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Quantum Interference, Complementarity and
Entanglement:
From Fundamentals to Applications

Tedros Tsegaye



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Royal Institute of Technology (KTH)
Department of Electronics

Quantum Interference, Complementarity and Entanglement: From Fundamentals to Applications

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In memory of my father...

Abstract

A fundamental entity in quantum mechanics is the quantum mechanical state. The only connection between the theory of quantum mechanics and our observable world is provided by state measurements, and in this interference between quantum states plays a key role.

Recent interference experiments probing the world of quantum mechanics have started to resolve paradoxes and give new insights. While the classical concepts of phase and polarization are well established, the understanding of their quantum mechanical counterparts is not complete. While examining those concepts the increased understanding of quantum interference give rise to new applications: In quantum cryptography, secure (protected by the laws of physics) secret quantum key distribution has been set-up between places tens of kilometers apart. Quantum computers can be viewed as complex quantum interferometers. This emerging technique anticipates the construction of a new class of computers that can process data (superposition states) in parallel. Certain algorithms exist that can solve problems that grows exponentially for classical computers on a much faster polynomial growing time using quantum computers.

The thesis is focused on the generation and detection of some non-classical few-photon states, and in particular on entangled states. A common aspect between the experiments of the thesis is the use of quantum interference. In paper A, the complementary wave-particle duality of light is examined. Paper B, C and D implements relative phase and polarization rotation experiments based on analogous theories. Using two photons, three orthogonal states of the relative phase operator and the polarization rotation operator can be generated. The techniques give a linear increase of the sensitivity of relative phase shifts and polarization rotations with the number of available photons. The sensitivity of classical measurement techniques are limited to the square of the number of available photons. Paper E uses the complementary wave-particle duality of light in an interference experiment. The technique called interaction free measurements enables (at least in principle) the perfect detection of an absorbing object without the object absorbing any photon. Our method is based on the principle that a Fabry Perot interferometer tuned to resonance transmits an impinging photon. In contrast, when placing an object between the mirrors of the Fabry Perot Interferometer, the impinging photon will be reflected from the first mirror. This technique using quantum objects could be used to produce entangled multi-photon states that can be used to improve the schemes of papers A, B, C, and D by going to an higher manifold (using a higher number of photons).

Preface

This thesis is divided into three parts. Part I contains of five chapters that introduce the reader into the aspects of quantum optics considered in the original work. There are no new results in these chapters. Instead, I have written these chapters to give an intuitive and simplified background of the research area in order to give an introduction to the original work. Most of the contents could be refereed to as “common knowledge” in the quantum optics community, and other parts come from the cited sources.

In Part II I give a review of the new theory and the experimental setups that are presented in the thesis. The relationship between the various experiments is also shown in greater detail.

Part III contains two chapters. In chapter 9, a summary of the each paper in the original work is provided, including a description of my own contribution. The focus has been put on the scientific relevance of the results and on the relation to the work of other groups. Finally conclusions of the work are drawn in chapter 10.

The scientific news, that is, my contribution, through my thesis work to the scientific progress, is found in the original work. The reprinted papers are found at the end of the thesis.

Acknowledgements

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Contents

Preface	vii
Acknowledgements	ix
List of papers	xiii
I Background	1
1 Introduction	3
2 Coherence and quantum interference	7
3 Complementarity and entanglement	9
3.1 Complementarity	9
3.2 Entanglement	10
4 Quantum mechanical phase and polarization	15
4.1 Classical description using Stokes parameters	15
4.2 Quantum mechanical description	19
4.3 Heisenberg limited interferometry	20
4.4 Heisenberg limited polarometry	21
5 Spontaneous parametric down conversion	23
II Review of original work	27
6 Relative phase states and polarization states	29
6.1 Relative phase states	29
6.2 Polarization	30
6.3 The First Manifold	31
6.4 Higher manifolds	32

7	Complementarity with Schrödinger kitten states	33
8	Interaction free measurements	35
III Epilogue		41
9	Summary of the original work	43
10	Conclusions and outlook	45

List of papers

The thesis is based on the following papers, which will be referred to by their respective letters:

- A** T. Tsegaye, G. Björk, M. Atatüre, A. V. Sergienko, B. E. A. Saleh,
and M. C. Teich
**Complementarity and quantum erasure with entangled-photon
states**
To be published in Phys. Rev. A.
- B** A. Trifonov, T. Tsegaye, G. Björk, J. Söderholm and E. Goobar
Experimental Demonstration of the Phase-Difference Operator
Opt. Spectrosc.(USSR) **87**, no. 4, pp. 611-615, (1999).
- C** A. Trifonov, T. Tsegaye, G. Björk, J. Söderholm, E. Goobar, M. Atatüre,
and A. V. Sergienko
Experimental demonstration of the relative phase operator
J. Opt. B, **2**, pp. 105-112, (2000).
- D** T. Tsegaye, J. Söderholm, M. Atatüre, A. Trifonov, G. Björk, A. V.
Sergienko, B. E. A. Saleh, and M. C. Teich
**Experimental demonstration of three mutually orthogonal polar-
ization states of light**
Submitted to Phys. Rev. Lett.
- E** T. Tsegaye, E. Goobar, A. Karlsson, G. Björk, M.Y. Loh, and K.H. Lim
**Experimental demonstration of an efficient interaction-free mea-
surement in a Fabry-Perot Interferometer**
Phys. Rev. A, **57**, pp. 3987-3990, (1998).

Related work not included in the thesis:

- * G. Björk, J. Söderholm, A. Trifonov, T. Tsegaye, and A. Karlsson
Complementarity and the uncertainty relations
Phys. Rev. A, **60**, no. 3, pp. 1874-1882, (1999).
- * J. Söderholm, A. Trifonov, T. Tsegaye, and G. Björk
Quantized Phase-Difference
Quantum Communication, Computing, and Measurement 2,
eds. P. Kumar, G. M. D'Ariano, and O. Hirota, Plenum, New York, (2000).
- * G. Björk, A. Trifonov, T. Tsegaye, and J. Söderholm
Quantum phase resolution and phase distribution
Quantum Semiclass. Opt. **10**, pp. 705-721, (1998).
- * J. Söderholm, G. Björk, T. Tsegaye, and A. Trifonov
States that minimize the evolution time to become an orthogonal state
Phys. Rev. A **59**, no. 3, pp. 1788-1790, (1999).
- * A. Karlsson, G. Björk, and T. Tsegaye
Theory of interaction-free measurements in cavity resonators
J. Opt. Soc. Am. B **15**, pp. 2958-2966, (1998).
- * M. Bourennane, F. Gibson, A. Karlsson, A. Hening, P. Jonsson, T. Tsegaye, D. Ljunggren, and E. Sundberg
Experiments on long wavelength (1550 nm) “plug and play” quantum cryptography systems
Opt. Express **4**, no. 10, pp. 383-387, (1999).
- * A. Trifonov, G. Björk, J. Söderholm and T. Tsegaye
A comprehensive experimental test of quantum erasure
submitted to Phys. Rev. A

Part I

Background

Chapter 1

Introduction

Quantum theory has been the basis for most of the modern twentieth century physics. Now, at the beginning of the twenty-first century, several conceptual questions still puzzle physicists. This thesis will not try to give a broad view of all the troublesome conceptual issues. However, some of these issues form the basis for the original work and in this part I will try to give a very brief introduction to the concepts of entanglement and complementarity. In quantum optics the aspect of complementarity most often examined, namely wave-particle duality, is closely interconnected with quantum interference and thus with the quantum description of phase properties. Recently there has been some work on the resemblance between, the polarization properties of an electro-magnetic field and the relative phase properties between two fields [1, 2].

A large part of this thesis is devoted to novel experiments closely related to the generator of relative phase shift between two modes (Paper B and C) and rotation of polarization (Paper D) respectively. Paper A is a recent experiment on the wave-particle duality and quantum erasure [3, 4, 5]. Paper E describes a high efficiency “interaction-free” measurement [6] that utilizes the wave-particle duality of optical fields. An alternative approach uses the rotation of polarization [7].

The techniques presented in the thesis are all based on quantum interference. The power of quantum interference can be exemplified by giving a few application examples.

- *Quantum interference forms the basis behind absolutely secure quantum cryptography systems* [8, 9, 10].
- *Quantum interference allows the construction of qualitatively new types of logic gates* [11, 12], *which in the future can open a whole new arena of quantum computing and information* [13, 14].
- *Quantum interference makes novel communications schemes in the future possible. E.g. quantum dense coding where two bits of information are sent by one*

physical bit [15, 16] and *quantum teleportation where an unknown quantum bit (state) is reproduced exactly on another location after sending two classical bits* [17, 18].

It will be assumed that the reader is familiar with linear algebra, Dirac notation and basic quantum mechanics. Below I will give a brief review of the notations and elementary concepts that are used in the rest of the thesis.

I prefer to treat light and atoms using the same basic theory, namely quantum mechanics. Such a consistent treatment makes certain optical phenomena easier to interpret and classify. An alternative approach is a semiclassical theory, where light is described by the classical Maxwells equations. In some applications this is an adequate description while, as should be noted below, other applications and optical phenomena are impossible to interpret using only classical concepts.

The step “ladder” operators (\hat{a} and \hat{a}^\dagger) and the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$ are of special interest. The number state $|N\rangle$, $N = 0, 1, \dots$ is an eigenstate of the number operator (with eigenvalue N) and the coherent state $|\alpha\rangle$ is the eigenstate of the annihilation operator \hat{a} (with eigenvalue α). The number states form a complete orthonormal basis which span the single mode Hilbert space. The superposition principle simply says that the wave function of any other pure state can be expressed as a superposition of several of those eigenstates. For example the coherent state can be expressed as a superposition of the number states:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle. \quad (1.1)$$

States that can not be described by a wave function are called statistical mixture states or, in short, mixed states. One state often referred to, as an example is the thermal state. The mixed states are described by the density matrix ρ . The density matrix can also be used as an alternative description for states that can be described by a wave function, i.e. pure states. The relationship between the density matrix and the wave function (for pure states) is $\rho = |\Psi\rangle\langle\Psi|$. The density matrix for a thermal state is,

$$\rho = (1 - e^{-\lambda}) e^{-\lambda \hat{a}^\dagger \hat{a}}, \quad (1.2)$$

where λ is a real number > 0 . The thermal state describes radiation from a blackbody source and the constant $\lambda = \hbar\omega/k_B T$, where ω is the frequency and T is the temperature of the blackbody.

