VIBRO-DRIVEABILITY

-A FIELD STUDY OF VIBRATORY DRIVEN SHEET PILES IN NON-COHESESIVE SOILS

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To Soli who made it possible

“Like all young and fresh PhD students I set out to solve my task like a genius, but with time mercifully laughter intervened.”

*after* Lawrence Durrell
SUMMARY

The most commonly used method to drive sheet piles is the vibratory driving technique; main reasons being the shorter installation time, less disturbance to the surroundings, and reduced damages to the driven sheet pile compared to impact driven sheet piles. It has become a desire to better predict the driveability, i.e. determine if it is possible to drive a certain sheet pile profile to desired penetration depth in a certain soil profile. The main problem to fulfil this desire lies in the lack of understanding of the fundamental mechanism behind the degradation of the penetrative soil resistance due to the continuous sheet pile motion.

This thesis constitutes the final report in the research project *Vibro-driveability and dynamic soil resistance in non-cohesive soils* within the Swedish Building Contractors Foundation (SBUF). This thesis presents the results of a study of full-scale, vibratory-driven sheet piles in non-cohesive soils. The primary objective of the study has been to develop a better understanding of the different mechanism and dynamic pile-soil interaction during vibratory installation of steel sheet piles. This has been achieved by dividing the present study into the following three parts: (i) a literature review, (ii) an experimental part, and finally (iii) an analytical part, where the results of two pre-existing prediction (simulation) models were compared with the results of the experimental study.

The thesis presents the results of a limited series of full-scale field tests where both the driveability and the ground vibrations generated during driving have been continuously monitored. In the light of these results, the thesis discusses how the complexity of vibro-driveability and the prediction of the induced vibration can be broken down and described in three subparts, namely: vibrator-related, sheet-pile-related, and soil-related parameters.

The fundamental mechanisms behind the shear strength reduction in cohesionless soils using the vibratory-technique to drive sheet piles have been explained. It appears as though the key phenomena behind the shear strength reduction observed during the vibratory installation of piles is not related solely to the liquefaction induced in satu-
rated granular soils, since the shear strength reduction has also been found in laboratory tests on air-dried granular soils.

Previously neglected, vibrator-equipment-related parameters, as well as sheet-pile-related parameters significantly affecting the vibro-driveability have been discussed in the light of the effects revealed during the field tests.

The vibro-driveability results from the field studies have been compared with the two vibro-driveability models, Vibdrive and Vipere, both of which were developed at University Louvain-la-Neuve, Belgium. The semi-empirical Vibdrive model has been used to study the predicted magnitude of the soil shear-strength reduction (the penetrative resistance) during vibratory driving, and how this is affected by variation in the two fundamental mechanisms. The semi-numerical Vipere model has been used to study the predicted magnitude of (i) the variation of soil resistance over time, and (ii) penetration speed versus depth during vibratory driving. These results have been correlated with the field test results.
Föreliggande avhandling berör ämnet vibrodrivning av spont i friktionsjord. Vibreringsteknikens ökande användning bottnar i huvudsak i att den i de flesta fall är en överlägsen metod att installera spont jämfört med konventionell fallhejare. Den begränsade erfarenheten av vibreringstekniken både nationellt och internationell har i stort sätt begränsat användningen av tekniken till enbart installation av spont. En av de primära svårigheterna är förståelsen av jordens reducerade penetrationsmotstånd under själva drivförloppet.

Avhandling utgör slutrapportering av forskningsprojektet *Vibrodrivning och dynamiskt jordmotstånd i friktionsjord* inom (SBUF) Svenska Byggbranschens Utvecklingsfond. Denna studie har genomförts med syfte att skapa en djupare insikt i ämnesområdet, samt ökad förståelse av den dynamiska samverkan av vibro-driven spont i friktionsjord, samt identifiera de primära faktorerna som inverkar på vibro-drivbarheten (neddrivningshastigheten). Studien är uppdelad i följande tre huvuddelar: (i) en litteraturstudie, (ii) en experimentell del, samt en (iii) analytisk del, där resultaten av två existerande prediktionsmodellerna för vibro-drivbarhet har jämförts med resultaten från det experimentella arbetet.

Avhandlingen presenterar resultaten av de i fält uppmätta storheterna av; dynamiskt jordmotstånd, effekter av friktion i spontlås, samt uppmätta markvibrationer, vilka kontinuerligt dokumenterats i samband med de genomförda fältförsöken. Med bakgrund av tidigare genomförda studier, och i fält observerade resultat, diskuteras i avhandlingen hur komplexiteten med vibro-drivning kan delas upp i tre huvuddelar, nämligen; vibratort-relaterade, spont-relaterade, samt jord-relaterade faktorer.

De grundläggande mekanismerna bakom reducerat jordmotstånd i samband med vibro-installation av spont i friktionsjordar förklaras. Den grundläggande parameter bakom friktionsjordars reducerade skjuvhållfasthet är inte enbart relat erad till jordförvätsknings (liquefaction) i vattenmättat tillstånd, då fenomenet med skjuvreduktion också återfinns i laboratorieförsök i torr friktionsjord.
Sammanfattning

Tidigare försummade effekter av såväl vibrator-relaterade som spont-relaterade faktorer, båda med påtaglig inverkan på såväl vibro-drivbarheten som omgivningspåverkan, diskuteras utifrån de i fält registrerade resultaten.

De dokumenterade storheterna på dynamiskt jordmotstånd och neddrivningshastighet har korrelerats med simuleringsresultaten av två existerande modellers, predikterade värden av vibro-drivbarhet (neddrivningshastighet) och predikterat jordmotstånd.
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CHAPTER 1

INTRODUCTION

1.1 Background

Steel sheet-piling consists of interlocked steel sections driven into the ground to form a wall. Sheet piles are specially designed to be driven into soils, and provide structural support or barriers for soils and water. Steel sheet-piling is used all over the world for all types of structures, however some of the more traditional applications are as retaining walls, docks, jetties, trenches, and cofferdams.

There are three basic techniques for driving steel sheet-piles into soils: impact driving, vibratory driving and jacking. From a historical point of view, piles and sheet piles have been driven into the ground by conventional impact hammers. However the use of the vibratory-driving technique has provided an alternative to conventional impact hammers since early in the 1930s. Since the 1970s, vibratory drivers have become increasingly popular amongst pile and sheet-pile contractors all over the world, because they enable a significantly higher production capacity to be attained, with less noise and less damage to the sheet piles in favourable soil conditions (non-cohesive soils). However, it is not always obvious which of the many vibratory driving systems available should be chosen for a given application, or the way the sheet piles should be driven in order to achieve the highest production capacity (highest vibro-driveability) and at the same time minimise environmental impacts such as noise and ground vibrations.

Surprisingly few publications associated with full-scale testing of vibratory-driven sheet piles are available. This is remarkable considering the numerous kilometres of sheet piles driven every year. Today’s limitation in engineering knowledge relating to the vibratory-driving technique presents an impediment for being able to utilise the full potential of the technique. Although past studies are important, very little has been done regarding a detailed description of the vibratory characteristics of full-scale sheet piles that have been vibratory-driven into the ground. The engineering issues related to the use of the vibratory-driving technique fall into the following three categories:

1. Vibro-driveability aspects
   - the driving and resisting forces generated and acting on the sheet pile,
- the penetration speed achieved versus depth, and
- the risk of damaging the sheet pile profile used.

2. Environmental aspects
   - the level of ground vibrations induced, and
   - the expected settlement in the vicinity of the sheet-pile wall.

3. Long-term aspects
   - the bearing capacity of vibro-installed bearing pile or sheet-pile wall, and
   - the prediction of bearing capacity from the driving log.

In this context, the thesis focuses on the vibratory characteristics of full-scale vibratory-installed sheet piles, since this has not been described systematically in detail to date. The objective has been to improve knowledge about the behaviour of vibratory-driven sheet piles by identifying the primary factors that significantly influence vibrowobble. This research project was undertaken within the Division of Soil and Rock Mechanics at Royal Institute of Technology, Stockholm, with grants from the Development Fund of the Swedish Construction Industry (SBUF), in co-operation with the two soil engineering and earthworks companies, the former Stabilator AB and Hercules Grundläggning AB. This research work has previously involved a thorough literature study of the driveability and bearing capacity of vibratory-driven piles and sheet-piles, as well as laboratory studies of the dynamic behaviour of the dynamic shaft resistance during vibratory driving of model piles.

1.2 The main objective and scope of the study

One of the main objectives of the study was to clarify the primary factors influencing the ability to install sheet piles in non-cohesive soils using modern vibratory-driving systems. For this reason, a limited series of full-scale field tests was performed on a heavily-sensored sheet pile and vibrator. In addition, the term vibro-driveability was defined, and the complex kinematics were divided into three main parts, designated vibrator, soil and sheet pile. The primary factors affecting vibro-driveability were also divided into the same three parts. The primary factors were studied during a limited series of full-scale field tests performed on a sensor-fitted sheet pile in a low-to-medium density sand.

Another primary objective of the study was to compare the vibro-driveability results with a simulation model. Two of the prediction models from the literature survey were selected for this. The results obtained in the field tests were compared with those predicted by the two models.
1.3 Structure of the thesis and related comments

Compared with what is known about impact driving, the understanding of vibratory-driving techniques is still in its infancy. This is mainly due to the limited amount of systematic studies performed, the lack of research generally, and the complexity of the technique.

This being the case, there is adequate justification for undertaking a comprehensive literature review and presenting it as a significant part of the thesis in Chapter 2. Chapter 2 commences with a historical review of the application of vibrators to drive piles and sheet piles. The principles behind the application of vibratory equipment and the key parameters generally used to characterise the performance of this equipment are also described. Chapter 2 also contains a review of some of the existing models published in the literature, along with previously conducted field and laboratory tests.

Chapter 3 is divided into three sections based on the conclusions drawn from the literature survey. It can be concluded that current research into the use of vibratory techniques for driving sheet piles can be described analogously as a number of isolated islands of knowledge. In Chapter 3, the author has attempted to link these islands of knowledge together using a common language and soil mechanics, and presenting a hypothesised overview of the vibro-driveability of sheet piles in non-cohesive soils.

Chapter 4 includes in-depth descriptions of the following: the instrumentation system developed for the field test, the chronology of the full-scale field test, the vibrator and sheet piles used in the test, the methods used to analyse the dynamic data obtained from the field tests, and the calibration procedures and constants.

Chapter 5 presents the results of the field study and comparisons with the two prediction models selected, namely Vibdrive and Vipere.

Finally, Chapter 6 contains a summary of the primary conclusions, as well as suggestions for future work.

The results of the study have to date been presented in five papers, the titles of which can be found in the list of references. Two of these papers deal with the ground vibrations generated in the vicinity of vibratory-installed sheet piles and have been presented at the following conferences:

1. The 25th Deep Foundation Institute’s Annual Meeting and Conference in New York City, October 2000.
The other papers deal with vibro-driveability related issues and have been (or will be) presented at the following conferences:

2. The 6th International Conference on the application of Stress-wave Theory to Piles in Brazil, September 2000.
3. The 25th Deep Foundation Institute’s Annual Meeting and Conference in New York City, October 2000.

Some of the raw data from the field measurements and the computer analyses have not been described in this thesis (the reasons for this are described later in the thesis). However all of the data has been compiled in a companion CD-ROM entitled, “Vibro-driveability and environmental studies of vibratory-installed sheet piles”. This is available for lending to interested parties for an administration fee of €30.00, on written request from the Division of Soil and Rock Mechanics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden.
CHAPTER 2

LITERATURE REVIEW

2.1 Introduction to vibratory driving

Vibratory-driving techniques are used all over the world primarily for driving and extracting sheet piling. Even though the technique is global, there are very few publications relating specifically to using vibratory driving, compared to the number relating to using impact driving, for installing piling or sheet piling. The lack of publications in the area reveals the potential for further research.

A major limitation with using the vibratory technique for driving piles is a lack of guidelines for vibratory driving in relation to driving refusal and bearing capacity. This has lead to the current situation where vibratory-driving techniques are used primarily for driving and extracting sheet piles.

From the literature survey, it was found that the vibratory characterisation of full-scale sheet piles, driven into the ground using vibratory techniques, has not been published systematically in detail. A decision was therefore made to summarised all the known publications as part of this thesis. The summaries follows this introductory section, and are briefly introduced below.

Section 2.2 consists of a brief history of the vibratory technique from both an international and a Swedish perspective. Parts of the international background have come from Rodger (1967), Rodger and Littlejohn (1980), and O’Neill and Vipulanandan (1989a).

Section 2.3 provides a brief review of the functioning of modern vibratory-equipment and various components, mechanical action, and the most important vibratory-parameters.

Section 2.4 addresses the definition of vibro-driveability and key parameters recognised as important in the vibro-driveability of piling and sheet piling. Key parameters affecting vibro-driveability have been categorised into the following three sections: vibratory-related, pile-related and soil-related.

Section 2.5 reviews some vibro-driveability tests conducted previous, and Section 2.6 presents previous attempts at predicting vibro-driveability.
Chapter two

2.2 Historical development

2.2.1 International developments

The idea of pile driving using the vibratory techniques originated almost simultaneously in both Germany and Russia at the beginning of the 1930s.

The first experiments were conducted in Germany in 1930, (Lorenz, 1960), followed by the first commercial application by Hertwig in 1932, using a vibratory driver to drive timber piles for the foundations of the Technische Hochschule in Berlin Charlottenburg. Together with Losenhausenwerk, Hertwig registered the first patents relating to the principle of the vibratory-driving technique in Düsseldorf.

In the former USSR, the concept of vibratory-driving was formulated by Pavlyuk as a by-product of his soil-dynamics research on footing vibrations. This was in 1931 and described later by Barkan (1957, 1962). The initial research carried out in the former USSR was continued by Barkan, who strengthened the use of the vibratory-driving technique. Barkan showed that the use of the vibratory-driving technique to install piles affected the soil differently than traditional methods such as impact driving. Barkan specifically pointed out that the shaft resistance of vibratory-driven piles was drastically reduced compared with impact driven piles.

After interruption to vibratory research during World War II, the Russians resumed their research investigating the possibility of extending the method to cohesive soils. Rusakov and Kharkevich (1942) and Tsaplin (1953) reported that a vibratory driver run at very low frequencies worked like an impact hammer. The first major post-WWII application of the technique was reported in 1949 by the former USSR, when vibratory drivers were used to drive sheet piles in the construction of the Gorky Hydroelectric Power Plant.

Research continued on both practical and theoretical levels. Use of the vibratory technique gradually became established, and several reports were published about the high rates of production associated with the technique. The use of vibratory-driving techniques was subsequently combined with water-jetting, allowing piles to be driven into denser soils. Up until that time, all vibratory drivers had been free-hanging systems, but in 1953 the first leader-mounted vibratory-driver systems were introduced onto the market by Savnoy and Luskin. These were called VPP vibratory drivers. These new vibratory-driving systems could not only vary the static surcharge force, but also the static moments by moving the position of the eccentric weights relative to each other. With all these new ideas, the uses of vibrators increased, and by the beginning of the 1960s, vi-
Vibrator units were being commercially produced in Germany, France, the USA, the USSR and Japan.

Barkan came up with the idea of increasing the capacity of the vibratory drivers by using the resonance effects of the vibratory-pole system. This resulted in the production of three different high-resonant-frequency vibratory-drivers in the former USSR in 1959. At the same time in the USA, Bodine started to develop vibratory drivers working at high frequencies with a capacity of up to 150 Hz. In 1961, one of the first practical applications of the Bodine Resonant Driver (BRD), was reported on a C.L. Guild Construction site, at East Province, Rhode Island (USA). These high frequencies allowed vibratory driving close to the longitudinal resonant frequencies of the piles being driven, which increased the rate of penetration even further. However, there were still no theories that could adequately predict the bearing capacity of a vibratory-driven pile. In 1970, Davisson (1970) proposed a formula for bearing capacity, developed for exclusive use with the BRD. The proposed formula was based on “The Engineering News” formula used for predicting the bearing capacity of impact-driven piles. Davisson’s formula used an energy-equilibrium method together with an empirical-loss factor, for predicting the bearing capacity with the possibility for altering the formula for predicting either the rate of penetration (the driveability) or the bearing capacity.

During the 1960s, a number of different research projects were initiated in the USA in order to study the penetration resistance and bearing capacity of vibratory-driven piles. This resulted in several PhD theses, including Hill (1966), Ghahramani (1967), Yang (1967) and Griggs (1967).

At the same time as the research projects commenced in the USA, German and French technicians began to show an interest in the vibratory technique, and developed vibrator units with a working frequency of 50 Hz. However, due to problems relating to the durability of the bearings, the driving frequency was lowered to about 25 Hz. This resulted in a tremendous increase in the reliability of the technique.

During the 1970s, various computer programmes were developed to predict the bearing capacity of vibratory-driven piles. These programmes used the same pile and soil model as Smith's lumped-mass formula for impact driving.

In 1980, Rodger and Littlejohn presented a comprehensive driveability study of model piles in dry, non-cohesive soils. The authors likened the movement of the pile to that of a rigid body, where the pile shaft is modelled by a viscous-Coulomb shaft resistance, and the pile-toe resistance is modelled by an elastoplastic resistance under a combined sinusoidal excitation with a static surcharge force.
From the end of the 1980s to the middle of the 1990s, at the University of Houston, Texas (USA), a more comprehensive series of research studies was conducted by O'Neill and Vipulanandan (1989a, b, 1990). They conducted extensive laboratory studies on three different geometrically-related parameters (vibratory-driver, soil and pile) that influenced both the driveability and the bearing capacity. The results of the vibratory-driven model piles were partially compared with the results of the impact-driven piles. The vibratory-driver-related parameters studied included for example: dynamic force, displacement amplitude, driving frequency, static surcharge force and eccentric moment. The soil-related parameters studied included: particle size, the degree of compaction, in situ stresses and the degree of saturation. The pile-related parameters studied included: open-ended and close-ended pipe piles and various low-displacement piles.

At the beginning of the 1990s, a research project was initiated at the University of Karlsruhe (Germany), resulting in several PhD theses and articles, including Dierssen and Gudehus (1992), Dierssen (1994), and Cudmani (2000). Dierssen (1994) presented a new soil model for predicting the driveability of vibratory-driven piles in non-cohesive soils. This model used the German heavy-probing tests as geotechnical input for estimating dynamic soil resistance.

In the Department of Structural Dynamics at the Netherlands Organisation for Applied Scientific Research (TNO) and the Department of Soil Mechanics and Execution Techniques at the Belgian Building Research Institute (BBRI), research projects relating to the driveability of sheet piles were conducted during the early 1990s. However only a few publications were published. At the end of the 1990s, another developmental research project was conducted, referred to as the Soil Investigation Procedure Determining the Installation of Steel Sheet Piles or SIPDIS. This project involved several partners from different countries (Profile ARBED, (L); British Steel, (UK); MRC Technique Services (Krupp Int.), (G)), and aimed to develop a concept for the definition of the most efficient and environmentally-friendly sheet-pile-installation method. However, the SIPDIS project has not yet resulted in any publications.

2.2.2 Developments in Sweden

In the middle of the 1970s, the vibratory technique started to become more frequently used by Swedish contractors. This was basically because of the higher driveability in non-cohesive soils, the lower ground vibrations, and the reduced noise levels associated with the technique. At the time, there were about 25 vibratory-driving units in the country,
mostly of Japanese origin. Most of the vibratory machines at that time could, if required, vary the static moment in different ways. Some could even vary the driving frequency.

In Sweden, the vibratory-driving technique never really established itself as an alternative method for driving load-bearing piles for several reasons (see below), and although the technique was frequently used during the 1970s, little Swedish documentation on the method can be traced. Eresund (1977) contains a brief overview of a considerable number of Swedish case studies where vibratory-driven piles can be found. One of the few publications by a Swedish contractor is the report by Mazurkiewicz (1975), who performed full-scale load-test comparisons between vibratory-driven and impact-driven load-bearing piles during a contract in northern Poland. These results showed clearly that vibratory-driven piles tended to develop a lower bearing capacity than impact-driven piles. This case together with several others, resulted in the abandonment of the vibratory-driving technique in Sweden. Other reasons for abandoning the technique, were the problems encountered attaching the concrete piles rigidly to the vibrator, and the fact that standard Swedish concrete piles were not prestressed in nature (that is, not able to cope with the tensions developing within the pile during driving).

Two other well-documented publications of Swedish origin that have come to light, are the study of the driveability of slender profiles by Westin (1978a, b), and the study of ground vibrations induced with sheet piles near existing structures by Hermansson and Åstrand (1974).

Another application for these vibrator units is the deep-soil-compaction method. In the beginning of the 1980s, two MSc theses (Jönsson and Wintzell, 1983; Avermalm and Callne, 1983) were conducted in co-operation with a major Swedish contractor. This method was called the VibroWing-method and after further development became the MRC compaction method (Sandberg and Törnbom, 1996; Krogh and Lindgren, 1997).

One of the most recent Swedish reports relating to the various application areas for the vibratory technique, is Massarsch (2000), which contains an extensive description of that author’s experience in this field.

## 2.3 Modern vibratory-systems

This section briefly describes the different parts of modern vibratory-driving systems, vibratory drivers, and important vibratory parameters relating to driveability and important environmental aspects.
2.3.1 Machine systems

There are two main types of modern vibratory-driving systems commercially available. These are free-hanging and leader-mounted vibrator systems (see Figures (2-1) and (2-2)).

The free-hanging vibratory-machine system is traditionally used in the Netherlands and Belgium. In Germany, the leader-mounted vibratory-machine system is preferred. In Sweden, there are no clear traditions concerning these choices, however lately the leader-mounted system tends to be more commonly used.

Free-hanging systems

The free-hanging vibratory-driving system basically consists of five main parts:

- a hydraulic-power unit,
- a power-transmission system,
- the vibrator,
- a hydraulic clamp or clamps, and
- a carrier for the vibrator unit (see Figure (2-1)).

The hydraulic-power unit is usually a diesel engine (for example a Caterpillar) connected to a reliable hydraulic pump with a fluid regulator. Sometimes even an electric motor can be used to regulate the hydraulic pump in order to minimise noise and exhaust emissions. A control panel enables the driver to operate all the hydraulic functions, such as oil flow, oil pressure, eccentric moment and the cylinder of the clamping device.

The power-transmission system consists of an open-loop hydraulic system with hydraulic hoses and electrical cables. The return oil flow from the vibrator is normally cooled in order to facilitate maintaining maximum oil pressure and flow.

The vibrator, which generates the movement, can weigh anywhere between 150 and 16,000 kg, and can therefore require a powerful carrier. A mobile crane is normally used to lift and manoeuvre the free-hanging vibrator and sheet pile into position. Smaller vibrators are sometimes connected directly to an existing excavator, and driven by the excavator’s own hydraulic system.

Drawbacks with the free-hanging system referred to include difficulties in positioning the object being driven, and changing the surcharge force \((F_o)\) during driving. The difficulties in positioning the sheet pile are due to the system’s in-built limitations in both controlling and guiding the sheet pile into the desired position during the installation phase. The static surcharge \((F_o)\) can be reduced by holding the vibrator with a crane.
However $F_0$ cannot be increased in the same way as in leader-mounted systems, where the vibrator can be pushed downwards.

Advantages with the free-hanging system include lower costs and greater reach. Production costs are much lower using free-hanging systems than using leader-mounted systems. In addition, the carrier can also be used for purposes other than holding the vibrator unit. With a mobile crane as the platform for the vibrator, it is possible to reach a larger area from a single spot. This can be beneficial when the bearing capacity of the underlying soil is low. When the ground stability beneath a carrier supported by legs is poor, the risk of serious incidents increases.

![Figure 2-1](image)

**Figure 2-1** The main parts of a free-hanging vibratory-machine system (after the ICE vibrator manufacturer’s brochure).

**Leader-mounted systems**

Telescopic leader-mounted vibratory-machine systems and free-hanging systems share many common features. The main difference is that the vibrator carrier is replaced by a telescopic leader with a parallelogram-based kinematic system. Otherwise the main parts of a leader-mounted system are essentially the same as the free-hanging system (see Figure (2-2)).

The vibrator itself is carried by a telescopic leader, usually mounted on a base machine such as rebuilt excavator chassis, equipped with special motors, pumps and hydraulic-oil coolers.
The advantages of the leader-mounted system include the high linkage-point of the leader mast, which increases its manoeuvrability, allowing positioning of the vibrator to any pre-desired angle of inclination during the pile-installation phase. The static surcharge force can also be adjusted and controlled across a wider range compared to the free-hanging system. The control panel of the vibrator and carrier system is integrated into a single unit, which enables the driver to operate the vibrator system with greater precision with respect to positioning, driving and extraction forces.

The drawbacks of the leader-mounted system are usually the costs; plus they can seldom be used for purposes other than vibratory driving and augering. Other drawbacks are the weight of the system, which can cause ground stability problems at the construction site, as well as transportation problems such as exceeding maximum bridge loads.

![Figure 2-2 The main parts of a leader-mounted vibratory-machine system (after the ABI vibratory-machine manufacturer’s brochure).](image)

**Accessories**

Modern vibratory-machine systems can be supplemented by various pieces of additional equipment, such as jetting aggregates, clamping devices, biodegradable oils and special silencing accessories for noisy power units.

Jetting aggregates are used in denser soil conditions in order to smoothen the installation process. There are at least three different types of jetting procedures:
• water jetting at low pressure,
• water jetting at high pressure, and
• air jetting.

The principle behind jetting procedures is to channel water or air in small tubes along the pile shaft to the pile toe, where the jetting medium then loosens the soil or “pre-digs” a hole at the pile toe. Each jetting technique works differently and careful consideration must go into method selection, as the purpose of using this technique is to loosen the surrounding soil.

Hydraulic clamping-devices are used to transfer the vibrations axially to the object intended to be installed by driving. Different types of clamps exist, specially developed in order to transfer the driving force axially into the object and at the same time minimise damage to the object being driven. There are at least five main types of vibratory clamping-devices, being:

• single clamps,
• double clamps,
• Caisson clamps,
• double sheeting clamp jaws, and
• wooden and concrete pile clamping-devices.

These five types along with their typical arrangements are shown in Figure (2-3).

![Figure 2-3 Types and arrangements of vibratory clamping-devices (after Tünkers brochure).](image-url)
Using biodegradable vegetable oils in the vibratory system instead of more toxic lubricants can be justified from an environmental point of view, and should be taken into consideration as there is a risk of oil spillage into the surrounding ground due to frequent oil changes.

Silenced power-units are advisable for construction sites situated in urban areas.

### 2.3.2 Vibratory drivers

The two main types of commercial vibratory-drivers are hydraulic and electrical. However, hydraulic vibratory-drivers predominate on the market because they are both lighter in weight, and allow driving-frequency adjustment.

Five different types of hydraulic vibratory-drivers can be distinguished, based on the driving frequency \( f_d \) \([\text{Hz}]\) used, and their being equipped with the facility to adjust the unbalanced moment \( M_e \) \([\text{kgm}]\). The classification of hydraulic vibrators, summarised in Table (2-1), has been proposed by several authors and tends to vary depending on different ranges.

\[
\begin{array}{|c|c|c|}
\hline
\text{Type of vibrator} & \text{Range of } f_d \text{ [Hz]} & \text{Range of } M_e \text{ [kgm]} \\
\hline
\text{Standard frequency} & 21 - 30 & > 230 \\
\text{High frequency} & 30 - 42 & 6 - 45 \\
\text{Variable eccentricity} & 40 & 10 - 54 \\
\text{Excavator mounted} & 30 - 50 & 1 - 13 \\
\text{Resonant driver} & > 100 & 50 \\
\hline
\end{array}
\]

Modern hydraulic vibrators (free or leader systems) consist of the following four main parts:

- a suppressor housing,
- elastomer pads,
- an exciter block, and
- a clamping device.

The vibrators used for free-hanging systems are constructed with the rotating eccentric weights laid flat, and the vibrators mounted on leader systems have their weights placed upright (see Figure (2-4)).
The suppressor housing, to which the exciter block is mounted via a number of elastomer pads, is either fitted with an eye for connecting the hook of a crane to, or with a guide-frame for connection a leader mast to it. The weight of the suppressor housing is usually called the bias mass or static surcharge force \( F_o \) [kN].

The elastomer pads are usually made of steel-reinforced rubber bounds, connecting the suppressor housing with the exciter block. The main purpose of these pads is to prevent vibrations being transferred to the crane or leader mast (that is to isolate the vibrations in the moving exciter block from that part of the vibrator held by the crane or leader mast.

The exciter block is where vertical vibrations are generated, and it contains the hydraulic motor, the gearbox and the counterrotating eccentric weights. The eccentric weights are rotated by the hydraulic motors and synchronised by a gear system in such a way that the horizontal components of the centrifugal forces are eliminated and the vertical component added to each other.

Finally the hydraulic clamping-device at the bottom of the vibrator forms the rigid connection between the driven object and the vibrator. Its only purpose is to transmit the vertical motion of the vibrator to the object being driven. It can be replaced to fit any type of pile, including wooden piles, concrete piles, H beams, wide-flange beams, flat steel plates, sheet piles or even plastic sheet piles (see Figure (2-3)). Clamping and releasing the jaw pads and holding the object being driven is achieved hydraulically. It is of the utmost importance that the jaw pressure is correct in these applications and that it is held constant during the installation phase.

**Figure 2-4** Diagram representing the four main parts of modern hydraulic vibrators. A free-hanging vibrator is shown on the left and a leader-mounted vibrator is shown on the right.
2.3.3 Main parameters of modern vibratory systems

The mechanical action in modern vibratory-equipment is governed by the driving force \((F_d [kN])\) generated (shown in Figure (2-7)), and consists of the following two parts:

- a stationary part called the static surcharge force \((F_o [kN])\), and
- a vibratory part called the sinusoidal vertical force \((F_v [kN])\).

The driving force \((F_d)\) generated can be seen as the theoretically-generated driving capacity of the vibratory-driving system under consideration. The driving force \((F_d)\) is usually described as the sum of the surcharge force and the sinusoidal vertical force \((F_d = F_o + F_v)\).

The theoretical performance of modern vibratory-equipment is governed by several vibratory parameters, and the most important of these are listed in the following:

- static surcharge force \((F_o)\),
- eccentric moment \((M_e)\),
- driving frequency \((f)\),
- unbalanced force amplitude \((F_c)\),
- dynamic mass \((m_{dyd})\),
- free-hanging (single) displacement amplitude \((S_o)\), and
- theoretical power \((P_t)\).

These vibrator-related properties are presented in more detail in the following sections.

**Surcharge force \((F_o)\)**

The stationary action imparted to the vibrator-driven object is represented by the operation of the vibratory-equipment carrier used; usually called the surcharge force in the case of a leader system, and even bias mass in the case of a free-hanging system. The stationary action (static surcharge force) is applied to the vibrator’s exciter block via the elastomer dampers that couple and isolate the suppressor housing to and from the exciter block (see Figure (2-3)).

In free-hanging vibrator-equipment, the surcharge force \((F_o)\) is represented as the sum of the weight of the bias mass plus the suspension force of the carrier (a crane), according to the following expression:

\[
F_o = g m_o - T \tag{2.1}
\]
where \( F_o \) = static surcharge force [N], 
\( g \) = gravity \([m/s^2]\), 
\( m_o \) = bias mass [kg], and 
\( T \) = suspension force of the carrier (crane), [N].

The static surcharge force \( (F_o) \) of a leader-mounted vibrator-system consists of the hydraulically-applied pre-stress pressure, generated by the hydraulic cylinders on the leader mast, together with the weight of the suppressor housing, given theoretically by:

\[
F_o = g m_{sp} + P_o A_{cyl}
\]  
(2.2)

where \( F_o \) = static surcharge force [N], 
\( g \) = gravity \([m/s^2]\), 
\( m_{sp} \) = weight of the suppressor housing [kg], 
\( P_o \) = hydraulic oil pressure \([N/m^2]\), and 
\( A_{cyl} \) = area of hydraulic cylinder \([m^2]\).

**Eccentric moment \( (M_e) \)**

The eccentric moment of a single eccentric mass \( (M_{ei} [kgm]) \) is the product of the weight of the eccentric mass \( (m_{ei}) \) and the radial distance \( (r_{ei}) \) measured from the centre of the motor shaft to the centre of gravity of the eccentric mass (see Figure (2-5)). The eccentric moment is given by:

\[
M_{ei} = m_{ei} r_{ei}
\]  
(2.3)

where \( M_{ei} \) = eccentric moment \([kgm]\), 
\( m_{ei} \) = weight of the eccentric mass [kg], and 
\( r_{ei} \) = eccentric radius [m].

The specified eccentric moment \( (M_e) \) of a modern vibrator-unit is the sum of each single eccentric mass moment, given by:

\[
M_e = \sum_{i=1}^{n} m_{ei} r_{ei}
\]  
(2.4)
Driving frequency \( (f_d) \)

The driving frequency \( (f_d) \) is specified by the number of revolutions of the eccentric masses per second, sometimes also expressed as rotations per minute [rpm]. A third way of expressing the speed at which the eccentric masses rotate, is the angular frequency \( (\omega) \). The three different ways can be combined and expressed by:

\[
\omega = 2\pi f_d = \frac{2\pi n}{60}
\]  

where \( \omega \) = angular frequency [rad/s], \( f_d \) = driving frequency [Hz], and \( n \) = rotations per minute [rpm].

Unbalanced force \( (F_v) \)

The vibratory action generated by the vibrator is represented by the vertical component \( (F_v) \) of the centrifugal forces \( (F_c) \) generated. The centrifugal forces are a result of inertial effects due to the counter-rotating eccentric masses within the exciter block of the vibrator.

The direction and magnitude of the vibratory action is assessed by applying Newton’s second law \( (\Sigma F = ma) \) to the counter-rotating eccentric masses. The acceleration directed toward the centre of the curvature \( (a_{ci}) \) of a singular rotating eccentric mass is shown in Figure (2-6), and given by:

\[
a_{ci} = r_{ei} \omega^2
\]  

where \( a_{ci} \) = centrifugal acceleration \([m/s^2]\), \( r_{ei} \) = radial distance to gravity centre \([m]\), and \( \omega \) = angular frequency \([rad/s]\).
The tangential component of the acceleration ($a_{ti}$) of the rotating eccentric masses is zero when the angular frequency is kept constant. The radially-directed centrifugal force ($F_c$) of each rotating eccentric mass is then calculated according to Newton’s second law, using Equation (2.3) and Equation (2.6), expressed as follows:

\[
F_c = m_{ei} a_{ci} = m_{ei} r_{ei} \omega^2 = M_{ei} \omega^2
\]

where

\begin{align*}
F_c &= \text{maximum unbalanced centrifugal force of the vibrator [N]}, \\
m_{ei} &= \text{weight of eccentric mass [kg]}, \\
a_{ci} &= \text{centrifugal acceleration [m/s}^2\text{]}, \\
r_{ei} &= \text{eccentric radius [m]}, \\
\omega &= \text{angular frequency [rad/s]}, \text{ and } \\
M_{ei} &= \text{eccentric moment [kgm]}.
\end{align*}

Figure 2-6  Description of the unbalanced forces generated by the counter-rotating eccentric masses.

The maximum unbalanced centrifugal force ($F_c$) generated by the vibrator can be assessed when both the eccentric moment ($M_e$) and the driving frequency ($f_d$) are known from Equation (2.7). The vertical unbalanced driving force ($F_v$) is given by taking the vertical component of ($F_c$) according to:

\[
F_v = F_c \sin \theta = M_e \omega^2 \sin \theta
\]

where

\begin{align*}
F_v &= \text{vertical driving force [kN]}, \\
F_c &= \text{unbalanced centrifugal force [kN]}, \\
M_e &= \text{specified eccentric moment [kgm]}, \\
\omega &= \text{angular frequency [rad/s]}, \text{ and } \\
\theta &= \text{rotation angle of eccentric mass [°]}. 
\end{align*}
Driving capacity \((F_d)\)

The theoretical driving force delivered to the head of the object being driven is the sum of the surcharge force \((F_o)\) and the vertical driving force \((F_v)\) generated. \(F_d\) can be viewed as the theoretical driving capacity of the vibratory equipment considered, given by:

\[
F_d = F_o + F_v
\]

where \(F_d\) = theoretical driving force \([N]\), \(F_o\) = static surcharge force \([N]\), and \(F_v\) = unbalanced force \([N]\).

Theoretical performance curves of modern vibratory-driving equipment are shown in Figures (2-7), and (2-8). The theoretical driving force \((F_d)\) is a downwardly-directed, time-dependent harmonic force that describes a sinusoidal path in time based around a value represented by the static surcharge force \((F_o)\), (see Figure (2-7)).

The maximum theoretical driving-capacity \(\hat{F}_d\), which is the value of the amplitude of the theoretical driving force, is shown in Figure (2-8) as a function of a variable driving frequency \((f_d)\) and static moment \((M_e)\).

Dynamic mass \((m_{dy})\)

The movement of the vibratory-driven object depends on what is called dynamic mass and the dynamic soil resistance. Dynamic mass \((m_{dy})\) is defined as the sum of the masses that move in the longitudinal direction of the driven object, consisting of the weight of the exciter block and the hydraulic clamping-device, and the weight of the sheet pile, given by:
where $m_{dy} = m_{eb} + m_{cl} + m_p$.

Figure 2-8 Illustration of vertical driving force amplitude as a function of the vibrator’s static surcharge force, unbalanced moment, and angular frequency.

**Free-hanging (double) displacement amplitude ($S_o$)**

Vibrator specifications often list a parameter called the maximum specified displacement amplitude ($S_sp$), represented in Figure (2-9). This parameter corresponds to the total (double) amplitude of the movement of a free-hanging vibrator; in other words the quotient of the maximum specified unbalanced moment ($M_e$), and the dynamic mass of the free-hanging vibrator ($m_{dy}$) given by:

$$S_{sp} = 2s_o = 2 \frac{M_e}{m_{eb} + m_{cl}}$$

where $S_{sp} =$ specified double-displacement amplitude [mm], $s_o =$ specified single-displacement amplitude [mm], $M_e =$ specified unbalanced moment [kgm], $m_{eb} =$ weight of exciter block [kg], and $m_{cl} =$ weight of the clutch [kg].

It should be noted that the double-displacement amplitude specified is not dependent on the operational driving frequency ($f_d$) since the dynamic mass of the free vi-
brating system remains stationary. However if the vibrator is equipped with the possibility of varying the specified unbalanced moment \( (M_e) \) during operation, this provides the driver with an optimum operational regime (with regard to the actual geology), for minimising environmental harm and maximising energy savings.

Figure 2-9 Definition of double amplitude, free-hanging displacement.

The double-displacement amplitude \( (S_o \text{ [mm]}) \) of a free-hanging sheet pile to be vibrated will always be smaller than the vibratory-driver manufacturer’s specified double-displacement amplitude \( (S_{op}) \) given by:

\[
S_o = \frac{m_{eb} + m_{cl}}{m_{eb} + m_{cl} + m_p} S_{sp}
\]  

(2.12)

where \( S_o \) = free-hanging double-displacement amplitude [mm], 
\( m_{eb} \) = weight of exciter block [kg], 
\( m_{cl} \) = weight of clamping device [kg], 
\( m_p \) = weight of sheet pile [kg], and 
\( S_{sp} \) = specified double-displacement amplitude [mm].

The main reasons for the actual double-displacement amplitude \( (S_o) \) being less than specified are two-fold. Firstly, the dynamic mass is increased by the weight of the sheet pile according to \( m_p = \rho AL_o \), and secondly, the sheet pile driven is subject to penetrative resisting forces, such as soil and clutch resistance.

**Theoretical power \( (P_t) \)**

Vibrator manufacturers often specify the theoretical power \( (P_t \text{ [kW]}) \) parameter. The specified power generally corresponds to the nominal power of the hydraulic motor that rotates the eccentric masses. The theoretical power is a vibratory parameter that does not correspond to any standardised operational conditions of vibrator equipment during actual driving conditions. Therefore, in the author’s opinion, the specified power limitations found in the vibratory-driver manufacturers’ brochures are both insufficiently
characterised, and incompletely accounted for in vibratory-driving analyses. It is well known that power consumption is in fact dependent upon the actual driving conditions.

In O’Neill and Vipulanandan (1989a), there is however a method provided for calculating the theoretical power, which expresses the power required to maintain the vibrating amplitude of a dynamic mass in the absence of soil reactions, but accounts for the presence of the isolation spring and bias mass of a free-hanging vibrator. These authors also state that the actual power ($P_a$), or energy delivered to the pile head by the vibrator, is always less than the power or energy theoretically produced ($P_t$) by the vibratory driver. This is mainly due to the energy dissipation induced by the movements of the suppressor housing, the mechanical energy losses in the exciter block, the energy losses at the pile-clamp connection due to sliding friction, and the flexural energy of long slender piles and sheet piles. The ratio between the delivered power ($P_a$) and theoretical power ($P_t$) expressed as $P_a/P_t$ was considered by the same authors to be an efficiency factor.

The theoretical power ($P_t$) is calculated by the same authors as the product of the net-force function generated by the vibratory-driving equipment ($F(t)$) and the vibrational velocity ($v(t)$), derived from a simple single-degree-of-freedom system, and mathematically expressed as follows:

$$P_t = \frac{1}{T} \int_{0}^{T} F(t) \cdot v(t) \, dt$$

(2.13)

$$\frac{1}{T} \int_{0}^{T} \left[ F_o + (M_e \omega^2 + M \zeta \omega^2) \sin \omega t \right] \cdot \zeta \omega \cos \omega t \, dt =$$

$$\zeta \left[ 4F_o + 2(M_e \omega^2 + M \zeta \omega^2) \right]$$

where $P_t$ = theoretical power [kW],
$T$ = period of time [s],
$F(t)$ = net force acting on the vibrator (function over time) [kN],
$v(t)$ = vertical velocity of the vibrator (function over time) [mm/s],
$t$ = time [s],
$F_o$ = weight of bias mass, i.e. static surcharge force [kN],
$M_e$ = unbalanced moment [kgm],
$\omega$ = angular frequency [rad/s],
$M$ = weight of exciter block (excluding eccentric masses) [kg], and
$\zeta$ = amplitude of the dynamic motion of the vibrator, expressed according to Equation (2.14).
The symbol $\varepsilon$ used here is the amplitude of the dynamic motion of the vibratory driver, which can be expressed as:

$$\varepsilon = \frac{\omega^2 M_e}{M(\omega_n^2 + \omega^2)}$$  \hspace{1cm} (2.14)

where $\omega$ = angular frequency [rad/s],
$M_e$ = unbalanced moment [kgm],
$M$ = weight of exciter block (excluding eccentric masses) [kg], and
$\omega_n$ = natural angular frequency of the primary mass and spring system $(k/M)^{0.5}$ [rad/s].

### 2.4 Vibro-driveability tests

The most reliable method for determining the driveability of a pile or a sheet pile is to perform a pre-driveability test. Ideally, the procedure should be a systematic programme where several of the profiles being considered are driven to various depths and with different vibratory parameters. The parameters of interest are then documented, as it is reasonable to assume that other profiles at the same site would develop approximately the same driving “history”. However as full-scale tests are costly, the proposed test-procedure should duplicate the conditions in the field as closely as possible. Unfortunately, there are only few well-documented and systematic tests found in the literature. However, based on the ambitions and complexity of these few cases, they can be categorised as follows:

- conceptual-model testing,
- pile-soil interface testing,
- large scale tests, and
- full scale tests.

As the testing categories above have been performed systematically and in considerable detail, it can therefore be concluded which parameters significantly affect driveability.

Before proceeding to the vibro-driveability tests found in the literature, a clarification of the term “driveability” is warranted, with respect to modern vibratory-driving equipment, and its use for the installation of different profiles.
2.4.1 Vibro-driveability

It is the author’s opinion that the term “vibro-driveability” should be used instead of “driveability” in conjunction with vibratory drivers, in order to be clear about what is meant.

The term “vibro-driveability” is normally used in relation to vibratory-driven piles or sheet piles, and normally relates to the rate of penetration with which the driven object moves through the soil strata. The rate of penetration, or penetration speed \( (v_p) \) is normally given in mm/s. The term “vibro-driveability” is further discussed by the author in Section (3.2.2).

In extensive research conducted at the University of Houston, Texas (USA), Rao (1993) subdivided the penetration speed \( (v_p) \) of his reduced-scale tests of vibratory-driven “model” piles into three driving states, ranging from hard to easy (see Table (2-2)).

<table>
<thead>
<tr>
<th>Driveability states</th>
<th>Corresponding penetration speeds ( (v_p) ) in [mm/s] (after Rao, 1993)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard</td>
<td></td>
</tr>
<tr>
<td>( v_p &lt; 22 )</td>
<td>22 &lt; ( v_p ) &lt; 60</td>
</tr>
</tbody>
</table>

In the Technical European Sheet Piling Association (TESPA) brochure (1995), it was stated that a value of approximately 8 mm/s for \( v_p \) could be adopted as a limiting value for vibro-driveability that would serve as a guide for limiting possible vibration-related nuisance factors. According to TESPA, vibratory driving below this recommended limit \( (v_p \sim 8 \text{ mm/s}) \) requires careful monitoring to avoid excessive heat production in the interlocks.

2.4.2 Conceptual-model testing

Conceptual-model testing can be characterised as the first attempt to reveal the fundamentals behind the soil-structure interaction mechanisms; in other words, the vibro-viscous shear-resistance of soils within a vibratory framework. Some of the earliest research programmes including these conceptual-model tests were conducted in the former USSR.

Barkan (1962) reported on how the speed that steel balls subject to a surcharge force sank into a sand-filled vessel that was vibrated. The penetration speed of the steel balls was presented in the form of a logarithmic plotting of the penetration depth \( (z \text{ [m]}) \) versus time \( (t \text{ [s]}) \), (see Figure (2-10)). The results and major findings of these conceptual-model tests are presented in more detail in Section (2.5).
2.4.3 Pile-soil-interface testing

The shear strength of soil that resists the extraction of a vibrating steel profile in a normal stress-controlled medium of sand (called a vibratory, direct shear test), has been investigated by the former Soviet researchers Levchinsky and Savtchenko, and reported by Barkan (1962). The friction coefficient was reported to be reduced depending on the displacement amplitude and frequency applied. This is presented in more detail in Section (2.5).

Another well-quoted article relating to pile-soil-interface testing is the vibratory direct shear test by Hereema (1979), carried out on both cohesive and noncohesive soil samples.

![The log of penetration versus time in sphere experiments (after Barkan, 1962).](image)

The relationship of dynamic shear resistance to velocity in sand samples was reported to be independent of displacement velocity. Further conclusions and findings relating to these pile-soil-interface tests are presented in Section (2.5).

In the author’s opinion, the pile-soil-interface testing does not represent actual field conditions, mainly for the reason that dynamic effects were not introduced into the soil sample.
2.4.4 Reduced-scale tests

Procedures for testing model profiles vibratory-driven into a soil-filled vessel are usually called reduced-scale tests. The early attempts most often referred to are those by Bernhard (1967), Hill (1966), Schmid and Hill (1967), Rodger (1976a,b); and later on, those by Rodger and Littlejohn (1980), Billet and Siffert (1989), together with O’Neill et al. (1989a,b); and even more recently those by Viking (1998) and De Cock (1998).

Reduced-scale tests generally consist of a lightweight vibrator driving a model pile fitted with sensors into a sand-filled cylinder, with either dry or saturated sands of different densities, grain sizes and lateral stresses, and even sometimes featuring pore-pressure transducers in the soil.

In the author’s opinion, reduced-scale testing is useful, as it enables the isolation and further study of the different parameters either understood or found to affect vibro-driveability. Findings from reduced-scale tests are presented further in Section (2.5). However, reduced-scale tests are known to suffer from improper boundary conditions that prevent the induction of vibration energy and generation of pore pressure, as well as the induction of volume changes away from the source. The experimental work can also be somewhat tedious, as preparing the soil-filled vessel requires a lot of effort. Developing models of vibratory drivers with the same properties as real ones is both costly and challenging.

2.4.5 Full-scale tests

Full scale vibratory-driveability tests have been conducted in several countries. Some of the earliest programmes were conducted by Barkan (1962) and Davisson (1970). Some research programmes have been conducted by manufacturers on their own equipment, which has led to a limited diffusion of both data and conclusions. Publications from these research programmes along with the results are not widely available, nor have they been fully analysed.

More recently, several collective European research programmes have been conducted, which have led to some results about actual penetration speed (the vibro-driveability \( v_p \) [mm/s]). These were however, carried out in soil conditions that can be characterised by their different investigation methods but not controlled, as is normally the case during research programmes. Publication of these results would be greatly appreciated by the geotechnical profession world-wide.
Monitoring in full-scale tests nowadays generally consists of three parts: the vibratory equipment, the profile, and the soil-related part (further discussed in Section (4.3)). The vibratory part normally consists of sensors that monitor oil pressure, the position of eccentric masses, driving frequency and in the case of leader-mounted vibratory-drivers, even the penetration depth. The profile-sensors nowadays involve both acceleration and strain gauges along the whole length of the profile, which more-or-less transforms the tested profile into a sensored probe. The soil-sensors usually consist of geophones or accelerometers, monitoring ground vibrations on the surface or at different depths, as well as settlement sensors, and sometimes even pore-pressure sensors.

