Vibration in metal cutting is a common problem in the manufacturing industry, especially when long and slender tool holders or boring bars are involved in the manufacturing process. Vibration has a detrimental effect on machining. In particular, the surface finish is likely to suffer, but tool life is also most likely to be reduced. Tool vibration also results in loud noise that may disturb the working environment.

The first part of this thesis describes the development of a robust and manually adjustable analog controller capable of actively controlling boring bar vibrations related to internal turning. This controller is compared with an adaptive digital feedback filtered-x LMS controller and it displays similar performance with a vibration attenuation of up to 50 dB.

A thorough experimental investigation of the influence of the clamping properties on the dynamic properties of clamped boring bars is also carried out in second part of the thesis. In relation to this, it is demonstrated that the number of clamping screws, the clamping screw diameter size, the screw tightening torque and the order the screws are tightened, have a significant influence on a clamped boring bar's eigenfrequencies as well as on its mode shape orientation in the cutting speed - cutting depth plane. Also, an initial investigation of nonlinear dynamic properties of clamped boring bars was carried out.

Furthermore, vibration in milling has also been studied in relation to milling tool holders with a long overhang. A basic investigation concerning the spatial dynamic properties of the tool holders of milling machines, both when not cutting and during cutting, has been carried out. Also, active control of milling tool holder vibration has been investigated and a first prototype of an active milling tool holder was implemented and tested. The challenge of transferring electrical power while maintaining good signal quality to and from a rotating object is addressed and a solution to this is proposed.

Finally, vibration is also a problem for the hydroelectric power industry. In Sweden, hydroelectric power plants stand for approximately half of Sweden's electrical power production and are also considered to be a so-called green source of energy. When renovating water turbines in small-scale hydroelectric power plants and modifying them to optimize efficiency, it is not uncommon that disturbing vibrations occur in the power plant. These vibrations have a negative influence on the production capacity and will wear various components quickly. Occasionally, these vibrations may cause severe damage to the power plant. To identify this vibration problem, experimental modal analysis and operating deflection shape analysis were utilized. To reduce the vibration problem, active control using inertial mass actuators was investigated. Preliminary results indicate a significant attenuation of the vibrations.
Analysis of Structural Dynamic Properties and Active Vibration Control Concerning Machine Tools and a Turbine Application

Henrik Åkesson
Abstract

Vibration in metal cutting is a common problem in the manufacturing industry, especially when long and slender tool holders or boring bars are involved in the manufacturing process. Vibration has a detrimental effect on machining. In particular the surface finish is likely to suffer, but tool life is also most likely to be reduced. Tool vibration also results in loud noise that may disturb the working environment.

The first part of this thesis describes the development of a robust and manually adjustable analog controller capable of actively controlling boring bar vibrations related to internal turning. This controller is compared with an adaptive digital feedback filtered-x LMS controller and it displays similar performance with a vibration attenuation of up to 50 dB.

A thorough experimental investigation of the influence of the clamping properties on the dynamic properties of clamped boring bars is also carried out in second part of the thesis. In relation to this, it is demonstrated that the number of clamping screws, the clamping screw diameter size, the screw tightening torque and the order the screws are tightened, have a significant influence on a clamped boring bar’s eigenfrequencies as well as on its mode shape orientation in the cutting speed - cutting depth plane. Also, an initial investigation of nonlinear dynamic properties of clamped boring bars was carried out.

Furthermore, vibration in milling has also been studied in relation to milling tool holders with a long overhang. A basic investigation concerning the spatial dynamic properties of the tool holders of milling machines, both when not cutting and during cutting, has been carried out. Also, active control of milling tool holder vibration has been investigated and a first prototype of an active milling tool holder was implemented and tested. The challenge of transferring electrical power while maintaining good signal quality to and from a rotating object is addressed and a solution to this is proposed.

Finally, vibration is also a problem for the hydroelectric power industry. In Sweden, hydroelectric power plants stand for approximately half of Sweden’s electrical power production and are also considered to be a so-called green source of energy. When renovating water turbines in small-scale hydroelectric power plants and modifying them to optimize efficiency, it is not uncommon that disturbing vibrations occur in the power plant. These vibrations have a negative influence on the production capacity and will wear various components quickly. Occasionally, these vibrations may cause severe damage to the power plant. To identify this vibration problem, experimental modal analysis and operating deflection shape analysis were utilized. To reduce the vibration problem, active control using inertial mass actuators was investigated. Preliminary results indicate a significant attenuation of the vibrations.
Preface

This thesis summarizes my work at the Department of Electrical Engineering at Blekinge Institute of Technology. The thesis is comprised by an introduction followed by six parts:

Part

I  On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry.

II  Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions.

III  Estimation and Simulation of Nonlinear Dynamic Properties of a Boring bar.

IV  Investigation of the Dynamic Properties of a Milling Tool Holder.

V  Preliminary Investigation of Active Control of a Milling Tool Holder.

VI  Noise Source Identification and Active Control in a Water Turbine Application.
Acknowledgments

Firstly, I would like to express my sincere gratitude to professor Ingvar Claesson for giving me the opportunity to begin a PhD candidacy. Special thanks to my supervisor and friend professor Lars Håkansson, for his profound knowledge within the field of both applied signal processing and mechanical engineering, but also for all his help in improving this thesis. I would also like to thank former and present colleagues at the department, especially my co-worker and friend Tatiana for all the constructive discussions, her help and support.

I would like to express my sincere gratitude to Dr. Thomas L Lagö, Adjunct Professor, who provided an industrial touch to my candidacy by introducing me to different vibration problems in industry. Special thanks to the president of Acticut International AB, CEO Rolf Zimmergren, for believing in my abilities. Furthermore, thanks go to the colleagues at Acticut International AB for their help, discussions and good collaboration.

I am indebted to my parents for providing me with a good foundation, giving me the freedom to follow my interest, letting me form my own ideas and supporting me in the pursuit of knowledge. My warmest gratitude is directed to my sisters for their support.

I would also like to thank my wife Lisa for her love, support and patience throughout the years and for being the person she is. Finally, my greatest thanks and love go to the most important people in the world, Pontus and Ebba for their unconditional love.

Henrik Åkesson
Ronneby, November 2009
Contents

Preface ................................................................. vii
Acknowledgments ................................................... ix
Publication list ..................................................... xiii
Introduction ......................................................... 1

Part
I  On the Development of a Simple and Robust Active Control System for
   Boring Bar Vibration in Industry ........................................ 10
II  Analysis of Dynamic Properties of Boring Bars Concerning Different
    Clamping Conditions .................................................. 54
III  Estimation and Simulation of Nonlinear Dynamic Properties of a
    Boring bar ............................................................. 98
IV  Investigation of the Dynamic Properties of a Milling Tool Holder ....... 130
V   Preliminary Investigation of Active Control of a Milling Tool Holder ...... 134
VI  Noise Source Identification and Active Control in a Water Turbine
    Application ............................................................ 152
Publication List

Part I is published as:


Part II is published as:


Part III is published as:


Part IV is published as:


Part V is based on the publication:


Part VI is based on the publication:

Other Publications


H. Äkesson, A. Brandt, T. Lagö, L. Håkansson, I. Claesson, "Operational Modal Analysis of a Boring Bar During Cutting" published at the 1st IOMAC conference, April 26-27, 2005, Copenhagen, Denmark


H. Åkesson, T. Smirnova, L. Håkansson, I. Claesson and T. Lagö, "Analog versus Digital Control of Boring Bar Vibration", Accepted for publication in proceedings of the SAE World Aerospace Congress, WAC, Dallas, Texas, USA, October 3-6, 2005.


H. Åkesson, T. Smirnova, L. Håkansson, I. Claesson and T. Lagö, "Comparison of different controllers in the active control of tool vibration; including abrupt changes in the engagement of metal cutting", Sixth International Symposium on Active Noise and Vibration Control, ACTIVE, Adelaide, Australia, 18-20 September, 2006.


Introduction

Vibration concerns the repetitive motion of an object or objects relative to a stationary frame referred to as the equilibrium of the vibration. Vibrations may be measured in terms of displacement, velocity or acceleration. Vibrations exist everywhere and may have a great impact on the surrounding environment. One general phenomenon of vibration is "self-oscillation" or resonance [1], meaning that a system exposed to even a weak force that excites a resonance, may causes a substantial vibration level that eventually results in damage to or failure of the system. Thus, it is of great importance in engineering design to consider the dynamic properties of the system from a vibration point of view. In Fig. 1, an example of the simplest possible vibrating system, a single degree of freedom (SDOF) system, is presented in conjunction with a diagram describing the displacement $x(t)$ of the mass. An equation according to

$$m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) = f(t)$$

where $f(t)$ is an external force acting on the mass. In general, the dynamic response of a structure cannot be described adequately by a SDOF model. The response usually includes time variations in the displacement shape as well as its amplitude. Thus, a multiple degree of freedom system or a distributed parameter system is usually required to describe the motion or response of a structure [1]. Generalizing to include several masses connected to each other and to the ground by springs and dampers, results in multiple degree of freedom (MDOF) system whose equation of motion may in matrix form be written as [1]

$$[M] \frac{d^2 \{x(t)\}}{dt^2} + [C] \frac{d\{x(t)\}}{dt} + [K] \{x(t)\} = \{f(t)\}$$

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix, $\{x(t)\}$ is the displacement vector for all masses and $\{f(t)\}$ is the force vector representing all external forces applied to each mass.
This way of representing systems is referred to as lumped parameter representation, since each mass in the system is modeled as a concentrated mass (single point). Also, distributed parameter system theory may be utilized for the modeling of structures, resulting in a model which represents an infinite dimensional continuous system in both space and time [1]. The two latest methods have the ability to model real structures, including their spatial dynamic properties, mainly in terms of natural frequencies $f_r$, relative damping coefficients $\zeta_r$ and mode shapes $\{\psi_r\}$.

An application where vibration is a frequent problem may be found in the manufacturing industry, particularly in the workshop, where metal cutting operations such as external and internal turning, boring, milling etc., take place. Vibrations affect the surface finish of the workpiece, tool life and the noise level in the working environment. In order to increase productivity and tool life, and improve the tolerance of machined workpieces, it is necessary to develop methods which increase stability and restrain tool vibration in metal cutting.

In many cases, the tooling structure may be considered a bottleneck with regard to the achievable accuracy imposed by static deflections and cutting regimes, but also in respect of surface finish due to forced and self-excited vibrations [2,3]. A long overhang cantilever tooling structure (e.g. boring bars) is often the critical component [2, 3]. Fig. 2 presents a typical configuration for internal turning using a boring bar with a long overhang, in which a tube is clamped to one side in the chuck by three jaws while the boring bar is clamped to the clamping house.

![Figure 2: Typical configuration for internal turning, illustrating the long boring bar overhang required to turn deep holes.](image)

There are several possible sources of boring bar vibrations, including transient excitations due to rapid movements or to the engagement phase of cutting; periodic excitation related to residual rotor mass unbalance in the spindle-chuck-workpiece system or random excitation from the material deformation process [4]. The two most widely used theories for explaining self-excited chatter or tool vibration are the regenerative effect and the mode coupling effect [5–8]. These theories generally assume that the dynamic interaction of the cutting process and the machine tool structure constitute the basic causes of chatter [5–8].

During cutting, the cutting force $F_r(t)$ is generated between the tool and the workpiece, see Fig. 3 b). The cutting force applied by the material deformation process during turning will strain the tool-boring bar structure and may introduce a relative displacement of the tool and the workpiece, changing the tool and workpiece engagement. This relation between cutting force and tool displacement is commonly described by the feedback system pictured in Fig. 3 a). The causes of instability
are generally considered to derive from mechanisms providing energy, along with the regenerative effect and the mode coupling effect. The regenerative effect is considered to be the most frequent cause of instability and chatter, and may appear when the tool removes an undulation on the workpiece surface that was cut during the previous revolution of the workpiece. Fig. 3 b) illustrates this scenario, where \( h_0(t) \) is the desired cutting depth or chip thickness, \( h(t) \) the actual chip thickness, \( y(t) \) is the displacement of the tool at the time \( t \) and \( y(t - T) \) is the displacement of the tool during the preceding revolution of the workpiece.

Figure 3: a) Block diagram describing the cutting process-machine tool structure feedback system and b) the principle for regenerative chatter where the \( h_0(t) \) is the desired cutting depth or chip thickness, \( h(t) \) the actual chip thickness, \( y(t - T) \) the previous cut shape and \( y(t) \) the present cut shape.

The classic self-excited chatter models fail to explain some types of vibration which might exist in the system [9]. For instance, chips produced by the chip formation process - which is a part of the material deformation process - may indicate a presence of narrow band excitation [10] (see Fig. 4 where a saw-toothed continuous fragmentary chip process is presented). The shape of the chips depends on many factors such as tool geometry, the workpiece material, the cutting speed, and the cutting depth.

Figure 4: a) A model of the chip formation of a saw-toothed continuous fragmentary chip and b) a photo of the chip formation, during continuous cutting [10].
white noise excitation” [9]. The irregularities of the workpiece surface, the chemical composition, the inhomogeneities, the microstructure and the spatial stochastic variation of the hardening [11] result in a cutting force, which may be considered as a stochastic process [12, 13]. Fig. 5 illustrates the material structure of one common work material: chromium molybdenum nickel steel SS 2541-03 (AISI 3239).

![Material Structure of SS 2541-03](image)

Figure 5: The material structure of chromium molybdenum nickel steel SS 2541-03 (AISI 3239).

A number of methods have been proposed to reduce harmful tool vibration. Three of these methods are as follows:

- “trial and error” - the operator tries to adapt the cutting data in an iterative fashion;
- passive control - constructional enhancement of the dynamic stiffness, can be achieved by increasing the structural damping and/or stiffness of the boring bar;
- active control - selective increase of the dynamic stiffness of a fundamental boring bar’s natural frequency.

The ”trial and error” approach requires continuous supervision and control of the machining process by a skilled operator. Passive control is frequently tuned to increase the dynamic stiffness at a certain eigenfrequency, (for example, that of a particular boring bar), making it an inflexible solution [14,15].

Active control, which is based on an adaptive feedback controller and a boring bar with an integrated actuator and vibration sensor, can easily be adapted to various configurations. Thus, active control provides a more flexible, and therefore preferable, solution. Active control of a boring bar can be implemented using either a digital or an analog approach. The use of a digital controller based on a feedback filtered-x LMS algorithm [16] results in substantial attenuation of vibrations, and exhibits stable behavior. However, it should be noted that an analog controller with the corresponding vibration attenuation performance is able to avoid unnecessary delay in control authority and eventual tool failure in the engagement phase of the tool. A digital controller always introduces delay associated with controller processing time, A/D-and D/A- conversion processes and anti-aliasing and reconstruction filtering.
Other benefits with an analog controller may include, low complexity, reduced cost and flexible bandwidth.

Both the analog and digital domains may be utilized for the implementation of feedback controllers [17–19]. Implementation of an active control solution requires a modification of the boring bar structure, and thus of the bar’s dynamic properties. Modifications should be carried out with care to avoid undesired problems that may result from making the boring bar too flexible, or moving boring bar resonance frequencies so that they coincide with other structural resonance frequencies of the machine tool system.

PART I - On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

The application of the active control of boring bar vibration in industry requires reliable, robust adaptive feedback controllers or manually tuned feedback controllers, that are easy to adjust on the workshop floor by the lathe operator. This part of the thesis presents the development of a simple adjustable robust analog controller, based on a digitally controlled, analog design that is suitable for the control of boring bar vibration in industry. Fig. 6 presents a block diagram of a feedback control system.

![Figure 6: Block diagram of a feedback control system.](image)

Initially, a digitally controlled, analog and manually adjustable lead compensator was developed. This manually adjustable controller approach was further developed to provide controller responses appropriate for the active control of boring bar vibration. The controller relies on a lead-lag compensator and enables manual tuning by the lathe operator. The emphasis has been on designing a controller that enables simple adjustment of its gain and phase. This should provide the robust control appropriate for industry application.

Furthermore, this part of the thesis features a comparative evaluation of the two analog controllers and a digital controller based on the feedback filtered-x LMS algorithm [18, 20] capable of providing good performance and robustness in the active control of boring bar vibration. This evaluation includes the consideration of a number of different dynamic properties of the boring bar, produced using a set of different clamping conditions likely to occur in industry. Both the developed analog controller and the adaptive digital controller are able to reduce the boring bar vibration level by up to approximately 50 dB.
PART II - Analysis of the Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

Successful implementation of active control, such as in the method presented in Part I, requires substantial knowledge concerning the dynamic properties of the tooling system. Furthermore, the interface between the boring bar and the lathe (i.e. the clamping house) has a significant influence on the dynamic properties of the clamped boring bar. Part II of this thesis presents the dynamic properties of boring bars for different clamping conditions, based on experimental and analytical results. The different cases reflect on the variations that may be introduced in the clamping conditions of a boring bar when the operator mounts the boring bar in the clamping house and tightens the clamp screws, as illustrated in Fig. 7. Thus, this section focuses on those dynamic properties of a boring bar which arise due to different clamping conditions of the boring bar introduced by a clamping house. In connection with this, Euler-Bernoulli modeling of a clamped boring bar with emphasis on the modeling of the clamping conditions was considered.

Figure 7: A lathe operator, mounting a boring bar in a clamping house using a standard wrench, accomplishing an arbitrary tightening torque. The boring bar may be expected to exhibit different properties when clamped or mounted in the clamping house by different operators.

PART III - The Estimation and Simulation of the Nonlinear Dynamic Properties of a Boring bar

The third part addresses the nonlinear behavior, frequently observed, in the dynamic properties of a clamped boring bar [21,22]. Two nonlinear SDOF models with different softening spring nonlinearity were introduced for modeling the nonlinear dynamic behavior of the fundamental bending mode in the cutting speed direction of a boring bar. Also, two different methods for the simulation of nonlinear models were used.

PART IV - Investigation of the Dynamic Properties of a Milling Tool Holder

In the previous parts, vibration in turning operations was discussed. This part addresses the vibrations that appear in the metal cutting operation called milling. An introduction to the milling operation and the basic terminology are presented. In
Fig. 8, central parts and components of a milling machine during milling and a milling operation are pictured. Furthermore, an extensive experimental investigation, involving an analysis of the dynamic properties of a certain milling tool holder, both during cutting operation and when not cutting, is conducted. The angular vibrations of the rotating tool, the vibrations of the machine tool structure, and the vibration of the workpiece were examined during cutting. The focus was on identifying the source or sources of the dominant milling vibrations and on determining which of these vibrations are related to the structural dynamic properties of the milling tool holder. Also, basic distributed parameter system models of the milling tool holder were developed in order to simplify, for example, the measurement configuration and the sensor setup.

**PART V - Preliminary Investigation of the Active Control of a Milling Tool Holder**

A thorough investigation of milling dynamics is conducted in Part IV. Large vibrations of both the spindle frame structure, the milling tool holder and the workpiece was observed during experimentation and this is presented in this study. In order to reduce the vibration levels observed on the workpiece and spindle frame, an active control system was proposed and implemented. This section analyzes and investigate the challenges of implementing active control of milling tool holder vibrations. Basically, the strategy is the same as for the active boring bar application, that is, to increase the dynamic stiffness of the milling tool holder by introducing secondary vibrations with the opposite phase for a certain mode of interest. However, moving from an application using a non-rotating tool to an application using a rotating tool, implies several challenges that must be resolved in order to successfully implement the proposed solution.
PART VI - Noise Source Identification and Active Control in a
Water Turbine Application

Another application with vibration problems is presented in Part VI. This part
discusses vibration generated by a turbine in the production of electrical power in a
hydroelectric power plant. See Fig. 9 for an overview of the principle of a hydroelectric
power plant.

Figure 9: a) Sketch of a hydroelectric power station. b) The generator zoomed in
presenting how the water makes the generator shaft rotate by applying force on turbine
blades.

In the considered hydroelectrical power plant, the vibrations become severe even
when the operating condition is characterized by a moderate load. This operating
condition results in a power production far from the maximum production capacity
of the power plant. These vibrations thus have a negative influence on the production
capacity, but also on the wear of various components such as bearings, hydraulic
connections, etc. In order to reduce the vibration problem, active control is proposed
also in relation to this application. Active control using inertial mass actuators was
investigated and preliminary results indicate a significant attenuation of the vibrations.

References


PART I

On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry
This part is published as:

On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

H. Åkesson, T. Smirnova, I. Claesson and L. Hákansson
Blekinge Institute of Technology
Department of Signal Processing
372 25 Ronneby
Sweden

Abstract

Vibration in internal turning is a problem in the manufacturing industry. A digital adaptive controller for the active control of boring bar vibration may not be a sufficient solution to the problem. The inherent delay in a digital adaptive controller delays control authority and may result in tool failure when the load applied by the workpiece on the tool changes abruptly, e.g., in the engagement phase of the cutting edge. A robust analog controller, based on a lead-lag compensator, with simple adjustable gain and phase, suitable for the industry application, has been developed. Also, the basic principle of an active boring bar with embedded actuator is addressed. The performance and robustness of the developed controller has been investigated and compared with an adaptive digital controller based on the feedback filtered-x algorithm. In addition, this paper takes into account those variations in boring bar dynamics which are likely to occur in industry; for example, when the boring bars is clamped in a lathe. Both the analog and the digital controller manage to reduce the boring bar vibration level by up to approximately 50 dB.

