Asset Pricing with an Excess Volatility Factor

A Multi-Index Model Approach

Tommaso Luigi Valli Fassi

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Abstract

This paper evaluates the impact that the integration of an excess volatility factor has on the asset-pricing performance of the Fama-French (1992, 1993) and of the Carhart (1997) models, with reference to the retrospective and prospective explanation of the time series and of the cross section of excess returns on stocks. Specifically, the research intends to determine whether or not a new excess volatility factor is able to capture common sources of excess returns on stocks, related to excess volatility, and left unexplained by the available asset-pricing models. The key motivation behind this research is to put in contact the current multi-index asset-pricing models with the behavioural finance perspectives over the determinants of speculative asset prices fluctuations. According to Shiller (2013) and Wang and Ma (2014) the excess volatility which speculative asset prices fluctuate through time is explained by the behavioural principle of investors' overreaction. The constancy with which this phenomenon affected stock prices for more than a hundred years suggests us its likely persistence through time. Hence, asset-pricing models should evaluate its role in explaining stock’s excess returns. The analysis is performed according to Fama and French’s (1993) methodology. The results indicate that the excess volatility factor is a statistically significant determinant of excess returns on stocks, and that its introduction into fundamental multi-index models ameliorates their asset-pricing and predictive performance. The research concludes with a theoretical contextualization of its findings and several examples of their practical implications for the financial industry.

Keywords: asset-pricing, excess volatility, excess returns, variance difference, Fama-French model, Carhart model, Capital Asset Pricing Model
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List of Abbreviations
(Alphabetical Order)

AMEX  American Stock Exchange
BE/ME  Book to Market Equity Value Ratio
CAPM  Capital Asset Pricing Model
GRS  Gibbons Ross Shanken F Test
HML  High Minus Low Book to Market Equity Fama-French Factor
M  Market Portfolio
MoM  Carhart Asset-Pricing Model Momentum Factor
NASDAQ  National Association of Securities Dealers Automated Quotation
NYSE  New York Stock Exchange
P.E.  Pricing Error
P/E  Price to Earnings Ratio
P1  Portfolio 1
P10  Portfolio 10
P2  Portfolio 2
P3  Portfolio 3
P4  Portfolio 4
P5  Portfolio 5
P6  Portfolio 6
P7  Portfolio 7
P8  Portfolio 8
P9  Portfolio 9
PRD  Theil Prediction Realization Diagram
RM – RF  Market Premium Fama-French Factor
RMSE  Root Mean Squared Error
SMB  Small Minus Big Market Capitalization Fama-French Factor
TR  Total Return
VD  Variance Difference
VD  Excess Volatility New Factor
VIF  Variance Inflation Factor
1. Introduction

The importance of the various asset-pricing models arising from the economic and financial literature goes beyond their practical application in the asset management industry. One of the key features that distinguishes contemporary market economies, in fact, is the attribution to a decentralized price system of the role to deliver information regarding the efficient allocation of scarce resources. Hence, explaining returns on stocks signifies that we are able to explain the profitability and the price evolution of residual claims on the economy’s major corporations. This kind of knowledge is of vital importance for investors and for the well being of the overall economic system.

In this context, stock markets represent a special and potentially more complex case. As illustrated by Shiller (2013, p.488), the excess volatility of stock prices relative to the trendiness of their dividends payments, might illustrate the fact that the economic problem of efficiently allocating scarce resources, is not between “man and nature”, but between “man and himself”. In other words, the process by which information is conveyed into prices and prices signal information to investors might be mainly a case of behavioural finance.

Shiller (2013) found that the excess volatility puzzle have been affecting speculative assets’ prices for more than a hundred years. In turn, Wang and Ma (2014) documented that the excess volatility phenomenon is mainly driven by fluctuations in investors’ overreaction. In other words, a persistent feature of irrational asset pricing was discovered. This challenges the common viewpoint for which the significance of a certain factor’s effect on asset prices is less likely to persist if asset pricing is irrational.

The fact that asset prices are persistently affected by investor’s irrationality means that the effects of overreaction are very difficult to arbitrage. Therefore, we may consider the fluctuations in investors’ overreaction as a source of undiversifiable risk that needs to be taken into consideration by the relevant asset-pricing models. In order to do so, I relied both on the existing literature for the presence of a multidimensional systematic risk (priced by multi-index asset-pricing models), and on the existing literature over the most relevant developments of behavioural finance.

Prominent researchers discovered several empirical contradictions over the asset-pricing model of Sharpe (1964) and Lintner (1965). The Sharpe-Lintner model predicted that securities’ expected returns had a positive linear relation with their systematic risk (generally referred to as the “market beta” or “β”, more formally defined as the expected change of an asset return given
a change in that of the market) and predicted that market betas had the exclusive power to describe the cross-section of expected returns. Empirical contradictions against the Sharpe-Lintner model consisted predominantly of limited explanatory power of the market factor, and of significant explanatory power of newly discovered other factors. This body of research culminated with the introduction by Fama and French (1992, 1993) of two new factors: one capturing the market capitalization effect, and one capturing the book-to-market equity effect. Similarly, Carhart (1997) finally introduced a fourth factor on the basis of previous studies by Jagadeesh and Titman (1993) that proved the profitability of momentum strategies. This repeating pattern consisting in explanatory power tests, and in the identification of profitable trading strategies considered inexplicable with the existing asset-pricing models, led over time to the introduction of new common risk factors. It is precisely with reference to that pattern that this paper intends to evaluate the explanatory power of an excess volatility factor on the cross-section of stock returns.

The justification for the introduction of an excess volatility factor is based on Wang and Ma’s (2014) findings. Wang and Ma proved the presence of significant abnormal returns in the costless investment strategy based on a simultaneous long position on a portfolio of extremely high variance-difference stocks, and a short position on a portfolio of minimum variance-difference stocks. Additionally, they proved the existence of a monotonic positive relation between the Sharpe-Lintner and Carhart alphas, and the portfolios of ascending variance difference. Finally and most importantly, they have demonstrated that over the long sample period from 1963 to 2010, the abnormal profitability of variance difference investment strategies cannot be explained by the size effect, the book-to-market equity effect and the momentum effect.

This research builds on Wang and Ma (2014) and adopts Lo and MacKinlay’s (1988) variance difference as a proxy for Shiller’s (1981) excess volatility with the intent to study its explanatory power over the distribution of stock returns. The method with which the analysis is conducted follows that of Fama and French (1993), but it also includes time-varying regression parameter models and prediction performance evaluations. The research concludes with the theoretical contextualization of its empirical findings and an assessment of its usefulness for the financial industry. In particular, I briefly considered both the theoretical and practical contribution of this research for portfolio passive management, for portfolio active management and for the evaluation of the fund managers’ investment strategy.
2. Literature Review

A comprehensive survey of the existing financial research explaining common stocks returns is beyond the scope of this paper. What instead is highly relevant for a full understanding of the following empirical research section is a review of the existing literature consistent with its theoretical framework and its research design. In particular, Paragraph 2.1.1 provides a rationale for the introduction of a new asset-pricing factor in the context of multi-index models. Paragraph 2.1.2 introduces to the behavioural background of the excess volatility related factor in question.

2.1 A Survey of the Literature

2.1.1 The Capital Asset Pricing Model: Basic Structure and Shortcomings

The introduction of the Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Lintner (1965) not only gave birth to modern asset-pricing principles and techniques, but also offered an equilibrium theory of asset-pricing related with the existing literature on portfolio selection. The Sharpe-Lintner model of asset-pricing in fact, builds on Harry Markowitz (1959) by adding two important assumptions according to which all investors have access to unlimited borrowing and lending at a risk-free rate clearing the risk-free borrowing and lending market (hereinafter referred to simply as “risk-free asset”), and all investors have homogenous expectations with respect to expected returns, variance of returns and the joint distribution of asset returns within the relevant period of time. Given these two key assumptions, the CAPM provides an equilibrium market-clearing framework where all investors choose to invest a different combination of the risk-free asset and that risky asset located on the tangency point between the mean-variance efficient frontier with a riskless asset, and the minimum variance frontier for risky assets. Investors’ different risk preferences in fact, will not cause them anymore to invest in different mean-variance efficient portfolios, but will simply drive them to invest in different combinations of the same risky and risk-free asset. Since the tangent portfolio is hold by all investors and it includes all traded risky assets, in equilibrium the risky asset on which the CAPM relies must be the Market portfolio (hereinafter also referred to as “portfolio $M$”).

A very interesting insight regarding the CAPM is that its asset pricing properties are derived from the implications of building on a portfolio selection background. As formally described by Fama and MacBeth (1972), for
an investor holding portfolio $M$, the risk of each of the assets pertaining to portfolio $M$ corresponds to each asset’s contribution to the overall risk of the portfolio. Given portfolio $M$, portfolio $M$’s standard deviation $\sigma$, portfolio assets’ weights $x_{im}$ with $i = 1, 2, ..., N$ and normal return distributions, for portfolio $M$ to be efficient its assets’ weights must maximize expected portfolio return:

$$E(R_m) = \sum_{i=1}^{N} x_{im}E(R_i)$$  \tag{1}$$

subject to constraints (2) and (3):

$$\sigma(R_m) = \sum_{i=1}^{N} x_{im} \frac{\text{cov}(R_iR_m)}{\sigma(R_m)}$$  \tag{2}$$

$$\sum_{i=1}^{N} x_{im} = 1$$  \tag{3}$$

Using Lagrangian optimization, I obtain the slope of the efficient set according to which the difference between the expected return on the single asset and the expected return on portfolio $M$, is proportional to the difference between the risk of the asset and the risk of the portfolio:

$$S_m = \frac{E(R_i) - E(R_m)}{\frac{\text{cov}(R_iR_m)}{\sigma(R_m)} - \sigma(R_m)}$$  \tag{4}$$

By rewriting our result in a convenient form and by defining beta as the risk of portfolio's asset $i$ relative to the total risk of the portfolio,

$$\beta_i = \frac{\text{cov}(R_iR_m)}{\sigma^2(R_m)}$$  \tag{5}$$

and $E(R_0)$ as the return on an asset of zero-$\beta$, I can rewrite the equation for $S_m$ as the equation usually representing the CAPM:
The above derivation formally introduces to what Fama and French (2004) describe as a structural weakness of the CAPM. According to them, the CAPM is simply a specific application to the market portfolio of a common property that holds for any mean-variance efficient portfolio: the linear relation between the expected return of an asset and its portfolio \( \beta \). A relevant issue arises at this point since the efficiency of the market portfolio is based on the three unrealistic assumptions of unrestricted short selling, of homogeneous expectations and of unrestricted risk-free borrowing and lending. The inefficiency of the market portfolio, due to the untenable character of its assumptions, would break the linear relation between expected return and market beta, and hence the validity of the CAPM would be lost. Ross (1977) demonstrated that if short selling is subject to “real world” related restrictions than the CAPM doesn’t hold.

Nevertheless, the fact that the domain of applicability of the Capital Asset Pricing Model relies on a set of very stringent assumptions is not a sufficient reason to consider the CAPM as an unreliable model. What does indicate, instead, is that its predictions arise from an extensive abstraction from reality and hence its reliability should be empirically tested. One of the testable implications particularly related to this research regards the CAPM’s degree of completeness of its explanation of assets’ excess returns. According to the CAPM, the portfolio \( \beta \) is a complete measure of risk and no other variable has any explanatory power over securities’ risk and return. To this day the vast existing research produced inconclusive results. Furthermore, among researchers and academics there is no agreement whether the existing empirical tests were correctly designed to perform a true test of the CAPM.

Given such an extensive and variegated literature on the topic, this survey focuses primarily on those studies consistently linked to my objective. More precisely, this study may be contextualized as being part of an on-going process by which researches providing evidence of CAPM shortcomings are followed by studies providing a less parsimonious formulation of the CAPM: Basu (1977), Banz (1981), Rosenberg, Reid and Lanstein (1985) evidence of variation in expected returns unrelated to market beta culminated in Fama and French’s (1993, 1996) three-factor model; Jagadeesh and Titman’s (1993) proof of the unexplained profitability of momentum strategies culminated in Carhart’s (1997) four-factor model. Hence, this study attempts to provide an empirical evaluation over the appropriateness of an excess-volatility-based fifth factor, following Wang and Ma’s (2014) evidence of the unexplained profitability of excess volatility strategies.
Basu's (1977) objective was to empirically evaluate whether or not the returns on common stocks are related to stocks' price to earnings ratios (P/E). The sample of firms taken into consideration consisted in about 1400 firms from all industrial sectors listed on NYSE between 1956 and 1971. The P/E ratios were calculated as the ratio between each security's market capitalization and reported annual earnings to shareholders. The stocks were then ranked according to their P/E and distributed into five quintiles and made into corresponding portfolios. Given the accounting nature of the information used to compute the ratios, long positions on the mentioned P/E based portfolios were taken on the first of April of each year. This stratagem was implied to ensure that, three months after the end of the previous fiscal year, all investors had the time to elaborate published accounting information. Returns on stocks were then calculated on a monthly basis over the following year. The procedure was repeated for each of the fourteen years of the time period taken into consideration. Basu's findings were contrary to capital market theory in the sense that the two quintiles of lowest P/E had higher annual returns and lower systematic risk if compared to the first quintiles of highest P/E. Low P/E quintiles had in fact an annual return of 13.5% and 16.3% with betas of 0.9401 and 0.9866, while high P/E quintiles earned only between 9.3% and 9.5% with betas of 1.1121 and 1.0387. Hence, low P/E portfolio had a positive Jensen annual abnormal return between 2% and 4.5%, while high P/E portfolios had a negative Jensen annual abnormal return between -2.5% and -3%. Assuming data to be normally distributed, all results were statistically significant at 0.05% level. At last, Basu mentioned a possible behavioural interpretation of its findings, according to which biased overly optimistic or pessimistic investors’ expectations move securities far from true P/E ratios.