**BBRI tests**

The Belgian Building Research Institute (BBRI) research programme ran from 1992-94, and featured 21 vibro-driveability case studies at 10 different sites in Belgium (see Table (2-3)).

The aims, complexity and importance of these field tests are contained in the few published reports that have emerged from the research programme to date, and the programme has not yet been fully analysed. Most of the information in this section of the thesis has come from personal communication with those involved in the BBRI project.

The general aim of the BBRI research programme was to improve the theoretical understanding of the penetration resistance, along with the vibration propagation through the ground generated during the installation phase, the influence of different sheet pile profiles, and the different types of vibratory-driving equipment (classic, high-frequency and variable-vibratory drivers).

The few reports relating to the BBRI tests that are generally available are those by Holeyman and Legrand (1994), Holeyman et al. (1996), and Holeyman (2000).

The following five computer programmes for predicting vibro-driveability have been developed in conjunction with the BBRI tests; (i.) HYPERVIB 1, (ii.) HYPERVIB II, (iii.) HYPERVIB IIa, (iv.) VIPER I, and (v.) VIPER II.

The few published reports available relating to these computer prediction programmes are those by Holeyman et al. (1996), De Cock (1998), Vanden Berghe and Holeyman (1997), and Vanden Berghe (2001).
Table 2-3 Summary of the BBRI research programme (after Holeyman et al., 1996).

| Driftline (m) | Corrected (m) | Driftline to 0.5 m | 21 | 3 | LBR(s/m², 0.87 m) | Predicted (Hm, 1.4 m) | Treeheight (m) | Lumber | (b) | Sampled
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>8.5</td>
<td>8.5</td>
<td>0.5 m</td>
<td>21</td>
<td></td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
</tr>
<tr>
<td>9.5</td>
<td>9.5</td>
<td>1.1 m</td>
<td>21</td>
<td></td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
</tr>
<tr>
<td>11.5</td>
<td>11.5</td>
<td>1.1 m</td>
<td>21</td>
<td></td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
</tr>
<tr>
<td>12.5</td>
<td>12.5</td>
<td>1.1 m</td>
<td>21</td>
<td></td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>1.1 m</td>
<td>21</td>
<td></td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
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<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
<td>20.6 20.6 m</td>
</tr>
</tbody>
</table>

*The real penetration line was not measured but does more or less correspond to the predicted penetration line based on controller information.*
SIPDIS tests

The SIPDIS\textsuperscript{1} research programme ran from 1996-98, and featured approximately 30 vibro-driveability cases, carried out at two different sites (see Table (2-4)).

As with the BBRI tests already mentioned, it is astonishing given the aims, complexity and the scientific value of the field tests in this research programme, that there are no reports published to date about the programme.

It is the author’s understanding, that this is basically due to fact that the considerable amount of data generated has not yet been fully analysed. Most of the information regarding this section of the thesis has emerged through personal communication with people involved in the SIPDIS project.

The general aims and final goal of the SIPDIS project were:

• improving the theoretical understanding of penetrative resistance, as well vibration levels generated during the installation of steel sheet piles with both vibratory-driving and impact-driving equipment;
• developing a new field-investigation tool for determining the most efficient installation method in different soil conditions; and
• developing computer programmes for making predictions and recommendations for applying results effectively, thereby producing more reliable installation and driving procedures for final planning and construction.

Two different specific probes, the vibratory-impact probe (VIP) and the jetting probe (JEP), were tested with the objective of determining the shaft and toe resistance during vibratory impact and vibratory jetting. However no data are available or published yet, however during the fall of 2002 the following three publications will be available, Seiffert (2002), and Borel et al. (2002), and Vié (2002).

<table>
<thead>
<tr>
<th>Type</th>
<th>Section</th>
<th>Vibrating</th>
<th>Vibratory jetting</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No lock friction</td>
<td>With lock friction</td>
<td>No lock friction</td>
</tr>
<tr>
<td>VIPv</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>VIPi</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>JEP</td>
<td>-</td>
<td>-</td>
<td>4</td>
<td>-</td>
</tr>
</tbody>
</table>

\textsuperscript{1} Development of a new Soil Investigation Procedure Determining the Installation of Steel Sheet Piles
The Dynamic Penetration research programme financed by Deutsche Forschungsge- 
meinschaft, conducted at the Friedericiana University in Karlsruhe (Germany) has been 
researching vibro-driveability throughout the 1990s.

The research programme consisted of the following five groups and areas: (i) soil-mechanical aspects of toe and shaft resistances (Gudehus), (ii) irreversible displace-
ments in dynamical systems (Vielsack), (iii) safe numerics (Alefeld), (iv) experimental sys-
tem identification (Prange), and (v) control concepts for vibratory pile-driving (Gehbauer).

The field tests were all been performed in Germany at the following four sites: Karlsruhe, Berghausen, Hochstetten and Hagenbach. The soil conditions at these four 
sites consisted of sand, with silty and gravelly layers. The field tests were conducted with an sensor-fitted, close-ended, pipe pile of length 6.8 m, with an outer and inner diameter of 160 and 140 mm. The pile was equipped with a special head, and a toe with a conical tip, bringing the total length to 7.10 m. Each test pile was vibratory driven into the soil using a Müller vibrator (model MS15HB) mounted on a Müller telescopic mast (model MS-M 10000T) under systematically-varying static surcharge force, driving frequency and static moment. Each pile was fitted with sensors at six levels including strain gauges (glued to the pile), vertically-positioned accelerometers at the toe and head of the pile and

<table>
<thead>
<tr>
<th>Type</th>
<th>Section</th>
<th>Vibrating No lock friction</th>
<th>Vibratory jetting No lock friction</th>
<th>Impact No lock friction</th>
<th>Impact With lock friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single U pile</td>
<td>LX16</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>LX32</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Double U pile</td>
<td>LX16</td>
<td>2</td>
<td>2</td>
<td>-</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>LX32</td>
<td>-(**)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Double Z pile</td>
<td>AZ26</td>
<td>2</td>
<td>2(**)</td>
<td>2</td>
<td>1(**)</td>
</tr>
<tr>
<td></td>
<td>AZ48</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

a. One test was envisaged initially, but was dropped on site.
b. Two tests were envisaged initially, but only one was satisfactory.
c. Two tests were envisaged initially; only one was carried out, and was unsatisfactory.

German tests

The Dynamic Penetration research programme financed by Deutsche Forschungsge-
meeinschaft, conducted at the Friedericiana University in Karlsruhe (Germany) has been researching vibro-driveability throughout the 1990s.

The research programme consisted of the following five groups and areas: (i) soil-mechanical aspects of toe and shaft resistances (Gudehus), (ii) irreversible displacements in dynamical systems (Vielsack), (iii) safe numerics (Alefeld), (iv) experimental system identification (Prange), and (v) control concepts for vibratory pile-driving (Gehbauer).

The field tests were all been performed in Germany at the following four sites: Karlsruhe, Berghausen, Hochstetten and Hagenbach. The soil conditions at these four sites consisted of sand, with silty and gravelly layers. The field tests were conducted with an sensor-fitted, close-ended, pipe pile of length 6.8 m, with an outer and inner diameter of 160 and 140 mm. The pile was equipped with a special head, and a toe with a conical tip, bringing the total length to 7.10 m. Each test pile was vibratory driven into the soil using a Müller vibrator (model MS15HB) mounted on a Müller telescopic mast (model MS-M 10000T) under systematically-varying static surcharge force, driving frequency and static moment. Each pile was fitted with sensors at six levels including strain gauges (glued to the pile), vertically-positioned accelerometers at the toe and head of the pile and
mounted on the isolator, and temperature sensors. Depth was also continuously registered with a rope-winching type measuring device.

Reports published relating to the soil-mechanics of the research programme above include Prange et al. (1992), Dierssen and Gudehus (1992), Dierssen (1994), Stortz (1994), Hubert (1997), and Cudmani (2000).

**CPAR tests**

The CPAR research programme ran from 1994-98, and featured 21 vibro-driveability cases with static load tests, conducted at the National Geotechnical Experimental Site near College Station, Texas (USA).

The general aim of the CPAR programme was to develop the ability to predict the ultimate bearing capacity of structural pilings from the responses of pilings during installation with vibratory-driving systems.

The experimental work has been carried out with 41 vibratory-driven H and open-ended pipe piles, divided into two groups consisting of 24 tests piles and 17 anchor piles. Each experimental pile was fitted with accelerometers and strain gauges at its head and toe. All of the piles were driven by an ICE vibratory driver, model 416L.

Two published reports were found relating to the CPAR test, being that of Menclowa (1996), and Bosscher et al. (1998).

**IREX tests**

The IREX research project ran basically from 1994-98, and featured approximately 21 vibro-driveability cases, conducted at 2 different, Bustamante et al. (1999). At present no available publication exists and the project is in its evaluation phase.

The information regarding the IREX project has emerged through personal communication with people involved in the research project.

**2.5 Factors found to affect the vibro-driveability**

It is usually preferred by the engineering community to be able to predetermine which vibratory-related parameters result in the highest vibro-driveability rate. However, it is not solely the vibratory parameters that determine whether high or low driveability-rates are achieved. The vibratory-driving profile undergoes a longitudinal-vertical motion of

---

1. Construction Productivity Advancement Research
2. IREX: Institut pour la Recherche appliquée et l'expérimentation en génie civil
displacement of amplitude $S_o$, which continuously communicates with the neighbouring soil as it penetrates each soil stratum. When driving sheet piles, each vibratory-driven sheet pile communicates with the previously-installed sheet piles (or piles) through the friction forces generated in the sheet pile clutch.

According to the author’s understanding of the complexity of predicting the vibro-driveability of sheet piles driven using these techniques, this predictability can be subdivided into three main parts:

- the vibratory equipment,
- the sheet pile to be driven, and
- the soil profile.

The previously documented effects of how various key parameters affect vibro-driveability and relate to the above three main parts of the system are addresses in Sections (2.5.1) to (2.5.3).

### 2.5.1 Vibratory-related factors

The set of optimum vibratory-related factors to look at for achieving both the highest driveability and at the same time the minimum environmental impact, has not yet been scientifically established. However, it can be noted that initial improvements of the vibratory-driving technique targeted the penetration speed, whereas more recent developments target mitigating the environmental impacts associated with the use of the technique.

The choice of vibratory equipment, together with magnitude of the vibratory-related parameters is generally based on years of engineering experience and field verification. However, it is generally recognised that the following vibratory-related parameters are strongly related to driveability:

- static surcharge force $F_o$,
- the driving frequency $f_d$,
- the eccentric moment $M_e$,
- the free-hanging double-displacement amplitude $S_o$, and
- the efficiency of the vibratory-driving equipment.

Scientific studies performed in systematic detail on how the vibratory-related parameters affect vibro-driveability are hard to find. There are only two articles known to the author, being Rodger and Littlejohn (1980), and O'Neill and Vipulanandan (1989).
One of the most often quoted articles concerning the choice of vibrator parameters is that by Rodger and Littlejohn (1980). These two authors have summarised their body of experience into a table of recommended ranges for driving frequencies, and acceleration and displacement amplitudes for different piles and soil types. Their recommendations are reproduced in Table (2-5).

O’Neill and Vipulanandan (1989) performed a similar vibrator-parametric test in conjunction with their large-scale laboratory tests on model piles. The purpose of the parameter test was to assess the effects of different vibratory-related parameters on the rate of penetration (driveability). Their findings are reproduced here in the following sections.

The same systematic detail relating to vibratory parameters carried out in conjunction with field-tests does not exist today, but if it did, it would be greatly beneficial to the engineering community.

Table 2-5  A summarized description of how to choose vibratory parameters (after Rodger and Littlejohn, 1980).

<table>
<thead>
<tr>
<th>Cohesive soils</th>
<th>Dense cohesionless soils</th>
<th>Dense cohesionless soils</th>
</tr>
</thead>
<tbody>
<tr>
<td>All cases</td>
<td>High point resistance</td>
<td>High point resistance</td>
</tr>
<tr>
<td>High acceleration. Low displacement ampl.</td>
<td>High acceleration. -</td>
<td>Low frequency. Large displacement amplitude.</td>
</tr>
<tr>
<td>Requires high acc. for either shearing or thixotropic transformation.</td>
<td>Requires high acc. for fluidization.</td>
<td></td>
</tr>
</tbody>
</table>

Recommended parameters

- $f_d > 40$ Hz
- $a = 6-20$ g
- $s = 1-10$ mm

- $f_d = 10-40$ Hz
- $a = 5-15$ g
- $s = 1-10$ mm

- $f_d = 4-16$ Hz
- $a = 3-14$ g
- $s = 1-10$ mm

- $f_d = 10-40$ Hz
- $a = 5-15$ g
- $s = 9-20$ mm
Effects of static surcharge force \( (F_o) \)

It is theoretically evident that an increase in the static surcharge force \( (F_o) \) should increase the driving force \( (F_d) \) and consequently increase the penetration speed \( (v_p) \). However it is not evident, at least not from experiments, how much an increase in \( (F_o) \) affects the \( (v_p) \), or how \( (F_o) \) should be determined in relation to the unbalanced force \( (F_v) \).

It is also well known that increasing the static surcharge force increases the rate of penetration, but it has not yet been established how \( (F_o) \) should be determined in relation to the unbalanced force \( (F_v) \).

From the laboratory tests conducted by Rodger and Littlejohn (1980) it was revealed that the relationship between the penetration speed and depth, showed a linear dependency over the range of surcharge force investigated \( (0.13 < F_o/F_v < 0.22) \). However, once an upper threshold value of \( F_o/F_v > 1.0 \) was exceeded, it was found that the vibratory motion was suppressed. The two authors stated that it appears as though the optimum value between the surcharge and unbalanced force is \( F_o/F_v = \frac{1}{2} \), and refer to the model tests conducted by Hill (1966) and field tests conducted by Savionov and Luskin (1960). This occurs when the following ratio is reached:

\[
\hat{F}_v = 2F_o
\]  

(2.15)

where \( F_o = \) static surcharge force [N], and \( F_v = \) unbalanced driving force amplitude [N].

O’Neill and Vipulanandan (1989a) reported similar results from their research on large vibratory-driven model piles. An increase in the bias mass from 172 to 907 kg caused the penetration speed to triple using the optimum driving frequency, which was found to be about 20 Hz.

Effects of driving frequency \( (f_d) \)

It is well known that the vibratory-parameter frequency significantly affects the driveability of vibratory-driven piles and sheet piles. But it has never been scientifically explained how the best choice of driving frequency should be determined in relation to the best driveability (that is the highest penetration speed \( (v_p) \)).

Heerema (1979) conducted laboratory interface tests on samples of both sand and clay, investigating the relationship between dynamic shaft friction, displacement velocity and horizontal stresses in a pile-driveability analysis. The test series consisted of two parts: (1) a series of horizontal-load variation tests, and (2) a velocity variation test.
The laboratory interface test series were carried out at the TNO Delft Laboratories, in The Netherlands, in 1976.

During the horizontal-load variation tests, the amplitude and frequency were kept constant at \( \sigma_0 = 12.5 \text{ mm} \) and \( f = 1.6 \text{ Hz} \), while the horizontal loads were varied between 50 - 240 kN/m\(^2\). The friction force was measured at the zero crossing, while the test setup generated a constant velocity of \( 2\pi f \sigma_0 = 0.126 \text{ mm/s} \). During the velocity variation test, the horizontal load was kept constant, while the velocity at the zero crossing was varied between \( 7 \times 10^{-4} - 0.6 \text{ mm/s} \).

The results of the dynamic friction to normal stresses relationship tests on samples of sand showed linear dependency (see the left of Figure (2-11)). The results of the dynamic friction to velocity relationship tests on samples of sand were found to be virtually independent of the displacement velocity applied (see the right of Figure (2-11)).

Dierssen (1994) referred to the results of Hereema (1979), stating that it was most likely that the dynamic shaft resistance of a vibratory-driven sheet pile would also be found to be virtually independent of the driving frequency applied.

In the opinion of the author (of this thesis), it should be noted however that Hereema’s tests do not duplicate actual field conditions, as the inertial effects on the sand grains were not present in the laboratory tests.

![Figure 2-11 Results of normal stress-variation tests (on the left), and velocity-variation tests (on the right), where \( \sigma_0 \) is kept constant at 85 kN/m\(^2\) on sand samples (after Hereema, 1979).](image)

During one of the parametric studies of the laboratory tests by Rodger and Littlejohn (1980), a series of penetration and extraction tests were conducted where the unbalanced moment \( (M_e) \) was kept constant while the frequency was varied between 20-50 Hz. During the steady state in the penetrative motion of the model pile, it was found that the relationship between frequency and penetration velocity was linear. This relationship
exhibited a threshold frequency below which no penetration occurred of 18 Hz, corresponding to a free-vibrational acceleration of 1.6 g. The threshold value mentioned is explained further in Section (2.5.3), relating soil resistance with the amplitude of vibrational acceleration. During the steady state in the extraction tests on the model pile, the extraction velocity was found to be independent of frequency over the frequency range investigated (above), while the unbalanced moment was held constant.

**Effects of unbalanced moment \( (M_e) \)**

During the parametric studies of the laboratory tests by Rodger and Littlejohn (1980), another series of penetration and extraction tests were conducted, where the frequency was kept constant at 30 Hz, while the unbalanced moment \( (M_e) \) was varied between 22-90 kgm. The relationship between penetration velocity and unbalanced moment was found to be linear, with a free-vibrating displacement amplitude threshold of 0.3 mm (below which no penetration occurred), corresponding to an acceleration of 1.06g. As with the frequency tests, the threshold value mentioned is explained further in Section (2.5.3), relating soil resistance with the amplitude of vibrational acceleration. In the variable unbalanced-moment extraction tests, the relationship between extraction velocity and increasing unbalanced-moment was found to be linear.

![Figure 2-12](image-url)  
*Figure 2-12 The effect of unbalanced moment \( (M_e) \) on \( v_p \) for H piles in dry sand (Wang, 1994).*
The results of Wang (1994) based on extensive research work at University of Houston, Texas (USA) (see Figure (2-12)), showed that when the unbalanced moment was doubled from 57.6 to 115.2 kgmm, the rate of penetration increased by approximately 50% when all other vibratory and soil parameters were kept the same. However from the few unbalanced-moment variation tests carried out, it is difficult to conclude whether or not the relationship between the rate of penetration and the increasing unbalanced-moment displayed the same linear relation as reported by Rodger and Littlejohn (1980).

**Effects of displacement amplitude ($S_o$)**

There are few scientific publications of studies showing how the double-displacement amplitude ($S_o$) (see Equation (2.12)) affects vibro-driveability, or how it might be appropriately determined or selected.

There are however a few statements found in the literature, for example by Rodger and Littlejohn (1980), who stated that the displacement amplitude should be as large as possible (20-30 mm) during vibratory-driving conditions with large toe-resistance, while smaller values (5-20 mm) should be used during conditions with low toe-resistance (see Equation (2-6)).

The same authors did not however provide any information or explanation for the choice of displacement amplitude ($S_o$) under different soil conditions.

**Effects of the efficiency of the vibratory equipment ($\xi$)**

Generally, the rated energy or theoretical power ($P_t$ [kW]) of the vibratory driver is specified by the manufacturers of modern vibratory-equipment, and indicates the capacity of the vibratory-driving equipment. However, it is not evident how much of this power is effective when driving the profile in situ, compared with the theoretical power output.

In research carried out by O’Neill and Vipulanandan (1989a), they pointed out that the power or energy delivered to the pile head was always less than the theoretically-produced power of the vibratory driver. The ratio between delivered and theoretical pile-head power ($P_d/P_t$) was shown by the authors to be the efficiency factor ($\xi$).

The energy losses detected were believed to be related to losses originating in for example, the hydraulic system, the connection between the pile and the vibrator, the clamp, the sliding friction between the leader and the vibrator, the coupling of vibrator energy to the flexural energy in the pile and other factors.
Recommendations about the choice of the efficiency factor are provided in Moulai-Khatir et al. (1994). According to these authors, field conditions could be estimated as being in the range \(20 < \xi < 25\) percent.

However until further research is conducted relating to the internal energy losses in every part of the vibratory equipment, accurate terms for power loss cannot be incorporated into the computation of the actual driving function.

### 2.5.2 Pile-type and profile-related factors

Pile types or profiles used mostly in combination with the vibratory-driving technique include the following:

- sheet piles for permanent or temporary containing and retaining walls, cofferdams and caissons,
- H beams used for deep foundations or as installation support for hydraulic barriers,
- open or closed-ended tubes for deep foundations in the form of cast-in steel-sheet (CISS) piles,
- precast prestressed concrete piles, and
- steel profiles for deep compaction of granular soils.

The few results found from earlier investigations and research conducted on profile-related factors affecting the vibro-driveability, can be divided into three types:

- geometric effects,
- effects due to the presence of lock friction, and
- longitudinal, transversal and flexural properties of the driven profiles.

**Geometric effects**

Piles are normally divided into the two main groups, displacement and non-displacement piles. A displacement pile can be defined as a solid or a hollow pile driven with its tip closed, which displaces an equivalent volume of soil by compaction, lateral displacement or vertical displacement of the soil. Non-displacement piles are piles usually formed by boring, augering or other excavation method. However there could possibly be a third type of pile, the so-called low-displacement pile, which normally relates to geometric sections such as sheet piles (U, Z, and I profiles), H beams, and sometimes even open-ended tubes.
Driven steel, concrete or wooden profiles (displacement piles) are normally cylindrical or prismatic, and can be characterised by the following geometrical and mechanical properties:

\[
L = \text{profile length [m]},
\]

\[
A = \text{cross-sectional area [m²]},
\]

\[
\chi = \text{profile perimeter [m]},
\]

\[
E = \text{Young’s Modulus of the material [MPa]}, \text{and}
\]

\[
\rho = \text{volumetric mass [kg/m}^3].
\]

The profile sections (low-displacement piles) can be more fully characterised by their shape (U, Z, I and H profiles), and inside and outside perimeters if closed.

The above-mentioned pile-related properties allow the calculation of the areas of the profile coming into longitudinal and transversal contact with the soil; that is the toe and the shaft area \((A_t \text{ and } A_s)\) respectively) once the embedded depth \((z [m])\) of the profile is known. This also allows calculation of the mass of the profile \((m_p)\) equalling \(\rho AL\) [kg], and the longitudinal bar velocity \((c_b)\) equalling \((E/\rho)^{-1} \text{[m/s]}\) in the profile being considered.

In a total of 30 large-scale model tests at the University of Houston, Texas (USA) conducted by Rao (1993) and Wang (1994), the effects of pile geometry on the driveability of both displacement and low-displacement piles were studied. Pipe piles with and without a closed toe, and H beams of two different sizes and materials (steel and aluminium) were driven into either dry or saturated sands of varying densities. Though there were not enough replications to enable conclusive results from the geometrical pile effects on driveability, the results did nevertheless indicate that the effect of pile size (smaller versus larger) produced identical trends in the driveability curves \((v_p-z)\) in the case of the open-ended pipe piles, but not in the H piles. The smaller-sized H piles showed a \((v_p-z)\) pattern that was contrary to the general trend of decreasing \(v_p\) with increasing depth. The rate of penetration increased initially in the case of the smaller-sized H piles and later decreased at the end of the installation phase. The same authors then speculated that the lateral vibrations of the more slender H beams could be one possible explanation for the diverging driveability curves. During the laboratory tests, the pile-geometry effects (H shape versus open-ended pipe piles) were found to have a dominating effect on the rate of penetration in saturated sand compared to dry sand. The larger-sized open-ended pipe piles produced a significantly higher \(v_p\) value compared to the larger-sized H piles. According to the authors, the results related to a higher build up of pore pressure as a consequence of driving in the case of the open-ended piles, but not as much in the case of the H piles.
A total of 24 full-scale field tests were conducted at the University of Wisconsin-Madison, Wisconsin (USA) by Bosscher et al. (1998), to study the effects of pile size and geometry (in H and open-ended pipe piles) on the bearing capacity of piles installed with vibratory drivers. From the results of the acceleration measurements on the driven piles (at toe and head) and on the vibrator (bias mass and exciter block), the authors concluded that pipe piles having the larger diameter (200-250 mm) developed a lower toe-acceleration amplitude than that measured at the pile head and the vibrator. This was reported to be related to the plugging that developed during driving, as large-diameter pipe piles develop soil plugs of larger weights than those developed by smaller pipe piles.

**Effects of interlock friction**

It is well known that the shear force generated in the interlock (clutch) of two jointed steel sheet piles needs to be considered when driving sheet piles. The magnitude of the interlock resistance (shear force transmitted from one clutch to the other) is known to influence the following three areas:

- ground vibrations during installation,
- vibro-driveability, and
- flexural stiffness of a retaining wall built up with U-shaped profiles.

However, publications and documented knowledge about the effects of interlock friction on the three above areas remain rather limited at present.

In Legrand et al. (1993), a series of full-scale driving tests relating to the cases numbered 1-5 in Table (2-3), were performed on vibratory-driven sheet piles fitted with sensors, with and without the presence of interlock friction. One of the many intentions of the tests was to study the effects of clutch friction on the forces generated in the sheet pile and the ground vibrations that were induced. The sheet pile was driven into the ground separately first, then later on into the clutch of a previously installed sheet pile. The test results, with and without the presence of clutch friction, were then compared to one another. The comparisons clearly indicated that the presence of interlock friction generated significantly higher forces and acceleration in the sheet pile fitted with sensors (not graphically presented in the report), and higher ground vibration amplitudes on the soil surface (see Figure (2-13)). The ground vibration amplitudes generated on the soil surface increased by over 100% due to the presence of interlock friction. Legrand et al. concluded that on sites where vibration amplitudes have to be low, then sheet piles with good jaws must be used.
In Vanden Berghe et al. (2001), a series of reduced-scale laboratory tests were performed on a model sheet pile with two clutches welded onto it, taken from an ARBED PU 16 sheet pile profile, in order to study and evaluate the interlock shear force transmission in the clutch between two sheet piles. The sheet pile specimen was first vibratory-driven into a sand-filled cylindrical container and into the two receptive sheet pile clutches (see Figure (2-14)). It was then extracted at a constant speed of 0.01 mm/s to a maximum displacement of 20 mm. The aim of the tests was to study the extraction force to displacement relationship (not the transversal or longitudinal bending, nor torsion) in order to be able to study the constitutive quasi-static interlock relationships.
The cylindrical container (height 1.5 m and diameter 0.63 m) was filled with one of type of sand (dry or saturated), at three different initial densities. The average grain size of the fine and coarse sand was chosen to be smaller than the size of a typical interlock gap, in this case about 3 mm. From the results, it was observed that the time needed to drive the sheet pile into dry sand had a considerable influence on the shear resistance developed in the interlock. The longer the driving time, the greater the amount of sand that was found jammed into the clutch. The solidified sand was jammed so hard into the clutch that a screwdriver was needed to remove it. The densification of the dry sand specimens found in the interlock was found to have been partly crushed during a grain size analysis of the sand.

The quasi-static, extraction interlock resistance was found to be reduced to about 80% when the sheet pile specimen was installed in saturated sand. Vanden Berghe et al. (2001) believed that due to the liquefaction that developed during installation, the sand grains were more mobile and thus able to avoid being jammed into the interlock.
Since it was believed that there was less sand in the clutch, the interlock resistance should consequently also be reduced.

**Longitudinal, transversal and flexural effects**

It is greatly importance to describe the phase relationships between the longitudinal penetrative motion functions of both the pile head and the pile toe, particularly for those wishing to use the information to develop or calibrate mathematical models of the penetrative behaviour of vibratory-driven profiles. It is also very importance to describe the transversal (lateral) vibrations of slender profiles, which at times can play an important role, particularly in light of the speculated effect that these may have on both driveability and the environmental effects induced. Another often neglected profile-related aspect that has lately become an increasing concern, is the so-called flexural effect (twisting of slender sheet pile profiles), which has its background in the constantly increasing widths of sheet pile profiles. However the three profile-related parameters, longitudinal, transversal and flexural effects, are generally ignored, because they have not yet been studied in great detail (though some mention of these has been made in a few publications).

Studies of the longitudinal effects of vibratory-driven piles normally relate to the mechanics of vibratory driving; that is the question of whether the pile vibrates as a rigid body or not. In the pioneering vibratory-driving studies of the 1930s, conducted by Soviet research workers (reported by Rodger, 1976a), solutions concentrated on the rigid-body vibratory-motion of the pile with respect to the surrounding soil. Assuming rigid
body motion was thought valid if the length of the pile could be considered to be appreciably smaller than that of the elastic waves generated within the pile.

In one of the series of studies by Bernhard (1968), the driving frequency response (at 25 - 6000 Hz) of a vibratory-driven model pile consisting of a brass tube (φ = 19 mm, L = 914 mm) was investigated. The resonance frequencies, nodes and wave forms of a “free-free” brass tube under steady-state conditions were theoretically computed and compared with experimental conditions. No significant changes to higher critical frequencies could be detected between those obtained during the laboratory tests and those obtained under “free-free” conditions.

Furthermore, Bernhard concluded that during low frequencies the pile vibrated longitudinally without any nodes and at almost equal displacement amplitudes along the length of the pile. In the low-frequency range, the critical frequency responds essentially to the combined “adjacent soil and pile mass system”, while in the high-frequency range, the critical frequencies are governed mainly by the natural frequencies of the pile alone.

Finally, Bernhard also concluded that flexural vibrations in a vibratory-driven pile might be considered to be a secondary effect. If the horizontal accelerations are of sufficient magnitude, some adjacent soil particles will be pushed away. The soil particles cannot recover their original position due to the fast rate of bending, causing an additional decrease in skin friction.

Berhard’s (1968) findings that a vibratory-driven pile tends to vibrate longitudinally with almost equal displacement amplitudes along the length of the pile, were later confirmed by Rodger and Littlejohn (1980) and Smith and To (1988), in their laboratory test series, who also concluded that no nodes existed within the pile length, and that the first node might exist in the soil immediately below the pile toe.

From the extensive work of O’Neill and Vipulanandan (1989a), it was inferred that the vibratory-driven test piles they used behaved like rigid bodies. By analogy (though without proof) it could be assumed that even a full-sized pile would behave essentially like a rigid body, if the driving frequency was chosen to be equal or less than 10% of the natural frequency of the full-sized pile as a freely-vibrating rod, expressed according to:

$$f_d \leq 0.1 f_n = 0.1 \frac{c_b}{2L} = \frac{\sqrt{(E/\rho)}}{20L}$$

(2.16)

where

- \(f_d\) = driving frequency [Hz],
- \(f_n\) = longitudinal natural frequency of a free slender bar [Hz],
- \(c_b\) = bar velocity [m/s],
- \(L\) = length of the pile [m],
- \(E\) = Young’s modulus [Pa],
- \(\rho\) = density [kg/m³].
\[ E = \text{Young's modulus} \,[\text{N/m}^2], \quad \rho = \text{density of the pile material} \,[\text{kg/m}^3]. \]

In the full-scale field tests conducted by Legrand et al. (1993) on Z (Arbed BZ17) and U (Arbed PU16) profiles (11.7 and 16.1 m respectively), longitudinally-directed accelerometers and strain gauges were placed at the head and toe of the vibratory-driven sheet piles. Accelerometers were also placed on the wings of the U profile, perpendicular to the vertical penetrative motion of the sheet pile being driven. The longitudinally-directed acceleration signals were found to be almost perfectly sinusoidal with no reports of major phase differences between the head and toe acceleration signals in the time domain. Typical longitudinal acceleration amplitudes of 200 m/s\(^2\) (20g) were recorded. However during the beginning of the driving process, amplification of the longitudinal acceleration amplitude at the lower end of the sheet pile was observed. These amplifications were sometimes found to be as high as 40\% (see Figure (2-16)). The amplitude value of the perpendicular acceleration signals was sometimes found to be as high as the vertical acceleration value. However, no conclusive analysis of these phenomena was either discussed or presented in the report.

![Figure 2-16](image)

*Figure 2-16  Measured and calculated acceleration at the head and toe of the pile, at the commencement of driving the 16.1 m PU16 sheet pile (Legrand et al., 1993).*

In a total of 30 large-scale model tests at the University of Houston, Texas (USA), conducted by Rao (1993) and Wang (1994), it was hypothesised that the vibra-
tions of the smaller-sized H piles (observed but not measured) may have been significant enough to explain the difference in the general trend of the documented driveability ($v_p-z$) curves. The rate of penetration ($v_p$) initially increased instead of decreasing with penetration depth. However, the rate of penetration ($v_p$) later decreased in the end phase of the driveability tests. The same authors implied that driving the smaller-sized H piles resulted in initially higher lateral vibrations in the pile. These lateral vibrations in the more slender H piles could, according to these authors, possibly produce higher penetration speeds, as a result of a higher degradation of the dynamic shaft resistance, until penetration of the pile was sufficient to seat the pile laterally. However, none of the tests conducted could provide any information on how great these lateral flexibilities were during driving, due to the fact that there was no instrumentation to document these effects.

In the full-scale field tests conducted at the University of Wisconsin-Madison, Wisconsin (USA), by Bosscher et al. (1998), a free-hanging vibratory-driver was used to drive 20 pipe and H piles, of lengths ranging from 6-13 m, fitted with sensors. The longitudinally-directed acceleration was measured in terms of peak acceleration and taken from four different places in the test equipment: (i) the bias mass, (ii) the exciter block, (iii) the head on each of the piles, and (iv) the toe on each of the piles. The results of the tests on the 20 piles were divided into three groups. The first group (12 piles) showed acceleration signals at both head and toe, ranging from 95-110% of the acceleration measured at the exciter block. The results presented in Figure (2-17) are taken from a 9.14 m H pile, belonging to this first group of piles, which were considered to behave like rigid bodies. The second group (five piles), showed lower acceleration signals at their pile toes, ranging between 5-25% of the signals recorded at the exciter block. The third group (three piles) contained piles that could not be classified into either of the first two groups due to inconsistent trends observed in the results.
Figure 2-17  Typical frequency and acceleration amplitudes recorded for the group of piles considered to vibrate like rigid bodies (Bosscher et al., 1998):

### 2.5.3 Soil-related factors

Soils are normally divided into two main categories, cohesive and non-cohesive, and the vibratory technique is known to perform best in loose non-cohesive soils. Vibratory-driving equipment may not work as well in moderately stiff, saturated clays, or in dense sand or gravel, since the dynamic soil resistance does not decrease as favourably as it does in loose sand during the installation phase.

The primarily non-cohesive soil mechanisms governing the favourable reduction of soil resistance along the shaft and at the toe during vibratory driving are undoubtedly influenced by several key mechanisms. Results from earlier investigations have revealed that the following soil-related factors significantly influence vibro-driveability:

- induced cyclic motion of soil grains,
- initial relative density,
• local sand liquefaction, and
• the higher density of dense soil layers.

Other soil-related parameters found to affect the driveability are: grain size \((d)\), the content of fines \((w_d)\), and the internal friction angle of the soil \((\phi)\). However, none of these parameters affects the vibro-driveability as profoundly as the four soil parameters listed above.

**Effects of the cyclic motion of soil particles**

In one of the experimental attempts to measure penetration velocity (see Figure (2-10)), steel balls were placed under a surcharge load in a container filled with air-dried sand. The container was rigidly positioned on a platform and subject to vertical vibrations of specifically selected displacement amplitudes and frequencies (see Figure (2-18)a). It was found that the penetration velocity of the steel balls at varying relative vertical acceleration amplitudes \((\dot{a}/g)\) and surcharge forces, obeyed Stokes’ Sedimentation Law, which allowed the author to determine an equivalent vibro-viscosity factor \((\mu)\), (see Figure (2-18)b). The equivalent vibro-viscosity factor \((\mu)\) developed, then became the concept fluidization, also named the dynamic-shear strength reduction factor, since it was found that the coefficient of vibro-viscosity essentially depends on the acceleration of vibrations. The inverse of this equivalent kinematic viscosity \((1/\mu \text{ cms/kg})\) was found by the author to vary approximately linearly with the relative level of acceleration amplitude \((\dot{a}/g)\), and

![Figure 2-18 Penetration test results for steel balls in sand (after Barkan, 1962).](image)
to have a threshold value of approximately 1.4 for dry sand (see Figure (2-18)c). The influence saturation on the inverse of the equivalent kinematic viscosity \((1/\mu)\) of vibrating sand was also found to have an optimal value of the water content, which resulted in an almost total loss of the shear resistance in the soil (see Figure (2-18)d).

Another comprehensive experimental study conducted by Savtchenko under the supervision of Barkan, used a direct shear box-apparatus positioned on a vibrating table to investigate the relationship between the internal soil friction \((\tan \phi)\) and the influence of vertical cyclic displacement amplitude \((A)\) and varying frequencies \((f)\). The mechanism driving the vibrating table was adjustable, allowing both the frequency \((f)\) and the vertical amplitude of motion \((A)\) to be controlled. Because the value of peak vertical acceleration \((\ddot{z})\) is the product of the motion amplitude and the frequency squared, \((\ddot{z} = A(2\pi f)^2)\), several amplitude values were used to develop the same acceleration amplitude for various motions and frequencies. It was evident from the test results that \(\tan \phi\) was significantly reduced by increasing the displacement amplitude. However, the reduction of \(\tan \phi\) was less in the range of frequency variations investigated (see Figure (2-19)).

![Figure 2-19 Correlations between the effects on the dynamic friction coefficient of (top) varying the displacement amplitude, and (bottom) varying the frequency of vibrations (Barkan, 1962).](image-url)
The influence of increasing the relative acceleration amplitude \((\dot{a}/g)\) on the vibratory-friction angle \((\tan \phi)\) was found to decrease exponentially from an initial (static) value to a reduced dynamic value (see Figure (2-19)).

Barkan later fitted an exponential relationship to these experiments:

\[
\tan \phi_d = \tan \phi_{\text{min}} + (\tan \phi - \tan \phi_{\text{min}}) e^{-\beta \eta}
\]  

(2.17)

where

- \(\phi_d\) = internal friction angle taken from dynamic tests,
- \(\phi_{\text{min}}\) = limiting value of the reduction of \(\phi\) taken from dynamic tests,
- \(\phi_{st}\) = internal friction angle taken from static tests,
- \(\beta\) = grain size coefficient depending on acceleration, Figure (2-20),
- \(\eta\) = relative acceleration amplitude, \((\dot{a}/g)\).

Barkan also investigated how an increase in grain size \((d \text{ [mm]}\) affected the reduction of the relative friction coefficient \((\delta)\) (see Figure (2-20)), using the following definition of the relative reduction of the friction coefficient:

\[
\delta = \frac{\tan \phi_{st} - \tan \phi_d}{\tan \phi_{st}}
\]

(2.18)

where

- \(\delta\) = relative friction coefficient,
- \(\phi_{st}\) = friction angle obtained from static tests, and
- \(\phi_d\) = friction angle obtained from dynamic tests.

The correlation indicates that a coarser cohesionless-soil will reduce the penetration speed of a vibratory-driven profile when the same amount of energy is supplied to the driven profile, indicating that bigger grains require a higher excitation level before they start to lose contact with surrounding grains.

O’Neill and Vipulanandan (1989a) made similar observations to Barkan, where coarser sand decreased the penetration speed when all other parameters were kept constant.

Several investigations using different apparatus later verified that the internal shear strength of air-dried, cohesionless soils is significantly reduced by the presence of high acceleration amplitudes together with high-amplitude loading (Richart et al., 1970).
Barkan’s view of modelling the reduction in shear strength by the angle of shearing strength \((\tan\ \phi)\) was not shared by Alyanak (1961) or Forssblad (1963), who considered the loss of shear strength in cohesionless soil to be attributed to the reduction of normal stress applied in the shearing plane due to the induced vibrations.

Forssblad (1963) proposed a qualitative mechanism for explaining the reduction of the internal shear strength in a model similar to the one proposed by Alyanak (1961), as being related to the cyclic variation of the normal pressure between the adjacent grain particles. The contact force \((k)\) between two grain particles was divided into a normal and a frictional component. In addition, there was a vibratory force \((P_{vib})\) and forces of inertia superimposed on the forces acting at the points of contact (see Figure (2-21)).

Forssblad also proposed an interparticle contact model, which explained why and how the contact and friction force varied periodically in both direction and amplitude. Rodger (1976) expressed some reservations about the application of Alyanak and Forssblad’s respective models, stating that they were only valid in the sub and trans-threshold states of the acceleration amplitudes generated, and even then only to a limited extent.

### Effects of initial relative density

It is well known that the installation of all kinds of pile and sheet piles alters the initial relative density of the soil (the initial void ratio). Vibratory-driven piles and sheet piles change the initial density of cohesionless soils in a fundamentally different way than impact-driven piles. However from the literature survey, it could be concluded that only
a limited number of studies have been carried out on how the volume changes induced by pile driving affect the vibro-driveability.

Despite the pioneering work of Barkan (1962), contributing to our better understanding of the mechanisms behind the reduction in shear strength, he ignored the effects of the initial void ratio and normal load, by not measuring volume changes during his tests. However Barkan did perform one of the first significant studies on the changes of the density of cohesionless soils due to vibrations. One important implication of his work was that loose sand, vibrated at a certain relative acceleration level ($\eta$), densifies to a corresponding void ratio ($e_i$). To densify the soil even further, an even greater relative acceleration amplitude than $\eta$ needs to be applied.

Youd (1967) experimentally confirmed the work of Barkan, also finding both $\tan \phi$ and the loose initial void ratio ($e_i$) to be monotonically-decreasing functions of the relative acceleration ratio ($\eta$), as described by Equation (2.17). Youd’s results also showed that the effects of vibrations decrease with an increase in normal pressure between the sand grains. Youd defined the densified void ratio ($e_d$), corresponding to a cer-
tain relative acceleration level ($\eta_d$), as the minimum equilibrium void ratio corresponding to the static critical void ratio. This term was first introduced by Casagrande (1936) to describe the limiting void ratio above which a soil tends to reduce in volume when deformed due to shear, and below which dilation occurs.

Rodger and Littlejohn (1980), showed that the relationship between the actual soil density and the critical soil density was the most significant parameter affecting vibro-driveability.

Extensive laboratory tests conducted by O'Neill and Vipulanandan (1989a; b; 1990) showed that the relative density ($D_r$) and the effective lateral stress ($\sigma_h$) acting in the soil were the two soil parameters found to have the most pronounced effect on the vibro-driveability. Lateral soil stress was found to have less effect on the driveability than varied relative density has.

**Effects of the degree of saturation**

Local sand liquefaction is not a primary issue for impact driving in non-cohesive soils, but becomes one when using vibratory-driving techniques in non-cohesive soils. This has led to speculation that the excess pore-pressure induced during the installation phase is the primary factor behind the favourable reduction of soil resistance. However, this is hypothesised not to be the case (see Section (3.3.4)).

The effects of pore-pressure buildup induced during vibro-driveability have been addressed by O'Neill and Vipulanandan (1989a). They showed that the presence of water in cohesionless soils had a very positive influence on driveability, since the oscillating motion of the pile momentarily increases the pore pressure (as shown in Figure (2-22)), and thereby reduces the effective stress. The same authors reported however that there is no significant increase in the pore pressure corresponding to the decrease in driving resistance. It was speculated by the same authors that the cyclic motion of the sand particles induced by vibratory driving was another important mechanism behind the favourable reduction in the soil resistance, and that this phenomenon only occurs near the oscillating pile shaft.

Extensive laboratory work conducted at the University of Houston, Texas (USA) by Rao (1993) and Wang (1994), included further analysis of the effects of excess pore-water pressure developed during vibratory driving. Two miniature pore-water pressure transducers were placed in the test cylinder prior to performing the driveability tests. These were positioned 1.27 m below the surface at distances of $1\phi$ and $2.25\phi$ from the penetrating pile shaft. The transducers were zeroed after the soil was completely saturated, in order to avoid registering the hydrostatic pressure of the pore water.
The test results indicated that the excess pore pressure played a greater role in the case of non-displacement piles than in the case of full-displacement piles. In the report by Wang (1994), who conducted pore-pressure measurements in combination with finite element analysis of vibratory-driven model piles, it was reported that the excess pore-pressure amplitude measured near the pile shaft was significantly higher than the mean excess pore-pressure, and that the excess pore-pressure amplitude decayed very quickly with increasing radial distance from the oscillating shaft surface (as shown in Figure (2-22)).

Figure 2-22 Excess pore-water pressure readings versus time, where the top graph shows readings taken at 1φ from pile shaft, and the bottom graph shows readings taken at 2.25φ from the pile shaft (after Wang, 1994).
However, the mean excess pore-pressure was found to be similar at different distances from the oscillating shaft surface. According to Wang, the phenomenon observed was an indication that the cyclic pore-pressure was related to the cyclic motion of the sand particles induced by the vibratory driving, and that this phenomenon is more predominant near the oscillating pile shaft.

However, in the opinion of the author of this thesis, the similarity in the value of the mean pore-pressure, regardless of the distance to the pile, could also be related to improper boundary conditions (at the tank limits), due to the fact that there was no dissipation of the excess pore-pressure in relation to the distance from the driven pile.

The mean excess pore-pressure (about 6-7 kPa) was found to be much larger than the excess pore-pressure amplitude (~1 kPa at 1 φ from the shaft and barely detectable at 2.25 φ from the pile shaft), indicating that the pore pressure was determined by a total volumetric deformation of the sand instead of cyclic strain.

Pore-pressure development during vibro-driving was found to be closely related to driveability; with maximum pore pressure corresponding simultaneously to the maximum rate of penetration.

**Effects of dense layers**

The effects of denser layers of sand on the vibro-driveability were analysed by Wang (1994), who reported that the existence of a denser layer of soil strata significantly reduced the rate of penetration ($v_p$). The average ratio of dynamic shaft and toe resistances, when vibrator driving from a medium-dense layered sand into a very-dense layer of sand, showed that the dynamic shaft resistance ($R_s$) had only a slight gain (17%). On the other hand, the dynamic toe resistance ($R_t$) increased significantly (>100%) when driving into the denser sand layer.

### 2.6 Vibro-driveability models

#### 2.6.1 Types of models

In the past, a number of notable attempts have been made to develop formulae to predict the driveability of vibro-driven piles and sheet piles. The predictive models differ not only in the way they account for mechanical engineering principles, but also in the way they model dynamic soil resistance and their geotechnical investigation input.

Extensive reviews of previous attempts at prediction can be found in Rodger (1976), Viking (1997), and Holeyman (2000). However, it is should be noted that none
of these models cover all the fundamental physical phenomena accompanying vibro-driveability prediction.

The few earlier attempts at predicting driveability that are available in the literature can be subdivided into several different types. The models that are reviewed here, have been subdivided into the following categories.

- **Parametric methods**: this includes some of the very earliest predictive methods, based on site observations and simple expressions.
- **Force-balance methods**: these represent driveability as a relationship between the driving force generated and applied by the equipment versus the penetrative resisting forces, along with empirical parameters.
- **Energy-balance methods**: these represent driveability as a relationship between the energy generated and applied versus the energy consumed, along with empirical parameters.
- **Momentum-conservation methods**: these involve the application of the principle of balancing the soil-resistance impulse with the total weight of the vibratory-driver system over the time taken for one cycle.
- **Integration of the laws of motion**: where driveability is derived by integrating the equilibrium conditions of the system at all time.

### 2.6.2 Parametric methods

Some of the earliest methods for predicting driveability (not presented here) have been reviewed by Rodger and Littlejohn (1980). They stated that the earliest models were either of a linear-viscous or a linear-elastoplastic soil-response type (refer to Table (2-6)). However practical experience has been shown that viscosity, elasticity and plasticity are expected to be non-linear.

### 2.6.3 Force-equilibrium models

Force-equilibrium models aim at predicting whether the capacity of a vibrator system can or cannot overcome the penetration-resistance forces. Force-equilibrium models do not provide an estimate of the penetration speed of a driven sheet pile, they just compare the amplitude of the driving forces with the amplitude of the penetration-resistance forces.

The beta formula was proposed by Jonker (1987) for predicting the vibro-driveability of pipe piles. This is based on Jonker’s experience from projects involving vibratory-driven huge offshore pipe piles, and is based on the empirical expression given by Equation (2.19).
For mentioned references in Table (2-6), see Rodger and Littlejohn (1980).

<table>
<thead>
<tr>
<th>Phenomenological solutions</th>
<th>Semi empirical solutions</th>
<th>Empirical solutions</th>
<th>Phenomenological solutions</th>
<th>Wave equations</th>
</tr>
</thead>
</table>

\[
F_v + F_i + F_o = \beta_o R_{so} + \beta_i R_{si} + \beta_t R_t + \beta_t R_t
\]  

(2.19)

where

- \( F_v \) = force generated by the vibrator,
- \( F_i \) = inertial forces of dynamic masses,
- \( F_o \) = surcharge force,
- \( \beta_o \) = empirical factor of the shaft resistance outside a pipe pile, according to Table (2-7),
- \( R_{so} \) = soil resistance outside a pile shaft,
- \( \beta_i \) = empirical factor of shaft resistance inside a pipe pile, according to Table (2-7),
- \( R_{si} \) = soil resistance inside a pile shaft,
- \( \beta_t \) = empirical factor of toe resistance, according to Table (2-7), and
- \( R_t \) = soil resistance at a pile toe.
The Tünkers method, presented by Warrington (1989a), is another force-equilibrium formula that has been widely used by for example Tünkers (a US company), featuring the following formula:

\[ F_v = \sigma A_s \]  

(2.20)

where \( F_v = \) force generated by a vibro-driver [kN], \( \sigma = \) soil resistance [kPa] according to Table (2-8), and \( A_s = \) shaft area of soil \([m^2]\).