1 Introduction

Degrading vibrations in metal cutting e.g. turning, milling, boring and grinding are a common problem in the manufacturing industry. In particular, vibration in internal turning operations is a pronounced problem. To obtain the required tolerances of the workpiece shape, and adequate tool-life, the influence of vibration in the process of machining a workpiece must be kept to a minimum. This requires that extra care be taken in production planning and preparation. Vibration problems in internal turning have a considerable influence on important factors such as productivity, production costs, working environment, etc. In internal turning the dimensions of the workpiece hole will generally determine the length and limit the diameter or cross-sectional size of the boring bar. As a result, boring bars are frequently long and slender - long-overhang cantilever tooling- and thus sensitive to excitation forces introduced by the
material deformation process in the turning operation [1–3]. The vibrations of a boring bar are often directly related to its low-order bending modes [4–6]. The vibration problems in internal turning can be addressed using both passive and active methods [1, 2, 7]. Common methods used to increase dynamic stiffness of cantilever tooling involve making them (in high Young's modulus) non-ductile materials, such as sintered tungsten carbide and machinable sintered tungsten, and/or utilizing passive Tuned Vibration Absorbers (TVA) [1, 2, 8]. These passive methods are known to enhance the dynamic stiffness and stability (chatter-resistance) of long cutting tools and thus, enable the allowable overhang to be increased [1, 2, 8]. The passive methods offer solutions with a fix enhancement of the dynamic stiffness frequently tuned for a narrow frequency range comprising a certain bending mode frequency that in some cases may be manually adjusted [1, 2, 8]. On the other hand, the active control of tool vibration enables a flexible solution that selectively increases the dynamic stiffness at the actual frequency of the dominating bending modes until the level of the chatter component in the feedback signal is negligible [2, 7, 9]. An active control approach was reported by Tewani et al. [10, 11] concerning active dynamic absorbers in boring bars controlled by a digital state feedback controller. It was claimed to provide a substantial improvement in the stability of the cutting process. Browning et al. [12] reported an active clamp for boring bars controlled by a feedback version of the filtered-x LMS algorithm. They assert that the method enables to extend the operable length of boring bars. Claesson and Håkansson [9] controlled tool vibration by using the feedback filtered-x LMS algorithm to control tool shank vibration in the cutting speed direction without applying the traditional regenerative chatter theory. Two important constraints concerning the active control of tool vibration involve the difficult environment in a lathe and industry demands. It is necessary to protect the actuator and sensors from the metal chips and cutting fluid. Also, the active control system should be applicable to a general lathe. Pettersson et al. [13] reported an adaptive active feedback control system based on a tool holder shank with embedded actuators and vibration sensors. This control strategy was later applied to boring bars by Pettersson et al. [6]. Åkesson et al. [14] reported successful application of active adaptive control of boring bar vibration in industry using an active boring bar with embedded actuators and vibration sensors.

During the process of machining a workpiece in a lathe, the boundary conditions applied by the workpiece on the cutting tool may exhibit large and abrupt variation, particularly in the engagement phase between the cutting tool and workpiece. These abrupt changes of load applied by the workpiece on the tool may result in tool failure. However when utilizing active adaptive digital control of tool vibration, the problem of tool failure in the engagement phase may remain. For instance, the time required for the adaptive tuning of the controller, the inherent delay in, controller processing time, A/D and D/A- conversion processes, and analog anti-aliasing and reconstruction filtering might impede an active adaptive digital control system to produce control authority sufficiently fast to avoid tool failure. To provide means to address the issue of delay in control authority in the active control of boring bar vibration in industry an analog controller approach is suggested.

This article focuses on the development of a simple adjustable robust analog controller, based on digitally controlled analog design, that is suitable for the control of boring bar vibration in industry. Initially a digitally controlled analog manually adjustable lead compensator was developed. To provide more appropriate controller responses a manually adjustable bandpass lead-lag compensator was developed. Gain and phase
of the controller response may, at a selectable frequency, be independently adjusted on the two developed controller prototypes. Also, the performance and robustness using the two analog controllers was evaluated and compared with a digital adaptive controller based on the feedback filtered-x LMS-algorithm [15,16] in the active control of boring bar vibration.

2 Materials and Methods

2.1 Experimental Setup

Experiments concerning active control of tool vibration have been carried out in a Mazak SUPER QUICK TURN - 250M CNC turning centre. The CNC lathe, presented by the photo in Fig. 1, shows the room in the lathe in which the machining is carried out. In this photo (Fig. 1(b)), the turret configured with a boring bar clamped in a clamping house, and a workpiece clamped in the chuck are observable.

![a) b)](image)

Figure 1: a) Mazak SUPER QUICK TURN - 250M CNC lathe and b) the room in the lathe where the machining is carried out.

A coordinate system was defined: z was in the feed direction, y in the reversed cutting speed direction and x in the cutting depth direction (see upper left corner of Fig. 1 b)).

2.1.1 Work Material - Cutting Data - Tool Geometry

The cutting experiments used the work material chromium molybdenum nickel steel SS 2541-03 (AISI 3239). The material deformation process of this material during turning excites the boring bar with a narrow bandwidth and has a susceptibility to induce severe boring bar vibration levels [4, 5], resulting in poor surface finish, tool breakage, and severe acoustic noise levels. The workpiece used in the cutting experiments had a diameter of 225 mm and a length of 230 mm. To enable supervision of the metal-cutting process during continuous turning, the cutting operation was performed externally, see Fig. 1 b). An active boring bar was firmly clamped in a clamping house rigidly attached to the lathe turret. Only one side of the workpiece shaft’s end was firmly clamped into the chuck of the lathe, see Fig. 1 b). As a cutting tool, a standard 55° diagonal insert with geometry DNMG 150608-SL and
carbide grade TN7015 for medium roughing was used. The following cutting data was selected: Cutting speed \( v = 60 \text{ m/min} \), Depth of cut \( a = 1.5 \text{ mm} \), and Feed \( s = 0.2 \text{ mm/rev} \).

### 2.1.2 Measurement Equipment and Setup

A block diagram of the experimental setup for the active control of boring bar vibrations is presented in Fig. 2.

The control experiments used an active boring bar equipped with an accelerometer and an embedded piezoceramic stack actuator. The actuator was powered with an actuator amplifier, custom designed for capacitive loads, and the accelerometer was connected to a charge amplifier. A floating point signal processor with Successive Approximation Register AD- and DA- converters were used. Two commercial signal conditioning filters were used in the control experiments. A VXI Mainframe E8408A with two 16-channel 51.2kSa/s cards were used for data collection.

### 2.2 Active Boring Bar

The active boring bar used in this experiment is based on the standard WIDAX S40T PDUNR15 boring bar with an accelerometer and an embedded piezoceramic stack actuator. By embedding accelerometers and piezoceramic stack actuators in conventional boring bars, a solution for the introduction of control force to the boring bar with physical features and properties that fit the general lathe application may be obtained.

#### 2.2.1 Active Boring Bar - Simple Model

A Euler-Bernoulli beam may be used as a simple model to illustrate the structural dynamic properties of a boring bar [4, 5]. The Euler-Bernoulli differential equation describing the transversal motion in the y direction of the boring bar may be written...
as: [17–19],

\[
\rho A(z) \frac{\partial^2 u(z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left[ EI(z) \frac{\partial^2 u(z,t)}{\partial z^2} \right] = \frac{\partial m_e(z,t)}{\partial z}
\]  

(1)

where \( \rho \) is the density of the boring bar, \( A(z) \) the cross-sectional area, \( u(z,t) \) the deflection in \( y \) direction, \( E \) the Young’s elastic modulus, \( I(z) \) the cross-sectional area moment of inertia, and \( m_e(z,t) \) the space- and time-dependent external moment load per unit length. The boundary conditions of the boring bar depend on the suspension of the boring bar ends, and a clamped-free model is suggested [4, 5, 20]. Fig. 3 illustrates a boring bar with a piezoelectric stack actuator embedded in a milled space in the underside of the boring bar.

Figure 3: The configuration of the piezoceramic stack actuator in the active boring bar.

Assuming that the actuator operates well below its resonance frequency, thus neglecting inertial effects of the actuator. Then, in the frequency domain, the force the actuator exerts on the boring bar, \( F_a(f) \) (the Fourier transform of \( f_a(t) \) and \( f \) is frequency), may approximately be related to the constraint expansion or motion of the actuator, \( Z(f) = Z_2(f) - Z_1(f) \), as \( F_a(f) = K_a (\Delta L_a(f) - Z(f)) \) [17]. Where \( Z_1(f) \) and \( Z_2(f) \) are the Fourier transform of the displacements \( z_1(t) \) and \( z_2(t) \), \( \Delta L_a(f) \) is the Fourier transform of the free expansion of an unloaded piezoelectric stack actuator [17] and \( K_a \) is the actuator equivalent spring constant. If the point receptance at the respective actuator end are summed to form the receptance \( H_B(f) = Z_1(f)/F_a(f) + Z_2(f)/F_a(f) \), the relative displacement \( Z(f) \) may be expressed \( Z(f) = H_B(f)F_a(f) \). With the aid of Newton’s second law, an expression for the actuator force applied on the boring bar as a function of the actuator voltage \( V(f) \) may now be written as [17];

\[
F_a(f) = H_{f_v}(f) V(f)
\]  

(2)

where \( H_{f_v}(f) \) is the electro-mechanic frequency function between input actuator voltage \( V(f) \) and output actuator force \( F_a(f) \). The distance between the actuator - boring bar interface center and the natural surface of the active boring bar in the \( y \)-direction is \( \alpha \) (see Fig. 3). Thus, the external moment per unit length applied on the boring bar by the actuator force \( F_a(f) \) may be approximated as:

\[
m_e(z,f) = \alpha F_a(f)(\delta(z-z_1) - \delta(z-z_2))
\]  

(3)

where \( \delta(z) \) is the Dirac delta function and \( z_1 \) respective \( z_2 \) are the \( z \)-coordinates for the actuator - boring bar interfaces. Based on the method of eigenfunction expansion [21],
if $z_1 = 0$ the generalized load of mode $r$ is $F_{load,r}(f) = \alpha F_a(f)\psi'_r(z_2)$, $\psi'_r$ is the derivative of the normal mode $r$. Expressing the generalized load with the aid of Eq. 2 and relying on the method of eigenfunction expansion [21], the frequency-domain dynamic response of the boring bar in the $y$ direction may be written as:

$$u(z, f) = \sum_{r=1}^{\infty} \psi_r(z)H_r(f)\alpha H_{f\psi}(f)V(f)\psi'_r(z_2)$$

(4)

where $H_r(f)$ is the frequency response function and $\psi_r$ is the normal mode, for mode $r$.

2.3 System Identification

The design of feedback controllers usually rely on detailed knowledge of the dynamic properties of the system to be controlled, e.g. a dynamic model of the system to be controlled [22, 23]. Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [24,25]. Welch spectrum estimator [25] was used and Table 1 column A gives the spectral density estimation parameters used in the production of plant frequency function estimates, with various clamping conditions for the boring bar. Also, the spectral density estimation parameters used in the production of plant frequency function estimates during continuous metal cutting can be found in Table 1 column B. In Table 1 column C, the spectral density estimation parameters used in the production of frequency function estimates for the controller responses are given, and in Table 1 column D, the spectrum estimation parameters used for the production of power spectral density estimates for boring bar vibration with and without active control are presented.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling frequency $f_s$</td>
<td>8192 Hz</td>
<td>10240 Hz</td>
<td>10240 Hz</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block length $N$</td>
<td>16384</td>
<td>20480</td>
<td>20480</td>
<td>10240</td>
</tr>
<tr>
<td>Freq. resolution $\Delta f$</td>
<td>0.5 Hz</td>
<td>0.5 Hz</td>
<td>0.5 Hz</td>
<td>1 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>160</td>
<td>265-635</td>
<td>160</td>
<td>100</td>
</tr>
<tr>
<td>Burst length</td>
<td>90%</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Freq. range of burst</td>
<td>0-4000 Hz</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Window $w(n)$</td>
<td>Rectangular</td>
<td>Hanning</td>
<td>Hanning</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
<td>50%</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 1: Spectral density estimation parameters used for the production of boring bar vibration spectra, with and without active control.

2.4 Controllers

The application of active control of boring bar vibration in industry requires reliable robust adaptive feedback control or manually tuned feedback control, which is simple to adjust at the shop floor by the lathe operator. A controller suitable for active control of boring bar vibrations might be implemented using different approaches.
However, the abrupt changes that occur in the turning operations, i.e. in the engagement phase, suggest that an analog control approach might be suitable with respect to, for example, the controller delay and thus delay in control authority. Simple and effective compensators or controllers that may be implemented in the analog domain are the lag and lead compensators that approximate the PI controller and the PD controller, respectively. Moreover, the PID controller may be approximated in the analog domain by combining a lag compensator with a lead compensator [26].

A block diagram of the active boring bar vibration feedback control system, a feedback control system for disturbance rejection, is presented in Fig. 4 where $-W$ is the controller, $y(t)$ is the controller output signal, $C$ is the plant or control path, $y_c(t)$ is the plant output vibration (secondary vibration), $d(t)$ is the undesired tool vibration, and $e(t)$ is the error signal.

![Block diagram of the active tool vibration control system.](image)

If the plant and the controller are linear time invariant stable systems, their dynamic properties may be described by the frequency response functions $C(f)$ and $W(f)$, respectively. Then, the feedback control systems’ open-loop frequency function can be written as $H_{ol}(f) = W(f)C(f)$ and the closed-loop frequency function for the active boring bar vibration feedback control system is given by $H_{cl}(f) = 1/(1 + W(f)C(f))$.

### 2.4.1 Controller performance and robustness

Principally, there are two important aspects of the behavior of feedback controllers in active control systems, their performance and their robustness; that is, the ability of the controller to reject disturbance and to remain stable under varying conditions [22]. Basically, good performance in feedback control requires high loop gain; while on the other hand, robust stability usually implies a more retained loop gain. Usually, the discussion concerning the performance and robust stability of feedback controllers is based on the sensitivity function $S(f) = 1/(1 + W(f)C(f))$ and the complementary sensitivity function $T(f) = W(f)C(f)/(1 + W(f)C(f))$ [22,23]. The sensitivity function $S(f)$ gives a measure on disturbance reduction of a feedback control system. In the determination of the stability properties of the system, the complementary sensitivity function $T(f)$ has a vital role. It also governs the performance of the control system regarding the reduction of noise from the sensor detecting the error $e(t)$. Generally, the design of controllers rely on a model of the system to be controlled or the so called plant. However, a model of a physical system is an approximation of the true dynamics of the system and it is therefore likely to affect the performance of the control system [22,23]. To incorporate the plant uncertainty in the design procedure of a controller, a model of the plant uncertainty is usually included. A common way to model the plant uncertainty is with a multiplicative perturbation, yielding a
frequency function model of the plant as \([22, 23, 27]\);
\[
C_{\text{actual}}(f) = C_{\text{nominal}}(f)(1 + \Delta C(f))
\]
where \(\Delta C(f)\) is an unstructured perturbation given by;
\[
\Delta C(f) = \frac{C_{\text{actual}}(f)}{C_{\text{nominal}}(f)} - 1
\]
and \(C_{\text{nominal}}(f)\) is a nominal plant model of the plant. Thus, assuming that a control system design based on the nominal plant model results in a theoretically stable control system, then the denominator \(1 + W(f)C_{\text{nominal}}(f)\) has no zeros in the right half complex plane. However, the actual control system’s frequency response function will have the denominator \(1 + W(f)C_{\text{actual}}(f)\). Thus, if \(|1 + W(f)C_{\text{nominal}}(f)| > |W(f)C_{\text{nominal}}(f)\Delta C(f)|\), \(\forall f\) is fulfilled, the actual control system is stable \([22, 28, 29]\). Hence, for stable control the unstructured perturbation is upper limited as \(|\Delta C(f)| < 1/|T_{\text{nominal}}(f)|\), \(\forall f\) where \(T_{\text{nominal}}(f) = W(f)C_{\text{nominal}}(f)/(1 + W(f)C_{\text{nominal}}(f))\). If \(\Delta C(f)\) is bounded as \(|\Delta C(f)| \leq \beta(f)\), \(\forall f\). The condition for robust stability of a control system is given by \([28]\)
\[
\beta(f) < \frac{1}{|T_{\text{nominal}}(f)|}, \forall f
\]
The performance and robustness of an active feedback control system may also be visualized by a polar plot of its open-loop frequency response function in a Nyquist diagram \([22, 28, 29]\). If the closed loop system is to be stable, the polar plot of the open loop frequency response for the feedback control system \(W(f)C(f)\) must not enclose the polar coordinate \((-1, 0)\) in the Nyquist diagram. The larger the distance between the polar plot and the \((-1, 0)\) point, the more robust the feedback control system becomes, with respect to variation in plant response.

2.4.2 Compensators

A lead compensators purpose is to advance the phase of the open loop frequency response \(W(f)C(f)\) for a feedback control system, usually by adding maximal positive phase shift in the frequency range where the loop gain equals 0 dB, i.e. at the crossover frequency \([26, 29]\). This will increase the phase margin and generally increase the bandwidth of a feedback control system \([26, 29]\). The characteristic equation or frequency function for a lead compensator may be written as \([30]\):
\[
W_{\text{Lead}}(f) = K_{\text{lead}} \frac{1}{\alpha_{\text{lead}} j 2\pi f + p_{\text{lead}}} = K_{\text{lead}} \frac{\tau_{\text{lead}} j 2\pi f + 1}{\alpha_{\text{lead}} \tau_{\text{lead}} j 2\pi f + 1}
\]
Where \(z_{\text{lead}} > 0\) and \(z_{\text{lead}} \in \mathbb{R}\) \((-z_{\text{lead}}\) is the compensator zero), \(p_{\text{lead}} > 0\) and \(p_{\text{lead}} \in \mathbb{R}\) \((-p_{\text{lead}}\) is the compensator pole), \(\alpha_{\text{lead}} = z_{\text{lead}}/p_{\text{lead}} < 1\) is the inverse lead ratio for a lead compensator, \(K_{\text{lead}}\) is the compensator gain, and \(\tau_{\text{lead}} = 1/z_{\text{lead}}\). By utilizing a lag compensator, the low frequency loop gain of a feedback control system may be increased as the phase-lag filter attenuates the high frequency gain. In this way, the gain margin of the open loop frequency response for the feedback control system can be improved, and the phase shift added by the compensation filter can be minimized \([26, 28, 29]\). The characteristic equation or frequency function for a
lag compensator is fairly similar to the lead compensator characteristic equation and may be expressed as [30]:

$$W_{Lag}(f) = K_{lag} \frac{1}{\alpha_{lag}} \frac{j2\pi f + z_{lag}}{j2\pi f + p_{lag}} = K_{lag} \frac{\tau_{lag} j2\pi f + 1}{\alpha_{lag} \tau_{lag} j2\pi f + 1}$$

(9)

Here $z_{lag} > 0$ and $z_{lag} \in \mathbb{R}$ ($-z_{lag}$ is the compensator zero), $p_{lag} > 0$ and $p_{lag} \in \mathbb{R}$ ($-p_{lag}$ is the compensator pole), $\alpha_{lag} = z_{lag}/p_{lag} > 1$ is the inverse lag ratio for a lag compensator, $K_{lag}$ is the compensator gain, and $\tau_{lag} = 1/z_{lag}$.

A lead-lag compensator is obtained by connecting a lead compensator and a lag compensator in series, thus by combining Eqs. (8) and (9). The lag compensator may be adjusted to provide a suitable low frequency loop gain of the feedback control system. Subsequently, the lead compensator may be adjusted to provide an additional positive phase shift in the frequency range, where the loop gain equals 0 dB, i.e. at the crossover frequency [26, 29].

### 2.4.3 Digitally Controlled Analog Controller

The intention is to develop an analog controller with response properties that can be easily adjusted manually, without necessitating the replacement of discrete components, i.e. resistors and capacitances. If a lead compensator is considered, its frequency response function is given by Eq. (8) [26, 29]. A lead compensator may be designed according to the circuit diagram shown in Fig. 5, where $R_{d,2}$ and $R_{d,f}$ are adjustable resistors, $R_{d,1}$ is a fixed resistor and $C_d$ is a fixed capacitor. The parameters in Eq. (8) are related to the discrete components as $K_{lead} = \frac{R_{d,f}}{R_{d,2} + R_{d,1}}$, $\tau_{lead} = C_d R_{d,2}$, and $\alpha_{lead} = \frac{R_{d,1}}{R_{d,2} + R_{d,1}}$.

![Circuit diagram of a lead compensator.](image)

If $R_{d,2}$ and $R_{d,f}$ are implemented by digitally controlled potentiometers, the phase and gain of the compensator response may be adjusted independently at one selectable frequency, in discrete steps, to successively increase or decrease phase and gain respectively. For instance, by using two knobs for the compensator tuning (one for phase adjustment and one for gain adjustment) a function of the two knob angles may be produced according to:

$$[a, g] = AG_{lead}(\text{gain knob angle}, \text{phase knob angle})$$

(10)

This function produces integers $a \in \{1, 2, \ldots, L_a\}$ and $g \in \{1, 2, \ldots, L_g\}$, selecting the appropriate analog compensator frequency response function in the set of $L_a \times L_g$. 
different analog compensator frequency response functions:

\[ W_{\text{lead};a,g}(f) = K_{a,g} \frac{\tau_{a,g}^2}{\tau_{a,g} \omega + 1}, \quad a \in \{1, 2, \ldots, L_a\} \quad \text{and} \quad g \in \{1, 2, \ldots, L_g\} \]  

The micro-controller realized the AG(gain knob angle, phase knob angle) function, by controlling the adjustable resistors \( R_{d,2} \) and \( R_{d,f} \), (the so called digital potentiometers that have a digital control interface and an analog signal path). Such a micro-controller will allow the implementation of an analog lead-circuit which enabling orthogonal adjustment of the phase function and magnitude function at one selectable frequency of the compensator response. If the frequency for orthogonal adjustment of the phase function and magnitude function is set to 500 Hz, the magnitude and phase functions of the frequency response function realized by this circuit may be adjusted with the phase knob according to the 3-D plots in Fig. 6 a) and b), respectively. Observe, the gain knob only adjusts the level of the magnitude function surface and has no influence on its shape or the shape of the phase function surface.

![3D plots showing magnitude and phase functions](image)

Figure 6: a) Magnitude function and b) phase function of lead compensator frequency response function as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

A lag compensator with orthogonal adjustment of the phase function and the magnitude function at one selectable frequency of the compensator response may be designed similar to the adjustable analog lead compensator. Thus, an adjustable lag compensator may be designed according to the circuit diagram shown in Fig. 7.

![Circuit diagram of a lag compensator](image)

Figure 7: Circuit diagram of a lag compensator.
In Fig. 7, $R_{g,1}$ and $R_{g,f}$ are adjustable resistors and $R_{g,2}$ is a fixed resistor and $C_g$ is a fixed capacitor. The frequency response function for the lag compensator is given by Eq. (9) [26,29]. The parameters in the equation are related to the discrete components according to, 

$$K_{lag} = \frac{R_{g,f}}{R_{g,1} + R_{g,3}}, \quad \tau_{lag} = C_g R_{g,2}, \quad \text{and} \quad \alpha_{lag} = \frac{R_{g,1} R_{g,2} + R_{g,1} R_{g,3} + R_{g,2} R_{g,3}}{R_{g,1} R_{g,2} + R_{g,2} R_{g,3}}.$$

In much the same way as the orthogonally adjustable lead compensator, a microcontroller realizes an $AG_{lag}(gain\ knob\ angle,\ phase\ knob\ angle)$ function suitable for steering the digital potentiometers implementing the adjustable resistors $R_{g,1}$ and $R_{g,f}$ for the lag compensator. If the frequency for orthogonal adjustment of the phase and magnitude functions is selected to 500 Hz, the magnitude and phase functions of the frequency response function realized by this circuit may be adjusted with the phase knob, according to the 3-D plots in Fig. 8 a) and b), respectively.