Banz (1981) focused on the relation between return and market value of common stocks. In his study he considered all common stocks listed on NYSE for a minimum period of time of five years between 1926 and 1975. The research design was highly oriented towards the consideration of the most important statistical pitfalls that empirical methods based on econometrical approaches might be exposed to: final estimates are derived from sequential differently specified regressions; error terms are allowed both as homoscedastic and heteroscedastic, the analysis considers both Ordinary Least Squares and Generalized Least Squares regressions; the best specification of the pricing model is evaluated through the repetition of the same regression but with reference to three different proxy for the market portfolio; regression effects like the overestimation of past betas of high-beta-stocks and the underestimation of past betas of low-beta-stocks are evaluated and contained. Banz's objective of such a cautious implementation of the needed econometrical methods was that of avoiding a wrong rejection of the
simple Sharpe-Lintner market model. Strong of the deployed precautions, Banz concluded that the CAPM is misspecified. In other words, market beta cannot provide a full description of the variation in stock returns. Controlling for risk in fact, small NYSE firms had significant greater returns than large NYSE firms over the entire forty years period. A phenomenon nowadays shortly referred to as “size effect”.

Proceeding in chronological order, the last key research that brought Fama and French to the formulation of their three-factors model was that of Rosenberg, Reid and Lanstein (1985) over the unexplained profitability of a book to price strategy. The book to price strategy is based on zero-investment portfolios (or “pure hedge”) of long and short positions specifically combined to be immune to a set of “risk indexes” including market variability, stocks’ past performance, stock size, share turnover, predicting indexes of stocks prospective growth, earning to price ratios and more. Despite the immunized nature of these portfolios long on stocks with high book value to market price and short on stocks with low book value to market price, the authors found that time regressions on the CAPM, specified with reference to excess return on the market, earned positive and significant average residual returns of 0.36% per month throughout the entire time period of 141 months between. The regression permitted also to evaluate the optimal immunization of the strategy that in fact produced market betas indistinguishable from zero.

The great contribution by Fama and French (1993, 1996) was not simply that of comprehensively summarize prior research into a new asset pricing model, but specifically that of analysing the existing similarities among the variables mentioned in prior research, and construct two new risk factors that could relate to them all. Hence, the Fama-French three-factors model takes into consideration the most relevant prior findings but maintains a relatively parsimonious formulation. The main reasoning behind this result was that Basu’s (1977) price to earning ratio effect, Banz’s (1981) size effect and Rosenberg, Reid and Lanstein’s (1985) book to market value effect are not key variables arising from a particular economic theory but rather simple factors derived from empirical experience. This common trait of prior research findings permitted Fama and French to jointly consider all previous variables and recognise that they were simply different formulation of stock price ratios. From this perspective the explanatory capabilities of the above factors didn’t appear surprising anymore. In fact, if I rearrange the simple formula for the total return \((TR)\) on a stock over a single period, we notice how price may provide useful information about expected returns. As stated by Fama and French (2004, p.) “in principle the cross-section of prices has information about the cross-section of expected returns: a high expected return implies a high discount rate and a low price”, as it may be easily noticed from the
following elementary formulas, where $P_0$ and $P_1$ are respectively the stock’s price at time $t = 0$ and $t = 1$, and $D_1$ is the dividend distributed at time $t = 1$.

$$TR = \frac{(P_1 - P_0) + D_1}{P_0}$$  \hspace{1cm} (7)

hence,

$$P_0 = \frac{P_1 + D_1}{1 + TR}$$  \hspace{1cm} (8)

More precisely, Fama and French (1993) concluded that the relevant variables found in previous research had only an indirect role in capturing returns left unexplained by the CAPM market beta. Those relevant variables, in fact, were simply reflecting the effect linked to unidentified economical factors for which stocks of similar size, similar book to market value or similar price to earning ratios had higher covariance between each other than with the general market portfolio. Hence systematic (undiversifiable) risks specific to stocks of a certain category were left unconsidered by the usual market factor but were captured by price-based factors. (A formal description of the Fama-French asset-pricing model is part of the theoretical framework of this study and will thus be provided in the last paragraph on this chapter)

Shortly after its introduction, the Fama-French three-factor model had to face what would have subsequently been recognised as an untenable challenge: Jagadeesh and Titman’s (1993) proof of the unexplained profitability of momentum strategies. In their study they included all NYSE and AMEX stocks over the period between 1965 and 1989, and performed empirical test reckoning on a simple one-factor pricing model. The strategy consisted in overlapping holding periods with monthly review of stocks’ ranking on the basis of prior performance. Ranked stocks were then distributed among ten decile portfolios with the tenth decile composed of “winners” (past months best performing stocks) and the first decile of “losers” (past months worst performing stocks). The position was long on winners and short on losers. When stocks are ranked on the basis of their prior six months performance and are subsequently hold only for the following six months after portfolio formation, the above strategy produces significant abnormal returns up to an average of 12.01% per year.

Three years later, Fama and French (1996) tested the performance of their recently introduced three-factor model over the most relevant so-called CAPM anomalies. The results were generally encouraging but with a critical
exception: the above described Jagadeesh and Titman’s momentum effect or, in other words, the continuation of short-term returns. The authors referred to this limitation of their three-factor model as of its “main embarrassment” and have called, among other interpretations, for future research “to look for a richer model, perhaps including an additional risk factor” (Fama & French, 1996, p. 81). Accordingly, Carhart (1997) performed a study with the attempt of describing the persistence of mutual funds performance on the basis of a four-factor pricing model that could have taken into consideration Jagadeesh and Titman’s (1993) “one-year momentum anomaly”. His findings proved the decision to include a fourth factor correct. More specifically, the return persistence of portfolios of mutual funds, sorted according to their returns over the prior year, showed a highly significant momentum factor explaining for most of the returns.

To conclude, the rationale for which I intend to evaluate the appropriateness of the introduction of an excess-volatility-related fifth factor, emerges from what has been so far described of the existing literature and builds on Wang and Ma’s (2014) findings. The objective in fact naturally follows from the evolution of the literature, from Wang & Ma’s results and from their indications for future research.

### 2.1.2 Behavioural Interpretations of CAPM Anomalies

Since the 1980s, mounting confirmations of market anomalies, alongside notions appertaining to early classical studies, as that of the Keynesian beauty contest theory (1936), led to a greater affirmation of theoretical conceptualizations according to which financial markets were built on the coexistence of rational and irrational traders, of “noise” and “information traders”, of “smart money” and “ordinary investors”. More formally, economic and financial research took greater distance from purely neoclassic positions of fully rational expected-utility-maximizing economic agents and perfect capital markets. In this regard, Grossman and Stiglitz (1980) redefined the concept of market informational efficiency by theorizing an innovative model of an “equilibrium degree of disequilibrium”. According to their model the presence of informed “information traders” and of uninformed “noise traders”, alongside that one of imperfect price transmission of costly information, and of endogenous belief differences arising from different expenditures on information, is needed to reach equilibrium. If information is costly, when markets are noiseless and prices fully reflect information, equilibrium cannot be reached. Hence, they argue that costless information is a necessary condition for market efficiency. Nevertheless, according to Hayek (1945) that
same condition is reachable only in the absurd case of market disappearance, because if respected, prices become meaningless.

On the basis of a redefined idea of financial markets’ structure, new perspectives had to be formulated over the dynamic character of the asset pricing process. It is with regard to this perspective that Shiller (2013, p. 482) starts the “Early History of Behavioural Finance”. As a first attempt of providing a more precise viewpoint on the time-related dynamics of asset mispricing, Shiller refers to Keynes’s greatly ahead of time vision over speculative markets. Keynes’s Beauty Contest Theory provides an example of how investors’ rationality may cause similar mispricing of speculative assets as those predicted by capital markets of irrational investors. In the beauty contest theory in fact rational investors do not price assets on the basis of their expected true value but on the basis of other investors’ conventional valuations. Conventional valuations result in asset over-pricing and under-pricing that will eventually be corrected only in the longer term. As Shiller (2013, p. 483) explains in fact, over time “if an asset’s returns are carefully tabulated and disappoint for long enough, people will eventually learn to change their views, but it may take the better part of a lifetime”.

Black (1986) reaches a similar conclusion. He defines “noise” as any variable preventing investors from knowing stocks’ true value and expected returns. In the same time, noise represents the basic reason for trading and the greater the noise, the greater market liquidity. Most interestingly, Fisher (1986) argues that stock prices tend to move toward their true value only over time, as the action of information traders is large enough to counteract that of noise traders.

The fundamental contributions on this perspective by De Bondt and Thaler (1985) and by Lakonishok, Shleifer and Vishny (1994) are specifically linked to this research. Both groups of researchers started their analysis from the behavioural principal of overreaction to define and detect market anomalies. Kahneman and Tversky (1982, p.417) formally defined the overreaction phenomenon as the tendency by which investors fail to adjust their stock prices’ growth forecast to the correct degree of mean reversion. More specifically, they define as a basic principle of statistical prediction the one for which “the lower the predictability, the closer the prediction should be to the class average”. De Bondt and Thaler found empirical results consistent with the overreaction phenomenon and with the mean reverting nature of stock price series. More specifically, overreaction was found to be asymmetrically stronger for past losers than for past winners but in both cases it respected the assumption for which (1985, p. 4) “the more extreme the initial price movement, the greater will be the subsequent adjustment”. Hence, over a
time period of three years after portfolio formation, the less risky portfolios of past losers are found to outperform more risky portfolios of past winners by about 25%. Both categories of stocks were affected by overreaction: past losers were under-priced and past winners were overpriced. This predictable pattern of stock returns helps explaining the existence of excess volatility as the result of investors' irrational tendency to attribute disproportional importance to short-term paths and news, rather than concentrating on fundamentals quantified in the trendiness of dividends payments over time. Similarly, Lakonishok et al. (1994) proved that the contrarian investment strategy long on stocks with low past growth and low expected future growth, and short on stocks with high past growth and high expected future growth, is able to gain significant abnormal returns. The profitability of their strategy was attributed to investors' erroneous belief that stock prices would have continued to grow on a long-term trend, while instead that same conviction was gradually abandoned.

In summary, various studies interpreting the origins of asset mispricing and the causes of CAPM anomalies, even though from different perspectives, agree on the fact that stock returns fluctuation reflect variation in economic fundamentals more closely in the longer term than in the short-term. Similarly, prices are most close to fundamental values only once conventional valuations are suppressed or behavioural reactions are corrected. Two conditions that, according to the above literature, take place only in the longer-term. As far as this study is concerned, in accordance with Wang and Ma’s (2014) findings, I believe that over-reactive short-term returns followed by an adjustment toward value in the longer-term determine structural differences between the volatility of short-term returns and the volatility of longer-term returns. Hence, the difference between the short and the long-term returns' volatilities helps interpreting and quantifying the presence of excess volatility in stock prices.

2.2 Theoretical Framework

This section describes the three core concepts and models on which the research is based: Lo and MacKinlay’s (1988) variance difference, Shiller’s (1981) excess volatility puzzle, and the Fama-French and Carhart asset-pricing models.
2.2.1 Variance Difference

By “variance difference” I generally refer to one of the measures utilized in the volatility based specification test originally formulated by Lo and MacKinlay (1988). Their research was originally intended to formulate a test capable to define the nature of the stochastic processes of stock returns. More specifically, they provided a test for the random-walk hypothesis of stock returns. Their major finding consisted in the refusal of the random-walk hypothesis because of its inconsistency with the stochastic behaviour of weekly stock returns in the twenty-three years period between 1962 and 1985. The method by which their conclusion was derived, exploited a specific property of random walks according to which the variance of the increments is a linear function of the relative time interval. In other words, if a daily stock return process consists in a random walk, being \( r_t \) and \( r_{t-1} \) the stock return at time \( t \) and at time \( t-1 \) respectively, the variance of \( r_t + r_{t-1} \) should be equal to twice the variance of \( r_t \). The variance difference measure permits to evaluate the validity of the random walk hypothesis by comparing the above two variances. As far as this research is concerned, the variance difference measure of interest corresponds to the adaptation formulated in Wang and Ma (2014, p.3) by which the variance difference (VD) “of \( q \)-period returns is given by the difference between \( q \) of the variance of one-period return and the variance of \( q \)-period return”. The exact equation for the variance difference considered in this research corresponds to equation (11) in the Method paragraph of the Research Design chapter.

In Lo and MacKinlay (1988) VD was meant to test the random-walk hypothesis of stock returns, in Wang & Ma (2014) and in this research VD is used as a proxy for Shiller’s excess volatility of stock returns. Therefore, in contrast to Lo and MacKinlay (1988), in this research I am not interested in VD’s statistical indistinguishability from zero but simply in its extent.

2.2.2 Excess Volatility

In parallel to De Bondt and Thaler’s (1985, p. 1) statement regarding the term “overreaction”, also the terms “excess volatility” carry with them an implicit comparison to some degree of volatility that is considered to be appropriate. Shiller’s formulation of excess volatility referred to the volatility of stock prices in excess to what is prescribed by the efficient market hypothesis. More specifically, Shiller (1981) defined \( p_t \) as the detrended real Standard and Poor’s Composite Stock Price Index, and \( p_t^* \) as the present discounted value of the actual subsequent real dividends. According to the
efficient market hypothesis $p_t$ should correspond to the optimal expectation conditional on all information available at time $t$:

$$\begin{align*}
p_t &= E_t(p_t^*) \quad (9)
\end{align*}$$

Having assumed $p_t$ to be an optimal forecast of $p_t^*$, Shiller follows the analysis by assuming the forecast error $u_t = p_t^* - p_t$ to be uncorrelated with the forecast itself. Hence, considering the statistical principle by which the sum of two uncorrelated variables is the sum of their variances, in terms of standard deviations I have that $\sigma(p) \leq \sigma(p^*)$. As Shiller mainly graphically showed, this last inequality is clearly violated by real data. Real stock prices in fact, widely oscillate around the present values of subsequent real dividends. More formally, with the present value of subsequent real dividends $p_t^*$, the model refers to the following summation:

$$\begin{align*}
\sum_{k=0}^{\infty} \gamma^{k+1} d_{t+k} \quad (10)
\end{align*}$$

where $\gamma$ corresponds to the real discount factor for the detrended series ($\gamma \equiv \lambda y$) with $[\lambda \equiv (1 + g)]$ being the trend factor for prices and dividends, $g$ corresponds to the long-run growth rate of prices and dividends, and $d_{t+k}$ corresponds to the real detrended dividend at time $t + k$. At last, $y$ represents the real discount factor $[\frac{1}{1+r}]$, where $r$ corresponds to the one period real discount rate. As the series for prices and dividends are limitless, an estimated terminal value must be selected so that $p_t^*$ can be defined recursively moving backward from the terminal to the present date.