According to Warrington, this formula is only applicable when the displacement amplitude \((s_d)\) is less than 2.38 mm. When using sheet piles, Tünkers recommends substituting the shaft area \((A_s)\) with \((2.81 \times \text{the width of the sheet pile})\).

---

**Table 2-7** Values of the empirical factor of shaft resistance \((\beta)\) for the Beta method (after Jonker, 1987).

<table>
<thead>
<tr>
<th>Type of soil</th>
<th>Value of ( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round coarse sand</td>
<td>0.10</td>
</tr>
<tr>
<td>Soft loam/marl; soft loess; stiff cliff</td>
<td>0.12</td>
</tr>
<tr>
<td>Round, medium sand; round gravel</td>
<td>0.15</td>
</tr>
<tr>
<td>Fine angular gravel; angular loam; angular loess</td>
<td>0.18</td>
</tr>
<tr>
<td>Round fine sand</td>
<td>0.20</td>
</tr>
<tr>
<td>Angular sand; coarse gravel</td>
<td>0.25</td>
</tr>
<tr>
<td>Angular/dry fine sand</td>
<td>0.35</td>
</tr>
<tr>
<td>Marl; stiff/very stiff clay</td>
<td>0.40</td>
</tr>
</tbody>
</table>

---

**Table 2-8** Values of soil resistance \((\sigma)\) for the Tünkers method (after Warrington, 1989a).

<table>
<thead>
<tr>
<th>SPT value (blows/30) ([cm])</th>
<th>Soil resistance ((\sigma)) [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>cohesion less soil</td>
<td>cohesive soil</td>
</tr>
<tr>
<td>0-5</td>
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<td>40+</td>
<td>30+</td>
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</tbody>
</table>
2.6.4 Energy-balance methods

Energy-balance methods assume that the power generated by the vibratory-driver system is equal to the power consumed by the penetration-resistance forces that prevent the vibratory-driven object from penetrating the soil.

Energy methods are usually based on the steady state, without considering any transient effects in the system, and are expressed in their simplest form by the following expression:

\[ R_u \frac{v_p}{\beta_t} = b_f W_t + (F_i + F_o) v_p \]  

(2.21)

where \( R_u \) = bearing capacity or soil resistance \([kN]\),
\( v_p \) = average rate of penetration \([m/s]\),
\( \beta_t \) = empirical-loss factor \([-\] \),
\( W_t \) = theoretical power delivered to the system \([kNm/s]\),
\( F_i \) = inertial forces of dynamic masses \([kN]\), and
\( F_o \) = static surcharge force (bias mass) \([kN]\).

The general expression for the energy methods (Equation (2.21) above) is usually rewritten in two ways for predicting either the bearing capacity (not presented here), or the rate of penetration, as in the following equation:

\[ \frac{v_p}{\beta_t} = \frac{b_f W_t}{(R_u - F_i - F_o)} \]  

(2.22)

The Davisson (1970) formula used to estimate the bearing capacity of piles driven with the BRD vibratory-driver, proposes the following expression for the empirical parameter \( \beta_t \) in Equation (2.21):

\[ \beta_t = 1 - v_p s_e \left( \frac{R_u}{1000} \right) W_t \]  

(2.23)

where \( s_e \) \([mm/cycle]\) is an empirically-determined permanent set representing all energy losses.

Warrington (1989a) called Equation (2.22) the Vibdrive formula, provided that the empirical loss factor \( \beta_t \) equals 0.1, and that the power \( W_t \) is calculated according to Warrington (1989b).
2.6.5 Momentum-conservation models

Momentum models apply the principle of balancing the soil-resistance impulse with the momentum of the total weight of the vibratory-driving system, over the time taken for one full cycle. One of the first to apply this approach to simulate vibro-driveability was Schmid (1969). The dynamic resistance force developed at the pile toe \( R_t \) was considered by the Schmid to be of an impulsive nature, assuming that the sum of dynamic forces integrated over an entire penetration cycle is zero \( (0 < t < T_c) \), expressed as follows:

\[
(m_o + m_{vib} + m_p)g = \int_0^{T_c} Rdt = \alpha R_t T_c
\]

where
- \( m_o \) = bias mass [kg],
- \( m_{vib} \) = mass of the vibrator [kg],
- \( m_p \) = mass of the pile [kg],
- \( g \) = gravity [m/s\(^2\)],
- \( f \) = driving frequency [Hz],
- \( T_c \) = period of contact between the pile toe and the soil during one cycle [s],
- \( \alpha \) = empirical coefficient, varying between 0.5-1.0, usually taken to be 2/3, and
- \( R_t \) = soil resistance developed at the pile toe [kN].

To evaluate the period of time that the pile toe makes contact with the soil during one cycle \( (T_c) \), tests need to be performed to determine the minimum pile-acceleration amplitude, which is also called the excess acceleration \( (a_e = a - a_{min}) \). The contact time \( (T_c) \) is calculated using the following expression:

\[
T_c = \frac{2v_p}{f\alpha e}
\]

where
- \( T_c \) = length of time contact exists between the pile toe and the soil during one cycle [s],
- \( v_p \) = rate of penetration [mm/s],
- \( f \) = driving frequency [Hz], and
- \( a_e \) = excess acceleration [m/s\(^2\)].

The penetration speed follows a linear trend and passes the threshold value of the acceleration at \( a_{min} \), which becomes the key parameter for successfully applying the
method to estimate the contact time. The penetration speed can then be obtained by using Equation (2.24) and Equation (2.25) above, in order to derive the following expression of the penetration speed:

\[
vp = \frac{(a - a_{\text{min}})}{2T} \left[ \frac{(m_o + m_{\text{vib}} + m_p)g/f^2}{\alpha R_t} \right]^2
\]  

\(2.26\)

where \(v_p\) = rate of penetration [mm/s],
\(a\) = actual driving acceleration [m/s\(^2\)],
\(a_{\text{min}}\) = minimum acceleration required to drive the pile [m/s\(^2\)],
\(T\) = time taken for one penetration cycle [s],
\(m_o\) = bias mass [kg],
\(m_{\text{vib}}\) = mass of the vibrator [kg],
\(m_p\) = mass of the pile [kg],
\(g\) = gravity [m/s\(^2\)],
\(f\) = driving frequency [Hz],
\(\alpha\) = empirical coefficient (between 0.5-1.0, usually 2/3), and
\(R_t\) = soil resistance developed at the pile toe [kN].

### 2.6.6 Integration models of the mechanical action

In the most recent methods used to predict driveability, the vibrator-profile system is considered to behave like a rigid body, and therefore modelled as an SDOF system. The integration models require that the penetration movement of the vibratory-driven profile be described at all times after inertial equilibrium has been achieved.

The simplest integration models of the vibrator-profile system suggest that the dynamic mass \((m_{\text{dyn}})\) should be the focus of attention. Where it is assumed that the vibrator-profile system behaves like a rigid body, it is possible to apply Newton’s second law \((\Sigma F = ma)\) to the vibrating masses, and usually according to the following expression:

\[
F_o + F_v + F_m - R_s - R_t - R_c = a m_{\text{dyn}}
\]  

\(2.27\)

where \(F_o\) = static surcharge force [N],
\(F_v\) = unbalanced force [N],
\(F_m\) = static force of the dynamic masses [N],
\(R_s\) = dynamic soil resistance along the shaft [N],
\(R_t\) = dynamic soil resistance at the toe [N],
\(R_c\) = dynamic resistance in the clutch between the sheet piles [N],
\(a\) = acceleration of the driven profile [m/s\(^2\)], and
\(m_{\text{dyn}}\) = dynamic mass (sum of all the masses that vibrate) [kg].
The above expression for the integration-based formulae is generally where the rate of penetration \( (r_p) \) is obtained by different procedures for the integration of the rigid system under consideration, over the time cycle \( T = 1/f_d \). The dynamic resistance forces at the pile toe, shaft and in the clutch, are all modelled differently. The difference in modelling both dynamic soil resistance forces depends on the geotechnical parameters selected as the input for deriving the magnitude of the dynamic soil resistance.

### The Vibdrive model

The Vibdrive model was initially developed by Holeyman (1993). This model and its empirical parameters have been further developed over subsequent years, undergoing refinement by De Cock (1998), and Vanden Berghe and Holeyman (1997). Development has proceeded through comparisons with both laboratory and full-scale test results (see the BBRI tests in Section (2.4.5)).

The Vibdrive model is a rather simple model, however it does take into account the following two soil-related phenomena that alter the characteristics of initial shear strength: (i) the cyclic motion of grain particles due to vibratory accelerations (fluidisation), and (ii) the induced pore-pressure build up (liquefaction). The Vibdrive model is based on the assumption that the vibrator and sheet-pile system can be treated as a rigid body (see Section (3.3.3)), and therefore applies the Newton’s second law of motion to the moving masses, and the penetration speed is obtained by integrating the net downward and upward acceleration over a complete cycle \( (T = 1/f_d) \). The parameters \( (\tau_d) \) and \( (q_d) \) characterising the dynamic soil resistance along the shaft and at the toe respectively, are obtained by the quasi-static values taken from cone penetration tests. During vibratory penetration, the values of \( (\tau_d) \) and \( (q_d) \) depend on their corresponding quasi-static and liquefied values, as well as the vibratory accelerations according to a decreasing exponential law.

The following paragraphs briefly describe the input parameters, the geometric model, assumptions and empirical equations used to evaluate the soil resistance along the shaft and at the toe, and finally the procedure of integrating the motion equation.

### Input parameters for the Vibdrive model

The parameters used in the Vibdrive model are grouped into three categories, relating to the various parts of the overall system: the vibrator, the sheet-pile profile, and the soil.
Chapter two

**Vibrator**

- $M_e$ unbalanced moment [kgm],
- $f_d$ driving frequency [Hz],
- $m_v$ vibrating mass of the vibrator [kg], and
- $m_o$ bias mass (stationary mass) of the vibrator [kg].

**Sheet-pile profile**

- $A_t$ section area [cm$^2$],
- $\chi$ perimeter [m],
- $m_p$ mass of pile ($\rho A_t L$) [kg],
- $z$ penetration depth [m], and
- $L$ length [m].

**Soil parameters**

- $q_d$ dynamic soil strength at the toe [kPa], and
- $\tau_d$ dynamic side friction [kPa],
- $\psi$ empirical liquefaction factor in the range of $4 < (1/\psi) < 10$ [-],
- $\alpha$ acceleration ratio ($\alpha = a/g$) [-].

**Forces present in the Vibdrive model**

The four forces acting on the vibrator, sheet pile and soil system have been graphed in Figure (2-23), and consist of the following.

- $M_e \omega^2 \sin(\omega t)$: this is the unbalanced force ($F_v$) produced by the vibrator and delivered to the head of the sheet pile being driven, and is characterised by the unbalanced moment and driving frequency, which are derived according to Equation (2.8).
- $R_s$: the resisting side friction force acting on the sheet-pile shaft describing either the soil resistance or the friction force (or both) generated in the lock that couples two sheet piles to each other.
- $R_t$: the resisting force on the sheet-pile toe, which does not exist during the upward motion.
- $g m_{tot}$: which is defined by the gravitational forces on the total mass $m_{tot}$ given by $(m_{tot} = (m_o + m_v + m_p))$, where $m_o$ is the bias mass of the vibrator, $m_v$ is the mass of the excitor block, and $m_p$ is the mass of the sheet-pile profile.
Dynamic soil resisting forces modelled by the Vibdrive model

The parameters \( \tau_d \) and \( q_d \) characterising the dynamic soil resistance along the shaft and at the toe respectively, are obtained by the quasi-static values taken from cone penetration tests. During vibratory penetration, the values of \( \tau_d \) and \( q_d \) depend on their corresponding quasi-static and liquefied values, as well as the vibratory accelerations according to a decreasing exponential law.

The amplitude of the dynamic toe and pile-shaft resistance forces are calculated at each depth \( z \) of the subsoil strata profile according to Equations (2.28) and (2.29). The dynamic toe resistance \( R_t \) is in its simplest form modelled by a step function, where \( R_t \) becomes zero when the sheet pile moves up and positive when the penetration speed \( v \) is greater than zero, given by:

\[
R_t = q_d A_t [1 - \text{sgn}(v)]
\]

with 
\[
\text{sgn}(v) = \begin{cases} 
1 & \text{if } v > 0 \\
0 & \text{otherwise}
\end{cases}
\]

Equation (2.28)

where \( R_t \) = dynamic toe resistance [kN],
\( A_t \) = sectional area of the vibratory-driven sheet-pile toe [m\(^2\)],
\( v \) = velocity of the vibratory-driven sheet-pile, and
\( q_d \) = maximum compression stress at the toe [kPa].
The dynamic shaft resistance \( R_s \) is also modelled by a step function, where the direction of the shaft resistance is always in the opposite direction to the movement:

\[
R_s = \text{sgn}(v) \chi \int_0^z \tau_d \, dz
\]

with \( \text{sgn}(v) = \begin{cases} 
1 & \text{if } v > 0 \\
0 & \text{if } v = 0 \\
-1 & \text{if } v < 0 
\end{cases} \) \hspace{1cm} (2.29)

where \( R_s = \) dynamic shaft resistance [kN], \( \chi = \) perimeter of the sheet pile [m], \( \tau_d = \) maximum shear stress at the shaft of the sheet pile [kPa], \( v = \) velocity of the vibratory-driven sheet-pile, and \( z = \) penetration depth of the sheet-pile toe [m].

The first step in calculating the dynamic driving unit resistances at the toe \( (q_d) \) and along the shaft \( (\tau_d) \) is derived based on the static \( (q_s, \tau_s) \) and liquefied \( (q_l, \tau_l) \) soil resistance counterparts according to Equations (2.30) through (2.33).

The empirical soil strength reduction mechanism in the Vibdrive model relating to the acceleration ratio \( (\alpha) \), has been modelled by an exponential expression \( (e^{\alpha}) \) in a similar way as Barkan (1962), see Equation (2.17). The exponential strength reduction of both toe and shaft, are expressed by:

\[
q_d = (1 - e^{-\alpha}) q_l + q_s \cdot e^{-\alpha}
\]

and

\[
\tau_d = (1 - e^{-\alpha}) \tau_l + \tau_s \cdot e^{-\alpha}
\]

where \( q_d = \) driving unit resistance at the toe [kPa], \( q_l = \) liquefied soil resistance at the toe [kPa], \( q_s = \) static toe resistance profile [kPa], \( \alpha = \) acceleration ratio \( (a/g) \) [-], \( \tau_d = \) driving unit resistance along the shaft [kPa], \( \tau_l = \) liquefied soil resistance along the shaft [kPa], and \( \tau_s = \) static shaft resistance profile [kPa].

The other empirical soil reduction mechanism in the Vibdrive model relating to effects of excess pore-pressure, has also been modelled by an exponential expression containing the friction ratio and the liquefaction parameter. The exponential strength reduction related to liquefaction effects at both toe and shaft, are expressed by:
Literature review

\[
q_l = q_s \left(1 - \Psi \right) e^{-R_f} + \Psi
\]  \hspace{1cm} (2.32)

and

\[
\tau_l = \tau_s \left(1 - \Psi \right) e^{-R_f} + \Psi
\]  \hspace{1cm} (2.33)

where

- \( q_l \) = liquefied soil resistance at the toe [kPa],
- \( q_s \) = static toe resistance profile [kPa],
- \( \Psi \) = empirical liquefaction factor, set between \( 4 < (1/\Psi) < 10 \), [-],
- \( R_f \) = friction ratio \((f_s/q_c \cdot 100)\) taken from the CPT results [%],
- \( \tau_l \) = liquefied soil resistance along the shaft [kPa], and
- \( \tau_s \) = static shaft resistance profile [kPa].

**Equilibrium and integration procedure in the Vibdrive model**

The Vibdrive model focuses on the dynamic masses and therefore assumes that the vibrator and sheet-pile system behaves like a rigid body; in other words that the masses vibrate with the same acceleration amplitude. Given this assumption, the equation of movement is therefore derived from Newton’s second law of motion, expressed by the following differential equation:

\[
am_{dyn} = M \cdot \omega^2 \sin(\omega t) + g_m + R_t - R_s - R_c
\]  \hspace{1cm} (2.34)

The Vibdrive model can take into account effects of clutch friction \((R_c)\), which is then usually combined with the dynamic shaft resistance \((R_s)\); however no documented guidelines exists for how this should be accomplished. An analytical integration of Equation (2.34) is somewhat difficult, therefore a numerical approach had to be developed. On the basis of the different strengths adopted in the Vibdrive model (see Figure (2-24)), three zones of parts (A, B and C) can be distinguished. In Parts A and C, it is assumed that the sheet pile accelerates, since the biased driving force \((g_m + m_o \omega^2 \sin(\omega t))\) is greater than the resisting forces. In other words, it is assumed that vibratory motion is only possible when the two following inequalities of the forces are fulfilled:

\[
F_v > R_s + R_t
\]  \hspace{1cm} (2.35)

\[
F_v > R_s + g_m\text{tot}
\]
The two inequalities provided in the equation pair above (Equation (2.35)) are represented by horizontal lines in Figure (2-24). Multiplying the two areas labelled Parts A and C with the dynamic mass ($m_{dyn}$) graphs the acceleration of the sheet pile. To obtain the penetration velocity, an integration with respect to time has to be performed. The penetration velocity ($v(t)$) is evaluated as the net settlement velocity (the difference between the upward and downward ($v^\uparrow$ and $v^\downarrow$) velocities) in accordance with Equations (2.37) and (2.38), in relation to the direction of the result of the forces applied accelerating the sheet pile.

To start the integration process, the initial speed must be known, which also is the trickiest part of the numerical approach applied. This is solved by comparing the first speed calculated, with the value initially assumed. When the iterative process presents two equal values of calculated speed with the initial pre-set speed, then the steady state is reached and the calculation is completed. The average penetration speed is then calculated by the displacement of one cycle divided by the period of one cycle.

The net penetrative velocity ($v(t)$) is defined as the difference between the upward and downward ($v^\uparrow$ and $v^\downarrow$) velocities in relation to the direction of the result of the forces applied. The net penetrative-velocity amplitude is estimated by the following expression:

$$v(t) = v^\uparrow - v^\downarrow = \int_{0}^{T/2} \frac{F^\uparrow(t)}{m_{dyn}} dt - \int_{T/2}^{T} \frac{F^\downarrow(t)}{m_{dyn}} dt$$

(2.36)

where expressions for the net effective upward and downward forces ($F^\uparrow(t)$ and $F^\downarrow(t)$ in Equation (2.36) are calculated by

$$F^\uparrow(t) = M_e \omega^2 \sin(\omega t) - g(m_o + m_v + m_p) - R_s - R_c \geq 0$$

(2.37)

and

$$F^\downarrow(t) = M_e \omega^2 \sin(\omega t) + g(m_o + m_v + m_p) - R_s - R_c - R_t \geq 0$$

(2.38)

where $F^\uparrow(t)$ = net effective upward force [kN],
$F^\downarrow(t)$ = net effective downward force [kN],
$R_s$ = dynamic soil resistance along the shaft [N],
$R_v$ = dynamic soil resistance at the toe [N],
$R_c$ = dynamic resistance in the clutch between the sheet piles [N],
$M_e$ = unbalanced moment [kgm],
$f_d$ = driving frequency [Hz],
\[ m_v = \text{vibrating mass of the vibrator [kg], and} \]
\[ m_o = \text{bias mass (stationary mass) of the vibrator [kg].} \]

**Figure 2-24** Illustration of the procedure for the integration of the downward and upward parts of the excess force (after Vanden Berghe and Holeyman, 1997).

**The Karlsruhe model**

As part of extensive research work at the University Friedericiana in Karlsruhe (Germany), Dierssen (1994) analysed and developed a scheme presenting how the dynamic toe resistance can be modelled during slow vibratory-motion (see Section (2.4.5)).

From field tests, Dierssen confirmed the existence of two types of vibratory motions, namely fast and slow vibratory-driving. Dierssen’s research work focused primarily on the so-called slow vibratory-motion.

The difference between slow and fast vibratory-motion was explained as follows. During slow vibratory-motion, the pile toe tended to lose contact with the soil below the toe during parts of the upward cyclic movement. Fast vibratory-motion on the other hand, tended to occur when the pile toe maintained contact during the cyclic motion of the pile toe. Dierssen used a numerical integration scheme that closely followed the displacement of both toe and shaft in the time domain (illustrated in Figure (2-26)).

**Dynamic toe resistance modelled by the Karlsruhe model**

Dynamic toe resistance during slow vibratory-motion was divided model-wise into four different phases to describe the observed kinematics of the pile-toe force and displacement curve \((R_{t-u})\).
Chapter 2.25
Schematic representation of the integration procedure in the Vibdrive model (after Vanden Berghe and Holeyman, 1997).

Input parameters
\( m, m_p, M, f_p, \tau, q, A, \zeta \), etc. see Table 5-2

Part A in Fig. 3-16
\( F_r + g m_p > R + R_s \)

Downward motion
\( r_t > 0 \)
\Rightarrow acceleration of the downward motion

Upward motion
\( r_t < 0 \)
\Rightarrow reduction of the upward motion

Part B in Fig. 3-16
\( F_r + g m_p < R + R_s \)

Downward motion
\( r_t > 0 \)
\Rightarrow reduction of the downward motion

Upward motion
\( r_t < 0 \)
\Rightarrow acceleration of the upward motion

Part C in Fig. 3-16
\( F_r + g m_p > R + R_s \)

Downward motion
\( r_t > 0 \)
\Rightarrow reduction of the downward motion

Upward motion
\( r_t < 0 \)
\Rightarrow acceleration of the upward motion

No motion
\( r_t = 0 \) or \( r_{t, \text{part}} < 0 \)
sheet pile imobile

Next time increment

\( r_{t+\Delta t} = (a_t + d_{t+\Delta t})\Delta t/2 + r_t \)

\( d_{t+\Delta t} = (r_t + r_{t+\Delta t})\Delta t/2 + d_t \)
In Phase 1-2 (see Figure (2-26)), the pile toe makes contact with the soil, but starts to lose it due to the upward movement of the pile toe. The toe force \( R_t \) decreases rapidly. The straight line of Phase 1-2 is expressed by:

\[
R_t = R_{t,1} - C_u \frac{A_t}{\phi_t} (u - u_1)
\]  \hspace{1cm} (2.39)

where \((C_u)\) represents the unloading modulus of the soil, \(A_t\) represents the sectional area of the pile toe, and \(\phi_t\) the diameter of the pipe-pile toe.

In Phase 2-3, the pile toe completely loses contact with the soil below, and both the pile toe and soil are subject to an upward movement, with the soil having a slower velocity. Therefore, a cavity is able to form underneath the pile toe. The toe force \( R_t \) is then set to zero. This cavity cannot be filled completely by soil flowing in from the area immediately adjacent to the cavity, as the interval of time between the loss of contact and the reversal of the movement of the pile toe (approximately 50 ms) is too short.

In Phase 3-4, the pile toe has changed direction and is on its way down, still without coming into contact with the soil, and the toe force \( R_t \) equals zero.

In Phase 4-1′, the pile-toe resistance begins to appear again and the pile toe starts to penetrate the soil again. At the time of impact (when contact between the pile toe and the underlying soil is re-established), the soil has regained its reversible part of deformation. But a small part of the deformation is irreversible. The toe force \( R_t \) increases rapidly, and the straight line part of Phase 4-1′ is expressed by:

\[
R_t = C_l \frac{A_t}{\phi_t} (u - u_4)
\]  \hspace{1cm} (2.40)

where \((C_l)\) represents the loading modulus of the soil, \(A_t\) the sectional area of the pile toe, and \(\phi_t\) the diameter of the pipe-pile toe.

From this field test, it was concluded that the unloading modulus could be equal to four times the loading modulus, and a performance coefficient for each cycle was introduced and defined as the ratio \(\beta\), according to the following:

\[
\beta = \frac{\Delta u}{u_4 - u_1} = \frac{u_4 - u_1}{u_4 - u_1} \hspace{1cm} (2.41)
\]

This ratio was found to be in the range \(0.3 < \beta < 0.4\). The performance coefficient \(\beta\) can also be defined from the characteristics of the soil, and expressed according to Equation (2.42).
Figure 2-26 Description of the four phases of the slow vibratory-motion of the pile-toe force, and shaft resistance versus displacement (after Dierssen, 1994).

\[ \beta = 1 - \frac{C_u}{C_f} \] (2.42)

where \( (C_f) \) is a fictitious rigid modulus, which can be considered to be equal to \( (1.6C_t) \).

Meanwhile, the outward radiation of waves generated at the pile toe was estimated according to Lysmer and Richart (1966), and modelled using a system of mass-spring shock absorbers, which uses the following expressions for the damping value \( (C) \) and stiffness \( (K) \) of the springs:
Dynamic shaft resistance modelled by the Karlsruhe model

As well as the dynamic toe resistance during slow vibratory-motion, the shear stress along the shaft of the driven pile can also be divided model-wise into different phases in order to describe the observed kinematics of the shear stress and the displacement curve by an idealised hysteresis cycle \((\tau_s-u)\), seen in Figure (2-26).

According to Dierssen, the motion of the rate of shear stress of the lateral stress-ess can be expressed according to:

\[\dot{\tau} = C_1 \zeta \dot{u} \left[1 - \left(\frac{\tau}{\tau_{\text{max}}}\right)^{2} \right] \text{sgn}(\dot{u}) \]  

(2.45)

where \(\tau\), \(\dot{\tau}\), and \(\tau_{\text{max}}\) are dependent on the depth \(\zeta\), and where \(\tau_{\text{max}}\) is proportional to \(4.33 \sigma'_p\), and the constant \(C_1\) is defined by Equation 2.39 below.

\[C_1 = \frac{\ln(1-0.9)}{2\times10^{-3}} \frac{\tau_{\text{max}}}{\zeta} \]  

(2.46)

Integration of Equation (2.45) results in the following expression of the shear stress:

\[\tau = \left(\tau_o - \tau_{\text{max}} \text{sgn}(\dot{u})\right) \exp\left(-C_1 \frac{\zeta}{\tau_{\text{max}}} (u - u_o) \text{sgn}(u)\right) + \tau_{\text{max}} \text{sgn}(\dot{u}) \]  

(2.47)

where \(\tau_o\) and \(u_o\) represent the initial shear stress and the initial depth respectively, at the beginning of the cycle being considered.

This shear-stress value (according to Equation (2.47)) is then corrected for the radiation of the waves generated along the shaft of the pile under consideration. The dynamic shear stress is expressed by the following:
\[ \tau_d = \dot{u} \sqrt{\rho G} \left| 1 - \frac{\tau}{\tau_{\text{max}}} \right| \]  

(2.48)

where \( \dot{u} \) = velocity of the vibro-driven pile, 
\( \rho \) = unit weight of the soil, and 
\( G \) = the shear modulus.

According to Dierssen, the second type of penetration movement, called “fast vibratory-driving”, can occur during vibratory driving when the cyclic discharge of the soil under the pile toe suddenly becomes incomplete. This situation occurs when the vibratory parameter called unbalanced moment (\( M_u \)) is lowered (that is when \( S_o \) is decreasing) and at the same time the static surcharge force (\( F_o \)) is increasing.

Cudmani (2000), Cudmani et al. (2002) was also involved in extensive research work at University Friederican in Karlsruhe (Germany), continuing Dierssen’s (1994) research work. The earlier work was expanded to include fast vibratory-motion in order to study the mechanisms causing the transfer between slow and fast vibratory-motion.

Based on field test results conducted by Hubert (1997), Cudmani proposed that a continuous transition mode existed between fast and slow vibratory-motion. The modelling of this transition is explained by starting with a pile toe in a slow vibratory-mode, according to a certain initial machine setting, and then suddenly changing the static surcharge force (\( F_o \)), which transfers the motion into a fast mode.

The modelling of the continuous transition mode proposed starts with a slow vibratory-motion of the pile toe. The pile toe has a reversal motion at Point 4 in Phase 1, shown in Figure (2-27)).

The motion of the pile-toe loss with the underlying soil is decreased by increasing the vibratory-parameter static-surcharge force (\( F_o \)). The transition from slow to fast vibratory-motion depends on the reversal point (Point 3 to 4 in Phase 2, Figure (2-27)).

The maximal dynamic-toe resistance reached during the transition range (\( R_{t,\text{max}} \)) depends on the position of the reversal point (Point 3) in respect to Points 2 and 4, in Phase 3, Figure (2-27).

When the vibratory-driving parameter, static surcharge force (\( F_o \)) is increased further, the reversal motion of the pile toe takes place earlier (at Point 2), and the limit force of the pile toe reaches the maximum toe-resistance value at fast vibratory-motion (\( R_{t,\text{max}} \)), (see Phase 4 in Figure (2-27)).
Further increase in the static surcharge force ($F_{\text{w}}$) does not cause any change in the toe resistance with fast vibratory-motion ($R_{t,\text{max}}$), (see Phase 5 in Figure (2-27)).

The following experimental observations have been used by the author to model the transition range between slow and fast vibratory-motion:

- the maximal toe-force can exceed the dynamic toe resistance ($R_{t,\text{max}}$) in each period of slow vibratory-driving (depending on the machine settings);
- if the downward motion begins exactly at Point 4, the maximal toe-force ($R_{l,\text{max}}$) is reached at slow vibratory-driving (depending on the machine settings);
- if the downward motion starts between Points 2 and 4, a limiting force is reached that is not equal to the maximal tip resistance reached using fast

*Figure 2-27  Modelling of the transition range between slow and fast vibratory-motion (after Cudmani, 2000).*
vibratory-driving. The limit force is modelled ranging between \( R_{l,max} \) and \( R_{s,max} \).

**The Vipere model**

The Vipere (VIbratory PEnetration REsistance) model is the most recent numerical model for simulating vibro-driveability. Vipere was developed by Vanden Berghe (2001) in a PhD project at the Division of Soil Mechanics at the Université catholique de Louvain (Belgium).

The hypoplastic constitutive behaviour of the soil has been implemented into a previously developed geometric model originally suggested by Holeyman and Legrand (1994). This model treats the pile as exhibiting rigid-body behaviour, and models the soil into one dimensional discretisation. The hypoplastic constitutive equations used in the Vipere model were originally developed by Bauer (1996) and Gudehus (1996). These equations were originally developed to simulate the soil resistance during undrained triaxial tests and with constant volume (direct simple shear tests (DSS)). The main assumption was the homogeneity of the stress and strain distribution in a tested sample, allowing modelling of a unique soil element whose state is described by the void ratio and the strain and stress tensor. For all of the hypoplastic equations, only one expression is required to simulate the elastic as well as the plastic state together with the yield surface. This equation, applied in the Vipere model, calculates the stress rate induced by a strain rate as a function of the current stress state and void ratio.

The following subsections briefly describe the input parameters, the geometrical model, assumptions and constitutive equations used to evaluate the soil resistance along the pile shaft and at the toe, and finally the procedure of integrating the motion equation.

**Input parameters for the Vipere model**

The parameters used in the Vipere model are grouped into five categories, relating to the various parts of the overall system, (the vibrator, the sheet-pile profile, and the soil), together with the hypoplastic-model-related parameters as well as integration-related parameters. The following parameters are used as input:

**Vibrator**

- \( M_s \), static moment, [kgm],
- \( f_d \), driving frequency, [Hz],
- \( m_v \), vibrating mass of the vibrator, [kg], and
- \( m_o \), stationary mass of the vibrator, [kg].
**Sheet-pile profile**

- $A$, section area, [m$^2$].
- $\chi$, perimeter, [m], and
- $L$, length, [m].

**Soil parameters**

- $\phi'$, effective friction angle [$^\circ$],
- $e$, initial void ratio,
- $e_{cr}$, critical void ratio, and
- $e_{mo}$, minimum void ratio.

**Hypoplastic model parameters**

- $h_s$, strength, [MPa],
- $n$ -
- $\alpha$, dimensionless positive constant,
- $\beta$, dimensionless positive constant, and
- $\lambda$, dimensionless positive constant.

**Integration parameters**

- $R_{\text{max}}$, maximum radial discretisation, [m],
- $P_{\text{geo}}$, dimensionless geometrical parameter and
- $P_{\text{time}}$, dimensionless time parameter.

**Forces present in the Vipere model**

The forces acting on the vibrator and sheet-pile system are the same as for the Vipere model, just labelled differently (see Figure (2-28)).

- $M_e \omega \sin(\omega t)$: this is the unbalanced force produced by the vibrator and delivered to the head of the sheet pile being driven, and characterised by the static moment and frequency as explained by Equations (2.7) and (2.8).
- $g m_{\text{tot}}$: this is defined by the gravitational forces on all masses of sheet pile and vibrator in which ($m_{\text{tot}} = (m_o + m_v + m_p)$).
- $F_{\text{shaft}}$: this is the resisting side friction force on the sheet pile, describing the soil resistance or the friction force (or both) generated in the lock that couples two sheet-piles to each other.
- $F_{\text{toe}}$: this is the resisting force on the sheet-pile toe, which does not exist during the upward motion.
Geometrical description in the Vipere model

The Vipere model has borrowed its geometrical configuration from the HYPERVIB 2 model developed by Holeyman (1994, 1996). The soil surrounding the pile or sheet pile has cylindrical symmetry (see Figure (2-29)), where the soil volume is slotted into concentric rings, and the behaviour of each soil volume is described by a hypo- elastoplastic law.

The coefficient of diffusion or damping ($\alpha$) is usually set to 0.03 times the magnitude of the waves induced along the shaft, and is taken into account by increasing the height difference ($\Delta h$) of the concentric rings in the soil with increasing radial distance ($r$) from the shaft, as illustrated in Figure (2-29) and expressed according to the following:

$$\Delta h = \alpha r$$  \hspace{1cm} (3.49)

The radial discretisation is characterised by the number of concentric rings ($N_r$), and the maximum radius ($R_{\text{max}}$). The absorption condition at the boundary of the modelled soil volume is taken into account (Novak 1989), and the complex vertical stiffness of the soil ($K$) can be written as:
where \( G_{\text{max}} \) = maximum shear modulus [kPa],
\( R_{\text{max}} \) = maximum radius of soil discretisation [m],
\( b_{N_r} \) = height of the last soil segment [m],
\( \omega \) = angular frequency [rad S\(^{-1}\)],
\( V_s \) = shear wave velocity [m/s], and
\( K_{0,1} \) = modified Bessel function of order, 0 and 1 respectively [-].

\[
K = 2\pi G_{\text{max}} b_{N_r} R_{\text{max}}^2 \frac{\omega}{V_s} \left( \frac{R}{V_s} \right) \frac{K_1 \left( \frac{R \omega}{V_s} \right)}{K_0 \left( \frac{R \omega}{V_s} \right)}
\]  

(2.50)

Figure 2-29 Description of the Vipere geometrical model, (Vanden Berghe, 2001).

**Constitutive law for shaft resistance of the Vipere model**

The constitutive relationship used to calculate the shear stress along the shaft of the different elements in the geometrical model graphed in Figure (2-29), is the hypoplastic...
The model considers each soil element as being simply sheared with the shear displacement imposed between the internal and external boundaries of the concentrically-arranged soil rings (Figure (2-30)).

In the axially-symmetrical distribution of stresses, the non-vanishing stress components in the cylindrical coordinate system \((r, \theta, z)\) are: \(\sigma_r\), \(\sigma_\theta\), \(\sigma_z\) and \(\tau_{rz}\). The non-vanishing velocity components are \(u\) and \(w\), perpendicular and parallel to the vertical \(z\)-axis. Stresses and velocities are independent of \(\theta\) and are functions of \(r\), \(z\) and time only; that is, symmetry where \(\partial (\cdot) / \partial \theta = 0\).

It is assumed that there is no stress or strain variation along the \(z\) axis \((\partial (\cdot) / \partial z = 0)\), and that the radial normal stress and the shear stress do not change as a function of the depth.

The soil is also assumed to be fully saturated, and the frequency of the oscillation is supposed to be high enough to avoid the excess pore-water dissipation during vibratory driving. Therefore the model considers the soil to be sheared during undrained conditions; in other words there are no volume changes \((\Delta e = 1 + \varepsilon_r + \varepsilon_\theta + \varepsilon_z = 0)\).

As the model considers only sheared elements, then neither axial nor radial normal strains are permitted along these boundaries. The non-vanishing strain in the strain tensor during cylindrical shearing is the shear strain \((\gamma_{rz} \neq 0 \text{ and } \varepsilon_r = \varepsilon_\theta = \varepsilon_z = 0)\), and therefore \((T_s)\) can be expressed by:

\[
T_s = \begin{bmatrix}
\sigma' & 0 & \tau_{rz} \\
0 & \sigma' & 0 \\
\tau_{rz} & 0 & \sigma'
\end{bmatrix}
\]  

(2.51)

Based on the above assumptions, the distribution of stresses and strains on each soil element interface is illustrated in Figure (2-30). Distribution can be expressed by the stress tensor \(T_s\) defined by Equation (2.53), and the strain tensor according to the matrix in Equation (2.52):

\[
D_s = \begin{bmatrix}
0 & 0 & \gamma_{zr} \\
0 & 0 & 0 \\
\gamma_{zr} & 0 & 0
\end{bmatrix}
\]  

(2.52)

The constitutive relationship evaluating the shear stress along the shaft of the different soil elements is the hypoplastic model. This hypoplastic constitutive law evaluates the stress rate tensor \((\dot{T}_s)\) as a function of the current stress state tensor and the strain rate tensor in Equation (2.53) below:
\[
\dot{T}_s = f_s \left[ a_1^2 \dot{D}_s + \dot{\tau}_s \text{tr}(\dot{T}_s \dot{D}_s) + f_{d \alpha_1} (\dot{T}_s + \dot{\tau}_s) \| \dot{D}_s \| \right] 
\]

(2.53)

where \( f_{s, \alpha_1} \) = scalars depending on the mean stress \( P' \) and void ratio \( e \),
\( a_1 \) = a dimensionless scalar,
\( \dot{D}_s \) = strain rate tensor of the granular skeleton, and
\( \dot{T}_s \) = stress ratio tensor of the granular skeleton.

\[ \Delta \tau_{rz} = f_s [\Delta \gamma_{rz}] \left[ a_1^2 + \frac{2\tau_{rz}}{\sigma_r' + \sigma_\theta' + \sigma_z'} \right] \text{sgn} \Delta \gamma_{rz} + \quad (2.54) \]

\[ f_{d \alpha_1} \sqrt{2} \cdot \left[ \frac{2\tau_{rz}}{\sigma_r' + \sigma_\theta' + \sigma_z'} \right] \]

\[ \Delta \sigma' = f_s [\Delta \gamma_{rz}] \left[ \frac{2\sigma_r' \tau_{rz}}{\sigma_r' + \sigma_\theta' + \sigma_z'} \right] \text{sgn} \Delta \gamma_{rz} + \quad (2.55) \]

\[ f_{d \alpha_1} \sqrt{2} \cdot \left[ \frac{5\sigma_z' - \sigma_r' - \sigma_\theta'}{3(\sigma_r' + \sigma_\theta' + \sigma_z')} \right] \]

Based on previous assumptions (see Equation (2.53)), this can be simplified and expressed in an incremental shape defined by the following four equations:

Figure 2-30 Illustration of shaft resistance; on the left, simple shearing where the soil volume surrounding the shaft is in axis symmetry, and on the right, the state of stresses applied to the interface of each sheared soil volume (Vanden Berghe, 2001).
The Vipere model evaluates the dynamic shaft resistance \( R_s \) on the assumption that the loading conditions of the soil along the shaft behave similarly to the loading situations during cylindrical simple shearing. The dynamic shaft resistance \( R_s \) is evaluated according to Equation (2.65). The mean shear resistance \( \tau \) in Equation (2.65) that is modelled to act on the interface between the soil exposing the shaft of the vibro-driven pile is evaluated on the basis of the shear strain-shear stress \( (\gamma - \tau) \) relationship. Figure (2-31)a, describes the simulation of the \( (\tau - \gamma) \) hysteresis loops during vibratory driving, and looks very similar to those reported during previously conducted laboratory tests, (see Section 2-4).

Vanden Berghe (2001) stipulated that the shape of the \( (\tau - \gamma) \) hysteresis loops is a consequence of the fact that the soil volume close to the shaft experiences two phases of dilation and two phases of contraction during each cycle, (see Figure (2-31)b). It can also be observed that when the shear strain rate changes sign, the soil behaviour becomes initially contractive, as a rapid decrease towards zero, (Parts 1-2, in Figure (2-31)b), and is then followed by a dilative phase when the sheared soil volume is not able to sustain the imposed shear strains (section 2-3, Figure (2-31)b). This phenomena is also illustrated by the butterfly-shaped curve (see Figure (2-31)c), which shows the relationship between the effective radial normal stress and the vertical shear stress of the cyclically-sheared soil volume close to the shaft.
Constitutive law for toe resistance in the Vipere model

The dynamic toe resistance in the vibratory-driven pile has been represented by a cylindrical soil volume under the pile toe, according to Figure (2-32). The section of the soil cylinder is equal to the section of the pile toe, and its height is equal to 70% of the diameter of the pile diameter. The soil volumes at the pile toe are assumed to maintain contact with each other. This means that any possible gaps between the pile toe and underlying soil volume as experimentally observed by Dierssen (1994) and De Cock (1998) are not accounted for.

The behaviour of the cylinder below the pile toe (see Figure (2-32)) is assumed to be hypoplastic, and is estimated according to the same constitutive expression used for the shaft (Equation (2.53)). The soil element below the toe is assumed to be loaded as in tri-axial extension or compression with cylindrical symmetry, where $\sigma_\chi$ and $\sigma_r = \sigma_\theta$ are the principal stresses.
With respect to the shaft resistance, the frequency of the cyclic loading is assumed to be high enough that the excess pore-pressure cannot be dissipated during driving. Therefore the soil is assumed to behave as undrained ($\Delta e = 0$ and $e_r = e_0 = \frac{1}{2}e_\theta$). The pore pressure is calculated with the assumption that the total mean stress ($P_\sigma$) of the soil element under consideration stays constant ($\Delta u = P' - P'_o$).

The vertical normal strain ($e_z$) is calculated by dividing the pile displacement ($u_p$) by the height of the soil element below the pile toe ($0.7\phi_{pile}$), and the resulting toe resistance is calculated on the basis of the following expression:

$$F_t = (\sigma_z + u)A_t$$ (2.58)

where $\sigma_z$ = effective vertical stress [kPa],
$u$ = pore pressure [kPa], and
$A_t$ = section area of the pile toe [m$^2$].

The stress ($T_s$) and strain ($D_s$) tensors can then be simplified according to:

$$T_s = \begin{bmatrix} \sigma_r' & 0 & 0 \\ 0 & \sigma_r' & 0 \\ 0 & 0 & \sigma_z' \end{bmatrix}$$ (2.59)

and

$$D_s = \begin{bmatrix} -\frac{1}{2}\Delta e_z & 0 & 0 \\ 0 & -\frac{1}{2}\Delta e_z & 0 \\ 0 & 0 & -\frac{1}{2}\Delta e_z \end{bmatrix}$$ (2.60)
Based on the assumptions defined above, together with the stress \((T_\sigma)\) and the strain \((D_\varepsilon)\) tensors, the stress rate at the pile toe can be expressed in its incremental form, defined by the function of the deviator \((q)\) and the effective mean stress \((\sigma')\):

\[
\Delta \sigma'^\prime_s = f_s |\Delta \varepsilon_1|^3 \left[ \frac{q}{3} \right] \text{sgn} \Delta \varepsilon_1 + a_1 f_d \left[ \frac{3}{2} \right]
\]  

(2.61)

and

\[
\Delta q = f_s |\Delta \varepsilon_1|^3 \left[ \frac{q}{3} \right] \text{sgn} \Delta \varepsilon_1 + a_1 f_d \left[ \frac{3}{2} \right]
\]  

(2.62)

where the density \((f_d)\) and the stiffness \((f_d)\) factors are functions of the effective mean stress (see Vanden Berghe, 2001)). The deviator and the effective mean stress are given by:

\[
\sigma'^\prime = \frac{\sigma_z' + 2 \sigma'_r}{3}
\]  

(2.63)

\[
q = \sigma_z' - \sigma'_r
\]

The Vipere model evaluates the dynamic toe resistance \((R_t)\) based on the assumption that the loading conditions of the soil below the pile toe behave similarly to the loading conditions during an undrained tri-axial (TXT) test. The dynamic toe resistance \((R_t)\) is evaluated on the basis of the total normal axial stress which is applied to the pile toe, (see Figure (2-33)d). The total normal axial stress is the sum of the effective axial normal stress \((\sigma'_{toe})\) and the pore pressure \((u)\), according to Equation (2.58), where the effective axial normal stress \((\sigma'_{toe})\), in Figure (2-33)a, and the pore pressure \((u)\) in Figure (2-33)b, are evaluated with the hypo-plastic model as previously described.

Vanden Berghe (2001) stipulated that when the pile toe is thrust into the soil strata, the effective normal stress \((\sigma'_{toe})\) increases dramatically during the dilation phase of the soil volume, (Part 4-1’ in Figure (2-33)a). When the motion of the pile toe changes direction, (upward motion), the effective axial normal stress decreases rapidly to a constant low value, (Part 1-2), and stays low during the following contraction and dilation phase of the soil volume, (Parts 2-3-4, Figure (2-33)a). When the pile toe moves upwards,
the soil volume is modelled to becomes active, due to the fact that the lateral stresses push the soil towards the pile toe.

Equilibrium and integration procedure in the Vipere model

Equilibrium of the pile

Figure (2-34) illustrates the different forces acting on a vibratory-driven pile. The vibratory driver induces a theoretical driving force delivered to the head of the pile, given by the sum of the surcharge and the unbalanced force (see Equations (2.1) and (2.8)). The dynamic shaft and toe forces are calculated using the hypoplastic model based on the relative displacement between the pile and the soil elements, and the stress state in these soil elements.
The acceleration of the vibratory-driven pile is calculated according to Newton’s second law, resulting from the imbalance between these forces, given by the following expression:

\[
\ddot{u}_p(t) = \frac{gM_{tot} + m_e\omega \sin(\omega t) - 2\pi r_1 b_1 \tau_1(t) - F_{toe}}{M_{vib}}
\]

where \( \ddot{u}_p(t) \) = acceleration of pile \([m/s^2]\),
\( M_{tot} \) = total mass of the vibrator and the pile \([kg]\),
\( M_{vib} \) = vibrating mass (exciter block, clamp and pile) \([kg]\),
\( m_e \) = static moment \([kgm]\),
\( \omega \) = angular frequency \([rad/s]\),
\( r_i \) = equivalent radius of the pile \((\text{perimeter}/2\pi)\) \([m]\),
\( b_i \) = current penetration depth of the pile \([m]\), and
\( \tau_i \) = shear stress at the interface between the pile and the soil \([kPa]\), and finally
\( F_{toe} \) = dynamic toe resistance \([kN]\).

![Figure 2-34 Forces present on the vibratory-driven profile (Vanden Berghe, 2001)](image-url)
Equilibrium of the soil element

Figure (2-29) illustrates the inter-ring forces acting on each soil element, assuming a uniform distribution of the shear stress along the internal and external sides of the concentrically-spaced soil rings surrounding the pile shaft. The gravitational force of the soil element is not taken into account, since it is assumed to be balanced by the base resistance of the soil element. The expression of the inter-ring reaction \( T_i \) between the two soil elements \( (i) \) and \( (i-1) \) is given by:

\[
T_i = 2\pi r_i h_i \tau_i
\]

where
- \( T_i \) = inter-ring reaction force between two soil elements [kN],
- \( r_i \) = radial distance to the interface between the two elements [m],
- \( h_i \) = mean height of the interface between the two elements [m],
- \( \tau_i \) = shear stress at the interface between the two elements [kPa].

The acceleration of each inter-ring soil element is calculated according to Newton’s second law, according to Equation (2.66). The displacements of the soil elements are obtained by a double integration of the acceleration given by the following expression:

\[
\ddot{u}(t) = \frac{T_i + 1 - T_{i+1}}{M_i}
\]

where
- \( \ddot{u}(t) \) = acceleration of the soil element \( i \) [m/s\(^2\)],
- \( T_i, i+1 \) = inter-ring reaction forces [kN], and
- \( M_i \) = mass of the soil element.

Figure 2-35 The inter-ring forces acting on the concentric soil element surrounding the shaft, (Vanden Berghe, 2001).
**Integration procedure of the Vipere model**

The explicit integration procedure is summarised in Figure (2-36), where the equations of motion for the pile and each soil element consist of $Nr+1$ motion equations.

