Research Report 12/99

Modelling Flexible Bellows by Standard Beam Finite Elements

by

Göran Broman, Madeleine Hermann, Anders Jönsson

Department of Mechanical Engineering
University of Karlskrona/Ronneby
S-371 79 Karlskrona, Sweden

ISSN 1103-1581
ISRN HK/R-RES—99/12—SE

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Printed by Psilander Grafiska, Karlskrona 1999

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Göran Broman[1]
Madeleine Hermann[2]
Anders Jönsson[2]

Department of Mechanical Engineering
University of Karlskrona/Ronneby
Karlskrona, Sweden
1999

Abstract

A procedure for modelling flexible metal bellows by the standard beam finite elements in *I-DEAS Master Series 6* is presented. In spite of the geometry of the bellows being far from a beam, it is shown that the bellows dynamic behaviour can be modelled by beam elements with the suggested procedure. The model size is reduced by a factor of 100-1000 compared to a shell elements model. This is especially advantageous when the bellows is only a part of for example an exhaust system to be optimised with respect to overall design parameters. In comparison to existing “semi-analytical” methods the standard beam finite elements have the advantages that axial, bending and torsion degrees of freedom are included simultaneously and that the interaction between the bellows and the rest of the system, also modelled by beam or shell finite elements, is easily facilitated. The procedure is verified by experimental results from other investigators.

Keywords

Flexible Bellows, Beam Finite Elements, Dynamic Behaviour, Axial, Bending, Torsion, Exhaust Systems, Experimental Verification.

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[1] Assistant professor, Ph.D. Mech. Eng.
[2] Ph.D. Student, M.Sc. Mech. Eng.

Acknowledegements

This work was carried out at the Department of Mechanical Engineering, University of Karlskrona/Ronneby, Karlskrona, Sweden, in collaboration with AP Automotive Systems, Inc, Torsås, Sweden.

We wish to thank the research group at the department for valuable discussion and support. We are also grateful for the support from AP Automotive Systems, Inc, especially M.Sc. Kristian Althini, Research & Development Manager, M.Sc. Per Folkesson, Prediction Engineer, and Anders Svensson, Laboratory Manager.

Financial support from Blekinge Research Foundation is gratefully acknowledged.

Karlskrona, June 1999

Göran Broman
Madeleine Hermann
Anders Jönsson

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1 Notation

A	Area [m^2]
c	Torsion stiffness [Nm/rad]
E	Modulus of elasticity [Pa]
G	Shear modulus of elasticity [Pa]
h	Convolution height [m]
I	Area moment of inertia [m^4]
J	Mass moment of inertia per unit length [kgm]
K	Polar area moment of inertia [m^4]
k	Axial stiffness [N/m]
L	Length [m]
m	Mass per unit length [kg/m]
n	number of convolutions
p	Pressure [Pa]
R	Radius [m]
(r, θ, x)	Cylindrical coordinates
s	Material thickness [m]
t	Time [s]
u	Axial displacement [m]
V	Velocity [m/s]
w	Transverse displacement [m]
(x, y, z)	Cartesian coordinates
α	Shear coefficient
δ	Displacement [m]
θ	Torsion displacement [rad]

ν	Poisson's ratio
ρ	Mass density [kg/m^3]

Indicies

C	Crown
D	Defined
F	Fluid
M	Mean
P	Pipe (equivalent)
R	Root
T	Total
X, y, z, θ	Coordinate axes

2 Introduction

With the introduction of transverse engines and catalytic converters flexible metal bellows have become an important component in automotive exhaust systems. Examples of such bellows are shown in figure 2.1.

Figure 2.1. Examples of flexible metal bellows used in automotive exhaust systems.

A flexible connection between the manifold and the rest of the exhaust system is necessary to allow for the rolling movements of the engine. Since for transverse engine orientation the main direction of the exhaust system downstream the manifold is perpendicular to this rolling, considerable axial and bending deflections at the connection must be allowed for. Some torsion also takes place because of the curved path of the exhaust system. Using a rigid joint would give severe vibration of the exhaust system, with noise and quick failure due to exceeded material strength as consequences.

Furthermore, the connection must be gas-tight to assure successful combustion control, necessary for the catalytic converter to operate at optimal conditions. Any failure of the exhaust system upstream the converter affect emissions. Usually the converter is placed as close to the engine as possible in order to get a quick temperature rise of the catalytic material after starting the engine. The catalytic effect only takes place above a specific, so-called, light-off temperature of approximately 300-500 ^{o}C. Continuously tougher requirements on cutting emissions during the last decades have also driven up combustion temperatures, implying increased thermal expansion of the exhaust system components that must be allowed for to relief otherwise fatal stress.

Metal bellows have proven to fulfil the above combined requirements. So, although invented about hundred years ago they play today an important role in modern automotive engineering and emissions control [1].

Although seemingly simple, proper dimensioning requires deep understanding of the characteristics of the bellows and their interaction with the rest of the exhaust system. Off-the-shelf products will seldom fit a specific application. This was experienced by many car and component fabricators when bellows were first introduced into exhaust systems. Failures took place after rather short operation times and substitution into stronger - and much more expensive - materials did not solve the problem [2].

With this background a collaboration started between the Department of Mechanical Engineering at the University of Karlskrona/Ronneby, Karlskrona, Sweden and AP Automotive Systems, Inc, Torsås, Sweden, fabricating approximately two millions of welded manifolds and supplying six hundred thousand bellows per year for the automotive industry. A project was defined with the overall aim of deepened understanding of the dynamics of the exhaust system as a whole, including manifold, flexible connection, catalytic converter, pipes, mufflers, and hangers. It is desired to find a method of modelling and evaluating customer-proposed exhaust system designs at an early stage of product development. The models need to be rather simple so that the effect of design alterations on natural frequencies and other characteristics can be presented and discussed with the customer promptly. As a part of the overall project the bellows is studied in this work. The collaboration has so far also resulted in two master's theses on bellows [3,4]. Three master's theses are currently in progress including studies on hangers, first attempts of a complete systems analysis, and flow calculations (CFD) in a manifold, respectively.

Flexible metal bellows have been used for considerable time in other applications, such as piping systems of chemical industries, cooling systems, ships, and power plants. Numerous papers deal with various aspects of bellows, such as stresses due to internal pressure and axial deflection, fatigue life estimations, column instability and scrim, etc. A comprehensive bibliography including ninety titles is given by Snedden [5]. A good grasp of bellows research can also be gained from the conference proceedings of the 1989 ASME Pressure Vessels and Piping Conference, including twenty-four titles with many additional references [6]. The extraordinary work of Andersson [7,8], published in 1964-65, must be considered as a milestone. Among other things he derived correction factors relating the behaviour of the bellows convolution to that of a simple strip beam. This approach has subsequently been the basis of standards and other publications presenting formulae for hand-calculation for bellows design.

Such formulae have been included in national pressure vessel codes, among which the ASME code is the most well-known [9]. The most comprehensive and widely accepted text on bellows design is however the Standards of the Expansion Joint Manufacturers Association (EJMA), which has now reached its seventh edition [10]. A comparison of the ASME code and the EJMA standards is given by Hanna [11], concluding that the two conform quite well in most aspects. However, some significant differences exist and are discussed. The EJMA standards is found to comply better with experience, and is therefore recommended except when code approval is necessary.

