Magnetic fields of cool active stars

LISA ROSÉN
Magnetic fields are present throughout the universe and are very important for many astrophysical processes. Magnetic field influences a star throughout its life and affects nearby objects such as planets. Stellar magnetic field can be detected by measuring the Zeeman splitting of spectral lines in the intensity spectra (Stokes I) if the field is strong, or by analyzing polarization spectra if the field is weak. Magnetic fields in stars similar to the Sun are ubiquitous but, in general, relatively weak. Until recently these fields were detected through circular polarization (Stokes V) only since linear polarization (Stokes QU) is significantly weaker. The information embedded in different Stokes spectra is used for reconstruction of the surface magnetic field topology with Zeeman Doppler imaging (ZDI) technique. However, cool stars often have complex field geometries and this, combined with a low field strength, partial Stokes parameter observations and the presence of cool spots, makes accurate magnetic mapping difficult. We have performed numerical tests of ZDI to investigate some of the problems of magnetic inversions and ways to overcome them. The most reliable results were found when magnetic field and temperature inhomogeneities were modelled simultaneously and all four Stokes parameters were included in the reconstruction process. We carried out observations of active cool stars in all four Stokes parameters trying to find an object with linear polarization signatures suitable for ZDI. The RS CVn star II Peg was identified as a promising target, showing exceptionally strong linear polarization signatures. We reconstructed the magnetic field in II Peg using full Stokes vector observations for the first time in a cool star. Compared to the magnetic maps recovered from the Stokes IV spectra, the four Stokes parameter results reveal a significantly stronger and more complex surface magnetic field and a more compact stellar magnetosphere. Spectropolarimetric observations and magnetic inversions can also be used to investigate magnetic activity of the young Sun and its implications for the solar system past. To this end, we studied a sample of six stars with parameters very similar to the present Sun, but with ages of only 100-650 Myr. Magnetic field maps of these young solar analogues suggest a significant decrease of the field strength in the age interval 100-250 Myr and a possible change in the magnetic field topology for stars older than about 600 Myr.

Keywords: stars: magnetic field, stars: late-type, stars: activity, stars: imaging, polarisation, starspots

Lisa Rosén, Department of Physics and Astronomy, Observational Astronomy, 516, Uppsala University, SE-751 20 Uppsala, Sweden.

© Lisa Rosén 2016

ISSN 1651-6214
urn:nbn:se:uu:diva-283357 (http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-283357)
De ä bar ä äk
- Ingemar Stenmark
List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I  Rosén, L., Kochukhov, O., (2012)  
*How reliable is Zeeman Doppler imaging without simultaneous temperature reconstruction?*  
Astronomy & Astrophysics, *548*, A8

*Strong variable linear polarization in the cool active star II Peg*  

*First Zeeman Doppler Imaging of a Cool Star Using all Four Stokes Parameters*  

*Magnetic fields of young solar twins*  
Submitted to Astronomy & Astrophysics

Reprints were made with permission from the publishers.
List of papers not included in the thesis

The following are publications to which I have contributed but that are not included in this thesis.

1. The energy budget of stellar magnetic fields

2. Zeeman-Doppler imaging of active young solar-type stars
   Hackman, T., Lehtinen, J., Rosén, L., Kochukhov, O., Käpylä, M. J.
## Contents

1 Stellar magnetic fields ............................................. 9  
   1.1 Magnetic activity .............................................. 9  
   1.2 Importance of stellar magnetic fields ......................... 11  
   1.3 Surface magnetic field characteristics ......................... 12  

2 Zeeman effect in stellar spectra .......................... 15  
   2.1 Line splitting .............................................. 15  
   2.2 Polarization .............................................. 17  

3 Spectropolarimetric observations ......................... 20  
   3.1 Spectropolarimeter .......................................... 20  
   3.2 Extraction of polarized spectra ............................. 21  

4 Least-Squares Deconvolution ............................ 24  
   4.1 LSD profile analysis ....................................... 26  

5 Mapping a stellar surface ...................................... 28  
   5.1 Doppler imaging ............................................ 28  
   5.2 Zeeman-Doppler imaging .................................... 29  
   5.3 Numerical implementation of ZDI .......................... 30  

6 Numerical experiments with ZDI ........................ 33  
   6.1 Experimental set-up ......................................... 33  
   6.2 Magnetic spots with homogeneous temperature .............. 34  
   6.3 Cool magnetic spots ......................................... 36  
   6.4 Cool magnetic spots in all four Stokes parameters ........ 38  

7 Linear polarization in cool active stars .................. 42  
   7.1 Targets and observations ................................... 42  
   7.2 Linear polarization detections ................................ 43  
     7.2.1 Follow-up of II Peg ................................... 44  

8 Mapping a cool star magnetic field using all four Stokes parameters 46  
   8.1 Combining individual-line and LSD profile ZDI .............. 46
8.2 Line-profile comparison ........................................ 47
8.3 Temperature and magnetic maps .............................. 48
8.4 Magnetic energy distribution ................................... 48
8.5 Extended magnetic field topology ............................ 50

9 Magnetic field of young suns ................................. 53
  9.1 Sun in Time sample ........................................... 53
  9.2 ZDI procedure ................................................ 53
  9.3 Mean magnetic field strength ................................ 54
  9.4 Magnetic field topology ...................................... 54
  9.5 Magnetic activity cycles ..................................... 56

10 Summary and future plans ..................................... 59

11 Svensk sammanfattning ........................................ 61

12 Acknowledgements .............................................. 63

References .......................................................... 65
1. Stellar magnetic fields

There is a wide variety of stars in our Universe with temperatures ranging over two orders of magnitude and with masses from below 0.5 $M_\odot$ up to more than 100 $M_\odot$. Most stars, about 95%, have an effective temperature less than 7000 K and almost 85% are cooler and less massive than the Sun which has an effective temperature of about 5780 K. As with mass and temperature, the range in stellar magnetic field strength is also vast. Most of the hotter stars do not have any kind of magnetic field, but if they do, it is often dominantly dipolar-like and strong, a few kG and more. The strongest stellar magnetic fields can be found in magnetic neutron stars, or magnetars, where the fields can have strengths up to $10^{15}$G. On the other hand, most of the cooler stars do have a magnetic field although, in general, their global fields are not that strong, ranging from just a few G up to a few kG. They often also tend to have complex field structures, including magnetic spots where the field lines are more concentrated and hence the field is stronger. Inside these magnetic spots the temperature is cooler compared to the rest of the photosphere. All these factors make detecting and modeling the magnetic fields of cool stars challenging. Nevertheless, this challenge can be successfully addressed nowadays thanks to sophisticated instrumentation and analysis techniques.

1.1 Magnetic activity

When a star is formed parts of the galactic magnetic field can be woven into it. It is believed that this field can remain inside the stellar radiative zone, hence making the star magnetic even though it is not actively generating the field of its own. This is only one of the possible origins of the so-called fossil field. It has been shown that this field can remain stable in the course of stellar evolution on the main sequence (Braithwaite & Nordlund 2006). Typical characteristics of such a field are a relatively high strength and a simple structure because it can relax to its lowest energy state. This type of field is found in hot, massive stars, i.e. stars with a convective core and an outer radiative zone. However, only a small fraction of all hot stars are magnetic. The magnetic characteristics of cool stars, i.e. stars with a radiative core and an outer convective zone, are quite different. The origin of their initial magnetic fields could be the same as for more massive stars, but these seed fields will be disrupted or amplified since the star itself is actively generating a magnetic field in the outer envelope through a dynamo process. This means that all cool stars
are magnetic to some extent and that their fields are typically complex and constantly evolving. However, the details of how stars generate their magnetic fields are not fully understood and there does not exist a complete theory that can explain all the magnetic features even for the most studied star - the Sun. On the other hand, theories and models of stellar magnetic field generation mechanisms are continuously improved and can be tested using higher quality and more detailed observations of both the Sun and other stars.

The main mechanism of cool star magnetic field generation is a dynamo process, which basically is an interplay between rotation and convection (e.g. Charbonneau 2010). Hot plasma rises up to the surface where it cools down and sinks again. This convection is the cause of the granulation pattern seen on the Sun. The depth of the solar convection zone is about $0.3 \, R_\odot$, while some stars have a more shallow convection zone and others are fully convective. The Rossby number $Ro$ is the ratio between the rotational period $P_{\text{rot}}$ and the convective turnover time $\tau_c$. It is inferred both theoretically and observationally that the magnetic activity is stronger the faster the star rotates, or more precisely, the smaller is the Rossby number $Ro$ (e.g. Noyes et al. 1984; Vidotto et al. 2014).

Many cool stars are not rotating as solid bodies but are instead experiencing differential rotation since the rotational velocity changes both with radius and latitude. The Sun, for instance, has a radiative core rotating more or less as a solid body while the convection zone has differential rotation. This causes a shearing of the plasma giving rise to a magnetic field. The rotational velocity gradient is largest at the so called tachocline, which is the border between the radiative core and the convection zone (e.g. Charbonneau 2005). Fully convective stars do not have this layer and their magnetic fields have to be powered by a different type of dynamo mechanism.

Sunspots often appear in pairs of different polarities, so called magnetic bipolar regions as was first discovered by Hale and collaborators (Hale et al. 1919). These bipolar magnetic regions often occur on both hemispheres and appear mirrored, i.e. the leading spot on the northern hemisphere and the leading spot on the southern hemisphere are of opposite polarities (e.g. Fan 2004). The two regions also appear to be located equally far from the equator of the Sun and the zone of spot emergence migrates towards the equator over an 11 year period (Maunder 1922).

The cause of sunspots has for a long time been attributed to emerging flux tubes (Parker 1955). The idea is that strong magnetic flux tubes generated at the tachocline are elevated to the surface due to magnetic buoyancy, i.e. that the gas pressure inside the flux tube is lower compared to the gas pressure outside the flux tube. However, there is no conclusive observational or numerical evidence for such a mechanism. The flux tubes would have to survive all the way through the convection zone. Instead, other scenarios suggest spontaneous magnetic field generation closer to the surface (Brandenburg 2005). Recent simulations by Warnecke et al. (2013) demonstrated formation
of magnetic bipolar regions through one such spontaneous process, negative effective magnetic pressure instability (NEMPI) where the magnetic field is suppressing the total turbulent pressure and vertical magnetic field lines can emerge.

Not only the small-scale, local field changes during the 11-year solar activity cycle, but the global dipolar field also switches polarity in the middle of this period making the full solar cycle 22 years. Similar global field polarity reversals has also been found on other (e.g. Donati et al. 2008; Fares et al. 2009; Boro Saikia et al. 2015a). Activity cycles can be inferred from certain magnetic proxies besides the direct magnetic measurements. One of them is the variation in the CaII H&K line emission and another is the variation in X-ray luminosity. However, magnetic cycles inferred from proxy indicators do not always coincide with long-term global magnetic field variation.

1.2 Importance of stellar magnetic fields

When a star is formed the collapsing gas cloud has some initial non-zero angular momentum. As mentioned in section 1.1, parts of the galactic magnetic field are probably also embedded in the gas cloud. As the cloud contracts the angular velocity increases and the forming star eventually spins faster than the surrounding gas. The magnetic field lines connecting the rapidly rotating protostar with the rest of the gas would therefore be curved if not for the magnetic stress built up in the field lines. Instead of bending the field lines, the surrounding gas and the protostar spins with the same angular velocity due to the angular momentum transfer from the star to the surrounding gas and hence, the rotation period of the star is decreased. This process of angular momentum dissipation is called magnetic breaking. Another effect of a magnetic field connecting a protostar with the surrounding gas is that matter is funneled along the field lines thereby influencing accretion on the protostar (Bouvier et al. 2007).