![Diagram](image_url)

Figure 2-36  The integration procedure from the Vipere model, (Vanden Berghe, 2001).
Chapter two

The system is not linear and cannot be solved directly. Moreover, the shear stresses calculated with the hypoplastic model are calculated during small incremental time steps. The Vipere model uses an explicit method to integrate the calculated acceleration.

**Numerical stability during the running of the Vipere model**

The dimensionless time parameter \( P_{\text{time}} \) is defined as the number of time steps needed for a shear wave to pass a soil element having a thickness of \( \Delta r \), expressed by:

\[
P_{\text{time}} = \frac{\Delta t}{\Delta r V_{sw}} = \frac{R_{\text{max}}}{N_r} \frac{\Delta r}{\Delta t V_{sw}}
\]

(2.67)

where
- \( \Delta t \) = the time step increment [s],
- \( \Delta r \) = radial width of each soil element [m],
- \( R_{\text{max}} \) = maximum radial discretisation [m],
- \( N_r \) = number of rings in the discretisation [-], and
- \( V_{sw} \) = shear wave velocity in linear elastic media [m/s].

The dimensionless geometric parameter \( P_{\text{geo}} \) is defined by the number of soil elements encompassed by the wavelength of the input oscillation, defined by:

\[
P_{\text{geo}} = \frac{\lambda}{\Delta r} = \frac{V_{sw} N_r}{f_d R_{\text{max}}}
\]

(2.68)

where
- \( \lambda \) = wavelength of the input oscillation [m],
- \( \Delta r \) = radial width of each soil element [m],
- \( R_{\text{max}} \) = maximum radial discretisation [m],
- \( f_d \) = driving frequency of the vibrator [Hz],
- \( N_r \) = number of rings in the discretisation [-], and
- \( V_{sw} \) = velocity of the shear wave in linear elastic media [m/s].

**Longitudinal one-dimensional models**

Historically, the one-dimensional wave-propagation equations have been used primarily to model the behaviour of impact-driven piles. One of the first reported attempts to use the one-dimensional wave equation in pile driving is that by Isaacs (1931). The implementation of the theory in a finite differential numerical scheme is attributed to Smith (1960). It may be feasible to extend the one-dimensional wave equation from impact driving to vibratory driving, but in this respect few studies have been made. However,
those attempts that have been published are by Chua et al. (1987), Gardner (1987), Middendorp and Jonker (1987), Ligterink et al. (1990) and Moulai-Khatir et al. (1994).

Chua et al. (1987) and Gardner (1987) developed a modified wave-equation computer code called VIBEWAVE in order to explain the action of a vibratory-driven pile. These authors described how the mechanical action of the system can be represented by a two-mass system (bias mass and pile/exciter block) separated by a soft spring, where the lower mass is subject to a sinusoidal force function \( F(t) \), (see Figure (2-37)).

![Figure 2-37 Description of the principle behind the modified one-dimensional model with a harmonic forcing function (Gardner, 1987).](image)

The choice of the soil parameters in the Smith model was found to have a significant effect on the prediction of the behaviour of the piles. The soil behaviour is represented by a spring-slider-dashpot system according to Smith’s (1960) early suggestion. However, although the results looked promising, the two authors did not offer any information about the input values for the Smith soil parameters (that is, the quakes, damping constants and the ultimate soil resistance), or whether such parameters could be applied.
Middendorp and Jonker (1987), as well as Ligterink et al. (1990) used the TNOWAVE computer programme to analyse the driveability of vibratory-installed offshore pipe-piles. The two authors identified the need for a better soil model that was able to describe the soil resistance during the steady state, as many of the current methods used to determine the dynamic soil resistance of vibratory-driven piles are usually derived from the geostatic axial capacities calculated for the pile. The different static axial peak resistance expressions are then modified by different $\beta$ factors to account for the dynamic effects that take place during vibratory driving, as in the general expression, Equation (2.19). These authors also warned that the soil parameters used in the modified computer programme may depend on the driving frequency selected and pile displacement amplitude.

Moulai-Khatir et al. (1994), together with the University of Houston, Texas (USA) developed the VPDA (vibratory-pile-driveability analysis) computer programme based on the one-dimensional wave-propagation approach. The mechanical action of the vibratory driver is modelled in a similar manner to that described in Gardner (1987) together with an efficiency parameter. The soil model is a modification of Smith’s (1960) original approach, with two different hyperbolic mobilization curves, adapted for the unit toe and shaft resistances (see Figures (2-38) and (2-39)). A viscose damper was used to model damping along the shaft, whereas damping was not a consideration at the pile toe.

It should be noted that the research mentioned previously involving the CPAR tests (see Section (2.4.5)), used the VPDA computer programme in order to analyse the 24 full-scale tests. Attempts were made to predict the ultimate bearing capacity of vibratory-driven displacement piles from the responses recorded during the installation phase, which were then compared to the actual results obtained from static load tests. However no conclusive results have been reported by the CPAR programme.

Finally, it should also be noted that the well-known GRLWEAP computer programme has also been used in an attempt to model the mechanical behaviour of vibratory-driven piles in the latest version (GRL, 1998). However this has not been reviewed here.
Figure 2-38  Loading paths of the displacement-dependent hyperbolic formula describing the unit toe resistance during vibratory pile-driving (after Moulai-Khatir et al., 1994).

Figure 2-39  Loading paths of the displacement-dependent hyperbolic formula describing the unit shaft resistance during vibratory pile-driving (after Moulai-Khatir et al., 1994).
2.7 Conclusions from the literature review

The parameters found in this literature survey relate to vibro-driveability of vibratory-driven sheet piles, and can be subdivided into three categories (see Section (2.5)), and are each the subject of the following sections.

2.7.1 Vibratory-related factors

Some of the earlier investigations into how the main vibratory-driver-related parameters affected the driveability \( v_p - \zeta \) curve reached the following conclusions.

- **Driving frequency.** The choice of optimum frequency appears to be a vibro-related parameter that influences the rise of the two vibratory-driving states, fast and slow. These states arise due to the fact that the soil and pile toe move out of phase with each other. This indicates that the choice of driving frequency is important for dynamic toe resistance. In contrast however, dynamic shaft resistance has been found to be virtually independent of frequency selection. The range of driving frequencies selected in the literature reviewed appears to be around 20-30 Hz in virtually all non-cohesive soil conditions. It has not yet been established how the choice of optimum driving frequency should be determined. However, the choice of the driving frequency is usually determined at each specific site, and seems to be more related to the entire vibrating system including pile-vibrator and soil, than just the dynamic soil resistance.

- **Static-surcharge force.** How to determine the best value for optimum bias mass has not been established either. However, it has been stated that increasing the bias weight increases the global rate of penetration, however there is also indications that the magnitude of the surcharge is also partly related to the rise of either the fast or the slow vibratory state.

- **Eccentric moment.** The eccentric moment determines the amplitude of both the displacement and the driving force, but how to determine the best eccentric moment with respect to driveability has not yet been established. Several authors have stated that the displacement amplitude should be chosen (high or small) to drive the pile efficiently at the selected driving frequency but none of them have provided the underlying assumptions for this, nor explanations for why this should be so.

2.7.2 Sheet-pile-related parameters

The following are the main sheet-pile-related parameters found to affect the \( v_p - \zeta \) curve.
• **Lateral flexibility** of slender sheet piles has been observed to reduce the rate of penetration. A weak section modulus of sheet or H piles can lead to strong transverse vibrations. These transverse vibrations have been reported in field tests to be as large as the vertical ones. This leads to reductions in the vertical vibration amplitude at the pile toe or in the worst case, damage to the sheet or H-pile.

• **Clutch friction.** The friction resistance in the lock between two sheet piles has been reported to reduce the global rate of penetration and increase the nearby soil-vibration amplitudes. Problems with poor-quality sheet piles lead to significant increases in the clutch-friction resistance forces, which reduce the vertical-vibration amplitude at the pile toe. This in its turn, leads to earlier driving refusal. Field tests clearly reveal that the use of sheet piles of poor quality leads to a significant increase in soil vibration compared to using sheet piles exhibiting jaws in good condition.

### 2.7.3 Soil-related parameters

The following are the main soil-related parameters affecting the \( v_p \cdot \zeta \) curve.

• **Relative density** is the soil parameter that has been reported to have the most significant influence on the global rate of penetration. The rate of penetration has been found to decrease with increasing relative density. The volume changes (densification or dilation) induced in the soil have been found to relate to a characteristic driveability (\( v_p-z \)) curve.

• **Lateral effective stress** has also been reported to have a significant affect on the penetration rate, however these are not as pronounced as the effect of changes in the relative density of the sand. Increased confining pressure has been found to proportionally decrease the penetration rate.

• **Content of fines** greater than 12% is a soil parameter that usually dictates that the use of vibratory-drivers will not be successful. This has been observed by several authors, but none of them have provided an underlying explanation for it.

• **Degree of saturation** of sand has been observed to be closely related to the global rate of penetration recorded. The rate of penetration has been observed to be higher in saturated sand than in dry sand, but the difference seems to more pronounced with low-displacement piles. The acceleration recorded during driving has been higher in dry sand than in saturated sand, indicating that driving in dry sand is harder. Pore-pressure readings near the pile shaft (in the sand) have been observed to be higher nearer the oscillating pile shaft than further away. The mean pore-pressure readings during driving have been observed to be much higher than the cyclic amplitude, which indicates that pore pressure is primarily related to the total volumetric deformation of the sand rather than to cyclic strains.
- **Layers of different relative density.** The existence of layers with higher densities - has been found to result in a significant reduction in the rate of penetration. It has been concluded from laboratory tests that the dynamic pile-toe resistance increases significantly, while the dynamic pile-shaft resistance increases only slightly when the pile is vibratory driven from a medium-dense sand into a very dense layer of sand.

### 2.7.4 Predictive methods of vibro-driveability

The latest proposals for predicting the driveability of sheet piles take into account the behaviour of the whole vibrator-sheet pile-system as a rigid body, using Newton's second law covering the forces acting on the rigid system. The rate of penetration is obtained by numerically integrating the acceleration of the rigid system.

Modelling of the two dynamic soil resistance forces at the toe and along the shaft has been derived from different geotechnical field tests chosen as input to derive the magnitude of the dynamic soil-resistance forces.
CHAPTER 3

DISCUSSION REGARDING PRIMARY MECHANISMS AND SIMULATION OF VIBRO-DRIVEABILITY

3.1 Chapter introduction

This chapter aims to clarify the nature of the interactions within the vibrator, sheet pile and soil system, and to point out the primary mechanisms affecting vibro-driveability, and has been divided into the following main sections:

- Section 3.2, the kinematics of the vibrator, sheet pile and soil system,
- Section 3.3, the factors hypothesised to influence vibro-driveability,
- Section 3.4, vibro-driveability simulation, and
- Section 3.5, concluding remarks.

The chapter is divided into these particular sections based on the conclusions of the literature survey (see Section 2.7), from which it can be concluded that today’s research into the use of vibratory techniques can be described as a number of isolated islands of knowledge. This chapter is therefore an attempt by the author to present these islands of knowledge using a common language, as a hypothesised overview of the vibro-driveability of sheet piles.

After this introduction, Section 3.2 describes the kinematics and primary forces acting on the system, as vibro-driveability is affected by the overall interactive nature of the whole vibrator, sheet-pile profile and soil system.

Section 3.3 discusses the primary factors effecting the vibro-driveability (actual and hypothesised) of each part of the system. Some of this has been found previously, but has been generally ignored or believed to have had negligible impact on vibro-driveability.
Discussion regarding primary mechanisms and simulation of vibro-driveability

Section 3.4 describes the two predictive models selected from the literature survey (Vibdrive and Vipere model), which have been used in Chapter 5 to compare the results obtained from the field tests.

3.2 Kinematics of the vibrator, sheet pile and soil system

3.2.1 General nature of the system

The ease with which a sheet pile enters a soil stratum depends on the characteristics of the interaction between the soil and the sheet pile, as well as the characteristics of the sheet pile and the vibratory equipment. Hence it is imperative that attempts to attain optimum vibro-driveability take into account the mechanical interaction and nature of the whole vibrator, sheet-pile profile and soil system (see Figure 3-1). However before proceeding to the discussion about the hypothesised factors affecting vibro-driveability in Section 3.3, it is important to understand the mechanical interaction and nature of the whole vibrator, sheet pile and soil system, the magnitude and direction of the dynamic forces acting on the system and their principal variation in time, together with the penetrative behaviour of a vibratory-driven sheet pile.

The kinematic nature of the whole vibrator, sheet pile and soil system described here is based on the following assumptions and simplification:

- the bias weight is assumed not to vibrate,
- the sheet pile is simplified to behave as a rigid body (i.e. the head and toe have the same acceleration),
- the sheet pile and excitor-block of the vibrator are also simplified into a rigid body having the same acceleration as the sheet-pile head, and
- there are no soil and sheet-pile phase differences.

As a consequence of the above assumptions and simplification, the kinematics of the whole system can be simplified and “idealised” according to Figure (3-2); where the time-dependent part of the driving force \( F_v(t) \) is correlated with the time-dependent motion \( z(t) \), and the resisting forces \( R_s \) and \( R_t \) are correlated with the motion \( z(t) \) according to Figure 3-1.

The dynamic forces being referred to, act on the “idealised” vibrator, sheet pile and soil system mentioned above as illustrated in Figure (3-1), and can be obtained by establishing a dynamic equilibrium during the downward and upward stroke of the penetration motion.
Figure 3.1  The major components of the vibrator, sheet pile and soil system, illustrating the free-body diagram of the direction of the forces acting on the system during downward and upward stroke motion.
3.2.2 Penetrative motion of the system

There are few published results relating to penetration motion \(v_p z(t)\) or \(u(t)\) in the field, though several laboratory tests exist (see Section 2.4). From these published results it can be concluded that the penetration motion is best described by a downwardly-directed sinusoidal motion, in accordance with Figure (3-2)b, and the following expression:

\[
z(t) = tv_p + z_0 \cos(\omega t)
\]

where \(z(t)\) = global penetration motion of the vibrator-pile system [mm],
\(v_p\) = global penetration velocity of the vibrator-pile system [mm/s],
\(z_0\) = amplitude of harmonic penetration motion [mm],
\(\omega\) = angular frequency [rad/s], and
\(t\) = time [s].

As a consequence of the assumptions stated in Section 3.2.1 and from field tests, it can be assumed that both the surcharge mass of the system and the sheet pile, experience the same global penetrative speed \(v_p\) mm/s. In theory, the magnitude of the penetration speed \(v_p\) in Equation (3.1) corresponds to the dynamic equilibrium of the forces acting on the sheet pile, as graphed in Figure (3-1).

3.2.3 Driving forces

The forces normally referred to that drive the sheet pile (see Section 2.6), consist of the sum of the surcharge force \(F_o\), the unbalanced force \(F_v\) and the inertial force \((m_p + m_c)a\), in other words the product of the mass of the vibrator body times it’s maximum deceleration at the bottom of the downstroke, as illustrated by Figure (3-1).

The mechanical action (driving force) of a vibrator sitting on top of a driven sheet pile is normally derived from a simplified single-degree-of-freedom system (SDFS), but the ‘surcharge mass-spring-vibrator’ system is actually a two-degree system, consisting of two masses (the suppressor housing \(m_o\) and the excitor block \((m_p + m_c)\)), interconnected via the isolating elastomers. However the SDFS simplification is justified because the spring (elastomers) separating the two masses makes the vibratory motion of the excitor block essentially independent of the magnitude of the surcharge mass (see Appendix A). Here the dynamic part \(F_v\) together with the weights of the moving masses \((m_p\) and \(m_c\)) are derived in accordance with Figure (3-2)a, and mathematically by Equation (A.7).
This SDFS simplification has during favourable conditions been confirmed to represent the general case, based on results from full-scale field tests by Bosscher et al. (1998).

The influences of driving forces on vibro-driveability are discussed in the following subsections of this chapter.

### 3.2.4 Resisting forces

The forces normally referred to resisting the motion of the sheet pile (see Section 2.6), consist of the sum of the dynamic soil resistance at the toe \( (R_t) \) and along the shaft \( (R_s) \), together with the interlock friction developed in the sheet-pile clutch \( (R_c) \), as illustrated by Figure (3-1).

The influences of the driving forces on vibro-driveability are discussed in subsequent subsections of this chapter.

#### Dynamic shaft resistance

The few publications or plottings found describing the relationship of dynamic load transfer curves of the toe resistance, \( (R_s-z) \), are those by O’Neill and Vipulanandan (1989), Viking (1998) and De Qock (1998); all of which relate to slow vibratory-driving of model piles.

It can be assumed from these publications (and with a great deal of simplification) that the constitutive relationship \( R_s-z \) varies symmetrically between its positive loading amplitude value and its negative unloading amplitude value \( (\pm R_s_{\text{max}}) \), as illustrated by Figure (3-2)c. However, where there is a higher unloading stiffness \( (k_u) \) than loading stiffness \( (k_l) \), this could be explained by hysteresis resulting from the induced shear strains of the individual soil grains, which is largely irreversible.

From the illustration of the principles behind the constitutive relationship \( R_s-z \) in Figure (3-2)c, it is assumed that the amplitude values of the dynamic soil resistance \( (\pm R_s_{\text{max}}) \) during the steady state are reached as soon as the sheet-pile shaft starts to slide against the surrounding soil. Sliding of the shaft relative to the surrounding soil volume starts somewhere between the time points \( t_1 \) and \( t_3 \), during the downward part of the motion \( (z(t)) \). Since the sheet-pile motion changes direction at the point in time denoted by \( t_3 \), it might be reasonable to consider that the shaft once more slides relative to the surrounding soil somewhere between the time points denoted by \( t_3 \) and \( t_5 \), which correlate with the upward part of the cyclic motion of the pile.
Discussion regarding primary mechanisms and simulation of vibro-driveability.

Figure 3.2 Schematic description of the relationship between driving force, penetrative motion, shaft and toe resistance.

- $F$: unbalanced force amplitude, kN
- $F'$: static surcharge force, kN
- $F_d$: theoretical driving capacity, kN
- $z_n$: penetrative motion in time, mm
- $\Delta z$: permanent set, mm
- $v$: global rate of penetration, mm/s
- $M_e$: maximum eccentric moment, kNm
- $\omega$: angular frequency, rad/s
- $m_{dyn}$: dynamic mass, kg
- $R_s$: dynamic shaft resistance, kN
- $R_t$: dynamic toe resistance, kN

Amplitude value of dynamic shaft resistance during loading and unloading

Amplitude value of dynamic toe resistance during loading
Chapter 3

Sheet-pile interlock resistance

It is well known amongst practitioners that the dynamic friction force generated in the sheet-pile interlock \( (R_c) \) is of great significance. It is reasonable to assume that the magnitude of \( (R_c) \) quite frequently overshadows the sum of both the dynamic toe and shaft amplitude resistances (why and how is discussed in Section 3.3.3).

The direction and variation of the interlock friction force \( (R_c) \) generated should intuitively be correlated with the upward and downward penetration motion \( (z(t)) \), in accordance with the motion-dependent shaft resistance \( (R_s) \).

The amplitude of the interlock resistance \( (R_c) \) has been found to be influenced by factors such as the size and amount of soil grains that can sometimes be found jammed into the interlock (Vanden Berghe et al. 2001). Results from Vanden Berghe and co-worker’s laboratory tests on full-sized sheet-pile interlocks, showed that the quasi static amplitude values of \( R_c \) varied between 2-20 kN/m.

Dynamic toe resistance

The few published reports and plottings of the relationship of dynamic load transfer curves of the toe resistance, \( (R_t-z) \), relationship that do exist include O’Neill and Vipulanandan (1989), Dierssen (1994), and De Qock (1998); all of which relate to slow vibratory-driving of displacement piles. Dierssen (1994) however also presented one graph of \( R_t-z \) relating to fast vibratory-driving.

The general pattern of the few slow \( R_t-z \) curves reported, is a concave, upward loading-unloading pattern (as seen in Figure (3-2)d), indicating the development of strain hardening. The peak value \( R_{t,max} \) is generally reached during the lower end of the downwardly-directed penetrative phase, and the magnitude is proportional to for example the grain characteristics, the geometry of the pile toe, the initial relationship between stress state versus void ratio, the acceleration ratio, and the induced excess pore pressure according to Section 3.3.4.

Another observation from the few slow \( R_t-z \) curves published, is that they never tend to reach the typical plateau seen in classic elastoplastic patterns (similar to the Smith soil model of impact loading of the toe). These patterns might be explained by the fact that the downward displacement achieved never allows \( R_t \) to develop a completely plastic pattern before the toe is again forced upwards.

It has been hypothesised that slow \( R_t-z \) curves of vibratory-driven, low-displacement piles (sheet piles and H-beams) ought to display an \( R_t-z \) curve similar to Figure (3-2)d, but with a less developed, concave-shaped, upward loading and unloading curve.
This is explained by having a different penetration mechanism (low-displacement pile penetration) which is probably more related to plunging than displacement.

3.3 **A more detailed discussion on primary factors influence on vibro-driveability**

3.3.1 **Section introduction**

As stated earlier, it is evident that the optimum vibro-driveability is affected by each part of the vibrator, sheet pile and soil system. The influence of each part and its related parameters are dealt with in Section 3.3.2 to 3.3.4

3.3.2 **Primary vibrator-related factors affecting vibro-driveability**

The most important mechanisms relating to the vibrator part of the system are shown in Figure (3-1). From the tests conducted, together with previously published research results (see Section 2.5.1), the four most important factors have been found to be:

- the ratio of surcharge to unbalanced forces \( (F_o/F_v) \),
- the ratio of the actual to theoretical driving capacities \( (F'_d/F_d) \),
- the choice of driving frequency \( (f_d) \), and
- the choice of the vibrator-equipment.

**Choosing the optimum ratio of surcharge to unbalanced forces**

There appear to be an optimum value for the ratio of the surcharge force \( (F_o) \) and the unbalanced force \( (F_v) \), given by \( F_o/F_v = \frac{1}{2} \) (see Section 2.5.1). No explanation has been presented as to why this is so, nor whether the optimum value \( F_o/F_v = \frac{1}{2} \) can be applied generally.

The optimum value of the ratio of surcharge to unbalanced forces can be phenomenologically explained by the following view of the problem. An increasing value of \( F_o/F_v \) up to \( \frac{1}{2} \), keeping all the other parameters constant, also increases the penetration speed \( (v_p) \). As long as the ratio falls within the range \( 0< F_o/F_v = \frac{1}{2} \), then the cyclic displacement motion will display an equal amplitude \( (\zeta) \) in the upward and downward part of the penetration motion (see Figure (3-3)b). However a situation where \( F_o/F_v > \frac{1}{2} \), will lead to a state referred to as fast vibratory-driving, which is explained by the fact that the static part \( (F_o) \) of the driving force \( (F'_d) \) starts to exceed the upwardly-directed half of the
unbalanced force amplitude \( (F_p) \), (see Figure (3-2)c). This in turn changes the penetration motion to a process with a lower value of the upward part (\( z_{ou} \)) in relation to the downward part of the penetrative motion (\( z_{od} \)), illustrated in Figure (3-3)d.

It is the author’s understanding (however without proof), that the two different penetration functions illustrated in Figures (3-3)b-d are influenced by the vibratory-equipment machine settings, which in turn produce the two characteristic dynamic load transfer curves of the pile-toe curves (\( R_t-z \)), termed ‘fast’ and ‘slow’ vibratory-driving.

\[
F_o \approx \frac{1}{2} F_v
\]

\[
F_o + F_v = F_d \quad [\text{kN}]
\]

\[
T = \frac{1}{f}
\]

\[
\Delta z
\]

\[
F_o \approx \frac{1}{2} F_v
\]

\[
F_o + F_v = F_d \quad [\text{kN}]
\]

\[
T = \frac{1}{f}
\]

\[
\Delta z
\]

Figure 3-3  Different forms of the penetrative motion, depending on the choice of \( F_o/F_v \) ratio.

**Ratio of actual to theoretical driving forces**

From the literature study (see Section 2.5.1), it appears as though the efficiency of the vibrator-equipment significantly affects the optimum vibro-driveability. Moulai-Khatir et al. (1994) have stated that in field conditions, the efficiency can be estimated to be in the range \( 0.20 < \xi < 0.25 \). However, it should be noted that no other explanation than energy losses has been presented as to why this falls in such an extremely low range, or in which field tests this range is based.
The efficiency factor ($\xi = \frac{F_d'}{F_d}$) in this study has been defined as the ratio between the amplitude of the actual force delivered ($F_d'$) over the theoretical driving force ($F_d$). Where the theoretical force is calculated according to the vibrator specifications and operating range which is expressed by Equation (A.7), and the magnitude of the actual delivered (read measured) driving force delivered is taken from the readings of the strain gauges mounted near the sheet-pile head (see Section 4.3.2).

At this point, it must be noted that the actual peak force delivered to the sheet-pile head will always be less than the theoretical peak driving force, due to the simple fact that non-negligible energy losses exist in each part of the vibrator equipment, and also due to the flexural and torsional energy propagation in the relatively slender sheet-pile profile, which is not sensed by transducers that measure only axially-applied loads. However until further research is conducted into energy losses, accurate values of the loss term ($\xi$) cannot be applied to the computation of the axial driving force that actually drives the sheet pile into the ground.

**Choice of driving frequency**

The few published studies about how the choice of driving frequency relates to the highest driving speed ($v_p$), all relate to laboratory tests (see Section 2.5.1). No systematic field-study exists that relates the choice of driving frequency to the magnitude of the dynamic soil resistance developed ($R_t$ and $R_s$).

From the literature survey, it appears as though dynamic friction (read as dynamic shaft resistance or $R_s$) seems to be virtually independent of the driving frequency chosen ($f_d$). Instead, the dynamic toe resistance ($R_t$) tends to be strongly related to the choice of driving frequency, especially in the case of vibratory-driven full-displacement piles.

Even though laboratory tests conducted by Heerema (1979) clearly indicate that $R_s$ is independent of driving frequency, it should be noted that these tests did not duplicate the actual field conditions, since the inertial force affecting the sand grains was not present during the laboratory test. However, these considerations would most likely not change the fact that the dynamic shaft resistance ($R_s$) is virtually independent of the driving frequency chosen. The key mechanisms behind the shaft resistance developed are not significantly related to frequency. Instead, it is hypothesised that the magnitude of the $R_s$ developed is directly related to another mechanism taking place in the soil surrounding the vibrating sheet-pile shaft. It is the magnitude of the acceleration to which the sheet pile is vibrated that introduces inertial forces to the individual soil grains that surround the shaft. Further discussion covering why and how this is so is found in Section 3.3.4.
Even though the published results appear to link $R_t$ to $f_d$, it should be noted that field-related studies of this interesting mechanism have not been performed in great detail. Choice of driving frequency seems to be correlated to the dynamic toe resistance ($R_t$), especially in the case of vibratory-driven full-displacement piles (see Section 2.5.1). It reasonable from the results presented, to correlate driving frequency to the appearance of the two vibratory-driving states called ‘fast’ and ‘slow’ vibratory-driving. This is explained by the fact that the vibrating volume of soil beneath the pile toe most likely moves out of phase relative to the pile. However, it is doubtful whether the ‘fast’ and ‘slow’ vibratory-driving phenomenon also accounts for results in low-displacement piles, which could be explained by a different penetration mechanism, for example related to toe-plunging instead of soil displacement.

**Choice of vibratory equipment**

The two main types of modern vibratory-driving systems commercially available are: (i) free-hanging and (ii) leader-mounted vibratory-systems (see Section 2.3.1). In Sweden, there are no traditional preferences with respect to choosing one or the other system. However, lately the leader-mounted systems seem to have become used more commonly.

The major drawback using a free-hanging vibratory-system is the difficulty encountered when positioning the sheet-pile profile intended to be driven. Why and how these difficulties can give rise to undesirable laterally-induced motion in the sheet pile is discussed in detail in the next section.

**3.3.3 Primary sheet-pile-related factors affecting vibro-driveability**

The aim in the following subsections is to position the author’s overall view on the key factors related to the behaviour of a vibratory-driven sheet pile. It should be noted that the following subsections partly discusses and later on verifies the here hypothesised key factors of the sheet pile that affects the vibro-driveability.

The most important factors of the sheet-pile-profile part of the overall system (shown in Figure (3-1)) that significantly influence vibro-driveability, not only include geometric and mechanical properties of the chosen profile (as previously described in Section 2.5.2), but also the three factors shown below, which have been indicated in the results of the tests conducted, as well as in the limited amount of published field and laboratory research data available:

- the assumption of axial rigidity,
• the flexibility during driving, and
• the friction force in the sheet-pile interlock.

These three factors are elaborated on in the following subsections.

The axial rigidity assumption

One of the uncertainties that exists when applying vibro-driveability formulae, especially those derived from laboratory tests, is whether or not the assumption of pile rigidity is a justifiable, and especially whether these formulae can also be applied to long sheet-piles in the field. One of the primary objectives of this section of the chapter is to discuss why and when it can be appropriate to treat and model a full-scale pile or sheet pile as a rigid body during vibratory driving. In other words, to theoretically define when a vibratory-driven sheet pile can be assumed to behave as a rigid body, defining when there will be no significant longitudinal vibrations having any engineering impact on the vibro-driveability.

Viking and Bodare (1998) used the following “rule of thumb” when defining a system with a vibrator and model pile of length \( L \) behaving as a rigid body:

\[
\frac{1}{4f_d} = \frac{T}{4} \geq 2t_n
\]

where \( T = f_d^{-1} \) the time period of the unbalanced force \( F_v \), and
\( t_n = 2L/c_b \) the time taken for a stress wave to travel back and forth.

This can be explained phenomenologically by the following. A quarter of the period of time \( (T/4) \) for the driving frequency chosen \( (f_d) \), should be equal to or greater than twice the time it takes a stress wave to travel back and forth \( (t_n) \) along the length \( (L) \) of the sheet pile chosen. It could be assumed that a vibratory-driven sheet pile tends to behave like a rigid body when the time it takes for the driving force to change from zero to maximum \( (T/4) \) is greater than or equal to twice the time \( (t_n) \) it takes for a tentative stress wave to travel the distance \( (4L) \) along the sheet pile, having a bar velocity of \( (c_b) \), (see Figure (3-4)).

Only a few of the previously published reports (see Section 2.5.2) present how the resonance frequency of a vibratory-driven pile or sheet pile can be estimated, and seldom can the underlying assumptions and justifiable considerations behind the equations be found.

Nevertheless, analysis of the forced longitudinal vibrations of vibratory-driven sheet piles can be considered to closely follow the conditions of a free and longitudinally-
vibrating finite rod of length \( L \), constrained at the longitudinal position \( x = 0 \) by the vibrating force produced by the vibrator, and free at its other end \( x = L \). The resonance frequency of a vibratory-driven sheet pile, can be estimated by Equation (3.3), and the assumptions and considerations behind Equation (3.3) are found in Appendix B.

\[
f_n = \frac{n c_b}{2L}
\]

where \( n = \) vibrating modes \( 0, 1, 2,... \), \( n = 1 \) is the half wave mode \([-]\),
\( c_b = \) bar velocity [m/s], and
\( L = \) length of the sheet pile under consideration [m].

In order to illustrate this, assume that a vibratory-driven sheet pile is 18 m in length, with a \( c_b \) for steel of \( \sim 5100 \) m/s. The natural resonance frequency of the vibrating

![Illustration of the relationship for rigid-body behaviour expressed by Equation (3.2).](image-url)
sheet-pile can be estimated by Equation (3.3) as follows:

\[ f_n = \frac{n \times 5100}{2 \times 18} \]  \hspace{1cm} (3.4)

where the first mode \((n = 1)\) gives rise to a resonance frequency of \(f_1 \approx 142\) Hz.

Longitudinal resonance phenomena seem to generally occur at very high frequencies compared with the driving frequencies \((f_d)\) used by today’s modern vibratory-equipment (usually in the range of \(30 < f_d < 40\) Hz). It is therefore justifiable from an engineering point of view, to see a vibratory-driven sheet pile as behaving like a rigid body. Such an assumption would greatly simplify the problems associated with a single-degree of freedom vibrating system, where the inertial force of the vibrating masses and driving forces could be assumed to be essentially in phase with each other, providing that internal damping is also neglected.

**Effects of induced lateral vibrations on vibro-driveability**

Another important uncertainty that exists when applying vibro-driveability formulae to sheet piles, is whether or not neglecting the possible existence of dynamic lateral flexibility of vibratory-driven sheet piles is justifiable. One of the primary objectives with this section is to discuss why and when it is appropriate to consider the effects of dynamic lateral motions of vibratory-driven slender profiles. According to Holeyman (2000), these transversal and flexural mechanisms are generally ignored in vibro-driveability analysis, which usually confines itself to longitudinal behaviour.

In this section, situations in which these effects occur and should definitely be considered are defined, along with a theoretical quantification of these transversal effects. It is the author’s opinion that these lateral effects may at times have such a major engineering impact that they should be considered during pre-analysis of both vibro-driveability as well as the ground vibrations induced. It is also fairly unexpected that in the most often cited article in the context of induced ground vibrations (Clough et al. (1980)), that the existence of lateral effects is not even mentioned.

The non-negligible role of the induced lateral flexibility can be explained by looking at situations where the driving force \((F_d)\), is allowed to enter the driven profile at an eccentric distance \((e)\) from the neutral axis of the profile under consideration. A bending moment of \((M_d = eF_d)\) is induced at the head of the vibratory-driven profile, resulting in a lateral sinusoidal motion \((u)\) of the profile, which can cause a non-negligible amplitude. These laterally-induced motions can be illustrated by situations were U-shaped sheet piles are vibratory driven one at a time (see Figure (3-5)), using vibrators equipped
with a single clamping device that hold the profile at the web (see Figure (2-3)). In other words, induced lateral-flexibility is minimized simply by arranging things so that the driving force \( F_d \) produced by the vibratory equipment can enter the profile with a minimum eccentric distance \( e \) from the neutral axis of the profile under consideration.

The engineering consequences of neglecting the effects of lateral sinusoidal motions \( u_l \) could include a lower vibro-driveability as well as an increased amount of induced ground vibrations. Lower vibro-driveability can occur due to the fact that part of the driving force \( F_d \) is used to bend the profile during the installation phase, as a consequence of the eccentricity \( e \). Higher levels of induced ground vibrations can occur due to the fact that pre-installed sheet piles forming a sheet-pile wall, are coupled to the profile that is being vibratory-driven by the sheet-pile interlock. The previously installed sheet-pile wall, could be considered to act like the membrane of a loud speaker, sending out laterally-induced P waves, as discussed and visualised in Viking et al. (2000c).

![Figure 3-5 Illustration of laterally-induced vibratory motion of singularly-driven U-shaped profiles (after Viking et al., 2000b).](image)

Analysis of these dynamic lateral effects can be done with beam-column deflection theory, where the shape of curvature is defined by the axial compressive forces pro-
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Produced by the vibrator \((F_d)\) and the bending moment \((M_d = e F_d)\) induced at the sheet-pile head. \((F_d)\) and \((M_d)\) are balanced by the unknown support reactions; that is, the dynamic soil resistances \((R_s + R_p)\) and clutch friction if present, together with the moment reaction \((M_f)\) of the subsoil strata and lateral support \((V)\) of leader mast.

The shape of the curvature solution is very sensitive to the end restraints, which is the case when comparing a leader-mounted vibrator with a free-hanging vibrator system (see Section 2.3.1). A leader-mounted vibrator driving a U-shaped profile can be represented by a beam column with one fixed end, and one pinned, exhibiting lateral curvature like a vibratory-driven U-profile according to the schematic illustration to the left in Figure (3-6).

A free-hanging vibrator system is best represented by having one fixed end and one free end, with lateral curvature like a vibratory-driven U-profile according to the schematic illustration to the right in Figure (3-6). The bending moment \((M_d = e F_d)\) generates a curvature similar to the leader mounted system due to the same eccentricity \((e)\).

**Figure 3-6** Schematic of the lateral deflection curvature of an ideal, elastic U-shaped sheet pile with a leader-mounted vibratory-system to the left and a free hanging system to the right.
The lack of lateral support in the case of free hanging systems causes an increase of the lateral deflection curvature due to effects of the second order moment.

**The effects of friction force in sheet-pile interlock on vibro-driveability**

Dynamic interlock resistance \((R_c)\) is a sheet-pile-related factor, well known to have a tremendous impact on both the vibro-driveability and the ground vibrations generated. But again, amazingly few publications are available, with regard to the considerable impact of \(R_c\) and the number of kilometres of sheet piles driven each year (see Section 2.5.2).

One of the primary objectives of this section is to discuss why interlock resistance \((R_c)\) develops, as well as the effects of \(R_c\) on vibro-driveability, and the specific situations where the effects of \(R_c\) should definitely be considered. Another objective is to present the author’s view about possible steps that can be taken in order to minimise the negative effects of \(R_c\) during installation.

The primary factors relating to dynamic interlock resistance \((R_c)\) on vibro-driveability have, from the publications mentioned above, been found to consist of the following eight factors:

- friction in the interlocks,
- geometric shape and the size of the interlocks,
- manufacturing tolerance of the interlocks,
- impurities in the locks (such as corrosion and soil in reused piles),
- friction caused by the presence of soil grains in interlocks,
- the size of soil grains,
- saturation conditions of the soil, and finally
- the choice of installation method together with the mode of operation.

It should be noted that it is not currently possible to theoretically quantify the interlock friction force \((R_c)\) developed during installation, due to the complexity of the problem describing when, how, and to what extent the presence of soil grains in the interlock affects vibro-driveability and the ground vibrations induced. It is also challenging attempting to describe the whereabouts and the extent of the effects of obstacles such as boulders that may be present in the subsoil strata. These obstacles can cause damage to the leading interlock, drift from initial verticality, and sometimes clutch release. All of these are basic factors known to have significant impacts on the dynamic interlock friction force \((R_c)\).

From field-test-related results presented by Legrand et al. (1993) on effects of clutch friction \((R_c)\) on the induced ground vibrations, it was found that ground vibrations can be two to five times the magnitude of equivalent situations without \((R_c)\). It should
be noted that the conducted studies were of a purely research nature, featuring brand new sheet piles, and didn’t consider any production-capacity-related aspects, which concludes that the presence of \( R_c \) has in fact a no negligible impact on the ground vibrations generated.

Ferron (2001), being the most recent work on the subject, states that the dynamic interlock resistance \( R_c \) can be set to 1.0 kN/m, based on the results of experience, though this experience unspecified.

The geometrical shape and size of the interlock gaps, involves considering the somewhat opposing factors affecting the magnitude of the interlock resistance \( R_c \) or vibro-driveability (the strength of the interlock), and a close fit for the degree of watertightness. A close fit in this case equates to a small value for the interlock gap \( d_{il} \), (see Figure (3-7)), which minimises undesirable presence of grains in the interlock gap.

![Illustration of the interlock gap \( d_{il} \).](image)

However, it is not solely the geometrical shape and size of the interlock gaps through which grains can enter, that stipulates the magnitude of the interlock friction force that develops. The presence of impurities (such as corrosion) together with permanent deformations of brand new profiles (due to incorrect storage or handling) may also generate non-negligible unfavourable developments of \( R_c \), and even sometimes make it difficult or perhaps even impossible to interlock the sheet-pile profiles. These two non-desirable situations occur quite frequently with re-used sheet piles from temporary sites. Therefore it is strongly advised to use only new or undamaged profiles, or profiles with interlocks that are in very good condition, within urban and environmentally-sensitive areas. The consequences of neglecting these two factors (impurities and deformations) in relation to the magnitude of the interlock friction force developed, might not only include reduced productivity. In the worst case scenario, this might result in severe damage to nearby structures due to considerable settlement as a consequence of the increased ground vibration.
Chapter 3

The effects of the ($R_v$) developed, in relation to the choice of the vibratory-equipment, as well as the mode of operation, is another factor that ought to be considered. Irrespective of the choice between the two types of vibratory equipment, it is of great importance that care and attention be paid to ensure that the sheet piles are maintained in correct vertical (transversal & longitudinal) alignment with the pre-installed sheet wall during the whole vibratory-installation process (see Figure (3-8)). Correct vertical alignment is achieved by using efficient pile guides at least at two levels. In the case of leader-mounted vibratory-drivers, the leader mast can be seen to work simultaneously as a carrier for the vibrator and as the upper guide for the profile being driven. However, in the case of wire-suspended vibrators, it is unsatisfactory to drive sheet piles without fixed guides at the upper end. Problems may be encountered during driving (especially with long profiles) if the upper guide is neglected, for example drift from the desired vertical alignment can occur, causing dynamic flexing of the profile (see Figure (3-8)), which in the long run might cause the release of the clutches.

\[ F_d \]

Transversal alignment

Longitudinal alignment

Next profile to be installed

Driving direction

Leading interlocks

Pre installed sheet pile wall

Figure 3-8 Illustration of the difficulties involved maintaining a continuously-correct alignment of both the transversal and longitudinal position of the sheet pile during installation.
These problems are basically related to the practical difficulties encountered trying to maintain continuous correct driving-alignment with only the rope or wire holding the vibrator.

In all civil engineering projects there is a constant need to minimise the costs of the work. A consequence of this in relation to sheet piling, is the development and use of wider and deeper profiles, since fewer profiles are required to achieve the same given length. However the development of wider and deeper sections also means greater surface and sectional area, and therefore demands an equivalent increase in driving capacity ($F_d$) together with improved possibilities for making continuous adjustments and monitoring vertical alignment. However, the geometric effects of the sheet-pile profiles on vibro-driveability are not the issue in this case. Of even greater concern here is developing engineering awareness about the importance of the interacting phenomena involved in balancing the development of wider and deeper profiles versus the need to achieve greater driving capacity ($F_d$) together with the need for better control of the alignment. A lack of understanding of these interacting phenomena might immediately result in a significant increase in the interlock friction resistance ($R_c$), which in the long run could result in even more damage to nearby structures due to increased ground-vibration levels.

3.3.4 Primary subsoil-related factors affecting vibro-driveability

The aim in the following subsections is to position the author’s overall view on the key factors and fundamental mechanisms of the soil behaviour in the vicinity of a vibratory-driven sheet pile. It should be noted that the following subsections do not represent the final solution as to how the intrinsic soil behaviour should be addressed. This is due to the complexity of the problem, and the fact that the soil behaviour as it is hypothesised later on, has not been scientifically verified due to a lack of appropriate soil instrumentation during the field tests.

The primary subsoil-related factors, representing the third part of the overall system illustrated by Figure (3-1), which relate to the favourable reduction of soil shear strength (penetrative resistance) are most likely influenced by the following:

- acceleration of soil grains (inertial forces),
- large cyclic strain amplitudes,
- initial void ratio in relation to stress state,
- saturation conditions, and
- pore-pressure build up.
Chapter 3

Stresses within a subsoil mass

It was considered justifiable by the author to simply idealise the analysis of soil behaviour into studies on the interaction of individual soil particles for more easily pointing out and illustrating the author’s views about the key phenomena and kinematics behind the shear strength reduction.

The resistance of cohesionless soils to deformation is strongly related to the shear resistance at the points of contact between the individual soil particles. However, the interlocking of soil particles is also a very important contributing mechanism behind the resistance developed, and is significantly influenced by the packing of the individual grains. Knowledge about the factors that influence the shear strength is the key to forming a better understanding of soil behaviour under loading related to vibratory-driving and the magnitude of this reduced shear resistance.

An idealised granular soil volume can be described as a regular array of individual granular particles characterised by its dry weight ($\rho$), the relative density ($D_r$), void ratio ($\varepsilon$) and degree of saturation ($S_r$). The stresses developed within the idealised granular soil volume, either by external loads at the boundary or by the weight of the soil, generate intergranular normal ($N$) and tangential ($T$) contact forces between the individual soil grains, as illustrated by Figure (3-9).

The whole purpose of using the vibratory technique to drive profiles into cohesionless soils is to introduce external loads to the regular array of individual granular particles (see Figure (3-9)) for putting the soil volume into a state were it loses shear strength. Bernhard (1967) refers to this state as soil fluidisation, which can be defined as the change of an air-dry cohesionless soil from its initial state into a state were quasi-fluid characteristics predominate.

Several authors have implied that the shear-strength reduction induced is primarily related to the soil mechanism termed liquefaction. An explanation for this is that during the time it takes for a sand grain to move into its new position, it is temporarily and partially supported by the pore water; which means that the external loads are transmitted to the fluid and thereby change the initial pore-water pressure ($u$). However, the shear strength reduction is also present under artificial conditions in air-dried sand in laboratory tests performed by Barkan (1962), Bernhard (1967), O’Neill and Vipulanandan (1989a), and Viking (1998). From this it can be concluded that soil liquefaction is not the primary parameter for the induced shear-strength reduction in a vibratory-technique context.
Discussion regarding primary mechanisms and simulation of vibro-driveability

Hypothesised fundamentals behind the shear strength reduction

As the vibratory-driven sheet pile undergoes an essentially purely axial-vibratory motion, with a longitudinal acceleration amplitude \(a\) in the range of 10-20g, it interacts with the soil volume in the vicinity of the profile. The interaction between the sheet pile and the soil introduces inertial forces to the soil. These inertial forces cause a dynamic motion of the individual grains with the same frequency as the driving force, as illustrated by Figure (3-10).

When the peak acceleration of the soil volume exceeds \(\sim 1\)g (approximating gravity), the mechanical events taking place in the subsoil strata start to become very complicated to describe in detail. However for the sake of simplicity, at the point when the peak acceleration of an idealised granular soil volume located beside the sheet-pile shaft, exceeds a site-specific threshold value corresponding to the initial vertical confining stress \((\sigma_v')\), then the vertical confining stress \((\sigma_v)\) within the soil drops to nearly zero.

Figure 3-9  A cubically-packed assemblage of soil grains illustrating how effective confining and shear stresses \((\sigma'_{v,h} \text{ and } \tau'_{v,h})\) respectively are carried by the static inter-granular contact forces (N and T).

\(\sigma'_{v,h}\) : effective confining stresses
\(\tau'_{v,h}\) : effective shear stresses
N : inter-granular contact force
\(\tau\) : inter-granular shear force
Since granular soils cannot sustain tension, the grains are unable to follow the subsequent cyclic motion of the vibratory-driven profile and start to experience “free fall” until the individual grains interact again later in the loading cycle. During the free-fall phase of each steady-state loading cycle, the individual grains become separated from one another and are hence free to constantly move into a new position.

The almost total absence of vertical confining stress ($\sigma_v$) during parts of each steady-state loading of the soil volume (short-time drops in the inter-granular contact forces $N$ and $T$), appears to be the key phenomenon behind the shear-strength reduction during vibratory-installation of piles, and is illustrated in Figure (3-11).

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**Figure 3-10** Cubically-packed assemblage of soil grains subject to a cyclic confining pressure and subsequent varied shear stress due to introduced inertial forces, which drastically reduce both the number of contact points and the magnitude of individual particle contact forces $N$ and $T$. 

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The minimum soil-acceleration amplitude induced that is needed to force the individual soil grains to vibrate, and large enough to break most of the inter-granular contact forces, has been found to be in the range of 1.0-1.5 g; a range termed the threshold value (see Section 2.5.3). It should be noted that both an increase in relative density and confining pressure (increasing depth \( z \)) increases the level of acceleration required to produce a void ratio or a volume change in the soil volume, or both. It is reasonable to assume that the vibration amplitude of the individual soil grains decreases theoretically with both increasing penetration depth (\( z \)), due to increasing confining pressure, and with increasing radial distance (\( r \)) from the profile due to the increase in area and material damping (see Figure (3-10)). It should be noted that Figure (3-10) illustrates a situation were pile rigidity could be assumed; in other words where the acceleration of the sheet-pile head and toe vibrate with the same amplitudes \( a_h \sim a_d \).

Even though it appears as though the cyclic acceleration induced motion of the individual soil grains is the key mechanism behind the shear strength reduction, it is of course evident that induced excess pore-pressure is undoubtedly of great assistance in reducing the shear strength during the installation process in field conditions. It seems reasonable to correlate the large amplitude of cyclic accelerations (cyclic strains) that generate large volume changes (\( \Delta e \)) of a cyclic nature, which in turn produce a cyclic variation in the induced excess pore-pressure (\( \Delta u \)). As the cyclic acceleration amplitude (cyclic strain) in the soil is assumed to decrease with the radial distance (\( r \)), (see Figure (3-10)), it is most likely that the amplitude of the cyclic variation of pore-pressure changes (\( \Delta u \)) will decrease in the same manner, with increasing radial distance.