The operator associated with the a multi mode electrical field can be expressed as:

$$\hat{\mathbf{E}}(x) = \hat{\mathbf{E}}(\mathbf{r}, t) = \hat{\mathbf{E}}^{(+)}(\mathbf{r}, t) + \hat{\mathbf{E}}^{(-)}(\mathbf{r}, t) = \hat{\mathbf{E}}^{(+)}(x) + \hat{\mathbf{E}}^{(-)}(x), \quad (1.3)$$

$x = [\mathbf{r}, t]$ is a generalized space-time coordinate. The first term in the equation above is:

$$\hat{\mathbf{E}}^{(+)}(\mathbf{r}, t) = \sum_{l, \sigma} c_l \mathbf{e}_{l, \sigma} \hat{a}_{l, \sigma} e^{-i(\mathbf{k}_{l, \sigma} \cdot \mathbf{r} - \omega_l t)}, \quad (1.4)$$

The index l denotes the spatial mode and the index σ denotes the polarization mode. $\mathbf{k}_{l, \sigma}$ is the k-vector of the mode, ω_l is the frequency of the mode, c_l is a normalization constant, $\mathbf{e}_{l, \sigma}$ is the unit vector of the E field, and $\hat{a}_{l, \sigma}$ is the annihilation operator for the mode. The second component of eq. 1.3 corresponding to the creation operator is the hermitian conjugate of 1.4

$$\hat{\mathbf{E}}^{(-)}(\mathbf{r}, t) = (\hat{\mathbf{E}}^{(+)}(\mathbf{r}, t))^\dagger, \quad (1.5)$$

For a single mode field the electrical field can be expressed as:

$$\hat{E}(\mathbf{r}, t) = \hat{a} e^{-i(\mathbf{k} \cdot \mathbf{r} - \omega t)} + \hat{a}^\dagger e^{+i(\mathbf{k} \cdot \mathbf{r} - \omega t)}. \quad (1.6)$$

Since the harmonic oscillator is described by the “ladder” operators, a single mode of the electrical field can be viewed as an harmonic oscillator that can be in an arbitrarily state, e.g. in a number state, in a coherent state or in a thermal state mentioned above. The next chapter examines some correlation properties of those states.

Chapter 2

Coherence and quantum interference

The 1:st and 2:nd order correlation functions are defined by:

$$G^{(1)}(x_1, x_2) = \langle \hat{\mathbf{E}}^{(-)}(x_1) \hat{\mathbf{E}}^{(+)}(x_2) \rangle, \quad (2.1)$$

$$\begin{aligned} G^{(2)}(x_1, x_2) &= \langle : \hat{\mathbf{E}}^{(-)}(x_1) \hat{\mathbf{E}}^{(+)}(x_1) \hat{\mathbf{E}}^{(-)}(x_2) \hat{\mathbf{E}}^{(+)}(x_2) : \rangle \\ &= \langle \hat{\mathbf{E}}^{(-)}(x_1) \hat{\mathbf{E}}^{(-)}(x_2) \hat{\mathbf{E}}^{(+)}(x_1) \hat{\mathbf{E}}^{(+)}(x_2) \rangle, \end{aligned} \quad (2.2)$$

where x_i are generalized space-time coordinates. The notation $\langle : X : \rangle$ is called normal ordering. This means that all operators $\hat{\mathbf{E}}^{(-)}$ should be placed to the left of all operators $\hat{\mathbf{E}}^{(+)}$. The normal ordering follows from “the nature of the intensity measuring process” [19]. The definition of $G^{(1)}$ above directly relates to the intensity of the field for $x_1 = x_2$. The definition of $G^{(2)}$ is directly related to the correlation between two intensities at x_1 and x_2 respectively [20]. This kind of correlation technique, known as coincidence technique, where used in papers A, B, C and D of this thesis.

The 1:st order coherence function is the normalized, 1:st order normally ordered correlation function. It is defined by:

$$g^{(1)}(x_1, x_2) = G^{(1)}(x_1, x_2) / \sqrt{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2)}. \quad (2.3)$$

I will, in this chapter, limit my interest to single mode light. The first order coherence function thou reduces to:

$$g^{(1)}(\mathbf{r}_0, t, \mathbf{r}_0, t + \tau) = \langle \hat{a}^\dagger \hat{a} \rangle e^{-i\omega\tau} / \langle \hat{a}^\dagger \hat{a} \rangle = e^{-i\omega\tau}. \quad (2.4)$$

Since $|g^{(1)}| = 1$ regardless of the state, it can not be used to discriminate classical behavior from quantum mechanical. However quantum interference may

be seen in the intensity correlation. That is why the interferometers I will describe later use coincidence technique to measure correlation of photocurrents (i.e. correlation between intensities rather than between field amplitudes). Such measurements are described by the 2:nd order coherence function which is the normalized, 2:nd order normally ordered correlation function. It is defined by:

$$g^{(2)}(x_1, x_2) = \frac{G^{(2)}(x_1, x_2)}{G^{(1)}(x_1, x_1)G^{(1)}(x_2, x_2)}. \quad (2.5)$$

For the single mode light we have:

$$g^{(2)}(\mathbf{r}_0, t, \mathbf{r}_0, t + \tau) = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle / \langle \hat{a}^\dagger \hat{a} \rangle^2, \quad (2.6)$$

$$g^{(2)}(\mathbf{r}_0, t, \mathbf{r}_0, t + \tau) = (\langle N \rangle^2 + \langle \Delta N^2 \rangle - \langle N \rangle) / \langle N \rangle^2. \quad (2.7)$$

In a classical description the normal ordering is ignored and the classical coherence function is:

$$g_{classical}^{(2)} = \langle N^2 \rangle / \langle N \rangle^2 = (\langle N \rangle^2 + \langle \Delta N^2 \rangle) / \langle N \rangle^2. \quad (2.8)$$

Using the fact that both terms in the denominator of the right hand side of eq. 2.8 are positive we see that $g_{classical}^{(2)} \geq 1$.

Two examples of states that have a classical description are coherent states for which $g_{classical}^{(2)} = g^{(2)} = 1$ and thermal states for which $g_{classical}^{(2)} = g^{(2)} = 2$. For the number state with N photons on the other hand $g^{(2)} = 1 - 1/N$. This so-called sub-poissonian behavior is most clearly seen from single photon emitters. Large efforts have been undertaken on finding suitable single photon emitters for various applications. One goal of our group is to make single photon emitting quantum dots [21].

Chapter 3

Complementarity and entanglement

The principles of complementarity and entanglement led to many controversies in the early days of quantum mechanics. Bohr wrote about complementarity that “Complementarity: any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena” [22]. Regarding entanglement Feynman wrote “It [The superposition principle] contains the *only* mystery in [quantum mechanics]” [23]. The combined effects of complementarity and entanglement for space-time separated systems was highlighted in the Einstein-Podolsky-Rosen (EPR) paradox paper [24] where the authors exclaimed: “No reasonable definition of reality could be expected to permit this.”

I am not intending to give a complete coverage of all aspects of those principles. Instead I will introduce some properties that will be used in later chapters of this thesis.

3.1 Complementarity

Complementarity expresses the fact that any quantum system has at least two properties that cannot be measured simultaneously. One of those complementary property pairs, and perhaps the historically most important, is the wave-particle duality. A quantum system has both particle-like and wave-like behavior. However, observation of one property precludes the observation of the other. For example when observing the path of a particle in one arm of a Mach Zender interferometer any interference effects vanish. Complementarity is also closely interconnected with the uncertainty principle [25].

Two set of bases $\{|A_i\rangle\}$ and $\{|A'_j\rangle\}$ over an $N + 1$ -dimensional Hilbert space are said to be “maximally non-commutative” [26] or “mutually unbiased” [27] if

all inner products between pairs of vectors with one vector from each basis have the same magnitude,

$$|\langle A_i | A'_j \rangle| = \frac{1}{\sqrt{N+1}}, \forall i, j = 1, 2, \dots, N+1, \quad (3.1)$$

Schwinger called operators corresponding to such bases *complementary* and Kraus, Maassen and Uffink have shown that such bases yield the strongest conceivable bound on an information-theoretic statement of the *uncertainty relation* [28, 29]. The number of such complementary operators/mutually unbiased bases have been discussed by Wootters [27] and Ivanovic [30]. In this thesis we can limit the analysis to states with either one or two photons in any or both of two modes. For the case of one photon in one of two modes, where the bases are over one 2-dimensional Hilbert space, we have three mutually unbiased sets of bases and thus three mutually complementary operators. In the case of two photons photon in one of two modes where all such vectors defines a 3-dimensional Hilbert space, we have four mutually unbiased sets of bases and thus four mutually complementary operators.

3.2 Entanglement

When a multi-mode description is combined with the superposition principle interesting things occurs. For pure states it is always possible to describe both modes by a common wave function. It is sufficient for this discussion to consider only two two-level systems. The states can be expressed in a basis consisting of eigenstates $|A_1\rangle$ and $|A_2\rangle$ to some operator \hat{A} in mode A and of $|B_1\rangle$ and $|B_2\rangle$ to some operator \hat{B} in mode B . The superposition principle simply says that the wave function of a state can be in a superposition of the eigenstates (e.g. $|\Psi_A\rangle_A = c_1|A_1\rangle + c_2|A_2\rangle$, where c_1 and c_2 are arbitrary constants fulfilling $|c_1|^2 + |c_2|^2 = 1$). If the states of the two modes are independent of each other, the state is said to be in a product state

$$|\Psi\rangle = |\Psi_A\rangle_A |\Psi_B\rangle_B, \quad (3.2)$$

where $|\Psi_A\rangle_A$ and $|\Psi_B\rangle_B$ are arbitrarily superpositions of the states $|A_1\rangle$ and $|A_2\rangle$, and $|B_1\rangle$ and $|B_2\rangle$, respectively in mode A and B.

