Duality in Multi-Commodity Market Computations

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Overview

- Introduction to electronic commerce.
- A price-oriented any-time algorithm.
- A resource-oriented multi-commodity algorithm.
- Conclusions

Example 1 - Selling a Motor Cycle

\[ \begin{align*}
\text{motorcycle} & \succ \text{desk} \quad \text{if} \quad \$1000 \\
\text{motorcycle} & \succ \text{desk} \quad \text{if} \quad \$1000
\end{align*} \]

\( \succ \) means “is preferred over”

Example 2 - Selling Electricity

Thinkable Market Design

- Announce on a server, minimum price $1000.
- Collect bids.
- Select the highest.

Properties of the Design

The market algorithm is simple and easy to implement. Major obstacles for this kind of market are related to secure bids and transactions.
### Consumer Preferences

200W $\rightarrow$ 18C $\succ$ $0.1$

210W $\rightarrow$ 19C $\succ$ $0.15$

240W $\rightarrow$ 20C $\succ$ $0.20$

280W $\rightarrow$ 21C $\succ$ $0.15$

and everything in between, e.g., 217.3987W, i.e. an infinite number of points.

### Consumer Preferences (cont.)

The consumer preferences are modeled as *utility functions*.

\[ u(r, m) > u(r', m') \Leftrightarrow \langle r, m \rangle > \langle r', m' \rangle \]

From the utility function one can compute:

- a demand, i.e. how much a consumer is willing to buy at different prices, and
- a price for which the consumer is willing to buy an additional (small) amount of resource at the current allocation.

### Producer Preferences

The preferences of the producer are modeled by

\[ \max p \cdot r - \text{cost}(r), \]

i.e. it maximizes its profits given the costs for production. From this one can compute both the supply and the price as in the case of a consumer.

### Market Design

- Consumers
- Producers

? A Price-Oriented Approach

- Consumers
- Price
- Supply
- Auctioneer

Update price until supply meets demand (WALRAS, Newton methods). With many consumers and producers, they will typically reveal their true preferences (Sandholm and Ygge, IJCAI, 1997).

### Problem

The resource can not be reallocated as long as there is a mismatch between supply and demand, and there might be time constraints.
Contribution 1 of the paper

An algorithm that produces feasible allocations from intermediate prices, also for the case of multi-commodity markets. (This presentation has, for simplicity, only been on two commodities, electricity and money.)

PROPORTION - Basic Principles

Excess demand: 6.
Total demand: 16.
Total supply: 10.

Allocate what is demanded to all agents holding a negative demand, and allocate 10/16 times their demands to all agents having a positive demand.

PROPORTION - Outcome Quality

For obtaining a good approximation of the equilibrium in the analyzed examples, 85 iterations are typically required. After applying PROPORTION after 5 iterations and 99.47% of the average utility improvement is obtained. For 10 iterations the number is 99.96%. The total excess demand is smaller after having applied PROPORTION after 5 iterations than after 85 iterations without PROPORTION.

Suggestion 2 - A Resource-Oriented Approach

Price
Allocation
Consumers
Producers

Advantages of the Resource-Oriented Algorithm

- At each step of the algorithm, feasible allocations are obtained.
- The computation of the price functions is significantly easier than the computation of the demand from the utility functions etc. presented above.
- The algorithm has excellent convergence properties.

Contribution 2 - A Resource-Oriented Algorithm for the Multi-Commodity Case

\[ c_\alpha = -\sum \left( x_i \nabla \alpha \right) \times x_i \cdot \nabla \alpha \cdot \left( \sum x_i \mathbb{V} \nabla \alpha \cdot x_i \mathbb{V} \right) \]

This is not as bad as it looks! It is actually quite easy to implement and a C++ implementation is available on the web. Furthermore, for a wide class of problems the above formula can be reduced to:

\[ c_\alpha = -\sum \left( x_i \nabla \alpha \right) \times x_i \cdot \nabla \alpha \cdot \left( \sum x_i \mathbb{V} \nabla \alpha \cdot x_i \mathbb{V} \right) \]

(The latter result is not part of the presented paper, but is contained in a submitted paper.)
Conclusions

- In computational markets with continuous resources, price-oriented and resource-oriented approaches are conceivable.
- We argued for the duality between price-oriented and resource-oriented and introduced:
  - a novel algorithm to produce feasible allocations from intermediate prices, and
  - a novel resource-oriented algorithm suitable also for the multi-commodity case.