Wang and Ma (2014) documented that the variance difference approximation of the excess volatility phenomenon is mainly driven by the behavioural phenomenon of overreaction. Hence, the link between excess volatility, variance difference and overreaction is at the base of the behavioural interpretation of this research’s problematisation and empirical results.

### 2.2.3 The Fama-French and Carhart Asset-pricing Models

The asset-pricing model formulated by Fama and French (1992, 1993), commonly referred to as the “Fama-French asset-pricing model” or the “three-factors asset-pricing model”, consists in an expansion of the original single-index CAPM to a multi-index asset-pricing model. Fama and French introduced two new explanatory factors in the form of zero net investment portfolios specifically designed to produce returns capable to mimic for the
impact of stocks’ market equity value (“ME”) and stocks’ book equity value (“BE”). In prior findings, the cross section of average returns was found to have a stable negative relation with ME and a stable positive relation with BE, hence these two specific “dimensions of risk” were not fully grasped by the more simple single-index CAPM. The zero net investment portfolios were built considering all stocks trading on AMEX, NASDAQ and NYSE. For the ME mimicking portfolios stocks were divided into two groups, the first one containing all stocks with a market equity higher than NYSE median size, and the second one containing all stocks with a market equity lower than NYSE median size. Regarding BE instead, three groups were formed by sorting stocks in ascending order on the basis of their BE/ME ratio and then distributing them according to the following logic: the base 30% stocks formed the group of lowest BE/ME, the following 40% of stocks were assigned into a middle group, and the top 30% of stocks were considered as those having the highest BE/ME. The two ME portfolios and the three BE/ME portfolios were then intersected between each other in order to derive six portfolios. More specifically, within each group both the stocks pertaining to the small ME group and those pertaining to the big ME group, were redistributed into the three portfolios of ascending BE/ME.

The mimicking portfolio for stocks’ size was then formulated as the monthly difference between the simple average of the returns from the three small ME portfolios and the simple average of the returns from the three big ME portfolios. This mimicking portfolio, usually denominated as “SMB” (Small-Minus-Big), analyses the different return behaviours of big and small market capitalization stocks from a perspective meant to be free from the influence of the BE/ME ratio. Similarly, the mimicking portfolio for stocks’ book-to-market equity is derived as the difference between the simple average of the returns on the two high BE/ME (the first one pertaining to small ME stocks and the second one pertaining to big ME stocks) and the simple average of the two low BE/ME portfolios. The resulting mimicking portfolio for book-to-market equity is then usually referred to as “HML” (High-Minus-Low). At the time the model was introduced, the independency between the size and the book-to-market variables seemed to be well achieved as the correlation between them was a merely -0.08%.

The Carhart asset-pricing model builds on the Fama-French one and it extends it by one further factor. This fourth factor is constructed on a monthly basis including all AMEX, NASDAQ and NYSE stocks. More specifically, it is constructed as the difference between the return on an equally weighted portfolio of the stocks with the highest 30% of one month lagged past eleven months returns, and the return on an equally weighted portfolio of the stocks with the lowest 30% of one month lagged past eleven
months returns. As far as this research is concerned, I am not going to use Carhart’s original formulation of the momentum factor but rather Kenneth French’s one, that redesigned it to control for size effects. At last, I refer to the momentum factor with the abbreviation “MoM”.

The Carhart and Fama-French models represent the benchmark multi-index models of this research. The asset-pricing validity of an excess-volatility related factor is, in fact, assessed with regard to the contribution of its juxtaposition next to the factors of the Fama-French and Carhart models. Section 3.3.2 reports formal equations of all the models employed in this research.
3. Research Design

3.1 Research Objectives

The general research objective of this paper can be reformulated into a number of statistically testable hypotheses. In particular, I will evaluate whether or not an excess-volatility based factor,

- provides any time-series asset-pricing improvement to the Fama-French and Carhart models;
- corrects for evident patterns linking the asset-pricing models’ performance to the characteristics of the dependent variable;
- provides any advantage in terms of the frequency of insignificant pricing errors;
- has any advantage in the description of the cross-section of stock excess returns;
- sustains the stability through time of the models explanatory power;
- sustains the insignificance through time of the models’ pricing errors;
- is, through time, as persistently significant as the other regressors of the models;
- ameliorates the quality of the models predictions;
- ameliorates the distribution and the behaviour of the regression residuals.

3.2 Data

With reference to the data employed in this research a distinction needs to be made between raw data and final data. The distinction arises from the 12-months rolling window nature both of the VD calculation and of the investment strategy (see the next section for a detailed description of methods and their impact on data). As far as the raw data is concerned I include all end-of-day prices of AMEX, NASDAQ and NYSE stocks trading from May 1995 to March 2015 (for a total of 1929 stocks). Quotes were downloaded from Thomson Reuters Datastream. Stocks that started trading on the above exchanges on a subsequent date from the starting point of our time period were removed due to limited available computing power.

In addition to the above-mentioned raw data, needed for the first part of the calculations, the data required to perform our analysis consists in the time series, relative to the same time period, of all the explanatory factors pertaining to the Fama-French and to the Carhart asset-pricing models. More
specifically, the time series for the market premium factor $RM_t - RF_t$, the market equity value factor $SMB_t$, the book-to-market value factor $HML_t$, the momentum factor $MoM_t$ and the risk-free rate series $RF_t$, were all downloaded from the Data Library by Kenneth R. French available on the website of Dartmouth College - Tuck School of Business.

3.3 Method

3.3.1 The Variance Difference Investment Strategy

Building on Wang and Ma (2014) I initially deploy their method that, in turns, consists in an adaptation from Lo and MacKinlay (1988) and Jagadeesh and Titman (1993, 2001). At the beginning of every month they calculated individual stocks’ $VD(q)$ over the past 12 months, and then sorted stocks according to their individual $VD(q)$ into 10 decile portfolios with the first portfolio (P1) containing the least $VD(q)$ stocks and the tenth portfolio (P10) containing the greatest $VD(q)$ stocks, with $VD(q)$ defined as follows:

$$VD(q) = q\sigma_1^2 - \sigma_q^2 = q \sum_{k=1}^{n} \frac{(P_k - P_{k-1} - \hat{u})^2}{n - 1} - \sum_{k=q}^{n} \frac{(P_k - P_{k-q} - q\hat{u})^2}{m}$$ \hspace{1cm} (11)

With $P_k$, $P_{k-1}$ and $P_{k-q}$ respectively corresponding to a stock’s price at day $k$, at day $k - 1$ and at day $k - q$, where $q$ is equal to 22 days. At last, $n$ is the number of observations taken into consideration and $\hat{u}$ is the mean estimator of daily log stock returns.

$$\hat{u} = \frac{\sum_{k=1}^{n} P_k - P_{k-1}}{n}$$ \hspace{1cm} (12)

The first and second summation terms in the right-hand-side of equation (3) are the unbiased daily variance estimator and the unbiased variance estimator of $q$-days returns:

$$\sigma_1^2 \sum_{k=1}^{n} \frac{(P_k - P_{k-1} - \hat{u})^2}{n - 1}$$ \hspace{1cm} (13)

$$\sigma_q^2 \sum_{k=q}^{n} \frac{(P_k - P_{k-q} - q\hat{u})^2}{m}$$ \hspace{1cm} (14)
The denominator, \( m \), in the summation of equation (6) permits the unbiased variance estimator of \( q \)-days returns to consider overlapping observations:

\[
m = (n - q + 1)(1 - \frac{q}{n})
\]

(15)

The trading strategy according to which stocks’ \( \text{VD}(q) \) are calculated has a formation period of \( M \) months, a holding period of \( N \) months and \( q \)-days returns. More shortly, the trading strategy will hence be denominated according to its parameters as an \( M/N/q \) strategy. The parameters are selected on the basis of Wang and Ma’s (2014) findings, according to which the most interesting trading strategy is the 12/12/22. Stocks are valued and ranked according to their \( \text{VD}(q) \) at the beginning of each month. Each decile portfolio equally includes all stocks ranked in the same \( \text{VD}(q) \) decile in any of the latest \( N \) months. In other words, \( \text{P10} \) at month \( t \) is going to contain \( 1/N \)th of the \( \text{P10} \) selected at month \( t \), \( 1/N \)th of the \( \text{P10} \) selected at month \( t-1 \), \( 1/N \)th of the \( \text{P10} \) selected at month \( t-2 \), ..., \( 1/N \)th of the \( \text{P10} \) selected at month \( t-N+1 \). The same is valid for all other decile portfolios that in fact at each month hold a new position for \( N \) months and close the existing position initiated in month \( t-N \).

Wang and Ma were interested at the average profitability of each of the \( \text{VD}(q) \) decile portfolios. They derived the monthly returns of the zero-investment \( \text{P10-P1} \) portfolio but deployed them only for a graphical inspection over their volatility. The objective was most probably that of trying to explain the reasons for which the phenomenon of excess volatility is hard and potentially costly to arbitrage given the high volatility of portfolio \( \text{P10-P1} \).

As far as this research is concerned instead, I derive the monthly returns of each decile excess volatility portfolio and refer to the series of the costless portfolio \( \text{P10-P1} \) as that pertaining to a new explanatory factor: the excess volatility based fifth-factor (hereinafter referred to as “\( \text{VD}_t \)”) to be juxtaposed next to the Fama-French and the Carhart models. The intuition and justification of such an approach resides in the similarities with which \( \text{VD}_t \) and the Fama-French and momentum factors were originally built. As shown in the last paragraph of the theoretical framework, all of them consist in a series of monthly returns from costless investment portfolios designed as the difference between the extreme values of ranked overlapping observations of the variable of interest.


### 3.3.2 Time-Series Analysis

The performance of the $VD_t$ factor is evaluated both in a time-series perspective and in a cross-sectional perspective according to the same methodology employed by Fama and French (1993). With reference to the time-series analysis, Fama and French evaluated the performance of their newly specified $SMB_t$ and $HML_t$ factors through regression analysis on the monthly excess returns of 25 portfolios obtained by intersecting five quintiles of ascending market equity stocks portfolios and five quintiles of ascending book-to-market equity stocks portfolios. The performance of the newly specified $VD_t$ factor is assessed through regression analysis on the monthly excess returns of 10 portfolios of ascending variance difference stocks. I adopt portfolios formed on variance difference because I intend to determine whether or not the mimicking portfolio $VD_t$ captures common factors in stock returns related to excess volatility, and hence overreaction, that are left unexplained by the available asset-pricing models. The time series analysis is based on the comparison of four different asset-pricing models: the Fama-French model, the Carhart model, a new four-factors model and a new five-factors model structured as follows:

$$Y = X\beta + U$$

(16)

$Y$ is a $t \times 1$ dimensional vector of the monthly observations of the dependent variables in the form of the excess return on each portfolio: $y_t = R_t - RF_t$, where $R_t$ is the return of a certain portfolio at time $t$ and $RF_t$ corresponds to the risk-free rate of return at the same moment in time usually resembled by the Treasury bill rate. $X$ is a $t \times (k + 1)$ dimensional matrix of regressors where $k$ is equal to 3 for the Fama & French model, is equal to 4 for the Carhart model and is equal to 5 for the new fully specified five-factors asset-pricing model. $RM_t$ is the return for the market portfolio. $U$ is a $t \times 1$ dimensional vector of the error terms. $t$ is equal to 216 and corresponds to the 216 monthly observations from April 1997 to March 2015. $\beta$ is a $(k + 1) \times 1$ dimensional vector of the $k + 1$ regression coefficients, with $k$ equal to 3, 4, and 5 for the Fama-French, the Carhart and the five-factors model respectively. In addition I consider also a new four-factor model having an $X$ factor matrix composed of the first, the second, the third, the fourth and the sixth columns. In other words, a four factor model corresponding to the Fama-French model plus the $VD_t$ factor. By comparing the performance of the
Carhart asset-pricing model to that of the new four-factors model I attempt at confronting the performance of $VD_t$ against that of the momentum factor. The relevance of such a comparison is based on the close interpretation that $VD_t$ and $MoM_t$ have in the existing literature. Jagadeesh and Titman (1993) interpret the momentum anomaly as a measure of overreaction to short-term information and price paths. Wang and Ma (2014) interpret the excess volatility anomaly as a measure of the time fluctuation of investors' overreaction. It is hence of some interest to evaluate the nature of the relation of these two measures and assess whether they are correlated up to the point in which they are statistically significant only when taken separately, or maintain complementary relations even when employed simultaneously.

The principal indicators of the performance of $VD_t$ as an explanatory factor in the time series perspective are the adjusted $R^2$ and the statistic significance as measured by the $t$-statistic from the hypothesis test that the true coefficient relative to $VD_t$ equals zero. Comparing the adjusted $R^2$ from regression analysis using different asset-pricing models enables us to verify which model better captures the variance of the excess returns on the analysed portfolios. By definition in fact the adjusted $R^2$ is a measure corrected by the employed number of regressors, of the fraction of the total sum of squares of our response variables that is not left unexplained as part of the sum of the squared residuals.

