System Dynamics vs. agent-based modeling—comparing models and approaches: A literature review and a transformation procedure

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Abstract

Systems modeling and simulation methods such as System Dynamics (SD) and agent-based (AB) modeling have been used to foster a better understanding of the dynamics and complexity of natural, technical, and social systems. System Dynamics provides an aggregate-level perspective, highlighting thinking in feedback loops and employing differential equations to model the causal relations in a system, exploring the system's dynamics by numerically solving the equations. Agent-based modeling, in a bottom-up method, focuses on constituent units (agents) and their interactions to explore the emerging behavior at a system level by means of simulation. Comparing these modeling methods can help us understand their strengths and weaknesses in order to choose the right approach for a given modeling problem. It may also support the analysis of a given system to build multiple models using the different approaches and comparing them, in particular to treat fundamental uncertainties in systems modeling and simulation. In this paper, we review the existing studies comparing the SD and AB approaches and models, investigating the aims, methodology, and results of such comparative studies. We also highlight lessons learned for future model comparisons by examining how the corresponding SD and AB models are built for the purpose of comparison. A procedure for transforming System Dynamics models into agent-based models is presented and discussed using examples from the literature.

Keywords:

System Dynamics, agent-based modeling, modeling and simulation, complex systems modeling, individual-based modeling, ordinary differential equations, model comparison, literature review

Contents

1 Introduction

Systems modeling and simulation approaches, such as System Dynamics (SD) modeling, at a macro level, and agent-based (AB) modeling, at a micro level, have been widely employed in the natural and social sciences to describe the dynamics of complex phenomena, ranging from environmental assessment (Kelly et al. 2013), ecological and environmental systems (e.g., see Ford 1999 for SD; Hare and Deadman 2004 and Matthews et al. 2007 for AB), and sustainability (e.g., see Hjorth and Bagheri 2006 for SD; Köhler et al. 2009 for AB) to biology (e.g., see Ruth and Hannon 1997 for SD; An et al. 2009 for AB), economics (e.g., see Radzicki and Sterman 1994 for SD; Tesfatsion 2006 for AB), and social sciences (e.g., see Lane 1999 for SD; Gilbert and Troitzsch 2005 for AB). In general, SD modeling, an approach based on ordinary differential equations (ODEs), models a system at the aggregate level with a focus on causal relations and feedback loops, and describes the system in terms of state variables (stocks) and their rates of change with respect to time (flows). In contrast, AB modeling looks at a system not at the aggregate level, but at the level of its constituent units, or "agents" (Bonabeau 2002) and explores the macro behavior emerging from micro-level dynamic interactions among agents (agents can be individuals, groups, households, firms, vehicles, etc.).

Despite their differences, both SD and AB modeling approaches can address the same modeling problems and the same questions regarding the dynamics of a given system (Scholl 2001). There has been an academic discourse on the necessity and advantages of comparing the SD and AB paradigms, two modeling traditions with fundamentally different perspectives. Given the extensive use of SD and AB approaches in environmental modeling and sustainability assessment (Kelly et al. 2013), it seems useful to further elaborate on the strengths and weaknesses of each approach and investigate the possibility and advantages of building and comparing corresponding models using SD and AB to treat the uncertainty of models used in environmental and sustainability studies.

In this paper, we review previous studies which have compared these two approaches. Our review includes two groups of studies: (i) studies comparing SD and AB approaches without conducting simulation experiments and (ii) studies based on simulation case studies which involve building and comparing SD and AB models applied to the same modeling problem. Thus, the goals of our review of the comparative studies are as follows:

(a) To better understand the differences between the two modeling approaches in general, to clarify when these approaches are appropriate, and what must be taken into account when utilizing them;
(b) To clarify the comparison method employed by the reviewed studies, how we can design simulation studies to compare models based on these two approaches, and what procedures can be used to build corresponding SD and AB models in such comparative studies;
(c) To highlight the advantages of employing comparative studies in the field of sustainability assessment.

As for goal (b), such model comparisons—also known as alignment of computational models—not only provide rigorous results to inform decisions regarding the choice of the modeling approach that suits a given modeling problem better, but also can help to determine to what extent two models claiming to deal with the same phenomenon can produce the same results (Axtell et al. 1996). In other words, multi modeling—using multiple models to describe the same domain—can be used to address uncertainties associated with the system and model (Uusitalo et al. 2015). Model-to-model comparison and analysis also support the principle of scientific replicability in the field of simulation modeling (Rouchier et al. 2008; Halbe et al. 2015).

In the next section (Section 2), basic definitions of the two modeling approaches, SD and AB, will be described. After a brief overview of our research method and the studies reviewed (in Section 3), we will then present and discuss the main findings of reviewing two groups of comparative studies (Section 4), separating studies with a focus on the general approaches (without experimentation) and those based on case studies with detailed simulation experiments, taking into account aspects such as comparison aim, methodology, and results in the studies reviewed. Based on what we learn from the methodology of model-based comparison in the studies reviewed, we will highlight the transformation procedure used to build an AB model given an SD model using examples from the reviewed studies. In the final section (Section 5), we will draw conclusions from our review.

2 Definitions

2.1 What is System Dynamics?

The System Dynamics Society introduces SD as follows: "System dynamics is a computer-aided approach to policy analysis and design. It applies to dynamic problems arising in complex social, managerial, economic, or ecological systems—literally any dynamic systems characterized by interdependence, mutual interaction, information feedback, and circular causality"[1] (See Richardson 2011 for an overview of SD and suggestions for further reading).

With an emphasis on feedback thinking (Richardson 1991), System Dynamics is equipped with both qualitative and quantitative methods to help us better understand "the structure and dynamics of complex systems" (Sterman 2000). *Qualitative SD methods* include diagramming techniques, such as causal loop diagrams (CLDs) and stock/flow diagrams (SFDs, see Figure 1), which are employed—in a way similar to the systems thinking approach (Senge 2014; Mingers and White 2010)—to communicate the contents of models in processes of model conceptualization and model exposition (Lane 2008). *Quantitative SD methods* include techniques from dynamic systems theory (such as differential equations) and numerical methods (such as Euler's method, used in computer simulation for numerical approximations of differential equations) which are employed to "simulate logical consequences of models" (Lane 2008). Diagrams, although useful for learning and group model building, are insufficient for creating reliable policy insights because without formalisms and their numerical solution, the models lack the ability to reveal roughly correct timescales and to show the outcome of competing feedback loops. Therefore, they can lead to incorrect interpretations of system behavior (Lane 2008; Sterman 1994).

The main concepts of the SD method are as follows (Grösser and Schaffernicht 2012):

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[1] http://www.systemdynamics.org/what-is-s/

- The *feedback loop*, which is a “logically closed causal chain” where an initial change in a variable is fed back to its origin. The elements that constitute a feedback loop are variables and links.
- The *polarity* of a feedback loop is positive if it reinforces initial changes and is negative if it dampens them.
- There are three categories of *variables*: stocks, flows, and auxiliary variables. “Stocks and flows—the accumulation and dispersal of resources—are central to the dynamics of complex systems.” (Sterman 2001)
- *Causal links* include link polarity (positive or negative), delays in the links, and the shape of relationships (linear or nonlinear).

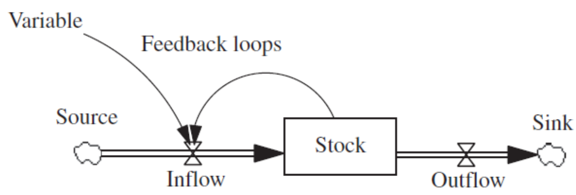

Figure 1. A stock/flow diagram used to visualize a simple SD model. Adapted from Sterman (2000)

Emphasizing the system structure as a cause of dynamic behavior, system dynamics is capable of identifying the behavioral patterns of complex systems. This is because the complexity of the system stems from the interactions of the components, not the components themselves (Sterman 2000; Phelan 1999).