Comparisons of the EJMA standards with finite element and experimental analyses can be found in many papers, for example Ting-Xin et al [12,13] and Osweiller [14]. The EJMA organisation itself has also conducted research to verify the standards. In summary, the EJMA formulae for stresses and stiffness for axial loading are found to be correct to within approximately 20 % for most bellows configurations. No comparisons regarding torsion loading have been found. An explanation of some of the EJMA formulae is given by Broyles [15].

Less has been found on the dynamic behaviour of bellows and most of these studies concentrate on axial vibrations, for example Gerlach [16] and Jakubauskas and Weaver [17]. Dealing also with transverse vibrations are EJMA [10] and Ting-Xin et al [13], both using Bernoulli-

Euler beam theory to simulate the bellows. Morishita et al [18] and Jakubauskas and Weaver [19,20,21] include rotary inertia in the beam model. The conclusion is that rotary inertia is important to consider, especially for bellows with a low length over diameter ratio. The error can otherwise amount to several hundred per cent.

The concept of fluid added mass to account for inertia effects due to the fluid inside the bellows has been established for long time. The latest developments is that of Jakubauskas and Weaver [19,20,21], which is the first study including the influence of convolution distortion on the fluid added mass. The resistance to motion because of the pumping of fluid in and out of the convolutions, when distorted due to the bending of the bellows, is modelled as added inertia. This is shown to be significant for the higher natural frequencies of bellows containing a high-density fluid such as water. As the fluid in exhaust systems is of low density, all influence of fluid mass can be neglected in the present study. Added masses and other complications of end caps, braid, and inner-liner, present in some bellows, are only briefly discussed in this work.

The drawbacks of all studies of the dynamic behaviour of bellows found in the literature are that they deal with only one type of vibration at the time and treat the bellows as an isolated unit with specified boundary conditions. For these cases various methods for hand-calculations are presented. No descriptions of a bellows model suitable for integrating in a finite element analysis of a whole dynamic system have been found.

The aim of this work is to propose a method of modelling flexible metal bellows by the existing beam element formulation of the computation software *I-DEAS Master Series 6*, in which the whole exhaust system is to be analysed later on. Axial, bending and torsion characteristics of the bellows itself, as well as the interaction with the rest of the system, should be possible to consider. Experimental results will be used for verification.

Of course the bellows could be modelled directly by shell elements. This would be straightforward, but the finite element representation of the bellows would then constitute a very large part of the overall model. Due to its special geometry thousands of elements would be required. It would consequently be very time consuming to perform the analysis of the whole system due to a rather small part of it. This would be especially disadvantageous when many simulations are to be performed when studying the influence of design alterations of the system. Hence the attempt of modelling the bellows by beam elements.

3 Characteristics of Bellows

3.1 Geometry

The basic design of a flexible connection for exhaust systems is shown in figure 3.1. The gas-tight bellows is the main part, and critical to the desired function. An inner-liner is sometimes used for heat protection and to reduce the risk of flow induced vibrations. A braid is sometimes used for protection from outer mechanical influence. The parts are connected at the ends with end caps. This work deals with the bellows itself. See section 4.8 for a brief discussion of some ideas of how the other parts could be modelled.

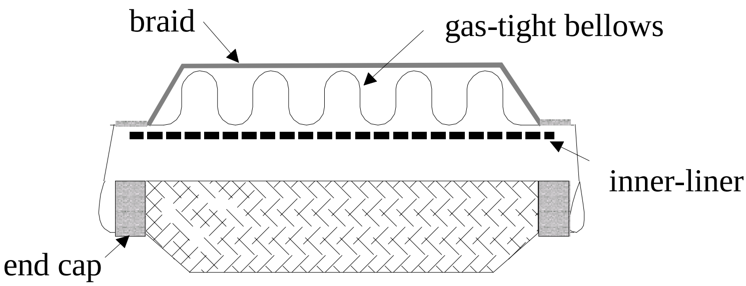

Figure 3.1. Basic flexible connection design.

The bellows convolutions are most commonly corrugated from thin-gage metal using mechanical or hydraulic forming. The geometry of a U-shaped convolution is defined in figure 3.2. R_r is the meridional radius of the convolution root, R_c is the meridional radius of the convolution crown and h is the convolution height. R_m is the mean radius of the bellows, that is, the distance from the bellows center-line to mid convolution height, and s is the material thickness.

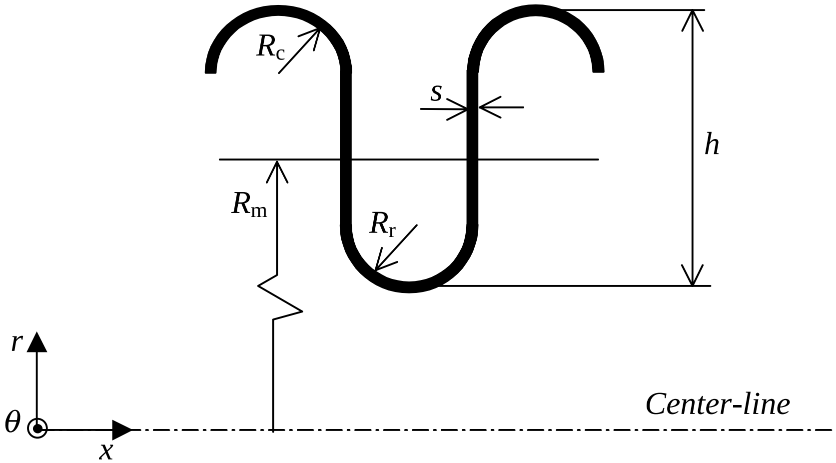

Figure 3.2. U-shaped convolution geometry.

It is assumed that $s << R_r$, R_c, $h << R_m$. With n as the number of convolutions the length of the whole bellows is

$$L = 2(R_r + R_c)n \tag{3.1}$$

With these assumptions it feels natural to think of the bellows as an equivalent thin-walled pipe of length L, radius R_m, and wall thickness s_p, according to figure 3.3.

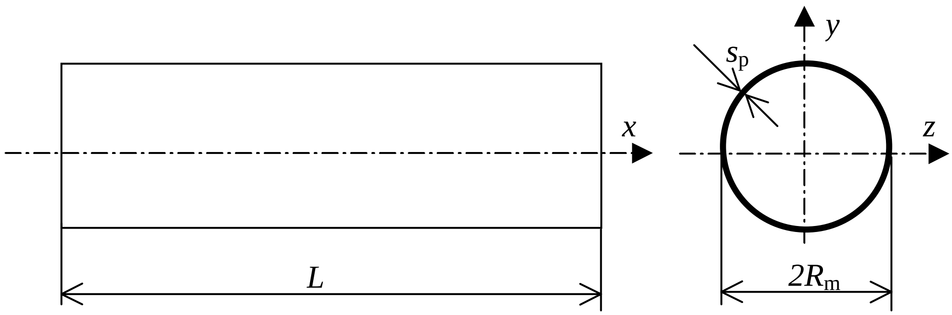

Figure 3.3. Thin-walled pipe analogy of bellows.

By defining special values for wall thickness and material properties this pipe can be a good model of the corresponding bellows.