Pure states that cannot be written as product states have some degree of entanglement. A subset of the form:

$$|\Psi\rangle = c_1|A_1\rangle_A |B_1\rangle_B + c_2|A_2\rangle_A |B_2\rangle_B, \quad (3.3)$$

where c_1 and c_2 are configuration dependent constants fulfilling $|c_1|^2 = |c_2|^2 = 1/\sqrt{2}$, is of special interest. Those states are called maximally entangled states. When measurements are performed on one of the modes (e.g. mode A) of the entangled two-mode state, the state in mode A is immediately reduced to one of the measurement eigenvalues (e.g. A_1) of the corresponding operator. The

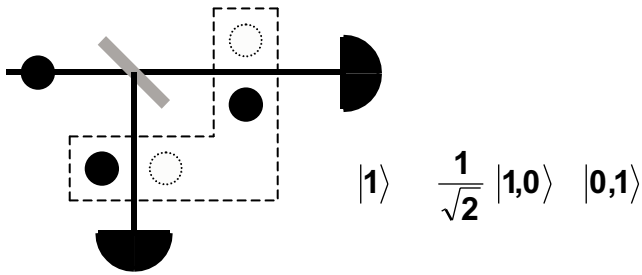


Figure 3.1. A single photon impinging on a beam splitter is transformed to the state (3.4). A measurement on the photon numbers of the transmitted beams will reveal that the photon is either transmitted or reflected.

total superposition state reduces in the same instant into the term containing the eigenstate $|A_1\rangle$ and thus the state in mode B collapse to the state $|B_1\rangle$.

A physical implementation of the state 3.3 is when a single photon state hits a beam splitter (fig. 3.1). The state after the beam splitter can be written as:

$$|\Psi\rangle = c_1|1\rangle_A|0\rangle_B + c_2|0\rangle_A|1\rangle_B \quad (3.4)$$

where c_1 and c_2 are arbitrary constants fulfilling $|c_1|^2 + |c_2|^2 = 1$. Observer A and B have a limited choice of possible measurements for the state above. They can measure whether the photon is transmitted ($\hat{A}\hat{B}|\Psi\rangle \Rightarrow |A_1 = 1, B_1 = 0\rangle$) or reflected ($\hat{A}\hat{B}|\Psi\rangle \Rightarrow |A_2 = 0, B_2 = 1\rangle$). It is not a big surprise that an observation of the (only) photon by e.g. observer A excludes the observation by observer B. Tan et. al. [31] shows in an elaborate scheme that the state 3.4 may be utilized to show non-classical correlation behavior.

A more direct example of this non-classical correlation behavior is achieved when allowing states for which measurements in different bases are “natural” (based on the simplest method to detect quantum systems, i.e. a measurement of the second order coherence function eq. 2.5). One example is the state below see fig. 3.2:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B) \quad (3.5)$$

The state consists of two photons, one in each of two spatial modes. The photons are orthogonally polarized, but the polarization of each photon is undetermined. By expressing the state in an arbitrarily orthogonal linear (or circular) polarization basis one can observe that the maximal entanglement is preserved. That is, if observer A measures the polarization of his photon to be in an arbitrarily

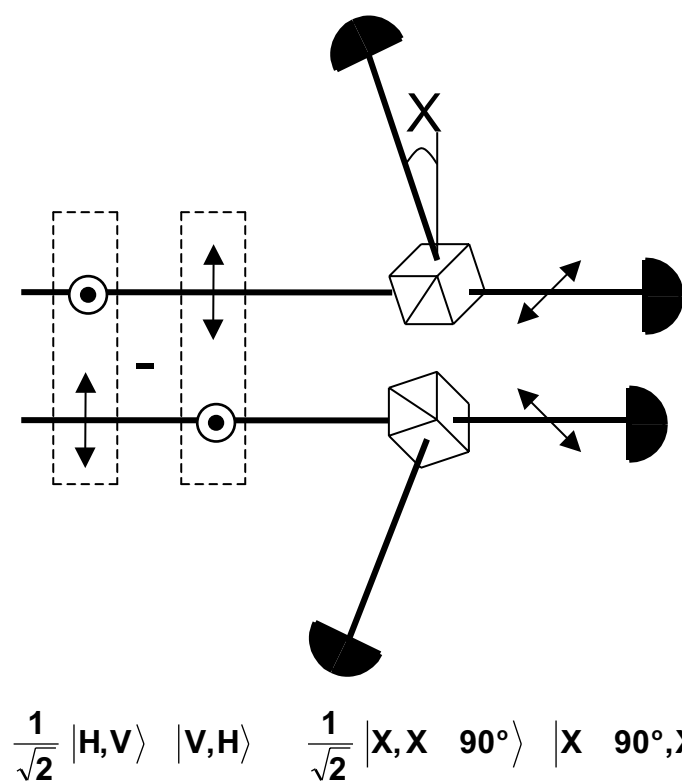


Figure 3.2. An entangled state produced by non - collinear degenerate Type II down conversion. The modes are selected so that the polarization of each individual photon in the two-mode system is undetermined. However the phase matching condition requires the two photons to have orthogonal polarization. Hence the state is entangled on the form (3.5). The entangled properties remain when measuring arbitrary orthogonal polarizations.

angle X , observer B's photon's polarization immediately collapse to $X + 90^\circ$ degree's. This effect for continuous variables (momentum and position) was called spooky action at a distance by Einstein, Podolsky and Rosen [24] and have since then gained a tremendous interest. Bohm [32] later considered discrete variables that shows the same effect. In 1964 Bell [33] derived a famous inequality on the allowed correlation effects that should be obeyed by any classical so called hidden variable theory, which obeys local realism. Following Aspect et. al. [34], many experiments have been performed where the results shows stronger correlation than allowed by locally realistic models. Most of the experiments have been done with parametric down conversion following the proposal of Ou, Hong and Mandel [35]. Even though some theoretical objection about loopholes exist [36], it is generally believed that nature follows the non-local (and in many people's opinion non-intuitive) predictions of quantum theory. Measurements of this effects have been conducted as a preparation for the experiments in papers A,B,C, and D.

The state (3.5) could alternatively be viewed as a four mode state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|1\rangle_{A_H} |0\rangle_{A_V} |0\rangle_{B_H} |1\rangle_{B_V} - |0\rangle_{A_H} |1\rangle_{A_V} |1\rangle_{B_H} |0\rangle_{B_V}) \quad (3.6)$$

Instead as expressing (or viewing) the state in terms of a pair of horizontally or vertically polarized photon in a spatial mode (e.g. A), the horizontally or vertically polarized modes are viewed as individual modes (e.g A_H and A_V). This method of notation will be used in the rest of the thesis since it will allow us to consider arbitrarily number of excitations in the horizontally or vertically polarized modes.

Chapter 4

Quantum mechanical phase and polarization

The “phase problem” has been discussed intensively since the early days of quantum mechanics [37, 38]. Various proofs of the non-existence or impossibility of a phase operator of a single optical mode were constructed in the sixties [38]. The problem is based on the fact that there is no operator that is canonically conjugate to the number operator.

Since phase is a well-established classical concept, a large effort has been put into the solution of the quantum mechanical phase problem. Various solutions used first the method of enlargement of the original Hilbert space, and later restriction of the original Hilbert space [38].

During recent years much effort have been put on a relative phase description [39] rather than the absolute phase description. The reason for this is the insight that phase must be defined in relation to a reference phase in order to be measured. Therefore all of the experimental implementations of phase measurement are two-mode (or two harmonic oscillator) schemes measuring the relative phase between the two modes (oscillators).

4.1 Classical description using Stokes parameters

In fig. 4.1 I show some examples of two mode systems. Fig. 4.1 (a), represents a two-mode system where the modes have the same frequency and polarization but are spatially different (e.g. the modes in the arms of a Mach-Zender interferometer). Fig. 4.1 (b), represents two orthogonal polarized modes (in the same spatial and frequency modes). Any linear loss-less bosonic two mode system can mathematically be described by the group $SU(2)$. I will here mainly look

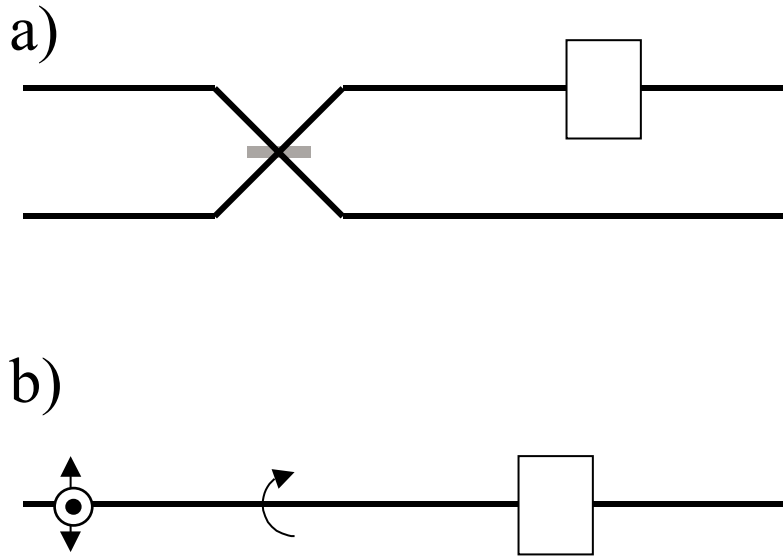


Figure 4.1. a) A spatial two-mode state, that undergoes mixing of modes through a beam splitter and a relative phase shift. b) A two-mode state in polarization, that undergoes mixing of modes through a rotation of polarization and a relative phase shift.

at polarization modes but the mathematical structure is analogous for any two-mode system. Only two kinds of operations are allowed. One corresponds to a rotation of the polarization modes (generally mixing of the modes). The second one corresponds to a differential phase shift inserted at some arbitrary angle (for a general two mode system this can be constructed as a specific mixing followed by a phase-shift in a fixed angle and thereafter another specific mixing of the modes).