Figure 8: Magnitude function and b) phase function of lag compensator frequency response function as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

By connecting the adjustable lead compensator in series with the adjustable lag compensator, a lead-lag compensator with orthogonal adjustment of the phase function and the magnitude function at one selectable frequency of the response may be realized. The magnitude and phase functions of the lead-lag compensator may be adjusted with the phase knob e.g. according to the 3-D plots in Fig. 9 a) and b) respectively.

Also, to utilize the capacity of the actuator amplifier, to limit the active control frequency range, a suitable high-pass filter, followed by a suitable low-pass filter was connected in series with the lead-lag compensator. The block diagram of the obtained analog band-pass lead-lag controller is shown in Fig. 10.

Selecting the frequency for orthogonal adjustment of the phase and magnitude functions to 500 Hz, this controller’s frequency response magnitude and phase functions are adjustable with the phase knob, according to the 3-D plots in Fig. 11 a) and b), respectively.

The implemented analog lead-lag controller consists of several blocks: high-pass filter, low-pass filter, lead compensator and lag compensator. Each block can be used separately or arbitrary combinations of the blocks can be used to enable controller flexibility.
Figure 9: a) Magnitude function and b) phase function of lead-lag compensator frequency response function, as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.

Figure 10: Block diagram of the implemented lead-lag circuit.

Figure 11: a) Magnitude function and b) phase function of band-pass lead-lag compensator frequency response function, as a function of phase knob adjustment range in % and frequency. The gain knob only adjusts the level of the magnitude function surface.
2.4.4 Feedback Filtered-x LMS Algorithm

The feedback filtered-x LMS algorithm is an adaptive digital feedback controller suitable for narrow-band applications [15, 16, 31]. This algorithm is based on the method of steepest descent and the objective of the control is to minimize the disturbance signal or desired signal in the mean square sense [15, 16, 31]. A block diagram of the feedback filtered-x LMS algorithm is shown in Fig. 12.

![Block diagram of the feedback filtered-x LMS algorithm.](image)

Figure 12: Block diagram of the feedback filtered-x LMS algorithm.

The feedback filtered-x LMS algorithm with leakage coefficient is defined by the following equations:

\[ y(n) = w^T(n)x(n) \]  
\[ e(n) = d(n) + y_C(n) \]  
\[ w(n + 1) = \gamma w(n) - \mu x_C(n)e(n) \]

\[ x_C(n) = \left[ \sum_{i=0}^{I-1} \hat{c}_i x(n-i), \ldots, \sum_{i=0}^{I-1} \hat{c}_i x(n-i-M+1) \right]^T \]

where \( \mu \) is the adaptation step size and \( \gamma \) is the leakage coefficient \( 0 < \gamma < 1 \), usually selected close to unity. By selecting \( \gamma = 1 \) the feedback filtered-x LMS algorithm is obtained. Furthermore, \( x_C(n) \) is the filtered reference signal vector, which usually is produced by filtering the reference signal \( x(n) \) with an \( I \)-coefficients FIR-filter estimate, \( \hat{c}_i \), \( i \in 0, 1, \ldots, I - 1 \), of the control path or plant. Furthermore, \( w(n) \) is the adaptive FIR filter coefficient vector, \( y(n) \) is the output signal from the adaptive FIR filter, \( e(n) \) is the error signal, \( y_C(n) \) the secondary vibration (the output signal from the plant), \( \hat{C} \) is an estimate of the forward path and \( d(n) \) is the primary disturbance. The reference signal vector \( x(n) = [x(n), x(n-1), \ldots, x(n-M+1)]^T \) is related to the error signal as \( x(n) = e(n-1) \) [15, 16, 31]. In order to select a step size \( \mu \) to enable the feedback filtered-x LMS algorithm to converge, the inequality \( 0 < \mu < 2/(E[x_C^2(n)](M+\delta)) \) may be used [15, 16, 31], where \( \delta \) is the overall delay in the forward path, \( M \) is the length of the adaptive FIR filter and \( E[x_C^2(n)] \) is the mean square value of the filtered reference signal to the algorithm. Incorporating a leakage factor \( \gamma \) in the feedback filtered-x LMS-algorithm causes the loop gain of the control system to be reduced, yielding a more robust behavior [15, 16].
3 Results

3.1 The Plant

The system to be controlled, $C$, is comprised of several parts: a signal conditioning filter, an actuator amplifier, an actuator, the structural path between the force applied by actuator on the boring bar, and the boring bar response (measured by an accelerometer mounted close to the tool-tip). In order to clamp the boring bar, it is first inserted into the cylindrical space of the clamping house. It is then clamped by means of four/six clamping screws; two/three on the tool side and two/three on the opposite side of the boring bar. The two standard versions of the clamping house are distinguishable only by the fact that one supports four clamping screws while the other supports six. It is obvious that the boundary conditions applied by the four-screw version of the clamping house will differ from the boundary conditions applied by the six-screw version of the clamping house. Also, to enable the boring bar to be inserted in the clamping house, the diameter of the clamping house’s cylindrical clamping space is slightly larger than the diameter of the boring bar. Thus, the exact spatial position of the clamped boring bar end in the clamping space of the clamping house is difficult to pinpoint. Furthermore, the tightening torque of the clamping screws, i.e. the clamping force, is likely to vary between the screws each time the boring bar is clamped and each screw is tightened. Thus, each time the boring bar is clamped it is likely that the clamped boring bar will have different dynamic properties.

Plant frequency function estimates were produced when the boring bar was not in contact with the workpiece, i.e. off-line. The control path was estimated off-line for 10 different possible clamping conditions with respect to the tightening torque of the screws and the spatial position of the boring bar in the clamping space of the clamping house. Two spatial positions within clamping space were selected. The first was that in which the upper side of the boring bar’s end (the tool side) was clamped in contact with the upper section of the clamping space surface. The second was that in which the opposite, underside of the boring bar’s end, was clamped in contact with the lower section of the clamping space surface. For each of these two spatial position configurations, five various tightening torques were used. The different off-line clamping conditions are presented by Table 2.

<table>
<thead>
<tr>
<th>Notation</th>
<th>$BC_1$</th>
<th>$BC_2$</th>
<th>$BC_3$</th>
<th>$BC_4$</th>
<th>$BC_5$</th>
<th>$BC_6$</th>
<th>$BC_7$</th>
<th>$BC_8$</th>
<th>$BC_9$</th>
<th>$BC_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torq. [Nm]</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Contact</td>
<td>Boring bar in contact with upper side to clamping housing</td>
<td>Boring bar in contact with under side to clamping housing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The ten different clamping conditions of the boring bar used for the production of plant estimates when the boring bar is not in contact with the workpiece (off-line). Four clamping screws were used, two on the tool side and two on the opposite side.

The spectrum estimation parameters and identification signal used in the production of off-line frequency function estimates are given in Table 1 column A. Fig. 13 shows frequency function estimates of the plant for five different clamping screw tightening torques ($BC_6 - BC_{10}$). These plant frequency function estimates are shown in the frequency range of the fundamental resonance frequency of the boring bar in Fig. 14.
Figure 13: Frequency function estimates of the plant, for the five different tightening torques of the clamping screws, when the underside, of the boring bar end, is clamped in contact with the lower part of the clamping space surface.

Figure 14: Frequency function estimates of the plant in the frequency range of the fundamental resonance frequency of the boring bar, for the five different tightening torques clamping screws, when the underside, of the boring bar end is clamped in contact with the lower part of the clamping space surface.
The five other plant estimates show similar differences but with the resonance frequency peak between 450 to 470 Hz. As opposed to a situation in which the boring bar is not in contact with the workpiece, contact with the workpiece during a continuous cutting operation will cause the boundary conditions on the cutting tool to change [4, 5]. Hence, the dynamic properties of the plant will be different when the boring bar is not in contact with the workpiece and during continuous turning. Also, different cutting data and work material are likely to affect the dynamic properties of the plant during continuous turning [4, 5]. Plant frequency function estimates were produced during continuous turning for a variety of different cutting data. The clamping conditions with respect to the tightening torque of the clamping screws and the spatial position of the boring bar in the clamping space of the clamping house were fixed and are given by clamping condition $BC_{10}$ in Table 2. The spectrum estimation parameters and identification signal used in the production of on-line frequency function estimates are given in Table 1 column B.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Cutting Depth</th>
<th>Cutting Speed</th>
<th>Feed Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-line 1</td>
<td>1.2 mm</td>
<td>80 m/min</td>
<td>0.2 mm/rev</td>
</tr>
<tr>
<td>On-line 2</td>
<td>1 mm</td>
<td>150 m/min</td>
<td>0.2 mm/rev</td>
</tr>
</tbody>
</table>

Table 3: Cutting data used for plant frequency response function estimation during continuous turning (on-line).

Fig. 15 presents two different plant frequency function estimates produced during continuous turning (on-line) with different cutting data (see Table 3). This diagram also presents a plant frequency function estimate produced when the boring bar is not in contact with the workpiece (off-line). The off-line frequency function estimate was produced using the spectrum estimation parameters and identification signal according to Table 1 column A.

The coherence functions corresponding to both the on-line control path frequency response function estimates and the off-line estimates are shown in Fig. 16 a). Estimates of the random error for the on-line and off-line frequency response function estimates in Fig. 15 are shown in Fig. 16 b).

### 3.2 Active Boring Bar Vibration Control Results

The cutting experiments utilized three different feedback controllers in the active control of boring bar vibration: first, an analog manually adjustable controller based on lead compensation; secondly, a manually adjustable analog stand alone controller, based on a lead-lag compensation; and finally an adaptive digital controller based on the feedback filtered-x LMS algorithm. To illustrate the results of the active control of boring bar vibration using the three different controllers, power spectral densities of boring bar vibration with and without active vibration control are presented in the same diagram. The spectrum estimation parameters used in the production of boring bar vibration power spectral density estimates are shown in Table 1 column D.

Initially, a simple analog manually adjustable lead compensator, based on digitally controlled analog design was developed. The adjustable lead compensator was tuned manually and it was possible to provide an attenuation of the boring bar vibration level by up to approximately 35 dB. However, using the manually adjustable...
On the Development of a Simple and Robust Active Control System for Boring Bar Vibration in Industry

Figure 15: Frequency function estimates of the plant during a continuous cutting operation (on-line) and when the boring bar is not in contact with the workpiece (off-line). The on-line estimation of the plant was produced using workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015. On-line 1: feed rate $s=0.2\text{mm/rev}$, cutting depth $a=1.2\text{mm}$, cutting speed $v=80\text{m/min}$ and On-line 2: feed rate $s=0.2\text{mm/rev}$, cutting depth $a=1\text{mm}$, cutting speed $v=150\text{m/min}$.

Figure 16: a) Coherence function estimates between input and output signal of the plant during continuous cutting (on-line) and when the boring bar is not in contact with the workpiece (off-line). b) Estimate of the random error for the on-line and off-line frequency response function estimates.
lead compensator in the active control of boring bar vibration frequently resulted in stability problems. By using the manually adjustable band-pass lead-lag controller in the active control of boring bar vibration, the vibration level was reduced by up to approximately 50 dB after a simple manual tuning of the controller (see Fig. 17). Furthermore in numerous cutting experiments, the active control of boring bar vibration based on the band-pass lead-lag controller after initial manual tuning has provided stable control with significant vibration attenuation.

Figure 17: a) Power spectral densities of boring bar vibration in the cutting speed direction with active control using the adjustable lead-lag controller (solid line) and without active control (dashed line). b) The corresponding spectra zoomed in to the three first resonance peaks. Workpiece material SS2541-03, cutting tool DNMG 150806-SL, grade TN7015, feed rate \( s = 0.24 \) mm/rev, cutting depth \( a = 2 \) mm, cutting speed \( v = 60 \) m/min.

By utilizing the feedback filtered-x LMS algorithm as controller in the active control of boring bar vibration, the adaptive controller will tune the adaptive FIR filter to de-correlate the error signal with the filtered reference signal vector. Therefore, high loop gain is provided in the frequency range of the resonance frequency that dominates the boring bar vibration. Frequently, at high boring bar vibration levels, the feedback filtered-x LMS algorithm in the active control of boring bar vibration yields an attenuation of the vibration by more than 50 dB at the dominating resonance frequency. However, feedback filtered-x LMS algorithm requires leakage to provide stable and robust control \([15,31]\) and the cost for improved robustness is a somewhat reduced vibration attenuation performance.

### 3.3 Stability and Robustness of the Controllers

The stability of a feedback control system requires that its open loop frequency response \( H_{ol}(f) \) does not violate the closed loop stability requirements, i.e. the Nyquist stability criterion \([22,28,29]\). A closed loop system is said to be stable if the polar plot of the open loop frequency response \( H_{ol}(f) \) for the feedback control system does not enclose the (-1, 0) point in the Nyquist diagram. The greater the shortest distance between the polar plot and the (-1, 0) point, the more robust the feedback control system is with respect to variation in plant response and controller response. The system fulfills the conditions for robust stability \([22,28,29]\) if there is no phase function present which (in combination with maximal magnitude of the possible plant
Figure 18: Nyquist diagram for a boring bar vibration control system based on a manually adjustable lead compensator for the four different plant frequency response function estimates corresponding to the clamping conditions: BC₁, BC₅, BC₆ and BC₁₀.

uncertainties at each frequency) can result in a feedback control system open loop frequency response that encloses the (-1,0) point in the Nyquist diagram.

An estimate of the open loop frequency function for a feedback control system may be produced based on the controller frequency response function and the plant frequency response function. The analog controller frequency response function was estimated after manually tuning for active control of boring bar vibration. In the case of the adaptive digital controller, the controller frequency response function was estimated after convergence with the step size set to zero. All the controllers were estimated with the spectrum estimation parameters and the identification signal shown in Table 1 column C. The open loop frequency responses for the active boring bar vibration control system were produced for the manually adjustable lead compensator and the off-line control path frequency function estimate for each of the 10 different clamping conditions of the boring bar (see Table 2). Also the open loop frequency responses for the manually adjustable lead compensator and the two different on-line plant frequency function estimates were produced. Observe that in order to facilitate interpretation of the Nyquist diagrams the number of the open loop frequency response functions plotted in the same diagram were limited to four. These open loop frequency response functions were selected in order to avoid redundancy in the Nyquist diagrams, and to form the open loop frequency response functions. The plant frequency response function estimates corresponding to the clamping conditions (BC₁, BC₅, BC₆ and BC₁₀) were selected (see Table 2). The Nyquist diagram in Fig. 18 shows polar plots of the selected open loop frequency responses. The corresponding Bode plot is shown in Fig. 19.

Observe, in Fig. 18, that the polar plots of the open loop frequency responses for the feedback control system based on the manually adjustable lead compensator are close to, or enclose, the (-1,0) point in the Nyquist diagram.

Fig. 19 demonstrates that open loop frequency responses for the boring bar vibration control system based on the manually adjustable lead compensator provide substantial loop gain at resonance frequencies above 2000 Hz. Open loop frequency responses for the boring bar vibration control system were produced based on the
Figure 19: Open loop frequency response function estimates for a boring bar vibration control system, based on a manually adjustable lead compensator for the four different plant frequency response function estimates corresponding to the clamping conditions: $BC_1, BC_5, BC_6$ and $BC_{10}$.

manually adjustable band-pass lead-lag compensator and the off-line plant frequency function estimate for each of the ten different boring bar clamping conditions (see Table 2).

Also, open loop frequency responses were produced for the manually adjustable lead compensator and the two different on-line control path frequency function estimates. The selected open loop frequency functions for the active boring bar control system based on band-pass lead-lag compensator are shown in the Nyquist diagram in Fig. 20 (these estimates were based on the clamping conditions: $BC_1, BC_5, BC_6$ and $BC_{10}$). The corresponding Bode plot of the open loop frequency response functions for the active boring bar control system are shown in Fig. 21.

Figure 20: Nyquist diagram for a boring bar vibration control system, based on a manually adjustable band-pass lead-lag compensator for the four different plant frequency response function estimates corresponding to the clamping conditions: $BC_1, BC_5, BC_6$ and $BC_{10}$.

Observe the distance between the polar plots of the open loop frequency response
function estimates for the boring bar vibration control system based on the manually adjustable band-pass lead-lag compensator and the (-1, 0) point in the Nyquist diagram in Fig. 20. In addition, the Bode plot (see Fig. 19) demonstrates low loop gain above 1000 Hz.

The adaptive digital control of boring bar vibration was carried out with and without a leakage factor in the feedback filtered-x LMS algorithm. The Nyquist diagram in Fig. 22 shows the polar plots of the four open loop frequency responses, which is based on the feedback filtered-x LMS algorithm. The four open loop frequency functions for the active boring bar control system based on the feedback filtered-x LMS algorithm are shown in the Nyquist diagram in Fig. 22.

The corresponding Bode plots of the open loop frequency response functions based on the feedback filtered-x LMS algorithm are shown in Fig. 23.

The feedback filtered-x LMS algorithm will automatically tune the adaptive FIR filter to de-correlate the error signal with the filtered reference signal vector. Thus, a high loop gain will be provided in the frequency range of the resonance frequency that dominates the boring bar vibration (see the Bode plot in Fig. 23). Finally, the leaky feedback filtered-x LMS controller yields a significantly lower loop gain compared to the case of no leakage.

If, for example, the plant frequency function estimate corresponding to the clamping condition $BC_{10}$ is selected as a nominal control path or plant model of the plant then an estimate of the upper bound $\beta(f)$ for the multiplicative perturbation modeling the plant uncertainty may be produced based on Eq. (6) using all the other plant frequency function estimates (not the plant frequency function estimate corresponding to the clamping condition $BC_{10}$). In Fig. 24 the magnitude of the inverse of the nominal complementary function for each of the controllers is plotted in the same diagram as the estimated upper bound $\hat{\beta}(f)$ for the multiplicative perturbation.
Figure 22: Nyquist diagram for a boring bar vibration control system, based on the feedback filtered-x LMS algorithm for the four different plant frequency response function estimates, corresponding to the clamping conditions: $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. Number of adaptive filter coefficients $M=20$, step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz.

Figure 23: Open loop frequency response function estimates for a boring bar vibration control system, based on the feedback filtered-x LMS algorithm for the four different plant frequency response function estimates, corresponding to the clamping conditions: $BC_1$, $BC_5$, $BC_6$ and $BC_{10}$. Number of adaptive filter coefficients $M=20$, step size $\mu = -0.5$, sampling frequency of the DSP $Fs=8$ kHz.
2 Discussion and Conclusions

The above results demonstrate that boring bar vibration in internal turning may be reduced by utilizing active control based on active boring bars with embedded actuator and sensor and a suitable feedback controller such as an analog manually adjustable band-pass lead-lag controller or an adaptive digital controller based on the feedback filtered-x LMS algorithm. It has been established that for each time the boring bar is clamped, it is likely that the clamped boring bar will have different dynamic properties (see Figs 13 and 14). Also, the dynamic properties of the clamped boring bar will differ between these instances when the boring bar is not in contact with the workpiece and during continuous turning (see Fig. 15). In addition, different cutting data and work materials will likely affect the dynamic properties of the clamped boring bar during continuous turning [4]. Thus, the plant in the active boring bar vibration system may display significant variations in its dynamic properties. Hence, a robust controller that performs well for substantial variations in the dynamics of the plant is required for the active control of boring bar vibration.

The development of a simple adjustable analog controller, based on digitally controlled analog design, started initially with a lead compensator. However, the vibration to be controlled is related to the fundamental bending modes of the boring bar and not its higher order modes [4, 5]. Thus, the high loop gain (provided by a controller above the fundamental bending modes eigenfrequencies) will likely be an issue concerning the stability and robustness of the active boring bar vibration control system. However if the manually adjustable lead control is utilized for active boring bar vibration control, it will (when stable) perform a broad-band attenuation of the tool-vibration. Therefore, the vibration level is reduce by over 35 dB at 460 Hz and harmonics of the 460 Hz boring bar resonance frequency are attenuated. The reduction of the harmonics of the dominating fundamental resonance frequency is probably a consequence of a linearization of the boring bars dynamic response imposed by the active vibration control. Polar plots of the open loop frequency responses (for the feedback control system based on the manually adjustable lead compensator), which approach or enclose the (-1, 0) point of the Nyquist diagram, also demonstrate problems with robustness and stability (see Fig. 18). According to the Bode plots (Fig. 19), the manually adjustable lead compensators provides (as expected) a substantial loop
gain at the boring bar resonance frequencies above 2000 Hz. Thus, to provide high boring bar vibration attenuation, high loop gain is only required in the frequency range of the boring bar’s fundamental bending modes eigenfrequencies. Basically, a manually adjustable controller should be able to provide adjustable band-pass gain and adjustable phase enabling to control the plant in the frequency range of the boring bar’s fundamental bending modes eigenfrequencies. This will produce adequate anti-vibration canceling the original vibration excited by the material deformation process.

The adaptive digital feedback control based on feedback filtered-x LMS algorithm (with and without leakage) performs a broad-band attenuation of tool-vibration and is able to reduce vibration levels by over 50 dB at 460 Hz, as well as to attenuate the harmonics of the 460 Hz boring bar resonance frequency. However, slightly lower vibration attenuation might be observed for the leaky feedback filtered-x LMS algorithm. The introduction of a leakage factor or a "forgetting factor" in the feedback filtered-x LMS algorithm’s recursive coefficient adjustment algorithm will provide a restraining influence on the loop gain of the control system and may thereby cause a somewhat reduced attenuation of the boring bar vibration.

The active boring bar vibration control system based on the manually adjustable band-pass lead-lag control performs a broad-band attenuation of tool-vibration and is also able to reduce the vibration level by over 50 dB at 460 Hz, as well as to attenuate the harmonics of the 460 Hz boring bar resonance frequency (see Fig. 17). Thus, the band-pass lead-lag controller provides attenuation performance comparable to that of the adaptive controller by tuning the adaptive FIR filter to de-correlate the error signal with the filtered reference signal vector. It thereby provides high loop gain in the frequency range of the resonance frequency that dominates the boring bar vibration (see Fig. 23). By comparing the loop gains provided by the adaptive digital controller with the loop gains provided by the band-pass lead-lag controller (see Fig. 19), it follows that the analog lead-lag controller is tuned to provide high loop gain over a broader frequency range as compared to the loop gain resulting from the adaptive digital control. On the other hand, by examining the Nyquist plots for the open loop frequency response functions concerning the band-pass lead-lag controller (see Fig. 20) and the feedback filtered-x LMS controller (see Fig. 22), it follows that the shortest distance between the polar plots and the (-1, 0) point is greater for the analog controller as compared with the feedback filtered-x LMS controller.