3.2 Mass and Rotary Inertia

For the real bellows the mass per unit length is varying periodically. The mean value can be determined either by measuring the mass of the bellows and divide by the length of it, or, if the bellows do not yet exist physically, according to

$$m = \frac{\rho 2\pi R_{\mathrm{m}} \left[\pi (R_{\mathrm{r}} + R_{\mathrm{c}}) + 2(h - R_{\mathrm{r}} - R_{\mathrm{c}})\right] s}{2(R_{\mathrm{r}} + R_{\mathrm{c}})} \tag{3.2}$$

where ρ is the mass density of the bellows material. In this work the mass per unit length will be considered constant and equal to this mean value, which is a good approximation except for very low numbers of convolutions. Furthermore, all mass is assumed located at the mean radius of the bellows, in accordance with the thin-walled pipe analogy.

For bending rotations the rotary inertia of the cross-section per unit length is then

$$J = J_{\mathrm{yy}} = J_{\mathrm{zz}} = \frac{mR_{\mathrm{m}}^2}{2} \tag{3.3}$$

and for torsion rotations the rotary inertia of the cross-section per unit length is

$$J_{\mathrm{xx}} = mR_{\mathrm{m}}^2 \tag{3.4}$$

With the product of wall thickness and mass density equal to

$$s_{\mathrm{p}} \rho_{\mathrm{p}} = \frac{m}{2\pi R_{\mathrm{m}}} \tag{3.5}$$

the equivalent pipe will have a mass per unit length, and the cross-section will have rotary inertia per unit length, equal to that of the corresponding bellows.

For example, the real bellows material mass density can be used ($\rho_p = \rho$) and the wall thickness calculated. Alternatively the wall thickness can be set to a small value and the mass density of the fictive pipe material calculated. The *I-DEAS* concept of *Non-structural Mass* can also be involved, see section 4.2. Which method to use is of less importance. The important thing is to simulate the bellows mass distribution, both axially and within the cross-section.

3.3 Axial Vibrations

The axial stiffness can be determined in one of the following ways:

(i) By shell finite element calculations.
(ii) By measurements on the actual bellows.
(iii) By using EJMA formulae.

The advantage of method (i) is that arbitrarily good accuracy can be obtained for the theoretical bellows design. The necessary number of elements is low because only one half convolution has to be modelled and axial symmetry can be utilised.

The advantage of method (ii) is that influences of tolerances in fabrication, material etc. will be automatically included. A disadvantage is that the bellows must already exist, which is often not the case at an early stage of product development and when different design configurations are to be tested. Another disadvantage is of course that measurement inaccuracies will also be included.

The advantage of method (iii) is that it is quick and simple. Only hand-calculations are necessary. A disadvantage is that it may be too inaccurate, depending on the specific bellows at hand. For most commonly used bellows configurations the axial stiffness is however predicted to within approximately 20 % and often better. Another disadvantage is that tolerances in fabrication, material etc. are not included, which is true also for method (i). Some EJMA calculations are given in appendix A.

The relation between the total axial stiffness, k_T, and the axial stiffness of one half convolution, k, is

$$k_{\mathrm{T}} = \frac{k}{2n} \tag{3.6}$$

The axial stiffness of the equivalent pipe is

$$k_{\mathrm{p}} = \frac{E_{\mathrm{p}} A_{\mathrm{p}}}{L} \tag{3.7}$$

where E_p is the modulus of elasticity of the pipe material and

$$A_{\mathrm{p}} = 2\pi R_{\mathrm{m}} s_{\mathrm{p}} \tag{3.8}$$

is the cross-sectional area of the pipe. Now, putting k_p equal to the total axial stiffness of the bellows gives

$$E_{\mathrm{p}} = \frac{k_{\mathrm{T}} L}{A_{\mathrm{p}}} = \frac{k_{\mathrm{T}} L}{2\pi R_{\mathrm{m}} s_{\mathrm{p}}} = \frac{k(R_{\mathrm{r}} + R_{\mathrm{c}})}{2\pi R_{\mathrm{m}} s_{\mathrm{p}}} \tag{3.9}$$

Thus, with this modulus of elasticity the equivalent pipe will have an axial stiffness equal to that of the corresponding bellows.

As the bellows, modelled as the equivalent pipe, is a kind of a rod the differential equation governing the bellows axial vibrations is the same as that of a uniform rod given by for example Weaver et al [22],

$$\frac{\partial^2 u}{\partial t^2} - \frac{E_{\mathrm{p}}}{\rho_{\mathrm{p}}} \frac{\partial^2 u}{\partial x^2} = 0 \tag{3.10}$$

where u is the axial displacement, t is time, and x is the axial coordinate, see figure 3.4.

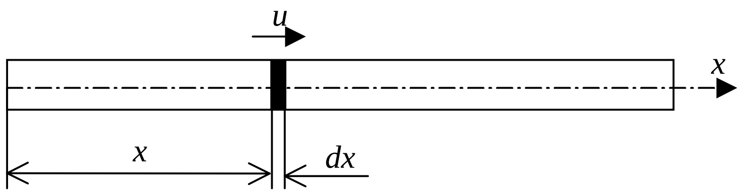

Figure 3.4. Displacements of bellows (pipe/rod) in axial vibration.

For specified boundary conditions analytical solutions for natural frequencies and mode shapes of equation (3.10) can be obtained [22]. Such solutions can be used for verification and convergence test of the finite element model.

3.4 Bending Vibrations

The special geometry of the bellows makes it very flexible in overall bending but very stiff against radial deflections. This means that significant transverse deflections and rotations of cross-sections perpendicular to the bellows axis can take place, with ovalling and shear deformations remaining negligible. This is confirmed by for example Jakubauskas [19]. Thus, although being far from a beam in appearance, a beam representation of the bellows should be relevant.

The bending stiffness of this beam can be expressed in terms of the axial stiffness. The cross-sections of the bellows are assumed to remain plane so the bending stiffness can be expressed in terms of the modulus of elasticity from equation (3.9), and the area moment of inertia for the pipe cross-section, which is

$$I_{\mathrm{p}} = I_{\mathrm{yy}} = I_{\mathrm{zz}} = \pi R_{\mathrm{m}}^{3} s_{\mathrm{p}} \tag{3.11}$$

Thus,

$$E_{\mathrm{p}} I_{\mathrm{p}} = E_{\mathrm{p}} \pi R_{\mathrm{m}}^{3} s_{\mathrm{p}} = \frac{k_{\mathrm{T}} L R_{\mathrm{m}}^{2}}{2} = \frac{k(R_{\mathrm{r}} + R_{\mathrm{c}}) R_{\mathrm{m}}^{2}}{2} \tag{3.12}$$

The equivalent pipe will automatically have this bending stiffness.

The differential equation governing the bending vibrations of the bellows can be obtained from the general differential equation for bending vibrations of a pipe conveying fluid, given by for example Paidoussis and Li [23] or Blevins [24],

$$
\begin{aligned}
&E_{\mathrm{p}} I_{\mathrm{p}} \frac{\partial^4 w}{\partial x^4} + (\rho_{\mathrm{p}} A_{\mathrm{p}} + \rho_{\mathrm{f}} A_{\mathrm{f}}) \frac{\partial^2 w}{\partial t^2} \\
&- \rho_{\mathrm{p}} I_{\mathrm{p}} \left(1 + \alpha \frac{E_{\mathrm{p}}}{G_{\mathrm{p}}}\right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + \alpha \frac{\rho_{\mathrm{p}}^2 I_{\mathrm{p}}}{G_{\mathrm{p}}} \frac{\partial^4 w}{\partial t^4} \\
&+ 2 \rho_{\mathrm{f}} A_{\mathrm{f}} V \frac{\partial^2 w}{\partial t \partial x} + (p \pi R_{\mathrm{m}}^2 + \rho_{\mathrm{f}} A_{\mathrm{f}} V^2) \frac{\partial^2 w}{\partial x^2} = 0
\end{aligned}
\qquad (3.13)
$$

where w is the transverse displacement, see figure 3.5, ρ_{f} is the fluid density, A_{f} is the flow area, V is the flow velocity, and p is the is the fluid pressure.