### 3.3.3 Cross-Section Analysis

The inspection of the intercepts from time series regressions may reveal the presence of unexplained cross-sectional patterns. For instance, Fama and French (1993) found that when the excess returns of portfolios sorted by ascending size are regressed only on the excess market return, the resulting intercepts from the regressions showed evident Banz (1981) size and BE/ME effects. In other words, the intercepts were monotonically increasing in their amount and statistical significance from bigger to smaller portfolios and from lower to higher BE/ME portfolios. This pattern was finally broken thanks to the introduction of Fama and French’s $SMB_t$ and $HML_t$ new factors. Hence, I assess the cross-sectional performance of the above asset-pricing models according to two consequential steps. At first I assess if the employed asset-pricing models produce practically small intercepts, statistically insignificant small intercepts and intercepts free from evident patterns correlated with the ascending variance difference nature of the analysed portfolios. Secondly, I perform a GRS statistical test, which corresponds to a multivariate $F$-test of the null hypothesis that all the intercepts obtained from the time-series regression analysis are jointly equal to zero: $H_0: a_i = 0 \forall i$. The reasoning
behind this approach relies on the fact that this method, designed by Gibbons, Ross and Shanken (1989), permits us to evaluate which asset-pricing model is closest to the non-rejection of the null hypothesis. The intercepts $\alpha_i$ correspond to the expected value of the assets’ unexplained return. Hence, in asset-pricing terms they are usually referred to as “pricing errors”.

The following is the formula required for the derivation of the GRS $F$-statistic:

$$ J = \left( \frac{T}{N} \right) \left( \frac{T - N - L}{T - L - 1} \right) \left[ \frac{\bar{\alpha}' \bar{\Sigma}^{-1} \bar{\alpha}}{1 + \bar{x}' \Omega^{-1} \bar{x}} \right] \sim F(N, T - N - L) \quad (17) $$

where $\bar{\alpha}$ is a $N \times 1$ vector of estimated intercepts, $\bar{\Sigma}$ is the $N \times N$ covariance matrix of the unbiased estimate of the residuals $\tilde{u}_i$, where the unbiased estimate corresponds to:

$$ \bar{\Sigma} = \frac{\bar{\alpha}' \bar{\alpha}}{T - L - 1} \quad (18) $$

$\bar{x}$ is a $L \times 1$ vector of the factors’ sample mean returns and $\Omega$ is a $L \times L$ matrix of the unbiased estimate of the covariance of the factors:

$$ \hat{\Omega} = \frac{(F - \bar{F})(F - \bar{F})'}{T - 1} \quad (19) $$

where $\bar{F}$ is equal to a $L \times L$ matrix of the factors sample mean returns.

Under the assumptions of identically and normally distributed error terms, and non singular covariance matrixes, $J$ follows an $F$ distribution with $N$ degrees of freedom in the numerator and $(T - N - L)$ degrees of freedom in the denominator.

### 3.3.4 Rolling-Window Analysis

The rolling-window analysis of time-series aims at evaluating the constancy over time of the conclusions derived at the time-series and cross-sectional stages, and to compare the predictive abilities of the four asset-pricing models taken into consideration. In particular, it is necessary to focus over the stability of the parameters because any regression analysis on time-series data is potentially valid only for a specific time frame.
3.3.4.1 **Parameter Stability**

Appraising whether or not the parameters and the key statistics obtained from the time-series analysis are valid through time, permits to derive a conclusion about the overall stability of the asset-pricing models’ performance. If the range by which coefficients, statistical significance, pricing errors and *adjusted R*\(^2\) fluctuate over time is large, then the asset-pricing model may not be “universally” valid but valid only with reference to specific periods in time or to specific conditions. Furthermore, if the specific conditions by which the asset-pricing model is valid are unknown, then the model is of no interest both in theoretical and practical terms. However, given the persistent instability of the economic environment, it’s not reasonable to expect completely time-invariant parameters. A parameter is considered to be stable if its volatility is reasonably limited and if extreme fluctuations in its value are rare.

To assess parameter stability I perform a rolling-window regression analysis for each asset-pricing model and on each portfolio with an estimation sample of 108 monthly observations, a single month increment, and 109 iterations. The formal representation of the regression analysis for each asset-pricing model provided in section 3.3.2 is still valid, but in this case, for each of the 109 iterations the time period of the regression analysis moves one step further in time. More formally, for \(i = 1,2,...,109\):

\[
Y_i = X_i \beta_i + U_i
\]  

\[
Y_i = \begin{pmatrix} y_i \\ \vdots \\ y_{107+i} \end{pmatrix}, X_i = \begin{pmatrix} 1 \\ (RM_i - RF_i) \\ \vdots \\ (RM_{107+i} - RF_{107+i}) \end{pmatrix}, \beta_i = \begin{pmatrix} \alpha_i \\ \beta_i \\ \delta_i \\ \gamma_i \\ \mu_i \\ \nu_i \end{pmatrix}, U_i = \begin{pmatrix} u_i \\ \vdots \\ u_{107+i} \end{pmatrix},
\]

As it may be noticed from the above notation, the analysis is performed on 109 different \(Y\) subsamples in order to derive 109 observations for each of the parameters pertaining to vector \(\beta\). In addition the parameter stability analysis will also focus on other regression outputs as the adjusted \(R^2\) and the significance level of the various parameters through time.
3.3.4.2 *Pseudo-Out-of-Sample Prediction*

Given the multivariate time series nature of the analysed asset-pricing models, an unconditional forecast of the response variable would depend on the forecast of predictors’ new data. By definition in fact, unconditional forecasts differ from conditional forecasts because they do not depend on real future values of the predictors, but instead they depend on estimated future values on the predictors. Hence, in the case of unconditional forecasts, the quality of the forecast of the response variable depends both on the quality of the predictor variables’ forecast and on the quality of the model by which response and predictor variables are related. In other words, the use of forecasted values for the predictors introduces an additional source of forecast error next to that depending solely from the forecasting model.

Given the ultimate objective of this research, the most appropriate method to confront the predictive performance of four different asset-pricing models, is to perform a conditional prediction of step-ahead forecasts for the response variable. The conditional prediction in fact permits to focus and quantify the forecast errors arising uniquely from the asset-pricing models.

I will perform a short horizon forecast of one month ahead. The predicted values are calculated as the product of the coefficients $\beta_i$ estimated at the rolling-window stage by the one-month ahead value of the predictors. Formally, for $i = [1,2,...,108]$:

$$\hat{Y} = X\beta_i$$

The quality of the prediction is then assessed through the comparison of the different root mean squared errors (RMSE) obtained from the different asset-pricing models on each portfolio and by visually inspecting Theil (1964, 1966) Prediction Realization Diagram (PRD).
4. Analysis and Findings

4.1 Descriptive Statistics

Table 1

Note: this table reports, in the upper part, the cross-correlations of the five explanatory factors and, in the lower part, the 10 portfolios’ standard deviations and correlations with $V_D_t$.

<table>
<thead>
<tr>
<th>Factors</th>
<th>MktRft</th>
<th>SMBt</th>
<th>HMLt</th>
<th>MoMt</th>
<th>VDt</th>
</tr>
</thead>
<tbody>
<tr>
<td>MktRft</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMBt</td>
<td>0.26409</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HMLt</td>
<td>-0.2112</td>
<td>-0.33653</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MoMt</td>
<td>-0.29129</td>
<td>0.09038</td>
<td>-0.15403</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>VDt</td>
<td>0.09984</td>
<td>0.1432</td>
<td>-0.08363</td>
<td>-0.04149</td>
<td>1</td>
</tr>
</tbody>
</table>

Portfolios’ Standard Deviations and Correlations with VDt at the first and second line respectively

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.049954</td>
<td>0.051391</td>
<td>0.046545</td>
<td>0.04612</td>
<td>0.047845</td>
<td>0.050813</td>
<td></td>
</tr>
<tr>
<td>0.109</td>
<td>0.167</td>
<td>0.134</td>
<td>0.242</td>
<td>0.266</td>
<td>0.297</td>
<td></td>
</tr>
<tr>
<td>0.054247</td>
<td>0.06504</td>
<td>0.069283</td>
<td>0.17018</td>
<td>0.297</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.178</td>
<td>0.477</td>
<td>0.216</td>
<td>0.956</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The final data with which I’m going to perform the analyses covers a time period of 216 monthly observations between April 1997 and March 2015. The returns to be explained refer to 10 decile portfolios of equally weighted stocks of ascending past-12-months variance difference as illustrated above. Table 1 reports the cross correlations between $V_D_t$ and the remaining factors valid for the mentioned time period. The clear relation between $V_D_t$ and size, already illustrated by Wang and Ma for a different time period, remains as the main correlation between $V_D_t$ and the other factors mimicking for stocks specific characteristics. As further proof against possible relevant linear relationships among the explanatory factors I quantified the severity of multicollinearity using the variance inflation factor (VIF) rule, according to which 1 minus the $R^2$ of the linear regression between any of the explanatory factors on the others (denominated as “tolerance level”) should remain above 0.2. Having obtained with our data a tolerance level above 0.6, I concluded that a fully specified model containing all explanatory factors is suitable for analytical purposes. The attention on multicollinearity is oriented at avoiding the erroneous rejection of statistically significant $t$-statistics. The increased probability of this type II error is usually related to multicollinear regressions as their standard errors are typically inflated.
The second part of the table reports each portfolio's volatility and their correlations with $V_D_t$. The relation between portfolios' volatility and excess volatility is monotonically positive only starting from the fourth portfolio. Surprisingly in fact, the portfolio with the least excess volatility is still more volatile than most of the other portfolios belonging to the first half of the group. On the other side, portfolios P7 and P9 that have high and very high excess volatility stocks, have no more correlations to $V_D_t$ as other low and middle excess volatility portfolios. In other words, high volatility does not always coincide with high correlation to excess volatility, and $V_D_t$ doesn't seem to always correlate better with portfolios of higher excess volatility stocks. It is unclear if these ambiguous relationships between excess and plain volatility and these apparent limitations of $V_D_t$ in correlating with excess volatility are resulting from our sample of data or from the method itself.

**Figure 1**

*Note: this graph plots the return series of factor $V_D_t$ throughout the entire time period of 216 observations: from April 1997 to March 2015.*

Figure 1 reports the time series of $V_D_t$ across the entire time period. As it may be noticed even from a simple visual inspection, the series contains 4 outlying observations. By going back to the raw data and repeating the calculation needed to derive the VD series separately for NYSE, Amex and Nasdaq, it emerged that most of the extreme variations came from stocks
listed on the Amex market. Nevertheless, even if down to a smaller scale, also the other two separately obtained series manifested similar peaks located at the same point in time. Given this shared feature of the different subsamples of our data, given the fact that Amex attracts many stocks that cannot trade on NYSE because they don’t meet their requirements, and considered Carhart’s (1997) perspective according to which high-variance-low-correlation factors are likely to explain much of the time-series variation, I opted for performing the analysis using the original series including all markets. In any case I am aware of the possible distorting effect that the presence of extreme values may have on the regression coefficients. Most probably $VD_t$ will better explain series also affected by greater variation. In the attempt of evaluating the extent of this possible bias, I ran some of the regressions included in the analysis with the alternative NYSE and Nasdaq-only $VD_t$. Contrary to expectations, results remained significantly similar.

Lastly, after having clarified which series to include in the analysis, I proceeded by testing them for the presence of a unit root through sequential Augmented Dickey-Fuller test, and concluded that all series were integrated of order zero (stationary).

### 4.2 Results from Time Series and Cross-Sectional Analysis

Asset-pricing models that produce practically small pricing errors, statistically insignificant small pricing errors and pricing errors free from correlations with the relative weight given to a specific investment strategy, are more likely to offer a complete descriptions of assets’ returns and prices. In other words, asset-pricing models having the above-mentioned features are more likely to have taken into consideration all rewarded sources of risk linked to the assets’ returns.

Figure 2 summarizes two relevant statistics from the time-series analysis using the four different asset-pricing models on 10 portfolios of ascending past-12-months excess volatility. The pricing errors curves derived from the employment of the Fama-French and of the Carhart models, show relevant patterns related to the ascending excess-volatility nature of the 10 different portfolios. The higher the excess volatility of the portfolios, the greater the distance of the Carhart and Fama-French pricing errors from zero. In addition, the Carhart and the Fama-French models have downward sloping adjusted $R^2$. 

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Note: these graphs plot the adjusted $R^2$ series and the pricing errors from the time-series analysis of the 10 portfolios of ascending past-12-months excess volatility using four different asset-pricing models: one for each plot as indicated on the titles. The adjusted $R^2$ series is scaled down by 10, and the pricing errors are in absolute value (all pricing errors had same sign), in order to be graphically comparable.

The juxtaposition of $V_D_t$ to the left of the Fama-French and Carhart models’ factors permits to break the link between the decreasing performance of the asset-pricing models and the increasing excess volatility of the analysed portfolios. Additionally, the regression summaries (see Appendix A) confirm the significance of $V_D_t$ in all portfolios starting approximately from portfolios P2 and P3. Finally, with the New Five-factors (“fully specified”) asset-pricing model the practical size of the pricing errors reaches its minimum distance from zero, and the adjusted $R^2$ is most stable.

Having evaluated the practical size of the pricing errors, Figure 3 reports the absolute values of their t-statistics. In order to evaluate their statistical significance, the histogram’s columns are divided into two parts according to the horizontal line representing the t-statistics’ critical value of 1.97 (210, 211 and 212 degrees of freedom and 0.05 significance level).
Figure 3

Note: this histogram reports the absolute value of the pricing errors’ $t$-statistics derived from the time-series analysis of 10 portfolios of ascending past-12-months excess volatility, using four different asset-pricing models as indicated in the legend (“Fully Specified Model” stands for the “new five-factors model”). The horizontal blue line corresponds to the $t$-statistics critical level of 1.97.