The Stokes parameters are widely used for the description of two mode fields in classical physics. Even though they are most often considered for description of the polarization state of a transverse field (which is a particular two-mode system), they can be used to describe arbitrary two mode states. The time independent Stokes variables $(S_0, \mathbf{S}) = (S_0, S_x, S_y, S_z)$ are defined as:

$$\begin{aligned} S_0 &= a_1^* a_1 + a_2^* a_2, \\ S_x &= a_1^* a_2 + a_2^* a_1, \end{aligned}$$

$$\begin{aligned} S_y &= i(a_2^* a_1 - a_1^* a_2), \\ S_z &= a_1^* a_1 - a_2^* a_2, \end{aligned} \quad (4.1)$$

where a_1 and a_2 are the complex amplitudes of the two field modes and $*$ denotes complex conjugation. The Stokes parameters $(s_0, \mathbf{s}) = (s_0, s_x, s_y, s_z)$ are defined as the time and ensemble average of eq. 4.1, $s_j = \langle S_j \rangle$.

If the modes 1 and 2 represents horizontally and vertically polarized light, the degree of polarization V is given by

$$V = \sqrt{\mathbf{s}^2}/s_0, \quad (4.2)$$

and $0 \leq V \leq 1$. When completely polarized ($V = 1$), the polarization can be described by two angles, θ and ϕ

$$\begin{aligned} \cos(\theta) &= S_z/S_0, \\ \phi &= \arg(a_1^* a_2) = \arg(S_x + iS_y). \end{aligned} \quad (4.3)$$

ϕ is not defined for $\theta = 0$ and $\theta = \pi$.

A scheme for simultaneously measuring the Stokes parameters is given in fig. 4.2. The field amplitudes a_1 and a_2 are split and part of the light is guided into a measurement scheme for each of the Stokes parameters S_j . The measurements for polarization modes are given below. S_x corresponds to a measurement with a polarizing beam splitter separating light polarized at 45 degrees and -45 degrees. S_y corresponds to a measurement with a circularly polarizing beam splitter separating right hand- and left hand- polarized light. S_z corresponds to a measurement with a polarizing beam splitter separating light polarized at 0 degrees and 90 degrees. S_0 is extracted as the sum of the intensities detected in all detectors. Since we know the splitting ratios of the beam splitters we can also easily normalize the values for the other Stokes parameters.

The generator of a relative phase shift between the two modes 1 and 2 is S_z . Using the transformation from horizontally/vertically polarized modes to modes polarized along angles ± 45 degrees,

$$\begin{cases} b_1 = \frac{1}{\sqrt{2}}(a_1 - a_2) \\ b_2 = \frac{1}{\sqrt{2}}(a_1 + a_2) \end{cases}, \quad (4.4)$$

we see that S_x generates a relative phase shift between those modes,

$$\begin{aligned} S_x(a) &= (a_1^* a_2 + a_2^* a_1) = \\ &= \frac{1}{2} ((b_1^* - b_2^*)(b_1 + b_2) + (b_1^* + b_2^*)(b_1 - b_2)) = \\ &= (b_1^* b_1 - b_2^* b_2) = S_z(b). \end{aligned} \quad (4.5)$$

A rotation of linear polarization modes is generated by S_y which transforms light polarized along one mode into another polarization mode at another angle. Light invariant under rotation is called circularly polarized light. Using the

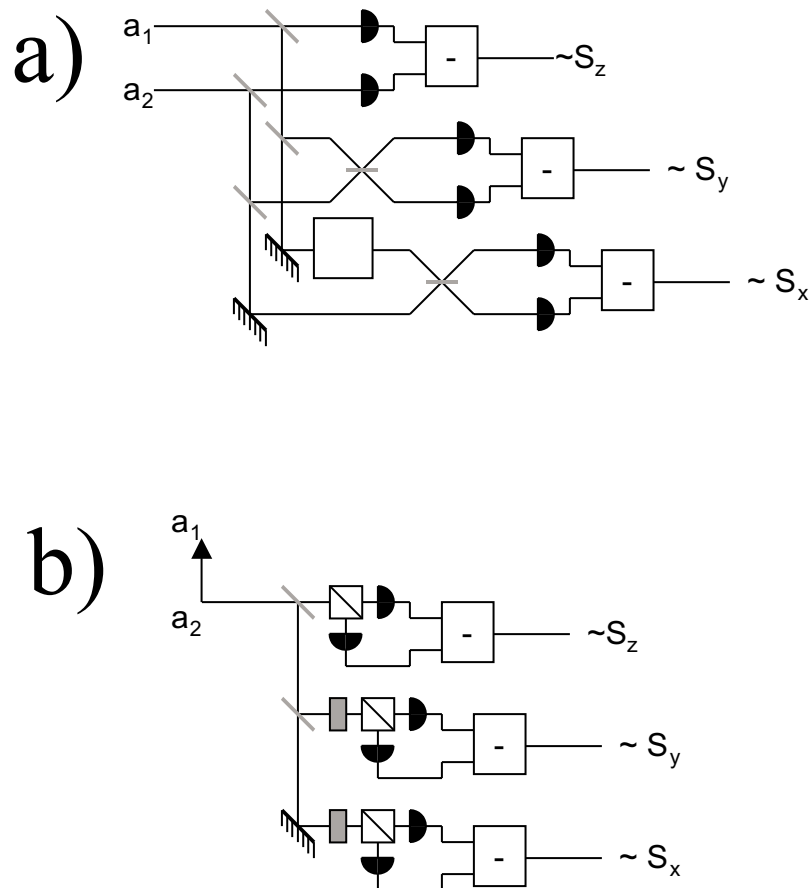


Figure 4.2. Measurement of the Stokes parameters. The incoming beams are split in three different measurement schemes, each measuring a signal proportional to one of the Stokes parameters (S_x, S_y, S_z). Knowledge of the splitting ratios of the beam splitters gives the absolute value. a) A spatial two-mode state. b) A two-mode polarization state.

transformation from horizontally/vertically polarized modes to circularly polarized modes,

$$\begin{cases} c_1 = \frac{1}{\sqrt{2}}(a_1 - ia_2) \\ c_2 = \frac{1}{\sqrt{2}}(a_2 - ia_1) \end{cases}, \quad (4.6)$$

we see that S_y generates a relative phase shift between the circularly polarized modes,

$$\begin{aligned} S_y(a) &= i(a_2^* a_1 - a_1^* a_2) = \\ &= \frac{i}{2}((c_2^* - ic_1^*)(c_1 + ic_2) - (c_1^* - ic_2^*)(c_2 + ic_1)) = \\ &= (c_1^* c_1 - c_2^* c_2) = S_z(c). \end{aligned} \quad (4.7)$$

A phase shift between two circularly polarized modes is the same as a rotation of the linearly polarized modes.

If we instead start from mode 1 and 2 being the circularly polarized modes the set of Stokes parameters are,

$$\begin{aligned} S'_0 &= c_1^* c_1 + c_2^* c_2, \\ S'_x &= c_1^* c_2 + c_2^* c_1, \\ S'_y &= i(c_2^* c_1 - c_1^* c_2), \\ S'_z &= c_1^* c_1 - c_2^* c_2, \end{aligned} \quad (4.8)$$

Here S'_z generates a phase shift between the two circularly polarized modes (or a rotation of the linearly polarized modes). S'_x and S'_y generates a phase shift between the linearly polarized modes in the horizontal/vertical bases and the $\pm 45^\circ$ bases respectively.

4.2 Quantum mechanical description

In a quantum mechanical description the complex amplitudes in eq. 4.1 are replaced by the operators \hat{a}_1 and \hat{a}_2 . This transforms the Stokes parameters S_j to the Stokes operators \hat{S}_j ,

$$\begin{aligned} \hat{S}_0 &= \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2, \\ \hat{S}_x &= \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1, \\ \hat{S}_y &= i(\hat{a}_2^\dagger \hat{a}_1 - \hat{a}_1^\dagger \hat{a}_2), \\ \hat{S}_z &= \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2, \end{aligned} \quad (4.9)$$

which satisfy the commutation relations,

$$[\hat{S}_x, \hat{S}_y] = 2i\hat{S}_z \quad (cycl.), \quad (4.10)$$

$$[\hat{\mathbf{S}}, \hat{S}_0] = 0. \quad (4.11)$$

This operators are closely related to the Pauli Spin operators that are commonly used to describe another two-level system namely spin 1/2 particles. Eq. 4.10 implies that the values of \hat{S}_x , \hat{S}_y , and \hat{S}_z can not be known simultaneously in contrast to the classical Stokes parameters. The condition eq. 4.11 can be viewed as the conservation of the total excitation number in the two modes upon rotation or differential phase shifting. The total Hilbert space $H_1 \otimes H_2$ of the two-mode field, can be split into a direct sum of Hilbert subspaces H_N , each subspace forming a closed space under the action of the Stokes operators \hat{S}_j ,

$$H_1 \otimes H_2 = \bigoplus_{N=0}^{\infty} H_N, \quad (4.12)$$

All the subspaces (manifolds) H_N are finite dimensional Hilbert spaces of dimension $N + 1$ and they are spanned by the basis vectors $|n, N - n\rangle$, where $n = 0, 1, \dots, N$.

4.3 Heisenberg limited interferometry

The generator of a relative phase shift between two linearly polarized modes is the Hamiltonian

$$\hat{H}_\phi = \hbar\phi\hat{S}_z, \quad (4.13)$$

which is just the number difference operator of the two modes multiplied by a constant. The eigenvectors to the operator \hat{H}_ϕ in a given manifold N are the number-difference states $|n, N - n\rangle_+$, with eigenvalues $2n - N$. The subscript $+$ symbolize that the two modes are orthogonal linearly polarized (in the classical sense). E.g. the first index represent the occupation of the horizontal polarized mode and the second index represent the occupation of the vertical polarized mode.

