_{\text{s}})} \\ T^{(01)} \frac{V_{\text{s}} + Z_{\text{c}} I_{\text{s}}}{2} e^{-\gamma(L_{01} + L_{02} - x_{\text{s}})} \\ T^{(01)} \frac{V_{\text{s}} + Z_{\text{c}} I_{\text{s}}}{2} e^{-\gamma(L_{01} + L_{0l} - x_{\text{s}})} \end{bmatrix}.$$

Without loss of generality, for a junction with $N + 1$ branches, the reflection coefficient $\rho^{(0i)}$ and transmission coefficient $T^{(0i)}$ is given by [5]

$$\rho^{(0i)} = \frac{1 - N}{1 + N} \quad \text{and} \quad T^{(0i)} = 1 + \rho^{(0i)}.$$

Particularly, in Fig. 2d), we have $N = 2$ and $i = 1, 2, l$.

Besides, to calculate the load response, we need to correct the reflection coefficient at the first junction, by using the following equation [4]

$$\rho_1' = T^{(0l)} \left(1 - \sum_{i=1}^{N} \frac{T^{(0i)}}{A_{0i}} \right)^{-1} - 1, \tag{2}$$

with

$$A_{0i} = 1 + \frac{1}{\rho_{0i} e^{-2\gamma L_{0i}}}.$$

As the input, the distance between the interference source and the junction x_s is assumed to be a random variable, which definitely results in random fluctuations in the frequency responses of the targeted load. To address the issue with a stochastic input, we apply the Monte Carlo method for analysis.

III. Monte Carlo Method

The basic idea of the Monte Carlo method is to generate a sequence of random values of all inputs, and to produce the corresponding random outputs. Here, we only have one input and one output, which are x_s and V, respectively, as shown in Fig. 3. The application of the Monte Carlo method is popular in power systems [6], [7]. The objective of our work is to apply the Monte Carlo method to our specific network and estimate the expectation value of the output for analyzing the average impact of IEMI.

In the Monte Carlo simulations, a scenario consists of a set of inputs. For a scenario, every input value is generated randomly, using the inverse transform method [6]. For example, given a random number u_i from a $U(0,1)$ distribution, the random value for input x_s is generated by

$$x_{\mathrm{s}i} = F_{x_\mathrm{s}}^{-1}(u_i),$$

where $F_{x_\mathrm{s}}^{-1}$ is the inverse of cumulative distribution function of the input x_s.

Generally, if $v_1, \ldots, v_n$ are n independent observations of the random variable V, the estimated expectation value of the output V, namely, m_V, is expressed by

$$\mathbb{E}[V] \approx m_V = \frac{1}{n}\sum_{i=1}^{n} v_i.$$

Let T be the number of iterations. The variance of the estimated expectation value m_V, denoted by $\mathrm{Var}[m_V]$, can be obtained by

$$\mathrm{Var}[m_V] \approx \frac{1}{T-1}\sum_{j=1}^{T} m_{V_j}^2 - \left(\frac{1}{T}\sum_{j=1}^{T} m_{V_j}\right)^2,$$

and the variance is related to the precision of the simulation.

To improve the precision, it is crucial to reduce the variance. Hence, we take the simple sampling as a baseline, and introduce three variance reduction techniques, which are widely used in the Monte Carlo simulations and suitable for solving the problem presented in this paper, to investigate their performance. In what follows, we briefly introduce mechanisms of above sampling techniques, and omit the mathematical details due to the limited space. Interested readers can refer to the related literature [6], [7].

A. Simple sampling

Simple sampling is the basis of other sampling methods in the Monte Carlo simulation. It is the most straightforward approach to collect samples, which are completely random. Each sample unit is drawn in equal probability, and they are completely independent of each other.