If robust stability is considered (see Fig. 24), it follows that the active boring bar vibration control system based on the leaky feedback filtered-x LMS controller is the only system that fulfills the conditions for robust stability. It is also the controller that results in the greatest shortest distance between the polar plots and the (-1, 0) point. However, the active boring bar vibration control system based on the band-pass lead-lag controller, tuned initially, remained stable during the course of numerous trials with varied clamping conditions and cutting data.

Acknowledgments

The present project is sponsored by the Foundation for Knowledge and Competence Development and the company Acticut International AB.
References


Part II

Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions
This part is published as:
Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

H. Åkesson, T. Smirnova, and L. Håkansson
Blekinge Institute of Technology,
Department of Signal Processing
372 25 Ronneby
Sweden

Abstract

Boring bars are frequently used in the manufacturing industry to turn deep cavities in workpieces and are usually associated with vibration problems. This paper focuses on the clamping properties' influence on the dynamic properties of clamped boring bars. A standard clamping housing of the variety commonly used in industry today has been used. Both a standard boring bar and a modified boring bar have been considered. Two methods have been used: Euler-Bernoulli beam modeling and experimental modal analysis. It is demonstrated that the number of clamping screws, the clamping screw diameter sizes, the screw tightening torques, the order the screws are tightened has a significant influence on a clamped boring bars eigenfrequencies and its mode shapes orientation in the cutting speed-cutting depth plane. Also, the damping of the modes is influenced. The results indicate that multi-span Euler-Bernoulli beam models with pinned boundary condition or elastic boundary condition modeling the clamping are preferable as compared to a fixed-free Euler-Bernoulli beam for modeling dynamic properties of a clamped boring bar. It is also demonstrated that a standard clamping housing clamping a boring bar with clamping screws imposes non-linear dynamic boring bar behavior.

1 Introduction

In industry where metal cutting operations such as turning, milling, boring and grinding take place, degrading vibrations are a common problem. In internal turning operations vibration is a pronounced problem, as long and slender boring bars are usually required to perform the internal machining of workpieces. Tool vibration in internal turning frequently has a degrading influence on surface quality, tool life and production efficiency, and will also result in severe environmental issues such as high noise levels. Boring bar vibrations are usually directly related to the lower order bending modes of the clamped boring bar [1, 2]. Also, the dynamic properties of a boring bar installed in a lathe are influenced by the boundary conditions imposed by the clamping of the bar [2, 3]. A number of theories concerning the machine tool chatter and the behavior of the dynamic system has been developed explaining tool vibration during
turning operations [4–7]. In 1946 the principles of the traditional theory of chatter in simple machine-tool systems was worked out by Arnold [4] based on experiments carried out in a rigid lathe, using a stiff workpiece but a flexible tool holder. In this way he was able to investigate chatter under controlled conditions. Later in 1965 Tobias [5] presented an extensive summary of results from various researchers concerning the dynamic behavior of the lathe, the chatter theory, and further developed the chatter phenomena considering the chip-thickness variation and the phase lag of the undulation of the surface. Also, in the same year, Merritt et al. [6] discussed the stability of structures with n-degrees of freedom, assuming no dynamics in the cutting process; they also proposed a simple stability criterion.

Parker et al. [8] investigated the stability behavior of a slender boring bar by representing it with a two-degree-of-freedom mass-spring-damper system and experimenting with regenerative cutting. They also investigated how the behavior of the vibration was affected by coupling between modes, by using different cutting speeds, feed rates and angles of the boring bar head relative to the two planes of vibrations. Pandit et al. [9] developed a procedure for modeling chatter from time series by including unknown factors of random disturbances present in the cutting process, they formulated self-excited random vibrations with white noise as a forcing function. Kato et al. [10] investigated regenerative chatter vibration due to deflection of the workpiece, and introduced a differential equation describing chatter vibration based on experimental data. Furthermore, various analytical models/analysis methods relating to the boring bar/cutting process have been continuously developed, assuming various conditions. For example, Zhang et al. [11] who’s model is derived from a two-degree-of-freedom model of a clamped boring bar and four cutting force components. In addition, Rao et al. [12] produced a continuous system model of boring dynamics based on a dynamic boring force model, including variation of chip cross-sectional area, and a uniform Euler-Bernoulli cantilever beam, while Kuster et al. [13] developed a computer simulation based on a three-dimensional model of regenerative chatter. A time series approach was used by Andrén et al. [1] to investigate boring bar chatter and the results were compared with an analytical Euler-Bernoulli model. Also, Euler-Bernoulli beam modeling, experimental modal analysis (EMA) and operating deflection shape analysis were used by Andrén et al. [2] to investigate the dynamic properties of a clamped boring bar. The issue of modeling the boundary conditions of a clamped boring bar is discussed briefly. Results obtained demonstrate observable differences concerning the fundamental bending modes. Scheuer et al. [3] made a preliminary investigation of the dynamic properties of a boring bar, for two different clamping housings based on experimental modal analysis. Their study indicates that different clamping conditions using a clamping housing with clamp screws may affect the fundamental bar bending modes slightly.

Following the literature review, it appears that little work has been done concerning the modeling of the clamping in the dynamic models of clamped boring bars. Also, when it comes to experimental investigations concerning the clamping properties’ influence on the dynamic properties of a clamped boring bar, it appears that little work has been done. Thus, it is of importance to investigate the clamping properties’ influence on the dynamic properties of the clamped boring bar in order to gain further understanding of the dynamic behavior of clamped boring bars in the metal cutting process. Also, the modeling of the boring bar clamping has to be addressed in order to further improve dynamic models of clamped boring bars.

This paper discusses Euler-Bernoulli modeling of a clamped boring bar with em-
phasis on the modeling of the clamping conditions, i.e. the boundary conditions. Also the variation in the dynamic properties of a clamped boring bar imposed by controlled discrete variations in the clamping conditions produced by a standard clamping housing commonly used in industry today is investigated experimentally. The focus is on the bars first two fundamental bending modes. Firstly, the Euler-Bernoulli theory for the modeling of a clamped boring bar using "fixed-free" boundary conditions is considered. In order to incorporate clamping flexibility in the distributed-parameter models, three-span Euler-Bernoulli boring bar model with free-pinned-pinned-free and free-spring-spring-free are considered. The three-span model is used to model clamping conditions of the boring bar when screws are clamping the bar at two positions along the bar. A four-span model with free-pinned-pinned-pinned-free and free-spring-spring-spring-free are considered to model the clamping conditions when screws are clamping the bar at three positions along the bar. Furthermore, derivations of the spring coefficients used in the elastic support models are presented for various screw sizes. An experimental investigation, based on experimental modal analysis, of dynamic properties of boring bars for a comprehensive set of different clamping conditions has been carried out. The investigation has included two standard clamping housings with different number of clamping screws, two different clamping screw diameter sizes, a number of different screw tightening torques, the order in which the screws were tightened and a "linearized" clamping setup. Both a standard boring bar and a modified boring bar have been considered. Experimental modal analysis of the boring bars has been carried out for the different clamping conditions using a number of different excitation force levels. The results from the Euler-Bernoulli modeling of a clamped boring bar and from the experimental investigation of dynamic properties of boring bars for a comprehensive set of different clamping conditions have been compared and discussed.

2 Materials and Methods

2.1 Experimental Setup

The experimental setup and subsequent measurements were carried out in a Mazak SUPER QUICK TURN-250M CNC turning center. The CNC lathe has 18.5 kW spindle power and a maximal machining diameter of 300 mm, with 1005 mm between the centers, a maximal spindle speed of 400 revolutions per minute (r.p.m.) and a flexible turret with a tool capacity of 12 tools. Fig. 1 a) illustrates some of the basic structural parts of internal turning, i.e. a workpiece clamped in a chuck and a boring bar clamped in a clamping housing. The room in the Mazak SUPER QUICK TURN-250M CNC lathe where machining is carried out presented by the photo in Fig. 1 b).

Initially, a right-hand cartesian coordinate system was defined. Subsequently a sign convention was defined for use throughout this paper. The coordinate system and sign convention are based on the right-hand definition where the directions of displacements and forces in positive directions of the coordinate axes are considered positive. Moreover, moment about an axis in the clockwise direction (when viewing from the origin in the positive direction of the axis) is considered positive. The boring bar was positioned in its operational position in all setups, that is, mounted in a clamping housing attached to a turret with screws, during all measurements. The turret may be controlled to move in the cutting depth direction, x-direction, and in the feed direction, z-direction, as well as to rotate about the z-axis for tool change.
The turret, etc. is supported by a slide which in turn is mounted onto the lathe bed. Even though the turret is a movable component, it is relative rigid, rendering the dynamic properties of the boring bars observable.

2.1.1 Measurement Equipment and Setup

The following equipment was used in the experimental setup;

- 12 PCB Piezotronics, Inc. 333A32 accelerometers.
- 2 Brüel & Kjær 8001 impedance head.
- 1 Brüel & Kjær NEXUS 2 channel conditioning amplifier 2692.
- OSC audio power amplifier, USA 850.
- Ling dynamic systems shaker v201.
- Gearing & Watson electronics shaker v4.
- Hewlett Packard VXI mainframe E8408A.
- Hewlett Packard E1432A 4-16 channel 51.2 kSa/s digitizer.
- PC with I-DEAS 10 NX Series.

Twelve accelerometers and two cement studs for the impedance heads were attached onto the boring bars with X60 glue. The sensors were evenly distributed along the centerline, on the under-side and on the backside of the boring bar; six accelerometers and one stud on the respective side (see Fig. 2 a). To excite all the lower order bending modes, two shakers were attached via stinger rods to the impedance heads, one in the cutting speed direction (y-) and the other in the cutting depth direction (x-). The shakers were positioned relatively close to the cutting tool.

2.1.2 Boring Bars

Two different boring bars were used in the experiment. The first boring bar used in the modal analysis was a standard "non-modified" boring bar, WIDAX S40T PDUNR15F3 D6G.
Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

Figure 2: a) Drawings of the boring bar including clamp screws, cement studs and sensors. The sensors are attached along the underside and the backside of the boring bar. The threaded holes denoted $B_1$, $B_2$ and $B_3$ are screw positions for clamping the boring bar from top and bottom. The dimension are in mm, where $l_1 = 10.7$, $l_2 = 18$, $l_3 = 101$, $l_4 = 250$, $l_5 = 35$, $l_6 = 100$, $l_7 = 18.5$, $l_8 = 25$. b) The cross section of the boring bar where $C_C$ is the center of the circle and $M_C$ is the mass center.

The second boring bar used in this experiment was a modified boring bar, based on the standard WIDAX S40T PDUNR15 boring bar, with an accelerometer and an embedded piezo-stack actuator, see Fig. 3. This boring bar is designed for active control of boring bar vibration [14]. The accelerometer was mounted 25 mm from the tool tip to measure the vibrations in the cutting speed direction ($y$). This position was as close as possible to the tool tip, but at a sufficient distance to prevent metal-chips from the material removal process from damaging the accelerometer.

Figure 3: The modified boring bar with an accelerometer close to the tool tip and an embedded piezo-stack actuator in a milled space below the centerline.

The standard WIDAX S40T PDUNR15 boring bar is manufactured in the material 30CrNiMo8, (AISI 4330) which is a heat treatable steel alloy (for high strength).

2.1.3 Clamping Housing

The clamping housing is a basic 8437-0 40 mm Mazak holder, presented to the right in Fig. 4. The clamping housing has a circular cavity that the boring bar fits easily into; the clamping is then carried out by means of screws on the tool side and on the opposite side of the boring bar by means of either four or six screws of size M8: two/three from the top and two/three from bottom. The basic holder itself is mounted onto the turret with four screws. Furthermore, one clamping housing was rethreaded in order to be able to use screws of size M10.
### 2.1.4 Clamping Conditions

Six different setups were considered using the different boring bars described in section 2.1.2 in conjunction with the different clamping housings.

**In the first setup**, the standard boring bar was clamped using four M8 class 8.8 screws. The clamp screws were first tightened from the top and then from the bottom using the following five different torques (10 Nm, 15 Nm, 20 Nm, 25 Nm, 30 Nm). The recommended torque for this screw class is 26.6 Nm, but from evaluations of the screws used, the threads stayed intact and the screws did not break for 30 Nm. As only four clamp screws were used, the clamping housing center screw positions were not used.

**The second setup** involved the same five torques as for the previous setup, but with six screws of size M10 class 8.8, which were, again, tightened first from the top and then from the underside. The use of six screws involved the use of all clamping housing screw positions.

**The third setup** involved the same five torques as for the previous setup, six screws of size M10 class 8.8, which were, however, tightened first from the underside and then from the top.

**Setup four and five** are configured in the same way as setup one and two, respectively, with the difference that the modified boring bar was used instead of the standard boring bar.

**The last setup (setup 6)** used a modification of the standard clamping in order to accomplish a more rigid clamping condition. A boring bar WIDAX S40T PDUNR15F3 D6G, the same model as the standard boring bar, was used together with three steel wedges produced of the material SS 1650 (AISI 1148). The steel wedges were glued with epoxy on the flat surfaces of the boring bar along the clamping length of the bar end. The steel wedges were shaped geometrically to render a circular cross-section on the boring bar along its clamped end. After the epoxy was set; the boring bar end with circular cross-section was pressed into the clamping housing and glued to it with epoxy to make the clamping rigid, see Fig 4. Table 1 summarizes the setup configurations used in the experimental modal analysis.

![Figure 4: The linearized boring bar-clamping housing setup.](image)
### Setup Configuration

<table>
<thead>
<tr>
<th>Setup number</th>
<th>Boring bar</th>
<th>Number of Screws</th>
<th>Screw Size</th>
<th>Tighten first from</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Standard</td>
<td>four</td>
<td>M8</td>
<td>top</td>
</tr>
<tr>
<td>2</td>
<td>Standard</td>
<td>six</td>
<td>M10</td>
<td>top</td>
</tr>
<tr>
<td>3</td>
<td>Standard</td>
<td>six</td>
<td>M10</td>
<td>bottom</td>
</tr>
<tr>
<td>4</td>
<td>Modified</td>
<td>four</td>
<td>M8</td>
<td>top</td>
</tr>
<tr>
<td>5</td>
<td>Modified</td>
<td>six</td>
<td>M10</td>
<td>top</td>
</tr>
<tr>
<td>6</td>
<td>Linearized</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: The configuration of the different setups from the experimental modal analysis.

### Analytical Models of the Boring Bars

To approximately model a boring bars lower order bending modes, Euler-Bernoulli beam theory may be utilized [1, 2]. The Euler-Bernoulli beam theory is generally considered for slender beams having a diameter to length ratio above 10 as it ignores the effects of shear deformation and rotary inertia [15]. As a result, it tends to slightly overestimate the eigenfrequencies; this problem increases for the eigenfrequencies of higher modes [15]. The Euler-Bernoulli differential equation describing the transversal motion of the boring bar in the cutting speed direction or y-direction is given by [15]:

$$ \rho A(z) dz \frac{\partial^2 u(z,t)}{\partial t^2} + \frac{\partial^2}{\partial z^2} \left[ EI_x(z) \frac{\partial^2 u(z,t)}{\partial z^2} \right] = f(z,t) $$

where $A(z)$ is the boring bars cross section area, $E$ is Young’s elastic modulus for the bar and $I_x(z)$ is the cross-sectional area moment of inertia about the ”$x$ axis.”, $\rho$ is the density, $t$ is the time, $u(z,t)$ is the deflection in the y-direction and $f(z,t)$ is the external force per unit length.

It is assumed that both the cross-sectional area $A(z)$ and the flexural stiffness $EI_x(z)$ are constant along the boring bar. The boring bar dimensions provided by Fig. 2 b) result in a cross-sectional area $A$ and a moment of inertia $I_x, I_y$ presented by Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$1.19330295 \cdot 10^{-3}$</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$I_x$</td>
<td>$1.13858895 \cdot 10^{-7}$</td>
<td>$m^4$</td>
</tr>
<tr>
<td>$I_y$</td>
<td>$1.13787080 \cdot 10^{-7}$</td>
<td>$m^4$</td>
</tr>
</tbody>
</table>

Table 2: Cross-sectional properties of the boring bar, illustrated in Fig. 2 b), where $A$ is the cross-sectional area and $I_x, I_y$ are the moments of inertia around the denoted axis.

### 2.2.1 Single Span Boring Bar Model

The simplest and most straightforward model of a boring bar is the Euler-Bernoulli model, which consists of a homogenous single span beam with constant cross-sectional
area \( A(z) = A \) and constant cross-sectional moment of inertia \( I_x(z) = I_x \). The beam has four boundary conditions, two at each end. One end is fixed and the other is free \([15]\), see Fig. 5.

![Figure 5: Model of a Fixed - Free beam, where \( E \) is the elasticity modulus (Young’s coefficient), \( \rho \) the density, \( A \) the cross-sectional area, \( I \) the moment of inertia and the length of the beam is \( l = 200 \).](image)

### 2.2.2 Multi-span Boring Bar with Elastic Foundation Model

The boring bar may be clamped with either two screws on the top and two on the underside or three screws on the top and three on the underside. If the clamping housing is consider to be a rigid body, and the screws to be rigid in the transverse direction, a number of boundary conditions are yielded, i.e. approximated as pinned. The pinned boundary condition assumes an infinitely stiff spring in the transverse direction but no rotational stiffness. Letting the screws assume more realistic properties as deformable bodies will yield “elastic supports” \([16]\) boundary conditions, instead of the pinned condition. The elastic support can be seen as two springs applied to one point, one spring acting in the transverse direction applying transverse stiffness resistance and one spring acting in the rotational direction applying rotational stiffness resistance. The configurations of the “elastic support” condition are presented in Fig. 6.

Two types of boundary conditions may be categorized from the models presented in Fig. 6, where \( z_{pos} \) denotes the position of the boundary condition along the beam. One is the “free” boundary condition where there is no bending or shear forces present \([15]\). The other boundary conditions derive from the “elastic support” condition and may be expressed as \([16]\)

\[
EI \left. \frac{\partial^2 u(z,t)}{\partial z^2} \right|_{z=z_{pos}} = -k_R \left. \frac{\partial u(z,t)}{\partial z} \right|_{z=z_{pos}}, \quad EI \left. \frac{\partial^3 u(z,t)}{\partial z^3} \right|_{z=z_{pos}} = k_T u(z,t) \tag{2}
\]

where the transverse spring produces a transverse force proportional to the displacement, and the rotational spring produces a bending moment proportional to the beam slope. However, if we let the rotational spring coefficient equal zero \( k_R = 0 \), and the transverse spring coefficient go to infinity \( k_T = \infty \), we will have a third boundary condition termed “pinned” \([15]\).

The stiffness coefficients of the screws were calculated by modeling the screws as a beam rigidly clamped at one end, and free at the other. The beam was considered to be homogeneous, having a constant cross-sectional area \( A \), a constant cross-sectional moment of inertia \( I \) and a length of \( l \). When a screw is threaded in the clamping housing and is clamping the boring bar, a part of the screw’s tip will not be in contact with the clamping housing; thus yielding both transverse, lateral and bending elasticity. This is due to the fact that the inside of the clamping housing is circular.
Figure 6: a) A model of a three span beam with elastic support, b) a model of a four span beam with elastic support, where $E$ is the elasticity modulus (Young’s coefficient), $\rho$ the density, $A$ the cross-sectional area, $I$ the moment of inertia, $k_T$ the transverse spring coefficient, $k_R$ the rotational spring coefficient the length of the different spans in mm are $l_1 = 35$, $l_2 = 50$, $l_3 = 215$ and $l_4 = 25$.

with a radius of 40 mm plus tolerance and the boring bar has a thickness of 37 mm, plus tolerance where the boring bar is clamped.

The screw clamping model is presented in Fig. 7, where a) shows the clamping configuration, b) illustrates the beam model of transverse vibrations and the transverse spring coefficient, and c) illustrates the beam model of the rotational spring coefficient.

Figure 7: a) Sketch illustrating screw clamping of the boring bar, via the clamping housing, b) the transverse stiffness model, and c) the rotational stiffness model

The transverse spring constant $k_T$ and rotational spring constant $k_R$ were calculated as $[15,17]$

$$k_T = \frac{EA}{l}, \quad k_R = \frac{EI}{l} \quad (3)$$

The calculated spring coefficients and the spring parameters used in the spring models are presented in Table 3 together with the dimensions and elasticity modulus.
and are based on dimensions and properties defined by MC6S norm "DIN 912, ISO 4762".

<table>
<thead>
<tr>
<th>Size</th>
<th>$A$ [m$^2$]</th>
<th>$I$ [m$^4$]</th>
<th>$E$ [N/m$^2$]</th>
<th>$l$ [m]</th>
<th>$k_T$ [N/m]</th>
<th>$k_R$ [Nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M8</td>
<td>$3.661 \cdot 10^{-3}$</td>
<td>$1.067 \cdot 10^{-10}$</td>
<td>$200 \cdot 10^9$</td>
<td>$1.5 \cdot 10^{-3}$</td>
<td>$4.881 \cdot 10^9$</td>
<td>$1.422 \cdot 10^4$</td>
</tr>
<tr>
<td>M10</td>
<td>$5.799 \cdot 10^{-3}$</td>
<td>$2.676 \cdot 10^{-10}$</td>
<td>$7.732 \cdot 10^9$</td>
<td>$3.568 \cdot 10^4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: The longitudinal and rotational spring coefficients and the spring parameters used in the spring models.

2.3 Experimental Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties of the system under examination; i.e. to determine the natural frequencies, mode shapes, and damping ratios from experimental vibration measurements. The procedure of modal analysis may be divided into two parts: the acquisition of data and the parameter estimation or parameter identification from these data, also known as curve fitting [18].

2.4 Spectral Properties

Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [19]. By using the Welch spectrum estimator [20], the cross-power spectral density $\hat{P}_{yx}(f_k)$ between the input signal $x(n)$ and the output signal $y(n)$ and the power spectral density $\hat{P}_{xx}(f_k)$ for the input signal $x(n)$ may be produced [2, 19], where $f_k = \frac{k}{N} F_s$ is the discrete frequency, $k = 0, \ldots, N - 1$, $N$ is the length of the data segments used to produce the periodograms and $F_s$ is the sampling frequency.

In the case of a multiple-input-multiple-output system (MIMO system) with $P$ number of responses and $Q$ number of references, an estimate of the cross-spectrum matrix $[\hat{P}_{yx}(f_k)]$ between all the inputs is produced, where the diagonal elements is power spectral densities for the respective input signal and the of-diagonal are cross-spectral densities. Also a cross-spectrum matrix $[\hat{P}_{yx}(f_k)]$ between all the inputs and outputs may be estimated in the same way.