The aim of system dynamics modeling is to “explain behaviour by providing a causal theory, and then to use that theory as the basis for designing policy interventions into the system structure which then change the resulting behaviour and improve performance” (Lane 2008).

Previous studies have pointed to a number of fields as theoretical origins of the SD method, including differential equations (Gilbert and Troitzsch 2005, p.28), control theory and the modern theory of nonlinear dynamics (Sterman 2002), dynamic systems theory (Grösser and Schaffernicht 2012), servomechanisms engineering (Richardson 2011), and systems theory (Phelan 1999).

Mathematically, a formal system dynamics simulation model is a system of coupled, nonlinear, first-order differential (or integral) equations of the form (Richardson 2011),

$$\mathrm{d}S/\mathrm{d}t = \text{inflow} - \text{outflow} = g(S, P), \qquad S(t_0) = S_0 \ \text{(initial value)} \tag{1}$$

where S is a stock variable (level or state variable), P is a set of parameters, and g is a nonlinear function. Simulation of such systems is conducted by partitioning the simulated time into discrete intervals of length h and stepping the system through time one h at a time. Each stock variable is computed from its previous value and its net rate of change $S'(t)$: $S(t) = S(t - h) + h \,.\, S'(t - h)$

2.2 What is agent-based modeling?

Agent based modeling (also known as agent based simulation or—not exactly equivalent—individual-based modeling[2]) is a simulation modeling method which describes the system in terms of its constituent units—called agents—with autonomous behavior. The interactions of agents with each other and the environment result in behavior emerging at the system level—see Bonabeau (2002) for an overview on AB modeling in human systems; see Macal and North (2010) for a brief tutorial on AB modeling; see Hare and Deadman (2004) for an overview of AB modeling in environmental modeling; see Heath et al. (2009) for a survey of AB modeling practices. A synonym of AB modeling would be "microscopic modeling, and an alternative would be macroscopic modeling" (Bonabeau 2002).

In terms of theoretical foundations, AB modeling can be traced back to complexity theory (Phelan 1999), complex systems (and complex adaptive systems) (Macal and North 2010), artificial life (and distributed artificial intelligence) (Hare and Deadman 2004), object-oriented software engineering (Macal and North 2005), cellular automata (Macy and Willer 2002), graph theory, and category theory (Borrill and Tesfatsion 2011).

A typical AB model has three elements (based on Macal and North 2010):

1. A set of agents (agent instances which are created in the run time according to agent types) with attributes or state variables and behavior rules;
2. A set of agent relationships and methods of interaction: an underlying topology of connectedness defines how and with whom agents interact;
3. The agents' environment: agents interact with their environment in addition to other agents.

Mathematically, we can assume that at each time t an individual agent i, $i \in \{1,\dots, n\}$ is well described by a state variable $x_{i,t} \in R^k$, and let the evolution of the agent's state variable be specified by the difference equation:

$$x_{i,t+1} = f_i(x_{i,t}, x_{-i,t}; \alpha_i) \qquad (2)$$

- where we assume that the behavioral rules may be individual-specific both in the functional form of the phase line $f_i(.)$ and in the parameters α_i, and may also be based on the state x_{-i} of all individuals other than i.
- where x_{-i} is the state of all individuals other than i and α are some structural parameters.

[2] In this paper we consider "individual-based modeling" (IBM) a subset of agent based modeling. IBM, which is mainly used in ecology (Grimm 1999; Heckbert et al. 2010), "stipulates that populations of organisms should be disaggregated and thus represented in terms of discrete individuals which are unique only in terms of characteristics" (Hare and Deadman 2004). Many studies use the two terms as if they were interchangeable (see Grimm 1999 and Grimm et al. 2006). There is little difference (if any) between agent-based modeling and individual-based modeling (Nielsen 2012). For example, "all models of a certain type (concentrated on and dealing with individuals) can be said to be individual-based if that is literally the case, but they need to perform with a certain level of agency of the 'actors' in the model before it is correct also to refer to them as agent-based" (Nielsen 2012).

3 Research method

3.1 Review method

We reviewed studies comparing AB and SD models and approaches. The studies were selected based on searching the following keywords in the topic (title, abstract, and keywords) of English-language articles indexed by Scopus and Web of Science. The search keywords included “agent-based” / “individual-based,” “System Dynamics” / “differential equations,” and comparison/compare. A brief overview of the studies selected is presented in the following section. The studies selected were then analyzed with regard to the comparison aim, criteria, methodology, and results. Moreover, using examples from the studies reviewed, we clarified a transformation procedure used to produce equivalent models for the comparison aims in the reviewed studies.

3.2 Overview of the studies reviewed

The studies reviewed can be classified in two groups: *Group A*, including the studies that compare SD and AB approaches in general, without investigating the differences in terms of concrete simulation experiments; *Group B*, including the comparative studies which build equivalent models and run simulations using AB and SD approaches. Table 1 lists the studies reviewed and their subject areas.

Table 1. List of the studies reviewed and their subject areas

Group A		***Group B***	
Study	**Subject area**	**Study**	**Subject area**
Phelan (1999)	General	Wilson (1998)	Ecology, population dynamics
Scholl (2001)	General	Parunak et al. (1998)	Management, industrial supply networks
Lorenz (2009)	General	Rahmandad and Sterman (2008)	Epidemiology
Behdani (2012)	Supply chains	Macal (2010)	Epidemiology
Kelly et al. (2013)	Integrated environmental assessment	Figueredo et al. (2013)	Immunology
Ip et al. (2013)	Public health		
Ouyang (2014)	Critical infrastructure systems		

3.2.1 Group A: Comparative studies without experimentation

Phelan (1999), in a theoretical analysis, discusses similarities and differences of SD and AB modeling approaches in the context of comparing systems theory and complexity theory and addressing their common vocabulary and concepts.

Scholl (2001), calling for "cross study and joint research" between SD and AB modeling approaches, highlights the overlaps between the two techniques, the modeling principles of each technique, and their respective strengths and weaknesses.

Lorenz (2009) proposes that three aspects be compared and that this helps with the choice between SD and AB: *structure* ("How is the model built?"), *behavior* ("What are the central generators of behavior?"), and *emergence* ("Can the model capture emergence?" Note: Lorenz (2009) uses the following definition of *emergence*: "the arising of novel and coherent structures, patterns and properties during the process of self-organization in complex systems"). He argues that the structure of SD models is static, which means that differential equations initially defined do not change during the simulation run. The structure of AB models, on the other hand, is dynamic, because "agents are created or destroyed and interactions are defined through the course of the simulation run." The second aspect (behavior) focuses on the central generators of behaviors in the model. For SD the behavior generators are feedback and accumulations, while for AB they are micro-macro-micro feedback and interaction of the systems elements. The third aspect lies in their capacity to capture emergence, which differs between the two approaches. AB modeling is capable of capturing emergence, while the single-level structure of SD models does not provide that possibility.

Heckbert et al. (2010), in their review of AB models in ecological economics, briefly compare AB modeling with other complex systems modeling tools including SD modeling. They note that while SD models, as "the most used modeling tool for complex systems," has been useful in modular modeling of interconnected systems of the biosphere, hydrosphere, atmosphere, and anthroposphere, and also in participatory modeling leading to participant learning and awareness, SD models fail to describe the "decisions and actions of multiple actors" and multiple spatial relationships; SD models are "fundamentally not adaptive"; and the ability to evolve in SD equations and feedbacks is "limited to variations in parameter values" (Heckbert et al. 2010).