The pioneering concept of Casagrande (1936), which later resulted in one of the cornerstones of soil mechanics, termed critical-state soil mechanics (CSSM), was further developed by Castro and Polus (1977), who related the steady state of deformation to describe how the soil locus of the initial point (state) described by the void ratio (\( e \)) and the effective confining pressure (\( \sigma' \)), in the steady state of deformations, principally approaches the steady-state line (SSL). Using this concept, it is possible to both explain and visualise the primary mechanisms behind the shear-strength reduction of cohesionless soils subjected to the steady-state load situations ascribed to vibratory driving. This is done by applying the most general form of the SSL, which can be viewed as a three-dimensional curve in either the \( e-\sigma'-\tau \) or the \( e-\rho'-q \) space. Where the SSL can be considered to mark the boundary between the contractive and dilative behaviour, it is also the boundary between the initial states that might or might not lead to the induction of flow liquefaction.
The SSL schematically visualised in Figure (3-11) represents a two-dimensional projection of the three-dimensional SSL onto the $e - \sigma'_c$ plane with constant $\tau$. The SSL therefore describes the principal state toward which any soil specimen would migrate irrespective of the position of the initial point and volume changes ($\Delta e$) under drained conditions; and the changes in the effective confining stress ($\sigma'_c$) under drained conditions, or some other combination under partially-drained conditions.

The shear-strength reduction of an idealised regular array of individual granular particles, from an initial strength ($\tau$) to the steady-state strength ($\tau_{su}$) within the framework of SSL, results from the induced volumetric strains and reduced effective confining pressure ($\sigma'_c$). The phenomenon of shear-strength reduction is illustrated in Figure (3-11), where it is initially assumed that the idealised soil element is in a drained equilibrium situation, either in a loose A or dense B state. The cubically-packed assemblage of soil grains undergoes the following three principal states between the initial undisturbed state, to the disturbed post state after finishing the vibratory sheet pile installation process.

1. **The initial undisturbed state:** the idealised soil element is in a drained equilibrium situation, either in a loose A or dense B state depending on the initial relation between void ratio and confining pressure.

2. **The steady-inertial shearing state:** the vibratory installation process generates an extensive mobilisation of the individual soil grains due to high acceleration levels which introduces inertial forces to the soil grains. The idealised soil element is cyclical sheared in partially undrained conditions which produces a hydraulic gradient in the soil element that drives the pore water out of the voids during the active vibratory driving state, causing a continuously cyclic varied excess pore-pressure, and thereby reducing the effective confining pressure ($\sigma'_c$) to either point $A'$ or $B'$ depending on the initial state.

3. **The final disturbed state:** As the vibratory-driving process is stopped (leaves the steady-inertial shearing state), the volume (void ratio) is decreased as the individual grains leave the free-fall state and the excess pore water dissipates, and falls into their disturbed static post position (state). The effective confining pressure ($\sigma'_c$) therefore slightly increases when the grains in the idealised soil element reaches their new final state, $A''$ or $B''$ respectively.
Figure 3-11 Illustration of the process of how initially loose or dense soil specimens in vibratory driving of sheet piles might change the initial stress state of the soil.

3.4 Vibro-driveability simulations applied in the discussed Vibdrive and Vipere models

3.4.1 Section introduction

Difficulties in applying vibro-driveability formulae arise from the trade-off that has to be made between the accuracy of each candidate formula (justified assumptions of fundamental mechanisms) and the practicability of applying it to real problems. With today’s knowledge, this means that the key issue is to access and implement the prime factors of each part of the system (the vibrator, the sheet pile, and subsoil strata).

In the past, a number of notable attempts have been made to develop models and formulae for predicting the penetration speed ($v_p$) as well as the dynamic toe-penetration resistance ($R_t$) and the shaft penetration resistance ($R_s$) using the vibratory technique to install sheet piles (see Section 2.6). However, few of the existing driveability models have explicitly described how the prime factors assumed to affect the penetration process have been incorporated. Incorporating the cyclic mobility of the individual soil particles in combination with partial liquefaction seems to present difficulties (see Section 3.3.4).

At first glance the Vibdrive and Vipere models appear to be the forerunners of tomorrow’s tools for simulating the vibro-driveability of sheet piles. This is mainly due to the fact that these two models explicitly incorporate the two prime soil-related factors...
that affect the shear strength reduction of cohesionless soils during vibro-installation of sheet piles which are: (i) shear strength reduction of the soil due to the high acceleration of the sheet pile (induced cyclic mobility of the individual soil particles due to inertial effects) and (ii) shear strength reduction due to induced partial liquefaction (the cyclic induced excess pore pressure due to cyclic void ratio changes).

This section presents and discusses two typical simulation results of vibro-driveability simulations applied in the Vibdrive and the Vipere models, which has previously been extensively described in Section 2.6.6. Within this context, the purpose of this section is to describe the simulation procedures of the two models in more detail, by quantifying the input parameters needed by the two models, and presenting two typical simulation results generated by the two models. This is done in order to discuss parts of the results in relation to the assumptions of the models, together with the hypothesised factors affecting the vibro-driveability as discussed in Section 3.3.

It should be noted that input parameters applied here, described in Table (3-1), neither correlate with the Vårby field study, nor take into account the effects of friction forces in the sheet pile interlock.

It should also be noted that the simulation results presented and discussed relating to the Vibdrive model have been produced by the author, however the simulation results related to the Vipere model have been derived by Vanden Berghe (2001). The reason for this is that the Vipere model has been recently developed, and the complexity of running the Vipere model at this current stage.

### 3.4.2 Input parameters for simulations

The parameters needed as input in the two models consist of the following four parts; vibrator, pile-profile, soil, and integration parameters (see Table (3-1)).

The two initial parts, the vibrator and the pile related input parameters are the easiest to access since they are usually specified by the manufacturers. The two final parts, the soil and the integration-related input parameters are more difficult to access since they are related to the constitutive assumptions of each soil model and the integration procedures applied.
### Table 3.1 Input parameters for the Vibdrive and Vipere models for simulating vibro-driveability.

<table>
<thead>
<tr>
<th>Type of parameter</th>
<th>Vibdrive model</th>
<th>Vipere model</th>
</tr>
</thead>
</table>
| **Vibrator parameters** | Unbalanced moment $M_e = 46.0$ kgm  
Driving frequency $f_d = 33$ Hz  
Dynamic mass of vibrator $m_v = 6700$ kg  
Stationary mass of vibrator $m_o = 3500$ kg |                                                                              |
| **Pile parameters** | Circular shaped displacement pile  
Section area $A_t = 167$ cm$^2$  
Pile perimeter $\chi = 288$ m  
Penetration depth $\zeta = 10$ m  
Pile length $L_p = 12$ m | Mass of pile $m_p = \rho A_t L_p \approx 1600$ kg |
| **Soil parameters** | Dynamic toe resistance pre set to $R_t = 100$ kN  
Dynamic shaft resistance pre set to $R_s = 400$ kN | Friction angle $\phi' = 30^\circ$  
Initial void ratio $e = 0.60$  
Critical void ratio $e_{cr} = 0.88$  
Minimum void ratio $e_{min} = 0.52$  
Strength $b_s = 200$ MPa  
Dimensionless constant $n = 0.35$  
Dimensionless constant $\alpha = 0.25$  
Dimensionless constant $\beta = 1.10$  
Dimensionless constant $\lambda = 1.38$ |
| **Integration parameters** | Initial penetration speed $v_{ini} = 76$ cm/s | Maximum radial discretization $R_{max} = 100$ m  
Number of soil elements $= 100$  
Time parameter $\Delta t = 15.10^{-6}$ s |

### 3.4.3 Evaluation of the driving forces

Both the Vibdrive and Vipere models evaluate the driving force according to the same general expression (Equation (2.9) and Figure (2-7)). The peak amplitude value of the driving force according to specified parameters in Table (3-1) is given by $(gm_{tot} + M_e \omega^2 \sin(\omega t))$ in which $(m_{tot} = (m_o + m_v + m_p))$. The total mass $(m_{tot})$ of the vibrator sheet pile system is approximately 11800 kg, and the peak amplitude value evaluated to approximately $(9.81\times11800 + 46\times(66\pi)^2 = 116 + 1978 \approx 2100$ kN).

Figure (3-12)a shows the sinusoidal driving force simulated by the Vibdrive model in the time domain, biased around $gm_{tot}$, and Figure (3-13)a shows three cycles of the same driving force modelled by the Vipere model, both according to the same specified vibrator parameters in Table (3-1).
3.4.4 Evaluation of the dynamic soil resistance

The Vibdrive and Vipere models evaluate the soil resisting forces differently. The Vibdrive model evaluates the dynamic soil resisting forces, \( R_t \) and \( R_s \), on the basis of CPT test results, and Vipere on the basis of both tri-axial and simple direct shear test results of soil samples taken from the site where the sheet pile is intended to be vibratory driven.

The more elaborate model Vipere require results of laboratory tests as input parameters (previously described in Section 2.6.6) in order to evaluate the time dependent behaviour of the dynamic soil resistance forces, \( R_t \) and \( R_s \). Figures (3-13) and (3-14) presents typical results of a vibro-driveability simulation of the Vipere model, with input parameters according to Table (3-1). Vanden Berghe (2001) evaluated the peak values of the dynamic compressive stress and shear stress to \( q_d = 1257 \text{ kPa} \) and \( f_d = 110 \text{ kPa} \) respectively (see Figures (2-31)a and (2-33)a). The peak values of the dynamic soil penetrative resistance forces can then be evaluated to approximately \( R_t = A_t q_d = 167 \times 10^{-4} \times 1257 \approx 21 \text{ kN} \) and \( R_s = A_s \tau_d = 2.88 \times 10.5 \times 110 \approx 3400 \text{ kN} \). Figure (3-13)b presents the sum of the soil penetrative resisting forces within the time domain, \( R_t + R_s \approx 3421 \text{ kN} \).

It can be noted by a comparison of the simulated results of the Vipere model, that there is a phase shift of approximately 10 mS (120°) between the peak values of the driving force \( F_d \approx 2100 \text{ kN} \) in Figure (3-13)a, and the peak value of the sum of resisting forces \( R_t + R_s \approx 3421 \text{ kN} \) in Figure (3-13)b. Another thing to be noted, is that even though the peak value of \( R_t + R_s \) is approximately 60 per cent higher than the peak value of \( F_d \) it is possible to drive the pile due to the large phase shift between the peak values of the driving and resisting forces. Why and how this taken into account in the Vipere model is not explicitly described by Vanden Berghe (2001).

The simpler simulation model Vibdrive requires results of CPT-tests, in order to evaluate the peak values of the dynamic soil resistance forces, \( R_t \) and \( R_s \). Unfortunately, such results were not available for the Vibdrive model. Initially, it was believed by the author possible to correlate the choice of soil parameters for the Vibdrive model with the dynamic compressive stress and shear stresses evaluated for the Vipere model, \( q_d = 1257 \text{ kPa} \) and \( f_d = 110 \text{ kPa} \). However, it was immediately discovered that no such correlation exists between the two models in evaluating the dynamic soil parameters \( q_d \) and \( f_d \), further discussed in Section 3.4.6. When it was realised that no correlation of the soil characteristics related to the Vibdrive model was available, the soil parameters were then preset to \( R_t = A_t q_d = 100 \text{ kN} \) and \( R_s = A_s \tau_d = 400 \text{ kN} \) in order to develop a better understanding of the assumptions adopted behind the Vibdrive model.
3.4.5 Evaluation of the penetration speed

Figures (3-12)a to (3-12)d present the results of a vibro-driveability simulation by Vibdrive, with input parameters according to Table (3-1). The integration procedure represented by the four curves in Figure (3-12) have been performed in accordance with previously-described integration procedures (see Section 2.6.6). The average penetration speed \( \dot{v}_p \) is simulated to \( \sim 27 \text{ mm/s} \), which is evaluated on the basis of the permanent displacement of one cycle divided by the time period of the cycle \( \Delta n/f_d^{-1} = 0.81/33^{-1} \).

Figure (3-12)a shows the forces applied over time, the two resistance forces represented by the dynamic toe and shaft resistance are graphed as horizontal lines, together with one active force represented by the sinusoidally-unbalanced force \( M_0 \omega^2 \sin(\omega t) \) which is biased around the total weight of the system \( g m_{tot} \). Figure (3-12)b shows the simulated acceleration of the rigid vibrator-pile system according to Equation (2.34). The two Figures (3-12)c and (3-12)d show the integrated velocity and displacement curves. Note that negative values of velocity and displacement both correspond to a downward penetrative motion.

Starting from time 1, at the first iteration the sheet pile is assumed to be initially instable \( (\nu \neq 0) \) and the initial velocity \( v_{in} \) is set to \( \sim 76 \text{ cm/s} \).

At time 2, the resisting forces and driving force become equal, however the speed causes an inertial force that is not equilibrated. In other words, between time 1 and 2, the sheet pile experiences an acceleration in the down direction.

At time 3, where the velocity changes direction and becomes zero, a jump in the acceleration curve is generated. The peculiar jump in the acceleration curve is due to the assumption of perfect plasticity according to Equations (2.28) and (2.29), where \( R_s \) is set to zero when \( \nu = 0 \) and \( R_t \) set to 100 kN when \( \nu > 0 \), in Equation (2.34).

Between times 2 and 4 the net downward-directed force \( F^p(t) \) expressed by Equation (2.38), generates an acceleration which increases the velocity and the downward motion.

At time 4, penetration speed is at its maximum, acceleration changes direction, and the active force is equal to the resisting forces. However, speed at this time creates an inertial force that is not equilibrated. The sheet pile decelerates until the energy stored between times 2 and 4 have dissipated.
Figure 3-12 Simulation results of the Vibdrive model in accordance with input parameters given in Table (3-1): (a.) applied forces, (b.) acceleration, (c.) velocity, (d.) displacement.
At time 5, the action load \((M, \omega^2 \sin(\omega t))\) becomes greater than the resisting force \((R_s)\), and the net upwardly-directed force \((F^i(t))\) expressed by Equation (2.37) starts to generate an acceleration which decreases the downward velocity but at the same time increases the displacement.

At time 6, velocity becomes zero again and another peculiar jump in the acceleration curve is generated, according to the same assumptions as for time 3, which is stipulated by Equations (2.28) and (2.29).

At time 7, once again the resisting force \((R_s)\) and the action load \((M, \omega^2 \sin(\omega t))\) become equal, however the speed causes an inertial force that is not equilibrated.

Ending at time 8, the final velocity should then be approximately equal to the preset initial velocity \((\sim 76 \text{ cm/s})\). Otherwise, a new initial velocity \((v_{in})\) is to be set and the calculation steps of time 1 through 8 performed again.

Figures (3-13) and (3-14) presents the results of a vibro-driveability simulation by the Vipere model performed by Vanden Berghe (2001), with input parameters according to Table (3-1). The integration procedure represented by the four curves in Figure (3-14) has been performed in accordance with the previously-described integration procedures (see Section 2.6.6). The average penetration speed \((v_p)\) is by the Vipere model is simulated to \(\sim 16 \text{ mm/s}\) which is given by the slope of the moving average in Figure (3-14)c, \((\Delta u/\Delta t = (18 - 16.6)/(1.5 - 1.41)).\)

![Figure 3-13 Forces acting on the vibrator, pile and soil system, modelled by the Vipere, according to the input parameters described in Table (3-1).](image-url)
Figure (3-13)a describes three cycles of the driving force, and Figure (3-13)b the sum of the two resisting forces \((R_s + R_t)\), which does not vary the same regular pattern due to the principally different phases of dilatance and contraction of the soil (see Figures (2-31) and (2-33)). Finally Figure (3-13)c describes the resulting force that is the sum of driving and resisting forces according to the expression given by the numerator in Equation (2.64).

Figure (3-14)a describes the same three cycles of the integrated resulting acceleration according to Equation (2.64), and Figures (3-14)b and (3-14)c the integrated velocity and displacement curves.

![Figure 3-14 Resulting acceleration of the vibrator, pile and soil system, modelled by the Viper: (a) acceleration, (b) velocity and finally the (c) displacement of the pile, (Vanden Berghe 2001).](image)

### 3.4.6 Discussion on the results of vibro-driveability simulations

The Vibdrive model is rather simple since it evaluates the resisting forces \((R_s \text{ and } R_t)\) as constant peak values both with respect to time and displacement, which is in contradiction to the discussion in Section 3.2.4. The perfect plastic soil behaviour modelled by Vibdrive, is the reason for the peculiar jumps in the acceleration curve (see Figure (3-12)). It would be more realistic to expect a smoother pattern of the modelled acceleration curve. This could be accomplished by introducing some kind of transaction phase for both the toe and shaft resistance model.

The Vibdrive model also evaluates the peak values of the toe \((R_t)\) and shaft \((R_s)\) resistance on the basis of the dynamic unit resistance stresses \((q_d \text{ and } \tau_d \text{ respectively})\) as
Discussion regarding primary mechanisms and simulation of vibro-driveability

previously described by Equations (2.30) and (2.31). Where \(q_d\) and \(\tau_d\) in turn are characterised by both the static \((q_s\) and \(\tau_s))\) and liquefied values \((q_l\) and \(\tau_l))\), which are all based on CPT test results (the cone resistance \((q_c))\) and the local unit skin friction \((f_s))\). The shear strength reduction of the soil by Vibdrive, is modelled by two empirical parameters, acceleration ratio \((\alpha = a/g)\) and the liquefaction index \((\psi))\). The two Figures (3-15) and (3-16) illustrate how \((q_d\) and \(\tau_d\) respectively) depend on the varied range of:

\[
\frac{1}{\psi} - \text{factor}
\]

\(\psi\)

Figure 3-15  Variation of the liquefied toe \((q_l)\) and shaft \((\tau_l)\) unit soil resistance with the friction ratio \((R_f = f_s/q_c)\) and the empirical liquefaction index \((\psi))\).
Figure 3-16 Dependence of toe resistance \( q_d \) and shaft resistance \( \tau_d \) on the acceleration ratio \( \alpha \) and the empirical liquefaction index \( \psi \).

the liquefaction factor \( 4 < (1/\psi) < 10 \), acceleration ratio \( 0 < (\alpha=a/g) < 20 \), and friction ratio \( 0 < (R_f = q_c/f_s) < 5 \).

From Figures (3-15) and (3-16) the following can be concluded; that it appears as if the peak value of \( q_d \) and \( \tau_d \) respectively) is not significantly affected by the acceleration ratio, since a field-related value of \( \alpha \) would approximately range between 15 < \( \alpha < 30 \). Instead, it appears as if the range of the liquefaction factor \( \psi \) has a more pronounced influence on \( q_d \) and \( \tau_d \) respectively. It can also be concluded that the same exponential strength reduction is applied to both toe and shaft, which could be discussed if it is appropriat.

3.5 Concluding remarks

3.5.1 General remarks

In principal, the phenomena associated with shear-strength reduction in cohesionless soils to be subject to vibratory-technique installations have been described and explained. The prime mechanisms causing most of the induced shear-strength reduction have been pointed out. It also seems evident, that strength reduction is related to factors such as inertial forces in the soil with large motion of particles as a consequence, which causes cyclic volume changes and a cyclic variation in excess pore-pressure. A better understanding of the three stress-states in the soil (the undisturbed stress-state, the vibratory-driving stress-state, and finally the disturbed stress-state) are believed to be the key to understanding the mechanisms related to vibro-driveability.

3.5.2 Kinematics of the vibrator, sheet pile and soil system

The author’s overall views about the kinematic nature of the vibrator, sheet pile and soil system’s nature have been presented and discussed in Section 3.2, and is based on the assumptions and justified simplifications presented in Section 3.3.

The term 'vibro-driveability’ is used to describe the ease with which a vibratory-driven sheet-pile penetrates a non-cohesive soil, and is defined in Section 2.4.1. The forces that both drive and resist the penetration behaviour are discussed from the overall kinematic nature of the system presented (see Figure (3-2)).
3.5.3 Primary factors influencing vibro-driveability

In light of the complexity associated with attempting to describe vibro-driveability, it is justifiable to subdivide the prime factors affecting vibro-driveability into three main parts: (i) vibratory-equipment, (ii) sheet pile, and (iii) soil-related factors.

The prime vibratory-equipment-related factors discussed to have a significant impact on vibratory-driven sheet piles are presented below.

- The efficiency of the vibratory-equipment: since any successful attempt to simulate the vibro-driveability should begin with a reasonably accurate value of the magnitude of the driving force generated that actually enters the sheet pile.
- Choice of ratio ($F_o/F_v$): this appears to be a factor that ought to be considered in “hard” driving conditions, and with displacement piles, because of the correlation with the undesirably ‘fast’ vibratory-driving state.
- Choice of vibratory equipment: this factor ought to be considered along with how the vibratory equipment should be operated, especially when environmental aspects need to be addressed.

The main profile-related factors that are seen to have a significant impact on vibratory-driven sheet piles are:

- Justification of axial rigidity-body assumption: from an engineering point of view this can be justified in most of the more favourable cases with favourable soil conditions.
- Effects of lateral flexibility in the sheet pile: since this factor can definitely cause unexpected problems, such as considerably lower penetration speed and the generation of considerably higher ground-vibration values.
- Effects of friction forces in the clutch: this might also cause considerably lower production capacity, generate considerably higher ground-vibration values, and in the long run, generate injurious settlement in neighbouring structures.

The primary non-cohesive, soil-related factors seen to have a significant impact on vibratory-driven sheet piles are:

- Cyclic mobility of the individual soil particles combined with partial liquefaction: these are seen as the fundamental mechanisms behind the shear-strength reduction in non-cohesive soils in conjunction with the using the vibratory-technique to drive sheet piles (see Section 3.3.3). The fundamental importance of the vibration caused by the individual soil grain motion as well as the pore-water pressure induced is a difficult task to simulate, analytically, empirically as well as numerically.
- Inertial forces but not primarily the effects of liquefaction: these have been phenomenologically explained as the hypothetical fundamentals of shear-strength
reduction. These are the primary soil-related factors that alter the initial shear strength of the soil to the favourably-reduced value during the vibratory installation process of sheet piles.

### 3.5.4 Vibro-driveability simulations

Difficulties in attempting to describe the penetration behaviour of a vibratory-driven sheet pile come from the fact that a trade-off must be made between the accuracy of the description process and its practical application to real problems. This means that the key issue is to obtain a better fundamental understanding of the vibratory sheet-pile driving process, and hence which parameters significantly affect the vibro-driveability.

The two models Vibdrive and Vipere have been described and used here for comparing simulated vibro-driveability with the actual test results obtained, because they appear to be the forerunners of tomorrow's tools for simulating the vibro-driveability of sheet piles.

**The Vibdrive model**

The Vibdrive model is relatively simple compared to the Vipere model. However, Vibdrive considers the two primary soil-related factors that alter the characteristics of the initial shear strength of the non-cohesive soil.

The simpler model named Vibdrive attempts to incorporate two prime soil-related characteristics which are important factors to consider during vibro-driving of sheet piles. The two factors affect the shear strength reduction of cohesionless soils during vibro-driving of sheet piles: (i) cyclic mobility of soil grains due to high acceleration leaves which introduces inertia forces to the soil, (ii) excess pore pressure due to cyclic void ratio changes. The model does not completely succeed to consider these factors since it incorporates some drastic assumptions. Even if it can be considered that the Vibdrive model is simple, it has its advantages since it allows one to understand the stipulated assumptions and how these influence the calculations.

The Vibdrive model does not consider: (i) friction forces in the sheet-pile interlock, (ii) the efficiency of the theoretical driving force, and (iii) the effects of lateral flexibility of slender sheet piles. The vibrating “body” (vibrator and sheet pile) is treated as a rigid body, and Newton’s second law of motion is applied in order to calculate the penetration speed.

The dynamic soil resistance at the pile toe and along the shaft is correlated with CPT tests. The Vibdrive model incorporates the effects of acceleration and liquefaction...
by the introduction of two empirical parameters in order to correlate the CPT results with the shear-strength reduction. These two empirical parameters have been correlated with both laboratory and full-scale field tests. However it is not explicitly described how these two empirical parameters should be viewed in relation to the shear-strength effect of acceleration and liquefaction, nor are guidelines available on how the CPT results should be treated in order to derive the liquified, and dynamic counterparts used as input to drive the dynamic soil resistance.

The Vibdrive model applies the same empirically-constitutive expressions of the dynamic soil resistance to both the pile toe and shaft. However the load situations at the pile toe and shaft are different. It might be reasonable to consider that the soil volume below a vibrating pile-toe is displaced during cyclic loading, and that the soil beside the shaft is constantly sheared in a cyclic manner. In this context, it seems evident that dynamic soil resistance should be modelled differently for the pile toe and shaft.

The Vipere model

The Vipere model is more elaborated than the Vibdrive model and takes into account the shear-strength reduction by introducing a hypoplastic constitutive soil model. Vipere takes into account the differences in the types of deformations of the soil at the pile toe and along the shaft.

The soil parameters used as input for shaft resistance are taken from direct simple shear (DSS) tests, and the soil parameters for the pile toe are taken from undrained tri-axial (TXT) tests. The input parameters are relatively complex in nature (many engineering applications) and the full extent of choices together with their impact on the vibro-driveability is not easy to comprehend.

The Vipere model makes similar assumptions to the Vibdrive model; for example it does not consider the friction forces in the sheet-pile interlock, the efficiency of the theoretical driving force, nor the effects of lateral flexibility of slender sheet piles. The vibrating body (vibrator and sheet pile) is treated as a rigid body, and Newton’s second law is applied to calculate the penetration speed, according to a well defined integrain scheme.

Some of the primary limitations of the Vipere model are the following: the toe model does not take into account the gap appearance between the toe and soil below, as observed by Dierssen (1994) and De Qock (1998), neither inertial effects introduced to the soil volume below the toe. The soil behaviour at the toe is assumed to be undrained, i.e. the induced excess pore pressure is not modelled to dissipate in the far field located soil elements. However, results of excess pore pressure measurements, Rao (1993), dis-
plays a sinusoidal time dependent nature biased around a mean value higher than the initial hydrostatic water pressure. An easier procedure to determine the necessary parameters of the hypoplastic soil model is needed, preferably correlated with field tests instead to the time consuming laboratory tests. Since a inconsistently performed calibration might significantly influences the simulated penetration speed, especially for small penetration depths.

The Vipere model does however make assumptions that the Vibdrive model does not. It assumes that the radial normal and shear stresses are equal along the whole length of the shaft, and do not change with increasing penetration depth. Soil is also assumed to be sheared in undrained conditions, and the soil volume below the pile toe is assumed to not lose contact with the toe.

Field-testing procedures to estimate dynamic soil resistance.

From a geotechnical point of view, subsoil characteristics are usually determined by means of standard investigation methods such as:

• probing tests (for example CPT and SPT),
• sampling tests (for example boring), and
• laboratory tests (such as tri-axial, resonant-column, and direct shear tests).

Generally speaking, all these methods are developed to produce input information for static design issues. It is obvious that such investigation methods are not well suited to characterise soil behaviour that exists during the vibratory-installation process of piles and sheet piles. However, it is obvious that the investigation method devised to evaluate the dynamic soil stress and strain characteristics that exist in cohesionless soils must attempt to duplicate the boundary conditions existing in vibratory-driven piles as closely as possible.

The process of obtaining representative values for the dynamic soil properties is the most difficult part of a tentative attempt to predict vibro-driveability. Soil samples must be obtained from the construction site under consideration, in order to be tested under conditions anticipated to represent the operating environment in the soil closest to the vibratory-driven profile. Since soil properties that influence the dynamic soil resistance of vibratory-driven profiles are established for a different order of magnitude of deformations compared to those involved in static soil properties, a new field-testing procedure is needed to enhance the process of obtaining more representative values of the dynamic soil resistance in the future.
CHAPTER 4

EXPERIMENTAL WORK

4.1 Introduction to the chapter

Chapter 4 includes a description of the field tests undertaken in conjunction with this thesis. Along with a chronology of the main test events (see Table (4-1)), this chapter also contains a description of the test site, the vibratory equipment (vibrator, sheet piles, and instrumentation), and finally the methods used in the analysis of data collected during the tests.

The full-scale field tests reported on here were performed in Vårby, a suburb of Stockholm (Sweden). The test site was chosen firstly for its relatively homogeneous soil conditions, and secondly because there was good probability of being able to keep the sensors in place for the entire duration of the planned field tests.

The two main objectives of the measurements were:

• to study and document the actual magnitude of field-related parameters such as driving force, dynamic soil resistance, and ground vibrations generated with and without the presence of clutch friction, with both the axial and lateral accelerations of the sheet pile; and
• to find out whether it was possible to correlate a prediction of sheet-pile driveability with the two driveability models previously presented in Section 3.4.

Section 4.2 of the chapter, describes the materials and methods used in the full-scale field tests.

Section 4.3 presents the instrumentation system and the calibration procedures used for the various sensors used in the tests.

Finally, Section 4.4 presents the methods used in analysing the data obtained during the field tests. The results of these analyses were subsequently compared with both the hypothesised behaviour, as described in Sections 3.2 and 3.3, together with the results provided by the two simulation models, Vibdrive and Vipere, described in Section 5.3.
4.2 Materials and methods

This section is divided into five subsections addressing the following five areas:

- location of the full-scale field tests,
- chronology of the main test events (see Table (4-1)),
- results of the soil investigations conducted,
- the vibratory-driver system used, and finally
- the sheet-pile profiles used.

4.2.1 Location of the field tests

The test site was situated approximately 50 m from the eastern shore of the steel-girder bridges crossing the Fittja Straits in the suburb of Vårby, approximately 20 km southwest of central Stockholm (Sweden). The work being done on the test site involved driving sheet piles between an abandoned summer cottage (approximately 100 years old) and a bridge on the eastern shore of the Fittja Straits (see Figure (4-1)).

Figure (4-2) is a representation of the test site plan at Vårby, showing the relative positions of the driven sheet piles marked by \( A1-A5 \) and \( B1-B3 \), together with the position of the soil investigations undertaken.

All the vibratory-driven sheet piles at the test site in Vårby were installed in cooperation with Stabilator AB (a Swedish foundation-construction company), using a leader-mounted ABI vibratory driver (MRZV 800V), with an ABI leader system (TM 14/17L), which can be seen in Figures (4-5) and (4-8).

![Map showing the location of the test site in relation to the existing steel-girder bridge over the Fittja straits, 20 km southwest of Stockholm.](image)
4.2.2 Field-test chronology

The full-scale test series consisted of two parts (Series A and Series B), conducted over a two-day period. Series A was conducted on Saturday 30th October, and Series B on Sunday 31st October, 1999. The same sheet pile fitted with sensors was driven a total of six times into the soil at the Vårby test site (denoted by A4-A6 and B1-B3 in Table (4-1)).

The initial part of Series A consisted of three driveability tests using three new sensor-fitted sheet piles, designated A1-A3 in Figure (4-2) and Table (4-1). These same three sheet piles (A1-A3) but without sensors were later used to thread the interlocks of the sensored sheet-piles, according to the test chronology shown in Table (4-1).
It should be noted that Tests A1-A3 feature three different LX-16 sheet-pile profiles, but Tests A4-B3 involved the very same sheet pile, reused and re-driven several times over.

Table 4-1 Field-test chronology at Vårby.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Profile</th>
<th>Sheet pile fitted with sensors (Yes/No)</th>
<th>Clutch friction present (Yes/No)</th>
<th>Radial distance of 3 tri-axial geophones from the C.L. of driven sheet-piles.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>V1/L1/T1</td>
</tr>
<tr>
<td>A1</td>
<td>LX-16</td>
<td>No</td>
<td>No</td>
<td>1.2</td>
</tr>
<tr>
<td>A2</td>
<td>LX-16</td>
<td>No</td>
<td>Yes</td>
<td>1.15</td>
</tr>
<tr>
<td>A3</td>
<td>LX-16</td>
<td>No</td>
<td>Yes</td>
<td>1.15</td>
</tr>
<tr>
<td>A4</td>
<td>PU-16</td>
<td>Yes</td>
<td>No</td>
<td>1.0</td>
</tr>
<tr>
<td>A5</td>
<td>PU-16</td>
<td>Yes</td>
<td>Yes</td>
<td>1.2</td>
</tr>
<tr>
<td>A6</td>
<td>PU-16</td>
<td>Yes</td>
<td>No</td>
<td>1.2</td>
</tr>
<tr>
<td>B1</td>
<td>PU-16</td>
<td>Yes</td>
<td>No</td>
<td>1.2</td>
</tr>
<tr>
<td>B2</td>
<td>PU-16</td>
<td>Yes</td>
<td>No</td>
<td>1.1</td>
</tr>
<tr>
<td>B3</td>
<td>PU-16</td>
<td>Yes</td>
<td>Yes</td>
<td>0.85</td>
</tr>
</tbody>
</table>

The full-scale field test studies can be summarised into the following steps.

1. *Positioning of the tri-axial geophones:* the soil crust was removed at the sites for the three geophones. The surface was first levelled by sight then the geophones were positioned at three radial distances according to Table (4-1) and illustrated in Figure (4-2).
2. *Positioning of the cables between the pre-positioned geophones and a DAT tape recorder:* the cables were positioned and parameters including cable number and position, time, radial distances from the geophones to the C.L. of the sheet pile, file name and position of the coming data, were recorded on for example the DAT tape.
3. *Positioning of the video-camera:* the camera was positioned on a spot directly in front of the sheet pile intended to be driven, prior to each driveability test. This was done to document and provide a backup recording of the depth markers painted on the sheet-pile surface.
4. *Marking and raising the sheet pile:* the sheet pile to be driven was depth-marked at every 0.5 m, then raised vertically, so that it was hanging free in a vertically position in the vibratory-driver’s hydraulic clamp, then finally positioned in the plane were it was intended to be driven into the ground. The
signal cable from the vibratory sensors (geophones) was connected to the main switchbox (see Figure (4-10)).

5. **Zeroing the depth sensor:** the depth sensor was zeroed when the vertically-positioned sheet-pile toe was barely touching the surface of the ground. Just before the sensor-fitted sheet pile was driven, signals from the full Whetstone bridges were zeroed when the sheet pile was hanging freely in the hydraulic clamping-device, using a digital multimeter to measure when zero voltage outputs were obtained from the strain-gauge circuit.

6. **Commencement of recording:** after the signal was given from those involved, both the video camera and the DAT tape recorder were started. Then the go-ahead was given to operator of the vibratory driver to begin pile installation.

7. **Readings taken:** during the penetration of the sheet-pile phase, verbal notes were recorded on microcasette at different pile-penetration depths, as well as visual readings of both the lateral and axial displacement amplitude of the sheet pile, by studying the stickers attached to the piles, as shown in Figure (4-28).

8. **Final recording:** when the installation of the sheet pile was complete and the vibratory-machine switched turned off, the video recorder and DAT tape recorder were also turned of. Final notes were then made of for example the time, the total driving time, the penetration depth, and file name on the DAT tape.

9. **Extraction and repeating:** after starting up the vibratory-machine and extracting the installed sheet pile, the procedure started once again from step one above again, until all the driveability tests were conducted. It should be noted that some of the extraction phases were also recorded in order to analyse the effects of the absence of dynamic toe resistance.

### 4.2.3 Soil conditions

Apart from the top 1.5-2.0 m layer of topsoil and clay, the soil conditions at the Vårby test site consisted of the more than 40 m of medium-dense to almost loose glacial sand. The groundwater table lies approximately 2.0 m below ground level, and the 40 m of glacial sand is relatively well-graded, varying between a silty and a gravelly sand, with depths shown in Table (4-2).

The soil investigations performed at the test site Vårby included:

- three CPTu type tests (abbreviated to CP1-3 in Figure (4-2)),
- 21 plus an additional 14 dynamic probing tests; the results of which are thoroughly described by Axelsson (2000), and
- soil sampling at six different levels.
Furthermore, pore pressures were measured using a piezometer and an open stand-pipe. The positions of the soil investigations are shown in Figure (4-2).

The results of the three CPTu tests, revealed a sandy deposit that is relatively homogeneous with respect to the range of areas investigated and penetration resistances.

Results of the dynamic-probing tests were obtained using the recommended Swedish standard equipment for dynamic probing. The weight of the hammer was 63.5 kg with a drop height of 0.50 m; the rods were 32 mm in diameter, and made of solid steel. The results of the dynamic probing test and further details can be found in Axelsson (2000).

The soil samples were taken at six different levels, corresponding to the results presented in Table (4-2). The soil profile at the test site was interpreted from the CPT tests and the soil sampling, according to Axelsson (2000), and are presented in Table 4-2.

<table>
<thead>
<tr>
<th>Depth range [m]</th>
<th>Soil type</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 2.5</td>
<td>clay</td>
</tr>
<tr>
<td>2.5 - 4.5</td>
<td>silty sand</td>
</tr>
<tr>
<td>4.5 - 8</td>
<td>sand</td>
</tr>
<tr>
<td>8 - 14</td>
<td>silty sand</td>
</tr>
<tr>
<td>14 - 19</td>
<td>gravelly sand</td>
</tr>
<tr>
<td>19 - 24</td>
<td>silty sand</td>
</tr>
<tr>
<td>24 -</td>
<td>sand</td>
</tr>
</tbody>
</table>

**Table 4-2  Soil profile (after Axelsson, 2000).**

**Basic soil properties**

According to measurements and calculations, such as mean grain-size ($d_{50}$), coefficient of uniformity ($C_u$) and grain-size distribution, the basic soil properties at the Vårby test site were determined from soil samples taken from each section of the tests site, and are presented below in Table (4-3).

The grain-size distribution of test site Vårby was determined from different samples taken at five different levels, and presented in Figure (4-3).
Mineralogical analysis of the soil samples (see Table (4-4) showed that the soil strata consisted mainly of hard minerals such as quartz and feldspar.

Table 4-3 Soil properties of test site Vårby, after Axelsson (2000).

<table>
<thead>
<tr>
<th>Sampling depth</th>
<th>Type of soil</th>
<th>Uniformity coefficient</th>
<th>Mean particle size</th>
<th>Relative density</th>
<th>Friction angle</th>
<th>Shear modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m]</td>
<td>[-]</td>
<td>[mm]</td>
<td>[%]</td>
<td>[°]</td>
<td>[MPa]</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>silty sand</td>
<td>15</td>
<td>0.16</td>
<td>35</td>
<td>33</td>
<td>50</td>
</tr>
<tr>
<td>10.1</td>
<td>silty sand</td>
<td>7</td>
<td>0.15</td>
<td>35</td>
<td>33</td>
<td>60</td>
</tr>
<tr>
<td>13.2</td>
<td>sand</td>
<td>7.5</td>
<td>0.25</td>
<td>40</td>
<td>34</td>
<td>70</td>
</tr>
<tr>
<td>15.8</td>
<td>gravelly sand</td>
<td>6</td>
<td>1.0</td>
<td>50</td>
<td>35</td>
<td>90</td>
</tr>
<tr>
<td>18.2</td>
<td>sand</td>
<td>5</td>
<td>0.55</td>
<td>40</td>
<td>32</td>
<td>85</td>
</tr>
</tbody>
</table>

Mineralogical analysis of the soil samples (see Table (4-4) showed that the soil strata consisted mainly of hard minerals such as quartz and feldspar.

Figure 4-3 Grain-size distribution of soil samples from Vårby (Axelsson, 1998).
Minimum and maximum densities

The initial relative densities together with the volume changes induced in the sand are considered to be one of the most significant soil-related parameters influencing driveability (the behaviour of the sheet-pile-soil system) during the installation phase. Unfortunately, neither the maximum and minimum dry densities, nor the corresponding maximum and minimum void ratio. The different sand samples were too small in order to perform the desired laboratory analysis.

CPT results

The soil profile at the Vårby test site was investigated using three individual cone penetration tests (CPTu) in order to form a detailed picture of the subsoil conditions. The three test locations can be seen in Figure (4-2).

The three CPT tests (CP1-CP3) were performed with a cylindrical cone with a cross-sectional area of 1000 mm$^2$, and cone angle of 60°. The penetration rate was a constant 20 mm/s, in accordance with the Swedish standard for CPT equipment (SGF, 1993). Uncorrected cone resistance ($q_c$), sleeve friction ($f_T$), and pore pressure ($u$) were recorded continuously during penetration through the soil.

The CPT test results denoted by CP3 are presented in Figure (4-4) below, and the results of the two other tests can be found in Appendix C. The three different CPT results describe a subsoil profile with a relatively homogeneous friction material.

### Table 4-4 Mineral composition of soil from Vårby (after Axelsson, 2000).

<table>
<thead>
<tr>
<th>Mineral type</th>
<th>Sampling depth [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10.1</td>
</tr>
<tr>
<td>Quartz</td>
<td>50%</td>
</tr>
<tr>
<td>Na - Ca - Feldspars</td>
<td>30%</td>
</tr>
<tr>
<td>K - Feldspars</td>
<td>15%</td>
</tr>
<tr>
<td>Poryxene - Amphibole</td>
<td>5%</td>
</tr>
</tbody>
</table>
4.2.4 The vibratory-driver system

The vibratory-driver system used to install the sheet piles during the full-scale field test at Vårby was a leader-mast-mounted type, defined earlier in Section 2.3.1. The vibrator equipment used in the tests was manufactured by ABI, and is illustrated in Figures (4-5) and (4-8). The equipment was owned and installed by Stabilator AB.

The basic performance data for the vibratory-driver system used is summarised in Table (4-5), and the system consisted of the following three parts:

- an ABI static moment variable vibrator (MRZV 800V),
- mounted on an ABI telescopic leader mast (TM 14/17 L),
- held by a remodelled Sennebogen excavator.

Figure 4-4  Results of CPT tests (test run CP3) from the Vårby test site.
Figure 4-5 Picture taken at the Vårby test site, where the vibratory system and the sheet piles were used.

Table 4-5 Basic performance data for the vibrator-driving equipment used (from ABI).

<table>
<thead>
<tr>
<th>Basic performance data</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum revolution</td>
<td>2460</td>
<td>rpm</td>
</tr>
<tr>
<td>Static moment</td>
<td>0 - 12</td>
<td>kgm</td>
</tr>
<tr>
<td>Maximum driving force</td>
<td>800</td>
<td>kN</td>
</tr>
<tr>
<td>Dynamic weight of vibrator unit</td>
<td>2450</td>
<td>kg</td>
</tr>
<tr>
<td>Maximum pile weight</td>
<td>2000</td>
<td>kg</td>
</tr>
<tr>
<td>Maximum usable pile length</td>
<td>15</td>
<td>m</td>
</tr>
</tbody>
</table>
The vibrator unit

The vibrator unit that was used is called a static moment variable vibrator, manufactured by ABI (MRZV 800V), and features eccentric masses that can be varied “on the fly” (without interruption to operation). Another name found in the literature for this type of vibrator is a “resonance-free vibrator”. The vibrator was always started with its static moment set to zero, which could subsequently be varied on the fly in the range $0 < M_e < 12$ kgm, as illustrated in Figure (4-6). Table (4-5) lists all the other basic performance data for this vibrator.

The driving frequency could not be varied during the driving phase (on the fly), however driving frequency could be set to the desired value prior to starting the vibrator. The driving frequency was kept constant during the entire field test, and set to 41 Hz.

Figure (4-7) shows the various parts of the vibrator used (ABI MRZV 800V) from two different angles. This model had variable in-line eccentric weight shafts driven by hydraulic oil motors. Rubber mounts sat between the vibrator guide (a non-vibrating part) and the gearbox (moving part), with the purpose of absorbing the vibrations generated by the rotating eccentric weights.

Table 4-5  Basic performance data for the vibrator-driving equipment used (from ABI).

<table>
<thead>
<tr>
<th>Basic performance data</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum driving force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leader cylinder</td>
<td>70</td>
<td>kN</td>
</tr>
<tr>
<td>Support cylinder</td>
<td>90</td>
<td>kN</td>
</tr>
<tr>
<td>Maximum extraction force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leader cylinder</td>
<td>140</td>
<td>kN</td>
</tr>
<tr>
<td>Support cylinder</td>
<td>175</td>
<td>kN</td>
</tr>
</tbody>
</table>

Figure 4-6  Illustration of the principles behind the variable eccentric moment mechanism (from ABI’s brochures).
The telecopic leader

The ABI telecopic-leader system (TM 14/17L) is shown in Figure (4-8). The telecopic mast to which the vibrator was connected, could be inclined in any number of vertically positions according to requirements, and at the same time monitor the angle of inclination using an ABI Mobilram System.

There are three components of the ABI leader system connected to the carrier:

- main boom (see 2 in Figure (4-8)),
- main-boom cylinder (see 4 in Figure (4-8)), and
- folding-down cylinder (see 5 in Figure (4-8)).

The folding up and down of the leader mast and the adjustment of the forward and backward inclination of the telecopic-leader system is made possible by the above three components.

The telecopic-leader mast is made of an inner part (see 7 in Figure (4-8)) and an outer part (see 6 in Figure (4-8)), whose movement relative to one another is guided by a pair of large erthalon bushings. The vertical position of the vibrator is actuated by the part of the telecopic-leader mast called the leader cylinder (see 8 in Figure (4-8)). The support cylinder (see 9 in Figure (4-8)) allows the useable length if the pile to be increased, as well as applications involving driving below ground level.
4.2.5 The sheet piles

There were two main criteria for selecting the sheet-pile profiles for the field tests. Both the length and the profile should be representative of the most common vibratory-driven sheet piles in Sweden. The typical sheet piles were determined to be steel sheet-piles from Profil ARBED S.A. (Luxembourg), 14 m in length.

The following is a summary of the number and type of sheet piles used:

1. Carrier
2. Main boom
3. Vibrator
4. Main boom cylinder
5. Folding down cylinder
6. Outer part of telescopic cylinder
7. Inner part of telescopic cylinder
8. Leader cylinder
9. Support cylinder

Figure 4-8  Carrier featuring an ABI Mobilram system (TM 14/17 L) (from the operators manual).
Experimental work

- three sheet piles of profile LX-16, 14 m in length, without sensors, designated A1-A3 in Table (4-1), with the positioning seen in Figure (4-2); and
- one sheet pile of profile PU-16, 14 m in length, fitted with sensors, driven several times, designated A4-A6 and B1-B3 in Table (4-1), with the positioning shown in Figure (4-2).

These dimensions and other relevant data about the two types of sheet-pile profiles used are provided in Table (4-6) and Figure (4-9).

The three brand new LX-16 profiles without sensors (A1-A3) were installed first, with the principle purpose of initially testing their vibro-driveability, and at the same time, putting them in place for later use as a pre-existing sheet-pile wall into which the clutch of the sensor-fitted PU-16 profiles could be jointed with the sequence shown in Table (4-1), and positions in Figure (4-2).

Table 4-6 Basic data for the sheet-pile profiles used at the Vårby test site (according to Profil ARBED and British Steel).

<table>
<thead>
<tr>
<th>Sheet Profile</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$ [mm]</td>
</tr>
<tr>
<td>PU-16</td>
<td>303</td>
</tr>
<tr>
<td>LX-16</td>
<td>365</td>
</tr>
</tbody>
</table>

Figure 4-9 The basic dimensions of sheet-pile profile used at the Vårby test site.
4.3 Instrumentation and calibration

The instrumentation system used to document vibro-driveability and the environmental effects generated during the tests consisted of the following five main parts:

- vibrator instrumentation,
- the sheet-pile sensors,
- soil instrumentation,
- the data-acquisition system, and
- the video camera.

The instrumentation system together with the dynamic data-acquisition system are illustrated in Figure (4-10). The instrumentation setup consisted of 25 channels in total, subdivided into the following three groups:

- the vibrator (Channels 1-3),
- the sheet pile (Channels 4-16), and
- the geophones (Channels 17-25).

A detailed description of the indexes for each channel, and the units for each monitoring sensor used during the field tests is found in Table (4-7).

The vibrator-related instrumentation is described in detail in Section 4.3.1, and monitored the following three parts: (i) the static surcharge force \( F_0 \) applied (the oil-pressure), (ii) the penetrative movement of the sheet pile \( \gamma \) (the penetration depth), and (iii) the adjusted static moment \( M_e \) (the position of the eccentric weights).

The sheet-pile-related instrumentation is described in detail in Section 4.3.2, and consisted of 10 strain gauges and three accelerometers, positioned as illustrated in Figure (4-14). Eight of the 10 strain gauges were mounted in holes cut into the sheet-pile section and later filled with a two-component resin in order to protect the sensors. The accelerometers were small enough to fit into the holes. One of the three accelerometers was mounted laterally in order to monitor the laterally-induced motions in the sheet pile.

The soil instrumentation is described in detail in Section 4.3.3, and consisted of three tri-axial geophones. The purpose of these was to monitor the ground vibrations induced during the installation phase.

The data-acquisition system was developed especially for these tests and is described in detail in Section 4.3.4. The system consisted of a switchbox, 3 four-channel strain-gauge amplifiers, and one DAT tape recorder (SONY™ PC216Ax) together with
a channel-extension unit (SONY™ PCCX32Ax). The dynamic data recorded from the
various sensors were analysed as discussed in Section 4.4.

Prior to the full-scale field tests, all the above mentioned sensors were calibrated,
and each part tested; this is described in detail in Section 4.3.5. The readings from the
calibration process were later compared with known standard calibration procedures.

![Figure 4-10 Illustration of the complete instrumentation and data-acquisition system used at the Vårby test site.](image-url)
### Table 4-7 Description of each instrumentation channel used in the Vårby field tests.