The least-square estimate for a (MIMO system) may be written as [19]

$$\hat{H}(f_k) = \left[\hat{P}_{yx}(f_k)\right] \left[\hat{P}_{xx}(f_k)\right]^{-1}$$  \hspace{1cm} (4)

In the case of a multiple inputs, the multiple coherence is of interest as a quality of the MIMO system estimates [19].

2.4.1 Parameter Estimation

There are several different methods for identification of the modal parameters [18, 21]. There are two basic curve fitting methods: curve fitting in frequency domain using measured frequency response function (FRF) data and a parametric model of the FRF; or curve fitting toward the measured impulse response function (IRF) data using a parametrical model of the IRF [18]. Many methods use both domains, depending on which parameter is estimated [18]. A parametric model of the FRF matrix,


\[ \hat{H}(f) = \sum_{r=1}^{N} \frac{Q_r \{\psi\}_r \{\psi\}_r^T}{j2\pi f - \lambda_r} + \frac{Q_r^* \{\psi\}_r^* \{\psi\}_r^H}{j2\pi f - \lambda_r^*} \]  

(5)

where \( r \) is the mode number, \( N \) the number of modes used in the model, \( Q_r \) the scaling factor of mode \( r \), \( \{\psi\}_r \) the mode shape vector of mode \( r \), and \( \lambda_r \) is the pole belonging to mode \( r \).

Due to the fact that two sources (references) were used during data acquisition, a method capable of handling multi-references is required. One such method is the Polyreference least square complex exponential method developed by Vold [22, 23]. This method is defined for identification of MIMO-systems with the purpose of obtaining a global least-squares estimates of the modal parameters. This method was used in this work; however, the mode shapes were estimated using the frequency domain polyreference method [24]. The used modal scaling was unity modal mass [21,25].

As quality assessment of the estimated parameters the FRF’s were synthesized using the estimated parameters and overlayed with the non-parametric estimates of the FRF’s. Furthermore the modal assurance criterion (MAC) [18] defined by Eq. 6.

\[ \text{MAC}_{kl} = \frac{\{\psi\}_k^H \{\psi\}_l}{\{\psi\}_k^H \{\psi\}_k \{\psi\}_l^H \{\psi\}_l} \]  

(6)

was used as a measure of correlation between mode shape \( \{\psi\}_k \) belonging to mode \( k \), and mode shape \( \{\psi\}_l \) belonging to mode \( l \), where \( H \) is the Hermitian transpose operator.

2.4.2 Excitation Signal

For the experimental modal analysis, burst random was used as the excitation signal. Based on initial experiments concerning suitable burst length and frequency resolution (data segment time or data block length time), a burst length of 90% of the data block length time was selected, see Table 4. Basically, the frequency resolution was tuned to provide high overall coherence in the analysis bandwidth and the burst length was tuned to provide high coherence at resonance frequencies.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Burst Random</td>
</tr>
<tr>
<td>Sampling Frequency ( f_s )</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block Length ( N )</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency Resolution ( \Delta f )</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages ( L )</td>
<td>200</td>
</tr>
<tr>
<td>Window</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
</tr>
<tr>
<td>Frequency Range of Burst</td>
<td>0-4000 Hz</td>
</tr>
<tr>
<td>Burst Length</td>
<td>90%</td>
</tr>
</tbody>
</table>

Table 4: Spectral density estimation parameters.
Furthermore, four different excitation levels with the proportion \{1, 2, 3, 4\} were applied for each of the clamping conditions of the boring bars. By using a number of different excitation levels and carrying out system identification for each of the excitation levels, differences between the estimates of the system may indicate nonlinear behavior of the system and might provide information concerning the structure of the nonlinearity or the nonlinearities involved.

3 Results

The results presented constitute a small part of an extensive investigation of the dynamic properties of boring bars for the different setups; however they represent the essence of the experimental results.

3.1 Results of the Analytical Models

This section presents results from the different Euler-Bernoulli models of the boring bar. The first model assumed rigid clamping of the boring bar by the clamping housing. The second model assumes that boring bar clamping is pinned at the positions where the actual clamp screws clamp the boring bar inside the clamping housing. Finally, the multi-span boring bar models with flexible boundary conditions (corresponding to the standard boring bar clamped using four clamp screws or six clamp screws) are considered.

The simplest model is the single span model with rigid clamping (fixed) at one end and no clamping (free) at other. The first three resonance frequencies in the cutting speed direction and in the cutting depth direction are presented in Table 5, and the three first mode shapes in Fig. 8 a).

Two multi-span Euler-Bernoulli boring bar models with pinned boundary conditions were considered: one corresponded to the boring bar clamped with four screws in the clamping housing, and one corresponding to the boring bar clamped with six screws in the clamping housing. The eigenfrequencies and mode shapes (eigenfunctions) for the two models were calculated. The first three resonance frequencies in both the cutting speed direction and the cutting depth direction for the two different models are presented in Table 5. Thus, when the fixed clamping model is changed to the pinned model, the first resonance drops by approximately 170 Hz for the four-screw-clamped boring bar, and approximately 140 Hz for the six-screw-clamped boring bar. The first three mode shapes for the two models are presented in Fig. 8 b) and c).

Two multi-span models with elastic foundation were calculated in the same way as for the multi-span models with pinned boundary condition, but now for the elastic boundary condition, using the stiffness coefficients in Table 3. The length of the clamp screw overhang was selected to 1.5 mm. Both eigenfrequencies and mode shapes were calculated for the two multi-span boring bar models with flexible clamping boundary conditions. The calculated eigenfrequencies are presented in Table 5, and mode shapes are shown in Fig. 8 d) and e).
Figure 8: The first three mode shapes for the Euler-Bernoulli boring bar model with boundary conditions a) fixed-free b) free-pinned-pinned-free (four clamp screws), c) free-pinned-pinned-pinned-free (six clamp screws), d) free-spring-spring-free (four clamp screws) and e) free-spring-spring-spring-free (six clamp screws).
Table 5: The first three resonance frequencies in cutting speed direction for the Euler-Bernoulli models

<table>
<thead>
<tr>
<th>Model</th>
<th>( f_1 ) [Hz]</th>
<th>( f_2 ) [Hz]</th>
<th>( f_3 ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-free</td>
<td>698.33</td>
<td>4376.36</td>
<td>12253.94</td>
</tr>
<tr>
<td>Pinned, four screws</td>
<td>527.47</td>
<td>3390.18</td>
<td>9539.40</td>
</tr>
<tr>
<td>Pinned, six screws</td>
<td>566.92</td>
<td>3575.59</td>
<td>10058.76</td>
</tr>
<tr>
<td>Spring, four M8 screws</td>
<td>519.43</td>
<td>3303.79</td>
<td>9257.16</td>
</tr>
<tr>
<td>Spring, six M8 screws</td>
<td>532.09</td>
<td>3335.17</td>
<td>9278.08</td>
</tr>
<tr>
<td>Spring, four M10 screws</td>
<td>525.24</td>
<td>3346.84</td>
<td>9404.65</td>
</tr>
<tr>
<td>Spring, six M10 screws</td>
<td>541.52</td>
<td>3398.74</td>
<td>9484.36</td>
</tr>
</tbody>
</table>

3.2 Experimental Modal Analysis

Shaker excitation was used for the experimental modal analysis (EMA) of the boring bars. The utilized spectrum estimation parameters and excitation signals’ properties are given in Table 4. A frequency range covering the significant part near the resonance frequencies was selected, i.e. \( \pm 100 \) to \( \pm 200 \) Hz around the resonance peaks. The coherence values for the involved transfer paths at each eigenfrequency were greater or equal to 0.996. A number of different phenomena were observed during the experimental modal analysis of the boring bars for various configurations and setups. For instance, large variations were observed in the first resonance frequencies of the boring bar for different tightening torques of the clamp screws. Also, the order in which the clamp screws were tightened (first from the upper side of the boring bar or first from the under side of the boring bar) had a significant impact on, for example, the fundamental bending resonance frequencies.

3.2.1 Standard Boring Bar

When clamping the standard boring bar so that the bottom side of the boring bar is clamped against the clamping housing (i.e. the screws are tightened from the topside first and subsequently from the bottom side) the fundamental boring bar resonance frequencies increase with increasing tightening, see Fig. 9 a). In this setup, screws of size M8 were used; the spectrum estimation parameters and excitation signal are presented in Table 4.

By changing the excitation levels, nonlinearities in the dynamic properties of the boring bar might be observable via changes in frequency response function estimates for the same input and output locations at the boring bar. Four different excitation levels were used with the proportion \( \{1,2,3,4\} \) for each of the torque configuration presented in section 2.1.4. As can be seen in Fig. 9 a) the fundamental boring bar resonance frequencies increases with increasing torque. And in Fig. 9 b) it can be seen that the fundamental boring bar resonance frequencies decreases slightly with increased excitation level. The estimated resonance frequencies and relative damping from all 20 measurements are presented in Table 6.

The clamp screws were replaced with M10 screws and the number of clamp screws was increased to six. Using these clamping conditions, experiments were performed which were identical to those carried out using a clamping housing with four M8 screws.

When clamping the standard boring bar so that the bottom side of the boring
Figure 9: The driving point accelerance magnitude function in cutting speed direction (y-) of the boring bar response using the standard boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side, a) using five different tightening torques and b) using two different tightening torques and four different excitation levels.

Table 6: Estimates of the fundamental boring bar resonance frequencies and its relative damping based on all the measurements using the setup with standard boring bar, clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Fig. 9 a). The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions Fig. 9 b) produced for the four different excitation levels.
bar is clamped against the clamping housing, the fundamental boring bar resonance frequencies increases with increasing tightening, see Fig. 10 a). As can be seen in Fig. 10 b), the fundamental boring bar resonance frequencies decreases slightly with increasing excitation level.

![Image of frequency response functions](image)

Figure 10: The driving point accelerance magnitude function in cutting speed direction (y-) of the boring bar response using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the upper-side of the boring bar. a) Using five different tightening torques and b) using two different tightening torques and four different excitation levels.

Clamping by first tightening the clamp screws from the boring bar’s underside changes the frequency response functions significantly as compared with the case where the clamp screws were tighten first from the upper side. This might be observed by comparing Fig. 11 with Fig. 10.

### 3.2.2 Modified Boring Bar

The modified boring bar has a cavity, a milled space, onto which an embedded actuator was placed. This space constitutes a change in the dynamic properties of the boring bar in comparison to the standard boring bar. This is obvious since the material, steel, is removed from the boring bar and replaced partly with an actuator with a lower Young’s module, etc. The actuator was kept passive during the experiments, thus, no control authority was applied. The same experiments were conducted with the modified boring bar as were performed with the standard boring bar.

From the results presented in Fig. 12 it is clear that the dynamic properties of the modified boring bar have changed significantly, mostly with regard to cutting speed direction (compare with the results from the standard boring bar Fig. 9). However, we can observe the same phenomenon that occurred in results obtained with the standard boring bar; i.e. increasing resonance frequency with increasing torque and decreasing resonance frequency with increasing excitation force. The estimated resonance frequencies and relative damping from all the 20 measurements are presented in Table 7.

When the modified boring bar is clamped with six, size M10 screws, results obtained resemble those derived from clamping the same bar with size M8 screws, see Fig. 12.
Figure 11: The accelerance magnitude function of the boring bar response in the driving point in cutting speed direction (y-) using the standard boring bar, six screws of size M10 and when clamp screws were tightened firstly from the underside of the boring bar. a) Using five different tightening torques and b) using two different tightening torques and four different excitation levels.

Figure 12: The driving point accelerance magnitude function in cutting speed direction (y-) of the boring bar response using the modified boring bar, four screws of size M8 and when clamp screws were tightened firstly from the upper-side of the boring bar. a) Using five different tightening torques and b) using two different tightening torques and four different excitation levels.
Table 7: Estimates of the fundamental boring bar resonance frequencies and its relative damping based on all measurements, using the setup in which the modified boring bar is clamped with four screws first tightened from the upper-side of the boring bar. The grey columns of mode 1 and mode 2 correspond to frequency response functions in Figs. 12 a). The grey rows of mode 1 and mode 2 correspond to the boring bar frequency response functions in Figs. 12 b), produced for the four different excitation levels.

### 3.2.3 Linearized Boring Bar

Finally, the results from the boring bar with a so-called "linearized" clamping condition are presented. Since no screws were used in this setup, only the excitation levels were changed. The results are presented in Fig. 13 and Table 8, which are the driving point frequency response functions in both the cutting speed direction and the cutting depth direction. Thus, only a slight variation in the boring bar’s resonance frequencies and damping might be observed. Unfortunately, both resonance frequencies coincide with periodic disturbances originating from the engines in the lathe producing the hydraulic pressure. One disturbance was at approximately 591 Hz and the other disturbance at approximately 600 Hz. These disturbances will have different influences on the estimates, depending on the excitation level, this may be observed near the peak in Fig. 13.

Table 8: Estimates of resonance frequency and the relative damping for the fundamental bending modes of the linearized boring bar.
3.2.4 Mode shapes

This section presents all the mode shapes estimated from the three different setups: the standard boring bar, the modified boring bar and the linearized boring bar. The mode shapes were estimated in I-DEAS using the frequency poly-reference method. First, results are presented from the standard boring bar, with size M8 screws, tightening clamp screws firstly from the upper-side. The shapes are presented in zy-plane and xy-plane in Fig. 14, a) and b) respectively. The angle of rotation around z-axis (relative the cutting depth direction for each measurement) is presented in Table 9. The mode shapes in xy-plane illustrated in Fig. 14 b) and the corresponding values in Table 9 show an average rotation of approximately 20 degrees.

<table>
<thead>
<tr>
<th>Torque</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Nm</td>
<td>-17.55</td>
<td>-17.10</td>
<td>-16.33</td>
<td>-17.81</td>
<td>-107.13</td>
<td>-106.89</td>
<td>-106.68</td>
<td>-106.43</td>
</tr>
<tr>
<td>30 Nm</td>
<td>-22.54</td>
<td>-22.27</td>
<td>-22.04</td>
<td>-21.84</td>
<td>-110.92</td>
<td>-110.68</td>
<td>-110.45</td>
<td>-110.35</td>
</tr>
</tbody>
</table>

Table 9: Angle of mode shapes for the standard boring bar, relative to cutting depth direction axis.

Measurements derived from the modified boring bar differ somewhat from those obtained with the standard boring bar. Mode shapes are presented in Fig. 15 and the values of the angle of rotation in Table 10. The shapes are almost identical in the yz-plane, but in the xy-plane the shapes rotate around the z-axis. Table 10, demonstrates a trend of clockwise rotation with increasing torque, as well as counter-clockwise rotation with increasing excitation level; this applies to both modes. The
Figure 14: The two first mode shapes of the standard boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy-plane and b) in the xy-plane.
angles lies between 20 and 73 degrees for the first mode, thus the first mode shifts from being most significant in cutting depth direction, to being most significant in cutting speed direction.

<table>
<thead>
<tr>
<th>Torque</th>
<th>Angle of Mode 1, [Degree]</th>
<th>Angle of Mode 2, [Degree]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Nm</td>
<td>-20.98</td>
<td>-112.06</td>
</tr>
<tr>
<td>15 Nm</td>
<td>-33.78</td>
<td>-116.87</td>
</tr>
<tr>
<td>20 Nm</td>
<td>-60.47</td>
<td>-149.03</td>
</tr>
<tr>
<td>25 Nm</td>
<td>-69.59</td>
<td>-155.52</td>
</tr>
<tr>
<td>30 Nm</td>
<td>-72.25</td>
<td>-156.19</td>
</tr>
</tbody>
</table>

Table 10: Angle of mode shapes for the modified boring bar, relative to cutting depth direction axis.

The results from the linearized setup are presented by Fig. 16 and in Table 11.

In this linear setup, the zy-plane shape is almost identical to those shapes produced from standard, and modified boring bar measurements, see Figs. 14, 15 and 16. In the xy-plane the shapes only have a rotation of approximately 10 degrees.

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Angle of Mode, [Degree]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>-7.98</td>
</tr>
<tr>
<td>Mode 2</td>
<td>-99.72</td>
</tr>
</tbody>
</table>

Table 11: Angle of mode shapes for the linearized boring bar relative to cutting depth direction axis.

After parameter estimation the Modal Assurance Criterion (MAC) was used to measure the correlation between the estimated modes shapes. A typical MAC diagram is presented in Table 12.

<table>
<thead>
<tr>
<th>Mode [Hz]</th>
<th>526.72</th>
<th>555.67</th>
</tr>
</thead>
<tbody>
<tr>
<td>526.72</td>
<td>1</td>
<td>0.000</td>
</tr>
<tr>
<td>555.67</td>
<td>0.000</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 12: The modal assurance criterion matrix coefficients for the two estimated mode shapes at the resonance frequencies 526.72 Hz and 555.67 Hz. The modes are estimated using the standard boring bar (clamped with four screws tightened firstly from the top), the lowest excitation level and the highest tightening torque.

Typical values of the MAC matrices of-diagonal elements are 0.000-0.001, few values reach 0.007.
Figure 15: The two first mode shapes of the modified boring bar clamped with four M8 screws, when the clamp screws were tightened firstly from the upper-side, for five different tightening torques and four different excitation levels. a) in the zy-plane and b) in the xy-plane.
Analysis of Dynamic Properties of Boring Bars
Concerning Different Clamping Conditions

Figure 16: The two first mode shapes of the linearized boring bar for four different excitation levels. a) in the zy-plane and b) in the xy-plane.
3.3 Comparison between the experiments and the Euler-Bernoulli models

To get an overview of results from the different setups in the experiments and the Euler-Bernoulli boring bar models relevant results are summarized in this section. In Fig. 17 a) the magnitude of the boring bar driving point accelerance functions -clamp screw tightening torque 30 Nm and maximum excitation signal level- for the six different setups are shown.

Estimates of eigenfrequencies, relative damping and mode shape angle relative to cutting depth direction for the fundamental boring bar bending modes, max excitation level and tightening torque 30 Nm, for the six setups, are presented in Table 13.

The multi-span boring bar models are supposed to approximately model the standard boring bar clamped with clamp screws in the clamping housing, thus it is the experimental and analytical models in cutting speed direction that are most relevant to compare. The experimental results that are of concern here are those for mode 2 in Table 13.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Parameters of mode 1</th>
<th>Parameters of mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. [Hz]</td>
<td>Damp. [%]</td>
</tr>
<tr>
<td>1</td>
<td>525.45</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>528.58</td>
<td>0.97</td>
</tr>
<tr>
<td>3</td>
<td>502.43</td>
<td>1.37</td>
</tr>
<tr>
<td>4</td>
<td>482.88</td>
<td>1.51</td>
</tr>
<tr>
<td>5</td>
<td>489.11</td>
<td>1.70</td>
</tr>
<tr>
<td>6</td>
<td>582.52</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Table 13: Eigenfrequencies, relative damping and mode shape angle relative to cutting depth direction for the six different boring bar setups. Clamp screw tightening torque, 30 Nm and maximum excitation signal level.

For the standard boring bar setups, estimates of eigenfrequencies from the analytical models and from the experiments (clamp screw tightening torque 30 Nm and maximum excitation signal level) for the fundamental bending mode in the cutting speed direction are presented together in Table 14.

The integer part of the fundamental eigenfrequencies calculated with the Euler-Bernoulli models of the standard boring bar setups is identical for both the cutting speed and the cutting depth direction. By using this information together with the fundamental eigenfrequencies for mode 1 given in Table 13 information similar to that given in Table 14 may be extracted for the fundamental bending mode in the cutting depth direction.

The mode shape estimates from the experimental modal analysis of the six different setups have components in three dimensions and does not seem to coincide with the planes defined by the chosen coordinate system (they have components in both the cutting speed and the cutting depth directions), while the analytical mode shapes from the analytical models do. However, the mode shapes may still be compared. A number of typical experimental mode shapes are compared with the Euler-Bernoulli beam models mode shapes, in the cutting speed direction-feed direction plane, in Fig. 17 b).
Analysis of Dynamic Properties of Boring Bars Concerning Different Clamping Conditions

Figure 17: a) The magnitude of the boring bar accelerance functions for the six different setups, from the driving point in cutting speed direction. b) Analytical mode shapes for five models; one single-span models with fixed-free boundary condition (black solid line), two multi-span models with pinned boundary condition and two multi-span models with elastic boundary condition. Also a number of experimental estimated mode shapes (circles) are overlayed on top of the analytical results.
Table 14: Estimates of the fundamental eigenfrequency in the cutting speed direction for the experimental setups of the standard boring bar (clamp screw tightening torque 30 Nm and maximum excitation signal level) and the fundamental eigenfrequencies calculated with the Euler-Bernoulli models of the standard boring bar setups.

4 Summary and Conclusions

Experimental modal analysis has been carried out on two boring bars; one original standard boring bar and one modified boring bar. The results from the experimental modal analysis of the two boring bars demonstrate that the different controlled clamping conditions in the experiments yield different dynamic properties of the clamped boring bars. Hence, the different clamping conditions result in different boundary conditions along the clamped part of the boring bar. It has also been established that a boring bar clamped in a standard clamping housing with clamping screws is likely to have a nonlinear dynamic behavior. The standard clamping housing with clamp screws is the likely source of the nonlinear behavior. Multi-span Euler-Bernoulli models of a clamped boring bar incorporating pinned or elastic support boundary conditions approximating the flexibility of the actual screw clamping of the boring bar end inside the clamping housing provide significantly higher correlation with experimental modal analysis results compared to a traditional fixed-free Euler-Bernoulli model.