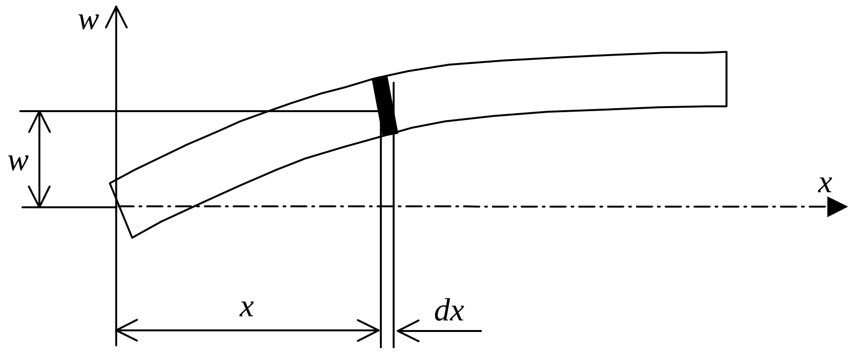

Figure 3.5. Displacements of bellows (pipe/beam) in bending vibration.

The non-dimensional factor α is the shear coefficient, or form factor for shear, in Timoshenko beam theory and depends on the shape of the cross-section [25,26]. The fifth term is the Coriolis force and the sixth term accounts for the pipe curvature effects of pressure and centrifugal force.

The fluid density in exhaust systems is low, so all terms including this is cancelled. The influence of transverse shear for the bellows is negligible as discussed above. The shear coefficient is therefore close to zero. For the bellows used in experiments by Jakubauskas and Weaver [21] the influence of a fluid pressure of 200 kPa on the fundamental frequency was approximately 7 %. For the higher natural frequencies this influence is smaller. Two bar was the maximum pressure allowed by the EJMA standards for that bellows according to Jakubauskas and Weaver. The mean pressure in exhaust systems is rather low, so also the pressure term is cancelled. What remain of equation (3.13) is thus

$$E_{\mathrm{p}} I_{\mathrm{p}} \frac{\partial^4 w}{\partial x^4} + \rho_{\mathrm{p}} A_{\mathrm{p}} \frac{\partial^2 w}{\partial t^2} - \rho_{\mathrm{p}} I_{\mathrm{p}} \frac{\partial^4 w}{\partial x^2 \partial t^2} = 0 \tag{3.14}$$

For specified boundary conditions approximate analytical solutions for natural frequencies and mode shapes of equation (3.14) can be obtained [22]. Such solutions can be used for verification and convergence test of the finite element model.

3.5 Torsion Vibrations

The torsion stiffness can be determined by the same methods as for the axial stiffness. The relation between the total torsion stiffness, c_{T}, and the torsion stiffness of one half convolution, c, is

$$c_{\mathrm{T}} = \frac{c}{2n} \tag{3.15}$$

The torsion stiffness of the equivalent pipe is

$$c_{\mathrm{p}} = \frac{G_{\mathrm{p}} K_{\mathrm{p}}}{L} \tag{3.16}$$

where G_{p} is the shear modulus of elasticity of the pipe material and

$$K_{\mathrm{p}} = I_{\mathrm{yy}} + I_{\mathrm{zz}} = 2I_{\mathrm{p}} = 2\pi R_{\mathrm{m}}^3 s_{\mathrm{p}} \tag{3.17}$$

is the polar area moment of inertia of the pipe cross-section. Now, putting c_{p} equal to the total torsion stiffness of the bellows gives

$$G_{\mathrm{p}} = \frac{c_{\mathrm{T}} L}{K_{\mathrm{p}}} = \frac{c_{\mathrm{T}} L}{2I_{\mathrm{p}}} = \frac{c_{\mathrm{T}} L}{2\pi R_{\mathrm{m}}^3 s_{\mathrm{p}}} = \frac{c(R_{\mathrm{r}} + R_{\mathrm{c}})}{2\pi R_{\mathrm{m}}^3 s_{\mathrm{p}}} \tag{3.18}$$

Thus, with this shear modulus of elasticity the equivalent pipe will have a torsion stiffness equal to that of the corresponding bellows.

As the bellows, modelled as the equivalent pipe, is a kind of a rod the differential equation governing the bellows torsion vibrations is the same as that of a uniform rod given by for example Weaver et al [22],

$$\frac{\partial^2 \theta}{\partial t^2} - \frac{G_{\mathrm{p}}}{\rho_{\mathrm{p}}} \frac{\partial^2 \theta}{\partial x^2} = 0 \tag{3.19}$$

where θ is the torsion angle, see figure 3.6.

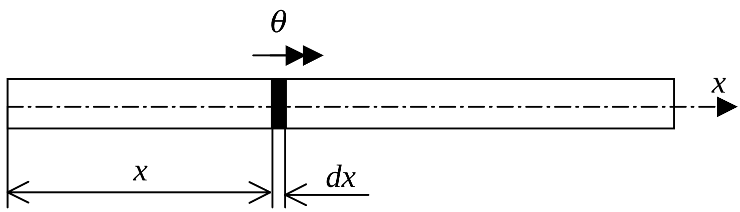

Figure 3.6. Displacements of bellows (pipe/rod) in torsion vibration.

For specified boundary conditions analytical solutions for natural frequencies and mode shapes of equation (3.19) can be obtained [22]. Such solutions can be used for verification and convergence test of the finite element model.

It should be pointed out that bellows are not primarily designed for torsion. The torsion stiffness is high compared to the others. So, the bellows will be highly stressed if torsion deflections are not kept small. Much of the problems when bellows were first introduced into automotive exhaust systems can probably be attributed to this fact. These bellows had rather few convolutions, and as seen from equation (3.15) the total torsion stiffness is inversely proportional to the number of convolutions. Today longer bellows are mostly used. This means on the other hand that axial and bending stiffness, and the corresponding natural frequencies, are also lowered. Although not designed for torsion it is important to consider also this degree of freedom when studying bellows as a part of a dynamic system.

4 Beam Finite Element Model of Bellows

4.1 Beam Elements

Analytical solutions to equations (3.10), (3.14) and (3.19) are difficult to make use of in a dynamic analysis of a whole system of which the bellows is only one part. It is therefore desired to simulate the bellows behaviour by a finite element representation, which is necessary anyway to analyse the whole system.

Of course the bellows could be modelled directly by shell elements. Due to its special geometry thousands of elements would however be required. In this chapter the possibility of modelling the bellows by the standard beam elements of *I-DEAS Master Series 6* [27] is discussed.

Either the two-noded linear or the isoparametric three-noded parabolic beam element included in the *I-DEAS Simulation* module is suitable for modelling the bellows (pipe). Each node has three translational degrees of freedom and three rotational degrees of freedom.

The cross-sectional properties of the beam elements can be defined in the *Beam Sections* task in two ways:

(i) By drawing the cross-section geometry. The software then calculates the cross-sectional properties from the given geometry.

(ii) By explicitly key in the cross-sectional properties in the *Key-In* dialogue frame.

Method (ii) together with the characteristics of the beam finite element formulation makes it possible to simulate the bellows as described in the following sections.