According to the obtained results, only the asset-pricing models containing $VD_t$ are able to produce insignificant pricing errors. The Fama-French and Carhart models in fact have only significant pricing errors. Furthermore, except for portfolio P10, the pricing errors’ $t$-statistics obtained with the New Five-factors’ model are persistently closer to the area of insignificance than those obtained with the Fama-French and Carhart models.

With regard to portfolio P10, the uniqueness of its results is partly explained in the rolling-window analysis. In fact, allowing parameters to vary through time revealed that models not including $VD_t$ suffer from misspecification, as their explanatory factors are insignificant for most of the time. In contrast, the resulting convex distribution of the pricing errors’ $t$-statistics is yet to be explained.

From a strictly statistical point of view, the validity of the juxtaposition of $VD_t$ next to the existing factors of the Fama-French and of the Carhart
models can be evaluated from the result of the F-test for the comparison between two pairs of nested models: the Fama-French against the new four-factors model, and the Carhart against the new five-factors model. Table 2 reports the results from this comparison.

Table 2

Note: this table reports the Analysis of Variance F-ratio tests of the pairwise comparison of the Fama-French (i), the Fama-French + 𝑉𝐷𝑡 (ii), the Carhart (iii) and the New Five-factors models.

<table>
<thead>
<tr>
<th>Models compared</th>
<th>Portfolios</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) against (ii)</td>
<td>F-ratio</td>
<td>0.011</td>
<td>3.2667</td>
<td>0.7174</td>
<td>22.064</td>
<td>34.691</td>
<td>47.044</td>
<td>6.1198</td>
<td>145.3</td>
<td>9.007</td>
<td>&gt;1000</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.9166</td>
<td>0.0721</td>
<td>0.399</td>
<td>4.75e-06***</td>
<td>1.5e-08***</td>
<td>7.55e-13***</td>
<td>0.01456*</td>
<td>2.2e-16***</td>
<td>0.003**</td>
<td>2.2e-16***</td>
</tr>
<tr>
<td>(ii) against (iv)</td>
<td>F-ratio</td>
<td>0.0739</td>
<td>3.0005</td>
<td>0.4753</td>
<td>22.398</td>
<td>35.676</td>
<td>49.176</td>
<td>5.8405</td>
<td>163.45</td>
<td>8.985</td>
<td>&gt;1000</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.786</td>
<td>0.0847</td>
<td>0.4913</td>
<td>4.06e-06***</td>
<td>9.83e-09***</td>
<td>3.15e-11***</td>
<td>0.01652*</td>
<td>2.2e-16***</td>
<td>0.003**</td>
<td>2.2e-16***</td>
</tr>
</tbody>
</table>

The results in Table 2 permit to conclude that, for assets of excess volatility levels higher than those of portfolios P2-P3 (lower second and third deciles), asset-pricing models that include factor 𝑉𝐷𝑡 are statistically superior to the traditional Fama-French and Carhart models.

With regard to the cross-sectional perspective, the results from the joint tests on the time-series regression intercepts are summarized in Table 3.

Table 3

Note: this table reports the GRS evaluation of the intercepts from the regression analyses performed with the Fama-French (i), the Fama-French + 𝑉𝐷𝑡 (ii), the Carhart (iii) and the New Five-factors models.

<table>
<thead>
<tr>
<th>Asset-Pricing Model</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRS F-statistic</td>
<td>10.6935</td>
<td>10.0463</td>
<td>10.136</td>
<td>9.6055</td>
</tr>
<tr>
<td>GRS Critical Value at 0.05 %</td>
<td>1.877</td>
<td>1.878</td>
<td>1.878</td>
<td>1.8782</td>
</tr>
<tr>
<td>P-value of the F-statistic</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

All the above F-statistics are distributed according to F(N,T-N-L) degrees of freedom. All regressions have equal N (10, number of portfolios analysed) and equal T (216, number of observations for each portfolios). With regard to parameter L, it corresponds to the number of explanatory factors employed plus 1 (L=4 for (i), L=5 for (ii) and (iii), L=6 for (iv). The resulting F-statistics are reported with reference to their significance probability.
Given such small $p$-values, the probability of having derived by chance a statistic as adverse to the null hypothesis as the one computed is very small. Hence, the null hypotheses are rejected in all of the above cases. Nevertheless, the new five-factors asset-pricing model is the closest to the non-rejection of the null hypothesis. Furthermore, among the other models, once again the one “behaving” the best is that containing $VD_t$. Hence, despite all asset-pricing models reject the null hypothesis, I obtained supporting evidence to the effect that the inclusion of $VD_t$ improves the cross-sectional explanation of portfolios’ excess returns.

In conclusion, according to the obtained results, $VD_t$ enhances the time-series and cross-sectional descriptive power of the Fama-French and the Carhart models by enhancing and stabilizing their explanatory power, by breaking the patterns between the characteristics of the dependent variables and the performance of the asset-pricing models, by lowering the practical size of the pricing errors and by sustaining their statistical insignificance.

4.3 Parameter Stability and Prediction Performance

I evaluated, for each portfolio, the constancy of the asset-pricing models’ explanatory power, the time-varying significance of their explanatory factors, the time-varying significance of their pricing errors and the quality of their predictions. The results obtained from the rolling-window time-series analysis are to be compared to those obtained from the static time-series analysis.

4.3.1 Time-Varying Explanatory Power

Fluctuations in the explanatory power of the four different asset-pricing models are reported in a series of graphs plotting the time-varying adjusted $R^2$ series derived from the rolling-window regression of each asset-pricing model on the returns of the ten portfolios of ascending excess volatility. This section presents all the empirical results obtained from the analysis, however, it reports only the 4 most essential graphs out of a total of 10. For a visual inspection of the other graphs, see Appendix B.

Up to the third portfolio the five-factors and the Carhart asset-pricing models are persistently the best models in terms of explanatory power. Their performance is approximately equal and the differences between all the asset-pricing models’ performances are very small. Figure 4 reports the fluctuations of the adjusted $R^2$ obtained with the four asset-pricing models for portfolio P3.
Figure 4

Note: this graph plots the 4 series of Adjusted $R^2$ fluctuations throughout the rolling-window time period of 109 observations for portfolio P3: one curve for each asset-pricing model.

From portfolio P4 to portfolio P9 the five-factors asset-pricing model is the one performing the best and the Fama-French model is the one performing the worst. The degree by which the performance of the four asset-pricing models differs is not constant and it does not seem to be always related to the ascending degree of excess volatility of the portfolios. The adjusted $R^2$ series get increasingly distant from each other for increasingly excess volatile portfolios with the exception of portfolios P2, P7 and P9 that, despite the increasing excess volatility of their stocks, produce adjusted $R^2$ akin to one another. The new four-factors asset-pricing model outperforms the Carhart one in four out of six cases. Nevertheless, once more the variation between the performances is quite limited. The following graphs (Figure 5 and 6) plot the adjusted $R^2$ fluctuations obtained on portfolios P4 and P8.
Figure 5

Note: this graph plots the 4 series of Adjusted $R^2$ fluctuations throughout the rolling-window time period of 109 observations for portfolio P4: one curve for each asset-pricing model.
Figure 6

Note: this graph plots the 4 series of Adjusted $R^2$ fluctuations throughout the rolling-window time period of 109 observations for portfolio P8: one curve for each asset-pricing model.
The results obtained with reference to portfolio P10, which is composed of stocks with the highest excess volatility, are unequalled. The performances of the four asset-pricing models show extreme differences. The two models containing the new $VD_t$ factor outperform the Fama-French model and the Carhart one throughout the entire sub-period. The variance of the adjusted $R^2$ is the least for all asset-pricing models if compared to that obtained with the other portfolios. The new asset-pricing models maintain a stable adjusted $R^2$ of approximately 0.98 while the Fama-French and the Carhart models reach a maximum adjusted $R^2$ of 0.2 and obtain an average of about 0.1. The following graphs in Figure 7 plot the adjusted $R^2$ fluctuations series from portfolio P10.

**Figure 7**

Note: this graph plots the 4 series of Adjusted $R^2$ fluctuations throughout the rolling-window time period of 109 observations for portfolio P10: one curve for each asset-pricing model.

Up to this point the static analysis and the rolling-window one are generally consistent to each other. The adjusted $R^2$ of the fully specified model are in general slightly superior to those obtained with the other analysed asset-pricing models. Nevertheless, introducing a variance difference based explanatory factor is of greatest advantage mainly when employed to portfolios of higher than average excess volatility. At last, consistently with the static analysis, the rolling-window time-series analysis confirmed the poor performance of the Fama-French model and of the Carhart model when
employed to describe the returns of portfolio P10. This is easily interpreted given the high correlation of the returns on portfolio P10 with those of the $VD_t$ as reported in Table 1.

The volatilities of the adjusted $R^2$ are closely related in most of the considered cases. In all of the adjusted $R^2$ series a common pattern is detected between the 15th and the 40th observation of the rolling-window subsample. When compared to the portfolios’ original return series, the common fall in explanatory power detected approximately between the 15th and the 40th observations corresponds to a period of higher volatility common to all ten portfolios approximately between observations 120 and 150. The asset-pricing models that include the $VD_t$ factor, if compared to the Fama-French and to the Carhart models, maintain a higher explanatory power also during periods of higher volatility in the portfolio returns. However, they fail to fully explain the increased volatility and hence follow a common decay in explanatory power.

In contrast, for portfolio P10 the adjusted $R^2$ series obtained from the different asset-pricing models manifest an opposite behaviour throughout the entire sub-period. The adjusted $R^2$ of the asset-pricing models including $VD_t$ falls when the Fama-French and Carhart models gain explanatory power, and vice versa. The extreme excess volatility character of the returns on portfolio P10 seems to have revealed a competing relation between the different asset-pricing models. However, the different scale of the opposite movements in the adjusted $R^2$ series also reveals a certain degree of complementarity between the explanatory factors. Looking at the fluctuations between observation 80 and 90 (Figure 7) in fact, an approximate 100% increase in the cumulative explanatory power of the Fama-French and Carhart factors corresponds to a mere 0.6% decrease in that of the new asset-pricing models that include $VD_t$. 
Table 4

Note: this table reports each portfolio’s average adjusted $R^2$ and volatility of the adjusted $R^2$ obtained from the employment of the Fama-French (i), the Fama-French + $VD_t$ (ii), the Carhart (iii) and the New Five-factors asset-pricing models (iv).

<table>
<thead>
<tr>
<th>Model</th>
<th>Portfolios</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Mean AdjR2</td>
<td>0.8733</td>
<td>0.7537</td>
<td>0.8453</td>
<td>0.8051</td>
<td>0.796</td>
<td>0.7704</td>
<td>0.8131</td>
<td>0.5942</td>
<td>0.7728</td>
<td>0.0714</td>
</tr>
<tr>
<td>Std.Dev AdjR2</td>
<td>0.0265</td>
<td>0.0327</td>
<td>0.0307</td>
<td>0.0379</td>
<td>0.0381</td>
<td>0.0516</td>
<td>0.051</td>
<td>0.0801</td>
<td>0.068</td>
<td>0.0233</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Mean AdjR2</td>
<td>0.8722</td>
<td>0.7577</td>
<td>0.8445</td>
<td>0.8327</td>
<td>0.841</td>
<td>0.8316</td>
<td>0.8184</td>
<td>0.8023</td>
<td>0.7832</td>
<td>0.9824</td>
</tr>
<tr>
<td>Std.Dev AdjR2</td>
<td>0.0268</td>
<td>0.0316</td>
<td>0.0308</td>
<td>0.0283</td>
<td>0.0256</td>
<td>0.0315</td>
<td>0.0496</td>
<td>0.0329</td>
<td>0.065</td>
<td>0.0019</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>Mean AdjR2</td>
<td>0.8789</td>
<td>0.7622</td>
<td>0.8575</td>
<td>0.822</td>
<td>0.8117</td>
<td>0.786</td>
<td>0.8345</td>
<td>0.6156</td>
<td>0.7991</td>
<td>0.0968</td>
</tr>
<tr>
<td>Std.Dev AdjR2</td>
<td>0.0238</td>
<td>0.0412</td>
<td>0.029</td>
<td>0.0378</td>
<td>0.0407</td>
<td>0.053</td>
<td>0.0446</td>
<td>0.0768</td>
<td>0.0492</td>
<td>0.0237</td>
<td></td>
</tr>
<tr>
<td>(iv)</td>
<td>Mean AdjR2</td>
<td>0.8779</td>
<td>0.766</td>
<td>0.8566</td>
<td>0.8482</td>
<td>0.8551</td>
<td>0.8453</td>
<td>0.839</td>
<td>0.82</td>
<td>0.8081</td>
<td>0.9827</td>
</tr>
<tr>
<td>Std.Dev AdjR2</td>
<td>0.024</td>
<td>0.04</td>
<td>0.0292</td>
<td>0.0297</td>
<td>0.03</td>
<td>0.0345</td>
<td>0.0438</td>
<td>0.0286</td>
<td>0.047</td>
<td>0.0017</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 reports the average and the volatility of the adjusted $R^2$ series of each asset-pricing model on the ten portfolios of ascending excess volatility. In terms of average adjusted $R^2$, the introduction of the $VD_t$ factor offers interesting improvements only starting from portfolio P4. In particular, the new five-factors model improves the adjusted $R^2$ obtained with the Carhart model by about 3.2% on portfolio P4, 5.35% on portfolio P5, 7.54% on portfolio P6, 33.2% on portfolio P8, 1.12% on portfolio P9 and 915.2% on portfolio P10. On the rest of the portfolios the percentage change is only of some basis points both in negative and in positive terms. Similar improvements are obtained with respect to the volatility of the adjusted $R^2$ through time. Most importantly $VD_t$, when insignificant, does not substantially hamper the explanatory power of the asset-pricing models. When its presence is significant instead, it substantially improves the explanatory power of the asset-pricing models. In conclusion, the new five-factors asset-pricing model contributes at achieving high and stable through time adjusted $R^2$ from portfolios of low excess volatility to portfolios of high excess volatility.