The number difference states are invariant under a differential phase shift and can therefore not be used to resolve a phase shift. The smallest phase shift that can be resolved with certainty using N photons is π/N . By constructing a symmetrical superposition state of the eigenstates of \hat{S}_z with the maximal difference in eigenvalues,

$$|\psi\rangle_+ = \frac{1}{\sqrt{2}} (|0, N\rangle_+ + e^{i\eta}|N, 0\rangle_+), \quad (4.14)$$

where η is an arbitrary real number, this phase resolution sensitivity can be achieved. In ordinary interferometry the phase resolution is scaled as $\sim 1/\sqrt{N}$.

This limit is known as the standard quantum limit. For a large number of photons, the scaling $\sim 1/N$, known as the Heisenberg limit, provides a significantly higher resolution.

States of the form (4.14) have been constructed for 1 [40] and 2 [41] photons. For more than two photons a highly nonlinear device such as the Davidovich switch [42] is required, which transforms the excitation in an impinging mode into a superposition of all the excitation either being transmitted or reflected in two different modes.

The increased phase resolution can also be viewed as an aspect of de Broglie wave interference [43, 44]. The N -photon state is viewed as an virtual particle with a de Broglie wavelength of λ/N , and hence the resolution increases from π for an individual particle to π/N for the de Broglie wave-package.

4.4 Heisenberg limited polarometry

A rotation of a linearly polarized mode is generated by the Hamiltonian

$$\hat{H}_\theta = \hbar\theta\hat{S}_y, \quad (4.15)$$

which is also known as the Jaynes-Cummings interaction Hamiltonian. It is used extensively in atomic physics (in this case one mode refers to the state of an atom and the other mode to the state of a light field) and in cavity quantum electro-dynamics. The eigenstates to \hat{H}_θ are rotationally invariant.

It is tedious (but straightforward) to calculate the eigenstates of \hat{S}_y in the linearly polarized bases, in paper D we do calculate them for $N = 2$. Since circularly polarized light is a more appropriate base to describe rotation invariance I will here do a transformation from linearly polarized bases into circularly polarized bases,

$$\begin{cases} \hat{c}_1 = \frac{1}{\sqrt{2}}(\hat{a}_1 - i\hat{a}_2) \\ \hat{c}_2 = \frac{1}{\sqrt{2}}(\hat{a}_2 - i\hat{a}_1) \end{cases} \quad (4.16)$$

The equation above describes the transformation from the linearly polarized annihilation operators \hat{a}_1 and \hat{a}_2 onto the circularly polarized annihilation operators \hat{c}_1 and \hat{c}_2 .

Classically, a transformation of two linearly polarized modes along mode 1 and 2 into circular polarized modes transforms S_y to S_z (4.7). Below I show that \hat{S}_y is transformed to \hat{S}_z under the transformation from the linearly polarized modes base onto the circularly polarized modes base (4.16),

$$\begin{aligned} \hat{S}_y(\hat{a}) &= i(\hat{a}_2^\dagger\hat{a}_1 - i\hat{a}_1^\dagger\hat{a}_2) = \\ &= \frac{i}{2} \left((\hat{c}_2^\dagger - i\hat{c}_1^\dagger)(\hat{c}_1 + i\hat{c}_2) - (\hat{c}_1^\dagger - i\hat{c}_2^\dagger)(\hat{c}_2 + i\hat{c}_1) \right) = \\ &= (\hat{c}_1^\dagger\hat{c}_1 - \hat{c}_2^\dagger\hat{c}_2) = \hat{S}_z(\hat{c}). \end{aligned} \quad (4.17)$$

The eigenvectors, to the operator \hat{H}_θ in a given manifold N are the number difference states of the circularly polarized modes $|n, N-n\rangle_\circ$ (e.g. the first index represents the excitation of the right hand polarized mode and the second index the excitation of the left hand polarized mode).

The number difference states are invariant under rotation and can not be used to resolve a rotation. The smallest rotation that can be resolved with certainty using N photons is $\pi/(2N)$. By constructing a state of the form,

$$|\psi\rangle_\circ = \frac{1}{\sqrt{2}} (|0, N\rangle_\circ + e^{i\eta}|N, 0\rangle_\circ), \quad (4.18)$$

where η is an arbitrary real number this rotation resolution sensitivity can be achieved. In ordinary polarometry the rotation resolution is scaled as $\sim 1/\sqrt{N}$.

The state (4.18) has been constructed for 2 [45] photons.

Chapter 5

Spontaneous parametric down conversion

When light passes through a transparent homogenous media some photons may be scattered by the atomic structure of the material. One example of scattering is normal elastic scattering where the frequency of the atoms is not changed. At low temperatures the atoms are fixed and light only changes it's direction of propagation. Another example is Raman scattering where the frequency of the phonons (normal vibrations of the atom lattice in the medium) in the material is added or subtracted from the frequency of the incident light. The scattered light consists of several discrete components at the sum or difference frequencies.

In parametric scattering [46] the spectrum is continuous. The dispersion of the refractive index of the material governs the direction and energy of the radiated light. The scattered light has to fulfill the energy and momentum conservation laws¹:

$$\omega_1 + \omega_2 = \omega_3, \quad (5.1)$$

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3. \quad (5.2)$$

For optically isotropic (non-birefringent) materials the above equations (the energy conservation (eq. 5.1) and phase-matching conditions (eq. 5.2)) are not possible to fulfill simultaneously because of normal dispersion. Since ω_1 and $\omega_2 < \omega_3$, due to normal dispersion the refractive indexes n_1 and $n_2 < n_3$ and hence $\mathbf{k}_1 + \mathbf{k}_2 < \mathbf{k}_3$.

Like other multi-photon phenomena the intensity of parametric down conversion is governed by the (optical) nonlinearity of the material,

¹The conservation laws are limited by the uncertainty principle, they are strictly true only for modes with infinite extent ($\Delta t \rightarrow \infty, \Delta x, \Delta y, \Delta z \rightarrow \infty$)

$$P(E) = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots \quad (5.3)$$

The strength of the parametric photon pair creation process is governed by the second order nonlinear dielectric susceptibility $\chi^{(2)}$.

In uniaxial crystals a principal axis, called the z-axis, exists. The z-axis and the wave-vector \mathbf{k} defines a plane called the principal plane. Light with polarization normal to the principal axis is said to be ordinary (o) polarized. Light with polarization in the principal plane is said to be extraordinary (e) polarized. The refractive index for the o-beam (n_o) does not change with the angle between the z-axis and the wave-vector whereas for the e-beam it does. The refractive index for the e-beam as function of the angle θ between the \mathbf{k} vector and z axis is

$$n^e(\theta) = n_o \sqrt{\frac{1 + \tan^2 \theta}{1 + (n_o/n_e)^2 \tan^2 \theta}}. \quad (5.4)$$

The difference between the refractive index for the e-beam and o-beam is $\Delta n(\theta) = n^e(\theta) - n_o$. When the light travels along the optical axis the refractive index is independent of the polarization so that $\Delta n(0^\circ) = 0$. The difference in refractive index between the e-beam and the o-beam is largest when the z-axis and the wave-vector are orthogonal $\Delta n(90^\circ) = n_e - n_o$.

Most crystals used in applied nonlinear optics (including the crystal we used, $\beta - BaB_2O_4$ Beta-Barium Borate (BBO)) are uniaxially negative ($n_e < n_o$) birefringent crystals. The dependence of the refractive indexes (at $T = 20^\circ C$) on the wavelength for BBO are approximately given by the dispersion relations [46]

$$n_o^2 = 2.7359 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2, \quad (5.5)$$

$$n_e^2 = 2.3753 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2, \quad (5.6)$$

where λ is to be expressed in μm units.

There are thus two possibilities for phase matching $\mathbf{k}_{o1} + \mathbf{k}_{o2} = \mathbf{k}_3^e(\theta)$ and $\mathbf{k}_{o1} + \mathbf{k}_2^e(\theta) = \mathbf{k}_3^e(\theta)$. The first case is called Type I phase matching condition and the down converted photons have the same polarization. In the second case (Type II phase matching) the down converted photons have mutually orthogonal polarization.

Type I phase matching was used in the down conversion experiment of paper B. We used a phase matching angle so that the conversion become degenerate ($\omega_1 = \omega_2$) for specific angles of the down converted light. There are several solutions to the phase matching condition with different frequencies travelling in different directions (fig. 5.1). By using pinholes and filters specific temporal modes of light with the same frequency, can be chosen for certain rotation angles

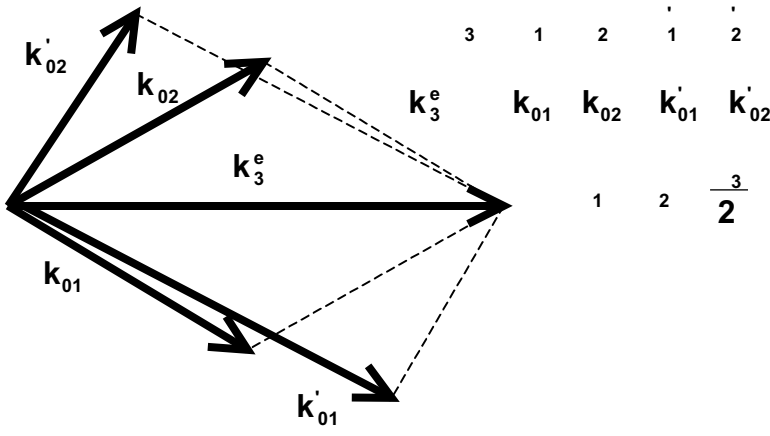


Figure 5.1. Several combinations for wave vectors/energies of the down converted light fulfill the phase matching condition given a certain pump. We can also find certain angles where the down converted photons have the same energy.