Kelly et al. 2013 review five modeling approaches including SD and AB modeling (the other three are Bayesian networks, coupled component models, and knowledge-based models). Regarding AB and SD modeling, they note the uncertainty in interpretation of data, measurements, or conditions, in which both SD and AB methods "require comprehensive testing of the model to allow this understanding to be developed." They criticize both approaches because the level of testing required to develop this understanding (which is dependent on the modeling objective) is rarely carried out. However, Kelly et al. (2013) note that time and other resource constraints contribute to this weakness.

Ouyang (2014), with a focus on simulation and modeling in the field of critical infrastructure systems, compares SD and AB approaches (together with other approaches such as empirical approaches, economic theory based approaches, and network based approaches), using comparison criteria including quantity of input data, accessibility of input data, types of interdependencies, computation complexity, maturity, etc.

3.2.2 Group B: Comparative studies with experimentation

Wilson (1998) compares an AB model and two versions (i.e., deterministic and stochastic versions) of differential equation models for population dynamics in a predator-prey system, demonstrating the role of stochasticity in spatial processes.

Comparing equations with agents, Parunak et al. (1998) illustrate how SD and AB models can be used to model industrial supply networks. To be able to choose between these two methods, they emphasize the importance of further development of a group of cases that demonstrate the respective strengths and weaknesses of the two approaches.

Rahmandad and Sterman (2008) use the spread of contagious diseases as an example to compare results of AB and SD modeling, analyzing the effect that heterogeneity and different network structures have on the differences between AB- and SD-based results. They note that previous AB and SD models of the same phenomenon “sometimes agree and sometimes diverge, especially when compartments contain smaller populations.” Rahmandad and Sterman (2008) find that “AB models relax aggregation assumptions, but entail computational and cognitive costs that may limit sensitivity analysis and model scope. Because resources are limited, the costs and benefits of such disaggregation should guide the choice of models for policy analysis.”

Macal (2010) argues that many systems can be modeled equally well by either method. He presents a formal specification for SD and AB models and uses the specification to build equivalent AB models, illustrating his work with an example of an SIR epidemic model (S: susceptible; I: infected, R: recovered).

Figueredo et al. (2013) investigate the potential contribution of AB models when compared to ordinary differential-equation models (a proxy for the SD method) in a biological domain, using the example of immune interactions with early-stage cancer. They find that it is possible to obtain equivalent AB models (i.e., implementing the same mechanisms), but not everything modeled in the SD equations can be implemented in the AB model, for example, no “half” agents can be modeled in the AB method. However, Figueredo et al. (2013) note that this does not matter if the population sizes in “the original model definition are large enough.” The simulation output in both SD and AB models in their studies differed depending on the attributes of the system being modeled. But the population size had a positive impact on result similarity—the bigger the population, the closer the simulation outputs. Figueredo et al. (2013) conclude that in some cases, “additional insight” from using AB models was obtained, mainly due to their stochastic nature which can produce different results (normal and extreme cases). They also note that in AB models, the variability in the output graph is closer to the real world, whereas in SD, output graphs better show the underlying pattern in the results (e.g., predator-prey pattern).

4 Main findings and discussion

4.1 The aim of the SD-AB comparison in the studies reviewed

The studies reviewed mention various reasons for comparing SD and AB approaches and models. Table 2 summarizes the aim of the comparison in the studies reviewed, including those without experimentation (Group A) and those with a simulation component (Group B). As shown in Table 2, the main purpose of comparing SD and AB approaches and models is to provide insights about when to choose an SD approach and when it is appropriate to use AB modeling. In addition to this purpose, studies comparing models (Group B) seek to enhance knowledge and inform modeling research communities in certain domains, for example immunology—where differential equation modeling (quantitative SD) is considered an established modeling approach—about the capabilities of the relatively new approach, i.e., AB modeling. So comparative studies, including experimentation and model comparison (Group B), employ the two different approaches to build multiple models in order to address the same problem and to better understand the system under study, (hopefully) with less uncertainty.

Table 2. The aim of comparisons of SD and AB approaches and models in the studies reviewed

Study	**Purpose of the comparison**
GROUP A:	
Scholl (2001)	Since both approaches have "produced rich bodies of research and literature on widely overlapping fields of application," "at the very least, it will be insightful to compare the aggregate behavior and emergent influence on the environment of agent-based models with the predictions of aggregate-level feedback models regarding the same subject area."
Schieritz and Milling (2003)	To give both AB and SD modeling communities "the opportunity to learn about each other's modeling paradigm, question own assumptions, see problems from a different viewpoint and probably identify potentials of integration that can overcome some of the pitfalls a single approach might have in certain areas."
Lorenz (2009)	To facilitate a "superior choice of simulation methodology" (to avoid "abductive fallacy," i.e., the fallacy of applying an inadequate simulation methodology to a given simulation task)
Behdani (2012)	To support "the choice of appropriate simulation paradigm," which is an "important step in the model development process"
Kelly et al. (2013)	To "assist modellers and model users in the choice of an appropriate modelling approach for their integrated assessment applications and that enables more effective learning in interdisciplinary settings."
Ip et al. (2013)	SD and AB are compared in the context of reconciling statistical and systems approaches.
Ouyang (2014)	To better understand critical infrastructure systems (CIS) to support planning, maintenance, and emergency decision making, modeling and simulation of interdependencies across CISs has recently become a key field of study.
GROUP B:	
Wilson (1998)	"the philosophy of this work is that […] insights about important processes and mechanisms can be ascertained through the comparison of a variety of modeling frameworks" The goal of this study was to determine an appropriate stochastic population-level SD model that captures generality to gain insight beyond the specific AB models examined in the study.
Rahmandad and Sterman (2008)	When is it better to use agent-based (AB) models, and when should differential equation (DE) models be used?
Macal (2010)	How to translate a deterministic SD model into an equivalent time-stepped, stochastic AB model.
Figueredo et al. (2013)	Pointing to limitations in the use of SD mathematical models in the field of cancer immunology such as spatial interactions and emerging properties, the authors motivate the use of alternative systems simulation modeling approaches such as AB models in immunology. Using a comparative approach, they aim to show the potential contribution of AB models to help cancer-related immunology studies.
Parunak et al. (1998)	To understand the relative capabilities and advantages of SD and AB approaches. (They motivate this by pointing to ethical and practical importance of the comparison question for modelers: ethical aspects are associated with the duty of modelers to prioritize the domain being modeled, not a given modeling approach, and practical aspects are related to the funding sources interested in spending resources on modeling approaches that will provide the best results for the domain-specific problems.)

4.2 The methodology of the SD-AB comparison in the studies reviewed

Here we present the comparison methodology employed in the studies reviewed with a simulation component (Group B). Table 3 presents the main stages followed in the studies comparing SD and AB models given the same modeling problem. In order to build corresponding models for purposes of comparison, some studies start from a given AB model and create at least one equivalent SD model, and some start from the SD model and then build an AB model. For example, Wilson (1998) and Rahmandad and Sterman (2008) start from an AB model, but Macal (2010) and Figueredo et al. (2013) follow an SD-to-AB modeling path.