4.3.2 Time-Varying Significance of the Explanatory Factors

Fluctuations in the significance of the five explanatory factors are reported in a series of graphs, plotting the factors’ $P$-value over time. In any of the reported observations, the significant factors are those having a $P$-value below 0.05. Hence, all plots are divided into two distinct areas: a non-significant area and a significant area located respectively above and below the 0.05 line. The series consists of 40 graphs, 4 for each portfolio (10 for each asset-pricing model). This section presents all the empirical results obtained
from the analysis, however, it reports only the 4 most essential graphs out of a total of 40. For a visual inspection of the other graphs, see Appendix C.

Factors $HML_t$ and $MoM_t$ are those most frequently insignificant. In particular, $HML_t$ is the only factor not remaining significant throughout the entire sub-period in any of the ten portfolios and with none of the asset-pricing models employed. Figure 8 reports the time-varying statistical significance of the explanatory factors included in the new five-factors asset-pricing model when employed to explain the returns of portfolio P1.

**Figure 8**

Note: this graph plots the 5 series of $P$-value fluctuations throughout the rolling-window time period of 100 observations for portfolio P1: one curve for each factor’s time-varying statistical significance.

Factor $VD_t$ is insignificant throughout the entire sub-period in the first three portfolios. This result is consistent with the output from the static time-series analysis and with those derived at the previous stage of the rolling-window analysis. However, for the remaining portfolios $VD_t$ is persistently significant in both of the new asset-pricing models employing it. More specifically, $VD_t$ is significant over the entire sub-period in portfolios P4, P5, P6, P8, and P10 both in the new four-factors and in the new five-factors asset-pricing models.
With respect to the returns of portfolio P7 and P9, $VD_t$ is significant in approximately 90 of the 109 observations of the sub-sample. In Figure 9 is plotted the time-varying statistical significance of the explanatory factors included in the new five-factors asset-pricing model when employed to explain the returns of portfolio P6. As previously mentioned, $MoM_t$ and $HML_t$ are the most unstable factors.

**Figure 9**

Note: this graph plots the 5 series of $P$-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P6: one curve for each factor's time-varying statistical significance.

All asset-pricing models not including $VD_t$, when employed to explain the returns of portfolio P10, seems to be affected by specification errors. The rolling-window analysis on portfolio P10 reveals that all the explanatory factors included in the Carhart model are statistically insignificant. The persistent insignificance of the Carhart factors explains the severe residual standard errors obtained in the static time-series analysis. The Fama-French and the Carhart models produced poor predicted values and large residuals that, in turn, inflated the numerators of the standard errors of the regression coefficients. These large standard errors, when used to compute the $t$-statistics indicated the insignificance of regressors’ coefficients. The
introduction of $VD_t$ captured most of the variation left unexplained by the previous models and produced valid predictions. The important reduction of the sum of squared residuals resulting from the introduction of $VD_t$ finally revealed more familiar significance patterns in the other factors. Figure 10 and 11 plot the time varying statistical significance of the explanatory factors included in the Carhart and in the new five-factors asset-pricing model when employed to explain the returns of portfolio P10. As it may be noticed from the graphs, the introduction of $VD_t$ brings all other factors into the area of significance and the greater instability of the $MoM_t$ and $HML_t$ parameters becomes comparable to that found with respect to the other portfolios.

Figure 10

Note: this graph plots the 4 series of $P$-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P10: one curve for each factor’s time-varying statistical significance.
Figure 11

Note: this graph plots the 5 series of P-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P10: one curve for each factor's time-varying statistical significance.
At this juncture, $VD_t$ seems to act as a suppressor variable or a moderator variable. In particular the inclusion of $VD_t$ seems to increase the predictive ability of the other factors and seems to strengthen the relation between the other factors and the dependent variable. However, according to the definition given by P. Horst (1941), a suppressor is a variable that, despite its uncorrelation with the dependent variable, increases the predictive validity of the other factors by suppressing their response-related irrelevant variance. Recalling the high correlation between $VD_t$ and the excess returns on portfolio P10, as reported in the cross-correlation section of Table 1, $VD_t$ does not strictly respect all the features related to suppressor variables.

By definition moderator variables are those affecting the strength and/or the direction of the relation between a dependent and an independent variable having zero-order correlation. The excess return series of portfolio P10 has the following correlations: 0.347 with ($RM_t - RF_t$), 0.2727 with $SMB_t$, -0.0974 with $HML_t$, -0.1359 with $MoM_t$ and 0.9565 with $VD_t$. Therefore, the zero-order correlation assumption for moderator variables is not respected.

In light of these results, I believe the higher predictive validity of $VD_t$ and hence its ability to reduce the sum of squared residuals, to explain the important developments in the factors’ series reported in Figures 10 and 11. In conclusion, for excess volatilities levels above that of portfolio P3 $VD_t$ has only moderate P-value fluctuations mainly in the area of significance. Furthermore, for portfolios of high excess volatility, the best specified models are only those that include the $VD_t$ factor.

### 4.3.3 Time-Varying Significance of Pricing Errors

Fluctuations in the significance of the pricing errors are reported in a series of graphs plotting the pricing errors’ P-value over time. In any of the reported observations, the significant pricing errors are those having a P-value below 0.05. Hence, all plots maintain the same logic as those pertaining to the previous paragraph. The series consists of 10 graphs, 1 for each portfolio. This section presents all the empirical results obtained from the analysis, however, it reports only the 3 most essential graphs out of a total of 10. For a visual inspection of the other graphs, see Appendix D.

Building on the results obtained with the analysis of the time-varying significance of the explanatory factors, fluctuations in the significance of the pricing errors from portfolio P10 are evaluated only with reference to the new asset-pricing models. The Fama-French and the Carhart models are excluded from the analysis of portfolio P10 because their factors are insignificant.
approximately throughout the entire rolling-window period; hence the regression model would be erroneously specified.

All the asset-pricing models have statistically significant pricing errors for the entire time period in portfolios P1, P9 and P10. This result is consistent with the distribution of the $t$-statistics derived with the static time-series analysis. However, with the remaining portfolios all asset-pricing models perform relatively well. Pricing errors are insignificant for most of the time with the majority of the portfolios. Figure 12 reports the fluctuations in pricing errors' significance for portfolio P5.

**Figure 12**

Note: this graph plots the 4 series of $P$-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P5: one curve for the time-varying statistical significance of each asset-pricing model's pricing error.

The series obtained from the different asset-pricing models are generally highly correlated. In addition, the common falls to the area of significance are related to the common falls in the models explanatory power (adjusted $R^2$). As it may be noticed from the plots in fact, the time varying adjusted $R^2$ series and the pricing errors significance series have similar dips around observation 30. Nevertheless, in all portfolios between and including P2 and
P8, the new asset-pricing models are persistently deeper in the area of insignificance if compared to the Fama-French and Carhart models. Therefore, the new models’ errors are more frequently insignificant than those obtained with the traditional models, as it can be seen from Figure 13 and 14, plotting the results obtained with portfolio P7 and P8.

**Figure 13**

Note: this graph plots the 4 series of P-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P7: one curve for the time-varying statistical significance of each asset-pricing model’s pricing error.
Note: this graph plots the 4 series of $P$-value fluctuations throughout the rolling-window time period of 109 observations for portfolio P8: one curve for the time-varying statistical significance of each asset-pricing model's pricing error.
Being the pricing errors not statistically insignificant at all times, it is relevant to evaluate if introducing factor $V_D_t$ contributes also at reducing the average size of the pricing errors. According to my results, the mean pricing errors from the Fama-French and from the Carhart models are persistently bigger than those from the new five-factors model except for portfolio P1. In its first row, Table 5 reports the percentage difference between the Fama-French mean pricing errors and the new five-factors model ones. In its second row instead, the same percentage difference is calculated between the Carhart model and new five-factors model. The results for portfolio P10 are in brackets because the two traditional models suffer from misspecification.

### Table 5

Note: this table reports, for each portfolio, the percentage difference between the mean pricing error obtained from the Fama-French model and the new Five-factors model (first row), and the percentage difference between the mean pricing error obtained from the Carhart model and the new Five-factors model (second row). The last results are in brackets because the Fama-French and the Carhart models, when employed with portfolio P10, suffer from misspecification.

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Five Factors to Fama &amp; French Model</strong></td>
<td>-3.87%</td>
<td>-16.20%</td>
<td>-25.13%</td>
<td>-70.85%</td>
<td>-83.02%</td>
<td>-215.88%</td>
<td>-19.63%</td>
<td>-110.73%</td>
<td>-15.94%</td>
<td>(-337.46%)</td>
</tr>
<tr>
<td><strong>Five Factors to Carhart Model</strong></td>
<td>0.21%</td>
<td>-12.83%</td>
<td>-6.41%</td>
<td>-49.32%</td>
<td>-63.16%</td>
<td>-168.98%</td>
<td>-9.40%</td>
<td>-91.66%</td>
<td>-7.81%</td>
<td>(-324.5%)</td>
</tr>
</tbody>
</table>

In summary, according to the above results, introducing $V_D_t$ into the traditional Fama-French and Carhart asset-pricing models notably reduces the average pricing errors and sustains their statistical insignificance through time. Coherently with the results from the static time-series analysis $V_D_t$ has limited effect on the insignificance of the pricing errors for portfolios situated at both extremes of the excess volatility ranking. The reason for which $V_D_t$ has most of its effect on median portfolios instead of portfolios of higher excess volatility is yet to be explained. Nevertheless, the effectiveness of $V_D_t$ in a wide range of excess volatility levels, promotes the importance of its use.
4.3.4 Analysis of the Prediction Errors

This section presents all the empirical results obtained from the analysis, however, it reports only the 3 most essential graphs out of a total of 40. For a visual inspection of the other graphs, see Appendix E.

Prediction errors arise both from unknown future values of the residuals $U_i$, and from errors in the estimation of the regressions’ coefficients. Comparing the RMSE of the predictions computed with different asset-pricing models permits to compare the different magnitudes of the average prediction mistake obtained from their employment. However, this comparison doesn’t provide any information regarding the nature of the error made in the prediction. Theil’s (1964, 1966) Prediction Realization Diagram (PRD) reveals the nature of the prediction error according to the position that the error takes in the plot. The following two PRDs plot the prediction errors from portfolio P8 obtained from the employment of the Fama-French model in Figure 15, and of the new five-factors model in Figure 16. PRDs are constructed as scatter plots of the realized returns against the predicted returns. In case of perfect predictions with zero errors the PRD distributes all observations on the horizontal line at the centre of the plot. The horizontal line and the two diagonal lines divide the plot into six sectors. Moving clockwise starting from the upper-left sector the following is the logic of the PRD diagnostic: observations located in the,

- upper-left sector correspond to underestimated decreases in earnings;
- upper-central sector are erroneously predicted increases in earnings when they, in fact, decreased;
- upper-right sector correspond to overestimated increases in earnings;
- lower-right sector correspond to underestimated increases in earnings;
- lower-central sector are erroneously predicted decreases in earnings when they, in fact, increased;
- lower-left sector.

In the following two graphs I noticed that most of the prediction errors consisted in overestimations and underestimations of increases and decreases in earnings. In Figure 15 only 14 observations out of 108 are located in the upper-central and lower-central sectors. By comparing the two graphs it can be noted how the five-factors model, in this case, slightly improved the precision of the prediction. In particular, observations 2, 3, 29 and 46, with the Fama-French model were situated far from the horizontal line and deep in the sectors accounting for predictions of erroneous direction. With the new five-factors asset-pricing model instead these observations get closer to the rest of the group.
Note: This Prediction Realization Diagram plots the 108 rolling one-month-ahead return predictions for portfolio P8, obtained with the Fama-French model.
From the analysis of all 40 graphs I was able to notice several common features in the prediction errors. In the great majority of the cases the prediction errors consisted in underestimations and overestimations of otherwise correctly predicted movements in future returns. In other words, the asset-pricing models rarely predicted opposite movements in future returns. The errors’ distributions are mostly unaffected by a change in asset-pricing model. Hence, switching asset-pricing model doesn’t cause major changes in the nature of the errors. For all portfolios and with all the asset-pricing models, the worst predictions were those between the 30th and the 40th observations. The location of the worst performance is coherent with the rest of the rolling-window analysis both in terms of time-varying adjusted $R^2$ and of the insignificance of pricing errors. The Carhart model and both the new asset-pricing models produce predictions closer to the horizontal line than those produced by the Fama-French model. However, their greater precision
was not sufficient to indicate with certainty which of them was performing the best. At last, the best prediction was that obtained with the new five-factors model for portfolio P10 as reported in Figure 17.

**Figure 17**

Note: This Prediction Realization Diagram plots the 108 rolling one-month-ahead return predictions for portfolio P10, obtained with the New Five-Factors model.

After having evaluated the nature of the errors for each model and at all levels of excess volatility, as a second step towards the identification of each model's prediction performance I computed and compared each models' RMSE. In its first four rows the following table (Table 6) reports the RMSE obtained with the different models for each portfolio. In its last four rows instead it reports the RMSE percentage differences from the most relevant models pairwise comparisons. The RMSEs from portfolio P10 for the Fama-French and for the Carhart models are reported in brackets because of the models' misspecification.

Keeping the Fama-French as the benchmark model, the new five-factors model has persistently lower RMSE in all portfolios except P1 and P9. For portfolios P2 and P8 the new model's advantage is particularly important.
The RMSE reduction obtained with the Carhart model is less important with regard to its percentage extent but more stable as it maintains smaller RMSE in all portfolios with the only exception of portfolio P1. The new four-factors model is performing exactly as the Fama-French model in the first two portfolios. This is probably due to the insignificance of $VD_t$ in portfolios of low excess volatility stocks. In the rest of the portfolios it obtains RMSE reductions only in 4 cases out of 7.