B. Complementary random numbers

To ensure that the random numbers evenly spread over the population, such that the effect of random fluctuations can be reduced, the technique of complementary random numbers [6], which creates a negative correlation between samples, is used. For each input value generated by (1), we can find the complementary random number of u_i, i.e., $1 - u_i$. Then, we can generate the negative correlated random value of the input

$$x_{\mathrm{s}i}^{*} = F_{x_\mathrm{s}}^{-1}(1 - u_i).$$

C. Correlated sampling

By using the technique of correlated sampling [6], we can compare two slightly different versions of the same model. For instance, in the mathematical model presented in Section II, we can vary the frequency distribution function $f_1(x_\mathrm{s})$ to $f_1'(x_\mathrm{s})$, and compare the difference of the two estimated expectation values of the targeted load response.

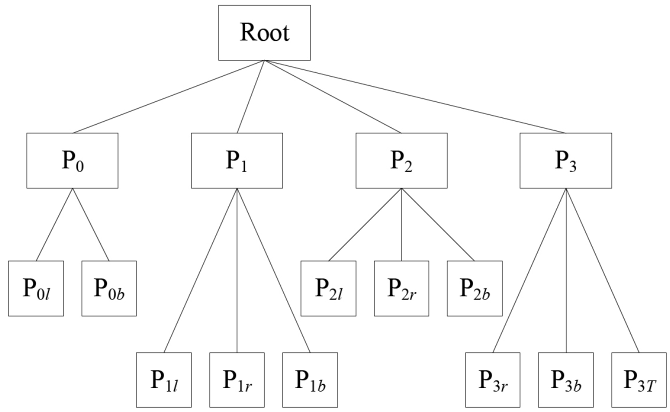

Fig. 4. Strata tree.

D. Stratified sampling

In stratified sampling, the population is divided into several parts, which are referred to as strata. Each stratum has its own stratum weight ω_h, $h = 1, \ldots, L$ (L is the total number of strata), which is the probability that a randomly chosen unit from the population belonging to the specific stratum [6]. For the problem studied here, the strata tree is given in Fig. 4, the total number of strata $L = 11$.

The expectation value m_{V_h} of each stratum is estimated independently. The estimated expectation value of the output is weighted in terms of the stratum weight

$$m_V = \sum_{h=1}^{L} \omega_h m_{V_h}. \tag{3}$$

The variance of the estimate from (3) is minimized if the samples distributed to each stratum n_h are according to the Neyman allocation [6]

$$n_h = \frac{\omega_h \sigma_{V_h}}{\sum_{k=1}^{L} \omega_k \sigma_{V_k}} n,$$

where σ_{V_h} is the standard deviation of stratum h.

The standard deviation for each stratum is unknown, which needs to be estimated in a pilot study, by using a fixed number of samples per stratum. Based on the values of standard deviation obtained in pilot study, we can calculate the optimal sample allocation for each stratum, and distribute the rest samples in the simulation.

IV. Case Study

The mathematical model described in Section II, and the simulation methods described in Section III are applied on a practical case, which is used to investigate the potential risks of the target quantitatively, when the network is exposed to two different probability distribution functions of the position of the IEMI source, where one has poor security and another has good security, regarding the critical system. In the Monte Carlo simulation, we jointly use the three variance reduction techniques, and perform a comparison with the simple sampling, which is treated as a baseline, to investigate advantages of the joint scheme in improving the simulation precision.

TABLE I
PARAMETERS OF INPUT AND MODEL CONSTANTS.

Symbol	f_{int}	V_{s}	I_{s}	Z_{c}	γ
Value	17 MHz	100 V	0 A	50 Ω	0.36
Symbol	L_{mb}	L_{md}	Z_{mb}	x_{s}	
Value	3 m	6 m	100 Ω	$\sim U(0,3)$ m	

TABLE II
TWO PROBABILITY DISTRIBUTIONS OF IEMI SOURCE BY ZONES.