Table 3. Summarizing the methodology of comparison between SD and AB models in the reviewed studies

Study	Comparison methodology
Wilson (1998)	- Start from a given AB model (for a predator-prey system). - Describe agents (predator and prey) and the discrete-time equations for their behavioral rules (prey reproduction, predation, predation reproduction, predator mortality, dispersal). - Build a series of SD models assuming that the AB model is closer to the real process, and that the SD differential equations can be adjusted to bring the two models into agreement: The initial SD model using ordinary differential equations describes reactions between the two species, but the second SD, which is to represent dispersal through space, requires extension to a set of spatio-temporal integrodifferential equations.
Rahmandad and Sterman (2008)	- Develop the agent-based SEIR model and ten AB scenarios (two conditions of heterogeneous and homogenous; for each condition they examine five network topologies, including fully connected, random, Watts-Strogatz small world, scale-free, and lattice networks.) - Derive the classical differential equation SD model from the AB model. - Calibrate the SD model to the trajectory of the infectious population in 200 randomly selected simulations from each of the ten AB scenarios. - Perform sensitivity analysis with regard to population size, the basic reproduction number (R0), and disease natural history. - Compare results of AB means with those of uncalibrated base case SD and also calibrated SD.
Macal (2010)	- Formulate the SD model, assuming the state equations are in reduced form and devoid of auxiliary variables and auxiliary equations. - Build the compartmental AB model as an illustrative artifact to guide us towards a fully individual-based AB model (the compartmental AB model is a kind of "naïve" AB model in which the agents—though satisfying the criteria to be an AB model as defined by Macal (2010) and providing equivalent results to the SD model—provide no additional (heterogeneous) information or implementation advantages over the SD model. - Build an individual-based AB model.
Figueredo et al. (2013)	- Start from the established SD differential equations for the interaction of immune cells and molecules with tumor cells. - Develop the AB model based on the SD model: 1. Identify the possible agents (immune cells, tumor cells, and molecules); 2. Identify the behavior and rules of each agent (die, kill tumor cells, suffer apoptosis); 3. Implement the agents and add them to an environment; 4. Establish connections and run the simulation. - Compare the results: Statistically compare the outcome samples obtained by SD and AB simulation using the Wilcoxon rank-sum test (to formally establish whether they are statistically different from each other). This test is applied as it is robust when the populations are not normally distributed; this is the case for the samples obtained by the SD and AB models.)
Parunak et al. (1998)	- Construct an experiment with an AB model of a supply network. (The AB model includes three types of agents: Company agents, Production Planning and Inventory Control agents, and Shipping agents.). - Build the SD model corresponding the AB model (the procedure is not described in the paper).

4.3 Results of the SD-AB comparison in the studies reviewed

4.3.1 Criteria for choosing between SD and AB modeling approaches

The question of criteria for choosing an appropriate approach for addressing a given modeling problem drives all the studies reviewed. For example, Kelly et al. (2013) consider three main questions to be answered when choosing a modeling approach: (i) what is the modeling purpose? (ii) what types of data are available for developing and specifying the model? (iii) who are the model users and what requirements concerning the scales and formats of model outputs exist? Regarding the modeling purpose, Lorenz (2009) notes that the characteristics of the method have to fit both the phenomenon (what is being modeled?) and the purpose (why is it being modeled?).

Table 4 summarizes both the criteria and the associated results of the comparison in the studies reviewed. Below we take a closer look at some of these criteria.

Modeling purpose

As shown in Table 4, the modeling purpose is one of the criteria for choosing the right modeling approach. Kelly et al. (2013) identify five main purposes for using models in the field of integrated environmental assessment: (i) *prediction*, where the purpose is to estimate "the value of a system variable in a specified time period given knowledge of other system variables in the same time period;" (ii) *forecasting*, which involves "predicting the value of a system variable in future time periods (short-, medium- or long-term), without knowledge of the values of other system variables in those periods;" (iii) *management and decision-making under uncertainty*, where models are used in problem formulation and can be incorporated into decision support systems and integrated assessment tools; (iv) *social learning*, which refers to the capacity of a social network to communicate, learn from past behavior, and perform collective action, e.g., dealing with complex technical tasks and at the same time the social relational activities; (v) *developing system understanding/experimentation*, where the purpose is to summarize and "integrate available knowledge on system components in order to improve understanding of the entire system and the way it may react to changes in system drivers" (Kelly et al 2013). They then argue that both SD and AB approaches are more appropriate for the social learning and improving system understanding/experimentation purposes, and not for the other three purposes. Kelly et al. (2013) explain this by noting the emphasis of both methods on "exploring the plausibility of assumptions and outcomes, rather than on accurate prediction, forecasting or decision-making. Such models are often developed to allow decision-makers and stakeholders to experiment with the model and try out differing assumptions about poorly understood processes. These models do not tend to be highly prescriptive about policy implications" (Kelly et al. 2013).

In contrast to this view of policy-support uses of modeling and simulation, one of the studies reviewed, Rahmandad and Sterman (2008), has a clear a focus on policy support, building SD and AB simulation models in order to compare the results of the simulations in response to policy interventions in public health. Policy support is mentioned as the modeling purpose of many SD studies. For example, Lane (2008) sees "designing policy interventions into the system structure" as the purpose of SD modeling, and Hilty et al. (2006), in their SD model of the impact of information and communication technology (ICT) on the environment, while stating that forecasting was not their modeling purpose, highlight the function of system experimentation and note that the "final goal of the project was to formulate policy recommendations based on new insights about the relative relevance of ICT application fields for environmental sustainability" (Hilty et al. 2006). As for AB models, there are many studies aiming at supporting decision making and policy options (e.g., Berger 2001; Sopha et al. 2011).

Input data

As presented in Table 4, another criterion mentioned in the studies reviewed concerning the choice of modeling approach is the input data and its various aspects. Ouyang (2014) highlights two aspects: (i) the quantity of input data needed for a simulation modeling task, for which that study ranks the SD approach as medium to large, and the AB approach as large (given the large amount of disaggregate data needed to describe the heterogeneity in individual attributes and network structures in the AB approach; see Rahmandad and Sterman 2008); and (ii) accessibility or availability of input data, for which that study ranks the SD approach as medium level of access, and the AB approach as difficult to access (given the better access to statistical data on aggregate level of a population in SD than individual level data in AB).

The type of available data is another aspect of input data considered by Kelly et al. (2013), who describe two main types of data available to build a model: quantitative data (such as time series, spatial, or survey data) and qualitative data (such as expert opinion and stakeholder beliefs derived from workshops and interviews). Both SD and AB modeling approaches require quantitative data for simulation purposes. Though qualitative data can be used throughout the modeling process, incorporating qualitative data into SD and AB models and "assessing the impacts of soft variables is challenging" (Kelly et al. 2013).

Feedback effect

Both SD and AB models capture the feedback effect (Rahmandad and Sterman 2008; Lorenz 2009). However, as seen in Table 4, feedback loops are recognized as the focus and basic building block of the SD approach (Scholl 2001; Lorenz 2009). SD is equipped with explicit diagramming methods, such as causal loop diagrams, which makes it easier to communicate and think about feedbacks in the system. Feedback loops enable an SD model to endogenously represent dynamic aspects of a system (Richardson 1999). AB models, however, have feedback at more than one level of modeling, and micro-macro-micro feedback is central in generating behavior (Lorenz 2009). However, AB models may hide, within the complexity of agent interactions, the feedback mechanisms key to dynamics of interest (Fallah-Fini et al. 2013). On the other hand, feedbacks in SD are structural, and their ability to evolve is limited to variations in parameter values.