4.2 Mass and Rotary Inertia

Mass can be applied to the beam element model in three ways:

(i) By defining a mass density of the beam material in the *Material Property Table*.

(ii) By defining a *Non-structural Mass* per unit length in the *Physical Property Table*.

(iii) By defining lumped mass in the *Create Individual Element* command.

The software combines methods (i) and (ii) so that the mass per unit length will be

$$m = \rho A + m_{\mathrm{d}} \tag{4.1}$$

where ρ is the mass density of the beam material, A is the cross-sectional area, and m_{d} is the defined *Non-structural Mass* per unit length. For bending rotations the rotary inertia of the cross-section per unit length will be

$$\begin{aligned} J_{\mathrm{yy}} &= (\rho + \frac{m_{\mathrm{d}}}{A}) I_{\mathrm{yy}} \\ J_{\mathrm{zz}} &= (\rho + \frac{m_{\mathrm{d}}}{A}) I_{\mathrm{zz}} \end{aligned} \tag{4.2}$$

where I_{yy} and I_{zz} are the area moments of inertia of the cross-section with respect to the y- and z-axis, respectively. For torsion rotations the rotary inertia of the cross-section per unit length will be

$$J_{\mathrm{xx}} = (\rho + \frac{m_{\mathrm{d}}}{A})(I_{\mathrm{yy}} + I_{\mathrm{zz}}) \tag{4.3}$$

Thus, the software uses information about the cross-section inherent in the area and the area moment of inertia to calculate the mass moments of inertia for the cross-section. Choosing for example method (i) ($m_{\mathrm{d}} = 0$ as default) for the pipe cross-section, equations (4.1), (4.2) and (4.3) together with equations (3.8), (3.11) and (3.17) give

$$m = \rho_{\mathrm{p}} A_{\mathrm{p}} = \rho_{\mathrm{p}} 2\pi R_{\mathrm{m}} s_{\mathrm{p}} \tag{4.4}$$

$$J = J_{\mathrm{yy}} = J_{\mathrm{zz}} = \rho_{\mathrm{p}} I_{\mathrm{p}} = \rho_{\mathrm{p}} \pi R_{\mathrm{m}}^3 s_{\mathrm{p}} = \frac{\rho_{\mathrm{p}} A_{\mathrm{p}} R_{\mathrm{m}}^2}{2} = \frac{m R_{\mathrm{m}}^2}{2} \tag{4.5}$$

$$J_{xx} = \rho_p 2I_p = \rho_p 2\pi R_m^3 s_p = \rho_p A_p R_m^2 = mR_m^2 \quad (4.6)$$

in agreement with equation (3.3), (3.4) and (3.5). The same will result from method (ii) if instead ρ is set to zero and m_d is set to the desired m. A warning from the software can be avoided by giving a small value for ρ instead of exactly zero. For simplicity it is recommended to use only one of the methods, although it is possible to combine them (both $\rho \neq 0$ and $m_d \neq 0$) for the same result. Which method to choose is of less importance. Method (iii) simulates a point mass. Thus, by that method the rotary inertia would not be included.

In conclusion it is possible to correctly simulate the bellows' (pipe's) mass and mass moments of inertia in the beam finite element model.

4.3 Axial Vibrations

The *I-DEAS* beam finite element formulation of axial displacements is based on the theory of a uniform rod. Thus, for axial vibrations of the equivalent pipe the software produces approximate solutions to the differential equation (3.10).

In conclusion the beam finite element model will therefore correctly simulate the axial vibrations of the bellows (pipe), provided that the pipe properties have been chosen according to chapter 3.

For more about the element formulation, see the interactive documentation *Smart View* in *I-DEAS Master Series 6* [27] and background texts by Przemieniecki [28] and Hinton and Owen [29].

4.4 Bending Vibrations

The *I-DEAS* beam finite element formulation of bending is based on Timoshenko beam theory, which includes deformation due to transverse shear and in dynamics the influence of rotary inertia [22,27,28,29]. Thus, for bending vibrations of the equivalent pipe the software produces approximate solutions to the differential equation

$$E_{\mathrm{p}} I_{\mathrm{p}} \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} - J\left(1 + \alpha \frac{E_{\mathrm{p}}}{G_{\mathrm{p}}}\right) \frac{\partial^4 w}{\partial x^2 \partial t^2} + \alpha \frac{\rho J}{G_{\mathrm{p}}} \frac{\partial^4 w}{\partial t^4} = 0 \tag{4.7}$$

Equations (3.14) and (4.7) are identical if the shear coefficient is set to zero in the beam element model. In the *I-DEAS* beam finite element formulation the corresponding parameter is called *Shear Area Ratio*. Using the *Key-In* method to define the cross-sectional properties the influence of transverse shear can be cancelled by setting this *Shear Area Ratio* to a small value, for example 10^{-10} (zero is not accepted).

In conclusion the beam finite element model will correctly simulate the bending vibrations of the bellows (pipe), provided that the pipe properties have been chosen according to chapter 3 and software parameters according to above.

4.5 Torsion Vibrations

The *I-DEAS* beam finite element formulation of torsion displacements is based on the theory of a uniform rod. Thus, for torsion vibrations of the equivalent pipe the software produces approximate solutions to the differential equation (3.19).

To simulate both the low axial stiffness and the high torsion stiffness of the bellows by setting the shear modulus of elasticity greater than the modulus of elasticity the standard equation of a real material

$$G_{\mathrm{p}} = \frac{E_{\mathrm{p}}}{2(1+\nu)} \tag{4.8}$$

must be overridden. This is possible in *I-DEAS* by defining Poisson's ratio, ν, as a *Null Property* in the *Material Property Table*. Had transverse shear been significant for the bellows, bending vibrations would have been affected by the value of the shear modulus of elasticity. However, with the *Shear Area Ratio* set to zero according to above the value of the shear modulus of elasticity is irrelevant to the bending behaviour.

Alternatively equation (4.8) could have been fulfilled for a reasonable value of Poisson's ratio and instead the polar area moment of inertia K_{p} changed according to equation (3.18). This fictive value could then be given in the *Key-In* dialogue frame in the *Beam Sections* task. The mass moment of inertia for torsion would not be affected since in the software this is based on the area moment of inertia according to equation (4.3) and not the polar area moment of inertia. Which method to use is of less importance.

Thus, the thin-walled pipe analogy do not hold entirely. It should be remembered that the bellows is not a pipe, rod or beam. With the special geometry and characteristics of the bellows it is hardly surprising that trying to model it in such a way will lead to some contradiction. Of course, no real beam can combine all bellows characteristics. It would then be unnecessary to fabricate bellows.

In conclusion the beam finite element model will correctly simulate the torsion vibrations of the bellows (pipe), provided that the pipe properties have been chosen according to chapter 3 and software parameters according to above.

4.6 Summary of Modelling Procedure

The modelling procedure for finding natural frequencies and mode shapes is summarised bellow. Some experience of *I-DEAS Simulation* is required since all details regarding software handling are not described.

1. Declare geometry and material properties of the bellows: mean radius, R_m, root radius, R_r, crown radius, R_c, convolution height, h, material thickness, s, number of convolutions, n, material mass density, ρ, modulus of elasticity, E, Poisson's ratio, ν.
2. Determine the length of the bellows according to equation (3.1) or by measurements.
3. Determine the mass per unit length of the bellows according to equation (3.2) or by measurements.
4. Determine the mass density, ρ_p and wall thickness, s_p of the equivalent pipe according to equation (3.5).
5. Determine the axial stiffness, k or k_T, and the torsion stiffness, c or c_T, of the bellows by one of the methods described in section 3.3.
6. Determine the modulus of elasticity, E_p, of the equivalent pipe material according to equation (3.9) and the shear modulus of elasticity, G_p, of the equivalent pipe material according to equation (3.18).