**Table 6**

Note: This table reports the Root Mean Square Errors of the one-month-ahead rolling predictions from month 109 to month 216 relative to the Fama-French (i), the Fama-French + $VD_t$ (ii), the Carhart (iii) and the New Five-factors model (iv). The asset-pricing models’ coefficients have an estimation period consisting of the 9 years monthly observations preceding each prediction.

<table>
<thead>
<tr>
<th>Model</th>
<th>Portfolios</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) RMSE</td>
<td>0.0182</td>
<td>0.0285</td>
<td>0.0195</td>
<td>0.0211</td>
<td>0.023</td>
<td>0.0241</td>
<td>0.0232</td>
<td>0.0339</td>
<td>0.028</td>
<td>(0.1494)</td>
<td></td>
</tr>
<tr>
<td>(ii) RMSE</td>
<td>0.0182</td>
<td>0.0285</td>
<td>0.0197</td>
<td>0.021</td>
<td>0.022</td>
<td>0.0232</td>
<td>0.0234</td>
<td>0.0281</td>
<td>0.0291</td>
<td>0.0182</td>
<td></td>
</tr>
<tr>
<td>(iii) RMSE</td>
<td>0.0183</td>
<td>0.0186</td>
<td>0.0191</td>
<td>0.0206</td>
<td>0.0224</td>
<td>0.0233</td>
<td>0.0224</td>
<td>0.0326</td>
<td>0.0274</td>
<td>(0.1498)</td>
<td></td>
</tr>
<tr>
<td>(iv) RMSE</td>
<td>0.0183</td>
<td>0.0172</td>
<td>0.0193</td>
<td>0.0206</td>
<td>0.0215</td>
<td>0.0227</td>
<td>0.0227</td>
<td>0.0273</td>
<td>0.0287</td>
<td>0.0183</td>
<td></td>
</tr>
</tbody>
</table>

| Five Factors to Fama & French Model | 0.55% | -65.70% | -1.04% | -2.43% | -6.98% | -6.17% | -2.20% | -24.18% | 2.44% | (-716%) |
| Five Factors to Carhart Model       | 0.00% | -8.14% | 1.04% | 0.00% | -4.19% | -2.64% | 1.32% | -19.41% | 4.53% | (-718%) |
| Carhart to Fama & French Model      | 0.55% | -53.23% | -2.09% | -2.43% | -2.68% | -3.43% | -3.57% | -3.99% | -2.19% | (0.267%) |
| New Four Factors to Fama & French Model | 0.00% | 0.00% | 1.02% | -0.48% | -4.55% | -3.88% | 0.85% | -20.64% | 3.78% | (-720.87%) |

Keeping the Carhart model as a benchmark, the new five factors model performs better in 4 portfolios out of 9. It performs as well as the Carhart model in portfolios P1 and P4, and it performs slightly worse in portfolios P3, P7 and P9.

The Fama-French and the Carhart models are misspecified and unable to produce predictions for portfolios of extremely high excess-volatility. Furthermore, the degree by which the new five-factors model outperforms the Carhart one is much more important than that by which, in certain portfolios, it underperforms it. Hence, $VD_t$ showed a positive role also in the amelioration of the quality of stocks’ returns predictions.
5. Diagnostics

In this section I check the robustness of the different asset-pricing models specifications. The diagnosis is oriented at evaluating the behaviour of the residuals for each pricing model and for each portfolio. The exact results of each test are reported in Table 11 (Appendix F).

Employing $V_D_t$ ameliorates the models fit and hence it significantly reduces the standard deviation of the residuals. Table 7 reports the percentage difference between the residuals’ standard deviation obtained from the new five-factors model to that derived from the Fama-French, the new four-factors, and Carhart models. Furthermore, according to the Jarque-Bera test for normal skewness in the residuals’ distribution, the new five-factors model produces symmetrically distributed residuals in 8 portfolios out of 10. The same test for the other asset-pricing models revealed symmetric distributions in 6 portfolios out of 10 with the Carhart model and only in 5 portfolios out of 10 with the Fama-French and the new four-factors models.

**Table 7**

Note: this table reports the percentage difference in the residuals’ standard deviation between the new five-factors asset-pricing model and the Fama-French model, the new four-factors model, and the Carhart model in the second, third and last column respectively. Results are reported for all of the ascending past-12-months excess volatility portfolios as indicated in column 1.

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Five Factors to Fama-French model</th>
<th>Five Factors to New Four Factors model</th>
<th>Five Factors to Carhart model</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>-3.962%</td>
<td>-4.185%</td>
<td>-0.670%</td>
</tr>
<tr>
<td>P2</td>
<td>-1.593%</td>
<td>-1.035%</td>
<td>-0.757%</td>
</tr>
<tr>
<td>P3</td>
<td>-4.770%</td>
<td>-4.827%</td>
<td>0.114%</td>
</tr>
<tr>
<td>P4</td>
<td>-10.370%</td>
<td>-5.240%</td>
<td>-4.964%</td>
</tr>
<tr>
<td>P5</td>
<td>-12.912%</td>
<td>-4.916%</td>
<td>-7.942%</td>
</tr>
<tr>
<td>P6</td>
<td>-16.242%</td>
<td>-5.347%</td>
<td>-10.795%</td>
</tr>
<tr>
<td>P7</td>
<td>-9.466%</td>
<td>-8.201%</td>
<td>-1.125%</td>
</tr>
<tr>
<td>P8</td>
<td>-40.978%</td>
<td>-8.732%</td>
<td>-33.551%</td>
</tr>
<tr>
<td>P9</td>
<td>-11.797%</td>
<td>-9.759%</td>
<td>-2.625%</td>
</tr>
<tr>
<td>P10</td>
<td>-770.575%</td>
<td>-3.263%</td>
<td>-770.022%</td>
</tr>
</tbody>
</table>

However, beyond an important reduction in the overall standard deviation of the residuals and an improvement in the symmetry of their distributions, the employment of $V_D_t$ does not provide any further amelioration of the residuals’
“behaviour” if compared to that obtained with the Carhart model. The Ljung-Box Q-test, the Jarque-Bera test and the Breusch-Pagan test reveal that the new five-factors and the Carhart models have the same cases of autocorrelation in the residuals (in 1 portfolio out of 10), of high kurtosis in the residuals’ distribution (residuals have higher than normal kurtosis in all of the 10 portfolios) and of heteroscedasticity in the residuals (in 5 portfolios out of 10).
6. Critical Reflection, Conclusion and Contributions

According to Fama and French (2004), a theoretical weak point of the CAPM consists in the unrealistic assumptions for which the linear relation between the expected return of an asset and its portfolio $\beta$ provide a complete and exclusive measurement of risk and return. Despite academics' disagreement whether or not the existing empirical tests are correctly designed to perform a true test of the validity of the CAPM, in the last decades, research has produced mounting evidence of different asset-pricing anomalies. Most studies' common conclusion was that specific systematic risks were not linked to the usual market factor but were instead captured by price-based factors. Price-based factors, as those introduced by Fama and French (1992, 1993) and by Carhart (1997), have the advantage of avoiding the complex challenge of providing a coherent economic theory behind their effect. The new asset-pricing factors in fact were declared having only an indirect role in reflecting the effect of unidentified economic factors.

The pricing anomaly identified by Wang and Ma (2014) consisted in the incapability of the Fama-French and Carhart asset-pricing models to fully explain the returns on portfolios constructed according to the variance difference investment strategy. Coherently with the evolution of the literature, Wang and Ma’s suggestion for future research was to employ Lo and Mackinlay’s (1988) variance difference to build an excess-volatility-related asset-pricing factor. Accordingly to their suggestion, the objective of this research is to evaluate the asset-pricing contribution of the inclusion of an excess-volatility based factor ($VD_t$) into the Fama-French and Carhart asset-pricing models.

The empirical method by which I evaluated whether or not $VD_t$ captures common factors in stock returns related to excess volatility, and hence overreaction, that are left unexplained by the available asset-pricing models, is based on Fama and French (1993). In addition, it is expanded to the consideration of the time-varying nature of the asset-pricing models' performance, and of their predictive capabilities. This adaptation is particularly relevant now that the Great Moderation of the busyness cycles seems to have come to an end.

On the one hand, our findings support the positive role of $VD_t$ as an asset-pricing factor. First, from the static time-series analysis and from the analysis-of-variance $F$-ratio test we obtained statistically significant proof that $VD_t$ contributes to breaking the link between the decreasing performance of the asset-pricing models and the increasing excess volatility of the analysed portfolios.
Secondly, $V D_t$ was found significant in the majority of the cases, and its inclusion into the Carhart model permitted to obtain a number of insignificant pricing errors when all other asset-pricing models had only significant pricing errors. Thirdly, the pricing errors’ $t$-statistics obtained with the new five-factors model are most frequently closer to the area of insignificance than those obtained with the Fama-French and Carhart ones. Finally, the adjusted $R^2$ was found to be most stable across ascending past-12-months excess volatility portfolios with the new five-factors model.

From the cross-section analysis, the new five-factors model had the smallest GRS $F$-statistic, thus suggesting some degree of greater importance of $V D_t$ in explaining the cross-section of portfolios’ excess returns.

The results from the rolling-window analysis were mostly coherent with those obtained from the static one. First, $V D_t$ improved the extent and the stability of the explanatory power of the asset-pricing models. Secondly, the significance of $V D_t$ is one of the most stable through time if compared to that of the other explanatory factors. Thirdly, for assets pertaining to the highest decile of excess volatility, the Carhart factors are significant only in the presence of $V D_t$. Fourthly, $V D_t$ reduces the average pricing errors and sustains their statistical insignificance through time for a wide range of excess volatility levels. Finally, $V D_t$ showed a relevant role also in the reduction of the RMSE of the predictions obtained with the different asset-pricing models.

On the other hand, the results from the diagnostics tests showed that the juxtaposition of $V D_t$ next to the Carhart factors provides only two advantages in terms of well behaviour of the residuals: a reduction in the residuals’ standard deviation, and a normalization of the skewness of their distributions. $V D_t$ was not able to correct for the few remaining autocorrelations and heteroscedasticities. However, the similarity of the results obtained from the retrospective analysis of past returns with those obtained from the prospective prediction of future returns, supports the importance of the integration of $V D_t$ as an asset-pricing factor. Furthermore, in this context, the more frequent normal skewness in the residuals obtained from the employment of $V D_t$, means that the two new asset-pricing models have no over-prediction or under-prediction bias. Their residuals, in fact, have mean equal to zero.

In light of the above considerations and results, it can be concluded that the new excess-volatility-based factor $V D_t$ contributes to the improvement of the existing multi-index asset-pricing models’ retrospective and prospective performance.
6.1 Theoretical Contribution

As illustrated by Fama and French (1992), if asset pricing is rational, all explanatory factors must proxy for the exposure to some source of rewarded risk. Hence, their significance is linked to economic variables and phenomena likely to persist unless major changes to the economic system were to happen. If asset-pricing is irrational instead, the likely persistence of the significance of the asset-pricing factors is less certain, because there is no economic variable backing its existence and linking it to the economic system.

However, according to the behavioural finance literature, irrational anomalies are as likely to persist as rational ones. The logic behind this conclusion relies, once more, on the unrealistic assumptions of the CAPM. As explained by Elton et al. (2011), if speculators are credit-constrained, have limits to short assets and face costly transactions, they cannot always take positions that exploit noise traders’ irrationality and would drive assets prices back toward true values. Being that the above-mentioned arbitrage-limiting conditions fairly reflect the real-world conditions faced by investors, smart investors knowledge of asset mispricing is not always exploitable. As a consequence, stock prices stably reflect investors’ conventional valuations, overreactions, emotions, biases and heuristics.

Coherently with the behavioural perspective, Qiu and Welch (2004) found that the UBS/Gallup investor sentiment index had explanatory power over the cross-section of American stocks’ returns. Most importantly, they found that the stocks with the highest correlation to the sentiment index were the same usually avoided by speculators due to their illiquidity. If investors persistently take biased decisions under uncertainty, and informed traders are subject to multiple constraints, investors’ emotions and irrational behaviours become one of the major sources of undiversifiable risk. Hence, the asset-pricing literature should take behavioural risks into consideration.

The new excess volatility factor $V_D_t$ was proven to be linked to investors’ fluctuation in overreaction (for further details see Wang and Ma, 2014), which, in turn, could proxy for sentiment risk. Finally, regarding the likely persistence of investors’ overreaction, Shiller (2013) showed that stock returns series have persistently been excessively volatile since 1871.

In conclusion, previous research proved that behavioural pricing anomalies are persistent through time. This research provides some evidence of the capability of $V_D_t$ to detect such behavioural anomalies. Therefore, it suggests its integration into the existing fundamental multi-index asset-pricing models.
6.2 Practical Contribution

As demonstrated by Elton and Gruber (1988), multi-index models like the four asset-pricing models we have assessed in this research, have an advantage over single-index models in the field of passive management, of active management and of portfolio managers performance evaluation. In particular, passive investment strategies attempting at matching a specific index can now control for the different sensitivity that the fund is being set and the index they are matching have with $VD_t$. For example, looking at Table 10 (Appendix A) we notice that portfolio P1, P8 and P10 all have the same sensitivity of 0.84 with the market risk premium. However, portfolios P1, P8 and P10 have a sensitivity with the excess volatility factor of -0.001, 0.15 and 0.99 respectively. Hence, with regard to their systematic risk and from a single-index perspective, these portfolios equally match the market risk premium. However, in case of unexpected large changes in the returns on $VD_t$, they are going to increasingly deviate from the index they were attempting to match. In summary, considering $VD_t$ permits fund managers to control for their assets’ sensitivity to overreaction anomalies, and ameliorate the quality of their index matching.

Active managers may permit their clients to vary the exposure of their investments to $VD_t$ according to the expected future returns on the factor.