θ between the wave vector of the impinging light and of the optic axis of the crystal.

The down conversion experiments in papers A, C and D used collinear degenerate Type II down conversion. This directly produced the desired $|1,1\rangle$ state in co-propagating linear polarization modes. While for the method of paper B, where degenerate Type I down conversion with a non-collinear phase matching angle was chosen, a careful alignment is needed as shown in the paper.

Since we have dispersion in the crystal, orthogonal polarized photons produced (by Type II parametric down conversion) in the beginning of the crystal will exit the crystal at different times. To the first order, the time mismatch can be written as

$$\Delta t = \left(\frac{1}{u_e} - \frac{1}{u_o} \right) L, \quad (5.7)$$

where L is the length of the crystal and u_e and u_o are the group velocities for the e and o beam respectively.

This timing information will reduce the visibility, but by introducing a birefringent plate that compensates for the birefringence of the nonlinear crystal the time information vanishes and the visibility is increased (The optical time delay of the birefringent plate should be half of eq. 5.7, since eq. 5.7 refers to the case when a photon pair is produced in the beginning of the crystal).

Part II

Review of original work

Chapter 6

Relative phase states and polarization states

6.1 Relative phase states

In order to express the relative phase between two modes the natural basis to consider are the modes (1 and 2) that the relative phase is applied between. The generator of a relative phase shift between the two modes is as discussed in chapter 4.3, the Hamiltonian \hat{H}_ϕ is explicitly expressed in (4.13). We denote here the eigenvectors to the operator \hat{H}_ϕ in a given manifold N (the number-difference states) by $|\varphi_{n,N}\rangle$

$$|\varphi_{n,N}\rangle = |n, N - n\rangle_+, \quad (6.1)$$

The eigenvectors of an operator that is complementary to the operator \hat{H}_ϕ will form a suitable base in order to measure a relative phase between the modes. We denote the relative-phase eigenstates (the eigenvectors of such a complementary operator) by $|\phi_{n,N}\rangle$. A mutually unbiased base can be constructed by making the first vector to be any arbitrary equipartition of the eigenvectors $|\varphi_{n,N}\rangle$,

$$|\phi_{0,N}\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^N e^{i\varsigma_n} |\varphi_{n,N}\rangle, \quad (6.2)$$

where ς_n are arbitrary real numbers. The rest of the eigenvectors representing the relative-phase states can be constructed by operating with \hat{H}_ϕ ,

$$|\phi_{k,N}\rangle = e^{\frac{iH_\phi}{\hbar} \phi_k} |\phi_{0,N}\rangle, \quad \phi_k = \frac{2\pi k}{N+1}, \quad (6.3)$$

where $k = 1, 2, \dots, N$.

The relative-phase operator can then be expressed using the relative-phase states as,

$$\hat{\phi}_{n,N} = \sum_n \phi_{n,N} |\phi_{n,N}\rangle \langle \phi_{n,N}|, \quad (6.4)$$

6.2 Polarization

A mixing of the two modes (e.g. rotation of polarization modes or mixing the modes in a beam splitter) is generated by the Hamiltonian \hat{H}_θ (eq. 4.15). The rotation operator expressed in the circular polarized base (c) is the number difference operator of the circular polarized base. The eigenstates in a given manifold N is denoted $|\vartheta_{n,N}\rangle$ and expressed in the circular polarized base they are the number difference states,

$$|\vartheta_{n,N}\rangle = |n, N - n\rangle_\circ, \quad (6.5)$$

where $n = 0, 1, \dots, N$. The eigenvalues of \hat{H}_θ are $2(n - N/2)$. These states are invariant under a geometric rotation.

In order to measure a polarization rotation we should find a operator that is canonical conjugate/complementary to the operator \hat{H}_θ . We denote its eigenvectors, the polarization rotation eigenstates, by $|\theta_{n,N}\rangle$. A mutually unbiased base can be constructed by making the first vector be any arbitrary equipartition of the eigenvectors $|\vartheta_{n,N}\rangle$,

$$|\theta_{0,N}\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^N e^{i\varrho_n} |\vartheta_{n,N}\rangle, \quad (6.6)$$

where ϱ_n are arbitrary real numbers. The rest of the eigenvectors can be constructed by operating with the operator \hat{H}_θ ,

$$|\theta_{k,N}\rangle = e^{\frac{i\hat{H}_\theta}{\hbar}\theta_k} |\theta_{0,N}\rangle, \quad \theta_k = \frac{2\pi k}{N+1}, \quad (6.7)$$

where $k = 1, 2, \dots, N$.

The polarization-rotation operator can be expressed using the relative-phase states as,

$$\hat{\theta}_{n,N} = \sum_n \theta_{n,N} |\theta_{n,N}\rangle \langle \theta_{n,N}|, \quad (6.8)$$

Just like circularly polarized light in classical optics, the states $|\vartheta_{n,N}\rangle$ are invariant under rotation. Therefore we call them “circularly” polarized. It should be noted that the number of orthogonal circularly polarized states are $N+1$ in each manifold N . For classical light, right (left) hand circularly polarized light is transformed to left (right) hand polarized light under a differential phase shift (in any linear basis) of π . This can be accomplished experimentally by inserting a $\lambda/2$ -plate at any angle in a beam. Similarly under such transformation the eigenstates $|\vartheta_{n,N}\rangle = |n, N - n\rangle_\circ \Rightarrow |\vartheta_{N-n,N}\rangle = |N - n, n\rangle_\circ$. Note specifically

that in all odd $(N+1)$ dimensional two-mode Hilbert-spaces there exist one state $|\frac{N}{2}, \frac{N}{2}\rangle$ which lacks chirality although it is circularly polarized (invariant under rotation).

The “complementary” polarization’s to circular in classical optics are linear. A classical linearly polarized light evolves to an orthogonal polarized light under the rotation of $\pi/2$. The states $|\theta_{n,N}\rangle$ will evolve to mutually orthogonal polarized states for the rotations $\theta_k = \pi/(N+1)$, $k = 0, 1, \dots, N$. Therefore we call these states “linearly” polarized.

6.3 The First Manifold

The eigenstates to the Stokes operator \hat{S}_x , \hat{S}_y and \hat{S}_z in the horizontal vertical basis are,

$$\left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}, \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ 1 \end{pmatrix} \right\}, \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}, \quad (6.9)$$

respectively. The set of linearly polarized light are given by,

$$\begin{pmatrix} \cos v \\ \sin v \end{pmatrix}. \quad (6.10)$$

These are the eigenstates to,

$$\hat{S}_v = \cos(2v)\hat{S}_z + \sin(2v)\hat{S}_x. \quad (6.11)$$

A third unique Stokes operator is then:

$$\hat{S}_{v^\perp} = \cos(2v)\hat{S}_x - \sin(2v)\hat{S}_z, \quad (6.12)$$

with eigenstates,

$$\begin{pmatrix} \cos\left(v + \frac{\pi}{4}\right) \\ \sin\left(v + \frac{\pi}{4}\right) \end{pmatrix}. \quad (6.13)$$

The sets of linearly polarized states for an angle v are invariant under a relative phase shift at that angle. Thus the linearly polarized states are also the number-difference eigenstates along those modes.

The relative phase states are complementary to the number-difference eigenstates along a given direction v . Thus single photon circularly polarized light and light polarized along $v + \pi/4$ are simultaneously relative phase states. The solutions closely resemble the classical case. However, we are still limited to measure only one Stokes operator and need an ensemble of identically prepared states on which we perform the different measurements in order to fully characterize the state if it is initially unknown.

6.4 Higher manifolds

Already for two photons the situation is changed. Using (4.16), the circularly polarized states (6.5) can be expressed in linearly polarized modes as:

$$\begin{aligned} |\vartheta_{0,2}\rangle &= \frac{1}{2} \left(|0,2\rangle_+ - i\sqrt{2}|1,1\rangle_+ - |2,0\rangle_+ \right), \\ |\vartheta_{1,2}\rangle &= \frac{1}{\sqrt{2}} (|0,2\rangle_+ + |2,0\rangle_+), \\ |\vartheta_{2,2}\rangle &= \frac{1}{2} \left(|0,2\rangle_+ + i\sqrt{2}|1,1\rangle_+ - |2,0\rangle_+ \right). \end{aligned} \quad (6.14)$$

We see here that the circularly polarized states are not complementary to the number difference states (e.g. $|\langle\varphi_{1,2}|\vartheta_{1,2}\rangle| = 0$) where we recall the definition of complementarity in a finite Hilbert-space given by (3.1). Hence the number difference states are not “linearly” polarized states and the circularly polarized states are not relative phase states.

In paper B and C we construct a specific relative phase state. When introducing a phase shift of $\pm 2\pi/3$ the state is transformed onto orthogonal relative phase states. This is an example of Heisenberg limited interferometry, since the scaling of phase the resolution is $2\pi/(N+1) \sim 1/N$.

In paper D we construct a specific linearly polarized state. When introducing a geometric rotation of $\pm \frac{\pi}{3}$ the state is transformed onto orthogonal linearly polarized states. This is an example of Heisenberg limited polarimetry, since the scaling of the rotation resolution is $\pi/(N+1) \sim 1/N$.

A better resolution in interferometry ([47, 41] and Paper A) and polarimetry [45] is achieved by constructing “Schrödinger kitten states”,

$$\begin{aligned} |\psi\rangle_+ &= \frac{1}{\sqrt{2}} (|0, N\rangle_+ + e^{i\eta}|N, 0\rangle_+), \text{ or} \\ |\psi\rangle_\circ &= \frac{1}{\sqrt{2}} (|0, N\rangle_\circ + e^{i\eta}|N, 0\rangle_\circ), \end{aligned} \quad (6.15)$$

where η is an arbitrarily phase. However the method with the relative phase states/linearly polarized states allow in theory to construct a measurement where you can distinguish between $N+1$ different phase-shifts/rotation angles, while the method of constructing “Schrödinger kitten states” only allows distinction between two orthogonal states in every manifold.