<table>
<thead>
<tr>
<th>Channel no.</th>
<th>Index</th>
<th>Explanation</th>
<th>Part of the instrumentation system</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F_o$</td>
<td>Surcharge force: of the vibrator</td>
<td>Vibratory-instrumentation (see Section 4.3.1)</td>
</tr>
<tr>
<td>2</td>
<td>$F_v$</td>
<td>Unbalanced force: of the vibrator</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$z$</td>
<td>Penetration depth</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$a_h$</td>
<td>Axial acceleration: the sheet-pile head</td>
<td>Sheet-pile instrumentation (see Section 4.3.2)</td>
</tr>
<tr>
<td>5</td>
<td>$a_l$</td>
<td>Lateral acceleration: the sheet pile's midsection</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$a_t$</td>
<td>Axial acceleration: the sheet-pile toe</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$S_{hi}$</td>
<td>Strains: inside the sheet-pile head</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$S_{ho}$</td>
<td>Strains: outside the sheet-pile head</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>$S_{lm}$</td>
<td>Strains: left side of the sheet pile's midsection</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>$S_{rm}$</td>
<td>Strains: right side of the sheet pile's midsection</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>$S_{lhm}$</td>
<td>Strains: left side of sheet pile's lower midsection</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>$S_{rlm}$</td>
<td>Strains: right side of sheet pile's lower midsection</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>$S_{ltw}$</td>
<td>Strains: left side of the sheet pile's toe web</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>$S_{rtw}$</td>
<td>Strains: right side of the sheet pile's toe web</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>$S_{lbf}$</td>
<td>Strains: left side of the sheet pile's toe flange</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>$S_{rnf}$</td>
<td>Strains: right side of the sheet pile's toe flange</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>$V_1$</td>
<td>Geophone: position one, vertical direction</td>
<td>Soil instrumentation (see Section 4.3.3)</td>
</tr>
<tr>
<td>18</td>
<td>$L_1$</td>
<td>Geophone: position one, lateral direction</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>$T_1$</td>
<td>Geophone: position one, transversal direction</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>$V_2$</td>
<td>Geophone: position two, vertical direction</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>$L_2$</td>
<td>Geophone: position two, lateral direction</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>$T_2$</td>
<td>Geophone: position two, transversal direction</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>$V_3$</td>
<td>Geophone: position three, vertical direction</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$L_3$</td>
<td>Geophone: position three, lateral direction</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>$T_3$</td>
<td>Geophone: position three, transversal direction</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4-11 Description of the dynamic data-acquisition system.

<table>
<thead>
<tr>
<th>Channel No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Surcharge load, Unbalanced force, Penetration depth</td>
</tr>
<tr>
<td>2</td>
<td>Vertical acc. of sheet pile head, lateral acc. of midsection of sheet pile</td>
</tr>
<tr>
<td>3</td>
<td>Vertical acc. of sheet pile toe, lateral acc. of midsection of sheet pile</td>
</tr>
<tr>
<td>4</td>
<td>Strains, inside sheet pile head, Strains, outside sheet pile head</td>
</tr>
<tr>
<td>5</td>
<td>Strains, left midsection of sheet pile, Strains, right midsection of sheet pile</td>
</tr>
<tr>
<td>6</td>
<td>Strains, lower midsection of sheet pile, Strains, left lower toe web of sheet pile</td>
</tr>
<tr>
<td>7</td>
<td>Strains, right lower toe web of sheet pile, Strains, left lower toe flange of sheet pile</td>
</tr>
<tr>
<td>8</td>
<td>Strains, right lower toe flange of sheet pile, Strains, left lower midsection of sheet pile</td>
</tr>
<tr>
<td>9</td>
<td>Strains, right lower toe web of sheet pile, Strains, right lower toe flange of sheet pile</td>
</tr>
<tr>
<td>10</td>
<td>Strains, left lower midsection of sheet pile, Strains, right lower toe web of sheet pile</td>
</tr>
<tr>
<td>11</td>
<td>Strains, right lower toe flange of sheet pile, Strains, left lower midsection of sheet pile</td>
</tr>
<tr>
<td>12</td>
<td>Strains, right lower toe web of sheet pile, Strains, right lower toe flange of sheet pile</td>
</tr>
<tr>
<td>13</td>
<td>Strains, left lower midsection of sheet pile, Strains, right lower toe web of sheet pile</td>
</tr>
<tr>
<td>14</td>
<td>Strains, right lower toe flange of sheet pile, Strains, left lower midsection of sheet pile</td>
</tr>
<tr>
<td>15</td>
<td>Strains, right lower toe web of sheet pile, Strains, right lower toe flange of sheetpile</td>
</tr>
<tr>
<td>16</td>
<td>Strains, left lower midsection of sheet pile, Strains, right lower toe web of sheet pile</td>
</tr>
<tr>
<td>17 - 25</td>
<td>Strains, right lower toe flange of sheet pile, Geophones, see Fig. 4-10</td>
</tr>
</tbody>
</table>


Stationary computer for post analysis of collected data with computer programs PCscan and MATLAB according to Section 4.4.
4.3.1 Vibrator instrumentation

The instrumentation mounted on the ABI vibrator is manufactured by the German electronics company 1Loster GmbH, and is distributed as an accessory for the ABI units. The vibrator instrumentation consists of three sensors for monitoring the following vibratory parameters:

- Channel 1, for sensing the static surcharge force ($F_0$),
- Channel 2, for sensing the eccentric moment ($M_0$), and
- Channel 3, for sensing the penetration depth of the sheet pile ($z$).

Power was supplied to the three sensors mentioned above by the carrier’s electrical system. Signals from the sensors were all wired to a switchbox mounted in the vicinity of the carrier’s engine bay (see Figure (4-12)). The input signals to the switchbox were treated by the Loster-developed hardware system, in order to transfer the incoming voltage signals into physical values. The way these three input voltage signals were treated prior to being recorded by the DAT recorder is described in the following three sections.

![Figure 4-12 Picture of the switchbox mounted onto the vibratory equipment on the carrier.](image)

**Static surcharge force**

The static surcharge force ($F_0$) could be monitored by an oil-pressure transducer that recorded the oil pressure in the hydraulic cylinder during operation (see Figure (4-8)). The leader cylinder makes it possible to alter the vertical position of the vibrator along the

---

1. Loster GmbH, Unterholzenerstr. 27, D-94360 Mitterfels, Germany
leader mast. The oil-pressure sensor generated a physical output signal that varied between 1-5 V, corresponding to the 0-70 kN capacity of the leader cylinder (see Table (4-5)).

The procedure for the calibration of the oil-pressure sensor (the static surcharge force \( F_o \) applied) is described in Section 4.3.5.

**Static moment**

The static moment \( M_e [\text{kgm}] \) of the ABI vibrator used here could be varied on the fly. The principle of varying the static moment in the range \( 0 < M_e < \text{max.} \) is accomplished by shifting the phase of the relative position of the eccentric weights (see Figure (4-6)). The relative position \( \eta \) could be varied between \( 0 < \eta < 1.0 \) corresponding to zero and the maximum static moment. The sensor monitoring the relative position \( \eta \) generated a physical output signal of 0-5 V, corresponding to the 0-12 kgm of the eccentric weight capacity (see Table (4-5)).

The calibration procedure used for the position \( \eta \) of the eccentric weights (the static moment \( M_e \)) is described in Section 4.3.5.

**Penetration depth**

The penetration depth \( z [\text{m}] \) of the sheet piles being driven was monitored primarily using a Celesco\textsuperscript{1} PT9150 depth-measuring drum (see Figure (4-13)). The rotation of the measuring drum generated 500 rectangular output pulses per meter of steel wire extracted. The number of pulses were converted to a linearly-incremental electric signal, with an output range of +/-5 V, representing 0-20 m of penetration depth.

The primary depth-sensor (the depth-measuring drum) was mounted on the top of the leader mast as illustrated in Figure (4-10). The teflon-coated steel wire was attached at one end to the drum and at the other to a fixed part of the extendable leader mast (see Figure (4-10). Extension of the steel wire generated an output signal that could be correlated to the penetration depth, as illustrated in Figure (4-13).

There were two backup recording systems used: \( (i) \) video recordings of the depth markers painted on the sheet pile, and \( (ii) \) verbal notation of these depth marks on a microcassette recorder.

The vertical position of the vibrator can be altered by either the leader or the support cylinder, as mentioned earlier in Section 4.2.4 (see Figure (4-8)). Which one of the two cylinders that was actually responsible for changing the vertical position of the

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\textsuperscript{1} Celesco Transducer Products, Inc., 7800 Deering Avenue, PO Box 7964, Canoga Park, CA 91309-7964, USA
vibrator was sensed by the Unibox device mounted in the vibratory switchbox (see Figure (4-12)). The Unibox consisted of a universal microprocessor module that sensed which leader cylinder moved the vibrator, and in which direction the vibrator and sheet pile were moving.

A reset button was used to zero the output voltage signal of the Unibox, and was mounted on the main switchbox (see Figure (4-25)). The reset button made it possible to correlate the position of the sheet-pile toe with zero penetration depth when it touched the surface of the ground.

The procedure for calibrating the vertical position of the vibrator (that is the penetration depth (\( z \)) of the sheet pile being driven) is described in Section 4.3.5.

![Figure 4-13 Illustration of the function of the depth-measuring drum (from Celesco’s brochure).](image)

### 4.3.2 Sheet-pile instrumentation

The instrumentation on the sheet pile consisted of ten strain-gauge circuits and three accelerometers mounted at the positions illustrated in Figure (4-14). The sheet-pile instrumentation was mounted at the following four positions:

- near the toe (four strain gauges and one axially-directed accelerometer),
- near the head (two strain gauges and one axially-directed accelerometer),
- mid-depth (two strain gauges and one laterally-directed accelerometer), and
- lower mid-depth (two strain gauges).

The arrangement of the sheet-pile instrumentation was critical to the success of the full-scale field tests. Redundancy was purposely factored into the design of sheet-pile
sensor system, due to the inherent risk of losing sensors or cables during the installation phase.

At the sheet-pile toe (known to be the most critical position), four strain-gauge circuits were mounted in holes cut out for them (see Figures (4-16) and (4-17)). The purpose of the four circuits was to monitor the magnitude of the dynamic soil resistance \( R_t \) generated at the sheet-pile toe. The holes were cut out with precision and as close to the toe as possible. The holes also provided protection to the sensors but did not change the pile-surface geometry. The sensors were assembled as close to the toe as possible, but without exposing them to excessive risk of being lost due to encountering boulders or other obstructions during the test. The holes were later filled with a two-component resin in order to protect the sensors. It should be noted that all the strain gauges mounted on the sheet pile were wired as full Whetstone bridges.

At the sheet-pile head, two strain-gauge circuits were mounted on the inside and outside surfaces of the sheet-pile web. The purpose of these two circuits was to monitor the driving force \( F_d \) generated by the vibratory equipment. The two sensors were located at a specific distance from the head were the axial forces were believed to be equally spread throughout the section of the profile. The choice of two circuits on both the inside and the outside, was based on the desire to document the bending moment induced in the sheet pile (as illustrated in Figure (3-5)). This bending moment in the sheet pile (as hypothesised in Section 3.3.3) was believed to generate a lateral motion of the sheet pile. This motion was also believed to be detectable a laterally-mounted accelerometer.

The decision to use three accelerometers only was based on the fact that they are more robust than strain-gauge circuits. It should be noted that the accelerometers were small enough to fit into the holes cut into the sheet piles. The two axially-mounted accelerometers were positioned as near to the head and toe as possible, for being able to verify the axially-rigid behaviour hypothesised in Section 3.3.3. However, no accelerometers were positioned on the vibrating part of vibrator unit. Based on the work of Bosscher et al. (1998), it was assumed that the vibrator and the sheet pile vibrated essentially in phase. The laterally-mounted accelerometer was mounted midway along the length of the sheet pile, and was of the same type as the two axially-mounted ones.

Strain-gauge circuits were also positioned at mid and lower-mid positions along the sheet pile, with the purpose of acting as additional reserve gauges, and with the added bonus of providing the means to study the distribution of the dynamic soil resistance between the head and toe of the sheet pile being driven.
Figure 4-14: Positioning of the sensors on the 14 m sheet pile profile PU-16.

Fig. A: Instrumentation at sheet pile toe

I. \( a_x \): Axial acceleration near toe, channel 6.
II. \( S_{1w} \): Strain gauge circuit left web, channel 13.
III. \( S_{2w} \): Strain gauge circuit right web, channel 14.
IV. \( S_{1f} \): Strain gauge circuit left flange, channel 15.
V. \( S_{2f} \): Strain gauge circuit right flange, channel 16.

Fig. B: Instrumentation at lower midposition

VI. \( S_{3w} \): Strain gauge circuit left web, channel 11.
VII. \( S_{4w} \): Strain gauge circuit right web, channel 12.

Fig. C: Instrumentation at midposition

VIII. \( a_y \): Lateral acceleration, channel 5.
IX. \( S_{5w} \): Strain gauge circuit left web, channel 9.
X. \( S_{6w} \): Strain gauge circuit right web, channel 10.

Fig. D: Instrumentation at sheet pile head

XI. \( a_z \): Axial acceleration near sheet pile head, channel 4.
XII. \( S_{7w} \): Strain gauge circuit inside of web, channel 7.
XIII. \( S_{8w} \): Strain gauge circuit outside of web, channel 8.

Cut out holes, see Figure 4-17:
- 6 No. 100 x 30 [mm]
- 2 No. 100 x 20 [mm]

Cut out holes, see Figure 4-15:
- 4 No. 100 x 30 [mm]

Switch box on the sheet pile, positioned 1 500 [mm] from head, see Figure 4-22.

Signal cable to main switch box, see Figure 4-25.
Strain gauges

All 10 strain-gauge circuits mounted on the sheet pile were wired as full Wheatstone bridge circuits, with the positions shown in Figure (4-14). The strain gauges used during the full-scale field tests were foil type strain gages (N11-FA-5-350-11) from Showa Measuring Instrumentation Co. Ltd. (Japan), and were 5 mm in length with a resistance of 350 ohm (see Figure (4-16)). The passive (Poisson effect) gauges (N11-FA-2-350-11), were 2 mm in length with a resistance of 350 ohm (see Figure (4-16)).

The two strain-gauge circuits at the head of the sheet pile (see Figures (4-18) and (4-19)) were mounted on the surface of the web at an adequate distance from the head of 1.5 times the equivalent diameter. The reason for mounting these sensors at 1500 mm from the sheet-pile head was to minimise the influence of a concentration of stress near the hydraulic clamp holding the sheet pile. Positioning two gauges inside and outside the web near the head was decided because of the desire to study the sinusoidal time-dependent bending of the sheet-pile section, described in Section 3.3.3. This is induced due to the eccentric position of the hydraulic clamping device in relation to the centric line of the sheet-pile section used.

Figure 4-15 Positioning of the strain gauges used in the sheet-pile sections.
The gauges mounted along the sheet pile were placed inside specially cut holes in the sheet-pile section. The dimension of these holes were either 20 or 30 mm wide and 100 mm long (see Figure (4-16)). The power and output-signal leads running to and from the gauges in the holes and the two L-shaped (25 x 25 x 3) profiles were protected by grooves cut into the sheet-pile surface, as shown in Figure (4-16). The holes and grooves were later filled with a two-component resin (NM EL Epoxy 960 and NM 980 Accelerator) in order to protect the sensitive sensors and signal cables (see Figures (4-17) and (4-19)).

![Diagram](image)

**Figure 4-16  Positioning of the Whetstone-wired strain gauges in the hole cut out in the sheet-pile profile.**

The four strain-gauge circuits at the toe of the sheet pile (see Figure (4-17)) were mounted inside the profile section along the web and the flange of the U-shaped profile, as illustrated in Figure (4-16). The holes were cut out precisely and as close to the toe as possible. The holes served as protection for the sensors and did not change the geometry of the section used. The purpose of mounting the sensors 600 mm from the sheet-pile toe was primarily for protecting them from damage that could be caused by the toe encountering any obstructions in the subsoil strata. As mentioned before, the purpose of using the four circuits was to monitor the magnitude of the dynamic soil resistance ($R_t$) at the sheet-pile toe. However prior to the tests, it was feared that the penetrative resistance force ($R_p$) might be too small to be detected with the instrumentation and acquisition system used, so a discussion ensued as to how and where the sensors should be best positioned at the sheet-pile toe.
Figure 4-17 Pouring two-component resin into a protective hole for the strain gauges, which was cut out in the web near the toe.

Figure 4-18 Accelerometer and strain-gauge circuit mounted on the inside of the web, near the sheet-pile head.
Amplifiers

The output signal from the strain gauges needed to be amplified for better detecting the relatively weak output signal. Amplification was achieved with three PCP-241 amplifiers, each with four separate channels and individual power supply, that were rented from the Marcus Wallenberg Laboratory, Division of Sound and Vibrations, Department of Vehicle Engineering, at the Royal Institute of Technology (Sweden). The amplifiers have been especially developed for application with strain gauges, with an amplification range of 1 to 3160 times, in increments of 10 dB.

The read-out signals from the strain gauges were amplified 100 times and balanced to zero with an inbuilt potentiometer in each amplifier.

Accelerometers

The three low-gravity accelerometers mounted on the sheet-pile profile were specially designed and mounted on printed circuit boards, also specially developed for the application by a company called Geometrik AB. The three accelerometers (ADXL150AQC) were single-axial and surface micro-machined, manufactured by Analog Devices Inc., and had a range of +/-50 g. This differential capacitor sensor is composed of fixed and moving plates. The moving plates connected to the beam generate the differential capacitance, which is measured by the chip circuitry, as illustrated in Figure (4-20).

On the inner side of the sheet-pile web near the head of the sheet pile, one axially-mounted accelerometer was first glued to the surface of the steel (see Figure (4-18)) using an epoxy glue, and then mechanically protected (see Figure (4-19) by a two-component epoxy raisin (NM EL Epoxy 960 with Accelerator NM 980).

Measurements of the driving frequency and accelerations induced at the head could be made via the mounted accelerometer, which also later permitted evaluation of the velocity, the driving energy generated at the head, the calculation of inertial forces and also the study of any phase shifts between the acceleration signals recorded at the head and those recorded at the toe.

The laterally-mounted accelerometer was placed at a mid-position along the sheet pile in order to evaluate the magnitude of the lateral motion due to the eccentrically-induced driving force at the top (addressed earlier in Section 3.3.3).
This accelerometer was of the same type as the two-axially mounted ones and was mounted on a small enough circuit board to fit into the holes cut into the 12 mm thick sheet-pile web. The signal cables ran along recessions in the sheet-pile surface from the holes cut into it to the corners of the profile, where the two welded L-shaped profiles served as protection.

**Switchboxes on the sheet pile and the protection of signal cables**

At the head of the sheet pile a switchbox was mounted connecting all the cables from each sensor. The switchbox was bolted onto a non-corrosive steel plate, which was bolted onto the sheet pile (see Figure (4-22)). The switchbox consisted of a watertight electronics box made of plastic, containing the printed circuit board with a screw-type connection block fused to it, to which all the signal and power cables could be connected.

Each sensor cable leaving the two protective L-shaped profiles in the corner of the sheet-pile web (flange) entered the switchbox from the sides via a cable-tension relieving mounting bracket with a rubber bushing (see Figure (4-22)). The signal cables left the sheet-pile switchbox via a special steel-reinforced, rubber-coated signal cable that was connected to the main switchbox (see Figure (4-22)).

![Figure 4-22](image)

*Figure 4-22  The sheet-pile switchbox mounted on the inner side of the sheet-pile web, with signal cables from the sensors entering from both sides and leaving the box (and sheet pile) to the right.*
Experimental work

4.3.3 Soil instrumentation

The soil instrumentation consisted of three tri-axial geophones (see Figure (4-24)), positioned at three different radial distances from the sheet piles being driven, and illustrated by Figure (4-10). The various radial positions of the three tri-axial geophones are given in Table (4-1).

The geophones used in the tests were manufactured by Sensor Nederlands, (SM-1), and had a resonance frequency of 4.5 Hz, and an output signal of 33.33 V/m/s. The directions of the three tri-axial geophones relative the sheet pile being driven are shown in Figure (4-10), using the following three indexes:

- **V** representing the vertical direction.
- **L** representing the longitudinal direction, and
- **T** representing transversal direction.

![Figure 4-23 Protection of the signal cables by the two L-shaped profiles.](Figure4-23.png)

![Figure 4-24 The tri-axial geophone unit used in the test, bolted onto an oedometer weight with soil spikes welded on the plate.](Figure4-24.png)
The geophones chosen were determined to be the most suitable sensors for monitoring the vibrations generated in the ground during the installation phase. This was mainly due to their robustness and the fact that they did not need a power supply. The only extra equipment needed besides the geophones, were signal cables and enough free channels in the data-acquisition equipment.

### 4.3.4 Data-acquisition equipment

The data-acquisition system used to record and document the vibratory-installation of the sheet pile is illustrated in Figures (4-10) and (4-11), and consisted of the following three parts:

- the main switchbox,
- the DAT tape recorder, and
- the expansion unit.

#### Main switchbox

In order to be able to assemble the different parts of the instrumentation equipment as fast and simply as possible, as well as to minimise the risk of introducing human error, the “main switchbox” consisted of a standard electronics box (Figure (4-25)) with the following connections:

- a power supply for the accelerometers, strain gauges and amplifiers,
- the sheet-pile signals,
- the vibrator signals,
- the three amplifiers, and
- the DAT tape recorder.

The power supply connected to the main switchbox consisted of a 12 volt battery accumulator. However, the accelerometers and strain-gauge sensors required a lower power signal, so two power-supply regulators were built into the main switchbox, which generated:

- 5 V power for the accelerometers, using a 78L05 regulator, and
- 10 V power for the strain gauges, using a 78L10 regulator.

The main switchbox was also equipped with a reset button, used for resetting or zeroing the input signal coming from the depth-measuring drum, as mentioned previously in Section 4.3.1.
DAT tape recorder and expansion unit

The data obtained during the full-scale field tests were recorded on a portable 16-channel DAT tape recorder (SONY™ PC216Ax). To be able to monitor all 25 channels described in Table (4-7), a portable 16-channel expansion unit (SONY™ PCCX32Ax) was connected to the DAT tape recorder, as shown in Figure (4-26).

The following is a brief description of the digital tape recorder specifications:

- 16 separate channels,
- sampling frequency up to 6 kHz per channel,
- 240 AC, or 12 DC power supply, and
- one acoustic channel for audio (verbal) recording if desired.

During recordings that used all 32 channels, the data were sampled with a 2.5 kHz, 16-bit quantification mode. Verbal recordings were continuously made on the acoustic (voice) channel of the tape recorder, noting for example, the test event and the passage of the various depth markers on the sheet pile past the surface of the ground. The acoustic depth-markers on the DAT tape together with the video recordings of the driveability tests, served as a back-up system for the sheet-pile penetration depth, further discussed in the section called “Methods of analysis of data collected”.

Figure 4-26 A SONY™ PC216Ax DAT tape recorder, connected to a PCCX32Ax expansion unit.
Chapter 4

PCscan

A software package named called PCscan MK II accompanied the DAT tape recorder. This software package served as both an analysing and an interfacing tool between the tape recorder and the computer to which the data recorded were exported.

The recorded DAT tapes contained the output voltage signals from each sensor (the raw data). The PCscan software made it possible to view the data in real time during recording, as well as after downloading (see Figure (4-27)). In both view modes, PCscan offers six types of basic analysis methods, of which four are available in real-time mode. The six types of analyses are:

- power spectrum, plotting the frequency power spectrum,
- $1/3$rd octave band, plotting the $1/3$rd octave band analysis data,
- histogram, plotting the histogram,
- FIR filter, plotting the FIR filtered data,
- differentiation, plotting either the first or second derivative of the data,
- integration, plotting either the first or second integration of data, and
- $x$-$y$ plot, plotting data from selected channels on $x$ and $y$ axis respectively, were the $x$ axis is data from one of the selected channels as a variable.

Figure 4-27 Screen view showing how PCscan displays the data for each of the 25 channels taken from the whole driveability test of sheet pile B3.
Figure (4-27) shows a screen view of the amplitude values in the time domain for all the 25 channels recorded testing sheet pile B3. Initial studies of the input data displayed in Figure (4-27) utilised features such as zooming, viewing the numerical data, or performing the other basic analyses listed above.

The raw signals were subsequently subject to post-analysis treatment as described in Section 4.4.

**Video camera**

The main purpose for using a video camera was to provide a back-up system for monitoring the penetration speed ($v_p$). This was accomplished by video-recording how fast the depth markers painted on the sheet piles at every half a metre entered the soil strata. The penetration speed recorded on video was later compared with the results from the data analyses from the depth-measuring drum.

Since the video camera was used during the entire driving session, it also served as an independent source of information every time something that was unclear arose in the post-analysing phase of the driveability signals.

**Visual monitoring of displacement amplitudes**

Two kinds of stickers were stuck onto the sheet piles at every metre (see Figure (4-28)) in order to visually monitor both the vertical and lateral double-displacement amplitude. Visual readings were taken each metre as the stickers and sheet pile entered the soil strata. The axial and lateral double-displacement readings together with the penetration depth ($z [m]$) were noted on a microcassette recorder. The idea of using stickers for monitoring the axial displacement amplitude was obtained from the vibratory-machine manufacturer company ICE\(^1\).

Monitoring the lateral displacement amplitude, as previously described in Section 3.3.3, was accomplished by rotating the original sticker through 90 degrees.

The principle used here, is that the human eye is able to make visual readings when the velocity is zero, that is at the end points of the sinusoidal motion. The visual reading generates a fuzzy picture of two stickers instead of one. Readings of the double-displacement amplitude of 12 mm are taken at the cross-section of the two fuzzy stickers (horizontal scale and the slope) generated as illustrated in Figure (4-29).

---

1. International Construction Equipment BV, Hefbrugweg 6, 1332 AN Almere, The Netherlands
4.3.5 Calibration of the instruments

Prior to conducting the driveability tests, calibration of both sensors and instruments were done to determine the relationship between the input and output from the different devices. The readings were compared to standard known calibrations (Dunnicliff, 1988). The calibration procedures are described and presented in the subsections that follow, and the calibration constants obtained have been summarised in Table (4-8) at the end of this section.
Calibration of oil pressure and the position of the eccentric weights

The sensors for oil pressure and the position of the eccentric weights were calibrated by the vibrator manufacturer (ABI). Two sensors were mounted on the vibrator equipment with the assistance of an instrumentation technician from ABI. After the two sensors had been mounted they were function tested by the same instrumentation technician. The same kind of procedure was repeated prior to conducting the actual field tests. The function test consisted of a procedure of pressing the hydraulic clamping device; that is pressing the vibrator against a large steel plate lying on the ground, and at the same time monitoring both the oil pressure manometer and voltage readings from the output signal from the pressure transducer.

The static surcharge force \( F_o \) generated by the vibratory equipment was controlled primarily by adjusting and monitoring the oil pressure in the hydraulic-leader cylinder shown in Figure (4-8). Maximum oil pressure in the leader cylinder is 185 bar, corresponding to maximum static surcharge force \( F_o \) of 70 kN. The oil-pressure transducer could detect pressures in the range of 0-400 bar, corresponding to an output signal of 1-5 V.

The correlation between output voltage and static surcharge force has been calculated according to the following equation:

\[
F_o = \frac{70}{185} (x_{op} - 1) \cdot 100 \tag{4.1}
\]

where \( F_o \) = static surcharge force [kN],
\( x_{op} \) = output voltage from the oil-pressure sensor [V],
70 = maximum pull-down force [kN], and
185 = maximum oil pressure in the hydraulic cylinder [bar].

The position of the rotating eccentric weights (see Figure (4-6)) in relation to their maximum value was controlled by adjusting their relative positions on the fly, and at the same time monitoring the output voltage signal from the position sensor. The output signal varied between 0-5 V, corresponding to a range of \( 0 < \eta < 1.0 \) in the maximum static moment (12 kgm as shown in Table (4-5)).

The correlation between the output voltage of the position sensor and the variable static moment have been calculated according to the following equation:

\[
M_e = \eta M_{e,\text{max}} = \frac{x_p}{5} \cdot 12 \tag{4.2}
\]

where \( M_e \) = variable static moment [kgm],
\[ \eta = \text{relative positions of the eccentric weights [\%],} \]
\[ M_{e,\text{max}} = \text{maximum static moment that could be applied [kgm], and} \]
\[ x_p = \text{output voltage from the eccentric-weight position sensor [V].} \]

**Calibration of the depth-measuring drum**

The output signal provided by the depth-measuring drum (see Figure (4-13)), was calibrated using an ordinary tape measure. The vibrator-unit was moved through a range of positions in the vertical plane and readings from the tape measure and the voltage output were correlated.

As already mentioned in Section 4.2.4, the vertical position of the vibrator was changed using either one of the leader cylinders (see Leader cylinders I and II in Figure (4-8)). The actual leader cylinder responsible for changing the vertical position of the vibrator was detected by a device called a Unibox, containing a microcomputer capable of sensing the cylinder and the vertical direction of the vibrator as it was moved. The correlation between the output voltage of the Unibox (microcomputer) and the vertical position \( (z) \) was calculated to be in the order of \( \pm 4 \text{ m/V} \), which was in accordance with the specifications. The reset button mounted on the main switchbox (see Figure (4-25)) that was used to zero the output signal of the Unibox, was also tested and worked adequately.

**Calibration of the accelerometers**

The three accelerometers mounted on each sheet pile were calibrated by comparison with a reference accelerometer (Brüel & Kjær 4369, series no. 1016176), borrowed from the Marcus Wallenberg Laboratory, Division for Sound and Vibrations, Department of Vehicle Engineering, at the Royal Institute of Technology (Sweden). Both the reference accelerometer and each of the accelerometers to be calibrated were subject to the same sinusoidal vibratory motion using white noise as the input signal to an electrical vibrator. The output signals from the two accelerometers were connected to a frequency analyser (Hewlett Packard 3582A). The output signal from the reference accelerometer was first amplified by using an Brüel & Kjær (2635) amplifier before being connected to the HP frequency analyser (see the test setup in Figure (4-30)). The HP frequency analyser monitored the magnitude of the transfer function, phase, and coherence. Screen dumpings of the transfer function can be found in Appendix D.
Experimental work

Figure 4-30 The calibration setup for the accelerometers. The power supply for the vibrator and the digital oscilloscope sitting above it can be seen on the left side of the workbench, and the HP frequency analyser can be seen to the right of these. The power supply for the accelerometer and the vibrator can be seen on the floor, and the amplifier for the reference accelerometer can be seen sitting on top of the chest of drawers under the bench.

From the known voltage sensitivity of the reference accelerometer, the acceleration could be calculated. The voltage output recorded by each calibrated accelerometer was then compared to obtain the voltage sensitivity for each calibrated accelerometer.

From the calibrations, it was determined that the voltage sensitivity of the three accelerometers was 40 mV/g over the frequency range investigated (12-40 Hz), which was in agreement with the specifications (illustrated in Figure (4-31)).

Calibration of the load cell

A 20-tonne Bofors load cell was used in the calibration procedure for the strain gauges. On the right of Figure (4-33), the load cell used to monitor the axial load applied to the sensor-fitted sheet pile can be seen, and on the left the hydraulic jack used to apply the load.
Chapter 4

Figure 4-31  Screen dump of the calibration of the laterally-mounted accelerometer from the oscilloscope, showing the $a_1$ phase curve above and the transfer signal below, taken over the frequency range of 0-50 Hz.

The load cell was calibrated using the hydraulic press in the testing laboratory at the Division of Soil and Rock Mechanics, at the Royal Institute of Technology (Sweden). Loads of up to 200 kN were applied in increments of 10 kN. The relationship between the output signal from the load cell and the hydraulic pressure was found to be linear (see Figure (4-32)).

![Figure 4-32](image)

Figure 4-32  The calibration curve for the load cell used in the tests.
Calibration of the strain gauges

Foil strain gauges were used for the full-scale field tests, manufactured by Showa Measuring Instrumentation Co. Ltd. (Japan). The strain-gauges circuits glued and mounted onto the sensor-fitted sheet pile were all wired as full Whetstone bridge circuits, at the positions shown in Figure (4-14).

During the strain-gauge calibration procedure, the sensor-fitted sheet pile was axially compressed at one end by a hydraulic jack, and the load applied was monitored by a 20-tonne Bofors load cell. The sheet pile was not calibrated in tension as it was assumed that the calibration constants determined from compression loading could also be applied to tension loading. The calibration data related to each strain gauge are presented in Appendix D.

An illustration of the calibration assembly is shown in the Figures (4-33) and (4-34). The load cell and hydraulic jack were both positioned as close to the neutral axis of the sheet pile as possible in order to minimise the effects of bending. The 14-metre compression-loaded sheet pile rested on an HEA 400 beam 16 m in length, with two reaction plates at both ends (two HEA 200 beams welded onto the flange of the HEA 400 beam). Behind the two HEA 200 beams were two triangular-shaped steel plates, also welded on for extra precaution. The loads were applied incrementally, and maintained for about two minutes during the final calibration procedure in order to be able to complete all the 10 manual readings of the change of resistance. Loads were applied in increments of about 30 kN, up to 150 kN. Readings of the change in the resistance were obtained using the same power supply, amplifiers, cables and switchboxes belonging to the test setup as earlier shown in Figure (4-10), and which were subsequently used during the actual full-scale field testing.

Since the resistance of each glued strain gauge depends on so many variables (such as the quality of the bond to the surface and the direction of the stresses generated in relation to the placement of the gauge), calibration procedures were conducted for each one of the various gauges when they had been glued on and connected to the power supply and all of their respective signal cables. Several loadings and unloadings were repeated during the time it took to glue each gauge (a period of days) in order to “exercise” the sheet pile and mitigate the effects of residual stresses from the welded L-shaped profiles and the two HEA 200 reaction beams welded onto the flange of the HEA 400 beam.

The readings from the strain gauges were analysed in order to determine the relationship between the compressive load applied and the change in resistance of each strain gauge, in other words the relationship between the load applied (in kN) and the output reading (in mV or V).
The relationship between the applied and recorded loads was linear across the range of actual loads applied. Since amplifiers were used in the final calibration procedure, only the slope ($k$) or the calibration factor (in kN/mV or kN/V) for each strain-gauge circuit was of interest. An example of the calibration curve obtained for the strain-gauge circuit relative to the right toe-flange ($k_{rtf}$) position is shown in Figure (4-35). The calibration curves for all the strain gauges belonging to Channels 7-16 are shown in Appendix D, and summarised in Table (4-8). The procedure for the calibration of the strain

**Figure 4-33** Illustration of the test arrangement used for calibration of the strain-gauge circuits.

**Figure 4-34** Pictures of the set-up of the strain-gauge calibration. The picture on the left shows the jack and reaction beams at the toe, and the picture on the right shows the load cell and the reaction beams at the head of the sheet pile.
gauges were all performed in Stabilator AB’s pile factory, situated in Västerhaninge, 40 km south of Stockholm (Sweden).

Figure 4-35  Example of a calibration curve for strain gauge in the right toe-flange (krtf) on Channel 16.

Calibration of the geophones

The nine geophones used during the full-scale field tests were all calibrated in the laboratory of the Division of Soil and Rock Mechanics, Royal Institute of Technology (Sweden) prior to the commencement of actual field tests. The same electric vibrator and test set-up used for the accelerometers were also used here (see Figure (4-30)). From the calibrations, it was determined that the voltage sensitivity of the nine geophones was 33.33 V/m/s over the frequency range investigated; which agreed with the specifications.

Background measurements of ground vibrations were also conducted one week prior to the full-scale field test. This was deemed necessary as the test site is situated relatively close to a major European motorway (the E4), of the steel girder bridge across the Fittja Strait (see Figure (4-1)).

The background ground-vibration measurements were conducted before lunch on a normal working day using the three tri-axial geophones and the DAT tape recorder. The results showed that the vertical vibration amplitude could sometimes reach peak values as high as 0.2-0.4 mm/s when for example a heavy truck passed the bridge-joint. Figure (4-36) shows the filtered vertical ground-vibrations over a time period of 8.3 minutes. The seven to eight vibration peaks, were recorded as trucks and buses passed over the “broskarv” nearby.
Figure 4-36  Results of the vertical ground-vibration measurements taken over an 8.3 minute period, with an RMS and peak value of 0.026 and 0.25 mm/s respectively.

An FFT analysis of the above mentioned 8.3 minute period showed that most of the information was found around 50 Hz. This was believed to be related to electrical mains noise, which resulted in the development of a 6th order Butterwort band exclusion filter. The filter was applied with the two “roll-off frequencies” of 45 and 55 Hz. Further information is found in the MATLAB file called `bakgrundsvib.m` found in Appendix D. The FFT analysis conducted after filtering is shown in Figure (4-37), and the RMS and peak value of the filtered background vibrations over the 8.3 minutes were found to be 0.026 mm/s and 0.25 mm/s respectively.
Experimental work

The raw data in the Ascii file exported from the SONY™ DAT recorder were calibrated in MATLAB™ prior to the post-analysis procedures described in next section.

The calibration functions of the output voltage signals obtained have been summarised in Table (4-8). The sign convention used for the signals is that positive acceleration occurs during the bottom half of the downward motion and is read as deceleration, while positive section forces in the sheet pile correspond to compression.

**Table 4-8  Calibration relations for each sensor, where x denotes the output voltage signal of the sensor.**

<table>
<thead>
<tr>
<th>Channel</th>
<th>Calibration function</th>
<th>Explanation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F_o = 37.84(x - 1)$</td>
<td>Static surcharge force of the vibratory driver, 1-5 V/DV or 0-24 mA</td>
<td>kN</td>
</tr>
<tr>
<td>2</td>
<td>$F_p = 160x$</td>
<td>Percentage of the maximum static moment of the vibratory driver</td>
<td>kN</td>
</tr>
<tr>
<td>3</td>
<td>$z = -4.0x$</td>
<td>Penetration depth below the ground surface, rectangular impulse</td>
<td>m</td>
</tr>
</tbody>
</table>

**Figure 4-37  FFT analysis of the 8.3 minute recording of the background vibrations.**
Table 4-8  Calibration relations for each sensor, where $x$ denotes the output voltage signal of the sensor.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Calibration function</th>
<th>Explanation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$a_h = 9.81x/0.040$</td>
<td>Axial head acceleration</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>5</td>
<td>$a_l = 9.81x/0.040$</td>
<td>Lateral acceleration</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>6</td>
<td>$a_t = 9.81x/0.040$</td>
<td>Axial toe acceleration</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td>7</td>
<td>$S_{hi} = 1718.2x$</td>
<td>Head strains at the inside of the sheet-pile webb</td>
<td>kN</td>
</tr>
<tr>
<td>8</td>
<td>$S_{ho} = 1763.3x$</td>
<td>Head strains at the outside of the sheet-pile webb</td>
<td>kN</td>
</tr>
<tr>
<td>9</td>
<td>$S_{lm} = 1769.0x$</td>
<td>Left-side strains at mid-depth</td>
<td>kN</td>
</tr>
<tr>
<td>10</td>
<td>$S_{rm} = 1674.3x$</td>
<td>Right-side strains at mid-depth</td>
<td>kN</td>
</tr>
<tr>
<td>11</td>
<td>$S_{llm} = 999.3x$</td>
<td>Left-side strains at lower mid-depth</td>
<td>kN</td>
</tr>
<tr>
<td>12</td>
<td>$S_{rlm} = 1293.7x$</td>
<td>Right-side strains at lower mid-depth</td>
<td>kN</td>
</tr>
<tr>
<td>13</td>
<td>$S_{lw} = 1211.7x$</td>
<td>Left-side strains at the toe webb</td>
<td>kN</td>
</tr>
<tr>
<td>14</td>
<td>$S_{rww} = 1305.3x$</td>
<td>Right-side strains at the toe webb</td>
<td>kN</td>
</tr>
<tr>
<td>15</td>
<td>$S_{lhf} = 906.2x$</td>
<td>Left-side strains at the toe flange</td>
<td>kN</td>
</tr>
<tr>
<td>16</td>
<td>$S_{rfw} = 973.6x$</td>
<td>Right-side strains at the toe flange</td>
<td>kN</td>
</tr>
<tr>
<td>17-25</td>
<td>$V_1, L_1, \ldots, T_3 = 33.33 x$</td>
<td>Static surcharge force of the vibratory-driver, 1-5 V/DV or 0-24 mA</td>
<td>mm/s</td>
</tr>
</tbody>
</table>
4.4 Analysis and post-analysis methods

The following sections describe the analysis and post-analysis procedures that were used to reduce the digitised raw-data into the physical properties described in Sections 3.2 - 3.4.

All test records obtained from the vibrator, the sheet pile, and the soil instrumentation were analysed by extracting all the digitised raw-data records from each driveability test, using the MATLAB™ computer programme.

4.4.1 General data-reduction procedures

The files containing the raw data from the digitised test records were stored on magnetic tapes with the SONY™ DAT recorder. The raw data had a sample rate of 6 kHz for each of the 25 channels. However, the original raw data records were later reduced to one thousand samples per second, when exported as Ascii files. This was accomplished using the PCscan software. Though some may consider this level of data reduction to be heavy, it was chosen to be on the safe side. The sampling theorem stipulates that the sampling rate should ensure that the interesting information is to be found way below half the sampling frequency applied (500 samples/s). The interesting information is found way below a sampling rate of 1 kHz, since the vibrator worked around 40 Hz.

Though there is no theorem to determine the number of discrete levels of quantifications, it is normally understood that “the more the better” applies here. The number of discrete levels were of concern in relation to the relatively weak signals associated with the four strain-gauge circuits at the toe that were monitoring the dynamic toe resistance.

The number of discrete levels is \(2^n\), where \(n\) is the number of bits used to quantify the monitored signal. The DAT recorder used 16 bits, which is therefore \(2^{16}\). Since the DAT recorder had a range of sensitivity of +/- 5 V, this becomes 0.15 mV per discrete level according to the following expression:

\[
\frac{5 - (-5)}{2^n} = \frac{10}{2^{16}} = 0.153 \text{ mV}
\]  

(4.3)

However this never caused any concern, due to the simple fact that the strains developed near the toe were strong enough to be recorded.
4.4.2 Vibratory driver data

The theoretical driving force

The theoretical driving force \( F_d = F_o + F_v \) [kN] generated by the vibratory-driving equipment was derived from the digitised time histories of the voltage readings from the sensors mounted on the vibrator. \( F_d \) was derived by combining the results described by Equations (4.1) and (4.2) together with the FFT analysis of the driving frequency used, which can be expressed by the following:

\[
F_d = F_o + F_v = 37.84 \left( x_{op} - 1 \right) + \frac{12x_p}{5} \cdot \frac{\left( 2\pi f_d \right)^2}{1000} \tag{4.4}
\]

where

- \( F_d \) = driving force [kN],
- \( F_o \) = static surcharge force [kN],
- \( F_v \) = unbalanced force [kN],
- \( x_{op} \) = output voltage from the oil-pressure sensor [V],
- \( x_p \) = output voltage from the eccentric-weight position sensor [V],
- \( f_d \) = driving frequency [Hz].

The following assumptions were made in order to calculate the theoretical driving force \( F_d \) acting on the sheet pile:

- the mechanical energy losses in the vibrator were not considered,
- sliding friction between the vibrator and the leader was ignored, and
- additional factors were also excluded.

The post-analysis steps described above have led to the sinusoidal-biased driving force presented in the next Chapter.

Penetration depth

The penetration depth “histories” \( \zeta(t) \) for the sheet piles driven in the tests were derived from the digitised time histories of the voltage readings coming from the Unibox mounted in the vibratory switchbox (see Figure (4-12)). The following linear relationship was applied to the output voltage readings taken from the microcomputer working inside the Unibox:

\[
\zeta = \pm 4x
\tag{4.5}
\]

where \( \zeta \) = penetration depth [m], and
\( x \) = output voltage reading [V].

It should be noted that the primary way of analysing the penetration depth described by Equation (4.5) was checked initially by two back-up systems. As mentioned previously, the two back-up systems consisted of video-recording the depth markers painted on the sheet pile and verbal notes recorded on a microcassette recorder. However, the primary way of analysing the penetration-depth time histories proved satisfactory.

### 4.4.3 Sheet pile-related data

**Axial acceleration, velocity and displacement time histories**

The axial acceleration, velocity and displacement time histories were all derived from the digitised time histories of the voltage readings from the two axially-mounted accelerometers mounted on the sheet pile. The velocity \( v(t) \) and the displacement \( u(t) \) time histories were obtained through the procedure described below for the integration of the acceleration \( a(t) \) with respect to time, using the trapezoidal rule for numerical evaluation.

The following assumptions were made in order to calculate the acceleration \( a(t) \), velocity \( v(t) \), and displacement \( u(t) \) time histories:

- the sheet piles were treated as rigid bodies (see Section 3.3.3),
- sheet-pile penetration proceeded with a uniform global rate of penetration, and
- a simple force equilibrium was applied according to Figure (4-38).

The digitised acceleration-time histories \( a(t) \) of interest were collected at any depth of interest. In the analysis conducted (described below) these depths of interest were selected at 2, 6, and 10 m below the surface of the ground.

The time window for the acceleration-time histories considered interesting were generally chosen to be at \( T = 0.16 \) seconds. These time windows were located at a point in time where it could be considered that the sheet pile was penetrating with a uniform global rate of penetration; in other words where \( v_p = \text{constant} \). This assumption was confirmed by the video recordings taken during the field tests.

Before the actual integration process were performed, it was needed to ensure that the zero offset of the digitised, measured acceleration-time histories of interest was small. If not, the correction procedure for the acceleration signal described below was
applied. Further information relating to the zero-offset procedure can be found for example in Hudson (1979), Dowding (1996) and Johansson (1996).

The zero-offset-corrected acceleration signal \( a_{\text{corr}} \) was accomplished by correcting the acceleration measured \( a_m \) with the average value \( a_{av} \) over the time window considered \( T \), together with a manual fine tuning constant \( k_a \) according to the following expression:

\[
  a_{\text{corr}} = a_m - (a_{av} + k_a)
\]  

(4.6)

where:  
\( a_{\text{corr}} \) = zero-offset-corrected acceleration signal [m/s\(^2\)],  
\( a_m \) = measured acceleration signal [m/s\(^2\)],  
\( a_{av} \) = average acceleration in the time window considered [m/s\(^2\)],  
and  
\( k_a \) = manual constant for fine-tuning the zero offset [m/s\(^2\)].

The zero-offset corrected acceleration signal \( a_{\text{corr}} \) was then integrated into the desired velocity-time histories \( v(t) \) according to:

\[
  v(t) = \int_{0}^{T} a_{\text{corr}} \, dt
\]  

(4.7)

where:  
\( a_{\text{corr}} \) = zero-offset-corrected acceleration signal [m/s\(^2\)],  
\( a_m \) = measured acceleration signal [m/s\(^2\)],  
\( a_{av} \) = average acceleration in the time window considered [m/s\(^2\)],  
and  
\( k_a \) = manual constant for the fine-tuning of the zero offset [m/s\(^2\)].

The integrated velocity-time signal was then checked in order to ensure that the average velocity in the time window considered was zero. If this is not the case, then the manual fine-tuning constant \( k_a \) in Equation (4.6) was adjusted until the integrated velocity-time signal \( v(t) \) oscillated around the horizontal time axis.

The velocity-time signal was then integrated to the desired displacement time histories \( u(t) \). The same zero-correction procedure used for the measured acceleration signal was applied to the integrated velocity-time signal \( v(t) \). The velocity signal was zero-corrected with the average value \( v_{av} \) for the time window considered \( T \), together with a new manual fine-tuning constant \( k_v \) according to:

\[
  v_{\text{corr}} = v(t) - (v_{av} + k_v)
\]  

(4.8)
where: \( v_{corr} \) = zero-offset-corrected acceleration signal [m/s], 
\( v(t) \) = integrated velocity signal [m/s], 
\( v_{av} \) = average velocity in the time window considered [m/s], and 
\( k_v \) = manual constant for fine-tuning the zero offset [m/s].

The zero-offset-corrected velocity-time history \( (v_{corr}) \) was then finally integrated. However in order to obtain the downwardly-directed sinusoidal displacement-time history \( (u(t)) \), as previously described by Equation (3.1), it is necessary to add the integration constant \( (v_p) \), which is the constant global rate of penetration at the beginning of the time window considered \( (T) \). The integration constant \( (v_p) \) was taken to be the constant global rate of penetration at the beginning of the time window considered \( (T) \), and which was obtained from depth-measuring device readings, according to \( v_p = \Delta z/\Delta t \).

The displacement-time history \( (u(t)) \) was derived according to the following expression:

\[
T 
\int_{0}^{T} (v_{corr} + v_p) \, dt 
\]

where: 
\( u(t) \) = displacement-time history [m], 
\( v_{corr} \) = corrected velocity-time history [m/s], and 
\( v_p \) = integration constant (global rate of penetration) [m/s].

The signal treatment and analysing steps described above were performed using MATLAB™. The three specially-developed MATLAB™ files referred to as accA.m, accB.m and accplot.m, were used, and can be found in Appendix D. These have led to the displacement-time \( (u(t)) \) dynamic toe versus displacement \( (R_t-u) \), and the shaft resistance versus displacement \( (R_s-u) \) curves presented in the next chapter.

**Lateral and axial acceleration amplitude versus depth**

The amplitude values of both axial and lateral accelerations have been plotted against penetration depth. The main purpose of this was to be able to document the magnitude of the laterally-induced accelerations together with the axially-rigid behaviour, as hypothesised in Section 3.3.3.