Finally, multi-index models permit to assess which kind of decisions and strategies the fund manager is taking beyond their direct disclosure. For example, portfolio P3 and portfolio P4 have approximately the same volatility of 0.046. However, portfolio P4 is approximately 8 times more sensitive to changes in $VD_t$, thus signalling that the two portfolios have different sources of risk. In particular, the two portfolios have different exposure to fluctuations in investors’ overreaction. In addition, integrating $VD_t$ into the Fama-French and Carhart asset-pricing models provides a corrected estimate of the fund manager’s contribution to the actively managed fund performance (in terms of benchmark indexes underperformance or outperformance). The next graph (Figure 18) plots the Alphas that could have been attributed to a fund manager holding a short position on portfolio P6 according to the four different asset-pricing models.
As it can be seen from the above figure, not only the Fama-French and the Carhart models overestimated the fund manager's contribution, but they also failed to detect that for 3 years (36 monthly observations) the fund manager actually held the wrong position on the asset. He or she kept on holding a short position while the long one was actually outperforming the sum of the benchmark indexes.
7. Limitations

Because of limited computational power, the analysis was performed only on 1929 stocks of the 7101 trading on AMEX, NASDAQ and NYSE throughout the research time period. Therefore, the results derived may be partly sample-specific.

With regard to the static time series and to the cross section analyses, this research replicates the approach of Fama and French (1993). However, the diagnostics chapter highlighted that some of the assumptions of the GRS $F$-test are not respected. As suggested by Cochrane (2005), General Method of Moments can be employed to perform the same test and it is robust against the common features in financial data\(^1\).

\[^1\text{With reference to the data of this research the GMM test produced similar results and sustained the same conclusions. Results from the GMM test on the asset-pricing models' intercepts are available upon request.}\]
8. References


9. Appendices

9.1 Appendix A

Table 8

Note: this table reports the 10 regressions summaries for:
Response variables: monthly excess returns of 10 portfolios of ascending past-12-months excess volatility.
Predictor variables: the three factors from the Fama-French asset-pricing model.
Time period: from April 1997 to March 2015.
Significance levels: $0 \cdot *** \cdot 0.001 \cdot ** \cdot 0.01 \cdot * \cdot 0.05 \cdot \cdot 0.1 \cdot \cdot 1$

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>$b$</th>
<th>$s$</th>
<th>$h$</th>
<th>Coefficients</th>
<th>$b$</th>
<th>$s$</th>
<th>$h$</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>0.885682</td>
<td>0.467306</td>
<td>0.325488</td>
<td></td>
<td>31.065***</td>
<td>12.063***</td>
<td>8.007***</td>
<td></td>
</tr>
<tr>
<td>P2</td>
<td>0.840629</td>
<td>0.476031</td>
<td>0.284143</td>
<td></td>
<td>21.533***</td>
<td>8.974***</td>
<td>5.105***</td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>0.793723</td>
<td>0.489484</td>
<td>0.351866</td>
<td></td>
<td>28.118***</td>
<td>12.762***</td>
<td>8.742***</td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>0.764605</td>
<td>0.491056</td>
<td>0.327572</td>
<td></td>
<td>24.974***</td>
<td>11.805***</td>
<td>7.504***</td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>0.818414</td>
<td>0.451836</td>
<td>0.387477</td>
<td></td>
<td>25.586***</td>
<td>10.396***</td>
<td>8.496***</td>
<td></td>
</tr>
<tr>
<td>P6</td>
<td>0.846422</td>
<td>0.50427</td>
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<td></td>
<td>23.778***</td>
<td>10.426***</td>
<td>8.024***</td>
<td></td>
</tr>
<tr>
<td>P7</td>
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<td>0.469345</td>
<td>0.352866</td>
<td></td>
<td>26.479***</td>
<td>9.661***</td>
<td>6.922***</td>
<td></td>
</tr>
<tr>
<td>P8</td>
<td>0.977391</td>
<td>0.507057</td>
<td>0.238337</td>
<td></td>
<td>16.416***</td>
<td>6.808***</td>
<td>2.807***</td>
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</tr>
<tr>
<td>P9</td>
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<td>P10</td>
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<th>t-statistic</th>
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<td>0.1574</td>
<td>-0.03051</td>
<td>-2.810**</td>
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Table 9

Note: this table reports the 10 regressions summaries for:
Response variables: monthly excess returns of 10 portfolios of ascending past-12-months excess volatility.
Predictor variables: the four factors from the Carhart asset-pricing model.
Time period: from April 1997 to March 2015.
Significance levels: 0 ' *** ' 0.001 ' ** ' 0.01 ' * ' 0.05 ' . ' 0.1 ' 1

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<th>t-statistic</th>
<th>Intercepts</th>
<th>t-statistic</th>
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<td>0.485049</td>
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<td>-0.092721</td>
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<td>12.832 ***</td>
<td>7.391 ***</td>
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<tr>
<td>P2</td>
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<td>0.489756</td>
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<td>9.2420 ***</td>
<td>4.654 ***</td>
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<tr>
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<td>0.510525</td>
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<td>25.942 ***</td>
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<td>8.120 ***</td>
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<td>12.904 ***</td>
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<td>11.356 ***</td>
<td>7.867 ***</td>
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<td>6.209 ***</td>
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Residual Standard Error

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<tr>
<td>P3</td>
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<tr>
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<tr>
<td>P5</td>
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<tr>
<td>P6</td>
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<td>0.02217</td>
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<tr>
<td>P7</td>
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<td>0.02158</td>
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<td>P8</td>
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<td>0.03686</td>
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<tr>
<td>P9</td>
<td>0.7699</td>
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<td>P10</td>
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<td>0.1573</td>
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<td>-2.649 **</td>
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Table 8

Note: this table reports the 10 regressions summaries for:
Response variables: monthly excess returns of 10 portfolios of ascending past-12-months excess volatility.
Predictor variables: the four factors from the New four-factors asset-pricing model consisting of the Fama-French factors + $VD_t$.
Time period: from April 1997 to March 2015.
Significance levels: 0 ‘ *** ’ 0.001 ‘ ** ’ 0.01 ‘ * ’ 0.05 ‘ . ’ 0.1 ‘ 1

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<th>$h$</th>
<th>$v$</th>
<th>$t$-statistic</th>
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<td>0.4677469</td>
<td>0.3253619</td>
<td>-0.0008593</td>
<td>30.942 *** 11.976 *** 7.981 *** -0.105</td>
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<tr>
<td>P2</td>
<td>0.836379</td>
<td>0.465701</td>
<td>0.287107</td>
<td>0.02014</td>
<td>21.499 *** 8.7750 *** 5.183 *** 1.807</td>
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<tr>
<td>P3</td>
<td>0.792274</td>
<td>0.485962</td>
<td>0.352876</td>
<td>0.006865</td>
<td>27.997 *** 12.589 *** 8.758 *** 0.847</td>
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<tr>
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<td>0.756299</td>
<td>0.470868</td>
<td>0.333365</td>
<td>0.039358</td>
<td>25.853 *** 11.799 *** 8.003 *** 4.697 ***</td>
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<td>0.426078</td>
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<tr>
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<td>19.622 *** 6.871 *** 2.975 ** 3.001 **</td>
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<tr>
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<td>P4</td>
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<td>P5</td>
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<td>0.01942</td>
<td>-0.003274 -2.422 *</td>
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Table 10

Note: this table reports the 10 regressions summaries for:
Response variables: monthly excess returns of 10 portfolios of ascending past-12-months excess volatility.
Predictor variables: the five factors from the New five-factors asset-pricing model.
Time period: from April 1997 to March 2015.
Significance levels: 0 ' *** ' 0.001 ' ** ' 0.01 ' * ' 0.05 ' . 0.1 ' a 1

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9.2 Appendix B

Figure 19: time varying adjusted $R^2$ series for portfolio P1.
Figure 20: time varying adjusted $R^2$ series for portfolio P2.
Figure 21: time varying adjusted $R^2$ series for portfolio P5.
Figure 22: time varying adjusted $R^2$ series for portfolio P6.
Figure 23: time varying adjusted $R^2$ series for portfolio P7.
Figure 24: time varying adjusted $R^2$ series for portfolio P9.
9.3 Appendix C
Explanatory factors time-varying significance graphs series.

Figure 25: Carhart model on portfolio P1.
Figure 26: Fama-French model on portfolio P1.
Figure 27: New four-factors model on portfolio P1.
Figure 28: Carhart model on portfolio P2.
Figure 29: New five-factors model on portfolio P2.
Figure 30: Fama-French model on portfolio P2.
Figure 31: New four-factors model on portfolio P2.
Figure 32: Carhart model on portfolio P3.
Figure 33: New five-factors model on portfolio P3.
Figure 34: Fama-French model on portfolio P3.
Figure 35: New four-factors model on portfolio P3.
Figure 36: Carhart model on portfolio P4.
Figure 37: New five-factors model on portfolio P4.
Figure 38: Fama-French model on portfolio P4.
Figure 39: New four-factors model on portfolio P4.
Figure 40: Carhart model on portfolio P5.
Figure 41: New five-factors model on portfolio P5.
Figure 42: Fama-French model on portfolio P5.
Figure 43: New four-factors model on portfolio P5.
Figure 44: Carhart model on portfolio P6.
Figure 45: Fama-French model on portfolio P6.
Figure 46: New four-factors model on portfolio P6.
Figure 47: Carhart model on portfolio P7.
Figure 48: New five-factors model on portfolio P7.
Figure 49: Fama-French model on portfolio P7.
Figure 50: New four-factors model on portfolio P7.
Figure 51: Carhart model on portfolio P8.
Figure 52: New five-factors model on portfolio P8.
Figure 53: Fama-French model on portfolio P8.
Figure 54: New four-factors model on portfolio P8.
Figure 55: Carhart model on portfolio P9.
Figure 56: New five-factors model on portfolio P9.
Figure 57: Fama-French model on portfolio P9.
Figure 58: New four-factors model on portfolio P9.
Figure 59: Fama-French model on portfolio P10.
Figure 60: New four-factors model on portfolio P10.
9.4 Appendix D

Figure 61: time varying significance of pricing errors for portfolio P1.
Figure 62: time varying significance of pricing errors for portfolio P2.
Figure 63: time varying significance of pricing errors for portfolio P3.
Figure 64: time varying significance of pricing errors for portfolio P4.
Figure 65: time varying significance of pricing errors for portfolio P6.
Figure 66: time varying significance of pricing errors for portfolio P9.
Figure 67: time varying significance of pricing errors for portfolio P10.
9.5 Appendix E
Prediction Errors Graphs Series.

Figure 68: Carhart model on portfolio P1.
Figure 69: New five-factors model on portfolio P1.
Figure 70: Fama-French model on portfolio P1.
Figure 71: New four-factors model on portfolio P1.
Figure 72: Carhart model on portfolio P2.
Figure 73: New five-factors model on portfolio P2.
Figure 74: Fama-French model on portfolio P2.
Figure 75: New four-factors model on portfolio P2.
Figure 76: Carhart model on portfolio P3.
Figure 77: New five-factors model on portfolio P3.
Figure 78: Fama-French model on portfolio P3.
Figure 79: New four-factors model on portfolio P3.
Figure 80: Carhart model on portfolio P4.
Figure 81: New five-factors model on portfolio P4.
Figure 82: Fama-French model on portfolio P4.
Figure 83: New four-factors model on portfolio P4.
Figure 84: Carhart model on portfolio P5.
Figure 85: New five-factors model on portfolio P5.

Figure 86: Fama-French model on portfolio P5.
Figure 87: Carhart model on portfolio P5.
Figure 88: Carhart model on portfolio P6.
Figure 89: New five-factors model on portfolio P6.
Figure 90: Fama-French model on portfolio P6.
Figure 91: New four-factors model on portfolio P6.
Figure 92: Carhart model on portfolio P7.
Figure 93: New five-factors model on portfolio P7.
Figure 94: Fama-French model on portfolio P7.
Figure 95: New four-factors model on portfolio P7.
Figure 96: Carhart model on portfolio P8.
Figure 97: New four-factors model on portfolio P8.
Figure 98: Carhart model on portfolio P9.
Figure 99: New five-factors model on portfolio P9.
Figure 100: Fama-French model on portfolio P9.
Figure 101: New four-factors model on portfolio P9.
Figure 102: New four-factors model on portfolio P10.
Figure 103: Carhart model on portfolio P10.
Figure 104: Fama-French on portfolio P10.
### Table 11

Note: this table reports the results from 4 different robustness tests on the asset-pricing models’ residuals. Each test is reported according to its rejection or non-rejection of the tests’ null hypotheses (“ x ” = non rejection of the null hypothesis, “ - ” = rejection of the null hypothesis) and the relative \( P \)-value. The tests are performed on each asset-pricing model. The bold characters and the selected areas of the table highlight the best performing models in asset-pricing terms.

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<th>( P )-value</th>
<th>( P )-value</th>
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<td>Fama &amp; French</td>
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<td>-</td>
<td>x</td>
<td>x</td>
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<tr>
<td></td>
<td>Fama &amp; French + VDt (New Four-factors)</td>
<td>Test Result</td>
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<td>x</td>
<td>-</td>
<td>-</td>
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<td>Jarque-Bera test for Normal Skewness in the Residuals</td>
<td>Fama &amp; French</td>
<td>Test Result</td>
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<td>x</td>
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<tr>
<td></td>
<td>Fama &amp; French + VDt (New Four-factors)</td>
<td>Test Result</td>
<td>x</td>
<td>x</td>
<td>-</td>
<td>-</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Fully Specified (New Five Factors)</td>
<td>Test Result</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<td>Breusch-Pagan test for Conditional Heteroscedasticity in the Residuals</td>
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<td>Fama &amp; French + VDt (New Four-factors)</td>
<td>Test Result</td>
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<td>Carhart</td>
<td>Test Result</td>
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<td>Test Result</td>
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