Table 4. Summary of the results of comparisons in the studies reviewed (Group A)

Study	Criteria for comparing Approaches	Comparison	
		System Dynamics modeling	Agent-based modeling
Phelan (1999)	Research agenda Techniques Epistemology	Confirmatory[1] Circular flows Post-positivist[3]	Exploratory[2] Agent-based models Positivist[4]
Scholl (2001)	Focus Approach	Feedback loops Deductive[5]	Individuals interacting based on generally simple rules Inductive[6]
Lorenz (2009), Schieritz and Milling (2003)	Basic building block Unit of analysis Level of modeling Perspective Adaptation Handling of time Mathematical formulation Origin of dynamics	Feedback loop Structure Macro Top-down Change of dominant structure Continuous Integral equations Stocks	Agent Rules Micro Bottom-up Change of structure Discrete Logic Events
Behdani (2012)	Capturing emergence? Capturing self-organization? Capturing co-evolution? Capturing path dependency?	Debatable (lack of modeling more than one system level) Hard (lack of modeling individual decision making) Hard (system structure is fixed) Debatable (no explicit consideration of history to determine future state	Yes (via modeling system at two distinctive levels) Yes (via modeling autonomous agents) Yes (network structure is modified by agents' interactions) Yes (current and future state can be explicitly defined based on system history)
Kelly et al. (2013)	Reason for modeling/type of application Type of data available to populate model Focus on breadth of system Focus on depth of specific processes Aggregated effects Interactions between individuals	System understanding; Social learning Quantitative data (common); Qualitative data (possible) Common feature Possible feature Common feature No	System understanding; Social learning Quantitative data (common); Qualitative data (possible) Possible feature Common feature Possible feature Common feature
Ip et al. (2013)	Generality[7] Realism[8] Fit[9] Precision[10]	Moderate-High Low Low-Moderate Moderate-High	Moderate High Low-Moderate Low-Moderate
Ouyang (2014)	Quantity of input data needed Accessibility level of input data Computation cost (performance)	Medium, Large Medium level of access Medium	Large Difficult to access Slow

Notes: 1. *Confirmatory* analysis aims at understanding the behavior of an overall system when the behavior of the components is known with a high degree of accuracy; 2. *Exploratory* analysis aims at deriving a set of component relations that will yield an overall system exhibiting the observed behavior; 3. *Post-positivism* contests the notion of positivism that observation reflects reality; the closest we can get to ontological reality is a shared agreement about experiential reality (arising from our shared perceptions with others); 4. *Positivism* maintains that our sensory perceptions provide accurate knowledge of reality (Phelan 1999). 5. The SD approach is *deductive*, i.e., it describes dynamic systems by their feedback structure at an aggregate level without accounting for individual agents or events (the dynamics of the underlying structures are seen as dominant.). 6. The AB approach is *inductive*; i.e., the modeler may modify rules and parameters and then try to understand what the resulting outcomes are with regard to the emergent behavior of the overall system (Scholl 2001). 7. *Generality*: applicability of the model to phenomena other than that for which it was developed; 8. *Realism*: degree to which the model reflects reality as viewed by experts in the field; 9. *Fit*: degree to which the model output matches historical data and has predictive accuracy; 10. *Precision*: fineness of model and level of details specified (Ip et al. 2013).

4.3.2 Results of model comparisons

Table 5 presents a brief summary of the results of the comparison of AB and SD models in the studies with simulation experiments reviewed (Group B).

Table 5. Summary of the results of comparisons in the studies reviewed (Group B)

Study	Result of the SD-AB comparison
Wilson (1998)	- The study compares an AB model of a predator-prey system with a series of SD models, including a deterministic and stochastic SD model. - The deterministic SD leads to qualitatively different behaviors than the AB model. - The study finds good agreement between AB results and the stochastic SD results under various dispersal scenarios.
Rahmandad and Sterman (2008)	- The SD and mean AB dynamics differ for several metrics relevant to public health, including diffusion speed, peak load on health services infrastructure, and total disease burden. - The response of the models to policies can also differ even when their base case behavior is similar. - In some conditions, however, these differences in means are small compared to variability caused by stochastic events, parameter uncertainty, and model boundary.
Macal (2010)	- Probabilistic elements in the SD model are identified, isolated, and translated into probabilities that are used explicitly in the AB model. For the SIR epidemic model, the two probabilities are related to agent contact and to agent transmission of infection. - The equivalence of the model results is not exact in terms of numerical accuracy for the reasons noted. - The study shows that the AB model is able to provide information beyond what the SD model provides due to the explicitly stochastic nature of the AB model.
Figueredo et al. (2013)	- It is possible to obtain equivalent AB models from a given SD model (i.e., implementing the same mechanisms). - However, the simulation output of both types of models might differ depending on the attributes of the system to be modeled. - In some cases, additional insight from using AB modeling was obtained. - Overall, the authors confirm that AB modeling is a useful addition to immunologists' tool set, as it has extra features that allow for simulations with characteristics that are closer to the biological phenomena.
Parunak et al. (1998)	- The SD model shows the same periodicities as the AB model. - The SD model does not show many of the effects observed in the AB and in real supply networks, including the memory effect of backlogged orders, transition effects, or the amplification of order variation.

4.4 Lessons learned from comparison methodology—A transformation procedure

SD assumes homogeneity and perfect mixing within compartments (stock variables). For example, in an epidemic model using SD, a well-mixed population is assumed, i.e., the probability of any infected individual contacting any susceptible individual is reasonably well approximated by the average[3]. However, the perfect mixing assumption is relaxed in AB models, where heterogeneity is captured across individuals (in heterogeneous attributes and behavioral rules), in the network of interactions among them (in various network topologies), and in different mixing sites for population subgroups (Rahmandad and Sterman 2008).

[3] see Jones, James Holland. 2013. "Notes on R0." Department of Anthropological Sciences, Stanford University.

The studies reviewed (with simulation experimentation; Group B) build equivalent models to conduct the comparison. The result of that comparison is reported in Section 3. In this section, based on what we have learned from the methodology of model-based comparison in the studies reviewed, we highlight the transformation procedure used to build an AB model given an SD model using examples from the studies reviewed, which is a procedure that relaxes assumptions of homogeneity and perfect mixing within compartments of the SD model. The reasons why we highlight this procedure, which is a subordinate step in the studies reviewed (which were analyzed in the previous section), are that: (i) this procedure is one of the aspects of the studies reviewed here, and thus fits the scope of our review; (ii) it is insightful to formulate this procedure to better understand differences between the SD and AB modeling approaches; (iii) the procedure provides user guidance for future comparative studies; and (iv) even if we do not intend to compare the two models, given the availability of either an SD or AB model, it is useful to be able to reuse knowledge embedded in one type of model by following such a procedure to build a new model of the other type.

First we introduce the selected examples and then describe the procedure. Figure 3 visually presents the main elements of SD and AB, which are discussed in the following.