The histories of acceleration amplitude versus depth presented were derived by extracting the maximum value out of every 500 samples (that is a maximum from every 20th sinusoidal period). The main reason for choosing every 20th period instead of every period was simply to keep the calculation time as reasonable as possible.
The values of lateral displacement amplitude presented have not been derived through the double-integration procedure, as previously described, and which were applied to the axial displacement amplitude. Since it was only the magnitude of the sinusoidal time-dependent lateral motion that were of interest, it was decided to apply the following straightforward approach given by:

\[
\hat{u}_l = \frac{\hat{a}_l}{\omega^2} = \frac{\hat{a}_l}{(2\pi f_d)^2}
\]

where: \(\hat{u}_l\) = amplitude of lateral motion [m],
\(\omega\) = angular velocity [-],
\(f_d\) = driving frequency [Hz], and
\(\hat{a}_l\) = amplitude of lateral acceleration [m/s²].

The signal treatment and analysing steps described above, which have led to the graphs of lateral and axial acceleration amplitude versus depth presented were performed using MATLAB™. The specially developed MATLAB™ files referred to as accA.m, accB.m and accplot.m, were used, and can be found in AppendixD.

**Driving force, toe and shaft resistance histories**

The driving force and the dynamic soil resistance histories at the toe and the shaft have been plotted over time \(t\) and against both displacement \(u(t)\) and penetration depth \(z\).

These forces acting on the sheet pile as hypothesised in Section 3.2.3 and visualised in Figure (3-1), have been derived from the digitised time histories of the voltage readings taken from the 10 strain-gauge circuits mounted on the sheet pile.

When plotting the amplitude values of the forces against penetration depth \(z\), the extraction of the maximum value was derived in the same way as for acceleration amplitude versus depth; in other words the maximum was derived out of every 20th sinusoidal period. However this was not the case when plotting forces in either the time domain or against the displacement.

The following assumptions have been made in deriving these forces:

- the sheet pile was treated as a rigid body (see Section 3.3.3),
- the vibrator is assumed to have the same acceleration as the sheet pile,
- the suppressor housing is assumed not to vibrate, and
- the dynamic force equilibrium of the entire system is established according to Figure (4-38).
The sign convention used for the strain-gauge circuit signals is that positive section forces in the sheet pile correspond to compression (see Figure (4-38)).

The section force near the sheet-pile head (representing the actual driving force delivered to the sheet-pile head) for all tests was calculated as the average value of the calibrated voltage readings of the two strain-gauge circuits mounted near the head, according to:

\[ S_b = \frac{S_{bi} + S_{bo}}{2} \quad (4.11) \]

where:  
- \( S_b \) = average value of forces measured near the sheet-pile head [kN],  
- \( S_{bi} \) = force on the inside of the sheet-pile web, Channel 7 [kN], and  
- \( S_{bo} \) = force on the outside of sheet-pile web, Channel 8 [kN].

The section force \( S_t \) near the sheet-pile toe (representing the dynamic soil resistance at the toe) was calculated as the average value of the four functioning strain-gauge circuits near the sheet-pile toe.

It should be noted that during the three first tests (A1-A3), the section force \( S_t \) was calculated according Equation (4.12), and for the other tests (B1-B3) it was calculated according to Equation (4.13).

\[ S_t = \frac{S_{ltw} + S_{rtw} + S_{lrf} + S_{rtf}}{4} \quad (4.12) \]

where:  
- \( S_t \) = average value of forces measured near the sheet-pile toe [kN],  
- \( S_{ltw} \) = force measured at the left toe-web, Channel 13 [kN],  
- \( S_{rtw} \) = force measured at the right toe-web, Channel 14 [kN],  
- \( S_{lrf} \) = force measured at the left toe-flange, Channel 15 [kN], and  
- \( S_{rtf} \) = force measured at the right toe-flange, Channel 16 [N];

and  
\[ S_t = \frac{S_{ltw} + S_{rtf}}{2} \quad (4.13) \]

where:  
- \( S_t \) = section force near the sheet-pile toe [kN],  
- \( S_{ltw} \) = force measured at the left toe-web, Channel 13 [kN], and  
- \( S_{rtf} \) = force measured at the right toe-flange, Channel 16 [N].
The dynamic soil resistance at the sheet-pile toe \( (R_t) \) has been interpreted without taking into account the inertial force acting on the 600 mm section of the sheet pile below the position of the four strain-gauge circuits. This was accomplished by the above stipulated assumptions and the dynamic equilibrium of the entire system, as illustrated by Figure (4-38), and calculated according to:

\[
R_t = S_t - m_{mt}a_t
\]

(4.14)

where: 
- \( R_t \) = dynamic soil resistance at the sheet-pile toe [kN],
- \( S_t \) = average section force measured near the sheet-pile toe [kN],
- \( m_{mt} \) = mass of the 600 mm sheet of pile weighing \( \sim 45 \) [kg], and
- \( a_t \) = toe acceleration measured on Channel 6 \([\text{m/s}^2]\).

The dynamic soil resistance acting along the shaft \( (R_s) \) and sometimes also including the effects of clutch friction \( R_c \) have been evaluated by considering the inertial effects on the sheet pile 11.9 m in length, according to the dynamic equilibrium illustrated by Figure (4-38), and expressed according to the following equation of motion:

\[
(R_s + R_c) = S_b - S_t - m_Ma_t
\]

(4.15)

where: 
- \( S_b \) = section force near the sheet-pile head (Equation 4.11) [kN],
- \((R_s + R_c)\) = sum of dynamic soil resistance along the shaft and the effects of friction resistance \( R_c \) in the sheet-pile interlock when present [kN],
- \( S_t \) = section force near the sheet-pile toe (Equation 4.12) [kN],
- \( m_M \) = mass of the 11.9 m part of the sheet pile according to Figure (4-38) [kg], and
- \( a_t \) = toe acceleration measured on Channel 6 \([\text{m/s}^2]\).

### 4.4.4 The vibro-driveability-related data

The vibro-driveability-related data are in this case primarily related to the previously hypothesised relationships between forces and displacement as illustrated by Figure 3-2. Another way of visualising the documented vibro-driveability is to plot either the resisting forces or the penetration speed against the penetration depth.
Experimental work

Figure 4-38  Free-body diagram of the interaction between the sheet pile and the soil, used to calculate the section forces.
Rate of penetration versus penetration depth

The documented penetration speed of the sheet pile being driven \( (v_p \, [\text{mm/s}]) \) has been calculated as the time derivative of the penetration depth time-histories \( (z(t)) \), according to Figure (4-39) and Equation (4.16).

As described earlier, the penetration depth time-histories \( (z(t)) \) were derived from the digitised output voltage signals from the Unibox, according to Equation (4.5).

Since the output signal coming from the depth-measuring device is an amplitude-discrete signal (see Section 4.x), it was deemed necessary to filter and average the depth time histories \( (z(t)) \) prior to the integration process.

An average value was created over 2000 samples (2 seconds in steps of 100 samples), in order to filter the discrete \( (z(t)) \) time history. The penetration speed was then calculated as an average value in steps over 20 filtered samples, according to Equation (4.16). Since the data was reduced by 100 times during the filter process, this means that the interval of time \( (\Delta t) \) according to Equation (4.16) was exactly two seconds. The calculated penetration speed \( (v_p) \) was graphically summarised in the form of both \( (v_p) \) versus time \( (t) \) and depth \( (z) \). The signal treatment and \( v_p \) calculations were done using MATLAB\textsuperscript{TM}, described further in the two files called hast.m and hastplat.m, found in Appendix D.

The value of \( v_p \) was calculated as the time derivative of the penetration depth \( (z) \) according to Figure (4-39) and the following expression:

\[
v_p = \frac{\dot{z}_i + 1 - \dot{z}_i}{\Delta t} \quad \text{[mm/s]}
\]

where:
- \( v_p \) = penetration speed [mm/s],
- \( \dot{z} \) = time derivative of the two penetration depth readings [mm/s],
- and \( \Delta t \) = time interval between the two depth readings.

Corresponding digitised values of \( z(t) \) and \( v_p(t) \) were then graphed against each other as the \( v_p \) versus \( z(t) \) curves presented \( (v_p, z) \).

It should be noted that the primary way of analysing the penetration speed \( (v_p) \) described above, was initially checked with two back-up systems. The back-up systems consisted of video recordings of depth markers painted on the sheet pile and verbal notes made on a microcassette recorder. However the primary way of analysing the rate of penetration worked satisfactory.
Experimental work

Figure 4-39 Approximation of penetration speed \((v_p)\) from functions of depth readings \(f(z_i)\) over time.

**Dynamic toe and shaft resistance versus displacement**

The dynamic toe and shaft resistances \((R_t(t)\) and \(R_s(t)\)) corresponding to the sheet-pile motion \((u(t))\) that were plotted, were determined from the digitised time histories of the section forces and double-integrated acceleration signals measured.

Corresponding digitised values of \(u(t)\), \(R_t(t)\) and \(R_s(t)\) were then plotted against each other as the dynamic load transfer curves \(R_t-u\) and \(R_s-u\) presented.

The analysis steps described above have been performed in the following MATLAB™ files: \textit{kraftA.m}, \textit{kraftB.m} and \textit{kraftplot.m}, found in Appendix D.

**Driving force, toe and shaft resistances versus penetration depth**

The amplitude values of the driving force \((F_d(t))\), toe and shaft resistances \((R_t(t)\) and \(R_s(t)\)), corresponding to the sheet-pile penetration depth \((z(t))\) have been determined from the digitised time histories of the oil-pressure, position of eccentric weights, section forces and penetration-depth measurements.

**4.4.5 Ground vibrations due to sheet-pile installation**

The ground vibrations induced were derived from the digitised time histories of the voltage readings taken from the three different tri-axial geophones (see Figure (4-24)). It should be noted that not all the data obtained during the field tests conducted are presented here. Further information and results related to the ground vibrations induced are available in Viking and Bodare (2000) and the MSc thesis by Green and Nilsson (2000).

**Ground vibration versus time**

The ground vibrations plotted in the time domain were derived with the purpose of documenting and comparing the influence of the magnitude of vibrations due to clutch fric-
tion and the effects of introducing a bending moment on the sheet-pile head, as hypothesised in Section 3.3.3.

The time window for the ground vibration histories considered interesting were generally chosen to be \( T = 0.16 \) seconds. These time windows were mainly located in situations where it could be considered that the sheet pile was penetrating with a uniform global rate of penetration (where \( v_p \) is constant).

**Ground vibration amplitudes versus penetration depth**

The values of the amplitudes of the ground vibrations induced have been plotted against the penetration depth. The main purpose was to be able to study and compare the influence of vibration magnitudes due to the presence of clutch friction. Two additional reasons were: (i) to document the attenuation of the vibrations and derive the damping coefficient (\( \alpha [m^{-1}] \)) according to Equation (4.17), and (ii) to document the effects of introducing a bending moment of the sheet pile head, as hypothesised in Section 3.3.3.

The values of the amplitudes of the ground vibrations induced were derived in the same way as for the acceleration amplitude versus the depth; in other words the maximums were derived out of every 20\textsuperscript{th} sinusoidal period. The penetration depths were derived according to Equation (4.16) below:

\[
v_2 = v_1 \left( \frac{r_1}{r_2} \right)^n e^{-\alpha (r_2 - r_1)}
\]

(4.17)

where:  
\( v_1 \) = amplitude of motions at distance \( r_1 \) from the source [mm/s],  
\( r_1 \) = radial distances from the vibrating sheet-pile [m],  
\( n \) = power depending on the type of stress wave in the soil, [-], and  
\( \alpha \) = frequency-dependent coefficient of attenuation [m\(^{-1}\)].

**4.5 Concluding remarks on the experimental work**

The following factors and limitations have been concluded in relation to the experimental work conducted, the instrumentation system developed, and the methods for analysing the digitised data obtained.
4.5.1 Material and methods

The soil investigations conducted have not included advanced tri-axial testing on the soils samples, extracted from the six different sampling depths according to Table (4-3). This would have been necessary to derive accurate values for the soil parameters used as input for soil descriptions in the second vibratory-driveability simulation model Vipere.

4.5.2 Instrumentation and calibration

There was a great deal of disbelief prior to conducting the field tests that the instrumentation system developed should work as intended. This was especially so with respect to the sheet-pile-related part that had never been tested, except for the calibration procedures. However, the sheet-pile instrumentation proved to be surprisingly robust and worked better than expected, with only a few exceptions. The fact that the soil conditions at the test site chosen (Vårby) were favourable, most probably contributed a great deal to the functionality of the instrumentation system.

The sensor-fitted sheet piles were left in the ground between Days 1 and 2, and readings taken on Days A and B, with the consequences that two strain-gauge circuits on the sheet pile were lost (stopped working). A most likely explanation for this was probably short circuiting due to presence of groundwater.

The procedure of positioning both accelerometers and strain-gauge circuits in the holes cut out in the sheet-pile section, as illustrated by Figure (4-16), proved to work satisfactorily. However, the procedure of positioning and protecting both sensors and wires were very time consuming, and associated with a great deal of practical aspects.

The sheet piles were never calibrated for forces other than pure axial-compression loading. Later on, this led to discussions about whether the sheet-pile sections were actually exposed to loading situations related to bending or twisting or both, or whether the sensors did not work as they were intended to do, but obviously did during the extensive calibration procedure. These discussions are further developed in conclusions relating to Section 6.1.

4.5.3 Analysing methods and data collected

Since the relationships for the output readings taken from the 10 strain-gauge circuits assembled on the sheet pile, were calibrated only for pure axial-compression loading, it has not been able to separate the relationships obtained from the output voltage signals associated with the axial loading from those of the bending of the section. This means that
it has not been possible to actually scientifically relate the output voltage readings to the bending and twisting of the sheet-pile section. One possible way of separating axially-induced strains from bending, should theoretically be to position the sensors as close to the neutral axis of the profile as possible. However, the load distribution of the sheet pile section is most likely more complex than that.
CHAPTER 5

RESULTS AND ANALYSES

5.1 Chapter introduction

This chapter describes the results of the field study and the simulation of vibro-driveability of sheet piles. The study generated a huge amount of digital data, which is impractical to include in its entirety in this chapter. The decision was therefore made to summarise the primary results in this chapter, and include the rest of the results relating to the field study in an accompanying CD-ROM entitled, “Vibro-driveability and environmental studies of vibratory-installed sheet piles”. This CD-ROM is available for lending for an administration fee of €30.00, upon written request from the Division of Soil and Rock Mechanics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden. The primary results are presented in the sections following this introductory section, and are introduced in brief here.

Section 5.2 presents the primary results and an analysis of the field test. The results are presented and discussed in relation to the primary factors influencing the vibro-driveability that were mentioned earlier on in the thesis.

Section 5.3 presents the results of a preliminary comparison between the simulated results and the actual field results. Input parameters required for the two simulation models are related to the conditions found in the field tests, according to the following four groups: vibrator-related, sheet-pile-related, soil-related, and integration parameters.

Section 5.4 presents the concluding remarks based on this chapter.

5.2 Results and general analysis of the field tests

This section aims to present and summarise the primary results obtained during the field tests. The chronology of the main field test events has previously been presented in Table (4-1), and the relative position of the driven sheet piles in Figure (4-2).

Section 5.2.1 presents vibro-driveability results in two different ways, as previously discussed in the hypothesised kinematic behaviour of the system (see Section 3.2). This is followed by Section 5.2.2, which presents the results of the forces and accelera-
tions recorded. This is discussed in relation to the forces acting on the sheet pile (see for example Sections 3.2.3 and 3.2.4). Section 5.2.3 briefly presents the results of the ground vibrations generated, and discusses these in relation to the lateral flexibility of the sheet pile (see also Section 3.3.3). Section 5.2.4 then presents the load transfer curves derived for the dynamic shaft and toe resistances, in the context of the relationships previously discussed and graphed in Figure (3-2).

5.2.1 Vibro-driveability

Figures (5-1) to (5-5) display the results of vibro-driveability in two different ways, namely the $t$-$z$ curve (on the left-hand side), and the $v_p$-$z$ curves (on the right-hand side of each field test), from which the following observations can be made.

(i) From the results of the $v_p$-$z$ curves, it can be seen that results from both Day 1 and Day 2 display a similar pattern, where the curves are smooth with almost no variations, except for the distinct dip in the penetration speed ($v_p$) (read as lower values on $v_p$) during the initial depth range of approximately 1-3 m.

(ii) From the $v_p$-$z$ curves on the right-hand side in Figures (5-1) to (5-5), it can also be noted that the distinct dip in $v_p$ is present in all tests for the same depth range ($1 < z < 3$ m), with Test B1 being the only exception. Initially the $v_p$ dip was thought to be explained by initial difficulties in threading the leading interlock of the already installed pile into the profile being installed (as previously discussed and visualised in Figure (3-8)). However later on it was noted that the initial dip in $v_p$ is also present during the tests without the presence of interlock friction ($R_c$), such as in Tests A4 and B2. It is therefore speculated that the initial dip in $v_p$ could be explained by the presence of an initial lateral flexibility of the sheet pile, resulting in a lower vibro-driveability (read as lower values of $v_p$), since the dip is so pronounced during the initial 2 m. This speculation is partly supported by the recordings of significantly higher peak values by the laterally-mounted accelerometer, which for example correlates Figure (5-4)b and Figure (5-9)b. Another possible explanation for this is that the dip could be related to the initial approximately 2 m thick topsoil layer.

(iii) Overall, the penetration speed ($v_p$) is relatively constant during the whole installation phase (130-140 mm/s), which should be seen in light of the fact that all vibrator parameters were kept constant, and that the soil conditions at the Vårby site were relatively homogeneous. However from results seen in Figures (5-2)b and (5-4)b, it can be stated that the curves differ a fair bit, as they tend to show that the speed ($v_p$) starts to
decrease slowly from a relatively constant value of $\sim 140 \text{ mm/s}$ at $z \sim 8 \text{ m}$, down to $\sim 100 \text{ mm/s}$ at the final depth where installation is stopped.

(iv) It can be concluded from the results seen in Figures (5-1) to (5-5) that none of the vibro-driveability tests conducted reached refusal point. The installation of the sheet pile was simply aborted when the sheet pile reached a particular depth where it was considered that there was a risk of losing the instrumentation attached to the head of the sheet pile being driven was too great (see Figure (4-14)).

(v) From the $t$-$z$ curves on the left-hand side of Figures (5-1) to (5-5), it can be seen that all these curves display an almost identical pattern in the installation time and final penetration depth recordings. Information such as the continuous variations in the penetration speed ($v_p$) with depth, observed in the $v_p$-$z$ curves on the right-hand side of the figures, is difficult to detect from the $t$-$z$ curves plotted.

Figure 5-1  Vibro-driveability of Sheet pile A4, without clutch friction, presented as penetration depth versus (a) driving time and (b) global penetration speed.
Results and Analysis

Figure 5-2 Vibro-driveability of Sheet pile B2, without clutch friction, showing penetration depth versus (a) driving time, and (b) global penetration speed.

Figure 5-3 Vibro-driveability of Sheet pile B1, without clutch friction, showing penetration depth versus (a) driving time, and (b) global penetration speed.
Figure 5-4  Vibro-driveability of Sheet pile B3, with clutch friction, showing penetration depth versus (a) driving time, and (b) global penetration speed.

Figure 5-5  Vibro-driveability of Sheet pile A5, with clutch friction, showing penetration depth versus (a) driving time, and (b) global penetration speed.
5.2.2 Forces and accelerations

This section deals with the results of the forces and accelerations associated with the sensor-fitted sheet pile. The description and positioning of each sensor can be found in previous sections (see for example Section 4.3.2 and Figure (4-14)).

The first subsection below presents the recorded results of the two peak values of the sheet-pile section forces at the head ($S_{hi}$ and $S_{ho}$) and the peak values near the toe ($S_{rjf}$, $S_{rjf}$, $S_{rtw}$ and $S_{ltw}$) versus the penetration depth ($z$), together with the results of the three recorded peak accelerations ($a_h$, $a_t$ and $a_l$) versus depth.

The second subsection presents the results of the same forces and accelerations but plotted in the time domain, together with the calculated values of dynamic toe and shaft resistances ($R_t$ and $R_s$), and finally the results of integrated displacement versus time.

Peak head and toe forces, and peak accelerations versus penetration depth

The four Figures (5-6) to (5-9) display typical results for tests with and without the effects of clutch friction, featuring measured peak forces and accelerations at the head and toe plotted against penetration depth. By comparing the graphs on the left-hand side of these figures with those on the right-hand side, the influence of clutch friction becomes evident. The more jagged pattern of the curves on the right-hand side is due to presence of clutch friction in the sheet-pile interlock.

The two curves denoted by $S_{hi}$ and $S_{ho}$ in Figures (5-6) and (5-8) showing generally higher peak force values, represent the driving force amplitude ($F_d$) actually transmitted to the sheet-pile head. The four curves denoted by $S_{rjf}$, $S_{rjf}$, $S_{rtw}$ and $S_{ltw}$ in the same two figures, generally display a lower peak force amplitude, and represent the peak value of the dynamic soil resistance measured at the toe ($R_t$).

The two curves denoted by $a_h$ and $a_t$ in Figures (5-7) and (5-9), show a generally higher acceleration amplitude, and represent the axial accelerations of the sheet-pile head and toe respectively. The curve denoted $a_l$ represents the lateral acceleration of the vibratory-driven sheet pile.

It should be noted that two of the four strain gauges at the pile toe were lost from Day 1 (Test series A) to Day 2 (Test series B). In Figure (5-8), which relates to Day 2, only four forces are displayed compared to a total of six channels displayed in Figure (5-6). It is believed that the presence of groundwater most likely caused a short circuit in the gauges, since the sensor-fitted sheet pile was left in the wet ground from Day 1 to Day 2. The security system in the ABI vibratory-equipment did not allow the operator...
to release the grip of the clamping device holding the sheet pile, except when it was in close proximity to the ground.

Figures (5-7)a and b show only two of the total three curves displayed in Figures (5-9). The acceleration at the sheet-pile head was lost during the two initial tests (Tests A4 and A5). One of the signal cables associated with the lateral acceleration sensor \(a_{h}\) was found to have been disconnected from the switchbox mounted on the sheet pile.

The following observations can be made from Figures (5-6) to (5-9).

(i) It might be logical to assume that the presence of interlock resistance \(R_c\) would generate clearly detectable recordings of higher peak force values. However, by comparing Figure (5-6)b with Figure (5-8)b, it appears as though there are actually slightly higher recorded peak values for some of the four section forces. But this evidence is not conclusive due to the limited nature of the study, nor can the magnitude of the friction force present be determined. This means that the previously mentioned values of \(R_c\) (~1.0 kN/m by Ferron (2001), and 2.0~\(R_c\)~20 kN/m by Vanden Berghe et al. (2001)), could not be conclusively verified. This is discussed further in Section 5.2.5.

(ii) The peak acceleration versus depth curves have also been evaluated for situations with and without friction forces \(R_c\) present in the sheet-pile interlock. The more jagged pattern of the acceleration curves reflects the presence of friction forces in the sheet-pile interlock, compared with the test where interlock resistance was absent. The acceleration curves not only displayed a more jagged pattern, but they also displayed higher peak acceleration values. The peak axial accelerations in the case of freely-driven sheet piles (without the effects of friction force \(R_c\)) were in the vicinity of 15 g for the head acceleration \(a_{h}\), and 17 g for the toe acceleration \(a_t\) during the entire depth range investigated (see Figure (5-9)a). The difference observed between peak head and toe accelerations \(a_{h}\) and \(a_t\) during the most favourable conditions was found to be in the range of 2-3 g. The minor differences between the peak head and toe acceleration recordings \(a_{h}\) and \(a_t\) graphed in Figure (5-9), ought to be viewed as a confirmation of the assumption of pile rigidity, as discussed in Section 3.3.3. The slightly higher values of the toe acceleration \(a_t\) compared with the head acceleration, are most likely explained by inertial effects on the 0.6 m long sheet-pile mass below the position of the toe accelerometer (see Figure (4-14)).

(iii) The previous discussion about the non-negligible effects of lateral flexibility on vibratory-driven sheet piles in Section 3.3.3, has been able to be verified by the peak amplitude of the lateral acceleration \(a_l\) results. The peak lateral acceleration \(a_l\) in the freely-driven sheet piles (without friction force \(R_c\) effects) were found to be in the order of half the axial peak value in the depth range \(0 < z < 6\) m. The lateral acceleration curves
can be seen to distinctly increase up to about the same peak value as the axial accelerations, just before reaching the position \((a)\), when the sensor is in the same elevation as either the soil surface (~7 m) or the subsequent interlock (~6 m). In situations where clutch friction \((R_c)\) is present, the slightly earlier level of ~6 m is the level of subsequent interlock compared with the level of the soil surface. The amplitude values of \((a)\) can be seen to drop to ~2.5 g as a result of the surrounding soil, stiffening the lateral motion of the sheet pile.

(iv) As previously discussed in Section 3.3.2, the evaluated ratio of actual to theoretical driving force \((\xi = F'_d / F_d)\) exhibits values agreeing with Moulai-Khatir et al. (1994). The peak compressive force at the sheet-pile head have been evaluated for Test B2 resulting in Figure (5-8)a (without friction force \((R_c)\) effects), using both calculations and measurements. The theoretically delivered peak compressive force \((F_d)\) is calculated according to the Equation (A.5), and evaluated as \((70/2 + 6(82\pi)^2 + 15g2.450 ~ 794 \text{ kN})\), based on the assumption that the sheet-pile-vibrator system behaves as a rigid body, where the vibrator has the same acceleration as the sheet-pile head. The surcharge force \((F_o)\) used in these calculations is 35 kN, which is half the capacity of the push-down force of the leader cylinder as specified in the performance data of the vibrator used (see Table (4-5)). The peak unbalanced force \((F_v)\) is evaluated to 361 kN, with a static moment set to half of the maximum specified performance data in Table (4-5). From Test B2, the deceleration of the sheet-pile head measured ~15 g (see Figure (5-8)a), and the dynamic mass of the vibrator \((m_v)\) is also given by Table (4-5). The peak compressive force measurement \((F'_d)\) is approximately 310 kN and evaluated from Figure (5-8)a. The ratio \(\xi\) is then evaluated as \((310/794 ~ 0.39)\), which by Moulai-Khatir et al. (1994) was treated as an efficiency parameter and stated to be in the range \(0.20 < \xi < 0.25\). From these results here, it can be concluded that the measured value (read as the value actually delivered) of the peak force at the sheet-pile head was clearly less than the theoretically computed value. This fact is most likely explained by energy losses in the vibrator-sheet-pile system, and most likely to have been affected by the induced lateral motion of the vibratory-driven sheet pile.
Figure 5-6 Dynamic peak amplitude of section forces at the head and toe versus penetration depth; where (a) is Test A4, without $R_c$, and (b) is Test A5 with $R_c$ effects.

Figure 5-7 Dynamic peak amplitude acceleration versus penetration depth; where (a) is Test A4 without $R_c$, and (b) is Test A5 with $R_c$ effects.
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Figure 5-8  Dynamic peak amplitude of section forces at the head and toe versus penetration depth; where (a) is Test B2 without \( R_c \), and (b) is Test B3 with \( R_c \) effects.

Figure 5-9  Dynamic peak amplitude acceleration versus penetration depth; where (a) is Test B2 without \( R_c \), and (b) is Test B3 with \( R_c \) effects.
In other words, part of the theoretical driving force is used to bend the sheet pile instead of driving it completely vertically. Another possible explanation is the invalidity of the rigid-body assumption, even though it appears to be justified by \((a_b \sim a_i)\). A further explanation could be the invalidity of assuming that the suppressor housing of the vibrator does not vibrate. There may even be other undetected experimental errors in the instrumentation equipment.

**Toe and shaft resistances, acceleration, and displacement versus time**

This subsection presents some typical test results for the penetrative resistance forces, accelerations and displacement in time, with and without the effects of interlock resistance \((R_c)\), corresponding to the three penetration depths 2, 6 and 10 m.

The first four figures (Figures (5-10) to (5-13)) present typical results from Test A4 (without \(R_c\) effects) for the evaluated toe and shaft resistances \((R_t\) and \(R_s\)), the accelerations \((a_b, a_t, a_l)\) and the integrated displacement \((u)\) versus time \((t)\), all taken at penetration depths \((z)\) of 2, 6 and 10 m. The four figures following these (Figures (5-14) to (5-17)) present corresponding results from Test B3 (including \(R_c\) effects) in order to study the influence of friction forces in the sheet-pile interlock.

It should be noted that the procedure for evaluating the results has previously been described in Chapter 4 (see for example Section 4.4), and the following observations can be made of the results in Figures (5-10) to (5-17).

(i) In order to analyse the effects of clutch friction \((R_c)\) on the toe resistance generated \((R_t)\), Figures (5-10)a-b can be compared with Figures (5-14)a-b. The value of the amplitude of dynamic toe resistance \((R_t)\) appears to be equal at penetration depths of 2 and 6 m for both tests. From Figures (5-10)a-b, the peak value of the dynamic toe resistance can be seen to be approximately 42 kN at penetration depths of 2 and 6 m. From Figures (5-14)a-b, the peak value of the dynamic toe resistance can be evaluated as approximately 112 kN. This concludes that the presence of clutch friction forces \((R_c)\) in the sheet-pile interlock increased the evaluated peak toe resistance to approximately \(112 - 42\), which is fairly close to 60 kN. It should be noted, that performing the same comparative analysis is not advisable for the dynamic shaft resistance \((R_s)\) results presented here, which is further discussed in Section 5.2.5.

(ii) By comparing the acceleration curve patterns, it was found that the effects of clutch friction \((R_c)\) strongly influenced both the peak value as well as the pattern of the evaluated curves. By comparing Figures (5-12)a-c with (5-16)a-c, it can be seen that the presence of clutch friction \((R_c)\) produces acceleration curves that display a much noisier pattern, and with a higher peak amplitude of approximately 5 g. Without the presence
of clutch friction \( (R_c) \), the peak axial acceleration ranges approximately between 10-15 g, compared with 15-20 g or even sometimes higher when \( R_c \) is present.

(iii) In order to study the effects of lateral flexibility of the sheet-pile clutch friction, as previously discussed in Section 3.3.3, Figures (5-12)b-c can be compared with Figures (5-16)b-c. It can be concluded that the presence of clutch friction \( (R_c) \) significantly affects the peak value of the lateral accelerations generated (the lateral motions induced in the sheet pile). When \( (R_c) \) is present, the peak lateral acceleration is seen to be in the same order of magnitude as the axial peak values (see Figure (5-16)b). However, after the laterally-mounted accelerometer enters the soil stratum (at \( z = 7.0 \) m), the peak lateral acceleration signal drops to a minimum. However, the almost total disappearance of peak lateral acceleration seen in Figure (5-12)c is not seen when \( R_c \) is present (see Figure (5-16)c). Apparently, the three non sensor-fitted pre-installed sheet piles (A1 to A3 in Figure (4-2)), which together formed a pre-existing sheet-pile wall, laterally vibrated together with the sheet pile being threaded to them via the leading interlock. This statement is supported by the results of the peak ground vibration measurements versus depth, in Test B3 (shown in Figure (5-21)b). From this it can be concluded that the longitudinal ground vibration amplitude measured is larger than the vibration amplitudes registered by the vertically-positioned geophone. These observations are discussed further in Section 5.2.3.

(iv) Studies of the displacement versus time curves (Figures (5-13) and (5-17)), display a similar pattern to that previously discussed and illustrated by Figure (3-2), with respect to the presence or absence of clutch friction. The two first displacement \( (u-t) \) curves at a penetration depth of 2 m, both display a slightly lower slope over time than the correlating results at penetration depths of 6 and 10 m. This is in agreement with the previously mentioned \( v_d \) dip in the depth range 1-3 m. The axial displacement amplitude \( (\gamma_o) \) and the permanent set \( (\Delta \gamma) \), are evaluated to \( \sim 1.8 \) mm and \( \sim 3.1 \) mm respectively, with and without the effects of clutch friction. This leads to the conclusion that the effects of clutch friction cannot be detected from the integrated displacement \( (u-t) \) curves. The jagged pattern of the acceleration curves for Test B3, with clutch friction, disappears in the displacement \( (u-t) \) curves. This is explained by the procedure for the numerical integration of the discrete acceleration signals.
Figure 5.10 Dynamic sheet-pile toe resistance versus time for Test A4, without $R_c$ effects, corresponding to penetration depths ($z$) of (a) 2.0, (b) 6.0 and (c) 10.0 m.
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Figure 5-11 Dynamic shaft resistance versus time for Test A4, without $R_z$ effects, corresponding to penetration depths ($z$) of (a) 2.0  (b) 6.0 and (c) 10.0 m.
Figure 5-12  Acceleration of the sheet pile versus time for Test A4, without R_c effects, corresponding to penetration depths (z) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
Figure 5.13 Displacement of the sheet pile versus time for Test A4, without Rₐ effects, corresponding to penetration depths (z) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
Figure 5.14  Dynamic sheet-pile toe resistance versus time for Test B3 with $R_c$ effects, corresponding to penetration depths ($z$) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
Figure 5-15 Dynamic shaft resistance versus time for Test B3, with $R_c$ effects, corresponding to penetration depths ($z$) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
Figure 5-16  Acceleration of the sheet pile versus time for Test B3, with $R_e$ effects, corresponding to penetration depths ($z$) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
Figure 5-17  Displacement of the sheet pile versus time for Test B34, with $R_e$ effects, corresponding to penetration depths ($z$) of (a) 2.0 (b) 6.0 and (c) 10.0 m.
5.2.3 Ground Vibrations

This section presents some typical results for the ground vibrations generated with and without the effects of clutch friction force ($R_c$). Information from the tri-axial geophones and the radial distances from the sheet pile can be found in previous chapters (see for example Section 4.3.3 and Table 4-1).

Rather than presenting a thorough analysis of the ground vibrations generated in the field test (which at first may not appear to be related to the topic of the thesis anyway), the following two subsections present a limited analysis of the correlation that exists between the vibro-driveability achieved and the vibrations generated, as has previously been discussed in Section 3.3.3.

The first subsection presents the ground vibrations that were monitored in the time domain at three penetration depths. The second subsection presents the results of registered peak values versus penetration depth from the same tests occasions.

Ground Vibrations versus Time, at Penetration Depths (z) of 2, 6 and 10 m

Figures (5-18) through (5-20) present typical results for tests without clutch friction ($R_c$), and Figures (5-21) through (5-23) correspond to results including clutch friction ($R_c$). The following observations can be made based on these results.

(i) From an FFT analysis (not presented here) of the vertical signals of the geophone positioned at the second radial distance ($r_2$) from the driven sheet-pile, it can be concluded that the ground vibrations induced display a steady-state response with a frequency essentially the same as driving frequency applied.

(ii) The curves of the vibration recording where plotted in the time domain. All of these display a generally smooth and sinusoidal pattern, which is expected due to the favourable soil conditions, being relatively homogeneous, saturated sand.

(iii) From the results of laterally-induced ground vibrations together with the penetration speed and the measurements of the lateral accelerations of the sheet pile, it can be seen that these are sometimes of the same magnitude or even higher than the vertical vibrations. From these results, it appears quite clear that there is a correlation between the effects of lateral flexibility of slender vibratory-driven sheet piles and the magnitude of the disturbance induced in the surrounding soil volume, as well as an influence on the penetration speed.
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Figure 5-18  Ground vibration versus time for Test B2, without Rc effects, at a radial position (r_1) of 1.10 m and penetration depths (z) of (a) 2.0 m, (b) 6.0 m and (c) 10.0 m.
Figure 5-19  Ground vibration versus time for Test B2, without \( R_c \) effects at a radial position \( (r_2) \) of 3.90 m and penetration depth of \( (z) \) of (a) 2.0 m (b) 6.0 m and (c) 10.0 m.
Figure 5-20  Ground vibration versus time for Test B2, without $R_c$ effects at a radial position ($r_3$) of 20.0 m and penetration depth ($z$) of (a) 2.0 m (b) 6.0 m and (c) 10.0 m.
Figure 5-21 Ground vibration versus time for Test B3, with $R_c$ effects, at a radial position ($r_1$) of 0.85 m and penetration depth ($z$) of (a) 2.0 m (b) 6.0 m and (c) 10.0 m.
Figure 5-22  Ground vibration versus time for Test B3, with $R_c$ effects at a radial position ($r_2$) of 3.7 m and penetration depths ($z$) of (a) 2.0 m, (b) 6.0 m, and (c) 10.0 m.
Figure 5-23  Ground vibration versus time for Test B3, with R_c effects at a radial position (r_3) of 15.55 m and penetration depths (z) of 2.0 m (b) 6.0 m and (c) 10.0 m.
Peak particle velocity in measurements of ground vibration versus penetration depth

Figures (5-24) to (5-26) below present results of the continuously-monitored peak particle velocity of the same tests (Tests B2 and B3) as Figures (5-18) to (5-23) but plotted continuously against penetration depth ($z$). The three graphs denoted $(a)$ on the left-hand side of Figures (5-24) through (5-26), show the monitored peak vibrations without the effects of clutch friction, and the corresponding graphs denoted $(b)$ show the corresponding results including the effects of clutch friction forces in the sheet-pile interlock, from which the following primary observations can be made.

(i) From the comparison of the results shown in the three graphs denoted $(a)$ and the three denoted $(b)$, it appears as though the presence of clutch friction produces peak values that are at least twice as great as those without the effects of friction forces. This is in accordance with previously mentioned results reported by Legrand et al. (1993), shown for example in Figures (2-13).

(ii) From the general analysis of all the test results (not presented here), it was concluded that the previously mentioned decrease in penetration speed ($v_p$) during the initial depth range ($1 < z < 3$ m), corresponds to an equivalent observation of a significant increase in monitored peak particle velocities versus depth. Why these effects occur and how these observations should be interpreted is not immediately obvious, however these observations are discussed further in Section 5.2.5.

(iii) The transversal peak value does not appear to vary to the same extent as the vertical and even longitudinal values along the penetration depth. However a slight increase in both the longitudinal and transversal vibrations can be observed with increasing depth, which most likely correlates with an increasing shaft area that disturbs the surrounding soil as the sheet pile is driven deeper into the soil.

(iv) From the quite extreme value of the vertical peak particle-vibrations monitored approximately 50 mm/s (see Figure (5-24)b), it can be inferred that the individual soil grains in the vicinity of the measurement positions is exhibiting a vertical acceleration of approximately ($50 \times 10^{-3} \times (2\pi 41)$) which is approximately 1.3 g. The implication of this in relation to internal shear strength (the fundamental mechanism underlying the favourable penetrative resistance), is a reduction in the use of the vibratory-driving techniques for installing sheet piles in cohesionless soils (as previously discussed in Section 3.3.4 and illustrated by Figure (3-10)). However the scientific proof of the mechanism, that is the relationship between the effects of inertial forces on the individual soil particles and the internal shear strength reduction of cohesionless soils (discussed in Section 3.3.4), is somewhat vague and limited due to the adequate soil-instrumentation.
Figure 5-24  Peak particle velocity versus penetration depth in three directions: (a) in Test B1 with no \( R_c \) effects at a radial distance \( r_1 \) of 1.2 m and (b) Test A5 with \( R_c \) effects at an \( r_1 \) of 1.2 m.

Figure 5-25  Peak particle velocity versus penetration depth in three directions: (a) Test B1 with no \( R_c \) effects at a radial distance \( r_2 \) of 4.1 m and (b) Test A5 with \( R_c \) effects at an \( r_2 \) of 4.15 m.
5.2.4 Dynamic load transfer curves

This section deals with the evaluated results of the ‘dynamic’ load transfer curves developed during vibratory driving, and a comparison of these with the previously discussed relationships illustrated by Figure (3-2). Each part of the procedures for evaluating the dynamic load transfer curve presented here, has been thoroughly described in the previous chapter (see for example Section 4.4).

Figures (5-27) and (5-28) are taken from the evaluated time histories of Test B1, without the presence of clutch friction, simply because the phase lag between head and toe acceleration, and the section forces at both the head and the toe were found to be close to zero, and the time window chosen corresponds to a penetration depth of approximately 3.1 m.

The following observations and experiences were obtained during the evaluation procedure of the results shown in Figure (5-27) and (5-28).
(i) The curves \((a, a_h, a_t\text{ and } S_{lyf})\) in Figure (5-27) appears to be in phase and the peak value of the lateral displacement can be evaluated to about \((ûl = \ddot{a}/\omega^2 \sim 50/(2\pi 41)^2 \sim 1 \text{ mm})\), which at a first glance does not appear to be of significance, however is about 40 per cent of the evaluated peak axial acceleration.

(ii) It can be seen from the time ‘history’ of the dynamic section force \((S_{lyw})\) representing the dynamic toe resistance \((R_t)\) in Figure (5-27)f, that the curve is biased around a negative value. This is instead of varying in a pattern between approximately zero and a positive peak value that corresponds to a compressive loading during the downwardly-directed part of the penetration motion. Initially it was believed that the negatively biased values were attributable to inertial forces acting on the 600 mm long part of the sheet pile below the position of the transducers mounted near the pile toe. However another and more likely explanation for these values is the fact that the relationships for the output readings were only calibrated for pure axial-compression loading. This makes it very difficult to separate the relationships obtained from the output voltage signals associated with the axial loading from those of the bending and twisting of the sheet-pile section during the installation phase. This presents a problem for actually scientifically relating the output voltage readings to the bending and twisting of the sheet-pile section, even though from an engineers point of view, it is fairly obvious that the bending and twisting of the sheet pile is the only explanation for the results obtained. It can be seen how the biased level varies between positive and negative values during the depth range investigated.

(iii) Though the phase lag in the raw data during certain periods of time were found to be rather low, it was immediately noted that even the slightest phase lag has a tremendous impact on the pattern (read as shape) of the evaluated ‘dynamic’ load transfer curves. The results seen in Figure (5-27) and (5-28) correlate to a situation where the phase differences appear to be small but not zero, which caused problems in evaluating the ‘dynamic’ load transfer curve for the shaft resistance \((R_{s-u})\), and which therefore is not presented here. The load transfer curve shown in Figure (5-28)b has been evaluated from only one of the four monitored section forces near the pile toe (the section force \(S_{lyw}\)), and not according to the suggested average value given by Equation (4.12). It was observed that the phase relationships between forces and accelerations varied constantly during the entire installation phase. If for instance the toe resistance \((R_t)\) is compared with the double-integrated toe acceleration \((a)\), as is done in the results presented in Figure (5-28)b, it is found that the evaluated load transfer curve of the shaft \((R_{s-u})\), does not always necessarily display an expected value, due to the enormous impact of the slightest phase lag. The drift variation and the drift of the phase lag can be observed by comparing
the toe resistance ($R_t(t)$) curves shown in Figure (5-10) with the toe acceleration ($a_t(t)$) curves shown in Figure (5-12). It can be noted that the phase difference is approximately 6 mS at a penetration depth of 2 m and decreases at a depth of 6 m, and finally increases again at 10 m. This indicates that the phase difference is not constant, and therefore it was concluded to be difficult to derive a consistent phase correction procedure for the raw data signals.

**Figure 5-27** Time histories for acceleration, integrated velocity and displacement, together with dynamic toe force for Test B1 without clutch friction effects, at a penetration depth ($z$) of ~ 3.1 m.

**Figure 5-28** Typical time histories for sheet-pile B1, where (a) is the driving force recorded at the sheet-pile head, and (b) the evaluated dynamic load transfer curves at the sheet-pile toe.
5.2.5 Discussion of the field test results

This section contains a more detailed discussion of those results already mentioned, plus interesting observations, and draws a number of conclusions (though these ought to be interpreted cautiously due to the limited number of varied parameters and tests conducted).

Firstly, it should be stated that the specially developed instrumentation system worked better than expected, and with higher precision. In other words, when uncertainties occurred during the analysis work, they have generally not been related to the quality or correctness of the signals produced by the sensors. Secondly, the uncertainties that did occur have generally been related to how the observations recorded should be interpreted and correlated for obtaining results.

Vibro-driveability results

Presenting vibro-driveability in the form of driving time with depth ($t$-$z$ curves) provides a good overview of the whole penetration phase. Any phase where obstructions might be encountered should have been displayed as a horizontal line at a certain penetration depth where the obstructions could have been encountered, and the duration should have been given by the time. However, this information is undetectable when illustrating the vibro-driveability (speed) versus penetration depth. The penetration speed should of course experience some sort of decrease at a penetration depth where an obstruction is encountered. However the duration of this speed reduction is impossible to derive from a plotted $v_p$-$z$ curve. In the author’s opinion, this means that both curves ought to be used during a tentative analysis of the vibro-driveability being looked at, in order to draw conclusions from an analysis of the results.

From comparisons of the effects of the presence of clutch friction ($R_c$) and the extent to which this might affect vibro-driveability results, it could be concluded previously that no significant differences could be detected in the results presented, from either the $v_p$-$z$ curves or the $t$-$z$ curves. This could be due to the fact that all the conditions were as favourable as they could possibly, as the sheet-piles were new and the vibro-equipment was used in a quite homogeneous and water-saturated sand deposit. This is of course unlike the more interesting phase were a driven sheet-pile reaches a state of refusal. This state is where the capacity of the performance characteristics of the vibratory system are exceeded and the sheet pile cannot be driven any further; in other words where the penetration speed ($v_p$) approaches zero. It could therefore be worth discussing whether the amplitude values of the friction forces developed in the sheet-pile clutch at
Vårby were so small that the influence on penetration speed could not be clearly detected, simply due to the over-capacity of the vibrator used in relation to the low penetrative resistance at site.

**Force and acceleration results**

The field tests revealed that the relationship between the measured and theoretical axial driving forces ($\xi = F'_d / F_d$), which is an essentially vibratory-related parameter, ought to be considered. The ratio of peak compressive force measured during the downward motion at the sheet pile to the theoretically evaluated value, was found to be as low as 40 per cent, even during the most favourable field test. From the low value of the ratio ($\xi$) observed, it should be concluded that it is not just the difficulty encountered with estimating the dynamic soil and clutch resistances correctly that dictates whether a pre-vibro-driveability prediction will be accurate or not. At this stage it can only be speculated as to which part of the ‘vibrator, sheet-pile and soil system’ that these low values of the driving force ratio ($\xi$) should be attributed. However, the author has speculated that almost negligible losses ought to be associated with the slippage between the vibrator and the clamping device holding the pile as there were no characteristic slippage marks found on the sheet-pile surface where the clamp-jaws held the sheet pile. The biggest contributing factor is probably related to the observed flexural and torsional motion of the sheet pile, which could not be sensed by the transducers that were only intended to measure axial forces. However, each part of the vibratory equipment is also a non-neglectable source of the low driving force ratio ($\xi$) observed, and to date no accurate values can be applied to the computation of the actual driving force delivered to the head of the pile.

The discussion about whether or not it is possible or even justifiable to apply the assumption of pile rigidity during the use of vibrators, should according to the author, be concluded in the affirmative, due to the relative equal values of head and toe accelerations, which are also in agreement with the results of Legrand et al. (1993) and Bosscher et al. (1998). It can be concluded that it is justifiable to treat the vibratory-driven pile during favourable conditions such as found in relatively easy driving, such as soil without obstructions or boulders.

The peak lateral acceleration curves all show a similar overall pattern, characterised by high values in the initial penetration depth, which also correlates with the relationship between lower penetration speed and higher ground vibration. Why this was observed has not been possible to consistently conclude. At first glance, the peak values of the lateral acceleration results do not appear to be of significance, however in relation to the evaluated peak axial acceleration, it can be concluded that they are of significance
since they quite frequently appear to be approximately half of the recorded axial peak values, and sometimes even of equal value. It is therefore advisable to consider these effects in the future, which is most easily accomplished by attaching a paper sticker to the flange of the sheet pile (see Figure (4-28)). During the tests, these were found to be useful and are of course cheap compared with instrumentation systems used to monitor the lateral accelerations.

By far the most significant sheet-pile parameter is the effect of the interlock friction force ($R_c$), which Ferron (2000) and Vanden Berge et al. (2001) suggested was approximately 1.0 kN/m, or 2-20 kN/m respectively. It should be noted that the value suggested by Ferron is not supported by any references, however the work has been developed together with the world’s biggest producer of sheet piles, ProfileARBED in Luxembourg. This suggest that the figure might be related to in-house and unpublished research. Though the second value of the friction force ($R_c$) suggested is much larger, the work is also related to the same sheet-pile producer (ProfileARBED). This higher value is associated with laboratory tests of real interlocks and relates to a quasi-static-load situation, and not the dynamic state that governs the installation phase of interest here. In other words, something in between these values would be closer to reality.