A magnetic field line’s resistance to bending continues to slow down the stellar rotation rate throughout its entire life. The magnetic energy density will be larger than the kinetic energy density of the stellar wind up to a certain radius known as the Alfvén radius, \( r_A \). Within \( r_A \) circumstellar material rotates with the same angular velocity. Consider a star rotating with an angular velocity \( \Omega_* \) at the surface \( r_* \). If \( r_A > r_* \), the angular momentum per unit mass, \( \Omega_* r_*^2 \), is larger at \( r_A \) than at the stellar surface. The angular momentum carried away by the stellar wind is therefore larger than it would have been if the magnetic field would not have forced a corotation (Schatzman 1962; Choudhuri 1998). Also, in order to spin up all the matter within \( r_A \), angular momentum has to be transferred away from the star. The angular momentum loss will continue throughout the stars life due to the magnetic interaction with the stellar wind (e.g. Matt et al. 2012; Réville et al. 2015). This implies that the Sun was rotat-
ing more rapidly when it was younger. This in turn means that the magnetic field was stronger in the past compared to the present day Sun since a more rapid rotation generally powers a stronger dynamo.

A magnetic field will also cause mass ejections and an increase in emission of high energy radiation and particles. The increased activity of a star with a magnetic field affects the surrounding objects such as planets and their atmospheres. We can notice the effects of strong solar flares here on Earth today. If the magnetic field of the Sun was stronger at the time the Earth was formed, the bombardment by high energy radiation and particles would have been more intense. High energy particles could have interacted with molecules in the atmosphere and caused various chain reactions leading to changes in the chemical composition and various heating and cooling effects on Earth (e.g. Kulikov et al. 2007). High energy photons could even have reached the Earth’s surface and perhaps altered the DNA structure of the first organisms.

Many exoplanets have been discovered during the last decade. The next step would be to investigate whether rocky planets could be habitable. This depends on many planetary and stellar parameters. The location of the habitable zone is not only determined by the temperature of the star, but also by its magnetic activity and stellar wind. Therefore, it is important to study magnetic fields of planetary host stars in order to assess habitability of exoplanets.

1.3 Surface magnetic field characteristics

In order to compare one star to another, it is useful to characterise the magnetic field in more ways than just using the average strength. The orientation of the field is described by three different components: radial, meridional and azimuthal. A radial component represents the component of the field vector pointing in the direction normal to the surface of the star, a meridional component represents the field pointing in the tangential direction along the longitude of the star and an azimuthal component represents the field also pointing in the tangential direction to the surface, but along the stellar latitude (see Fig. 1.1).

Another common and useful characteristic is the relative contribution of poloidal and toroidal field components. Considering an axisymmetric field topology where the magnetic axis is aligned with the rotation axis, a poloidal field is aligned with the longitudes of the star and a toroidal field is aligned with the latitudes of the star, as illustrated in Fig. 1.1. It has been demonstrated that the field topologies in old, i.e. slowly rotating stars, are dominantly poloidal (Petit et al. 2008), which is the case for the Sun today. However, younger, more rapidly rotating stars do not seem to have a preferred configuration (Folsom et al. 2016) and the field topology in the same star can even switch between being dominantly toroidal and dominantly poloidal (Paper IV).
There are two ways to express the magnetic field characteristics on a stellar surface. One is to represent the magnetic field vector at each point on the surface defined by a latitude $\theta$ and longitude $\phi$ with individual radial, $B_r(\theta, \phi)$, meridional, $B_m(\theta, \phi)$, and azimuthal, $B_a(\theta, \phi)$, field components directly. This description was used for the numerical experiments in Paper I.

For Paper III and Paper IV we instead used another description of the magnetic field. The magnetic field is expressed as $B = -\nabla \psi$ where $\psi$ is a flux function. The condition of a divergence-free magnetic field implies that $\nabla^2 \psi = 0$. The solution to this equation describes the magnetic field by poloidal and toroidal components expressed as spherical harmonic expansions. The magnetic field vector distribution is now specified by the three sets of spherical harmonic coefficients $\alpha_{\ell, m}$, $\beta_{\ell, m}$ and $\gamma_{\ell, m}$, which represent the radial poloidal, horizontal poloidal and horizontal toroidal components respectively (Donati et al. 2006; Kochukhov et al. 2014). $\ell$ and $m$ is the angular degree and azimuthal order of each mode respectively. Using this formalism, it is possible to express $B_r(\theta, \phi)$, $B_m(\theta, \phi)$ and $B_a(\theta, \phi)$ as

\begin{align*}
B_r(\theta, \phi) &= -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \alpha_{\ell, m} Y_{\ell, m}(\theta, \phi) \\
B_m(\theta, \phi) &= -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \left[ \beta_{\ell, m} Z_{\ell, m}(\theta, \phi) + \gamma_{\ell, m} X_{\ell, m}(\theta, \phi) \right] \\
B_a(\theta, \phi) &= -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \left[ \beta_{\ell, m} X_{\ell, m}(\theta, \phi) - \gamma_{\ell, m} Z_{\ell, m}(\theta, \phi) \right]
\end{align*}
where $Y_{\ell,m}$, $Z_{\ell,m}$ and $X_{\ell,m}$ are

$$Y_{\ell,m}(\theta, \phi) = -C_{\ell,m} P_{\ell,|m|}(\cos \theta) K_m(\phi) \quad (1.4)$$

$$Z_{\ell,m}(\theta, \phi) = \frac{C_{\ell,m}}{\ell + 1} \frac{\partial P_{\ell,|m|}(\cos \theta)}{\partial \theta} K_m(\phi) \quad (1.5)$$

$$X_{\ell,m}(\theta, \phi) = -\frac{C_{\ell,m}}{\ell + 1} \frac{P_{\ell,|m|}(\cos \theta)}{\sin \theta} m K_{-m}(\phi) \quad (1.6)$$

and

$$C_{\ell,m} = \sqrt{\frac{2\ell + 1}{4\pi}} \frac{(\ell - |m|)!}{(\ell + |m|)!} \quad (1.7)$$

$$K_m(\phi) = \begin{cases} 
\cos(|m|\phi), & m \geq 0 \\
\sin(|m|\phi), & m < 0 
\end{cases} \quad (1.8)$$

and $P_{\ell,m}$ is the associated Legendre polynomial.
2. Zeeman effect in stellar spectra

Sunspots have been observed for over 2000 years, but not until 1908 had anyone proved they were also magnetic. George Ellery Hale was able to make this discovery (Hale 1908) thanks to the research by Pieter Zeeman, who discovered 12 years earlier that a magnetic field splits spectral lines and polarizes the light. The findings of Zeeman’s research was adequately named the Zeeman effect. Hale detected polarized light from the Sun and correctly used his knowledge of the Zeeman effect to determine that the cause of the polarization must have been a solar magnetic field. The same method is used today for detection and analysis of stellar magnetic fields, even though the instruments and analysis techniques have been significantly refined.

2.1 Line splitting

Spectral lines are the result of photons either being emitted or absorbed by an atom or molecule. The wavelength of the spectral line depends on the difference in energy, i.e. the difference in energy between one level and another in the atom. If no external magnetic field is present, a transition between an upper level \( E_u \) and a lower level \( E_l \) will result in one spectral line with the transition energy \( E_0 = E_u - E_l \) corresponding to a certain wavelength.

If an external magnetic field is present, the Hamiltonian of the atom is perturbed by the magnetic Hamiltonian \( H_B \) defined as

\[
H_B = \frac{e_0 h}{4\pi mc} (L + 2S) \cdot B + \frac{e_0^2}{8mc^2} (B \times r)^2
\]

(2.1)

where \( e_0 \) is the absolute value of the electron charge, \( h \) is the Planck constant, \( m \) is the electron mass, \( c \) is the speed of light, \( L \) and \( S \) are the total orbital angular momentum and total spin angular momentum respectively, \( B \) is the magnetic field vector and \( r \) is the sum of each electron’s position vector relative to the nucleus (Landi Degl’Innocenti & Landolfi 2004). Since the expectation value of \( r \) is of the same order of magnitude as the Bohr radius, the second term in eq. 2.1 will be small compared to the first term unless the magnetic field is extremely strong. This term can hence be neglected unless dealing with white dwarfs or magnetic neutron stars. \( H_B \) can then be simplified to

\[
H_B = \frac{e_0 h}{4\pi mc} (L + 2S) \cdot B = \mu_B (L + 2S) \cdot B
\]

(2.2)
where $\mu_B$ is the Bohr magneton. The addition of $H_B$ to the Hamiltonian will ultimately affect the energy levels of the atom. Each degenerate energy level will be split into $2J + 1$ sub levels. $J$ is the total angular momentum and each sub level can be represented by the magnetic quantum number $M$ which is the projection of $J$ along an arbitrary $z$-axis. The shift in energy between the sub levels and the unperturbed energy level can be calculated using

$$\Delta E = \mu_B g M$$

where $M = -J, -J + 1, \ldots, J - 1, J$ and $g$ is the Landé factor. The possible transition energies between a lower level $E_l$ and an upper level $E_u$ are now

$$\Delta E = (g_u M_u - g_l M_l) \mu_B B = (\Delta g M_u + g_l \Delta M) \mu_B B$$

where $\Delta g = g_u - g_l$ and similarly $\Delta M = M_u - M_l$. This means that the same transition as before now results in more than one spectral line, as shown schematically in Fig. 2.1. Due to selection rules for an electric dipole transition, $\Delta M$ can only be equal to -1, 0 or +1.

The distribution of the shifted lines around the unperturbed line depends on the value of $\Delta M$. The components corresponding to $\Delta M = 0$ are called $\pi$ components and are distributed symmetrically around the unperturbed line. Transitions corresponding to $\Delta M = -1$ are called $\sigma_r$ components and those corresponding to $\Delta M = 1$ are called $\sigma_b$ components. The $\sigma_b$ and $\sigma_r$ components together form a symmetric pattern around the unperturbed line.

As can be seen from eq. 2.3, $\Delta E$ is proportional to the magnetic field strength in such a way that the stronger the field, the larger is the shift in energy and hence also in wavelength, which means the Zeeman line splitting is more prominent for stronger magnetic fields. However, in practice a spectral line is broadened due to stellar rotation, thermal Doppler broadening and instrumental smearing. These effects greatly complicate detection of magnetic line

![Figure 2.1. Schematic illustration of an atomic transition from $J = 1$ to $J = 0$ where the left figure corresponds to a situation without any magnetic field while the right figure corresponds to a situation with an external magnetic field present.](image-url)
splitting, especially if a star is rapidly rotating. In other words, the splitting of spectral lines due to a magnetic field is clearly visible in an optical stellar spectrum if the mean surface field strength is at least several kG and the star is slowly rotating (Mathys et al. 1997). Somewhat weaker, 0.5-1.0 kG, fields can be diagnosed from high-quality spectra by modelling magnetic broadening and intensification of Zeeman sensitive lines (e.g. Saar 1996; Kochukhov et al. 2013a).

2.2 Polarization

As mentioned in the beginning of this chapter, it was not the line splitting that made it possible for Hale to detect the solar magnetic field. Instead, he used another aspect of the Zeeman effect: polarization caused by an external magnetic field. The advantage of polarization analysis is that it does not require kG field strengths for field detection. Field strengths as low as on the order of 0.5 G can be detected with today’s spectropolarimetric instruments and analysis techniques (Donati & Landstreet 2009).