The fundamental boring bar resonance frequencies decrease with increasing excitation level; see Tables 6 and 7. However, with regard to the behavior of relative damping as a function of excitation force level; the results from the standard boring bar indicate that the relative damping for the first mode increases with increasing excitation force level, while the relative damping for the second mode decreases with increasing excitation force level; see Table 6. Also, the results from the modified boring bar give an ambiguous indication of the effects on damping properties; see Table 7. The clamp screw tightening torque appears to affect the nonlinear behavior of the boring bar. Variation in the accelerance function estimates which was introduced by the four different excitation force levels seems to be larger for a low screw tightening torque (10 Nm) than for a high screw tightening torque (30 Nm), see, for example, Fig. 10 b). By examining, for example, driving point accelerances for the boring bar with "linearized" clamping for the four excitation force levels (see Fig. 13) it seems like the previously observed nonlinear behavior is almost removed. Thus this supports the conclusion that clamping conditions influence the extent of nonlinearities in boring bar dynamics. The boring bars resonance frequencies increase with increasing clamp screw torque; see the accelerance magnitude functions in Figs. 9.
When changing the number of screws used for clamping, or when using "linearized" clamping, changes in dynamic properties of the boring bar (clamping system) are expected. Hence new boundary conditions for the boring bar are introduced. Also, changing the standard boring bar to the modified boring bar will alter the dynamic properties of the boring bar - clamping system, i.e. a structural part of the system is different. The order in which the clamp screws were tightened (firstly from the upper side or firstly from the underside) had a major influence on the dynamic properties of the boring bar. This might be observed by comparing the boring bar driving point accelerances in Fig. 10 with the boring bar driving point accelerances in Fig. 11. If the clamp screws were tightened firstly from the upper side, the higher resonance frequency in Fig. 10 a) shows a variation from approximately 552 Hz to 562 Hz, for a change from the lowest to the highest clamp screws tightening torque. On the other hand, if the clamp screws were tightened firstly from the underside, the higher resonance frequency in Fig. 11 a) shows a variation from approximately 495 Hz to 530 Hz, for a change from the lowest to the highest clamp screws tightening torque. The discrepancies in the results when changing from which side the boring bars are tightened first, may be due to the difficulty in producing the exact same clamping conditions when tightening the clamp screws from the bottom first, compared to tightening the clamp screws from the top first.

Another interesting observation concerns mode shapes and, in particular angles of the different modes in the cutting depth - cutting speed plane (x-y plane). The standard boring bar clamped with four M8 screws, tightened firstly from the top has a first mode with an average mode shape angle or rotation of -20 degrees, relative to the cutting depth direction (x-direction). The second mode displays an average mode shape angle (or rotation) of -110 degrees relative to the cutting depth direction, see Table 9. Changing the clamp screw size or the number of clamp screws affects the so-called "mode rotation", both for the standard boring bar and in the case of the modified boring bar (see Figs. 14, 15 and 16). In addition, it should be noted that the boring bar in the linearized setup has rotated fundamental modes; the first mode has a mode shape angle or rotation of approx. -8 degrees relative to cutting depth direction (x-direction), and the second mode displays a mode shape angle or rotation of approx. -100 degrees relative to cutting depth direction (see Table 11).

When comparing the eigenfrequency estimates from the analytical results with the experimental results (see Table 13), some obvious discrepancy may be observed. The first and simplest model, the Euler-Bernoulli fixed-free model, overestimates the first fundamental bending resonance frequency substantially (with approx 100 Hz) compared to setup 6 and at least with 140 Hz compared to setups (1-3). The Euler-Bernoulli multi-span models produce eigenfrequency estimates, in the cutting speed direction, within approx. 6-40 Hz of the experimental eigenfrequency estimates for setups (1-3). Comparing the mode shapes from the Euler-Bernoulli models with the experimental mode shape estimates for the clamped standard boring bar, it is obvious that the fixed-free model shows a too rigid clamping at the base as the experimental results indicate some deflection adjacent to the clamping housing , see Fig. 17 b). While the mode shapes provided by the different multi-span models display a substantially higher agreement with the experimental results. Also, for the fundamental bending mode in the cutting depth direction, the Euler-Bernoulli multi-span models provide higher correlation with the standard boring bar clamped in the clamping housing with screws compared to Euler-Bernoulli fixed-free model.

The different boring bar clamping conditions result in different boundary con-
ditions for the boring bars in the experiments. Hence, if another boring bar with
different, e.g. length and/or cross-section and/or material as compared to the two
boring bars in this work would be considered. Different boundary conditions produced
by the boring bar clamping on this different boring bar would also result in different
dynamic properties of it. However, identical clamping boundary conditions applied to
the different boring bar and to one of the boring bars in this work would most likely
result in different dynamic properties of the two boring bars. Basically, the dynamic
properties of a clamped boring bar are related e.g. to its physical dimensions, the
material it is made of, the clamping conditions of it.

The following conclusions may be deduced from the results:

- The dynamic properties of a boring bar for different clamping conditions intro-
duced by a clamping housing with clamping screws may differ significantly.
- A clamping housing with clamping screws is likely to provide different clamp-
ing conditions/boundary conditions of a boring bar for different clamp screw
torques and if tightened firstly from the upper side or tightened firstly from the
underside.
- A boring bar is likely to exhibit different dynamic properties when clamped or
mounted in the clamping housing with clamping screws by different operators
and from time to time.
- Different dynamic properties of a clamped boring bar are likely to result in
different dynamic properties of the boring bar vibration during machining for
identical cutting data, insert and work material.
- The set of cutting data that enables stable cutting for a given insert and work
material may differ for different clamping conditions.
- A clamping housing with clamp screws is likely to introduce nonlinear dynamic
properties in the boring bar dynamics.
- A multi-span Euler-Bernoulli model compared to a fixed-free Euler-Bernoulli
model is more adequate for the modeling of dynamic properties of a boring bar
clamped in a clamping housing with clamping screws.

5 Future work

In order to further enhance the modeling of boring bars and the actual clamp-
ing/boundary conditions imposed by the clamping housing, the finite element method
(FEM) will be considered. Also, the possibility to improve accuracy of boring bar
models by the inclusion of simple nonlinearities will be investigated.

Acknowledgments

The present project is sponsored by the company Acticut International AB in Sweden
which has multiple approved patents and products covering active control technology
for metal cutting.
References


Part III

Estimation and Simulation of the Nonlinear Dynamic Properties of a Boring bar
This part is submitted as:
Estimation and Simulation of the Nonlinear Dynamic Properties of a Boring bar

Henrik Åkesson¹,², Tatiana Smirnova, Lars Håkansson and Ingvar Claesson
¹Blekinge Institute of Technology, Department of Signal Processing,
372 25 Ronneby, Sweden

Thomas T. Lagö
²Acticut International AB,
Gjuterivägen 6, 311 32 Falkenberg, Sweden

Abstract
In this paper, an initial investigation of the nonlinear dynamic properties of clamped boring bars is carried out. Two nonlinear single-degree-of-freedom models with different softening spring nonlinearity are introduced for modeling the nonlinear dynamic behavior of the fundamental bending mode in the cutting speed direction of a boring bar. Also, two different methods for the simulation of nonlinear models are used. The dynamic behavior in terms of frequency response function estimates for the nonlinear models and the experimental modal analysis of the clamped boring bar is compared. Similar resonance frequency shift behavior for varying excitation force levels is observed for both the nonlinear models and the actual boring bar.

1 Introduction
In industry where metal cutting operations such as turning, milling, boring and grinding take place, degrading vibrations are a common problem. In internal turning operations, vibration is a pronounced problem, as long and slender boring bars are usually required to perform the internal machining of workpieces. Tool vibration during internal turning frequently has a degrading influence on surface quality, tool life and production efficiency. At the same time, such vibrations result in high noise levels.

An extensive number of experimental and analytical studies have been carried out to study boring bar dynamics. However, most research has usually been carried out on the dynamic modeling of cutting dynamics and usually concentrates on the prediction of stability limits [1–5]. In a study concerning the motion of the boring bar, it was stated that clamped boring bars may display nonlinear dynamic behavior [6]. Later, a more thorough investigation concerning the clamping conditions of the boring bars [7] confirmed this assumption regarding nonlinear dynamic behavior.

When it comes to nonlinear modeling of the clamping of tools, Yigit et al. [8] examined and modeled a reconfigurable machine tool structure including weakly non-
linear joints in consideration of their cubic stiffness. They used a sub-structuring method called nonlinear receptance coupling and validated the method with experimental data from such a structure. Thus, referring to the literature review, it seems like little work has been done on the identification and modeling of the nonlinear dynamic properties of a clamped boring bar. Knowledge regarding nonlinear dynamic properties of a clamped boring bar may be utilized in the modeling of dynamic properties of boring bars in order to provide increased accuracy in such models. Hence, it seems to be important to be able to identify the nonlinear dynamic properties of a clamped boring bar.

Nonlinearities may be caused by several different factors. Common sources of nonlinearity are, for example; the contact phenomena, in which elements of a system during dynamic motion come into contact with the surrounding environment due to a large displacement, which, in turn, creates a new set of boundary conditions. Another example is friction in joints, or sliding surfaces and large forces and/or deformation that cause the properties of the material to behave in a nonlinear manner, for example plastic deformation [9]. This paper presents an initial investigation of the nonlinear dynamic properties of boring bars. Two nonlinear single-degree-of-freedom models with different softening spring nonlinearity have been introduced in order to model the nonlinear dynamic behavior of the fundamental bending mode in the cutting speed direction of a boring bar. Two different methods for the simulation of nonlinear models are used, i.e. the Runge-Kutta method, implemented in Matlab [10] and the digital filter method [11]. The dynamic behavior in terms of frequency response function estimates for the models and the actual clamped boring bar have been compared.

2 Materials and Methods

2.1 Experimental Setup

The experimental setup and subsequent measurements were carried out in a Mazak SUPER QUICK TURN - 250M CNC turning center. The CNC lathe has a spindle power of 18.5 kW and a maximal machining diameter of 300 mm. The distance between the centers is 1005 mm, the maximum spindle speed is 400 revolutions per minute (r.p.m.) and the center also has a flexible turret with a tool capacity of 12 tools. Fig. 1 illustrates some of the basic structural parts used during internal turning, i.e. a workpiece clamped to a chuck and a boring bar clamped to a clamping housing.

Initially, a right-hand cartesian coordinate system was defined. Subsequently, a sign convention was defined for use throughout the paper. The coordinate system and the sign convention are based on the right-hand definition where the directions of displacements and forces in positive directions of the coordinate axes are considered positive.

In all setups and during all measurements, the boring bar was positioned in its operational position, that is, it was mounted to a clamping housing attached to a turret with screws. The turret could be moved in the cutting depth direction, in the x-direction, in the feed direction, and in the z-direction, and it could also be made to rotate about the z-axis for tool change. The turret, etc. was supported by a slide which in turn, was mounted on the lathe bed. Even though the turret was a movable component, it was relatively rigid, rendering the dynamic properties of the boring bars observable.
2.1.1 **Boring Bar**

The boring bar used in the modal analysis was a standard boring bar, WIDAX S40T PDUNR15F3 D6G. In Fig. 2, a drawing of the boring bar is shown.

![Diagram of the boring bar](image)

**Figure 2:** a) Top-view of the standard boring bar "WIDAX S40T PDUNR15F3 D6G", b) the cross section of the boring bar where $C_C$ is the center of the circle and $M_C$ is the mass center of the boring bar.

The standard WIDAX S40T PDUNR15 boring bar is manufactured from the material 30CrNiMo8, (AISI 4330) which is a heat treatable steel alloy (for high strength), see Table 1 for material properties.

2.1.2 **Clamping Condition**

The boring bar was positioned in a clamping housing. The clamping housing was a basic 8437-0 40 mm Mazak holder, presented in Fig. 3 a) and b). The clamping housing has a circular cavity that the boring bar fits easily into. The clamping is then carried out by means of screws on the tool side and on the opposite side of the boring bar by means of either four or six screws: two/three inserted from the top and two/three inserted from the bottom. The basic holder itself is mounted onto the
### Material composition besides Fe, in percent

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>Si</th>
<th>Mn</th>
<th>S</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.26-0.33</td>
<td>1.80-2.20</td>
<td>1.80-2.20</td>
<td>0.30-0.50</td>
<td>&lt;0.40</td>
<td>&lt;0.60</td>
<td>&lt;0.035</td>
<td>&lt;0.035</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Young’s Modulus</th>
<th>Tensile Strength</th>
<th>Yield Strength</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>205 GPa</td>
<td>1250 MPa</td>
<td>1040 MPa</td>
<td>7850 kg/m³</td>
</tr>
</tbody>
</table>

Table 1: Composition and properties of the material 30CrNiMo8.

turret with four screws. In addition to the screws, the clamping housing also contains a guide that matches a track on the turret. This guide positions the clamping housing along the z-axis on the xy plane, whilst the guide pin positions the clamping housing on the z-axis.

![Screw positions for the clamping of the boring bar](image1.png)

Figure 3: The clamping housing. a) The guide and the guide pin may be observed on the underside of the clamping housing, whilst the threaded holes for the screws clamping the boring bar are shown on the right side. b) The screw positions for attaching the clamping housing to the turrets are shown from the top side.

Six screws of the size M10 were used to clamp the boring bar, three from the top and three from the bottom. The screws were of the type MC6S norm “DIN 912, ISO 4762”. The screws were zinc-plated, steel socket, head cap screws with the strength class 8.8, and a tensile yield strength of \( R_{p0.2} = 660 \text{MPa} \).

#### 2.1.3 Measurement Equipment and Setup

The following equipment was used in the experimental setup;

- 12 PCB Piezotronics, Inc. 333A32 accelerometers.
- 2 Brüel & Kjær 8001 impedance head.
- 1 Brüel & Kjær NEXUS 2 channel conditioning amplifier 2692.
- OSC audio power amplifier, USA 850.
- Ling dynamic systems shaker v201.
- Gearing & Watson electronics shaker v4.
• Hewlett Packard 54601B oscilloscope.
• Hewlett Packard 35670A signal analyzer.
• Hewlett Packard VXI mainframe E8408A.
• Hewlett Packard E1432A 4-16 channel 51.2 kSa/s digitizer.
• PC with I-DEAS 10 NX Series.

Twelve accelerometers and two cement studs for the impedance heads were attached on the boring bars with X60 glue (a cold hardener, two-component glue). The sensors were evenly distributed along the centerline, on the under-side and on the backside of the boring bar and six accelerometers and one stud were attached to the respective sides, see Fig. 4. To excite all the lower order bending modes, two shakers were attached via stinger rods to the impedance heads, one in the cutting speed direction (y-) and the other in the cutting depth direction (x-). The shakers were positioned relatively close to the cutting tool.

![Diagram of boring bar with dimensions and sensor locations](image)

Figure 4: Drawings of the boring bar including clamp screws, cement studs and sensors. The sensors are attached along the underside and the backside of the boring bar. The threaded holes denoted $B_1$, $B_2$ and $B_3$ are screw positions indicating where the boring bar has been clamped from top and bottom. The dimension are in mm, where $l_1 = 10.7$, $l_2 = 18$, $l_3 = 101$, $l_4 = 250$, $l_5 = 35$, $l_6 = 100$, $l_7 = 18.5$, $l_8 = 25$.

### 2.2 Experimental Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties or the modal parameters of the system under examination. In other words, the goal is to determine the natural frequencies, mode shapes, and damping ratios from experimental vibration measurements. The procedure of modal analysis may be divided into two parts: the acquisition of data and the parameter estimation or parameter identification of these data, a process also known as curve fitting [12]. Acquiring good data and performing accurate parameter identification is an iterative process, based on various assumptions along the way [12].
2.2.1 Partial Fraction Model

If the frequency response function estimates have well separated modes, the Partial Fraction Model technique [13] can be used to estimate the modal parameters. The partial fraction model \( H_L(f) \) is described as

\[
H_L(f) = \sum_{r=1}^{N} \frac{A_r}{j2\pi f - \lambda_r} + \frac{A_r^*}{j2\pi f - \lambda_r^*}
\]

(1)

\[
H_L, (f) = \frac{A_r}{j2\pi f - \lambda_r} + \frac{A_r^*}{j2\pi f - \lambda_r^*}
\]

(2)

Where \( A_r \) and \( \lambda_r \) are the residue and the system pole, respectively, belonging to the mode \( r \). The last fractional part is close to zero near the natural frequency \( f_r \). Thus, the model can be simplified into;

\[
H(f) \approx \frac{A}{j2\pi f - \lambda}
\]

(3)

\[
\hat{H}(f) = \frac{A}{j2\pi f - \lambda}
\]

(4)

\[
\hat{H}(f)(j2\pi f - \lambda) = A
\]

(5)

\[
j2\pi f \hat{H}(f) = \lambda \hat{H}(f) + A.
\]

(6)

This results in an over determined linear equation system like equation 7, easily solved by the least-square method using, for example, the the Moore-Penrose pseudo-inverse [14]:

\[
\begin{bmatrix}
\hat{H}(f_0) & 1 \\
\hat{H}(f_1) & 1 \\
\vdots & \vdots \\
\hat{H}(f_K) & 1 \\
\end{bmatrix}
\begin{bmatrix}
\lambda \\
A
\end{bmatrix}
=
\begin{bmatrix}
j2\pi f_0 \hat{H}(f_0) \\
 j2\pi f_1 \hat{H}(f_1) \\
\vdots \\
 j2\pi f_K \hat{H}(f_K)
\end{bmatrix}
\]

(7)

where

\[
\lambda = -\zeta 2\pi f_0 + j2\pi f_0\sqrt{1 - \zeta^2}.
\]

(8)

2.3 Spectral Properties

Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [15].

A non-parametric estimate of the power spectral density \( P_{xx}(f) \), where \( f \) is frequency, for a signal \( x(t) \) may be estimated using the Welch spectrum estimator [16], given by:

\[
\hat{P}_{xx}(f_k) = \frac{1}{LNF_s} \sum_{l=0}^{L-1} \sum_{n=0}^{N-1} w(n)x_l(n)e^{-j2\pi nk/N} \right|^2, \quad f_k = \frac{k}{N} F_s
\]

(9)

where \( k = 0, \ldots, N - 1, \) \( L \) is the number of periodograms, \( N \) is the length of the data segments used to produce the periodograms, \( x_l(n) \) is the sampled signal in segment \( l \) and \( F_s \) is the sampling frequency.
Thus, for each input signal \( x(t) \) and output signal \( y(t) \), a single-input-single-output system (SISO) system is simultaneously measured and the sampled signals \( y(n) \) and \( x(n) \) are recorded. By using, for example, the Welch spectrum estimator [16], the cross-power spectral density \( \hat{P}_{yx}(f_k) \) between the input signal \( x(n) \) and the output signal \( y(n) \) and the power spectral density \( \hat{P}_{xx}(f_k) \) for the input signal \( x(n) \) may be estimated [6,15].

A least-squares estimate of a frequency response function between the input signal \( x(n) \) and the output signal \( y(n) \) may be determined according to [15]:

\[
\hat{H}(f_k) = \frac{\hat{P}_{yx}(f_k)}{\hat{P}_{xx}(f_k)}
\]

(10)

and the coherence function as [15]

\[
\hat{\gamma}^2_{yx}(f_k) = \frac{\hat{P}_{yx}(f_k)\hat{P}_{xy}(f_k)}{\hat{P}_{xx}(f_k)\hat{P}_{yy}(f_k)}.
\]

(11)

The least-square estimate for the SISO system in Eq. 10, can be rewritten for the multiple-input-multiple-output (MIMO) system yielding the estimate of the system matrix \( \hat{H}(f_k) \) as [15]

\[
\left[ \hat{H}(f_k) \right] = \left[ \hat{P}_{yx}(f_k) \right] \left[ \hat{P}_{xx}(f_k) \right]^{-1}
\]

(12)

where \( \left[ \hat{P}_{xx}(f_k) \right] \) is an estimate of the cross spectrum matrix between all the inputs and \( \left[ \hat{P}_{yx}(f_k) \right] \) is an estimate of the cross spectrum matrix between all the inputs and outputs.

In the case of multiple inputs, the multiple coherence is of interest as an indication of the quality of the measurements. The multiple coherence function is defined by the ratio of that part of the spectrum which can be expressed as a linear function of the inputs to the total output spectrum (including extraneous noise), and the multiple coherence function is an extension of the ordinary coherence function from the SISO case [15].

### 2.3.1 Excitation Signal

For the experimental modal analysis, burst random was used as the excitation signal. Based on initial experiments concerning suitable burst length and frequency resolution (data segment time or data block length time), a burst length of 90% of the data block length time was selected, see Table 2. Basically, the frequency resolution was tuned to provide high overall coherence in the analysis bandwidth and the burst length was tuned to provide high coherence at the resonance frequencies.

### 2.4 Nonlinear Modeling Methods

The softening spring may be modeled in two different ways, yielding different properties with respect to the displacement. The first of these models yields a force proportional to a nonlinear stiffness coefficient, multiplied by the displacement, squared with sign. The equation describing a single-degree-of-freedom (SDOF) system with this type of a softening spring nonlinearity is given by [17]

\[
m\frac{d^2x(t)}{dt^2} + c \frac{dx(t)}{dt} + kx(t) - k_s x|\dot{x}(t)| = f(t)
\]

(13)
Table 2: Spectral density estimation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Burst random</td>
</tr>
<tr>
<td>Sampling frequency $f_s$</td>
<td>10240 Hz</td>
</tr>
<tr>
<td>Block length $N$</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency resolution $\Delta f$</td>
<td>0.5 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>200</td>
</tr>
<tr>
<td>Window</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
</tr>
<tr>
<td>Frequency range of burst</td>
<td>0-4000 Hz</td>
</tr>
<tr>
<td>Burst length</td>
<td>90%</td>
</tr>
</tbody>
</table>

where $m$, $c$ and $k$ are the mass, damping and stiffness coefficients of the underlying linear system, $x(t)$ is the displacement, $f(t)$ is the force, and $k_c$ is the nonlinear stiffness coefficient. The second model yields a force proportional to the nonlinear stiffness coefficient multiplied by the displacement cubed. Inserted into the equation of motion describing a SDOF system, this model results in [17]

$$m \frac{d^2 x(t)}{dt^2} + c \frac{dx(t)}{dt} + k x(t) - k_c x^3(t) = f(t)$$

where $k_c$ is the nonlinear stiffness coefficient.

In order to see if any of the proposed nonlinearities may explain the different results from the experimental modal analysis, a number of different simulations were carried out using different parameters.

2.4.1 Nonlinear Synthesis

The are different ways of simulating linear and nonlinear systems: the most common method to solve ordinary differential equations (ODE) is probably the Runge-Kutta method implemented in Matlab [18]. Another method is the digital filter method [19]. There are multiple advantages with using ODE solvers: they are rather straightforward to use and they are well known. The disadvantage, however, is that they are relatively time consuming if large amounts of data are involved. The filter method, on the other hand, while significantly faster than the ODE solvers [19], are not as well documented as the ODE solvers with regard to, for example, accuracy and the ability to handle nonlinear systems [10]. However, for linear systems, the limitations of the filter method are known [11] and depends on the sampling frequency and the transformation method used to convert the continuous time parameters to discrete time parameters [11].