Symbol	P_0	P_1	P_2	P_3
$f_1(x_{\mathrm{s}})$	0.1	0.2	0.3	0.4
$f_1'(x_{\mathrm{s}})$	0.4	0.3	0.2	0.1

TABLE III
PROBABILITY DISTRIBUTION OF IEMI SOURCE BY POSITIONS WITHIN DIFFERENT ZONES.

Symbol	P_{0l}	P_{0b}	P_{1l}	P_{1r}	P_{1b}	–
$f_2(x_s)$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	–
Symbol	P_{2l}	P_{2r}	P_{2b}	P_{3r}	P_{3b}	P_{3T}
$f_2(x_s)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

TABLE IV
METHOD COMPARISON.

Method	CS	CS'	CCS	SS
Mean [V]	53.94	36.72	17.22	36.60
Variance	0.05	0.01	0.02	0.96

To simplify the problem, we assume that all power lines in each zone have same the length of 3 m, and all the loads have the same impedance, which is a constant value of 100 Ω. The parameters of inputs and model constants are given in Table I. Here, we consider two groups of probability distribution function $f_1(x_{\mathrm{s}})$ and $f_1'(x_{\mathrm{s}})$, and their values are accordingly shown in Table II. Assume that the injected position is uniformly randomly distributed on the power lines, so the probability of the source injected on any power line is the same in each zone. In this four-junction, eight-branch network, the values of $f_2(x_{\mathrm{s}})$ are easily to obtain, as shown in Table III.

To test the effectiveness of different simulation methods, the model is simulated 100 times with each method. In the stratified sampling, 1100 scenarios are generated in each simulation, and 50 scenarios are generated for each stratum in the pilot study, which means the total number of samples is 550, with the rest 550 samples are distributed according to the results of Neyman allocation. To have a fair comparison, we also generate 1100 scenarios per simulation in simple sampling. The results are given in Table IV. For the sake of convenience, we have the following abbreviations:

- SS: simple sampling;
- CS: combination of complementary random numbers and stratified sampling, with $f_1(x_{\mathrm{s}})$;
- CS': combination of complementary random numbers and stratified sampling, with $f_1'(x_{\mathrm{s}})$;
- CCS: combination of complementary random numbers, correlated sampling and stratified sampling.

By adding the correlated sampling, CCS is used to calculate the difference of the two expectation values, and the expectation value and variance by CCS are estimated based on the results by CS and CS'. From Table IV we can see that, the expectation value of the load response associated with probability distribution function $f_1(x_{\mathrm{s}})$ is 53.94 V, and the expectation value is 36.72 V when the probability distribution function is $f_1'(x_{\mathrm{s}})$. The difference between them is 17.22 V, which is a reduction of approximately 32% from the worse case. This significant reduction in voltage indicates that, decreasing the probability of criminal accessing the innermost room, where the critical system is deployed, helps a lot in lowering the potential damage level on the target in the sense of average.

As a comparison reference, the expectation value of the targeted load response by SS is 36.60 V, which is almost the same as the result 36.72 V by CS. However, CS' provides a much higher accuracy, since its estimated variance is 0.01, which is one order of magnitude smaller than that of SS. Thus, the variance reduction techniques, complementary random numbers and stratified sampling, are proven to be valid. In addition, the variance estimated by CCS is less than the sum of estimated variances by CS and CS', i.e., $0.02 < 0.05+0.01$, which validates and highlights the effect of correlated sampling technique in solving the problem presented in the paper.

V. CONCLUSIONS

In this paper, based on the method of Monte Carlo, we analyze the impact of the probabilistic interference injection, in terms of the position in a low voltage power line network, on the voltage frequency responses of the targeted load. The difference of the estimated expectation value quantitatively proves that, reducing the probability of the criminal accessing the target is an effective way to reduce the potential risk it might suffer. Besides, the joint use of variance reduction techniques, which are complementary random numbers, correlated sampling and stratified sampling, in this problem is useful for enhancing simulation precision.

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