The type of polarization is determined by how the electric field vector oscillates. Assuming the direction of propagation is along the z-axis, the electric field vector is oscillating in the xy-plane and can be described by

\[ E_x = E_1 \cos(\omega t - \phi_1) \]
\[ E_y = E_2 \cos(\omega t - \phi_2) \]

where \( E_1 \) and \( E_2 \) are the respective amplitudes, \( \omega \) is the angular frequency and \( \phi_1 \) and \( \phi_2 \) are the respective phases. The tip of the electric field vector describes an ellipse, called the polarization ellipse.

There are some special cases of the polarization ellipse. If either \( E_1 \) or \( E_2 \) is equal to 0, the resulting electric field vector will not be rotating but instead oscillating along either the semi-major or semi-minor axis. The light will hence be linearly polarized. The light will also be linearly polarized if \( \phi_1 = \phi_2 \) or \( \phi_1 = \phi_2 \pm \pi \). The ellipse instead becomes a circle when \( E_1 = E_2 \) and \( \phi_1 = \phi_2 \pm \pi/2 \) and the light is hence called circularly polarized. Circular polarization can be either positive or negative, depending on which way the electric field vector is rotating. Clockwise rotation as seen by the observer corresponds to positive polarization and counter clockwise rotation corresponds to negative polarization.

The intensity and polarization of light can be fully described by the four Stokes parameters, \( IQUV \), where \( I \) is unpolarized light, \( QU \) characterizes linear polarization and \( V \) characterizes circular polarization. Stokes \( Q \) is defined as \( \langle I_{0^\circ} - I_{90^\circ} \rangle \), Stokes \( U \) as \( \langle I_{45^\circ} - I_{135^\circ} \rangle \) and Stokes \( V \) as \( \langle I_{0^\circ} - I_{90^\circ} \rangle \) where \( \langle \rangle \) denotes the temporal average. The quantities \( I_{0^\circ}, I_{90^\circ}, I_{45^\circ} \) and \( I_{135^\circ} \) represent the intensity of the light passing through an ideal linear polarizer with the
transmission axis set at the respective angles with respect to some reference direction. The quantities $I_{\sigma}$ and $I_{\pi}$ represent the intensity of the light passing through an ideal circular analyzer (Fig. 2.2).

In the case of line formation in a magnetic field, circular and linear polarization is connected to the $\sigma$ and $\pi$. In emission the $\pi$ components will always be observed as linearly polarized parallel to the magnetic field while $\sigma$ components can be observed as either circularly polarized or linearly polarized perpendicular to the magnetic field. These polarization properties of the Zeeman components are schematically illustrated in Fig. 2.3. For absorption the polarization of the $\sigma$ and $\pi$ components also depend on the polarization of the incident light. If the incident light is unpolarized, the $\sigma$ and $\pi$ components will absorb the same polarization as would have been emitted and hence the opposite polarization is present in the beam after the absorption. For Zeeman effect due to typical cool-star magnetic fields the polarization amplitude increases monotonically with increasing magnetic field strength. In such case, Stokes $V$ is several times stronger than Stokes $QU$.

![Figure 2.2. Definition of the Stokes QUV parameters.](image)
Figure 2.3. Illustration of polarization in the $\pi$ and $\sigma$ components.
3. Spectropolarimetric observations

Spectropolarimetric observations suitable for analysis of Zeeman signatures in spectral lines require a high resolution spectrograph and high signal-to-noise ratio. Depending on how the polarized spectra are derived, each exposure is divided into two or four sub-exposures which are combined to calculate the Stokes parameter spectra. The exposure time is determined by both the magnitude of the star and its rotation period. All sub-exposures comprising a single polarization observation should be taken at close rotational phases to avoid rotational smearing. The total exposure time should therefore not exceed $1 - 2\%$ of the stellar rotational period. This, in turn, implies that faint stars are not accessible for high-quality spectropolarimetric observations unless they have a relatively long rotation period. Observations from several rotational phases enable reconstruction of the stellar surface, which will be discussed in more detail in chapter 5. This typically implies that the star has to be observed during multiple nights. If the star has a relatively long rotation period, for instance similar to the Sun, the observations need to be obtained during almost one month. On the other hand, observations should not be too spread out since the magnetic field of an active cool star can change its structure on timescales of a few months. The optimal targets for high-resolution spectropolarimetry are therefore bright stars with relatively short rotation periods and strong magnetic fields.

3.1 Spectropolarimeter

The typical set-up of a polarimeter includes a retarder plate which causes a phase shift to the incoming light and a beam splitter which splits the incoming single beam into two beams with orthogonal polarization states. These two beams are then transported via optical fibers to a spectrograph where the light is dispersed and then recorded on a CCD detector. A polarimetric unit is usually mounted at the Cassegrain focus in order to minimize the number of reflections so that spurious polarization effects are avoided.

The observational data in this thesis have been obtained at three different telescopes using three spectropolarimeters: the Canada-France-Hawaii Telescope (CFHT) on Mauna Kea, Hawaii, using the ESPaDOnS spectropolarimeter (Donati 2003), the Telescope Bernard Lyot on the Pic du Midi, France, using NARVAL (Petit et al. 2008) and the ESO 3.6m telescope in La Silla, Chile, using HARPSpol (Snik et al. 2011; Piskunov et al. 2011). We could
therefore obtain observations of stars visible both from the northern or southern hemisphere.

The spectropolarimeters ESPaDOnS and NARVAL are twins and constructed in the same way. The polarimeter module consists of two rotating half-wave Fresnel rhombs and one fixed quarter-wave Fresnel rhomb placed between the two half-wave rhombs. Light entering a Fresnel rhomb will be reflected twice inside the rhomb which will cause a phase retardation of $\pi/2$. The half-wave Fresnel rhombs are therefore made up of a pair of Fresnel rhombs. A Wollaston prism is then used to split the beam into two beams with orthogonal polarization states. This set-up makes it possible to obtain both circular and linear polarization. The two beams are then fed into the bench-mounted echelle spectrograph via two fibers. For one single exposure the wavelength coverage is $3700 - 10500$ Å and the resolving power of the spectrograph is about $R = 65000$. The two spectra are finally recorded on the $2K \times 4.5K$ E2V CCD detector.

The spectropolarimeter HARPSpol has a slightly different design. In order to record both linear and circular polarization, it has one circular and one linear polarimeter. The light is phase-shifted by either an achromatic quarter wave-plate or a half-wave plate. The spectrograph is equipped with two $2K \times 4K$ CCD detectors, where one is covers the blue wavelengths between 3600 and 5260 Å and the other covers the red part of the spectrum between 5340 and 6910 Å. It has a resolving power of 110 000.

### 3.2 Extraction of polarized spectra

Before any analysis can be made the data has to be reduced, i.e. a 2D image from the detector has to be converted into a 1D spectrum. This is done by summing the stellar signal in the pixel columns normal to the dispersion direction and by assigning each pixel in the resulting 1D spectrum a wavelength. All the data from NARVAL and ESPaDOnS in this thesis has been reduced automatically by the Libre-ESpRIT software (Donati et al. 1997). For the HARPSpol data we used the REDUCE package (Piskunov & Valenti 2002).

During the reduction procedure, it is possible to correct for various spurious signals from both external sources such as cosmic rays and internal sources such as instrumental effects. There is, for instance, background offset in a detector which is corrected for by obtaining a so-called bias frame. A bias frame is taken with a zero exposure time and a closed shutter. The response of each pixel to incoming photons is not homogeneous across the detector, i.e. the same number of photons on two different pixels might not necessarily result in the same electron count. To correct for this, a relatively short exposure of a continuous source, a flat field lamp, is used.

During spectral extraction procedure it is also necessary to identify the different spectral orders on the detector. This can be done with the flat field
Finally, a science spectrum is extracted. First, the different calibration frames are applied to the science image. Then the 2D image is converted into a 1D spectrum. In the REDUCE package, this is done by fitting the 2D science frame with the product of the slit illumination function and a 1D spectrum.

As discussed in section 2.2, two orthogonal polarization states are needed to derive one Stokes parameter. All spectropolarimeters used in this thesis are able to record both spectra in a single exposure. If all the components in a spectropolarimeter were perfect, only one exposure would be necessary to derive a polarization spectrum. Instead, there are numerous internal errors and artifacts affecting the two spectra in a different way and therefore leading to spurious polarization features in a simple difference spectra. These problems can fortunately be efficiently corrected for by taking multiple exposures with different angles of the retarder plates, so that orthogonal polarization states exchange their position on the detector. For all the observations in this thesis we have used four sub exposures, i.e. eight spectra to derive each single Stokes parameter spectrum. The intensity spectrum Stokes $I$ is derivied by averaging spectra extracted from all the sub-exposures. Regarding the polarized spectra, there are two common methods of combining the exposures, one called the difference method and the other is called the ratio method (Bagnulo et al. 2009). The ratio method is used throughout this thesis. In this case a polarization spectrum is derived by

$$P = \frac{R - 1}{R + 1}$$

(3.1)

where $P$ is either Stokes $V$, $Q$ or $U$ and $R$ is defined as

$$R^4 = \frac{i_{1 \parallel / i_{1 \perp}}^1 \cdot i_{4 \perp / i_{4 \parallel}}^4}{i_{2 \perp / i_{2 \parallel}}^2 \cdot i_{3 \parallel / i_{3 \perp}}^3}$$

(3.2)

if there are four sub exposures (Donati et al. 1997). The upper indices 1 and 2 indicate the fiber, the lower indices $1 - 4$ indicate the sub exposure and $\perp$ and $\parallel$ indicate the two orthogonal polarization states. Sub-exposure 1 and 4 have identical set up in the sense that the same polarization state is transported through the same fiber and ends up on the same place on the CCD. In sub-exposure 2 and 3, the set up is changed compared to 1 and 4 and each fiber now transports the opposite polarization state. The spectra of the two polarization states will hence also switch position on the CCD. By dividing rather than subtracting the different sub-exposures, any artifacts caused by the fibers or detector will be canceled out and the difference in throughput between the fibers is corrected for.

