Minimal Aliasing Subband System Identification

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Abstract

Subband adaptive filters have been proposed to avoid the drawbacks of slow convergence and high computational complexity associated with time domain adaptive filters. While the computational complexity is reduced, other undesired properties, such as signal delays and signal aliasing, are introduced. Aliasing effects may result in loss of perception in speech applications. A method for the design of oversampled filter banks is proposed to reduce these effects. The design method aims at reducing the inband aliasing as well as the reconstruction aliasing for the reason of achieving robustness when weighting in the subbands alters the subband signal phase and magnitude.

1. Introduction

Subband adaptive filtering has arisen as an alternative for conventional time domain adaptive filtering, [1]. The main reason is the reduction in computational complexity and the increase in convergence speed for the adaptive algorithm, achieved by dividing the algorithm into subbands, [2]. The computational savings comes from the fact that time domain convolution becomes decoupled in the subbands, at a lower sample rate, [3].

Subband analysis and synthesis is often performed using multirate filter banks, [4]. Decimation and interpolation in the filter bank cause aliasing of the subband signals. This aliasing can be cancelled in the synthesis bank when certain conditions are met by the synthesis filters and in the subband processing. However, even if aliasing distortion in the filter bank output is cancelled in this way, the inband aliasing is still present in the subband adaptive filter input signals and, consequently, the adaptive filters are perturbed and the overall performance of the system is reduced, [5]. Also, a change of phase by any filtering of the subband signals will obviously alter the cancellation in the synthesis bank, and may result in large aliasing effects.

Several solutions to the subband filtering problem have been suggested in the literature. Non-critical decimation has been suggested in [1], where filter bank delay aspects, and amplitude distortions, have not especially been taken into consideration. The use of cross filters, [5], has been suggested to explicitly filter out the aliasing components. A delayless structure has been proposed in [6], where the actual filtering is performed in the time domain, with consequences of higher computational complexity. The computational complexity also increases significantly with cross band filters.

A uniform DFT-modulated FIR filter bank is used for the subband transformations. Modulated filter banks provide a computationally efficient implementation, due to the polyphase implementation [4], and great design simplicity. The main contribution in this paper is the suggested design method, where the filter bank response error and the inband and output aliasing errors are minimized simultaneously, while the total filter bank group-delay is pre-specified. The influence of the filter bank performance is evaluated on a subband identification system.

2. The Uniform DFT Modulated Filter Bank

In this section, an input-output expression will be derived for analysis-synthesis DFT filter banks with arbitrary decimation factor. Two sets of M filters, $H_m(z)$ and $G_m(z)$, form a uniform DFT analysis filter bank and synthesis filter bank, respectively, when they are related to prototype filters, $H(z)$ and $G(z)$, as

$$H_m(z) = H(zW_M^m) = h^T \phi(zW_M^m)$$
$$G_m(z) = G(zW_M^m) = g^T \phi(zW_M^m)$$

(1)

for $m = 0, \ldots, M - 1$

where $W_M = e^{-j2\pi/M}$, $h = [h(0), \ldots, h(L_h - 1)]^T$, $g = [g(0), \ldots, g(L_g - 1)]^T$ and $\phi(z) = [1, z^{-1}, \ldots, z^{-(L_g - 1)}]^T$.

Each subband signal is decimated by a factor $D$. An efficient implementation of such a filter bank is given in [8]. For simplicity of derivation, the direct form realization of the filter banks given in Fig. 1 is studied. The input signal $X(z)$ is filtered by the analysis filters $H_m(z)$ and decimated by the factor $D$ according to

$$X_m(z) = \frac{1}{D} \sum_{l=0}^{D-1} H(z^{\frac{1}{D}} W_M^m W_D^l) \cdot X(z^{\frac{1}{D}} W_D^l)$$
4. Synthesis Filter Bank Design

Given the analysis filter bank with an analysis prototype filter \( H(z) \) designed as described in Section 3, an optimal synthesis filter bank will be designed which minimizes the amplitude and phase distortion of the total filter bank system and also minimizes the aliasing distortion at the output.