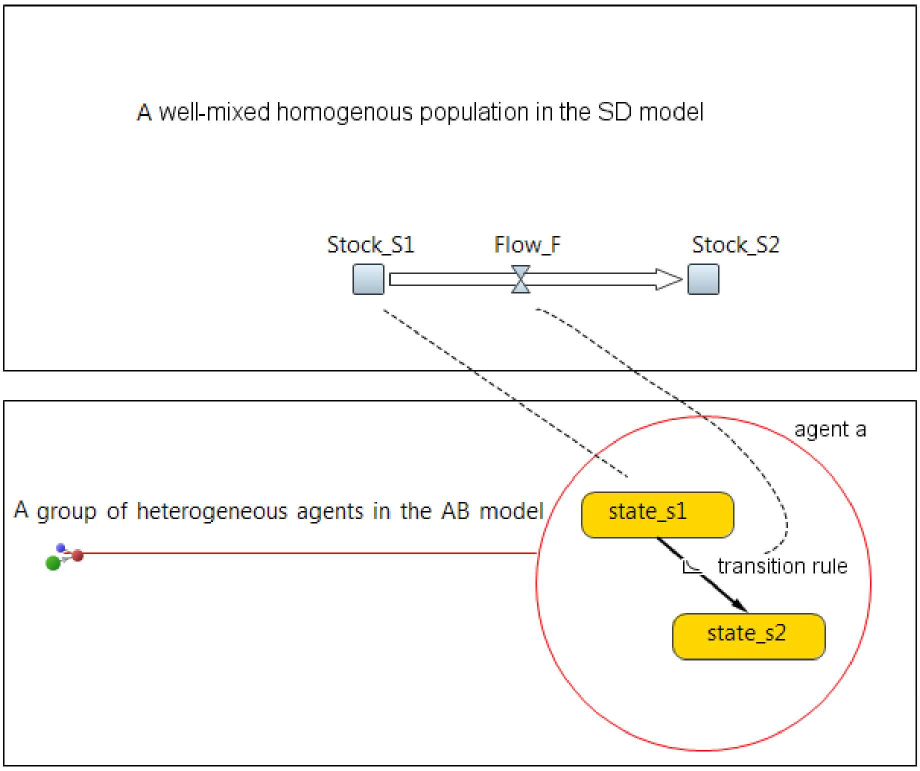

Figure 3. Visual demonstration for mapping the elements of an SD model onto the relevant elements in the equivalent AB model. The dashed line depicts the mapping of the stock variable and flow function in the SD model onto the agent state and transition rule, respectively, in the AB model. The homogenous value of a stock variable in the SD model at any point in time during the simulation corresponds to the average value of the respective agent state in the population of heterogeneous agents. The red circle represents only one agent ("instance") among the group of agents, corresponding to the population in the SD model.

4.4.1 An overview of examples taken from the studies reviewed

Since the procedure will be described using examples from the literature, we first introduce the examples here.

Example 1: An epidemic model—SIR and SEIR

Macal (2010) and Rahmandad and Sterman (2008) use a classic epidemic model which divides the population into three (SIR, as in Macal 2010) or four (SEIR, as in Rahmandad and Sterman 2008) compartments: susceptible (S), infected (I; symptomatic), and recovered (R; immune to reinfection)—and also exposed (E; asymptomatic infectious) in an SEIR model. The following system of differential equations describes an SIR model:

$$dS/dt = -\beta IS/N$$

$$dI/dt = \beta IS/N - \gamma I$$

$$dR/dt = \gamma I$$

Initial conditions: $S_0 = N\text{-}1$, $I_0 = 1$, $R_0 = 0$,

where N is the population size (assumed to be fixed); β is the likelihood that an infected individual transmits the infection to a susceptible individual upon contact; γ is the rate at which infected individuals recover from an infection (assuming permanent immunity).

Example 2: An immunological model—Tumor and effector cells

Figueredo et al. (2013) investigate a model of immuno-interactions with cancer using three case studies: the first SD model involves interactions between tumor cells and generic effector cells (two populations); the second model adds to the previous model the influence of IL-2 cytokine molecules in the immune responses of effector cells towards tumor cells (three populations); and the third model adds interactions between effector cells, tumor cells, and IL-2 and TGF-b molecules (four populations) (Figueredo et al 2013). The following system of differential equations describes a model involving only two populations of tumor and effector cells (the first case study in Figueredo et al 2013):

$$dT/dt = Tf(T) - d_T(T,E)$$

$$dE/dt = p_E(T,E) - d_E(T,E) - a_E(E) + \Phi(T)$$

where T is the number of tumor cells; E is the number of effector cells; f(T) is the growth of tumor cells, d_T(T, E) is the number of tumor cells killed by effector cells; p_E(T, E) is the proliferation of effector cells; d_E(T, E) is the death of effector cells when fighting tumor cells; a_E(E) is the death (apoptosis) of effector cells, Φ(T) is the treatment or influx of cells.

4.4.2 The procedure for creating an AB model corresponding to a given SD model

1. Identify populations and their respective states in the SD model

Consider the SD model defined earlier in Eq. (1) with a system of differential equations of the form:

$$\mathrm{d}S/\mathrm{d}t = \text{inflow} - \text{outflow} = g(S, P), \qquad S(t_0) = S_0 \ \text{(initial value)} \tag{1}$$

where one differential equation is associated with each stock variable S. This equation includes at least one inflow and/or outflow term, which in turn can be a function of S and other stock variables.

Step 1.1: Identify populations in the SD model.

For example, in Example 1, there is only one population, which is the population of individuals studied in the scope of the SEIR epidemic model. In Example 2, at least two populations can be identified: the population of tumor cells and population of effector cells.

Step 1.2: For each population identified in Step 1.1, list the states associated with the population. Stock variables, which are differentiated over time, guide us towards these states.

In the SEIR model in Example 1, there are four stock variables, so all members of a population are in one of four states—susceptible, exposed, infected, or removed. In Example 2, we have already identified two populations. The population of tumor cells can be associated with the stock variable, T, which indicates the number of tumor cells ("alive"). If the members of this population are not in the "alive" state, there should be another implicit state (not clearly expressed as a stock variable in the equations for Example 2), e.g., "non-alive" or "dead" state. The other population in Example 2 is the population of effector cells, associated with the stock variable E, which represents the number of effector cells ("alive"). As explained earlier for tumor cells, we can also consider an implicit state "dead" for the effector cells.

2. Define agents and agent states

Step 2.1: For each population identified in Step 1.1, define an agent (class) *a*, to represent the members of the population.

Step 2.2: For each population state *S* in the SD model (identified in Step 1.2), define a state variable *s* to the respective agent *a* (associated with the population) in a way that $s_a = S$.

In Example 1, following Step 2.1 and 2.2, for the population of individuals, we can define an agent called "individual" which can be in any of the following states: susceptible (S), exposed (E), infected (I), and recovered (R).

Similarly in Example 2, since we have two populations, we can define two agents: tumor cell and effector cell. Each of these agents can be either in the state "alive" or "dead."

3. Identify the flows between stocks in the SD model and define transition rules between agent states

Step 3.1: Identify the flows between stocks in the SD model

Note that the flows between stocks at the population level in the SD model are equal to the value of the sum of the underlying probabilistic transition rates for each population member.

Step 3.2: Define transition rules between agent states of the agent associated with the population.

In Example 1, looking at the equations in the epidemic model, following Step 3.1, we can identify these flow rates: the flow $\beta IS/N$ leaves the stock Susceptible and enters the stock Infected; the flow γI goes from the stock Susceptible to Recovered.

Then in Step 3.2, we can define two transition rules for the states of the agent "individual." At this step, we can use state charts to define and visualize the agents' states, transitions between the states, events that trigger transitions, timing, and agent actions. Most of the transitions occur according to the rates developed in Step 3.2 (e.g., see Figueredo et al. 2013). In Example 2, for the flow rate "*aT (1 - Tb)*" in the SD model we can define the transition "proliferation" with the rate "*a - (TotalTumor.b)*" for agent state "alive."

4. Add heterogeneity in individual attributes

Step 4.1: Identify individual attributes (agent parameters) based on the parameters used in formulating transition rules defined in Step 3.2.

Step 4.2: Add heterogeneity in individual attributes.

In the studies reviewed, various approaches were chosen to represent heterogeneity in AB models when the comparative experiments were designed.