The redundant information in the four sub-exposure observing sequence allows one to derive an additional diagnostic spectrum that can be used to assess various instrumental polarization effects. This spectrum is called a null spectrum and is obtained by instead suppressing the polarization signatures of
incoming light by taking the ratio

\[ R^4 = \frac{i_{1,\perp}/i_{1,\parallel}}{i_{4,\perp}/i_{4,\parallel}} \cdot \frac{i_{2,\perp}/i_{2,\parallel}}{i_{3,\perp}/i_{3,\parallel}}. \]  

(3.3)

Sub-exposure 1 is now divided with sub-exposure 4 and they should be identical, as well as sub-exposure 2 and 3 meaning the only signal that should remain in the null spectrum is random noise. An observation with a null spectrum showing any kind of signature should therefore be analyzed with caution. A signal in the null spectrum could either be due to instrumental artifacts, for example a crosstalk between circular and linear polarization and vice versa, or changes in the polarization spectra between the different sub-exposures.
4. Least-Squares Deconvolution

The degree of polarization in stellar light scales with the magnetic field strength. This implies that it can be difficult to detect any polarization, especially linear, in individual spectral lines if the field is weak. In order to overcome this problem a multi-line technique, called least-squares deconvolution (LSD Donati et al. 1997), can be used where all spectral lines are combined into one mean profile $Z$. The underlying assumption of this method is that all lines in the entire spectrum $Y(v)$ are self-similar, i.e. scaled versions of the same profile $Z(v)$,

$$Y(v) = \sum_i w_i \delta(v - v_i)Z(v_i)$$ (4.1)

where $v - v_i = c\Delta \lambda_i/\lambda_i$ is the position in velocity space. $\Delta \lambda_i$ is the shift in wavelength from the line center $\lambda_i$ and $w_i$ is the weight of each line $i$ determined by the depth, $d_i$, wavelength, $\lambda_i$, and effective Landé factor, $g_i$, of the line. $w_i$ is different depending on the Stokes parameter. The weight for Stokes $I$ is $w_i^I = d_i/d_0$, for Stokes $V$ $w_i^V = d_i\lambda_i g_i/d_0\lambda_0 g_0$ and for Stokes $QU$ $w_i^{QU} = d_i\lambda_i^2 g_i^2/d_0\lambda_0^2 g_0^2$ where $d_0$, $\lambda_0$ and $g_0$ are normalization values of line depth, wavelength and magnetic sensitivity respectively (Wade et al. 2000). The weight of each line at each position in velocity space can be represented by a line mask $M$ and eq. 4.1 can instead be expressed as a matrix multiplication

$$Y = M \cdot Z$$ (4.2)

where $Y$ is an $n$-element vector, $n$ is equal to the number of wavelength points in the observed spectrum. $M$ is a $n \times m$ matrix and $Z$ is an $m$-element vector. $m$ is equal to the number of velocity bins in the mean profile.

The line mask $M$ is calculated according to the expressions for weights given above. What is sought for is a mean profile $Z$ which, when multiplied with $M$, should give the observed spectrum $Y$. The mean profile $Z$ can be obtained by minimizing the $\chi^2$ function

$$\chi^2 = (Y - M \cdot Z)^T S^2 \cdot (Y - M \cdot Z) \rightarrow \min$$ (4.3)

where $S$ is a square diagonal matrix containing the inverse of the error bar $1/\sigma_{obs}$ of each point in the observed spectrum. The minimization is obtained through the least-squares solution

$$Z = (M^T S^2 M)^{-1} M^T S^2 Y.$$ (4.4)
Figure 4.1. A small segment of the Stokes IVQU spectra of II Peg are shown in the left panel and the corresponding LSD Stokes IVQU profiles obtained from the entire spectra are shown in the right panel. From top to bottom in both figures, the profiles represent Stokes $U$ (blue), Stokes $Q$ (green), Stokes $V$ (red) and Stokes $I$ (black). The Stokes VQU spectra and the LSD profiles have been shifted and magnified for display purposes.

An example of a segment of observed polarization spectra including three magnetically sensitive Fe I lines at wavelengths 5497.5, 5501.5 and 5506.8 Å and the corresponding LSD profiles derived from the entire spectrum can be seen in Fig. 4.1. No distinct polarization signatures can be seen in individual spectral lines, at least not for Stokes QU, while the corresponding Stokes VQU LSD profiles clearly display signatures.

There are cases when the assumption that all lines have the same shape but different scaling factors is not suitable. Lines which are strong or, for instance, have very broad wings should not be included in the line mask since their shapes deviate from the shape of ‘normal’ lines.

The LSD analysis is based on the weak-field assumption. The average separation of the $\sigma_r$ and $\pi$ components $\Delta \lambda_B$ in Å is expressed as

$$\Delta \lambda_B = \frac{\lambda_0^2 e_0 g B}{4 \pi m c^2} \approx 4.6686 \cdot 10^{-10} \lambda_0^2 g B$$

(4.5)

where $B$ is the magnetic field strength in kG, and $\lambda_0$ is the wavelength in Å of the unperturbed line and $g$ is the effective Landé factor. In order for the weak field approximation to be applicable, $\Delta \lambda_B$ must be much smaller.
than the thermal Doppler broadening or the intrinsic broadening. When this condition is fulfilled, Stokes $V$ is proportional to the derivative of Stokes $I$ and all circular polarization profiles are approximately self-similar. The situation for linear polarization is more complicated. As it turns out, linear polarization signatures are quite sensitive to line strength. The self-similarity for linear polarization not only requires a weak field, but also weak lines (Kochukhov et al. 2010). Nevertheless, linear polarization LSD profiles can be used for magnetic field modelling provided that they are interpreted with full polarized radiative transfer treatment.

4.1 LSD profile analysis

As mentioned in section 2.2, the polarization amplitude depends on the magnetic field strength. Therefore, information about the average magnetic field strength can be obtained from an LSD polarization profile. More specifically, the mean longitudinal magnetic field, $\langle B_Z \rangle$, is the sign and magnitude of the integrated magnetic field projected onto the line-of-sight (Kochukhov et al. 2010). It can be calculated from the first moment of a Stokes $V$ LSD profile

$$
\langle B_Z \rangle = -7.145 \cdot 10^6 \frac{\int V(v-v_0)dv}{\lambda_0 \cdot g_0 \int (1-I)dv},
$$

(4.6)

where $\lambda_0$ and $g_0$ are the normalization values of wavelength and Landé factor respectively, $\langle B_Z \rangle$ is in G and $v$ is velocity in $km s^{-1}$. $v_0$ is the velocity of the center-of-gravity of a Stokes $I$ LSD profile which can be expressed as

$$
v_0 = \frac{\int v(1-I)dv}{\int (1-I)dv}. 
$$

(4.7)

Since $\langle B_Z \rangle$ is calculated as the first moment of the disk-integrated Stokes $V$ profile, the resulting value not only depends on the amplitude of the profile but also on its shape. If the magnetic field is dominated by one polarity, either positive or negative, the corresponding Stokes $V$ profile will have one minimum and one maximum. In this case $\langle B_Z \rangle$ will significantly differ from zero. If, on the other hand, different parts of the star are dominated by different polarities, the corresponding Stokes $V$ profile can have more than one minimum and/or maximum and $\langle B_Z \rangle$ might be close to zero even though a Stokes $V$ signal is present inside spectral lines.

A flat Stokes $V$ profile does not necessarily mean there is no magnetic field. The signatures of two magnetic spots of opposite polarities on the same vertical strip on the star but on different latitudes would cancel each other and result in a flat Stokes $V$ profile.

It can also be useful to evaluate how probable it is that the polarization signature is real and not due to random noise. The so-called false alarm probability (FAP, Donati et al. 1992) is commonly used to assess the reliability of
polarization signature detection. It is calculated according to

\[ FAP = 1 - \Gamma(n/2, \chi^2/2) \]  

(4.8)

where \( \Gamma(n/2, \chi^2/2) \) is the incomplete gamma-function and \( n \) is the number of degrees of freedom equal to the number of velocity bins covering the profile. Commonly used detection limits are: definite detection if \( FAP < 10^{-5} \), marginal detection if \( 10^{-5} < FAP < 10^{-3} \) and no detection if \( FAP > 10^{-3} \) (Donati et al. 1997).
5. Mapping a stellar surface

A lot of information is embedded in a stellar spectrum. The type and number of lines give an indication of the stellar spectral type. Some lines are more sensitive to temperature or surface gravity than others and can hence be used to determine those stellar parameters. A visual inspection of the line profiles in a polarized and unpolarized spectra enable a qualitative assessment of horizontal inhomogeneities in the magnetic and temperature distributions. If there is a time-series of such observations, the analysis can be taken one step further. The detailed surface maps of temperature and magnetic field can be reconstructed using the Doppler imaging and Zeeman-Doppler imaging techniques.

5.1 Doppler imaging

For cool active stars Stokes I holds information about the mean surface temperature and its horizontal variation. A well known technique called Doppler Imaging (DI) can be used to interpret the line profile variation and transform it into a 2D map of the stellar surface (e.g. Rice et al. 1981; Vogt & Penrod 1983; Piskunov et al. 1990; Kochukhov et al. 2004b). A distortion, or bump, in the line profile corresponds to a temperature inhomogeneity on the star, see Fig. 5.1. The position of this inhomogeneity, or spot, can be directly deduced since each wavelength point in the profile corresponds to a single Doppler velocity, which in effect represents a vertical strip on the star. Since the star is rotating, the side of the star turning towards the observer is blueshifted and the other side turning away from the observer is redshifted. The latitude of the spot can be assessed by investigating where in the profile the corresponding profile distortion becomes visible and where it disappears and also how much its position changes as the star rotates. For example, a spot at a high latitude will only be visible near the center of the profile and the corresponding profile distortion will not change that much between consecutive rotational phases. If the spot, on the other hand, is closer to the equator, the corresponding bump can be seen moving all across the profile and its position will change quickly from one rotational phase to the next. This, of course, also depends on the inclination of the stellar rotational axis.

Depending on the magnetic field strength, the Stokes I profile will be broadened and distorted by the Zeeman effect. It can therefore be important to take the magnetic field into account when reconstructing temperature maps (Paper I).
5.2 Zeeman-Doppler imaging

The magnetic field can also be reconstructed using basically the same principles as for scalar DI. The magnetic mapping technique is called Zeeman-Doppler Imaging (ZDI, Semel 1989). Similar to signatures of temperature spots, the position of the magnetic signature in the polarized Stokes profile is translated to the position of the magnetic spot on the star. The magnetic signature of a certain spot distribution is different in Stokes $V$, $Q$ and $U$ depending on the field strength and orientation. In addition, the amplitude of the Stokes $V$ depends also on the continuum intensity or temperature of the magnetic region.

As was mentioned in section 2.2, circularly polarized light is sensitive to the line-of-sight component of the magnetic field. In Fig. 5.2 a radial, meridional and azimuthal magnetic spots and the corresponding Stokes $VQU$ profiles are shown at different rotational phases. The projection of a radial field component onto the line of sight will result in a circular polarization profile having the same shape (but varying amplitude) across the entire stellar surface since the projected vector will always either point towards or away from the observer as the star rotates. The same applies to a meridional field component. The Stokes $V$ signature of an azimuthal field component will, on the other hand, change both sign and amplitude as the star rotates. In addition to Stokes $V$ analysis, it is important to study linear polarization signatures. They are sensitive to the transverse component of the magnetic field and can hence differentiate between a radial and meridional field component which can be seen in Fig. 5.2. This figure also illustrates a big difference of the relative strength of Stokes $V$ and Stokes $QU$ profiles. The amplitude of the Stokes $V$ signatures is larger, in most cases, even though the Stokes $QU$ profiles have been magnified by a factor of 2.
Figure 5.2. Stellar surface maps with a single magnetic spot with different field orientation and the corresponding Stokes $V$, $Q$ and $U$ profiles at different rotational phases. From top to bottom, the surface maps and line profiles depict spots with a pure radial field, a meridional field and an azimuthal field.

5.3 Numerical implementation of ZDI

ZDI is an inverse problem since the information about temperature and magnetic field is embedded in the observed line profile time series. This information is extracted by fitting theoretical model profiles calculated as a function of temperature and magnetic field to the observed profiles. This is a general approach to the magnetic mapping problem, but there are some variations in how this is implemented numerically in specific ZDI codes.