7. Define the cross-section of the equivalent pipe in the *Beam Sections* task. The properties area, A_p, area moment of inertia, I_p, and polar area moment of inertia, K_p can by calculated by equations (3.8), (3.11), and (3.17), respectively, and given in the *Key-In* dialogue frame (method (ii) of section 4.1).

   Alternatively the cross-section of the equivalent pipe can be drawn (method (i) of section 4.1). The software then calculates these properties. Now calling the *Key-In* dialogue frame, with the drawn section active, will result in a dialogue frame already completed with these properties.

   In any case, set (change) the *Shear Area Ratio* to a small number. Name the *Beam Section*.
8. Define the material of the equivalent pipe. Use for example the *Quick Create* command, in the *Meshing* task, and change values of mass density to ρ_p, modulus of elasticity to E_p, and shear modulus of elasticity to G_p. Define Poisson's ratio as a *Null Property*. Name the *Material*.
9. Create linear or parabolic beam elements at the proper location in the *Meshing* task.

   In the case of a bellows being modelled as a part of a system the beam elements representing the bellows are of course located at the actual location of the bellows in the assembly. In the case of a bellows studied as a separate unit, beam elements are created on an arbitrary line of the length of the bellows.
10. Create boundary conditions in the *Boundary Conditions* task.

   In the case of a bellows being modelled as a part of a system the proper interaction (mutual boundary conditions) must be simulated. Since the rest of the system is also modelled by beam or shell elements this is readily fulfilled by simply connecting the end nodes of the bellows model to the corresponding nodes of the rest of the system. Boundary conditions for the whole system are then defined.

   In the case of a bellows studied as a separate unit, proper boundary conditions are applied to the end nodes.
11. Perform a *Normal Mode Dynamics* analysis and study the results.

4.7 Examples and Verification

The procedure above is exemplified for the bellows used for experiments by Jakubauskas and Weaver [21], and one of the bellows used for experiments by Ting-Xin et al [13].

4.7.1 Specimen from Jakubauskas and Weaver

1. Geometry and material properties.

R_m	=	84,2 mm
R_r	=	3,53 mm
R_c	=	2,48 mm
h	=	15,7 mm
s	=	0,368 mm
n	=	13
ρ	=	7860 kg/m^3
E	=	$2{,}07 \cdot 10^{11}$ Pa
ν	=	0,3

2. Length.

$$L = 2 \cdot (3{,}53 + 2{,}48) \cdot 13 = 156 \text{ mm.}$$

The measured length was 155,5 mm.

3. Mass per unit length.

$$m = \frac{7860 \cdot 2\pi \cdot 0{,}0842}{2 \cdot (0{,}00353 + 0{,}00248)} \cdot (\pi \cdot (0{,}00353 + 0{,}00248)$$
$$+ 2(0{,}0157 - 0{,}00353 - 0{,}00248)) \cdot 0{,}000368$$

= 4,87 kg/m.

The measured mass per unit length was 4,631 kg/m.

4. Mass density and wall thickness of the equivalent pipe.

Using the measured mass per unit length and setting the wall thickness to

$s_{\mathrm{p}} = 1$ mm,

the fictive density becomes

$$\rho_{\mathrm{p}} = \frac{4,631}{2\pi \cdot 0,0842 \cdot 0,001} = 8750 \ \text{kg/m}^3.$$

5. Axial and torsion stiffness.

One half convolution is modelled in *I-DEAS Simulation* according to figure 4.1.

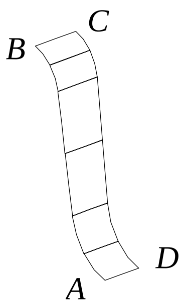

Figure 4.1. Shell element model of half convolution.

Because there is no variation in the circumferential direction it is only necessary to model a thin sector, using only one element in that direction, provided that proper boundary conditions are prescribed. These are given in table 4.1. In the radial direction six isoparametric eight-noded parabolic thin shell elements are sufficient. The difference in deflection result by using instead 12 elements is below 0,15 %, implying that the model has converged.

Table 4.1. Boundary conditions when determining axial stiffness.

Edge	Translation			Rotation		
	r	θ	x	r	θ	x
AB	Free	Zero	Free	Zero	Free	Zero
BC	Free	Zero	Zero	Zero	Zero	Zero
CD	Free	Zero	Free	Zero	Free	Zero
AD	Free	Zero	Free	Zero	Zero	Zero

A unit force is applied to edge *AD* in the axial direction and the corresponding deflection δ_x is calculated. With a sector angle of two degrees the axial stiffness for one half convolution becomes

$$k = \frac{360}{2} \cdot \frac{1}{\delta_x} = \frac{360}{2} \cdot \frac{1}{1{,}62 \cdot 10^{-4}} = 1{,}11 \text{ MN/m.}$$

in good agreement with k = 1,126 MN/m calculated by the finite element method by Jakubauskas and Weaver. The EJMA standards (see appendix A) gives k = 1,1 MN/m.

The same finite element model is used when calculating the torsion stiffness, but the boundary conditions are now according to table 4.2. The difference in deflection result by using instead 12 elements is below 0,1 %, implying that the model has converged.

Table 4.2. Boundary conditions when determining torsion stiffness.

Edge	Translation			Rotation		
	r	θ	x	r	θ	x
AB	Zero	Free	Zero	Free	Free	Free
BC	Zero	Zero	Zero	Zero	Zero	Zero
CD	Zero	Free	Zero	Free	Free	Free
AD	Zero	Free	Zero	Zero	Zero	Zero

A unit force is applied to edge *AD* in the circumferential direction and the corresponding deflection δ_θ is calculated. With a sector angle of two degrees the torsion stiffness for one half convolution becomes

$$c = \frac{360}{2} \cdot \frac{1 \cdot (R_\mathrm{m} - \frac{h}{2})^2}{\delta_\theta} = \frac{360}{2} \cdot \frac{1 \cdot (0{,}0842 - \frac{0{,}0157}{2})^2}{1{,}88 \cdot 10^{-7}} = 5{,}58 \text{ MNm/rad.}$$

The EJMA standards (see appendix A) gives c = 4,5 MNm/rad. Clearly the error in the EJMA standards prediction of torsion stiffness is rather large. Jakubauskas and Weaver do not deal with torsion for this bellows.

6. Modulus of elasticity and shear modulus of elasticity.

The modulus of elasticity of the equivalent pipe material becomes

$$E_\mathrm{p} = \frac{1{,}11 \cdot 10^6 (0{,}00353 + 0{,}00248)}{2\pi \cdot 0{,}0842 \cdot 0{,}001} = 12{,}6 \quad \text{MPa.}$$

The shear modulus of elasticity of the equivalent pipe material becomes

$$G_\mathrm{p} = \frac{5{,}58 \cdot 10^6 (0{,}00353 + 0{,}00248)}{2\pi \cdot 0{,}0842^3 \cdot 0{,}001} = 8{,}94 \text{ GPa.}$$

As seen $G_\mathrm{p} > E_\mathrm{p}$, implying a negative ν if equation (4.8) was to be fulfilled.