Chapter 7

Complementarity with Schrödinger kitten states

Paper A presents a quantum erasure experiment on a de Broglie wave-package in a superposition state of being in either of two modes. The state is of the form (4.14) with $N=2$. The state exhibits interference fringes, when a relative phase ϕ is introduced, with a period $\pi/N = \pi/2$. In the experiment the internal state of the wave-package is changed, by symmetrically tapping one photon from either of the two-modes, into a four-mode entangled state. Using a coincidence technique, only states in which one photon is tapped into the meter and the remaining photon (the object) is transmitted are post selected.

By observing the path of the tapped photon, knowledge of the path of the remaining photon is obtained. Thereby the possibility of observing interference between the two remaining modes is eliminated. When introducing a unitary rotation of the tapped two-mode states, the path information of the remaining photon is erased. Hence the interference fringes for the transmitted photon re appear. Compared to other quantum erasure experiments, a large and deliberate change is applied to the internal state of the impinging object (the 2 photon de-Broglie wave package). The outgoing object (a single photon) is fundamentally different than the impinging object. Half of the momentum of the initial state is lost to the meter mode. This large, and deliberate, object disturbance which distinguishes this experiment from previous quantum-erasure experiments is the principal interest Paper A.

Chapter 8

Interaction free measurements

The problem how to optically detect the presence of an absorptive object, without it absorbing any photons, was first discussed by Elitzur and Vaidman [48] who called the procedure an interaction-free measurement.

For an observer limited to classical mechanics this problem seems impossible. If she lets a classical wave hit the place where the object may or may not be, some amplitude will interact with (and hence be absorbed by) the object if it is present. If she does not shine light on the place she can not gain any information (at least not by optical means) about the absence or presence of the object.

Elitzur and Vaidman realized that since quantum mechanics allows the existence of superposition states they could use the complementary wave-particle nature of light to do better. They imagined a single photon impinging on an interferometer. When the interferometer is empty, the two possible ways to get to a detector are indistinguishable, and we get interference – the photon behaves like a wave.

Fig. 8.1 shows an Mach-Zehnder interferometer. The two beam splitters have equal reflectivity (R) and transmitivity (T). The upper and lower path lengths are set to be exactly equal. Any incident light will always exit to the detector D_{light} while there is complete destructive interference between the interferometer path probability amplitudes at the port with D_{dark} .

In the presence of an object in e.g. the upper arm, the interference is destroyed and the photon will act like a particle, the probability of a absorption of the photon $P(abs) = T$. The probability of a detection on D_{dark} is $P(D_{dark}) = TR = T(1 - T)^2$ and the probability of a detection on D_{light} is $P(D_{light}) = R^2 = (1 - T)^2$. When the object was absent we never got a click on D_{dark} , and the only photon that was sent in was not absorbed. So in the cases D_{dark} clicks we have succeeded, we can tell that the object is present and that it has not absorbed any photon.

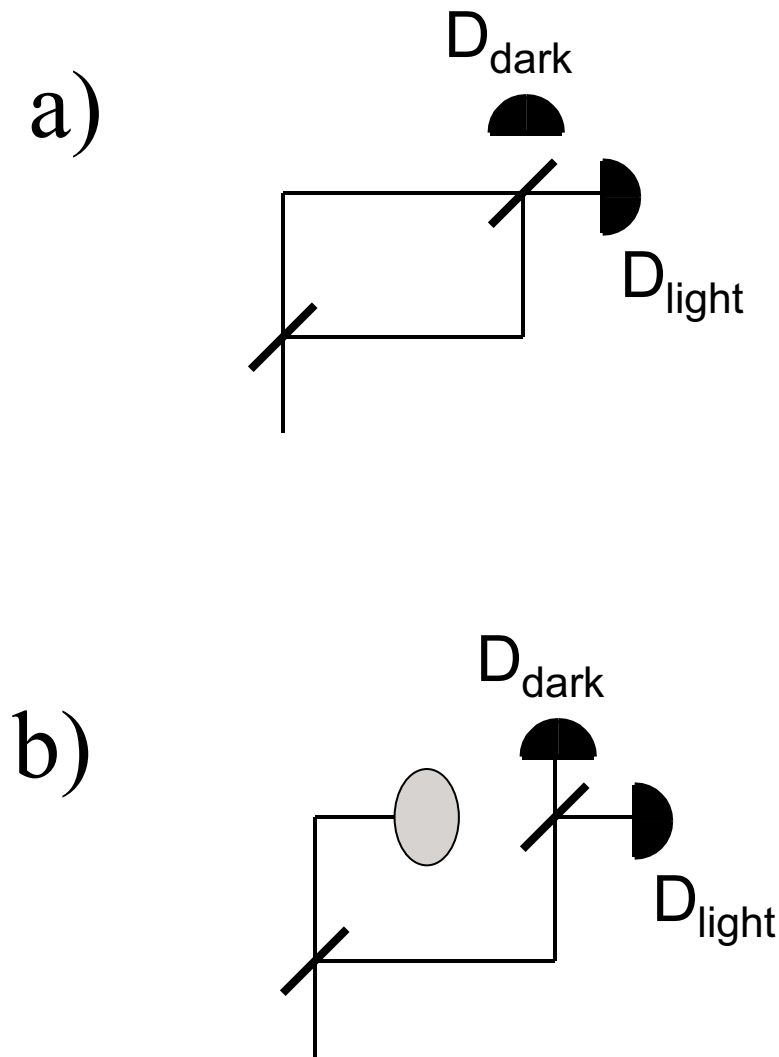


Figure 8.1. The original Elitzur and Vaidman interaction free measurement scheme. a) When no object is present the interferometer is adjusted so that any impinging photon from the lower arm will exit to the detector D_{light} . b) When the object is present the interference is destroyed, there is now a certain probability that the object absorbs the photon, but more importantly, if the photon “goes” through the other path there is a certain probability that the photon after the second beam splitter is detected by detector D_{dark} . Hence, for some outcomes we gain information about the presence of the object without it absorbing any photons.

How should one optimally choose the reflectivity and transmittivity of the beam splitters? In order to get perfect destructive interference the reflectivity and transmittivity of the beam splitters should be the same. The choice $R = T = 1/2$ gives a maximum value of $P(D_{dark}) = 1/4$. Then the probability of the object absorbing is $P(abs) = 1/2$ and the probability of the detector D_{light} clicking is $P(D_{light}) = 1/4$. However since we never get any “wrong” answers when the object is not present we can choose to redo this experiment many times (N) as long as detector D_{light} clicks until the photon is absorbed or detector D_{dark} clicks. A reasonable figure of merit η that gives what fraction of the measurements that are interaction-free is given by

$$\eta = \frac{P(D_{dark})}{P(D_{dark}) + P(abs)}, \quad (8.1)$$

For $R = T = 1/2$ $\eta = 1/3$, but when $R \rightarrow 0$ $\eta \rightarrow 1/2$. So in theory, using arbitrarily weak beam splitters and many trials, we achieve a success-ratio of $1/2$. This principle has been experimentally implemented by Kwiat et. al. [49].

Fig. 8.2 shows a scheme where the beam splitters in fig. 8.1 are replaced with polarizing beam splitters and where the polarization of an impinging horizontally polarized photon is rotated by an angle $\Delta\theta$. If no object is present the beam will exit with the same polarization and the second polarization rotator will rotate its polarization back to horizontal. Therefore the detector on the vertically polarized arm D_{dark} never detects the photon. Let $T = \sin^2(\Delta\theta)$ and $R = \cos^2(\Delta\theta)$. If the object is present in the horizontally polarized branch in fig. 8.2, there will be a certain probability $P(D_{dark}) = TR$ that the detector D_{dark} clicks. The probability that the object absorbs the photon is $P(abs) = T$ and the probability that the detector D_{light} clicks is $P(D_{light}) = R^2$. This scheme based on polarization rotation gives the same efficiency $\eta = 1/2$ as the original Elitzur-Vaidman scheme.

However we can do better than $\eta = 1/2$. In the clever experimental setup [7] shown in fig. 8.3 the Los Alamos National Laboratory (LANL) group recycled the photon into the same stage N times, in each stage the polarization of the light is rotated by an angle $90^\circ/N$. In absence of the object the impinging photon goes from horizontal to vertical polarization. However, if an object is inserted in the stage the probability that a photon is transmitted N times and remain horizontally polarized at the output is $P(H) = \cos^{2N}(90^\circ/N)$. When N is arbitrarily large, the probability $P(H) \Rightarrow 1$ and $\eta \Rightarrow 1$. The reported experimental result was $\eta = 0.73$.

Using multiple interference it also possible to achieve high efficiency interferometers. In paper E, we present an interference experiment based on a Fabry Perot interferometer. The interferometer is tuned for maximal transmittance and in the absence of an object the impinging photon is always transmitted. However when the object is placed between the mirrors of the cavity the interference condition is lost and the photon will either be reflected on the first high reflectivity mirror or absorbed by the object. We achieved $\eta = 0.92$, however

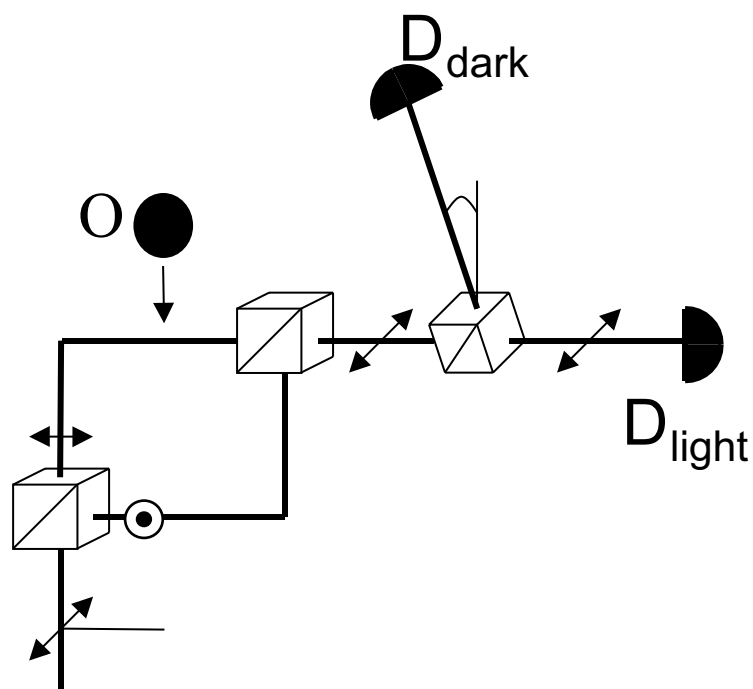


Figure 8.2. An alternative interaction free measurement scheme based on polarization rotation. The impinging light polarized at a certain angle α is divided in its horizontal and vertical components by a polarizing beam splitter. In absence of the object the beam is recombined in phase by a second polarizing beam splitter and thereafter analyzed in the basis of the original polarization α . One of the detectors (D_{dark}) will thus never register any of the impinging photons. If the object is present in the vertical path, the probability of absorption is proportional to the vertical polarized component of the impinging photon. There will also be a certain probability of that the detector D_{dark} fires and that the object is detected without it absorbing an impinging photon.