The stellar surface is first divided into many surface zones with each zone assigned a specific initial temperature and magnetic field. For the work in this thesis, the stellar surface is initially assumed to have a homogeneous temperature equal to the effective temperature of the star and a zero magnetic field. Local line profiles are then calculated according to the temperature and magnetic field of each zone. These local profiles are then convolved with a Gaussian profile in order to take instrumental and macroturbulent broadening into account. They are then Doppler-shifted and interpolated on the same wavelength or velocity grid as the observed spectra. Finally, local profiles from zones visible at a certain phase are summed up and normalized with the phase dependent continuum flux. These disk-integrated profiles are then compared with the observed profiles. If the deviation between the model profiles and
observed profiles is below a certain threshold value the iterations stop, if not, the local temperature and magnetic field values are adjusted and a new set of local profiles is calculated.

Essentially, ZDI is a regularized least-squares minimization problem,

\[
E = \sum \frac{(I_{calc}(T, B) - I_{obs}(T, B))^2}{\sigma_{obs}^2} + \sum \frac{(V_{calc}(T, B) - V_{obs}(T, B))^2}{\sigma_{obs}^2} + \sum \frac{(Q_{calc}(T, B) - Q_{obs}(T, B))^2}{\sigma_{obs}^2} + \sum \frac{(U_{calc}(T, B) - U_{obs}(T, B))^2}{\sigma_{obs}^2} + \Lambda_T R_T + \Lambda_B R_B \rightarrow \min
\]

where \(\Lambda_T\) and \(\Lambda_B\) are regularization parameters set by the user and \(R_T\) and \(R_B\) are regularization functions. Regularization is used in order to find a simplest, unique solution when the problem is ill-posed i.e. a given observational data set can be reproduced with a multitude of surface maps. When using only Stokes IV, ZDI has no unique solution even in the absence of observational noise since two magnetic field distributions can result in the same polarization profile, and also, that the same polarization profiles can be represented by different field vectors at the same point on the star (Piskunov & Kochukhov 2002).

There are different types of regularization functions. One of the most commonly used for magnetic field reconstructions of cool stars is maximum-entropy regularization (e.g. Brown et al. 1991; Donati & Brown 1997). Maximum entropy regularization minimizes information content of an image, effectively suppressing local deviations from a prescribed mean value. It can therefore fail to reconstruct global structures, for instance a dipolar field covering the entire star, where essentially all values are deviating from the mean value (Piskunov & Kochukhov 2002).

Another common type of regularization is Tikhonov regularization. Tikhonov regularization compares the value in a point with adjacent points and suppresses large gradients between neighbouring points. Therefore, Tikhonov regularization favours smooth solutions and struggles to reproduce small-scale, high-contrast features. However, it reconstructs global structures more reliably compared to the maximum entropy regularization.

If the magnetic field is represented with spherical harmonic components, another type of magnetic map regularization is applied. This regularization function is calculated as a sum of magnetic field energies contained in different harmonic components, with weights according to the square of the harmonic angular degree, \(\ell^2\). It will hence suppress unnecessary contribution of high-
order harmonics, i.e. complex field structures. For the ZDI analyses in this thesis, both Tikhonov regularization and the harmonic penalty function have been used. The former regularization method was employed for temperature reconstruction.

In this thesis, all magnetic field and temperature reconstructions were made using either the inverse code INVERS13 (Kochukhov et al. 2013b) which has been developed from the INVERS10 code (Piskunov & Kochukhov 2002; Kochukhov & Piskunov 2002) or the INVERS LSD code (Kochukhov et al. 2014). INVERS13 is designed to perform inversions using individual lines. The model profiles are calculated by solving the polarized radiative transfer equation using detailed line lists and realistic model atmospheres from the MARCS grid (Gustafsson et al. 2008).

Unfortunately, direct single line polarization modelling is not feasible for the majority of cool stars because their polarization signatures are too weak. Consequently, cool star ZDI studies uses LSD profiles for surface reconstructions. The LSD profiles are commonly modelled as single lines with some mean line parameters to describe their behavior (e.g. Brown et al. 1991; Marsden et al. 2011; Kochukhov et al. 2013b). Alternatively, the observed LSD profiles can be interpreted using theoretical LSD profiles derived from a synthetic spectrum (Kochukhov et al. 2014). The synthetic local Stokes spectra are calculated by solving the polarized radiative transfer equation using realistic model atmospheres and complete line list and then the synthetic LSD profiles are derived using the same line mask as was used for the observed spectrum. The INVERS LSD code relies on a precomputed table of local synthetic LSD profiles which can be produced with either the simplified individual line or the detailed multi-line approach. Only the latter is appropriate when dealing with the LSD Stokes $QU$ profiles (Kochukhov et al. 2010).

Both codes handle all four Stokes parameters self-consistently and simultaneously, meaning that both the temperature and magnetic field inhomogeneities are taken into account when calculating each Stokes parameter spectrum.
6. Numerical experiments with ZDI

ZDI is a widely used tool for magnetic reconstruction (e.g. Donati et al. 2003; Petit et al. 2004; Catala et al. 2007) and it is applied to different types of magnetic stars. Even though it is a very useful technique it has limitations. Cool stars do in general have relatively weak magnetic fields, which results in weak polarization. As discussed in section 5.2, circular polarization is stronger than linear polarization, up to 10 times. Stokes V is therefore usually the only detectable polarization parameter for cool stars. Because of this, magnetic fields of cool stars are currently studied using circular polarization only. We therefore investigated some of the problems with ZDI, focusing especially on simultaneous magnetic and temperature reconstruction in order to assess what errors are associated with magnetic field reconstruction when temperature variations are ignored. Previous studies of Ap/Bp stars have shown that linear polarization will bring out more complex features of the field (e.g. Kochukhov et al. 2004a; Kochukhov & Wade 2010; Silvester et al. 2014; Ronsomarov et al. 2015). We therefore also investigated the effect of including Stokes QU in the reconstruction process.

More details about this work can be found in Paper I (Rosén & Kochukhov 2012).

6.1 Experimental set-up

In order to investigate limitations and artifacts of magnetic inversions we set up an artificial stellar surface with a specific temperature and magnetic field configuration. The photospheric temperature was set to 5750 K, the projected rotational velocity $v \sin i = 40$ km s$^{-1}$ and the inclination $i = 50^\circ$. We then simulated observational data corresponding to the chosen magnetic field and temperature distributions by performing forward calculations to derive the rotationally modulated line profiles of three magnetically sensitive Fe I lines at wavelengths 5497.5, 5501.5 and 5506.8 Å. This was done for 20 rotational phases using atomic line data from the VALD database (Kupka et al. 1999) and MARCS model atmospheres (Gustafsson et al. 2008) to calculate the synthetic spectra. We assumed a resolving power of 65 000 and broadened the line profiles by convolving them with a Gaussian profile. No noise was added since we aimed to perform these tests under optimal conditions rather than test the influence of noise. These line profiles were then used as input for the inverse calculations where the temperature and magnetic field configuration was reconstructed.
Three different sets of numerical experiments were performed. In the first experiment we considered four magnetic spots at different latitudes on the surface but kept the temperature homogeneous across the stellar surface. The field strength of the umbra was set to 2 kG and the penumbra had a strength of 1 kG. Inversions were repeated for radial, meridional and azimuthal orientation of field vectors inside magnetic spots. In the two other experiments we used the same magnetic spot configuration and strengths, but also added temperature spots coinciding with the magnetic spots. The umbra and penumbra were assumed to have a temperature of 3750 K and 4750 K respectively. In the first two experiments we only used Stokes $IV$ profiles in the reconstruction process while in the third experiment all four Stokes parameters were used.

The $INVERS13$ code was used, described in section 5.2, which solves the polarized radiative transfer equation in a realistic model atmosphere in order to calculate all four Stokes parameters self-consistently. Unlike the treatment adopted by many currently used ZDI codes, the temperature variations are taken into account in calculation of polarization spectra and the magnetic field effects on the Stokes $I$ spectra also are taken into account.

### 6.2 Magnetic spots with homogeneous temperature

The resulting maps from the first experiment when the temperature of the magnetic spots was equal to that of the photosphere can be seen in Fig. 6.1 and Fig. 6.2. The maps in Fig. 6.1 were made using only Stokes $V$, keeping the temperature at a fixed value and the maps in Fig. 6.2 were made using both Stokes $I$ and $V$ to simultaneously reconstruct the temperature. Provided the temperature reconstruction is accurate, the reconstructed magnetic field maps should be relatively equal in the two cases since there should be no temperature differences between the two cases. However, large discrepancies between the magnetic field maps are found even though the temperature is correctly reproduced to within about 100 K.

To make a quantitative assessment of the quality of field reconstruction, the field value of surface zones from the middle of each spot was compared between the reconstructed magnetic field map and the true one. The field strength turned out to be underestimated by 21-78% when only Stokes $V$ was used for inversions, while the underestimation was only 6-20% when Stokes $IV$ was used, except for the lowest latitude radial spot which instead has been interpreted as a meridional spot. One of the biggest problems when only Stokes $V$ is used for magnetic inversions is the crosstalk between the radial and meridional field components. This is a well known issue, which has been investigated in several studies (e.g. Donati & Brown 1997; Kochukhov & Piskunov 2002). Crosstalk is also seen in both inversions here, and it is primarily between radial and meridional components at low latitudes. The azimuthal component seems to be most robust.
A magnetic field will affect Stokes $I$ by a broadening and intensification of the profiles meaning that information about the magnetic field is present in Stokes $I$. On a closer inspection, a difference can be seen between the Stokes $I$ profiles from a surface with magnetic spots compared to a surface without any magnetic spots (Fig. 6.3). Hence, extra information about the magnetic field can be gained from fitting Stokes $I$ even if the temperature is homogeneous.

This should, in principle, also affect temperature mapping. We tested this by reconstructing the temperature using only Stokes $I$ assuming a zero magnetic field and compared the results to inversions using Stokes $IV$. As it turns out, the temperature reconstruction will be more accurate if the magnetic field is taken into account. If only temperature is reconstructed the resulting map contained local temperature deviating by up to 500 K from the true map, compared to about 100 K when the magnetic field also was included in the inversion.
Figure 6.2. Inversions of magnetic spots with a homogeneous temperature. These inversions were carried out using Stokes $IV$ for simultaneous temperature reconstruction.

### 6.3 Cool magnetic spots

The resulting maps from the second set of ZDI experiments, when the magnetic spots coincided with the temperature spots, can be seen in Fig. 6.4 and Fig. 6.5. The maps in Fig. 6.4 were produced using only Stokes $V$, keeping the temperature at a fixed value and the maps in Fig. 6.5 were obtained using both Stokes $I$ and $V$ to simultaneously reconstruct the temperature and magnetic field. The field strength is strongly underestimated, by 82-95%, when temperature is kept constant, and the original spot configuration is very poorly reproduced as can be seen in Fig. 6.4. The situation is, however, improved when temperature is taken into account and the radial and azimuthal spots are now underestimated by 45-76% and the meridional spots by 70-84%. The original spot distribution is also quite recognizable. Crosstalk can be seen in both cases and, once again, primarily between the radial and meridional field components at low latitudes.

The meridional field components seem to suffer from the most severe underestimation. This is perhaps not so surprising since a meridional field vector will always be directed almost perpendicular to the line of sight, compared to...
Figure 6.3. Stokes $I$ profiles from a stellar surface with homogeneous temperature. The red dashed line represents a Stokes $I$ profile when the stellar surface contained magnetic spots and the black solid line represents Stokes $I$ profiles of the same stellar surface without any magnetic spots.

A radial or azimuthal field vector which will at some point during the stellar rotation be directed almost parallel to the line of sight. The circular polarization signature of a meridional field will hence not be as strong as that of a radial or azimuthal field purely due to geometry.