In Example 1 (Rahmandad and Sterman 2008), following Step 4.1, we can identify four individual attributes for an agent in the epidemic model: expected contact rate, infectivity, emergence time, and disease duration. In Step 4.2, Rahmandad and Sterman (2008) choose to add heterogeneity (to relax the perfect mixing assumption) on the expected contact rate for each individual to create heterogeneity conditions in individual attributes. They do so by defining the expected contact frequency for a link (defined based on network topology; see next step) between individuals i and j, $c[i, j]$, as a function of a random variable, $\lambda[i]$ and $\lambda[j]$, in which $\lambda[i]$ represents different propensities for each agent i to use its links. We should make sure that the average value of expected contact frequency for the population of agents in the AB model equals the value used in the SD model. (Note that in this example, the AB model uses the same values as the SD model for other individual attributes, i.e., infectivity, emergence time, and disease duration.)

In Example 2 (Figueredo et al. 2013), following Step 4.1, we can identify agent parameters of the tumor cell, such as a and b (associated with proliferation and death by age), m (related to damaging the effector cell), and n (for death caused by effector cells), as well as parameters of the effector cell, such as m for death due to age, d for death by apoptosis, p and g for proliferation, and s for its injection as treatment. In Step 4.2, Figueredo et al. (2013) choose to vary the following individual attributes in several scenarios defined in their analysis: different death rates of tumor cells (b, varying between 0.002 and 0.004), different effector cells apoptosis rates (d, varying between 0.1908, 0.3743, and 2), and different treatments (s, varying between 0.318, 0.1181, and 0).

5. Add network structure (heterogeneity) for agent interactions

The network of interactions between agents—also called contact network, relationship network, or social network—is a feature of modeling which is specific to AB modeling, and we cannot find anything explicitly equivalent in the SD model (even though the influence of the network is aggregated in macro-level parameters in the SD model).

Step 5.1 Add a network for the interaction of agents.

Among the studies reviewed on the comparison of SD and AB modeling, only one study, Rahmandad and Sterman (2008) examines the influence of various network structures in disease diffusion dynamics. That study explores five different network structures: fully connected, random, small world, scale-free, and lattice.

Unlike SD models, AB models have the ability to represent the network of interactions among agents. Various approaches can be used in modeling the interactions among agents (Bargigli and Tedeschi, 2014): from local interaction (where agents' behavior is directly affected by others' behavior, rather than being mediated by a centralized agent or market mechanism) to global interactions (where individual behavior depends on the behavior of all other agents), from deterministic to stochastic (e.g., switching from one partner to another depends on a probability), from exogenous to endogenous interactions, and from static—i.e., the neighboring sets are determined once and for all—to dynamic interactions—i.e., the interactive structure evolves over time depending on model assumptions (Bargigli and Tedeschi, 2014). The choice of approaches depends on the phenomenon to be modeled and on the model purpose. At this step, Macal and North (2010) suggest questions such as:

- *How do the agents interact with each other?*
- *How expansive or focused are agent interactions?*

This step defines possible network structures for the interactions of agents with each other. Several network structures have been widely used in AB modeling studies and also supported by simulation tools (Borshchev 2013):

- *Random networks*, where agents are connected randomly with a given average number of connections per agent;
- *Distance-based networks*, where any two agents are connected if the distance between them is less than a given maximum (in continuous space only);
- *Ring lattice networks*, in which agent connections form a ring where an agent is connected to a given number of closest agents;
- *Small-world networks*, which can be considered as ring lattice networks where some links have been "re-wired" to long-distance agents;
- *Scale-free networks*, where some agents are "hubs" with a lot of connections and some are "hermits" with few connections.

Another approach is to determine network structure from a field study. For example, Ahuja and Carley (1998) analyze email interaction among members of a virtual organization to determine the network structure associated with three tasks in the organization.

Mathematically, interaction networks in AB modeling are based on graph theory. Using AB modeling, Peres (2014) investigates the impact of network characteristics in 160 topologies on the diffusion of innovations.

6. Add spatial heterogeneity and mobility

Step 6.1 Add spatial heterogeneity and mobility to the AB model.

As discussed earlier, spatial heterogeneity has not been well addressed in the SD-AB comparisons—except for Wilson (1998), who uses the rules embodied by a spatially explicit AB model. Pérez and Janssen (2015), in a study of the effect of spatial heterogeneity and mobility in AB models on the performance of social-ecological systems, analyzes the system's outcomes (resources, agents' occupational level, and cooperation) under several scenarios in which the mobility of the agents and the landscape configuration (from homogeneous to very heterogeneous landscape) are varied.

4.4.3 Application of the procedure in sustainability assessments

Both SD and AB modeling and simulation approaches have been used (often in separate studies and sometimes in hybrid studies) in sustainability assessments (including research areas such as integrated assessment, environmental modeling, transition modeling, and social-ecological modeling as described in Halbe et al. 2015); mainly for purposes such as social learning, theory building, system understanding, and experimentation; for policy making under uncertainty; and to a lesser extent for prediction and forecasting (Kelly et al. 2013). Comparing SD/AB models in the field of sustainability assessment provides an opportunity to better understand the dynamics of the social, economic, and ecological systems by providing complementary macro and micro perspectives and managing the model uncertainty rooted in the modeling approaches (Uusitalo et al. 2015). Micro-level AB models would provide an analysis instrument different than the macro-level SD models in the analysis of the behavior of the base case system and also under various policy interventions. Comparison of analyses conducted using macro-level SD models and micro-level AB models would increase the quality of model-based sustainability assessment and the associated scenario analysis and policy simulations.

Consider the following SD example. Employing an SD approach, Hilty et al. (2006) model the enabling potential and rebound effects of ICT on environmental sustainability. This prospective study (revisited recently in Achachlouei and Hilty 2015c) assesses the impact of ICT on environmental sustainability in the European Union within a time horizon until 2020. To highlight the advantages of employing comparative studies in the field of sustainability assessment, here we will briefly outline the application of the transformation procedure described above to create an AB model corresponding to a submodel, namely the SD model in Hilty et al. (2006), with a focus on a small submodel on passenger transport and the rebound effect of ICT. One aspect of this SD model is that it uses feedback loops (closed causal chains) to model the direct rebound effects of cost and time efficiency provided by ICT—the increase in demand for a resource as a consequence of increasing the efficiency of using this resource in production or consumption is known as direct rebound effects. Applying the transformation procedure to the passenger transport submodel of this SD model (described in Achachlouei and Hilty 2015a), we can build a corresponding AB model (described in Achachlouei and Hilty 2015b). In our example of ICT effects on sustainability, for the case of rebound effects induced by time and cost efficiency provided by ICT applications, one can reuse the knowledge represented in the SD model and build a corresponding AB model to focus on individual level modeling of passengers (as agents) and their choices of transport modes characterized by reusing knowledge from an SD model and then adding heterogeneity (in individual attributes and network structures) relevant to the sustainability assessment purposes. And then, the collection of empirical data at the individual level can initialize the AB model based on the individual level of ICT adoption and use as well as choices of transport mode. Analysis of such AB models can provide new insights with regard to the sustainability impacts of increased use of ICT including different types of rebound effects. Detailed discussion of this example, i.e., the application of the transformation procedure in sustainability assessment of ICT can be found in (Achachlouei and Hilty 2015b).

4.5 Limitations of the studies reviewed and future research

4.5.1 Treatment of space

The treatment of space is considered an advantage of AB modeling over SD (Lorenz 2009), because AB models can be spatially explicit, i.e., “agents are associated with a specific location from which they may or may not be able to move” (Scholl 2001). However, Kelly et al. (2013) note that both SD and AB modeling can represent space, although the capability of SD is limited due to the immaturity of SD model building tools—not because of the method itself. For example, Riley (2007) adds spatial compartments to the SD model to capture heterogeneity by location in patch models.