7. Cross-section.

The cross-section of the equivalent pipe is drawn in the *Beam Sections* task. Then the *Key-In* dialogue frame is called and the *Shear Area Ratio* is changed to 10^{-10} for both the *y*- and *z*-direction. The section is named *Bellows Section 1*.

8. Material.

By the *Quick Create* command the mass density is changed into ρ_p, the modulus of elasticity is changed into E_p, and the shear modulus of elasticity is changed into G_p. Poisson's ratio is defined as a *Null Property*. The material is named *Bellows Material 1*.

9. Elements.

In this case the bellows is studied as a separate unit so a line of L = 156 mm is drawn on the workplane in the *Master Modeler* task. Beam elements are created on this line in the *Meshing* task.

10. Boundary conditions.

Jakubauskas and Weaver used fixed boundary conditions at both ends of the bellows. So in the *Boundary Conditions* task all degrees of freedom are fixed at the end nodes of this model.

11. Normal mode dynamics analysis.

The model is analysed in *Normal Mode Dynamics* using Lanczos method for solving the eigenvalue problem. The first four bending modes are given and compared to the results of Jakubauskas and Weaver in table 4.3.

Sufficient accuracy for the bending modes is achieved by ten to twenty linear beam elements. The difference in result for the fourth natural frequency between using ten and twenty elements is below 0,3 %, implying that the model has converged. The differences for the lower natural frequencies are even less.

The convergence for axial and torsion vibrations are a bit slower. The first four axial natural frequencies are analytically (from [22]) 122, 243, 365, and 487 Hz. The same predicted by twenty linear beam elements are, with the difference to the analytical result in parenthesis, 122 (0 %), 244 (0,4 %), 368 (0,8 %), 495 (1,6 %).

The first four torsion natural frequencies are analytically (from [22]) 3240, 6480, 9720, and 13000 Hz. The same predicted by twenty linear beam elements are, with the difference to the analytical results in parenthesis, 3240 (0 %), 6510 (0,4 %), 9810 (0,9 %), 13200 (1,5 %).

Table 4.3. Bending modes of sample bellows.
The first column includes frequencies calculated in this work, the second column includes frequencies calculated in [21] (adjusted for the slight difference in axial stiffness and length used in the calculations) and the third column includes frequencies measured in [21]. The differences between frequencies calculated in this work and measured frequencies from [21] are given in the fourth column.

Mode	Frequency [Hz]			
	Calculated (this work)	Calculated (from [21])	Measured (from [21])	Difference [%]
1	197	196	202	2,5
2	326	325	337	3,3
3	462	449	475	2,8
4	585	571	606	3,6

4.7.2 Specimen from Ting-Xin et al

1. Geometry and material properties.

R_m = 173,5 mm
R_r = 4,9 mm
R_c = 6,3 mm
h = 24,5 mm
s = 0,47 mm
n = 9
ρ = 7950 kg/m^3
E = $2{,}0 \cdot 10^{11}$ Pa
ν = 0,3

2. Length.

$$L = 2 \cdot (4{,}9 + 6{,}3) \cdot 9 = 202 \ \text{mm}.$$

In addition to this corrugated length a 15 mm straight pipe section is present at each end. The thickness of this is 0,49 mm.

3. Mass per unit length.

$$m = \frac{7950 \cdot 2\pi \cdot 0{,}1735}{2 \cdot (0{,}0049 + 0{,}0063)} \cdot (\pi \cdot (0{,}0049 + 0{,}0063)$$

$$+ 2(0{,}0245 - 0{,}0049 - 0{,}0063)) \cdot 0{,}00047$$

= 11,2 kg/m.

4. Mass density and wall thickness of the equivalent pipe.

Setting the wall thickness to

$$s_{\mathrm{p}} = 1 \ \text{mm},$$

the fictive density becomes

$$\rho_{\mathrm{p}} = \frac{11{,}2}{2\pi \cdot 0{,}1735 \cdot 0{,}001} = 10300 \ \text{kg/m}^3.$$

5. Axial and torsion stiffness.

One half convolution is modelled in *I-DEAS Simulation* in the same way as in the previous example. The axial stiffness for one half convolution becomes

$$k = \frac{360}{2} \cdot \frac{1}{\delta_{\mathrm{x}}} = \frac{360}{2} \cdot \frac{1}{1{,}47 \cdot 10^{-4}} = 1{,}22 \ \text{MN/m}.$$

The EJMA standards gives k = 1,2 MN/m.

The torsion stiffness for one half convolution becomes

$$c = \frac{360}{2} \cdot \frac{1 \cdot (R_{\mathrm{m}} - \frac{h}{2})^2}{\delta_\theta} = \frac{360}{2} \cdot \frac{1 \cdot (0{,}1735 - \frac{0{,}0245}{2})^2}{1{,}22 \cdot 10^{-7}} = 38{,}4 \text{ MNm/rad.}$$

The EJMA standards gives c = 25,4 MNm/rad. Clearly the error in the EJMA standards prediction of torsion stiffness is rather large. Ting-Xin et al do not deal with torsion for this bellows.

6. Modulus of elasticity and shear modulus of elasticity.

The modulus of elasticity of the equivalent pipe material becomes

$$E_{\mathrm{p}} = \frac{1{,}22 \cdot 10^6 (0{,}0049 + 0{,}0063)}{2\pi \cdot 0{,}1735 \cdot 0{,}001} = 12{,}5 \quad \text{MPa.}$$

The shear modulus of elasticity of the equivalent pipe material becomes

$$G_{\mathrm{p}} = \frac{38{,}4 \cdot 10^6 (0{,}0049 + 0{,}0063)}{2\pi \cdot 0{,}1735^3 \cdot 0{,}001} = 13{,}1 \text{ GPa.}$$

7. Cross-section.

The cross-section of the equivalent pipe is drawn in the *Beam Sections* task. Then the *Key-In* dialogue frame is called and the *Shear Area Ratio* is changed to 10^{-10} for both the *y*- and *z*-direction. The section is named *Bellows Section 2*. The cross section of the straight end sections is also defined. This is called *Ends of Bellows 2*.

8. Material.

By the *Quick Create* command the mass density is changed into ρ_p, the modulus of elasticity is changed into E_p, and the shear modulus of elasticity is changed into G_p. Poisson's ratio is defined as a *Null Property*. The material is named *Bellows Material 2*. The material of the straight end sections is also defined. This is called *Real Bellows Material 2*.

9. Elements.

In this case the bellows is studied as a separate unit so a line of L = 202 mm, plus one line of 15 mm in each end for the straight sections, are drawn on the workplane in the *Master Modler* task. Beam elements with cross-section *Bellows Section 2* and material *Bellows Material 2* are created on the line representing the convoluted part. Beam elements with cross section *Ends of Bellows 2* and material *Real Bellows Material 2* are created on the lines representing the end sections.

10. Boundary conditions.

Ting-Xin et al used fixed boundary conditions at both ends of the bellows. So in the *Boundary Conditions* task all degrees of freedom are fixed at the end nodes of this model.

11. Normal mode dynamics analysis.

The model is analysed in *Normal Mode Dynamics* using Lanczos method for solving the eigenvalue problem. The first four axial modes are given and compared to the results of Ting-Xin et al in table 4.4. The first four bending modes are given and compared to the results of Ting-Xin et al in table 4.5.

The first four torsion natural frequencies are 2630, 5280, 7950, and 10700 Hz.