Cool star magnetic fields are often complex and magnetic spots are common. These spots are expected to have a lower temperature compared to the rest of the photosphere and the difference can sometimes be as high as 1000-2000 K (e.g. Solanki & Unruh 2004). The spots therefore produce a local spectral contribution with a lower continuum intensity, leading to bumps in disk-integrated Stokes $I$ profiles. The decrease in local intensity will not only affect the Stokes $I$ spectrum, but also the polarized spectra if the temperature spot coincides with a magnetic spot. Just as with Stokes $I$, a cooler temperature will lead to a decreased continuum intensity and a lower amplitude of the magnetic spot signature in the disk-integrated polarization profile. In Fig. 6.6 two polarization profiles corresponding to the same magnetic field distribution but with different temperatures of magnetic spots are shown. As can be seen, there is a large discrepancy between the two profiles. If the local temperature is not taken into account when interpreting the polarization profiles, the polarization amplitude is effectively assumed to depend solely on the magnetic field strength. Hence, a small amplitude will be misinterpreted as a weak field, which leads to a severe underestimation of the magnetic field strength.
Temperature reconstruction with and without simultaneous magnetic reconstruction was also performed. The difference between the two cases was however not as large as for the homogeneous temperature distribution. A possible explanation could be that the effect of a cooler spot temperature dominates the Zeeman broadening of the Stokes I profiles. This implies that temperature maps derived in previous cool-star DI studies ignoring magnetic field still can be correct, although some caution should be exercised in case of very strong magnetic fields.

6.4 Cool magnetic spots in all four Stokes parameters
As a final set of experiments we used all four Stokes parameters for simultaneous magnetic and temperature reconstruction. The results of these inversions are presented in Fig. 6.7. When comparing to Fig. 6.4 and Fig. 6.5 a significant improvement can be seen. The field strengths are higher even though they are still underestimated by about 31-62%. The biggest improvement is found for
Figure 6.5. Inversions of magnetic spots coinciding with low-temperature spots. These inversions were carried out using Stokes IV for simultaneous temperature reconstruction.

The inner-most part of the magnetic spots cover a relatively small number of surface zones. The magnetic field is reconstructed zone-by-zone using Tikhonov regularization which tries to smoothen the maps. This is probably the reason the field strength in the middle of the spot is still underestimated. The increase in field strength from the outer to the inner region of the spot is 1 kG and Tikhonov regularization suppresses such steep changes between neighbouring surface zones. This results in a continuous ramp-up of the field strength towards the centre of the spot without quite reaching the initial strength.

What is most striking is the accuracy of the spot geometry and the absence of crosstalk. When linear polarization is included, it is possible to separate a meridional field component from a radial one.

These numerical tests indicate that magnetic imaging is most reliable when linear polarization is included.
Figure 6.6. Surface maps of the same magnetic field configuration but with different underlying temperature maps and the corresponding disk-integrated Stokes $V$ line profiles. The magnetic spots have a 2 kG umbra and 1 kG penumbra. In the upper left panel the magnetic spots have the same temperature as the photosphere and the Stokes $V$ profile is represented by the red dashed profile in the lower panel. In the upper right panel the umbra and penumbra of the magnetic spots are 2000 K and 1000 K cooler than the photosphere respectively and the Stokes $V$ profile is represented by the black profile.
Figure 6.7. Inversions of magnetic spots coinciding with low-temperature spots. These inversions were carried out using Stokes IQUV.
7. Linear polarization in cool active stars

As discussed in chapter 6 cool stars do in general have relatively weak magnetic fields, resulting in weak polarization signatures. Linear polarization is up to 10 times weaker than circular polarization making linear polarization even more difficult to detect. Because of this, the current studies of cool star magnetic fields are performed using circular polarization only. This provides limited information about the magnetic field geometry since circular polarization is only sensitive to the line-of-sight component of the magnetic field vector. Reconstructions of the magnetic field topology will therefore not be completely trustworthy when only circular polarization is used. On the other hand, linear polarization is sensitive to the transverse component of the magnetic field vector. By including linear polarization in the ZDI reconstruction the quality of the recovered magnetic maps is dramatically improved, as discussed in section 6.4. The study by Kochukhov et al. (2011) demonstrated that linear polarization induced by the Zeeman effect can be detected in cool stars. The S/N ratio of the linear polarization observations analyzed in that paper was, however, not sufficient for magnetic imaging. The purpose of our study was therefore to extend the work of Kochukhov et al. (2011) and to identify a cool star for which linear polarization could be detected at a level suitable for magnetic mapping.

More details about this work can be found in Paper II (Rosén et al. 2013).

7.1 Targets and observations

We performed an exploratory study including four RS CVn stars, II Peg, HR 1099, IM Peg and σ Gem. RS CVn stars are usually tidally locked close binaries. The two components are tidally locked and the primaries are rapidly rotating and are therefore known to be among the most active cool stars. All four stars were observed in four Stokes parameters at the CFHT, with the ESPaDOnS spectropolarimeter between 2012 February and July, (see section 3.1 for more details about the instrument, observing procedure and data reduction). During this time σ Gem was observed 4 times, II Peg and IM Peg were observed 3 times each and HR 1099 was observed once. As expected, we could not detect any linear polarization in individual spectral lines and therefore applied the LSD multi-line technique, (see chapter 4 for more details about LSD).
Figure 7.1. Representative sets of the LSD Stokes $IQUV$ profiles for II Peg, HR 1099, IM Peg and $\sigma$ Gem. The polarization profiles are magnified and shifted vertically.

7.2 Linear polarization detections

In Fig. 7.1 the set of observations with the highest detection probability for each star is shown. Our observations turned out to be quite successful since linear polarization was detected in at least one observation for each star.

$\sigma$ Gem was observed at four different rotational phases. The Stokes $V$ signatures were comparable in size and strength at all four phases while Stokes $Q$ was securely detected in one out of the four phases and no definite detection was made of Stokes $U$.

The three observations of IM Peg were obtained at different rotational phases. Similar to $\sigma$ Gem, one of the phases showed stronger linear polarization signatures compared to the others, even though the Stokes $V$ signatures were approximately equal in amplitude. Both Stokes $Q$ and $U$ were securely detected and the profiles can be seen in Fig. 7.1. A weak signature of the secondary star was detected in all Stokes $I$ profiles and can be seen on the blue side of the primary in Fig. 7.1.

The only observation of HR 1099 obtained in this study is shown in Fig. 7.1. The Stokes $QU$ signatures were securely detected. Linear polarization has previously been detected in this star by Kochukhov et al. (2011). In that study the observations were performed at the ESO 3.6 m telescope with HARPSpol.
which has a resolving power of about 110 000. Our observations were made with ESPaDOnS which has a resolving power of 65 000, i.e. significantly lower compared to HARPSpol. Despite the lower resolution, the detection levels in both studies were similar. A clear signature of the secondary can be seen in the Stokes $I$ profile and also a less prominent but still visible circular polarization signature can be seen in Stokes $V$.

Even though linear polarization was detected in all stars, the amplitude of the signatures for II Peg was significantly larger compared to the other stars. The achieved S/N ratio seemed sufficient for magnetic imaging. Never before has such a strong Zeeman linear polarization signature been measured for a cool star. All three observations of II Peg were equally promising and we therefore decided to carry out further observations of this target.

### 7.2.1 Follow-up of II Peg

Another 7 four Stokes parameter observations of II Peg were obtained during 2012 September and then 2 more during 2012 December – 2013 January. These observations are shown together with the 3 observations from 2012 July in the upper panels of Fig. 7.2. As can be seen in Fig. 7.2 the polarization amplitude seems to have been continuously high throughout the entire observation period while the initial observations from July actually yield some of the weakest polarization signatures.

For the complete set of profiles, the activity level is certainly high enough and the phase coverage is also sufficient for magnetic imaging. However, profiles obtained at approximately the same rotational phase but during different observation epochs differ in shape, see for example the Stokes $V$ profiles around phase 0.3 in Fig. 7.2. The three observation epochs span over 6 months and during this time the stellar magnetic field has evolved and the spot configuration changed. Even though we can not use all these observations together, the seven observations from 2012 September were obtained during seven consecutive nights. They do, therefore, have a relatively good phase coverage and can be used for magnetic mapping.

Further follow-up observations of II Peg were performed during 2013 June – July, see the lower panels of Fig. 7.2. We secured 12 complete Stokes $IQUV$ observations, resulting in a good phase coverage. The same ephemeris was used to calculate the rotational phase for all observations and, hence, phases can be directly compared between the different sets. The polarization amplitudes of the two sets are comparable, as evident from Fig. 7.2.
Figure 7.2. LSD Stokes $IQUV$ profiles of II Peg. The upper panels represent the observations obtained during 2012 July to 2013 January and the lower panels represent the observations taken in 2013 June – July. The green profiles were obtained during 2012 July, the red profiles during 2012 September the blue during 2012 December – 2013 January and the purple profiles are from 2013 June – July.
8. Mapping a cool star magnetic field using all four Stokes parameters

After obtaining two observational sets of II Peg four Stokes parameter data suitable for temperature and magnetic field mapping, we had the opportunity to reconstruct the magnetic field of a cool star using all four Stokes parameters for the first time. We could also compare maps produced from the same observational data using either Stokes $IV$ or Stokes $IQUV$.

More details about this work can be found in Paper III (Rosén et al. 2015).

8.1 Combining individual-line and LSD profile ZDI

It is always preferable to use individual lines instead of LSD profiles when performing temperature and magnetic field reconstructions. One reason is simply because more than one line of different strength can be included, which puts more constraints on the solution. Other important reasons are that lines with well-known line parameters can be modelled realistically and accurately, and lines of different magnetic and temperature sensitivity can be used. For Stokes $VQU$ we did not have a choice but to use LSD profiles. For Stokes $I$, however, there were visible signatures of temperature inhomogeneities in individual lines. We therefore decided to combine individual-line temperature mapping with LSD profile magnetic mapping.

The temperature reconstructions were made using INVERS13, while the magnetic field mapping was made using INVERSLS, see section 5.2 for more details on the codes. When using INVERSLS, a large grid of pre-calculated local LSD profiles was prepared using spectrum synthesis calculations. In this study, the grid contained 132525 unique LSD profiles covering a range of temperature, magnetic field strength and orientation, and the limb angle values.

The mapping process was performed by alternating between temperature and magnetic field reconstructions, starting from a homogeneous temperature and zero magnetic field. In each iteration, both the magnetic field distribution and local temperature variations were taken into account, using the respective maps from the previous step as a starting guess. In total, we performed, for each data set, three consecutive refinement steps for both temperature and magnetic field maps.
8.2 Line-profile comparison

A first step to uncover possible differences between the Stokes $IV$ inversions and the Stokes $IQUV$ inversions is comparing the line profiles. In Fig. 8.1 the observed line profiles are shown together with the model profiles from both inversions of the two datasets.

The model profiles from the Stokes $IV$ inversions match the observed Stokes $IV$ profiles, and the model profiles from the Stokes $IQUV$ inversions match the observed Stokes $IQUV$ profiles. However, the Stokes $QU$ profiles corresponding to the maps derived from the Stokes $IV$ inversions are significantly differ-
8.3 Temperature and magnetic maps

The surface maps derived from all inversions can be found in Fig. 8.2. The discrepancy between the model profiles of Stokes $QU$ from the Stokes $IV$ and Stokes $IQUV$ inversions directly corresponds to the difference in the magnetic maps. The Stokes $IQUV$ maps for both observational sets have magnetic features which are stronger and more complex compared to the corresponding Stokes $IV$ map. The agreement between the observed Stokes $V$ profiles and the model profiles of Stokes $V$ was similar independently if Stokes $QU$ profiles were simultaneously fitted, but there are clear differences in the magnetic maps. This implies that the same set of Stokes $V$ profiles can be fitted by very different magnetic field configurations and that the ZDI solution is not unique.