However, in the studies with simulation experimentation (Group B) reviewed, except for Wilson (1998), which studies the role of spatial dynamics in the comparison of SD and AB models in the predator-prey model, none of reviewed studies include a spatial aspect in their experiments to compare SD and AB models. Figueredo et al. (2013) note that since they have chosen SD models which do not consider spatial dynamics between effector cells and tumor cells, their corresponding AB models do not include space (distance) and how it would affect the simulation results. Similarly, Rahmandad and Sterman (2008) do not include spatial heterogeneity in their models, although they note the possibility of SD models disaggregating the population "more finely to account for clustering, heterogeneity, and other attributes that vary across individuals (e.g., age, sex, location)."

Future research can provide better insights on the comparison of the ability of SD and AB models to treat spatial attributes of complex systems.

4.5.2 Quantitative comparison

The studies reviewed note the increased computational costs when implementing AB models (see e.g., Rahmandad and Sterman 2008). However, no quantitative comparison between SD and AB modeling has been presented. It is common that studies comparing programming languages at the level of computational implementation investigate quantitative properties such as run time, memory consumption, program structure, the amount of effort required for writing them (Prechelt 2003), usability, readability, performance, and functional features (Purer 2009).

To better understand the computational costs of AB models compared to SD, one can conduct objective empirical comparisons between these two approaches using a number of implementations of the same set of well-defined requirements—e.g., in another computational context, Prechelt (2003) compares scripting languages using 80 implementations created by 74 different programmers.

4.5.3 Mental models of dynamic systems

Beyond quantitative simulation aspects, the SD and AB modeling approaches each support a certain perspective and a specific way of thinking about the system being modeled and its dynamics. SD highlights the causal links between variables and also the feedback loops in the system, whereas AB modeling starts with actors (constituent units) in the system and their network of interactions. These perspectives correspond to different types of mental models that humans build of dynamic systems. A mental model of a dynamic system (MMDS) is "a relatively enduring and accessible, but limited, internal conceptual representation of an external system whose structure maintains the perceived structure of that system" (Doyle and Ford 1998). The studies reviewed on SD-AB comparisons do not compare the mental models behind these approaches. The MMDSs in the context of SD models have been widely studied (for a review, see Grösser and Schaffernicht 2012), addressing the poor performance of humans in comprehending complex dynamic systems which involve feedback relations distant in time and space. Such studies aim to measure changes in the MMDSs and cognitive maps associated with the SD approach (Langfield-Smith and Wirth, 1992; Doyle et al., 2008). However, analogous studies are not available for AB models. One advantage of using AB modeling is argued to be the explicit representation of the actors and individuals (constituent units of the system), which helps "digital computers handle discrete systems in a natural way, and when the computational model, and the system it represents, are both built from easily identifiable discrete components, the mapping between system and model can be made simpler and easier to understand" (Bithell et al 2008).

Arguments about the AB approach's "natural way" and "easier to understand" manner can be evaluated in a comprehensive conceptualization and experimental measurement of the MMDS of AB models compared to that of SD models. Such studies, for example, should consider the limits of AB models when used without computer simulations; that is, the emergent behavior of complex systems modeled using the AB approach can only be observed if we run computer simulations over a populations of heterogeneous agents.

4.5.4 Diagramming techniques

In SD, the use of diagramming techniques such as causal loop diagrams (CLDs) and stock/flow diagrams (SFDs) extends the communicative power of this approach beyond the mathematical equations (differential equations) used in simulation modeling. Despite their limitations, such as their insufficiency for policy support, diagramming techniques help SD modelers communicate the contents of models with other stakeholders, supporting group decision making, participatory modeling, and feedback thinking (Lane 2008). In AB modeling, due to its roots in object-oriented analysis and design in software engineering, several object-oriented diagramming techniques such as UML[4] class diagrams and state charts have been employed (Borshchev and Filippov 2004; Grimm et al. 2006; Heath et al. 2012).

In terms of their capability for communication and conceptualization, these AB diagramming techniques have not been evaluated in the literature. To compare the capability of diagramming techniques in the SD and AB approaches, mental models of dynamic systems and also techniques from human-computer interaction (HCI) design research can provide insightful evaluations (for the reasons similar to those explained by Lane 2008 for the evaluation and improvement of SD diagramming techniques).

4.5.5 Decision-making agents

The studies reviewed comparing SD and AB models using simulation examples (Group B) choose systems where individuals do not make decisions—decision making here means that agents have a utility function based on which the agents evaluate various decision alternatives. For example, in the epidemic model by Rahmandad and Sterman (2008), agents do not make decisions, they simply change their state, e.g., from infected to recovered, according to the respective transition rates. Figueredo et al. (2013) also model cancer cells which do not make decisions, either.

One reason for this could be the higher complexity of AB models involving human decision making, which would complicate the comparison of AB and SD modeling. Since there are many fields of application (including human decision making) for which both SD and AB simulation models have been produced—for example, technology adoption and diffusion—future research can encompass comparisons of SD-AB models for such domains to provide new insights on the benefits and challenges of such comparative studies.

[4] Unified Modeling Language

4.5.6 Other approaches to disaggregation

It should be noted that AB modeling is not the only way to add heterogeneity to homogenous SD models. Fallah-Fini et al. (2013) present a method for disaggregation of aggregate compartmental models without the need to explicitly model every individual in AB modeling. This method, focusing on an attribute of interest, describes a method for connecting the micro-level dynamics associated with elements in a population with the macro-level population distribution in SD models. Fallah-Fini et al. (2013) find that their proposed method delivers accurate results with less computation than the AB model. In another study, Osgood (2009) demonstrates a technique for using dimensional analysis and scale modeling to reduce the computational burdens associated with AB modeling. Given a homogeneous compartmental SD model with a large population, Osgood (2009) formulates a "reduced-scale" AB model that simulates a smaller population. Future work combining the procedure from SD to AB modeling (detailed in this paper) with these midway disaggregation methods would be extremely valuable.

5 Conclusions

Reviewing the literature comparing AB and SD approaches and models, we answered the following questions:

1. *What are the advantages and disadvantages of the approaches? For what sorts of situations or research questions should AB models be used, and when are SD models appropriate?*

The studies reviewed suggest that both approaches are capable of representing temporal aspects of dynamic systems, but AB approaches are more appropriate for modeling spatially explicit complex systems. AB modeling is also a better approach for modeling heterogeneity in individual attributes and in the network of interactions among population elements; however, this means that AB modeling requires the collection of more data at the level of individuals, which in turn lead to a slower modeling process, higher computational costs, and more difficult calibration in the AB modeling, compared to the SD approach.

2. *How can we design simulation experiments to compare SD and AB models given a modeling question?*

The studies reviewed, as far as they compare models in simulation experiments, use the same parameters for both the AB and SD models. Some studies, starting from a given AB model, build a corresponding SD model by averaging over populations of agents, and others use an SD model as a starting point and build an equivalent AB model by adding heterogeneity in individual attributes and interaction networks among agents. In addition to reviewing the methods employed by the comparative studies, we also highlighted a transformation procedure as to how to disaggregate an SD compartmental model into a more heterogeneous AB model. This procedure, which is accompanied by two examples, may help future comparative or complementary SD-AB studies.

Based on our discussion of the limitations of the reviewed studies, we suggest future research to apply comparisons of SD and AB models to socio-economic and socio-ecological systems; to compare the potential of these approaches not only with regard to computer simulation, but also with regard to the mental models of dynamic systems they inspire, the support of communication in group-based modeling and analysis, and the usability of associated diagramming techniques.

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