The first part in the design of the synthesis filter bank is the minimization of amplitude and phase distortion. The System Response Error will be derived expressed in terms of the impulse responses of the prototype filters \( H(z) = h^T \phi(z) \) and \( G(z) = g^T \phi(z) \). The system response error is defined by

\[
\varepsilon_T = \frac{1}{2\pi} \int_{-\pi}^{\pi} |T(e^{j\omega}) - T_d(e^{j\omega})|^2 d\omega.
\]  

From Eq. (3), the total filter bank system response can be expressed in terms of \( h \) and \( g \), with \( \xi_m(z) = 1, m = 0, \ldots, M - 1 \)

\[
T(z) = \frac{1}{D} \sum_{l=0}^{D-1} \sum_{m=0}^{M-1} H(zW_M^l)G(zW_M^m) = h^T D(z) g,
\]

where

\[
D(z) = \frac{1}{D} \sum_{l=0}^{D-1} \sum_{m=0}^{M-1} \phi(zW_M^l) \phi(zW_M^m).
\]

The desired filter bank response is

\[
T_d(e^{j\omega}) = e^{-j\omega \tau_d}
\]

where \( \tau_d \) is the desired filter bank delay. Substituting Eq. (18) and Eq. (16) into Eq. (15) yields

\[
\varepsilon_T(g) = g^T E g - 2g^T f + 1
\]

where

\[
E = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_H(e^{j\omega}) h^T D(e^{j\omega}) d\omega
\]

and

\[
f = \frac{1}{2\pi} \int_{-\pi}^{\pi} \text{Re} \{ e^{j\omega \tau_d} D(e^{j\omega}) h \} d\omega.
\]

From Eq. (3), the aliasing terms \( Y(z) \) in the filter bank output signal \( Y(z) \) are described by the sum of all repeated spectra

\[
Y(z) = \frac{1}{D} \sum_{l=1}^{D-1} X(zW_M^l) \sum_{m=0}^{M-1} \xi_m(z^D) H(zW_M^m) G(zW_M^m).
\]

In the ideal case, the aliasing terms in the output signal are zero, i.e., \( Y(z) = 0 \). In this case the prototype filters are such that the products of \( H \) and \( G \) in Eq. (22) are zero for all terms. In the non-ideal case, the energy in all aliasing terms could be minimized. The sum of power magnitudes is defined

\[
D(e^{j\omega}) = \frac{1}{D} \sum_{l=1}^{D-1} \sum_{m=0}^{M-1} |H(e^{j\omega} W_M^m W_M^l) G(e^{j\omega} W_M^m W_M^l)|^2.
\]

Eq. (23) can be rewritten using the impulse responses \( h \) and \( g \)

\[
D(e^{j\omega}) = \frac{1}{D} \sum_{l=1}^{D-1} \sum_{m=0}^{M-1} |h^T \Phi_{m,l}(e^{j\omega}) g|^2
\]

where

\[
\Phi_{m,l}(z) = \phi(zW_M^m W_M^l) \phi^T(zW_M^l).
\]

The residual aliasing distortion is defined as

\[
\varepsilon_D = \frac{1}{2\pi} \int_{-\pi}^{\pi} D(e^{j\omega}) d\omega = g^T P g
\]

where

\[
P = \frac{1}{2\pi D} \sum_{l=1}^{D-1} \sum_{m=0}^{M-1} \int_{-\pi}^{\pi} \Phi_{m,l}^H(e^{j\omega}) h^T h \Phi_{m,l}(e^{j\omega}) d\omega.
\]

The optimal synthesis prototype filter in terms of minimal system response error and minimal energy in the output aliasing terms is found by minimizing the total error function

\[
\varepsilon_{tot}(g) = \varepsilon_T(g) + \varepsilon_D(g) = g^T (E + P) g - 2g^T f + 1
\]

that is, by solving the set of linear equations

\[
g = \arg \min_g \varepsilon_{tot}(g) = (E + P)^{-1} f.
\]

5. Evaluation

Two critical \((D = M)\) and two oversampled \((D = \frac{3}{2} M)\) decimated filter banks have been designed with \( M = 64 \) and \( M = 128 \) subbands. Prototype analysis and synthesis filter lengths, \( L_h = L_g \), are chosen as \( 2M \) and \( 4M \). The decimation factor is set to \( D = M \) and \( D = \frac{3}{2} M \). The group delay is specified as \( \tau_d = M \) and \( \tau_d = 2M \). This gives in total 16 scenarios. The group delay of the prototype analysis filter is set to \( \tau_h = \frac{1}{2} \tau_d \). The performance of the subband implementations is evaluated in the case of a real room impulse response estimation. A white noise sequence is emitted through a loudspeaker in a conference room and by using a microphone observation as a desired signal, the acoustic path is identified, see Fig. 2. Least squares estimation, [2], is used individually in each subband.

The system error \( \varepsilon_S \) is defined as the average difference between the power spectral density (PSD) of the desired signal \( d(n) \) and the PSD of the result of the system identification using the calculated weights, see Fig. 3. In the application of
Figure 3: The system error $e_k$.

Figure 4: System responses of the real room transfer function and the estimates for the oversampled subband implementations, $M = 64$ and $M = 128$ for $D = \frac{1}{2}M$, $\tau = 2M$ and $L = 4M$.

References