The meridional field seems to benefit most from including linear polarization. The rms of the meridional magnetic field shows the highest increase, 82–93%, compared to the change of the rms of the radial and azimuthal magnetic fields even though all of them are larger in the Stokes $IQUV$ case.

8.4 Magnetic energy distribution

Stronger magnetic features are seen in the Stokes $IQUV$ maps compared to the Stokes $IV$ maps. This also translates to a higher magnetic energy; it turns out that the total magnetic energy is 2.5-3.1 times higher for the full Stokes vector inversions. Since we are using a spherical harmonic decomposition of the magnetic field, it is possible to get a quantitative measure of how complex the magnetic field topology is. In Fig. 8.3 the energy corresponding to a particular angular degree $\ell$, $E_\ell$, is plotted as a fraction of the total energy, $E_{tot}$. This figure also shows the fraction of poloidal/toroidal field energy for each $\ell$ and the axisymmetry/non-axisymmetry of the poloidal and toroidal components. It is clear that the Stokes $IQUV$ maps are more complex than the Stokes $IV$ maps since a larger fraction of the magnetic energy is put into higher $\ell$ values. The fraction of total energy contained in $\ell = 1 - 5$ is 84% and 70% for the two Stokes $IV$ maps, but only 33% and 36% in the Stokes $IQUV$ maps. This means there is a shift in relative energy when all four Stokes parameters are included. Since Stokes $V$ is fitted equally well even though the field topology is more complex, it implies that Stokes $V$ is not sensitive to high-order $\ell$ values, i.e. complex field structures. A better agreement of the Stokes $IV$ and $IQUV$ inversion results is found for the total fraction of poloidal/toroidal and axisymmetric/non-axisymmetric field since all magnetic maps have a dominantly poloidal and axisymmetric field.
Figure 8.2. Temperature and magnetic field maps of II Peg. The two upper rows show the Stokes IV maps and Stokes IQUV maps derived using the observations from 25 September - 1 October 2012 and the two lower rows show the Stokes IV maps and Stokes IQUV maps corresponding to the observations from 15 June - 1 July 2013.

It is also interesting to investigate whether the absolute energy deposited in each $\ell$ is different between the Stokes IV and Stokes IQUV maps. The ratio between the energy in each $\ell$ from the Stokes IQUV maps, $E_{\ell}^{\text{IQUV}}$, to the corresponding parameter of the Stokes IV maps, $E_{\ell}^{\text{IV}}$, is given in Table 8.1. There is actually less energy in $l = 1 - 2$ in the Stokes IQUV case for both sets of observations. In the later set, this difference extends to $\ell = 3$.

<table>
<thead>
<tr>
<th>$E_{\ell}^{\text{IQUV}} / E_{\ell}^{\text{IV}}$</th>
<th>$l = 1$</th>
<th>$l = 2$</th>
<th>$l = 3$</th>
<th>$l = 4$</th>
<th>$l = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Sep - 1 Oct 2012</td>
<td>0.92</td>
<td>0.83</td>
<td>1.23</td>
<td>1.70</td>
<td>4.93</td>
</tr>
<tr>
<td>15 Jun - 1 Jul 2013</td>
<td>0.35</td>
<td>0.84</td>
<td>0.78</td>
<td>1.04</td>
<td>2.48</td>
</tr>
</tbody>
</table>
Figure 8.3. Magnetic energy distribution as a function of angular degree $\ell$. The two upper panels correspond to the energy distribution in the Stokes IV and Stokes IQUV maps respectively for the observations from 25 September - 1 October 2012. The two lower panels correspond to the magnetic energy distribution in the Stokes IV and Stokes IQUV maps respectively for the observations from 15 June - 1 July 2013.

8.5 Extended magnetic field topology

An increase of the surface field complexity for IQUV inversions should also affect the extended field topology which can be calculated using a potential field extrapolation of the radial surface magnetic field (Jardine et al. 2002). Since the strength of different harmonic field components has an inverse power law dependence on the radius depending on $\ell$ value, $B \propto 1/r^{(\ell+2)}$, fields with high $\ell$ will decay faster compared to fields with lower $\ell$ as $r$ increases. The result of potential field extrapolation calculations assuming a source surface radius, $R_S = 3R_*$, are presented in Figs. 8.4-8.5. In reality, the source surface is not necessarily a sphere and the value of $R_S$ is not constant (e.g. Vidotto et al. 2011). However, our calculations with a fixed $R_S$ should mostly be viewed as a comparison between the IV and IQUV inversion cases.

As expected, there are more open field lines in the Stokes IV case since more energy is put into low-order harmonic components. This means that
Figure 8.4. Extended field topology derived from ZDI maps with potential field extrapolation. The spherical surface plot shows the radial field component. The gray (magenta) curves show the field lines that are closed (open) within the $2.5R_\odot$ sphere.
the magnetic energy at $R_5$ is 2.5-5.3 times higher in the Stokes $IV$ cases. It can also be concluded that the magnetosphere is more compact in the Stokes $IQUV$ case. This will influence the stellar wind.
9. Magnetic field of young suns

The young Sun was rotating faster compared to today. Previous studies showed that the rotation rate of a cool star decreases with age and that the magnetic activity also decreases with age (e.g. Vidotto et al. 2014). This implies the magnetic activity of the young Sun was higher, which in turn means the impact of the solar wind on the planetary system was stronger. In order to investigate the conditions in the young solar system and the evolution of the solar magnetic field, we have studied the magnetic fields of six young solar analogue stars based on 16 observational sets in total. We derived and evaluated the surface magnetic field maps for all these sets.

More details about this work can be found in Paper IV (Rosén et al. (2016)).

9.1 Sun in Time sample

All stars in this work are part of the so-called ‘Sun in Time” sample (Güdel 2007). The Sun in Time sample contains stars with ages from 0.1 Gyr to 8.5 Gyr and they are all carefully selected to have all parameters except age close to the Sun. We have studied some of the youngest stars in this sample, with ages from 100 to 650 Myr, see Table. 9.1. They all have temperatures and masses close to the solar values which is important for the internal structure of the star and hence for the operation of magnetic dynamo mechanism.

Some of the observations were obtained from the Polarbase archive (Petit et al. 2014), which is an open archive with the observational data from NARVAL at TBL and ESPaDOnS at CFHT. Other observations were collected in the context of our own observational programme, Active Suns (Hackman et al. 2016), at the ESO 3.6m telescope.

9.2 ZDI procedure

These stars are expected to have stronger global magnetic fields compared to the Sun today. However, it was still not possible to detect linear polarization since the magnetic fields are not strong enough. We have therefore used Stokes \textit{IV} data in this study. Only one star, EK Dra, showed any distortion in Stokes \textit{I} and brightness mapping was performed for this star in addition to the magnetic field reconstruction.
Table 9.1. Information about the stars in this study.

<table>
<thead>
<tr>
<th>Star name</th>
<th>$T_{\text{eff}}$</th>
<th>Mass ($M_\odot$)</th>
<th>Radius ($R_\odot$)</th>
<th>$P_{\text{rot}}$</th>
<th>Age (Myr)</th>
<th>Membership</th>
<th>No. obs.</th>
<th>epochs</th>
</tr>
</thead>
<tbody>
<tr>
<td>EK Dra</td>
<td>5845 K</td>
<td>1.044</td>
<td>0.972</td>
<td>2.63</td>
<td>100</td>
<td>Pleiades</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>HN Peg</td>
<td>5974 K</td>
<td>1.103</td>
<td>1.042</td>
<td>4.65</td>
<td>250</td>
<td>Hercules-Lyra association</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$\pi^1$ UMa</td>
<td>5873 K</td>
<td>1.007</td>
<td>0.968</td>
<td>4.99</td>
<td>300</td>
<td>Ursa major stream</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\chi^1$ Ori</td>
<td>5882 K</td>
<td>1.028</td>
<td>1.052</td>
<td>5.088</td>
<td>300</td>
<td>Ursa major stream</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>BE Cet</td>
<td>5837 K</td>
<td>1.062</td>
<td>1.002</td>
<td>7.658</td>
<td>600</td>
<td>Hyades moving group</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>$\kappa^1$ Cet</td>
<td>5742 K</td>
<td>1.034</td>
<td>0.952</td>
<td>9.29</td>
<td>650</td>
<td>-</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

References: (1) Valenti & Fischer (2005); (2) Takeda et al. (2007); (3) Strassmeier & Rice (1998); (4) Montes et al. (2001a); (5) Boro Saikia et al. (2015b); (6) Eisenbeiss et al. (2013); (7) Gonzalez et al. (2010); (8) Güdel (2007); (9) Messina & Guinan (2003); (10) King et al. (2003); (11) Montes et al. (2001b).

All inversions were made using INVERS LSD with the simplified individual line approach. It has been shown that this approach is suitable for Stokes IV LSD profiles if the magnetic field is weak, $\leq 2$ kG, (Kochukhov et al. 2010). A pre-calculated table of local LSD line profiles was produced using the single-line approximation where the behavior of the observed LSD profiles was described by a set of average line parameters. The Milne-Eddington approximation was used in order to solve the polarized radiative transfer equation analytically.

9.3 Mean magnetic field strength

From the derived magnetic maps, it is possible to calculate the mean magnetic field strength. We did this for the total magnetic field, but also for each field component individually. The results can be seen in Fig. 9.1. The mean magnetic field, $\langle B \rangle$, is significantly reduced from the star of an age of 100 Myr to stars older than 250 Myr. The latter show a similar spread in the field strength between different stars as between different observation epochs of the same star. Considering the mean field strengths of individual field components, we find that the mean meridional field, $\langle B_m \rangle$, is weakest in 15 out of 16 observation epochs while the mean azimuthal field, $\langle B_a \rangle$, is strongest in 13 out of 16. This could imply that young solar-like stars have weak meridional fields, but a more likely explanation is that the meridional field component is underestimated. This is because only Stokes $V$ is used to reconstruct the magnetic field here and such inversions are known to have problems recovering the meridional field, see discussion in section 6.3.

9.4 Magnetic field topology

We are using spherical harmonic decomposition to describe the magnetic field and can therefore investigate the energy distribution of the magnetic field. In
Figure 9.1. Mean magnetic field as a function of age. Each star is represented by a specific shape and color table. The different observation epochs for the same star are represented by different color shades, where a lighter shade corresponds to an earlier epoch.
Fig. 9.2 the distribution between the $\ell$ values can be seen for each star and observation epoch. Most of the field energy, 89-97%, is contained in $\ell = 1 - 3$ for all stars. The four youngest stars with ages of about 100 to 300 Myr show a decreasing field energy as $l$ increases from 1 to 3 in 12 out of 13 observation epochs. This is not seen for the two stars older than about 550 Myr, since they have an octupole component which is approximately twice as large as the quadrupole component. This could be a sign of a change in the field topology as the star ages. More observations of these stars and other stars of similar age are needed to confirm this.