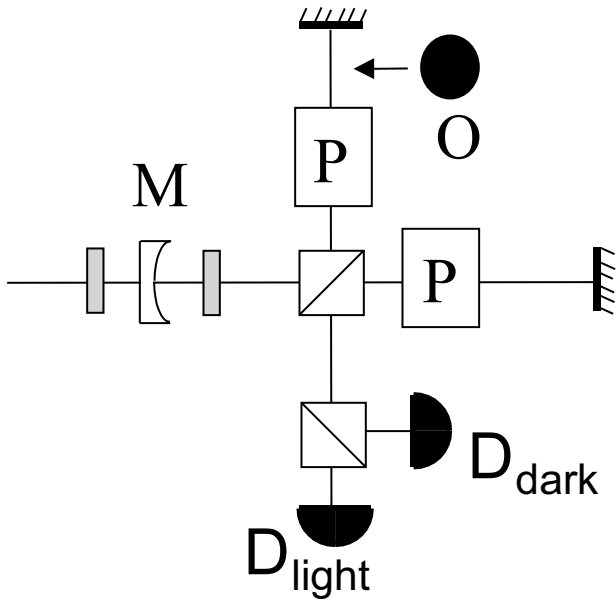


Figure 8.3. The ingenious scheme introduced by Kwiat et.al. A horizontally polarized pulse enters the setup from the left and is heavily attenuated by the mirror (M). The action of the two quarter wave plates are canceled in the first pass of the beam. In the absence of the object (O) the beam bounces back and fourth through the recycling system for a fixed number of times (N). Each time the quarter wave plate in the recycling system rotates the polarization by an angle $\Delta\theta = 90^\circ/N$, until the light is entirely vertically polarized. However, if the object is present there will be a probability $\sin^2(\Delta\theta)$ that an impinging photon is absorbed. The probability that an impinging photon remains horizontal polarized after N passes is then $P_H = \cos^2(\Delta\theta)$. After N roundtrips the pulse is coupled out from the system by rotating the polarizations in the two arms, utilizing the Pockels cells (P). Detector D_{light} clicks if the object was absent and detector D_{dark} clicks (with probability P_H) if the object was present.

the probability of transmission when the object was absent was only 76%. One reason for the reduction of the transmitivity was that the mirrors didn't have antireflection coatings on their backsides. Another reason was that the transmission line-width of the cavity was comparable with the relatively broadband laser we used. Using a narrow band laser and higher reflectivity mirrors with antireflection coatings on the backside, higher values of η , and the transmitivity of the cavity when the object is absent, can be achieved.

Part III

Epilogue

Chapter 9

Summary of the original work

The objective of this section is to make a short commentary of the publications consisting the scientific core of the thesis and to put the papers in the context of the work by other groups. The publications are results of a cooperation of several authors, and a short description of the author's contributions is given.

Paper A: Complementarity and quantum erasure have been observed for many systems. This experiment is the first complementarity and quantum erasure experiment on a “particle” (a two-photon state) where a large and deliberate change is induced to the internal-state of the object by the measurement. A quantum-erasure experiment was performed on superposition states of two photons in either of two spatio-temporal modes. Symmetrical tapping of energy from the two-mode state creates a four-mode entangled state. A direct measurement of one photon in two of the modes reveals the path of the second one, thereby eliminating the possibility of observing interference between the two remaining modes. It is shown that a unitary rotation of one of the two-mode states erases the path information; as a consequence the conditioned visibility of the other two-mode state can be resurrected.

Contributions by the author of the thesis:

The measurement set-up was constructed and the measurement was performed by the author.

Paper B: The first experiment on generation and detection of relative phase states. The observed visibility of 45 % is higher than can be explained without the two-photon interference effect i.e. it cannot be predicted with a “classical” model.

Contributions by the author of the thesis:

The initial set-up for spontaneous parametric down conversion was partially set-up by the author. The author constructed the automated measuring scheme and collected the data for the measurement on the relative phase eigenstate.

Paper C: An improved measurement on generation and detection of relative phase states. The observed visibility was 90 %. The paper also include the results of paper B. The relative phase distribution function was also measured for various weakly excited two-mode coherent states in addition to the relative phase eigenstate.

Contributions by the author of the thesis:

The author constructed the measurement set-up for the weakly excited two-mode coherent states and performed the measurements. The author performed the improved measurement of the relative phase distribution function of the relative phase eigenstate in addition to the contribution described under paper B.

Paper D: The first experiment on generation and detection of the polarization eigenstates. The observed visibility was 90 %.

Contributions by the author of the thesis:

The measurement was constructed and performed by the author.

Paper E: A high efficiency “interaction-free” measurement [48]. The first interaction free measurement was performed by Kwiat et. al. [49]. But the efficiency in this scheme was limited theoretically to maximum 50 %. Our experiment scheme based on interference of single photons in a cavity and the scheme reported in [7], which is based on rotation of polarization can theoretically asymptotically approach 100 % efficiency. The achieved experimental results are comparable for both schemes and were about 80%.

Contributions by the author of the thesis:

The experiment was partially constructed by the author. The author aligned the system and collected the data.

Chapter 10

Conclusions and outlook

To conclude, we have in Paper A performed a complementarity experiment with a peculiar kind of object. The initial object in our case, the Schrödinger kitten state, is in a superposition state of *two* quanta in either mode while previous quantum-mechanical experiments have all used “single-particle” objects. Furthermore, in our experiment the interaction Hamiltonian between meter and object is not of the Quantum-Non-Demolition type, and therefore the state of the object changes *radically* as a consequence of the object-meter interaction. Yet, as the composite post-selected system is a nearly maximally entangled pure state, quantum erasure still works well. As well as examining the particlelike and wavelike behaviors, we also examine the intermediate cases in which the which-path information of the particlelike behavior gradually is erased and is replaced by visibility and wavelike behavior.

Papers B, C, and D are relative phase and polarization rotation experiments based on analogous theories. The thesis shows the similarity between the Heisenberg limited interferometry and polarometry experiments, and it is also seen that the Stokes operators are not complementary operators for quantum polarization in higher manifolds.

In Paper E we have performed an interaction free measurement based on a Fabry Perot interferometer. The similarities between our interferometry approach and a polarometry approach are discussed in the Thesis.

Paper E is more applications oriented than the papers A-D. The ultimate, but at present unrealistic, goal for quantum state generation is to set up a “machine” in which any quantum optical state could be synthesized (“Dial a quantum state.” [50])

However, due to the limitations set by the existing technology it is hard to anticipate a completely general quantum state generator without limitations. To make the task realistic we have to restrict the state-space to states with fixed wavelength and a few number of photons (three photon entangled states [51] and four ion entangled states [52] have been experimentally realized). In addition we will tolerate a low success-rate. The desired states of the form

(6.1, 6.2, 6.5, 6.6, 6.15) will, however, be difficult or impossible to produce without nonlinear effects at the few photon level if we use light produced by parametric down conversion. By combining an interaction free measurement with an entangled atom, one is able to transfer the quantum superposition of the atom onto the photon. The interaction free measurement also works for several impinging photons. Superposition's of de Broglie wave packages of photons could be made. For example by preparing a pulse of light with 10 photons in it, a superposition of the form (6.1) or (6.5) could be produced. Thus by using the techniques of interaction free measurement high sensitivity interferometry and polarometry measurements could possibly be performed.

Another more direct application of interaction free measurements is X-ray imaging without absorbtion of harmful X-rays. However, this is a technique relying on interference of waves, which for X-rays would be very difficult to exploit in a controlled manner. In the optical regime we believe this technique perhaps could find some use in measurements on very sensitive samples. One example would be measurements of Bose-Einstein condensates without destroying them, since very few of the photons actually end up being absorbed by the ultra-cold condensate atoms. Two-dimensional absorption-free imaging is another example that recently been examined by our group. Another recently discussed proposal is what happen if the object is semi-transparent? It turns out that the above-described schemes do not work as well (after all, they rely on a "collapse" which in turn depends on the complete absorption or non-absorption of a photon). However, there are ways to improve them, and perhaps even to use them to make ultra-sensitive measurements of optical density.

Quantum interference is the basis behind the emerging fields quantum cryptography and quantum computing. Quantum cryptography provides means for two parties to exchange an enciphering key over a private channel with complete security of communication. The laws of physics (quantum interference, Heisenberg's uncertainty principle and the non-cloning theorem [53]) protect the information of encrypted messages. This is in contrast to classical cryptography that employs various mathematical techniques to protect encrypted messages from eavesdropping. Quantum computers are quantum interference system that performs calculations on a superposition of states simultaneously. The scaling for certain algorithms are smaller than for classical computers. If realized, they could constitute a threat to e.g. classical cryptography. The schemes presented in the original work could perhaps enable the construction of novel quantum gates, which could be used as the building block of a quantum computer.

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