2.4.2 Ordinary Differential Equation Methods

The simulation method used for simulating the nonlinear system is based on an explicit Runge-Kutta of order (4,5), based on the Dormand-Prince pair [18,20], referred to as ode45 in Matlab. The ode45 method combines a fourth order method and a fifth order method, both of which are similar to the classic fourth order Runge-Kutta [20, 21]. The numerical technique solves ordinary differential equations of the
form [20]
\[
\frac{dx(t)}{dt} = f(x(t), t), \quad x(t_0) = x_0. \tag{15}
\]

Since the Runge-Kutta method only solves first order differential equations, the second order differential equations in Eqs. 13 and 14 must be rewritten to coupled first order differential equations as in Eqs. 18 and 19.

The nonlinear models simulated with the differential equation solvers were based on the softening spring using the quadratic model in Eq. 16, and the cubed model in Eq. 17:
\[
g_q(x(t)) = k_s x| x(t) \tag{16}
g_c(x(t)) = k_c x^3(t) \tag{17}
\]

where \(g_q(x(t))\) and \(g_c(x(t))\) replace \(g(x_1(t))\) in Eq. 19 the for respective model. The model may then be described in state-space formulation as
\[
\frac{dx_1(t)}{dt} = x_2(t) \tag{18}
\]
\[
m \frac{dx_2(t)}{dt} = -c x_2(t) - k x_1(t) + g(x_1(t)) + f(t) \tag{19}
\]

where \(x(t)\) is the response of the system, and \(f(t)\) is the driving force.

### 2.4.3 Filter Method

The filter method is a time-discrete method for extracting digital filter coefficients from the analog system using an appropriate transformation method. Thus, the differential equation is transformed into a difference equation, represented by a digital filter [19]. The filtering procedure in the discrete time domain is given by
\[
x(n) = \sum_{k=-\infty}^{\infty} h(k) f(n - k) \tag{20}
\]

where, again, \(x(n)\) is the response and \(f(n)\) is the input to the system with the impulse response \(h(n)\), but in the discrete time domain.

The transformation may be performed by first dividing the total (multiple degree of freedom) system into subsystems using the modal superposition theorem and transforming each subsystem’s parameters into filter coefficients. The frequency response function for a dynamic system may be expressed in terms of modal superposition as [12]
\[
H(f) = \sum_{r=1}^{R} \frac{A_r}{j2\pi f - \lambda_r} + \frac{A_r^*}{j2\pi f - \lambda_r^*}, \tag{21}
\]

where \(R\) is the number of modes, \(A_r\) is the system’s residues belonging to mode \(r\), and \(\lambda_r\) is the pole belonging to mode \(r\). The poles and residues may be extracted from a lumped parameter system, or from a distributed parameter system, or estimated from experimental modal analysis [22]. Another approach is to directly express the system as in Eq. 22 and transform the analog filter coefficients into digital filter coefficients. The transfer function for an analog filter can be expressed as [23]
\[
H(s) = \frac{D(s)}{C(s)} = \frac{d_0 + d_1 s + \ldots + d_M s^M}{1 + c_1 s + \ldots + c_K s^K} \tag{22}
\]
where $M_a$ is the order of the polynomial $D(s)$ in the numerator, and $K_a$ is the order of the polynomial $C(s)$ in the denominator. Transforming the analog filter yields a digital filter whose $z$-transform may be expressed as [23]

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_K z^{-K}}$$

(23)

where $M$ is the order of the polynomial $B(z)$ in the numerator, and $K$ is the order of the polynomial $A(z)$ in the denominator. In the discrete time domain, the difference equation describing the filter may be written as [23]

$$x(n) = \sum_{m=0}^{M} b_m f(n-m) - \sum_{k=1}^{K} a_k x(n-k).$$

(24)

One of the most common transformation methods is the so-called ”impulse invariant” method, which allows the digital signal to represent the analog signal by an impulse at sampled intervals, i.e. $x(t) \rightarrow Tx(nT)$, where $T$ is the sampling period [24]. Other methods include the step invariant, the ramp invariant, the centered step invariant, the cubic spline invariant and the Lagrange method, each with different properties [11]. The ramp invariant method was used in the simulations and is defined as:

$$f(nT + t) = \frac{f(nT + T) - f(nT)}{T}t + f(nT), \quad \text{where} \quad 0 \leq t \leq T.$$  

(25)

This transform method produces zero error at DC, low error at the Nyquist frequency and low phase distortion [11]. At the same time, the ramp invariant method does produce a large error at a the resonance frequency in comparison to, for example, the impulse, step and cantered step invariant methods [11]. However, the error introduced in the area of the resonance frequency only becomes large when the resonance frequency approaches the Nyquist frequency. Therefore, an over-sampling of 20 times the highest resonance frequency was used in the simulations.

The filter method for simulation of nonlinear systems is carried out by using the digital filter coefficient for the linear system, and finding the solutions for the nonlinear difference equation. In the discrete time domain, the nonlinear difference equation may be written as [19]

$$x(n) = \sum_{m=0}^{M} b_m (f(n-m) - g(x(n-m))) - \sum_{k=1}^{K} a_k x(n-k).$$

(26)

Since Eq. 26 contains nonlinear terms, several solutions may exist for $x(n)$ [21]. The value of $x(n)$ may be found using any of the zero searching algorithms such as the secant method, bisection method or Newton-Raphson [21] (which was used in this synthesis).

The models with a nonlinear function $g(x(n))$ simulated with the filter method are defined as:

$$g_{Lin}(n) - g_s(x(n)) = f(n), \quad \text{where} \quad g_s(x(n)) = k_s x |x|(n)$$

(27)

and

$$g_{Lin}(n) - g_c(x(n)) = f(n), \quad \text{where} \quad g_c(x(n)) = k_c x^3(n).$$

(28)

(29)
The digital filter coefficients were based on the poles and residues estimated from data acquired during the experiments.

### 2.4.4 Error Analysis

To be able to draw any relevant conclusion from the simulated data, it is necessary to know approximately how large the error is. The two previously mentioned methods, i.e. the ODE and the filter method, introduce different errors. The ODE solver has both a local error and a global error [18]. The local error over a time step $h$ is of the order $O(h^{n+1})$. This results in an error less than $h^5$, where $h$ is the time step and $n$ the order of the ODE solver. The local error is then propagated with each time step until it results in the global error. Beside these errors there is of course the numerical precision which is limited by a 64-bit double-precision representation. Both the local and the global error can be set in matlab, or, if the step size $h$ is chosen small enough from the beginning, both the local and the global error will be kept under the default thresholds [20]. When using the filter method, one more error which is related to the Laplace to Z-transform conversion must be taken into consideration. To reduce the effects caused by this error, a suitable transformation method related to the problem formulation is selected along with a small step size. For the nonlinear simulations done in this paper, a number of different step sizes were tested including the final step size of $h = 1 \times 10^{-4}$. Furthermore, the threshold limit for the Newton-Raphson zero finder was set to $\varepsilon = 1 \cdot 10^{-14}$.

### 3 Computer Simulations of Nonlinear Systems

The simulation used a linear component of the models which was based on parameters derived from the experimental modal analysis of the standard boring bar. The standard boring bar was clamped with six screws that were first tightened from the bottom. The tightening torque was 10Nm and the excitation level was set at the lowest possible setting. This setup resulted in well separated modes. Thus, the Partial Fraction Model technique [13] can be used to estimate the modal parameters.

The estimated values are: the resonance frequency $f_0 = 504.177$ Hz, the damping $\zeta = 1.714$ % and the residue $A = -j1.357 \cdot 10^{-4}$. Thus, the linear system may be expressed as

$$H_L(f) = \frac{-j1.357 \cdot 10^{-4}}{j2\pi f - (-54.292 + j3.167 \cdot 10^3)} + \frac{j1.357 \cdot 10^{-4}}{j2\pi f - (-54.292 - j3.167 \cdot 10^3)}.$$  \hspace{1cm} (30)

In Fig. 5, the driving point accelerance functions of the boring bar in the direction of the cutting speed are presented together with the corresponding synthesized accelerance function.

These parameters are directly applicable to the filter-method when calculating the filter coefficients. However, when using the ordinary differential equation solvers, the partial fractions are collected into one polynomial fraction that may be expressed in terms of the mass, damping and stiffness coefficients $m$, $c$ and $k$. Theses parameters
were determined using the following relations

\[ A = \frac{1}{jm4\pi f_0} \quad (31) \]
\[ f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (32) \]
\[ \zeta = \frac{c}{2\sqrt{mk}} \quad (33) \]

which yield a mass of \( m = 1.632 \) kg, a damping of \( c = 126.307 \) Ns/m and a stiffness of \( k = 1.167 \cdot 10^7 \) N/m.

### 3.1 Excitation Signal

True random was selected for the excitation signal. The estimation parameters used in the nonlinear simulations are presented in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>True random</td>
</tr>
<tr>
<td>Sampling frequency ( f_s )</td>
<td>10000 Hz</td>
</tr>
<tr>
<td>Block length ( N )</td>
<td>20480</td>
</tr>
<tr>
<td>Frequency resolution ( \Delta f )</td>
<td>0.4883 Hz</td>
</tr>
<tr>
<td>Number of averages ( L )</td>
<td>200</td>
</tr>
<tr>
<td>Window</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 3: Spectral density estimation parameters and the excitation signal used in the simulated nonlinear system.
3.2 Softening Spring Model

The nonlinear softening stiffness coefficients $k_s$ and $k_c$ in the signed squared and cubic models were not obtained by direct parameter estimation. The resonance frequency shifting phenomenon always appears between the accelerance function estimates for the standard boring bar clamped with screws for different excitation force levels. Typically, a resonance frequency shift of 5 Hz and, for example, an initial resonance frequency of 500 Hz render a frequency deviation of 1%, which corresponds to a 10% deviation in the linear stiffness coefficient. By considering the stiffness deviation, the stiffness coefficient used in the linear model, the level of excitation force and the convergence rate in the simulation, the values for the nonlinear stiffness coefficients were selected as: $k_s = 5 \cdot 10^{11}$ N/m$^2$ and $k_c = 4 \cdot 10^{18}$ N/m$^3$ for the signed squared and cubic models, respectively. The levels of the excitation force were given the same ratios as for the experiments with the standard boring bar, and the signal type was normally distributed random noise, with the peak levels of 100, 200, 300 and 400 mN.

4 Results

The results from the experimental modal analysis and the simulations are presented as frequency response function estimates in form of accelerance, and the coherence to the corresponding frequency response function estimate is also presented. Furthermore, a table with the estimated undamped resonance frequency with corresponding damping is presented.

4.1 Experimental Results

The estimated accelerance response function from the experimental modal analysis is presented in Fig. 6 a) and the corresponding coherence function in Fig. 6 b). The estimated modal parameters are presented in Table 4.

![Figure 6](image)

Figure 6: a) Driving point frequency response function estimates of the boring bar in the cutting speed direction for four different excitation levels, and b) the corresponding multiple coherence.
Table 4: Resonance frequency and relative damping estimates for the frequency response functions based on the experimental modal analysis.

### 4.2 Simulation Results

Fig. 7 a) presents the frequency response function estimates that were produced based on simulations of the nonlinear model with signed squared stiffness, using the filter method and the four different excitation levels. Fig. 7 b) presents the corresponding frequency response function estimates that were produced based on simulations of the nonlinear model system with cubic stiffness, using the filter method and the four different excitation levels.

![Frequency response function estimates](image)

Figure 7: Frequency response function estimates based on simulations of the nonlinear models using the filter method and four different excitation levels, a) for the presented model with signed squared stiffness, and; b) for the model with cubic stiffness.

Table 7 gives the estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models simulated with the filter method for the four excitation force levels. The SDOF least-square technique [12] was used to produce estimates of the resonance frequency and the relative damping.

<table>
<thead>
<tr>
<th>Squared Model</th>
<th>Excitation</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
<td>503.73</td>
<td>503.42</td>
<td>503.11</td>
<td>502.80</td>
<td></td>
</tr>
<tr>
<td>Damping [%]</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cubic Model</th>
<th>Excitation</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
<td>503.98</td>
<td>503.79</td>
<td>503.46</td>
<td>503.01</td>
<td></td>
</tr>
<tr>
<td>Damping [%]</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
<td>1.71</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Resonance frequency and relative damping estimates for the frequency response functions based on the nonlinear models, simulated with the filter method.
The coherence function estimates are also presented for a narrow frequency range, including the resonance frequency, and are illustrated for the nonlinear model with signed squared stiffness in Fig. 8 a) and for the nonlinear model with cubic stiffness in Fig. 8 b).

![Figure 8: Coherence function estimates based on simulations of the nonlinear models, using the filter method and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.](image)

If the ordinary differential equation solver ode45 in Matlab is used for the four different excitation levels of the nonlinear model with signed squared stiffness, this results in the frequency response function estimates shown in Fig. 9 a). Fig. 9 b) presents the corresponding frequency response function estimates, based on simulations of the nonlinear model system with cubic stiffness, using the ordinary differential equation solver ode45 in Matlab and the four different excitation levels.

![Figure 9: Frequency response function estimates based on the simulation of the nonlinear models, using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness and; b) for the model with cubic stiffness.](image)

Table 6 presents estimates of the resonance frequency and the relative damping for the frequency response functions for the four excitation force levels, based on the
nonlinear models and simulated with the ordinary differential equation solver ode45 in Matlab. Also, in this case, the SDOF least-square technique [12] was used to produce estimates of the resonance frequency and the relative damping.

<table>
<thead>
<tr>
<th>Squared model</th>
<th>Excitation</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
<td>503.48</td>
<td>502.84</td>
<td>502.18</td>
<td>501.54</td>
<td></td>
</tr>
<tr>
<td>Damping [%]</td>
<td>1.70</td>
<td>1.70</td>
<td>1.70</td>
<td>1.71</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cubic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency [Hz]</td>
</tr>
<tr>
<td>Damping [%]</td>
</tr>
</tbody>
</table>

Table 6: Estimates of the resonance frequency and the relative damping for the frequency response functions based on the nonlinear models simulated with the differential equation solver ode45.

The coherence function estimates are also presented for a narrow frequency range (including the resonance frequency) and are illustrated for the nonlinear model with signed squared stiffness in Fig. 10 a) and for the nonlinear model with cubic stiffness in Fig. 10 b).

Figure 10: Coherence function estimates based on simulations of the nonlinear models using the ordinary differential equation solver ode45 in Matlab and four different excitation levels, a) for the model with signed squared stiffness presented and b) for the model with cubic stiffness.

4.3 Comparison between Experiment and Simulations

The comparisons between the synthesized accelerance functions based on the parameters estimated from the measured data and the simulated synthesized accelerance functions based on the simulated data are presented in Fig. 11. The driving point accelerance function estimates based on the experimental data are presented in Fig. 11 a) and the accelerance function estimates based on simulated data using the signed square nonlinear model and simulated with the filter method are presented in Fig. 11 b).
b). Resonance frequency estimates from the experimental results and all the simulated models are collected and presented in Table 7.

![Figure 11: Frequency response function estimates a) based on experimental data, using a standard boring bar clamped with six M10 screws and four different excitation levels, and b) based on simulated data from the signed square nonlinear model, simulated with the filter method.](image)

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental data</td>
<td>504.18</td>
<td>503.50</td>
<td>503.01</td>
<td>502.57</td>
</tr>
<tr>
<td>Filter method, $x</td>
<td>x</td>
<td>$ model</td>
<td>503.73</td>
<td>503.42</td>
</tr>
<tr>
<td>Filter method, $x^3$ model</td>
<td>503.98</td>
<td>503.79</td>
<td>503.46</td>
<td>503.01</td>
</tr>
<tr>
<td>ODE method, $x</td>
<td>x</td>
<td>$ model</td>
<td>503.48</td>
<td>502.84</td>
</tr>
<tr>
<td>ODE method, $x^3$ model</td>
<td>503.75</td>
<td>502.71</td>
<td>500.93</td>
<td>498.55</td>
</tr>
</tbody>
</table>

Table 7: Resonance frequency estimates for the frequency response functions based on the experimental data and the simulated data.

## 5 Summary and Conclusions

The experimental results obtained during the examination of the dynamic properties of a clamped boring bar (clamped in the clamping housing with screws), strongly indicate the presence of nonlinearity. Two different nonlinear single-degree-of-freedom models were simulated in order to investigate if they bear resemblance to the nonlinear dynamic behavior observed for a boring bar clamped in the clamping housing with screws. In addition, two different simulation methods were used to provide redundancy due to the fact that there are no explicit analytical solutions for the two different nonlinear single-degree-of-freedom models which can be used as benchmarks. Both the square with sign stiffness model and the cubic stiffness model show the similar trend in the frequency response function estimates as the experimental results do, see Tables 7 and 6. The trend is that the resonance frequency decreases with an increasing excitation level; see Fig. 7 and Fig. 9 (produced by the filter method and the
ODE solver method, respectively). The coherence function estimates from the data from the nonlinear systems, simulated with the filter method, display an expected dip at the resonance frequency, see Fig. 8. By using the filter method to simulate the comparatively nonlinear systems, the coherence function estimates yield higher levels in the resonance frequency range of the SDOF systems than those produced by the ODE solver. It is shown that the simulated results display a similar resonance frequency shift as the experimental data, as can be seen if comparing Fig. 6 with Figs. 7 and 9 and also Fig. 11 a) with Fig. 11 b). Thus, including a nonlinearity in the model of the clamped boring bar is likely to provide a more accurate model of the actual boring bar’s dynamic properties in comparison to a pure linear model.

Acknowledgments

The present project is sponsored by the company Acticut International AB in Sweden which has multiple approved patents and products covering active control technology for metal cutting.

References


Part IV

Investigation of the Dynamic Properties of a Milling Tool Holder
This part is published as:

Investigation of the Dynamic Properties of a Milling Tool Holder

Henrik Åkesson\textsuperscript{1,2}, Tatiana Smirnova, Lars Håkansson and Ingvar Claesson
\textsuperscript{1}Blekinge Institute of Technology, Department of Signal Processing, 372 25 Ronneby, Sweden
\textsuperscript{2}Acticut International AB, Gjuterivägen 6, 311 32 Falkenberg, Sweden

Abstract

Vibration problems during metal cutting occur frequently in the manufacturing industry. The vibration level depends on many different parameters such as the material type, the dimensions of the workpiece, the rigidity of tooling structure, the cutting data, and the operation mode. In milling, the cutting process subjects the tool to vibrations, and having a milling tool holder with a long overhang will most likely result in high vibration levels. As a consequence of these vibrations, the tool life is reduced, the surface finishing becomes poor, and disturbing sound appears. In this report, an investigation of the dynamic properties of a milling tool holder with moderate overhang has been carried out by means of experimental modal analysis and vibration analysis during the operating mode. Both the angular vibrations of the rotating tool and the vibrations of the machine tool structure were examined during milling. Also, vibration of the workpiece and the milling machine was examined during cutting. This report focuses on identifying the source/sources of the dominant milling vibration components and on determining which of these vibrations that are related to the structural dynamic properties of the milling tool holder.
## Contents

1 Introduction

1.1 Literature Review ........................................... 97
1.1.1 Chatter Theory ........................................... 97
1.1.2 Force Models ............................................ 98
1.1.3 Stability ................................................ 98
1.1.4 Vibration Control ........................................ 98
1.1.5 Motivation ............................................... 99
1.2 Basic Concepts of Metal Cutting in Milling ...................... 99
1.3 Measurement of Forces and Vibrations .......................... 102

2 Materials and Methods ........................................... 102
2.1 Experimental Setup ........................................... 102
2.1.1 Tool Holder and Tool ..................................... 103
2.1.2 Cutting Data ............................................ 103
2.1.3 Measurement Equipment and Setup ......................... 105
2.1.4 Spatial Measurements of the Acceleration During Milling ... 106
2.1.5 Modal Analysis Setup ..................................... 106
2.1.6 Excitation Signal for the Experimental Modal Analysis .... 107
2.2 Spectral Properties .......................................... 107
2.3 Operating Deflection Shape Analysis ........................ 109
2.4 Experimental Modal Analysis ................................ 109
2.5 Modal Parameter Estimation .................................. 109
2.5.1 Spectral Estimation Parameters ......................... 110
2.6 Distributed Parameter Model of the Milling Tool Holder ..... 112
2.6.1 A Geometrical Model of the Clamped Milling Tool Holder ... 112
2.6.2 A Model of Transverse Vibrations ....................... 113
2.6.3 A Model of Torsional Vibrations ......................... 114
2.7 A Finite Element Model of the Milling Tool Holder ........... 114

3 Results ......................................................... 114
3.1 Spatial Measurements of Vibration ............................ 115
3.1.1 Operating Deflection Shapes ............................. 117
3.1.2 Angular Vibrations ....................................... 120
3.2 Modal Analysis Results ...................................... 124
3.2.1 Mode Shapes ........................................... 124
3.3 Distributed Parameter Model Results ........................ 127
3.4 Finite Element Model ......................................... 130

4 Summary and Conclusion ......................................... 130
1 Introduction

Metal cutting is generally used in the manufacturing industry to machine, e.g., workpieces to desired geometries with certain tolerances. During the machining process, a number of different machining operations may be involved. There are several different machining operations including turning, milling, drilling, boring, threading, etc. [1]. Today, there are many advanced machines that have several axes and that can perform complex milling and turning operations about non-fixed axes [1] by, for example, rotating or leaning the axis of the spindle. Another example of the type of advanced operation that modern machines are capable of is the production of an oval or ellipsoidal cross-section of a workpiece by controlling the tool motion in the radial direction of the workpiece during turning.

The metal cutting operation may sometimes produce high server vibration levels. The cause of these vibrations can be attributed to many different factors such as the cutting parameters, the workpiece material and shape, the tooling structure, the insert, and the stability of the machine [2]. Thus, there are many different parameters that influence the stability of the cutting process in milling operations, and there has been a lot of research done in this area.

1.1 Literature Review

Turning has been one of the most studied metal cutting processes due to the fact that it is comparatively easy to monitor the forces applied to the tools under controlled conditions [3]. Turning has also been used to imitate the periodic excitation present in the milling operation by rotating a non-continuously shaped workpiece that produces an intermittent tooth pass excitation [4]. Many similarities between the cutting processes in turning operations and in milling operations may be observed, and chatter theory developed for turning operations is also used in milling theory to produce rough estimates of stability limits [5,6].