We also derived the poloidal and toroidal magnetic field energies. It has been suggested that old, slow rotators have a dominantly poloidal field (Petit et al. 2008) while young, rapid rotators do not seem to have a preferred field topology (Folsom et al. 2016). In our study the magnetic fields derived for half of the observation epochs have dominantly toroidal field, at three epochs the field is neither dominantly poloidal or toroidal, and at the remaining five epochs the field is dominantly poloidal. However, the same star switches between being dominantly toroidal and poloidal, indicating that these stars do not have a preferred magnetic field topology.

9.5 Magnetic activity cycles

Since we have multiple observations of four out of the six stars, we have the possibility to search for signs of magnetic polarity reversals, as seen for the Sun. Three of these stars, EK Dra, HN Peg and κ1 Cet show the same polarity in all maps, but one of the stars, χ1 Ori, does not. The magnetic maps of χ1 Ori can be seen in Fig. 9.3. In the 2007.1 epoch, the visible pole has a negative radial field while the meridional field is dominantly negative. One year later, the visible pole has a very weak, but positive field and the meridional field is now dominantly positive. For the 2010.8 epoch, the visible pole has a positive radial field and a dominantly meridional field. The maps from the 2011.9 epoch then show a similar structure as the 2007.1 epoch maps. The magnetic field reversals of χ1 Ori suggests that it has a magnetic cycle of either 2, 6, or 8 years.
Figure 9.2. Distribution of the magnetic field energy for different $\ell$ values. Each panel show the distribution for one star.
Figure 9.3. Magnetic field maps of $\chi^1$ Ori. The spherical plots show distribution of the radial, meridional and azimuthal field components at four different rotation phases. The field strength is given in G.
10. Summary and future plans

Cool star magnetic fields are important to understand and study since they strongly influence the star throughout its entire life. There is no complete theory that can explain the magnetic activity of the Sun, and the existing theories need to be tested on more than one star. A stellar magnetic field will also have an influence on the stellar environment and is therefore a crucial ingredient when determining the habitability of a planet.

Since stellar magnetic fields have such a large impact on stellar and planetary evolution, it is important to perform reconstruction of magnetic field topologies from polarization observations as accurate as possible. In this thesis, we have investigated the strengths and weaknesses of a widely used method for magnetic field reconstruction, ZDI, both numerically and observationally. Our numerical study showed that if temperature inhomogeneities coincide with the magnetic features, which is typical of the Sun, local temperature has to be taken into account when the magnetic field is reconstructed. Otherwise, the reduced continuum intensity can be misinterpreted as a weak magnetic field and reconstructed magnetic map bears little resemblance to the true field distribution.

We also showed that the magnetic mapping is more accurate when both circular and linear polarization is included compared to when only circular polarization is used, which is a common limitation of nearly all current cool star magnetic studies. We have carried out four Stokes parameter observations with the goal of finding a cool star for which linear polarization could be detected at a level sufficient for magnetic mapping. We were able to find such a target and could therefore perform the first ever ZDI analysis of a cool star using spectra in all four Stokes parameters. This study confirmed the results of our numerical analysis. The magnetic maps which were derived using both circular and linear polarization had stronger and more complex features, which are not recovered using circular polarization alone.

The past magnetic activity of the Sun in the context of the evolution of our own solar system can be investigated by studying young solar analogues. We performed such a study of six solar analogue stars with ages of 100-650 Myr. Our results suggest a decreasing magnetic field strength with an increasing age from 100 to 250 Myr and a relatively flat trend for stars between 250 and 650 Myr. There are also hints of a change in the magnetic field topology with increasing age. The conditions in the young solar system were therefore different compared to the current, less active, Sun.

In order to confirm these trends, it is crucial to carry out follow-up observations of the same stars, but also of more stars of similar age. It would also
be useful to extend the range in age up to the age of the Sun today. In order to learn more about the influence of stellar magnetic fields on the young solar system, our magnetic maps can be used to constrain the stellar wind models. Another interesting direction is to investigate the small-scale magnetic field of these stars using Zeeman broadening.

It would also be useful to extend four Stokes parameter ZDI to more cool active stars, by either finding more objects with strong linear polarization or obtaining higher quality observations with spectropolarimeters at large telescopes.

Det är också viktigt att veta vilka begränsningar ZDI har och hur man gör för att återskapa magnetfältet så bra som möjligt. Det vanligaste sättet att rekonstruera kalla stjärnoras magnetfält är att bara använda cirkulärpolariserat ljus. Vi ville därför göra en numerisk studie där vi undersökte några styrkor och svagheter med ZDI. Det visade sig att om stjärnan har fläckar av lägre temperatur jämfört med resten av ytan som sammanfaller med magnetiska områden så måste man ta hänsyn till att temperaturen är lägre där när man rekonstruerar magnetfältet. Om man inte gör det kommer den lägre intensiteten att misstolkas som ett svagt magnetfält. Det visade sig också att resultaten blev bättre om man inkluderade både cirkulärpolariserat och linjärpolariserat ljus.


Ett magnetfält ger ökad massförlust och strålning från en stjärna, alltså en starkare stjärnvind. Om det finns planeter runt en magnetisk stjärna kommer dessa att bombarderas med mer partiklar och strålning än vad de annars hade gjort om stjärnan inte hade haft ett magnetfält. När solen är väldigt aktiv har den många magnetfläckar och detta kan vi märka av på jorden genom norrsken.
men också genom störningar i satelliter och gps-system. Solen tros ha roterat snabbare när den var yngre vilket betyder att den förmodligen hade ett starkare magnetfält. Detta, i sin tur, innebär att solvinden var starkare och att påverkan på solsystemet var större.

För att undersöka om så var fallet och hur den tidiga solen kan ha utvecklats har vi studerat en samling soltvillingar, alltså stjärnor med liknande parametrar som solen, men betydligt yngre, bara 100-650 Myr. Från vår studie ser vi en drastisk minskning i magnetfältstyrka mellan 100 till 250 Myr, medan stjärnor mellan 250-650 Myr alla har liknande variation i magnetfältstyrka som en och samma stjärna kan ha vid olika observationstillfällen. Vi ser också en antydan till en ändring i magnetfälttopologin för stjärnor äldre än ungefär 600 Myr. Solens globala magnetfält byter polaritet vart elfte år, vilket ger den en magnetfältcykel på 22 år. Eftersom vi hade multipla observationer av vissa stjärnor kunde vi också undersöka om vi kunde se några ändringar i stjärnor- nas polaritet. För en av stjärnorna hittade vi sådana ändringar, men med en cykel på antingen 2, 6 eller 8 år – alltså betydligt kortare än dagens sol. För de andra stjärnorna kunde vi inte se någon cykel, men det betyder inte att de inte har någon. Detta kan betyda att längden på cykeln kan ändras allt efter som stjärnan utvecklas, eller att det finns individuella skillnader även mellan stjärnor som är väldigt lika varandra.
12. Acknowledgements

First of all, I would like to thank my supervisor, Oleg. Thank you for your guidance and helpfulness and for sharing your seemingly infinite scientific knowledge. Even though you are busy, you always take time to answer my questions. Thank you for involving me in interesting projects and introducing me to the scientific community. I appreciate that you have given me the opportunity to travel and go to conferences to present my work and to do observations in Chile. I am lucky and grateful to have had you as my supervisor.

I would also like to thank my co-supervisor, Eric. Thank you for your kind way and valuable comments and advice and for keeping track of my progress.

I also have to thank Nikolai. I have learned a lot from you. I also appreciate all the advice you have given me during these years. And all the anecdotes.

I would like to thank everyone at the department for creating a good working environment. I feel like I can have an interesting and friendly conversation with anyone and the atmosphere is very relaxed. There is also a very strong willingness to help each other. Everyone is busy, but there is never a problem to ask for help. I also like that people seem genuinely interested in their own and other people’s research. Keep it up!

It has been a lot of fun going to work during these years. There has been lots of fika, as it should, and various cheese, beer and wine tastings, also as it should. The lunch discussions are interesting, entertaining and random. The initiative to have a physics and astronomy volleyball team came from here. The most important thing is that it was a lot of fun, because who likes winning anyway, right? We have also been on adventures involving boat trips, caves, zip-lining and ice skating.

I want to thank my two office mates, Sara and Sara! Sara B made me feel welcome from day 1. Thank you for bringing joy to the corridor by being your happy self and for helping me out with teaching and other random questions. When Sara B finished her PhD and moved further south, Sara L moved in! Thank you for being an awesome office mate and for making our office more fun with plants and, of course, our joint installation of the aquarium. You are a fun and loyal person. Also, big thank you for the nice dinners and board game nights. I hope they will continue.

I also have to thank the people in our magnetic group. Naum, you are very smart and knows a lot of things. Thanks for all the interesting and funny conversations over the years and the friendly gestures. Alexis, thank you for being fun and social. There are few dull moments when you are around which is much appreciated. James, thank you for being such a kind person always helping out. You are also very funny!
I would also like to thank all the PhD students, you are a good group. There have been many glorious moments, wow moments, and we have shared so many laughs and I really enjoy your company. Sofie, for being a fun person with many other good skillz who enjoys the same awesome movies as me, and almost the same computer game, gg. Thomas N, a.k.a. Kalix, you have many good qualities. You are for example very brilliant, helpful and also hilarious, srsly. You also make the best french fries. Beatriz, for being a fun person, the travel advice, our common interest in Japan and other interesting conversations. Christian, for being friendly and social and for introducing me to bouldering. Erik A, a fellow northerner who made basic mechanics a bit more fun. He is also the only person I know who is literally a world champion! Lam, for your friendly and happy way. Samuel, for being ever positive. Thanks for also introducing many new board games and being the astro pub master!

I also want to thank Pieter for your fun and social way, and last minute help. Greg, you are kind and fun, and also brilliant and very good at making tasty beer. And Thomas M for being fun and friendly and your Extraordinary Exquisite Lego Technique.

Simply put, I have awesome colleagues.

I also want to thank all my other Uppsala friends. I am very happy to have you as my friends! We have had a lot of fun over the years and it is always nice to see you all whether its for dinner, after work, movie night or adventurous travel involving a minibus full of engineers fully capable of fixing said minibus without any help..., etc. Or just one of our classic parties which usually involve a quiz, some sort of race, stretching, or a snow ball fight. Or all things combined.

I also have to thank my Piteå friends. We have known each other for a very long time. Even though we don’t see each other that often, it is always so uncomplicated, fun and nice when we do. It feels good to come “home” and see you all.

Then I have to thank my wonderful family. You are the best anyone could ever ask for. My brother and sister, thank you for inspiring me in so many ways. To my mum and dad, you are the kindest most caring parents. Thank you for always, always believing in me and supporting me. It means a lot to me.

And finally, to John. Thank you for your love and support. Thank you for giving me so much and bringing me so much joy. Thank you for being you. I love you.
References

Boro Saikia, S., Jeffers, S., Petit, P., & Marsden, S. 2015a, IAU General Assembly, 22, 2256700
Charbonneau, P. 2005, Living Reviews in Solar Physics, 2, 2
Charbonneau, P. 2010, Living Reviews in Solar Physics, 7, 3
Fan, Y. 2004, Living Reviews in Solar Physics, 1, 1
Acta Universitatis Upsaliensis

Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology 1367

Editor: The Dean of the Faculty of Science and Technology

A doctoral dissertation from the Faculty of Science and Technology, Uppsala University, is usually a summary of a number of papers. A few copies of the complete dissertation are kept at major Swedish research libraries, while the summary alone is distributed internationally through the series Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology. (Prior to January, 2005, the series was published under the title “Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology”.)