Table 4.4. Axial modes of sample bellows.
The first column includes frequencies calculated in this work and the second column includes frequencies measured in [13]. The differences are given in the third column.

Mode	Frequency [Hz]		
	Calculated (this work)	Measured (from [13])	Difference [%]
1	86	83	3,6
2	173	164	5,5
3	261	-	-
4	351	-	-

Table 4.5. Bending modes of sample bellows.
The first column includes frequencies calculated in this work and the second column includes frequencies measured in [13]. The differences are given in the third column.

Mode	Frequency [Hz]		
	Calculated (this work)	Measured (from [13])	Difference [%]
1	157	-	-
2	240	-	-
3	338	325	4
4	421	-	-

4.7.3 Discussion of Results

As seen from tables 4.3-4.5 the agreement between theory and experiments is excellent, implying that the bellows can actually be modelled by standard beam elements according to the above procedure.

An interesting observation is that the measured frequency of 325 Hz in table 4.5 is claimed to be the first natural frequency by Ting-Xin et al. This must be a misunderstanding due to the simple beam theory used in their work. With Bernoulli-Euler beam theory the first mode becomes close to the measured value. However, including the rotary inertia of the cross-section it is clear that the first natural frequency (157 Hz) is far below the measured value, but the third natural frequency is close to it. Most likely Ting-Xin et al have actually measured the third natural frequency and have been mislead by their theoretical calculations to classify it as the first mode.

The torsion natural frequencies are much higher than the axial and bending natural frequencies. However, if the bellows is a part of a curved exhaust system it is important to include also this degree of freedom. In interaction with the rest of the system torsion modes may well appear at lower frequencies. No experimental results of torsion natural frequencies have been found in the literature.

4.8 Modelling of end caps, braid and inner-liner

If end caps, braid and inner-liner are present the dynamic behaviour of the flexible connection is of course influenced in some way.

The end cap can be modelled as a separate beam element with its actual pipe cross-section, which is connected to the neighbour element of the bellows model. This connection exists in reality for all degrees of freedom.

For the braid it is more complicated. In bending vibrations the braid is forced to move with the bellows, so in that case the mass and stiffness could be added to the beam elements representing the bellows. For axial and torsion vibration it is however not obvious that the braid must move in phase with the bellows for all modes. For axial and torsion vibrations bellows and braid should then be uncoupled. This could probably be facilitated by describing the braid by separate beam elements that are connected to the bellows elements only for some degrees of freedom. The *End Release* command available for the linear beam element in *I-DEAS Master Series 6* will then probably come in handy. Non-linearity of the braid due to friction is probably negligible. It may however imply different stiffness for push and pull deformations. If the braid is tightly wrapped around the bellows the stiffness for pulling will be much higher than for pushing. When the ends of the flexible connection is pulled apart the diameter of the braid strive to decrease. With the high radial stiffness of the bellows this is almost prevented. In this case the fibres in the braid must themselves be stretched. On the other hand, when the ends of the bellows are pushed towards each other, the diameter of the braid strives to increase. This is not prevented by the bellows. The fibres in the braid will then only change orientation, which they do without much resistance.

For the inner-liner it is also more complicated. Non-linearity due to friction is probably rather high. It is therefore thought to be better to model the inner-liner separately and connect it to the bellows in some way in the overall systems analysis. The complication regarding different connections to the elements representing the bellows for axial, bending and torsion modes discussed for the braid, holds also for the inner-liner.

The above components with their complications are not elaborated further in this work.

5 Conclusions

In this work a procedure for modelling flexible metal bellows by the existing beam finite element formulation of the computation software *I-DEAS Master Series 6* has been suggested. Experimental results from other investigators have been used for verification. The agreement is excellent.

In spite of the geometry of the bellows being far from a beam, it has been shown that the bellows dynamic behaviour can be modelled by beam elements. In this way the model size is reduced by a factor of 100-1000 compared to a shell elements model. This is especially advantageous when the bellows is only a part of for example an exhaust system to be optimised with respect to overall design parameters.

In comparison to “semi-analytical” methods the beam finite elements have the advantages that axial, bending and torsion degrees of freedom are included simultaneously and that the interaction between the bellows and the rest of the system, also modelled by beam or shell finite elements, is easily facilitated.

If braid and inner-liner are present the dynamic behaviour of the flexible connection will be influenced and the dynamic analysis more complicated. Most probably it will then be necessary to consider non-linear effects. Some ideas regarding these complications have been discussed in this work, but primarily these components are suggested to be subject for further investigations. Another suggestion for further work is the study of torsion of the bellows. Only little has been found about this in the literature and exciting formulae in for example the EJMA standards have been found considerably poorer than the corresponding formulae for axial deformations.

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Appendix A

EJMA Stiffness Calculations

The axial stiffness expression for one half convolution according to the EJMA standards, converted into SI-Units and the notation of this work, is

$$k = 6{,}80\frac{R_{\mathrm{m}}Es^{3}}{h^{3}C_{\mathrm{f}}} \tag{A.1}$$

where C_{f} is a non-dimensional coefficient given in a diagram in the standards (SECTION C/PAGE 128). This coefficient is a function of the non-dimensional groups

$$\frac{(R_{\mathrm{r}} + R_{\mathrm{c}})}{1{,}1\sqrt{2R_{\mathrm{m}}s}} \tag{A.2}$$

$$\frac{(R_{\mathrm{r}} + R_{\mathrm{c}})}{h} \tag{A.3}$$

For the bellows in section 4.7 these become

$$\frac{(0{,}00353 + 0{,}00254)}{1{,}1\sqrt{2\cdot 0{,}0842\cdot 0{,}000368}} = 0{,}694$$

$$\frac{(0{,}00353 + 0{,}00254)}{0{,}0157} = 0{,}383$$

giving C_{f} = 1,4. Thus, The axial stiffness for one half convolution for this bellows becomes

$$k = 6{,}8\frac{0{,}0842\cdot 2{,}07\cdot 10^{11}\cdot 0{,}000368^{3}}{0{,}0157^{3}\cdot 1{,}4} = 1{,}1 \quad \text{MN/m.}$$

The torsion stiffness expression for one half convolution according to the EJMA standards, converted into SI-Units and the notation of this work, is

$$c = 12{,}6 \frac{(R_\mathrm{m} - \frac{h}{2})^3 Gs}{1{,}14(R_\mathrm{r} + R_\mathrm{c}) + 2h} \tag{A.4}$$

where s is the nominal material thickness, that is the thickness of the material before the convolutions have been formed.

With

$$G = \frac{E}{2(1+\nu)} = \frac{2{,}07 \cdot 10^{11}}{2(1+0{,}3)} = 7{,}96 \cdot 10^{10} \quad \text{Pa}$$

the torsion stiffness for one half convolution becomes

$$c = 12{,}6 \frac{(0{,}0842 - \frac{0{,}0157}{2})^3 \cdot 7{,}96 \cdot 10^{10} \cdot 0{,}000386}{1{,}14(0{,}00353 + 0{,}00254) + 2 \cdot 0{,}0157} = 4{,}5 \quad \text{MNm/rad.}$$

HÖGSKOLAN
UNIVERSITY
KARLSKRONA
RONNEBY

Correspondence to:

Dr Göran Broman
Department of Mechanical Engineering
University of Karlskrona/Ronneby
371 79 Karlskrona, SWEDEN

Telephone: +46 455-38 55 04
Fax: +46 455-38 55 07
E-mail: Goran.Broman@ima.hk-r.se