It can be assumed that the 60 kN increase in the dynamic toe resistance previously reported in Section 5.2.2 can be directly related to the friction force ($R_c$) in the sheet-pile interlock of length 600 mm (as shown by Figure (5-29)), since the only change between Figures (5-29)a and b is the presence of the friction force ($R_c$). The corresponding value mentioned by Ferron ($r_c \sim 1$ kN/m) and Vanden Berghe et al. (2 < $r_c$ < 20 kN/m) can then be evaluated according to the expression ($R_c \sim 0.6r_c = 60$ kN), where $r_c$ is evaluated to be approximately ($60/0.6 = 100$ kN/m), which does not appear to be a reasonable value.
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Ground vibration results

Vibrations from construction activities have become more important due to the fact that they can sometimes cause damage to adjacent structures as well as lead to complaints from the people residing in them. The direct cause of the damage to structures is difficult to identify, simply due to the complexity of the vibration problems. It is also generally quite difficult to estimate the degree to which vibration amplitude is generated, as well as the attenuation at a given distance.

The decrease in penetration speed and corresponding increase in ground vibration generation observed during the initial depth range ($1 < z < 3$ m) appears to be correlated to initial difficulties. Some of the most likely explanations for this are the difficulties encountered, (i) while threading the leading interlock of the already installed sheet pile to the one about to be driven, or (ii) caused by the analog phenomenon of threading the slender sheet pile into the subsoil stratum until the driven sheet pile’s driving direction is stiffened up or thoroughly guided by the soil volume surrounding the increasing shaft area.

Finally, most of the ground vibration measurements are today measured only at the ground surface and usually only in the vertical direction ignoring the propagation path. However propagation characteristics of vibrations relating to vibratory-driven sheet piles seem at least from the results obtained here, to be dependent on the type of waves generated. In other words, the laterally-induced motion of the vertically-driven sheet pile not only generates shear waves along the vertically-vibrating shaft, it also ap-
pears that P waves are introduced at the soil surface due to the lateral motions of the slen-
der sheet pile (see Figure (3-5)). A better understanding of the propagation characteristics
of ground vibrations generated in relation to vibratory-driven sheet piles is most likely
dependent on the type of waves generated. It is therefore in the author’s opinion, very
important to monitor the particle motions in all three directions, not only on the surface
but also with depth, for better characterising propagating waves in the future, and it is
especially important to monitor all directions in situations where settlements of adjacent
structures need to be considered.

Plotted load transfer curve results

As previously mentioned, the output relationships of the voltage readings of the 10
strain-gauge circuits assembled on the sheet pile were only calibrated for pure axial-com-
pression loadings. It has therefore not been possible to separate the relationships ob-
tained from the output voltage signals associated with the axial loading from those of the
bending of the section.

It has been suggested that the sensor-fitted sheet pile should have been re-calib-
bered after the field test, in order to be able to correlate controlled bending with the
monitored (read calibrated) output signals from the different strain gauges. However, it
appears as though the slender U-profile stress is accompanied by quite a complex load
situation during the installation phase. This statement relates to how the clutch friction
force enters the sheet-pile section on one side of the neutral axis (seen in Figure (5-30)),
and the driving force on the other side of the same neutral axis, while at the same time,
recordings of the induced strains are made at four locations in the sheet-pile section
(shown by Figure (5-30)). Since the four gauges mounted near the sheet-pile toe are as-
sembled on each side of the neutral axis, the friction force entering the sheet-pile section
outside the neutral axis generates a moment on one side, and the driving force is applied
in the sheet-pile web on the other side of the interlock. It then becomes quite obvious
that the results of recording the four section forces near the toe will generate more ques-
tions regarding the toe and clutch resistance recordings than they will answer. However
if only one gauge were mounted at the toe, the effects of bending and torsion would not
have been detectable.

Another difficulty encountered in interpreting the recordings of the gauges at
the toe relates to the at times biased level of the recorded forces. The gauges were zeroed
prior to every test, while the sheet pile was hanging free in the jaws of the clamping de-
vice (under tension), and the oil pressure of the pull down force (read biased surcharge
force) always kept the sheet pile compressed by constantly pushing the sheet pile into the
ground. Pushing was monitored, and it appears quite logical that different parts of the sheet-pile section shown in Figure (5-30), would be subject to either compression or tension depending on how much friction force \( (R_c) \) developed in the sheet-pile interlock, or on which side of the neutral axis the driving force \( (F_d) \) enters the sheet-pile section.

![Diagram showing forces and neutral axis](image)

Figure 5-30 Position of the four section forces in relation to driving force and interlock resistance.

5.3 Comparison of the simulations with the field tests

5.3.1 Section introduction

This section compares the recorded results of vibro-driveability and the forces generated in the sensor-fitted sheet pile, with the results from the simulations of vibro-driveability and the predicted dynamic soil resistance of the Vibdrive and Vipere models. The primary reason for this section is not to validate the general credibility of the models in relation to actual field conditions, but rather to simply initiate the validation process of the two chosen models and then to assess the ability of the two models to take into account the primary parameters influencing vibro-driveability, as hypothesised in Section 3.3.

The input parameters applied here, described in Table (5-1), correlate with the field study conducted at Vårby, however the comparison performed does not take into account the effects of friction forces in the sheet-pile interlock.

It should also be noted that the simulation results presented and discussed that relate to the Vibdrive model have been conducted by the author, while the simulation results relating to the Vipere model have been derived by Vanden Berghe (2001). The reason for this is a combination of the Vipere model being a recent development and the complexity of running the Vipere model at this stage.
5.3.2 Evaluation of input parameters used in the comparisons

The parameters needed as input in the two models fall into the following four categories: vibrator, pile profile, soil, and integration parameters (see Table (5-1).

Both the Vibdrive and Vipere model evaluate the input data from the vibrator and sheet pile used according to the same expressions (see Section 2.6.6), on the basis of the previously described performance data for the vibrator and the characteristics of the sheet pile (see Section 4.2).

The evaluation process of soil parameters required for calculating the dynamic soil resistance is different for the two models. The Vibdrive model correlates the evaluation of the soil resistance with CPT test results, and the Vipere model requires results from laboratory tests (tri-axial and direct simple shear tests) to evaluate the hypoplastic characterisation of the dynamic soil resistance, which is further described in Section 5.3.4.

Table 5-1 Comparison of input parameters used in the Vibdrive and Vipere models and Vårby field tests.

<table>
<thead>
<tr>
<th>Type of parameter</th>
<th>The Vibdrive model</th>
<th>The Vipere model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vibrator parameters</strong></td>
<td>Unbalanced moment, ( M_e = 6.0 ) kgm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Driving frequency, ( f_d = 41 ) Hz</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Dynamic mass of the vibrator, ( m_v = 2450 ) kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stationary mass of the vibrator, ( m_o = 1020 ) kg</td>
<td></td>
</tr>
<tr>
<td><strong>Pile parameters</strong></td>
<td>One sheet pile without clutch friction</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Section area, ( A_l = 95.2 ) cm(^2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sheet pile perimeter, ( \chi = 150 ) cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sheet pile length, ( L = 14 ) m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mass of sheet pile, ( m_p = \rho A_l L \approx 1010 ) kg</td>
<td></td>
</tr>
<tr>
<td><strong>Soil parameters</strong></td>
<td>Accel. ratio, ( \alpha = 15 ) g</td>
<td>Friction angle, ( \phi' = 35^o )</td>
</tr>
<tr>
<td></td>
<td>Empirical liquefaction parameter, ( \psi = 10 )</td>
<td>Initial void ratio, ( e = 0.68 )</td>
</tr>
<tr>
<td></td>
<td>Dynamic peak toe resistance at ( z \approx 10 ) m</td>
<td>Critical void ratio, ( e_{cr} = 0.8 )</td>
</tr>
<tr>
<td></td>
<td>( R_{z,10} = 13.3 ) kN</td>
<td>Minimum void ratio, ( e_{min} = 0.5 )</td>
</tr>
<tr>
<td></td>
<td>Dynamic peak shaft resistance at ( z \approx 10 ) m</td>
<td>Strength, ( b_l = 200 ) MPa</td>
</tr>
<tr>
<td></td>
<td>( R_{z,10} = 103.6 ) kN</td>
<td>Dimensionless constant, ( n = 0.35 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dimensionless constant, ( \alpha = 0.25 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dimensionless constant, ( \beta = 1.10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dimensionless constant, ( \lambda = 1.38 )</td>
</tr>
<tr>
<td><strong>Integration parameters</strong></td>
<td>Initial penetration speed ( v_{ini} = 0.25 ) m/s</td>
<td>Radial discretization, ( R_{max} = 10 ) m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Geometrical parameter, ( P_{geo} = 10 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Time parameter, ( P_{time} = 20 )</td>
</tr>
</tbody>
</table>
5.3.3 Evaluation and comparison of driving forces

Both the Vibdrive and Vipere models evaluate the driving force according to the same general expression (see Equation (2.9) and Figure (2-7)). The peak value of the driving force is, in accordance with the specified parameters in Table (5-1), given by the expression \( \left( g m_{\text{tot}} + M_e \omega^2 \sin(\omega t) \right) \), where \( m_{\text{tot}} = (m_o + m_v + m_p) \). The total mass \( m_{\text{tot}} \) of the vibrator-sheet-pile system is approximately 4480 kg and the peak value of the driving force is evaluated to approximately \( 9.81 \times 4.48 + 6 \times (2\pi 41)^2 = 43.9 + 398.2 \approx 442 \text{ kN} \).

Figure (5-35)a shows the sinusoidal driving force simulated by the Vibdrive model in the time domain, biased around \( g m_{\text{tot}} = 43.9 \text{ kN} \), in accordance with the specified vibrator parameters in Table (5-1).

5.3.4 Evaluation and comparison of dynamic soil resisting forces

The Vibdrive and Vipere models evaluate the soil resisting forces \( (R_t, R_s) \) on different soil parameters. The Vibdrive model requires CPT test results, and the Vipere requires laboratory tests results for both tri-axial and simple direct shear tests.

Unfortunately the laboratory tests results required were not available for the Vårby site. Vanden Berghe (2001) states that the choice of input values for the soil parameters for the Vipere model was made based on the correlations existing between the CPT tests conducted and the specified soil parameters. When no correlation existed, this was estimated according to function values proposed in the literature.

The Vibdrive model correlates the evaluation of the soil resistance with CPT test results, however there are no guiding publications containing detailed information on how the dynamic soil characteristics \( q_d \) and \( \tau_d \) expressed by Equations (2.30) and (2.31), should be evaluated. It was therefore decided to evaluate \( R_t \) and \( R_s \) versus depth \( z \) according to the following four steps.

1. Figure (5-31) shows the evaluated quasi-static toe and shaft resistance \( q_t \) and \( \tau_t \) profile of the subsoil strata, which are required in Equations (2.30) to (2.34). These were derived from local cone resistance \( q_c \) and local unit-
sleeve friction ($f_s$) without correction for the pore pressure effects, and averaged according to the Equation (5.18).

2. The liquefied toe and shaft resistance ($q_l$ and $\tau_l$) profile was evaluated according to Equations (2.32) to (2.33), using the correlating friction ratio ($R_f = f_s/q_c$) and the empirical liquefaction factor ($\psi$) set to 1/10.

3. Figure (5-31) shows the evaluated dynamic-driving unit toe and shaft resistance ($q_d$ and $\tau_d$) profile according to Equations (2.30) and (2.31), with an acceleration ratio ($\alpha$) of ($a/g = 15$) and the evaluated liquefied toe and shaft resistance ($q_l$ and $\tau_l$) profile.

4. Figure (5-31) shows the dynamic peak toe and shaft resistance force ($R_t$ and $R_s$) profile for the subsoil strata, which were derived on the basis of the section area and the perimeter of the sheet pile according to Table (5-1), together with quasi-static, liquefied and dynamic toe and shaft resistance profile.

The empirical liquefaction factor ($\psi$) was set to 1/10, since it consisted of a relatively homogeneous and saturated soil strata of well-graded sand. The acceleration ratio ($\alpha$) was set to ($a/g = 15$), on the basis of the field test results of peak acceleration amplitude versus depth (see Figures (5-7) and (5-9)).

The mean values of the quasi-static toe and shaft resistance ($q_s$ and $\tau_s$) profiles used as input parameter of the Vibdrive model, were calculated using the following expressions:

\[
q_s = \frac{1}{2}(I+II) \\
\tau_s = \frac{1}{2}(2III)
\]  
(5.18)

where

\begin{align}
I &= \text{the average cone penetration resistance between the level of the base and a distance } (I = b/4) \text{ above the base,} \\
II &= \text{the average cone penetration resistance between the level of the base and a distance } (II = 3b/4) \text{ below the base,} \\
III &= \text{the average sleeve friction resistance between the level of the sleeve and an equal distance } (III = b/2) \text{ below and above the sleeve, and} \\
b &= \text{the width of the sheet pile } (b) \text{ in this case was 600 mm.}
\end{align}
Results and Analysis

Figure 5.31 Evaluated peak toe and shaft resistance forces ($R_t$ and $R_s$) in accordance with the Vibdrive model, where (a) shows a comparison of static and dynamic parameters ($q_s$ and $q_d$), (b) static and dynamic parameters ($\tau_s$ and $\tau_d$), (c) peak toe force ($R_t$), and (d) peak shaft resistance ($R_s$).

The simulated peak values of soil resistance forces for the Vibdrive where ($z = 10$ m), are approximately ($R_t = A_t q_d \sim 13.3$ kN) and ($R_s = \chi z \tau_d \sim 103.3$ kN), which can be seen in Figure (5-31).

Figures (5-32) through (5-34) compare the acceleration ($a$), the dynamic shaft resistance ($R_s$) and the dynamic toe resistance ($R_t$) in the time domain simulated by the Vipere model, with the field results from Test B2 at ($z = 10$ m), where the simulated peak values are ($a \sim 13$ g, $R_s \sim 12$ kN and $R_t \sim 173$ kN). From the comparison considered, it can be seen that both the simulated and measured acceleration in the time domain display the same shape and similar magnitude ($\sim 130$ m/s$^2$). The comparison of dynamic toe and
shaft resistances ($R_t$ and $R_s$) in the time domain, also displays a surprisingly good correlation, with respect to the evolution of the resistance. However, the predicted amplitude of dynamic toe resistance tends to be under-predicted by a factor of two.

Figure 5-32  Comparison between measured and predicted acceleration performed with the Vipere model, using Test B2 at $z = 10$ m, and readings from a toe accelerometer (Vanden Berghe, 2001).

Figure 5-33  Comparison between measured and predicted shaft resistance, performed with the Vipere model, using Test B2 at $z = 10$ m (Vanden Berghe, 2001).
5.3.5 Evaluation and comparison of simulated penetration speed

Figures (5-35)a-d present the results of a vibro-driveability simulation by the Vibdrive model at a penetration depth of approximately 10 m, with input parameters shown in Table (5-1). The integration procedure of the Vibdrive model has been performed in accordance with previously-described integration procedures (see for example Sections 2.6.6 and 3.4.5). The average penetration speed \( v_p \) is simulated to \(~17 \text{ mm/s}\), which is evaluated on the basis of the permanent displacement \( \Delta u \) of one cycle (see Figure (5-33)d) divided by the time period \( f_d^{-1} \) of the cycle, resulting in \( \Delta u/f_d^{-1} = 0.41/41^{-1} \).

Figure (5-35)a shows the driving and resisting forces applied over time according to the Vibdrive model. The two resistance strengths represented by the dynamic peak toe and shaft resistances are graphed as horizontal lines, together with one active force represented by the sinusoidally unbalanced force \( M_r \omega^2 \sin(\omega t) \), which is biased around the total weight of the system \( g m_{tot} \). Figure (5-35)b shows the Vibdrive simulated acceleration in accordance with Equation (2.64). Figures (5-35)c and d show the integrated velocity and displacement curves. Note that negative values of velocity and displacement both correspond to a downward penetrative motion.

Figure (5-36) shows a comparison of documented vibro-driveability \( v_p(t,z) \) with simulations performed with the Vipere model. The measured \( v_p(t,z) \) curve relates to Test B2, where \( R_c \) effects are not present or considered. It can be seen here that the Vipere model over-predicts the initial phase (the initial 6 m). However, it can be seen in the deeper penetration phase that the two \( v_p(t,z) \) curves display significant similarity.
Figure 5.35 Simulation results from the Vibdrive model at $z \sim 10\ m$, in accordance with input parameters given in Table (5.1), showing (a) forces applied, (b) acceleration, (c) velocity and (d) displacement.
The initial over-prediction of the penetration speed \( (v_p) \) is probably explained by the incorrect choice of input parameters, and the numerical difficulties encountered before the stationary state is reached. The observed dip in the penetration speed \( (v_p) \), which can be observed in Figure (5-36) just prior to a penetration depth of 2 m, is most likely explained by the lateral accelerations of the sheet pile, as previously discussed in Section 5.2.5, which at present is impossible to simulate.

### 5.3.6 Discussion on the comparison of simulations and field results

Section 5.3 compares the results of the Test B2, with the Vibdrive and Vipere models at a penetration depth of 10 m. The primary reason for choosing 10 m was due to the observed dips in penetration speed, which at present are not completely understood and therefore difficult to simulate. These two models propose two different methods for calculating the dynamic soil resisting forces for then calculating the penetration speed of a vibratory-installed sheet pile by integrating the same expressions of motion (Equations (2.34) and (2.64)). The following are the primary assumptions made in the two models.
• The sheet pile is treated as a rigid body (both models).
• Penetrative speed is evaluated from the integrated value of the unbalanced acceleration of the sum of forces acting on the rigid system (both models).
• The soil surrounding the sheet pile is discretised as a set of concentric rings (the Vipere model).
• The soil along the shaft is assumed to be deformed in a state similar to undrained simple shearing, whereas the soil at the toe is assumed to be deformed in a state similar to undrained tri-axial compression (the Vipere model).
• For each type of state, the hypoplastic constitutive model calculates the stress-strain relationships, which takes into the account the dilative and contractive behaviour of the soil (the Vipere model).
• The constitutive stress-strain relationships of the soil model are perfectly plastic; shaft resistance is directly mobilised in the opposite direction to the movement, and toe resistance is only mobilised during the downward direction (the Vibdrive model).

The results of the preliminary comparison between Test B2 and the two simulation models is summarised in Table (5-2). The following observations have been made from the preliminary comparison.

From the evaluation of the dynamic peak values of the two soil resistance forces ($R_t$ and $R_s$) relating to the Vibdrive model and shown in Figure (5-31), it was observed that the choice of acceleration ratio ($\alpha = a/g$) appeared to have a minor effect on the evaluated dynamic soil characteristics ($q_d$ and $\tau_d$) expressed by Equations (2.30) and (2.31), compared to the choice of the empirical liquefaction parameter ($\psi$). It was observed that $\alpha$ was evaluated as either 15 or 20 g, and it had a negligible effect on the evaluated values of $q_d$ and $\tau_d$ expressed by Equations (2.30) and (2.31), due to the more-or-less horizontal ($q_d/\tau_d - \alpha$) curves in Figure (3-16). Instead it can be seen that the choice of $\psi$ has a more pronounced effect on the evaluated values of $q_d$ and $\tau_d$.

From the comparison of results of the Vibdrive model and Test B2 at the penetration depth ($z$) of 10 m, it can be noted that even though both soil resisting forces ($R_t$ and $R_s$) appear to be slightly under-predicted (see Table (5-2)), the speed ($v_p$) is approximately six times lower in the model than the field test observations.

From the comparison of the results of the Vipere model and Test B2 at the penetration depth ($z$) of 10 m, it can be seen in Figure (5-36) that the measured and simulated penetration speeds ($v_p$) are quite similar (~120 mm/s). It can also be seen that both simulated and measured acceleration in the time domain display both the same shape and similar magnitude (~130 m/s²). The comparison of dynamic toe and shaft resistance ($R_t$ and $R_s$) in the time domain also displays a surprisingly good correlation, with respect to
the evolution of the resistance. However, the predicted amplitude of the dynamic toe resistance tends to be under-predicted by a factor of two.

Since a comparison of Test B2 with the Vipere model does not utilise the evaluated driving force results in the same time domain as Figures (5-32) to (5-34), (Vanden Berghe, 2001), it has not been possible to evaluate the magnitude nor the existence of the phase shift between the peak value of the driving force compared with the resisting forces, as previously observed in Figure (3-13). However as stated in the beginning of this section, the purpose of this preliminary comparison is simply to initiate the validation process of the two chosen models with actual field tests results.

<table>
<thead>
<tr>
<th>Penetration depth ($z$) = 10 m</th>
<th>$a$</th>
<th>$R_t$</th>
<th>$R_s$</th>
<th>$v_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/s$^2$</td>
<td>kN</td>
<td>kN</td>
<td>mm/s</td>
</tr>
<tr>
<td>Field Test B2 ($R_c = 0$)</td>
<td>120</td>
<td>23</td>
<td>205</td>
<td>120</td>
</tr>
<tr>
<td>Simulated by the Vibdrive model</td>
<td>136</td>
<td>13.3</td>
<td>103.3</td>
<td>17</td>
</tr>
<tr>
<td>Simulated by the Vipere model</td>
<td>125</td>
<td>12</td>
<td>173</td>
<td>125</td>
</tr>
</tbody>
</table>

5.4 Concluding remarks about the results

5.4.1 In general

The field-related values of actual dynamic soil resistance values presented are indeed encouraging. Several similarities amongst the primary parameters hypothesised in Chapter 3 can actually be observed in the field test results. However, the valuable analysis of the field-related properties of dynamic toe and shaft resistances is just in its infancy, compared to the work that has been done with impact-driven displacement piles. There is a lot of analysing remaining to be done, simply due to the complexity and the amount of data usually related to dynamic field tests.

5.4.2 Field observations of hypothesised factors

Several similarities in the primary parameters hypothesised in Chapter 3 can actually be observed in the field test results. The vibratory-related factor relating to how much of
the theoretically-generated driving force that actually enters the vibratory-driven sheet pile axially has been confirmed to be a factor that warrants better consideration in the future.

The sheet-pile-related presence of interlock friction forces have also been highlighted as being factors warranting better consideration in the future, due to the significant influence they have on the development of lateral acceleration in the pile, the forces generated in the pile, and the significantly higher ground vibrations. This is not only in the vertical direction, but also in the longitudinal direction due to the laterally-induced motion of the sheet pile. The assumption of pile rigidity has also been shown to be a justifiable assumption, especially during favourable driving conditions with relatively easy pile driving.

The limitations of the soil instrumentation, has not permitted proving nor investigation of the two fundamental mechanisms taking place in the soil during the use of vibrators to install sheet piles. These two primary soil-related factors control the extent of the induced shear strength reduction of cohesionless soils. It would have been advisable to have also had a number of tri-axial geophones or accelerometers together with pore-pressure transducers in the soil at different depths and radial distances in the vicinity of the vibratory-driven piles. This would have provided the possibility of accessing and studying the magnitude of inertial forces in the soil, and at the same time the excess pore-pressure variation in time at different depths and radial distances from the sheet pile.

5.4.3 Field test and simulation results

The results of the comparison between simulations with the Vipere model and the field-test related results (Test B2) presented are indeed encouraging. Several similarities in the simulated vibro-driveability and actual measurements can be seen. However the encouraging comparisons seem to be achieved only when the values of simulated and measured penetration speeds ($v_p$) display similar values. During these conditions, it appears as though the results of the comparisons display greater similarities than other situations. Why greater similarities in acceleration as well as dynamic penetrative resistances ($R_t$ and $R_s$) are achieved when ($v_{p,measured} \sim v_{p,simulated}$) is not completely understood at the present time.

The following are some of the difficulties that were encountered during the comparison phase of the study.
• The simulations of the initial penetration speed tend to be a difficult to accomplish at the present time.
• The parametric analysis of this difficulty indicated that small variations in an input parameter for the Vipere model had more impact on the penetration speed at smaller penetration depths, compared with depths greater than approximately 8 m.

The following are the primary limitations of the two simulation models studied, at the present time.

• The efficiency of the vibratory system is not taken into account in either of the models (that is the ratio between the theoretical driving force and the force actually delivered to the head of the vibratory-driven sheet pile).
• The effects of lateral flexibility in the vibratory-driven sheet pile are not taken into account in either model studied.
• The effects of interlock friction force \( R_c \) is not explicitly taken into account in the two models, however it is possible to address it by adding the magnitude of \( R_c \) to the dynamic shaft resistance.
• The ultimate build-up of excess pore-pressure, its dissipation and its effect on the shear strength reduction is not possible to model.
• The models do not explicitly take into account the inertial forces of the vibrating soil volume surrounding the driven pile and how this affects the reduction of the initial shear strength reduction of cohesionless soils.

Finally, it should be noted once more that this comparison of the field results with the two simulation models selected does not assume to validate the two models. The objective here has simply been to initiate the process of validation required between candidate prediction models of vibro-driveability with actual field-measurements of the vibratory-driven sheet piles. The validation process initiated here naturally needs to be continued with further test results, especially results approaching the point of refusal of penetration, as well using other potential vibro-driveability models.
CHAPTER 6

CONCLUSIONS

6.1 Conclusions

6.1.1 Chapter introduction

The process of obtaining representative values for the dynamic soil properties is the most difficult part of describing the vibro-driveability and the clutch friction developing in the sheet-pile interlock. The dynamic peak value of toe and shaft resistances preventing the sheet pile from entering the soil stratum are primarily a function of the initial soil conditions, the dynamic state during installation, and the vibrator parameters chosen. Based on the results of this study and the conclusions at the end of each chapter, the following paragraphs summarise the main results and conclusions.

6.1.2 Factors concluded to influence the vibro-driveability

In light of the complexity of attempting to describe vibro-driveability, it is justifiable to divide the prime factors affecting vibro-driveability into three main categories: (i) vibratory-equipment-related, (ii) sheet-pile related, and (iii) soil-related factors.

The main vibratory-equipment-related factors that have a significant impact on vibratory-driven sheet piles are summarised below.

- *Efficiency of the vibratory-equipment*: any successful attempt to simulate vibro-driveability should begin with a reasonably accurate theoretically-generated value for the driving force that actually enters the sheet-pile head axially.
- *Choice of vibrator parameters*: this appears to be an incompletely understood factor that ought to be considered in both “hard” driving conditions and driving with displacement piles. This is because of the correlation with the undesirably ‘fast’ vibratory-driving state.
- *Choice of vibratory equipment*: this ought to be considered along with how the vibratory equipment should be operated, especially when environmental aspects need to be addressed.
The main sheet-profile-related factors that are understood to have a significant impact on vibratory-driven sheet piles are summarised below.

- **Justification of axial rigidity**: from an engineering point of view this can be justified in most of the more favourable cases featuring favourable soil conditions.
- **Effects of lateral flexibility in the sheet pile**: this factor can definitely cause unexpected problems, such as considerably lower penetration speeds and the generation of considerably higher ground-vibration values.
- **Effects of friction forces in the clutch**: these could also cause considerably lower production capacity, generate considerably higher ground-vibration values, and in the long run, generate damaging settlement in neighbouring structures.

The main non-cohesive, soil-related factors understood to have a significant impact on vibratory-driven sheet piles are summarised below.

- **Inertial forces but not primarily the effects of liquefaction**: these have been phenomenologically explained as the hypothetical fundamentals of shear-strength reduction. They are the primary soil-related factors that alter the initial shear strength of the soil to the favourably-reduced value during the process of vibratory-installation of sheet piles.
- **Cyclic mobility of the individual soil particles combined with partial liquefaction**: the fundamental importance of the vibration caused by the individual soil-grain motion as well as the pore-water pressure induced is a difficult task to simulate analytically, empirically as well as numerically.

### 6.1.3 Field test results

The sheet-pile instrumentation was the most critical part of the study with respect to experimental success or failure. The procedures for positioning and protecting sensors and wires described, was both time consuming and involved a great number of practical considerations. However it proved to be surprisingly robust and exceeded functional expectations with all but a few exceptions.

The strain sensors used ought to be able to detect the effects of bending, twisting and flexurally-induced motions, however the sensor-fitted sheet piles were not calibrated for forces other than pure axial compression loading. Future attempts to monitor the development of stresses in sheet-pile profiles tentatively driven into specific soils strata using the vibratory-driving technique, should definitely include a calibration programme that includes a procedure for relating the voltage output to pure bending and twisting of the sheet-pile section. This would provide an improved setup for analysing
the complex stress situation developing in each sheet-pile section that is most interesting in this configurations.

Due to the limitations in the soil instrumentation, it has not been possible to scientifically prove or even investigate the two fundamental mechanisms taking place in the soil occurring during the installation of sheet piles using vibratory driving. These two primary soil-related factors control the extent of the reduction in the shear strength induced in cohesionless soils. In this regard, it would have been better to also have had a number of tri-axial geophones or accelerometers together with pore-pressure transducers located in the soil at different depths and radial distances in the vicinity of the vibratory-driven pile. This would have provided an opportunity to detect and study the magnitude of inertial forces in the soil, and at the same time the excess pore-pressure variation over time at different depths and radial distances from the sheet pile.

From the results obtained and taking into consideration the limitations mentioned above, the following conclusions have been drawn.

- *The efficiency of the driving force actually entering the pile* appears to be surprisingly low (less than half), which agrees with earlier publications.
- *The rigid-body (pile) assumption* definitely appears to be justifiable based on the study conducted, which is also in agreement with earlier publications, and can be readily applied during favourable conditions for this technique.
- *The lateral flexibility in the sheet pile* appears to be a factor that ought to be given consideration in the future, as it was found to be significant, ranging from half to equal the value of the recorded peak axial value.
- *Effects of friction forces in the clutch* have been confirmed to have a considerable effect on (i) the ground-vibrations induced (being twice as high or more), (ii) the flexibility generated in the profile (~twice as great), and (iii) on the magnitude of the stresses developed in the sheet pile (higher and more complex).
- *The dynamic peak toe resistance* varied between \(30 < R_t < 100 \text{ kN}\) during the range of penetration depths investigated.

### 6.1.4 Vibro-driveability simulations

The two models used for comparing simulated vibro-driveability with the actual test results obtained were Vibdrive and Vipere. Based on the results of the comparisons performed in this study, the following conclusions can be drawn.

It appears as though the Vibdrive model under-predicts the penetration speed, as well as the dynamic peak shaft resistance. However both peak values for the acceleration and dynamic toe resistance seem to be of the same magnitude.
Comparison of the Vipere simulation results with the field-test results are indeed encouraging. Several similarities can be observed between the simulated and the actual vibro-driveability measurements. However, the comparisons seem to be encouraging only when the penetration depth is approximately 8 m or greater.

Some of the difficulties with the Vibdrive model encountered during the comparison phase of the study are summarised below.

• The initial penetration speed was difficult to determine, and several iterations had to be conducted in order to achieve a satisfactorily consistent value for the final penetration speed.
• The Vibdrive model correlates the evaluation of the soil resistance with the results of CPT tests, however no published guide exists including detailed information on how to deal with the dynamic soil characteristics $q_d$ and $\tau_d$.

The difficulties with the Vipere model encountered during the comparison phase are summarised below.

• Simulations of the initial penetration speed tended to be difficult to achieve given the current stage of model development.
• The parametric analysis of these difficulties indicated that small variations in input parameters to the Vipere model had greater impact on the penetration speed at shallower penetration depths than depths greater than approximately 8 m.

The following is a summary of the primary limitations with the two simulation models at the present time.

• The efficiency of the vibratory system (the ratio between the theoretical and the actual driving force delivered to the head of the vibratory-driven sheet pile) is not considered in either of the two models.
• The effects of lateral flexibility in the vibratory-driven sheet pile is not taken into account in either of the two models.
• The effect of interlock friction force is not explicitly taken into account in the two models, however is possible to address by adding the magnitude of this force to the dynamic shaft resistance.
• The ultimate build-up of excess pore-pressure, its dissipation and its effect on the shear strength reduction is not possible to model.
• The models do not explicitly take into account the inertial forces of the vibrating soil volume surrounding the driven pile, nor how this affects the the reduction of the initial shear strength reduction in cohesionless soils.
6.2 Comments and scope for future work

It should once more be emphasised that there are surprisingly few publications available that present full-scale testing of vibratory-driven sheet piles, and is especially surprising considering the numerous kilometres of sheet piles driven each year. It can therefore be concluded that today’s engineering knowledge about the vibratory-driving technique is still in its infancy. Therefore there is a considerable need to perform more studies to allow definite conclusions to be drawn and adequate theories to be developed.

The difficulties encountered in attempting to simulate the penetrative behaviour of vibratory-driven sheet piles arose from the interactive complexity of the ‘vibrator, sheet-pile and soil system’. One of the key issues relates to a fundamental understanding of the vibrator-sheet-pile driving process, and involves the correct description of the dynamic soil resistance, as well as the magnitude of the friction forces developing in the sheet-pile interlock.

From a geotechnical point of view, subsoil characteristics are usually determined by means of standard investigation methods such as:

- probing tests (for example CPT and SPT),
- sampling tests (for example boring), and
- laboratory tests (such as tri-axial, resonant-column, and direct shear tests).

Generally speaking, all these methods have been developed to produce input information for static design issues. It is obvious that such investigation methods are not well suited to characterising the soil behaviour that exists during the vibratory-installation process of piles and sheet piles. However, it is obvious that the investigation method devised to evaluate the dynamic soil stress and strain characteristics that exist in cohesionless soils must attempt to duplicate the boundary conditions existing in the vicinity of vibratory-driven sheet piles as closely as possible.

The process of obtaining representative values for the dynamic soil properties is the most difficult part of a tentative attempt to predict vibro-driveability. It is the author’s opinion that a new field investigation method and analysing procedure is needed to enhance the process of obtaining more representative values for the dynamic soil resistance in the future.
REFERENCES


HOLEYMAN, ALAIN. E., LEGRAND, CHRISTIAN., (1994), Soil modelling for pile vibratory driving., Int. conf. on design and construction of deep foundation, Orlando, pp. 1165-1178.


HUDSON, D.E., (1979), Reading and Interpreting Strong Motion Accelerograms., Earthquake Engineering Research Institute, Berkeley, California., pp. 112.


LEGRAND, C., VAN ROMPAEY, D., MENTEN, J., (1993), A Comparison of Different Sheet-Pile Intallation Methods., (Obtained by personal correspondence.)

22nd annual Offshore Technology Conf. in Huston, Texas, May 7-10, pp. 315-326.


References


PRANGE, B., DIERSSSEN, G., HÜBERT, G., (1992), Forschungsbericht an die DFG zum TP 4 “Experimentelle Systemidentifikation” der Forschergruppe “Dynamische Penetration”.

RAO., PRAMOD M., (1993), Effect of pile geometry and soil saturation in the behavior of nondisplacement piles installed by vibration., MSc thesis presented to the Faculty of the Dep. of Civil and Environmental Engineering, University of Houston, Texas.


References


TSAPLIN, S., (1953), Vibratory impact Mechanisms for Road and Bridge Constructions. A Russian-English Translation by National Eng. Lab., No. 1003. (Original manuscript in Russian, Autotransizdat, Moscow.)

VANDEN BERGHE, J-F., (2001), Sand Strength Degradation within the Framework of Vibratory Pile Driving., PhD thesis presented to the Faculty of Applied Science, Université chatolique de Louvain., Belgium.


Viking, K., (1997), Vibratory driven piles and sheet piles -a literature survey., Report 3035, Dept. of Civil and Environmental Engineering, Royal Institute of Technology, 75 pp, ISSN 1400-1306.


Youd, T.L., (1967), Engineering Properties of Cohesionless Soils During Vibrations., Iowa University., USA.
Appendix A

JUSTIFIED SIMPLIFICATION OF THE VIBRATOR FROM 2 TO 1 DEGREE SYSTEM

A.1 The two-degree freedom system

The vibrator can be viewed as a two degree of freedom system moving in the longitudinal direction \(x\), see Figure (A-1), consisting of two masses interconnected via the elastomeer spring with stiffness \(k\). The lower mass is the excitor block with the attached clamping device, i.e. \((m_v + m_c)\), which is interconnected to the suppressor housing, sometimes also called bias mass.

In addition to the force generated by the spring \(k\), the mass \((m_v + m_c)\) is subjected to gravity \(g\), the unbalanced force \(F_v\) described by Equation (2.8), whereas the mass \(m_o\) is subjected to gravity and the suspension force \(T\). The net quasi stationary action on the head of the driven profile \(F_d\), is a result of the carrier operation of the vibro-equipment.

Figure A-1 Forces acting on the two degrees freedom model of the bias mass-spring-vibrator system.
A.1.1 Justified simplification into a SDOF system

If Newton’s second law (i.e. \( ma = \Sigma F \)) is applied on the two different masses of the suppressor housing-spring-excitor block system in Figure (A-1), the following expressions can be derived:

\[
\frac{m_o}{o} \ddot{u}_o = F_o + m_o g - k(u_o - u_v) \tag{A.1}
\]

\[
(m_v + m_c) \ddot{u}_v = (m_v + m_c)g + M_e \omega^2 \cos \omega t + k(u_o - u_v) - F_d(t)
\]

where \( m_v \) = mass of vibrator exclusive weight of eccentric masses [kg],
\( m_c \) = mass of clamping device [kg],
\( \ddot{u} \) = acceleration amplitude of motion [m/s²],
\( F_o \) = static surcharge force according to Equation (2.2) [N],
\( g \) = gravity [kgm/s²],
\( \omega \) = angular frequency [rad/s],
\( M_e \) = static moment according to Equation (2.4) [kgm],
\( t \) = time [s],
\( F_d \) = driving force measured at the head [N].

which redispersioned can be expressed as:

\[
\frac{m_o}{o} \ddot{u}_o + (m_v + m_c) \ddot{u}_v = T + (m_o + m_v + m_c)g + M_e \omega^2 \cos \omega t - F_d(t) \tag{A.2}
\]

and then applying the ‘position of masscenter’ on the acceleration of the two masses of the system.

\[
(m_o + m_v + m_c) \ddot{u} = T + (m_o + m_v + m_c)g + M_e \omega^2 \cos \omega t - F_d(t) \tag{A.3}
\]

If it can be proven that \( m_o \ddot{u}_o << (m_v + m_c) \ddot{u}_v \), it can then be justified to simplify the two degrees system (see Figure (A-1)) into a SDOF system (see Figure (A-2)), having the total mass \( (m_o + m_v + m_c) \), the following known relationship can be derived, which also have been graphed in Figure (A-2):

\[
(m_v + m_c) \ddot{u}_v = F_o + (m_v + m_c)g + M_e \omega^2 \cos \omega t - F_d(t) \tag{A.4}
\]

where \( F_o \) is given according to Equation (2.1).

The stiffness of the elastomeer pads are balanced in order to make them stiff enough to connect the to parts of the vibrator, and at the same time soft enough in order
to keep the natural frequency \( f_n = \frac{\pi}{2} \sqrt{\frac{k}{m_v + m_c}} \) of the bias mass/spring system as low as possible.

The magnitude and variation in time \( t \) of the theoretical force-function \( (F_d) \), generated by the vibro-equipment, i.e. the sum of static surcharge force \( (F_o) \) and unbalanced force \( (F_v) \) is rather easy to both estimate and describe. The force-function varies sinusoidally in time with applied driving frequency \( (f_d) \), adjusted eccentric moment \( (M_e) \) and the weights of the moving masses \( (m_v \text{ and } m_c) \) in accordance with Equation (A.7).

**Figure A-2 Forces acting on the SDOF model of the vibrator system.**

Strain gauges tentatively mounted on the in- and outside of a vibro-driven sheet pile weeb, see \( S_{hi} \text{ and } S_{bo} \) in Figure (4-38), should theoretically register the biased sinusoidal force function \( F_d \) expressed by Equation (2.9).

The theoretical driving force \( (F_d) \) generated by vibro-equipment and delivered to the head of the driven profile are normally derived from a simplified single degree of freedom system (SDFS) which models the mechanical action of the vibrator. However, the surcharge mass-spring-vibrator system are actually a two degree system, consisting of two masses interconnected via the isolating elastomers. However, since the SDFS simplification is justified due to the fact that the spring separating the two masses, i.e. excitor block from the surcharge mass, makes the vibratory motion of the excitor block essentially independent of the magnitude of the surcharge mass.

Since vibratory driving is a dynamic event, interpretations of the driving and resistive forces, can only be accessed if the inertial effects is isolated from the total resistance recorded by the strain gauges. This can be accomplished by establishing a dynamic equilibrium of the entire vibrator-profile-soil system, Equation (3-2), and then applying Newton's second law (i.e. \( ma = \Sigma F \)) on the different masses of the system which can be rewritten into the following more detailed form:
where \( m_v \) = mass of vibrator exclusive weight of eccentric masses [kg],
\( m_c \) = mass of clamping device [kg],
\( \ddot{u} \) = acceleration amplitude of motion \([m/s^2]\),
\( F_o \) = static surcharge force according to Equation (2.2) \([N]\),
\( g \) = gravity \([kgm/s^2]\),
\( \omega \) = angular frequency \([rad/s]\),
\( M_e \) = static moment according to Equation (2.4) \([kgm]\),
\( t \) = time \([s]\),
\( F_d \) = driving force measured at the head \([N]\).

where the expression of the acceleration amplitude of motion \((\ddot{u})\) can be replaced by the double integrated expression of the displacement amplitude of motion \((u)\), together with a redisposition of the expression of Equation (A.1) in order to derive the expression of the theoretical driving force time history, expressed as:

\[
F_d(t) = F_o + (m_v + m_c)g + M_e \omega^2 \cos(\omega t) + (m_v + m_c)u_o \omega^2 \cos(\omega t + \phi)
\]  

(A.6)

where \( F_o \) = driving force measured at head of driven profile \([N]\),
\( M_e \) = static moment according to Equation (2.4) \([kgm]\),
\( u_o \) = displacement amplitude according to Equation (2.11) \([m]\),
\( \phi \) = phase angle between motion of vibrator and profile \([rad]\).

Prior to an tentative field test, the electrical output signal from the strain gauges are usually zeroed. Normally in conjunction with a tentative field test, this is accomplished by letting the profile hang freely in the air held by the clamping device, O’Neill et al. (1989). The “zeroing” of the strain gauges signal, means that the gravitational forces acting on the clamping device and excitor block, are not accounted for in the analysis. The peak compressive force measured my strain gauges mounted at the head of the driven profile will then measure the theoretical driving force time history, expressed as:

\[
F_d(t) = F_o + (M_e + (m_v + m_c)u_o) \omega^2 \cos(\omega t)
\]  

(A.7)
which is the sum of the sum of the static surcharge force \( (F_{o}) \) (which does not accelerate), the maximum unbalanced force \( (F_{v}) \), and the inertia force of the vibrator masses, \( (m_{v} \cdot a_{v}) \). The peak compressive force measured at the head of the drive profile will most likely be less than the value expressed by Equation (A.7) due to energy losses in the vibro equipment (see Section 3.3.2), and due to propagation of flexural and torsional energy of the driven profile (see Section 3.3.3) which is not sensed by the gauges that only measure axial loads.

![Diagram](image)

Figure A-3  Forces acting on the two degrees freedom model of the bias mass-spring-vibrator system.
APPENDIX B

JUSTIFIED SIMPLIFICATION OF THE SHEET PILE BEHAVIOUR AS AN AXIAL RIGID BODY

B.1 Vibration in rods

Let us consider an elastic free longitudinal vibrating rod of length $L$, constrained in at the longitudinal positions ($x = 0$) by the unbalanced force ($F_v$), and free at ($x = L$) illustrated in Figure (B-1).

B.1.1 Displacement solution

The unbalanced force ($F_v$), produced by the vibrator can be expressed in complex form given by:

\[ F_v = \hat{F}_v e^{i\omega t} \]  

(B.1)

where  
\[
\hat{F}_v = \text{peak value of unbalanced force (i.e. } M_v \omega^2) \text{ [kN]},
\]
\[ \omega = \text{angular frequency } (2\pi f_d) \text{ [rad/s]}, \]
\[ f_d = \text{driving frequency [Hz]}, \]
\[ t = \text{time}. \]

For finite rods with various boundary conditions, the general expression of the displacement solution (here given in complex form) of a longitudinally vibrating bar in its natural mode is expressed according to:

\[ u(x, t) = Be^{i(kx - \omega t)} + Ce^{-i(kx - \omega t)} \]  

(B.2)

where  
\[ B, C = \text{constants [-]}, \]
\[ k = (k = \omega / \xi) \text{ [rad/m]}, \]
\[ \omega = \text{angular frequency } (2\pi f_d) \text{ [rad/s]}, \]
\[ \xi = \text{longitudinal position [m]}, \]
Appendix B

\[ f_d = \text{driving frequency [Hz]}, \]
\[ t = \text{time}. \]

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure.png}
\caption{Waves in a rod of length \( L \).}
\end{figure}

\section*{B.1.2 Boundary condition}

Boundary conditions of the rod illustrated in Figure (B-1) are as follows: at the constrained end \( (x = 0) \) the expression of the force is defined by the product of the strains to section area \( (A) \) and Young’s modulus \( (E) \) of the bar, given by:

\[ F_v(t) = AE \frac{\partial u(0, t)}{\partial x} \quad (B.3) \]

at the free end \( (x = L) \) the expression of the force is zero, given by:

\[ F_v(t) = AE \frac{\partial u(L, t)}{\partial x} = 0 \quad (B.4) \]

Substitution of Equation (B.2) into Equations (B.3) and (B.4), gives the solution of the two constants \( A \) and \( B \), according to:
The general form of displacement solution \( u(x,t) \) for the free vibrating bar constrained by the unbalanced force \( (F_v) \) at \( (x = 0) \) is obtained by substituting the expressions of the constants \( B \), and \( C \) into Equation (B.2) to get:

\[
B = \frac{1}{2AEk} \cdot \frac{\hat{F}}{e^{i k L}} \cdot \frac{e^{-i k L}}{\sin(kL)} \tag{B.5}
\]

\[
C = \frac{1}{2AEk} \cdot \frac{\hat{F}}{e^{i k L}} \cdot \frac{e^{i k L}}{\sin(kL)} \tag{B.6}
\]

The general form of displacement solution \( u(x,t) \) for the free vibrating bar constrained by the unbalanced force \( (F_v) \) at \( (x = 0) \) is obtained by substituting the expressions of the constants \( B \), and \( C \) into Equation (B.2) to get:

\[
u(x, t) = \frac{1}{2AEk} \cdot \frac{\hat{F}}{e^{i k L}} \cdot \frac{1}{\sin(kL)} \left[ e^{-i k (L - x)} + e^{i k (L - x)} \right] e^{-i \omega t}
\]

\[
= \frac{\hat{F}}{AEk} \cdot \frac{\sin[k(L - x)]}{\sin(kL)} e^{-i \omega t}
\]

Resonance of the longitudinally vibrating bar is related to solutions of Equation (B.6), when \( (u(x, t) \rightarrow \infty) \). The solution is given by the denominator of Equation (B.6), and by substitute \( (k = \omega / \epsilon_b) \) into Equation (B.6), we get:

\[
\sin \left( \frac{\omega n L}{\epsilon_b} \right) = 0 \tag{B.7}
\]

to satisfy Equation (B.7), we get:

\[
\frac{\omega n L}{\epsilon_b} = n \pi \tag{B.8}
\]

or

\[
f_n = \frac{n \epsilon_b}{2L} \tag{B.9}
\]

\[
n = 1, 2, 3, \ldots
\]

The expression of Equation (B.7) is the frequency equation from which the frequencies \( (f_n) \) of the natural modes of vibration can be derived, and assuming only harmonic solutions, \( (i.e. \ n \neq 0) \), the first resonance frequency is then given by:

\[
f_1 = \frac{\epsilon_b}{2L} \tag{B.10}
\]
The resonance frequency of a free vibrating steel sheet pile with length 14 m, with a bar velocity of 5100 m/s, can be estimated to \( f_1 \sim \frac{5100}{28} = 182 \) Hz. To be compared with today driving frequencies of modern vibro-drivers \( (30 < f_d < 40) \) Hz.

Comparisons of the displacement amplitude of sheet pile head and toe of a free vibrating bar, i.e. \( (x = 0) \) and \( (x = L) \) respectively. The expression of Equation (B.11) displays the fact that amplitude of the toe will always theoretically display a higher value compared to the constrained head.

A comparison of the displacement amplitude of the head with the toe a free vibrating steel sheet pile, with \( L = 14 \) m, \( c_b \sim 5100 \) m/s, can be estimated according to:

\[
\frac{u(0, t)}{u(0L, t)} = \cos(kL) = \cos\left(\frac{\omega L}{c_b}\right) \tag{B.11}
\]
APPENDIX C

RESULTS FROM: CALLIBRATIONS

C.1 Accelerometers

C.1.1 Accelerometer at sheet pile head, channel 4

phase and transferfunction

Figure B-1  Screen dump taken from the oscilloscope of the calibration of the accelerometer $a_j$, phase curve above and transfer signal below taken over the frequency range 0-50 [Hz].

Coherrens and transferfunction
Figure B-2  Screen dump taken from the oscilloscope of the calibration of the accelerometer $a_x$, coherence curve above and transfer signal below taken over the frequency range 0-50 [Hz].
APPENDIX D

THE COMPANION CD-ROM, ENTITLED:

VIBRO-DRIVEABILITY - AND

ENVIRONMENTAL STUDIES OF VIBRATORY

INSTALLED SHEET PILES

D.1 The MATLAB™-files

D.1.1 Work-space files

D.2.2 M-files

D.2 The raw-data Binary-files

D.1.1 Helgeansholmen

D.2.2 Vårby

D.3.3 PCscan program