1.1.1 Chatter Theory

Some of the earliest studies on the principles of chatter in simple machine tool systems were produced by Arnold [7] in 1946. In 1965, Tobias [8] presented an extensive summary of results from a large number of research studies concerning the dynamic behavior of the lathe application and the chatter theory, and he further developed the research into the chatter phenomena in consideration of the chip thickness variation and the phase lag of the undulation of the surface. The same year, Merritt [9] discussed the stability of structures with n-degrees of freedom, assuming the absence of dynamics in the cutting process. He also proposed a simple stability criterion. Together with Tobias and Merritt, Koenigsberger and Tlusty are also considered to be the pioneers of chip regeneration formulation for basic chatter theory [10]. Furthermore, Pandit et al. [11] developed a procedure for modeling chatter from time series by including unknown factors of random disturbances present in the cutting process. They formulated self-excited random vibrations with white noise as a forcing function. Finally, Kato et al. [12] investigated regenerative chatter vibration caused by the deflection of the workpiece and introduced a differential equation describing chatter vibration based on experimental data.
1.1.2 Force Models

There are many force models of various complexity and properties that describe the cutting process [13–17]. Tlusty [13] presented the relationship between maximum depth of cut, stiffness of the structure, and a specific cutting coefficient of the cutting process for turning, where the maximum width of cut is proportional to the static stiffness and the damping ratio at the cutting tool point of the machine tool. Later, in 1991, Smith and Tlusty [18] summarized the force models and simulation methods of the milling process currently used so far. In 1996, Altintas presented a force model focusing on the helical end mill geometry [14]. In addition to this, Tlusty et al. presented numerical simulations of the milling dynamics, including saturations such as the tool jumping out of cut [19]. Also, Engin and Altintas [20] presented a generalized mathematical model of inserted cutters for the purpose of predicting cutting forces. The model is capable of considering various insert geometries, angles, and positions relative to the cutter body.

1.1.3 Stability

In 1981, Tlusty and Ismail [19] studied the basic non-linearity of the cutting processes by analyzing the vibrations that occur when the tool leaves the workpiece for a part of the cycle. This was done for both turning and milling and took into consideration the mode coupling self-excitation. No further conclusions with respect to turning were made, however, they found that stability boundaries related to the milling were calculated erroneously by a factor of two to three by the methods known at the time, thus opening up possibilities for the improvement of stability methods. Furthermore, an improved method for obtaining stability lobes was presented in 1983 by Tlusty et al. [21]. In the beginning of the 1990s, the milling operation received more consideration, which resulted in more accurate stability lobes/diagrams based on various additional properties related to the milling operation [5,22–27]. A practical example of how to increase cutting performance by considering stability lobes was presented by Tlusty et al. [28]. They studied the performance of a long and slender tool in high speed milling and increased the metal removal rate by choosing the appropriate tool length with respect to stability lobes that allowed maximum spindle speed in high speed milling. However, more contributions can be done to this field, since the variation of machines, tool configuration, inserts, workpiece material, etc., seem to be virtually unlimited.

1.1.4 Vibration Control

The common methods used to control vibrations in milling systems utilize the control mechanisms of the cutting parameters related to the machine. Other methods are either based on passive vibration absorption or active control that applies a secondary control force.

Several methods have been developed to find cutting parameters that can be used to avoid instabilities [29–32]. The most commonly used method is to change some of the parameters during cutting, i.e., changing the spindle speed or the feed rate of the workpiece. The goal to be achieved by changing one of the cutting parameters is to reduce the dynamic feedback into the system and thus to avoid instability. The most common parameter to change is the spindle speed.

An example of a passive solution was put forth in 2008 by Rashid et al. [33]. They presented the development and testing of a tuned viscoelastic dampers in a milling
operation that were able to attenuate vibrations.

Active solutions have also been proposed, for example, by J.L. Dohner et al. [34] who developed an active structural control system able to alter the dynamics of the system. Furthermore, active solution based on embedded piezoelectric actuators in a palletized workpiece holding system for milling was presented by Rashid et al. [35] in 2006. Another active solution focusing on the spindle was implemented by Denkena et al. who used a contactless magnetic guide in a milling machine prototype to sense and actuate harmonic disturbances [36].

1.1.5 Motivation

A major part of the research within metal cutting technology concerns methods for improving the cutting performance and increasing the tool life [3]. Cutting performance may be defined in terms of, e.g., the material removal rate, the surface roughness, and the forces the cutting process applies to the different machine parts [3]. Usually, work concerning methods for improving cutting performance focuses on maximizing the material removal rate, while keeping the surface finish below a certain roughness limit [3].

Methods that focus on tool life basically aim at the development of technology able to increase the time the tools continue to have the desired cutting performance during machining [3]. Thus, it is important to maintain the required chip formation/chip breakage and cutting forces, etc., for as long as possible [1].

From the literature review, it can be seen that much work has been done on both turning and milling theory. Many force models and methods for producing stability diagrams have been developed. Also, different methods for handling vibration problems have been presented in various articles, see for example section 1.1.4 concerning the review on vibration control. However, due to the complexity of different cutting operations, there is still much work to be carried out in order to identify and understand the causes of the problems that arise during machining. It appears that little experimental work has been done on the identification of the dynamic properties of milling vibration and spatial dynamic properties of the milling tool holder and the spindle house. Dynamic modeling of the cutting dynamics is an important research area for the manufacturing industry. Developments in this area are dependent on, among other things, knowledge of the dynamic properties of milling vibration during cutting and the spatial dynamic properties of the milling tool holder and the spindle house. In order to gain further understanding of the dynamic behavior of the milling tool holder and the spindle house in the metal cutting process, both analytical and experimental methods may be utilized. This paper investigates the dynamic properties of the milling vibration and the spatial dynamic behavior of the milling tool holder and the spindle house during milling and mode shapes and corresponding resonance frequencies for the first two modes of a milling tool holder clamped to milling machines. For the purpose of the investigation, spectrum analysis, operating deflection shapes analysis (ODS), experimental modal analysis (EMA), FE-modeling, and distributed-parameter system modeling have been utilized.

1.2 Basic Concepts of Metal Cutting in Milling

A large number of different types of milling cutters, designed for different milling operations, are available today. Some of the most common types of cutters are end
mill cutters, ball nosed cutter slot drills, side and face cutters, gear cutters, and hobbing cutters, see Fig. 1.

Since there are many different types of milling cutters, the understanding of the cutting parameters and their influence on the machining process is important in order to be able to use them properly [3]. The cutting parameters control the basic properties of the cutting process where chip formation is one of the crucial parts. As the milling cutter rotates, the material to be cut is fed into the cutter at a certain speed denoted as the feed rate, and each tooth of the cutter cuts away small chips of workpiece material. During the machining of a workpiece, the chip formation process and chip breakage are of vital importance for maintaining an efficient cutting process. The size and shape of the chip depend on many parameters. The most significant ones are: depth of cut, the feed rate, the cutting speed, the number of teeth, insert/tooth geometry, and the workpiece material [1]. A simplified drawing of the material removal process and the cutting setup in milling is presented in Fig. 2.

The cutting speed $v_C$ (m/min), is related to the spindle speed $n$ (r.p.m) according
Investigation of the Dynamic Properties of a Milling Tool Holder

\[ v_c = \frac{n \pi D_c}{1000} \]  \hspace{1cm} (1)

where \( D_c \) is the diameter of the cutter or tool (mm). The relation between the feed speed \( v_f \) (mm/min), and the feed per tooth \( f_z \) (mm/tooth) is

\[ v_f = n z_n f_z \]  \hspace{1cm} (2)

where \( z_n \) is the number of efficient teeth used in the cutter during machining. One of the parameters which is usually considered in the overall process for optimal efficiency of the production line is the rate at which material is being removed. The material removal rate \( M_{RR} \) (mm\(^3\)/min) depends on the three main cutting parameters: the feed rate \( v_f \), the depth of cut \( a_p \) (mm), and the width of cut \( a_e \) (mm). It may be expressed as

\[ M_{RR} = a_e a_p v_f. \]  \hspace{1cm} (3)

Another important configuration of the cutting setup involves the entrance and exit phases of the tool to and from the workpiece. The configurations used are usually referred to as conventional milling, slot milling and climb milling. These configurations are illustrated in Fig. 3.

Figure 3: Three different cutting configurations. To the left is the climb configuration, in the center the slot configuration and to the right is the conventional configuration presented. The thin arrows represent the counterclockwise rotation of the milling tool seen from the under side of the tool while the thicker strait hollow arrows represent the feed direction of each cutter.

Conventional milling starts with a thin chip thickness at the entrance phase and ends with a larger chip thickness at the exit phase. In order for the insert to start to cut, a sufficient chip thickness must be built up and before the actual cutting starts workpiece material will slide along the surface [1]. This may result in a deformation hardening of the surface and also poor surface finish. At the exit phase, the insert will be exposed to severe tensile stress and the workpiece material might also remain on the edge of the insert. By contrast, climb milling starts with a large chip thickness and exits with a thin chip thickness. The insert does not slide or rub the material, which allows for longer tool life and better surface finish when compared to the conventional setup. However, climb milling usually expose the machine to larger loads compared to conventional milling [2].
1.3 Measurement of Forces and Vibrations

The most common method of analyzing the properties and the performance of milling tools are done by measuring a number of different forces during cutting operations with the help of dynamometers. The measurement of the forces is carried out by either using a table-mounted dynamometer or using a spindle-mounted dynamometer. The table-mounted dynamometer is mounted on the table of the milling machine and any component to be milled can be fixed over the dynamometer. Forces in x, y and z directions may be measured and the coordinate systems of the measured signals stay fixed relative to the milling table. There are also tables/fixtures that measure the ”feed force”, the ”deflection force” and the moment applied to the table. The spindle-mounted dynamometer, which is mounted between the spindle and milling tool, usually measures the cutting forces in the x, y and z directions and moment applied to the spindle, but in this case the x-y coordinate system is rotating relative to the table. In other words, the x-y coordinates rotate with the milling tool. Examples of these types of dynamometers are presented in Fig. 4.

![Dynamometers](image)

Figure 4: Three types of dynamometers; in a) a table-mounted dynamometer measuring forces in the x-y-z directions is presented, b) shows a table-mounted dynamometer measuring torque and c) is a spindle-mounted dynamometer.

When measuring the vibrations of a milling tool holder or a milling tool, laser vibrometers are usually used [37]. This method requires a line of sight and may limit the conditions for the machining. For example, the use of cooling liquids may not be possible. Also, the chips removed during the cutting process might interfere with the measurement. Other types of vibration sensors that may be utilized for the measurement of milling tool vibration are the strain gauge and the piezo film. Such sensors usually require amplifiers mounted on the tool holder and wireless communication such as telemetric equipment to transfer the sensor signals to data acquisition systems. For non-rotating parts, accelerometers are commonly used for vibration measurements.

2 Materials and Methods

2.1 Experimental Setup

The first milling machine used in the experiments was a Hurco BMC-50 vertical CNC machining centre. The spindle was of the ATC type, which means that the spindle
speed can be varied between 10-3000 rpm in steps of 20 rpm, and the maximum torque was 428 Nm, see Fig. 1. The second machine used in the experiments was a DMU 80FD Duoblock which is a 5-axis milling machine, see Fig. 6. In addition to boring and milling operations, this machine can also carry out turning operations in a single machine setup. This is possible because it has a rotary table which can rotate with up to 800 rpm. It has a maximal a torque of 2050 Nm and a holding torque of 3000 Nm. The spindle has a maximal rotation speed of 8000 rpm and a maximal torque of 727 Nm.

Figure 5: The Hurco BMC-50 milling machine.

2.1.1 Tool Holder and Tool

The milling tool holder is the interface between the spindle and the tool which holds all the inserts. The milling tool holder used in the experiments was of the type E3471 5525 22160 which has an overhang of 140mm and a diameter of 48mm, see Fig. 7. Mounted on the tool holder was the tool R220.69-0050-12-7A presented to the left in Fig. 7. This tool has a cutting diameter of \( D_c = 50 \text{mm} \) and a seven-teeth, \( z_n = 7 \) (inserts) configuration. The insert used in the tool configuration was XOMX120408TR-M12 T250M.

The material type of the milling tool holder is SS-2511 (EN-16NiCrS4) and the material composition and properties are presented in Table 1.

2.1.2 Cutting Data

Three cutting parameters were considered in the experiments: cutting depth, spindle speed and table feed rate. While two of the cutting parameters were kept constant, the third was changed in five small steps. This was done for each parameter. In Table 2,
Figure 6: The DMU 80FD Duoblock milling machine.

Figure 7: The tool holder E3471 5525 22160 is illustrated with the tool R220.69-0050-12-7A mounted. The tool is configured with seven inserts of the type XOMX120408TR-M12 T250M.
Material composition besides Fe, [%]

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Ni</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.13-0.18</td>
<td>0.15-0.40</td>
<td>0.7-1.1</td>
<td>0.035</td>
<td>0.050</td>
<td>0.60-1.00</td>
<td>0.80-1.20</td>
</tr>
</tbody>
</table>

Material properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>210 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Mass density</td>
<td>7850 kg/m³</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>207 MPa</td>
</tr>
</tbody>
</table>

Table 1: Composition and properties of the material EN-16NiCrS4.

the cutting data used in the experiments are given. In the table, it is also observable that the width of the cut $a_p$ (how much of the workpiece is removed in the y-direction per tool pass) varied slightly, see Fig. 8. These variations were, however, inevitable due to the settings of the cutting data used in the experiments. The influence of these small changes is likely to be insignificant in respect to the degree of forces expected from the overall setup.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Spindle speed $n$ [rev/min]</th>
<th>Table feed $v_f$ [mm/min]</th>
<th>Cutting depth $a_p$ [mm]</th>
<th>Width of cut $a_e$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1401</td>
<td>1401</td>
<td>1</td>
<td>26.0</td>
</tr>
<tr>
<td>2</td>
<td>1401</td>
<td>1401</td>
<td>2</td>
<td>25.6</td>
</tr>
<tr>
<td>3</td>
<td>1401</td>
<td>1401</td>
<td>3</td>
<td>25.2</td>
</tr>
<tr>
<td>4</td>
<td>1401</td>
<td>1401</td>
<td>4</td>
<td>24.4</td>
</tr>
<tr>
<td>5</td>
<td>1401</td>
<td>1401</td>
<td>5</td>
<td>24.0</td>
</tr>
<tr>
<td>6</td>
<td>1401</td>
<td>1401</td>
<td>2</td>
<td>26.0</td>
</tr>
<tr>
<td>7</td>
<td>1465</td>
<td>1401</td>
<td>2</td>
<td>25.8</td>
</tr>
<tr>
<td>8</td>
<td>1528</td>
<td>1401</td>
<td>2</td>
<td>25.4</td>
</tr>
<tr>
<td>9</td>
<td>1592</td>
<td>1401</td>
<td>2</td>
<td>25.2</td>
</tr>
<tr>
<td>10</td>
<td>1656</td>
<td>1401</td>
<td>2</td>
<td>25.0</td>
</tr>
<tr>
<td>11</td>
<td>1401</td>
<td>1401</td>
<td>2</td>
<td>24.6</td>
</tr>
<tr>
<td>12</td>
<td>1401</td>
<td>1501</td>
<td>2</td>
<td>24.2</td>
</tr>
<tr>
<td>13</td>
<td>1401</td>
<td>1601</td>
<td>2</td>
<td>23.8</td>
</tr>
<tr>
<td>14</td>
<td>1401</td>
<td>1701</td>
<td>2</td>
<td>23.4</td>
</tr>
<tr>
<td>15</td>
<td>1401</td>
<td>1801</td>
<td>2</td>
<td>23.0</td>
</tr>
</tbody>
</table>

Table 2: The cutting data used during the milling measurements.

### 2.1.3 Measurement Equipment and Setup
- 12 PCB Piezotronics, Inc. 333A32 accelerometers.
- 2 Bruel & Kjær 8001 impedance head.
- 1 Bruel & Kjær NEXUS 2 channel conditioning amplifier 2692.
- OSC audio power amplifier, USA 850.
- Ling dynamic systems shaker v201.
- Gearing & Watson electronics shaker v4.
- Hewlett Packard VXI mainframe E8408A.
- Hewlett Packard E1432A 4-16 channel 51.2 kSa/s digitizer.
Figure 8: The side milling configuration of the cutting setup during the milling measurements.

- PC with I-DEAS 10 NX Series.
- Custom designed slit disk for measuring angular frequency.
- Rotec 5.3.
- Autodesk Inventor.

2.1.4 Spatial Measurements of the Acceleration During Milling

To examine the spatial dynamic behavior of milling machine components during the milling process, the acceleration at a number of different spatial locations on the structure was measured simultaneously. The accelerometers had to be positioned on non-rotating parts. Thus six accelerometers were positioned on the spindle frame and three accelerometers on the workpiece, see Fig. 8 (for the setup on the Hurco milling machine). The sensor setup on the DMU 80FD Duoblock milling machine was almost the same. The only difference was that instead of using nine accelerometers for the four positions defined in Fig. 9, twelve accelerometers were used. Thus, all directions were measured in the four nodes in the DMU 80FD Duoblock milling machine setup. Furthermore, the angular velocity along the tool holder was also measured at three positions, see Fig. 9. Two disks with 500 uniformly distributed gaps on the tool holder and a reflector tape on the spindle, together with optical scanning, was used to handle the measurements of angular velocities. All data from the milling measurements were collected with a sampling frequency of $f_s = 51200$ Hz, using a VXI Mainframe, Matlab and VibraTools SuiteTM.

The workpiece material was carbon steel SS1312 (EN 10 025) and the different workpieces used in the experiments had approximately the dimensions of 70x60x530 mm (y, z, x), see Fig. 9. The workpiece was clamped to the milling table which moved in the x-direction, resulting in a continuous cutting process along the workpiece, see Fig. 8.

2.1.5 Modal Analysis Setup

The next step was to examine the dynamic properties of the milling tool holder mounted in the spindle. This was done using two shakers that excited the tool holder close to the tool in two orthogonal directions, see Fig. 10 a). Each shaker excited
the tool holder via a stinger rod connected to an impedance head, thus measuring the driving point in the respective direction, see Fig. 10 b). At the same time, the acceleration at 11 other locations along the tool holder and spindle frame was measured, see Fig. 10 where the modal analysis setup on the Hurco BMC-50 milling machine is presented.

The modal analysis setup in the DMU 80FD Duoblock milling machine was almost identical to the setup in the Hurco BMC-50 milling machine. The differences between the sensor setups concerns the number of sensors and positions are presented in Fig. 11.

2.1.6 Excitation Signal for the Experimental Modal Analysis

All the measurements that were performed as a basis for the experimental modal analysis were done using the excitation signal burst random, 80% noise and 20% silent.

2.2 Spectral Properties

Non-parametric spectrum estimation may be utilized to produce non-parametric linear least-squares estimates of dynamic systems [38]. By using the Welch spectrum estimator [39], the cross-power spectral density $\hat{P}_{yx}(f_k)$ between the input signal $x(n)$ and the output signal $y(n)$, and the power spectral density $\hat{P}_{xx}(f_k)$ for the input signal $x(n)$, may be produced [38, 40], where $f_k = \frac{k}{N}F_s$ is the discrete frequency, $k = 0, \ldots, N - 1$, where N is the length of the data segments used to produce the periodograms and $f_s$ is the sampling frequency.
Figure 10: The experimental modal analysis setup in the Hurco BMC-50 milling machine.

Figure 11: Cross-section view in the y-z plane of the spindle house and the tool holder illustrating the positions of the sensors measuring acceleration and force in the y direction for the EMA setup in the two milling machines, the sensor configurations are identical for the x-z plane. The black squares represents the accelerometers and the black rectangular represents the impedance heads. In a) the EMA setup in the Hurco milling machine is presented where the positions of the sensor one to six are placed along the z-axis according to \{260, 180, 140, 100, 60, 20\} mm from the tool tip and in b) the EMA setup for the DMU 80 milling machine is shown and the positions of the sensor one to five are placed along the z-axis according to \{560, 370, 100, 60, 20\} mm from the tool tip.
In the case of a multiple-input-multiple-output (MIMO) system with $P$ number of responses and $Q$ number of references, an estimate of the cross-spectrum matrix $\hat{P}_{xx}(f_k)$ between all the inputs is produced, where the diagonal elements are power spectral densities (PSDs) for the respective input signal and the off-diagonal consists of cross-spectral densities. Also, a cross-spectrum matrix $\hat{P}_{yx}(f_k)$ between all the inputs and outputs may be estimated in the same way.

The least-square estimate for a MIMO system may be written as [38],

$$\hat{H}(f_k) = \left[ \hat{P}_{yx}(f_k) \right] \left[ \hat{P}_{xx}(f_k) \right]^{-1}. \quad (4)$$

In the case of multiple inputs, the multiple coherence is of interest as a measure of the quality of the MIMO system’s estimates [38].

### 2.3 Operating Deflection Shape Analysis

The spatial motion of a machine or a structure during real operating conditions may be investigated using operating deflection shapes analysis (ODS). By simultaneous measurements of $N$ responses at discrete points on a structure the forced spatial motion of the machine or structure, either at a moment in time, or at a specific frequency may be estimated [40]. Thus, by considering the phase and amplitude of the response signals from e.g. $N$ accelerometers distributed on an operating structure, it is possible to produce estimates of operating deflection shapes for the operating structure. The amplitude is measured by either power spectrum or power spectral density estimates depending whether the signal is tonal or random [38, 41]. And the phase between each spatial position is estimated from cross-power spectra or cross-power spectral densities [38, 41]. An estimate of a frequency domain operating deflection shape may be constructed as follows [40]:

$$\{ODS(f)\}_{RMS} = \left\{ \sqrt{\hat{P}_{11}(f)} \sqrt{\hat{P}_{22}(f)} e^{j\hat{\theta}_{21}(f)} \cdots \sqrt{\hat{P}_{NN}(f)} e^{j\hat{\theta}_{N1}(f)} \right\}^T. \quad (5)$$

Where $\hat{P}_{nn}(f)$ are e.g. estimated power spectra and $e^{j\hat{\theta}_{n1}(f)}$ are phase functions of cross-power spectra $\hat{P}_{n1}(f), n \in \{2, \cdots, N\}$.

### 2.4 Experimental Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties of the system under examination or the modal parameters. In other words, the purpose is to determine the natural frequencies, mode shapes and damping ratios from experimental vibration measurements. The procedure of modal analysis may be divided into two parts: the acquisition of data followed by the parameter estimation or parameter identification that can be determined with these data, a process also known as curve fitting [42]. Acquiring good data and performing accurate parameter identification is an iterative process, based on various assumptions along the way [42].

### 2.5 Modal Parameter Estimation

There are several different methods for the identification of the modal parameters [42, 43]. There are two basic curve fitting methods. One consists of curve fitting in the frequency domain using measured frequency response function (FRF) data and a
parametric model of the FRF. The other method employs curve fitting toward the measured impulse response function (IRF) data using a parametrical model of the IRF [42]. Many methods use both domains, depending on which parameter that estimated [42]. A parametric model of the FRF matrix, \( \hat{H}(f) \), expressed as the receptance between the reference points, or the input signals, and the responses or the output signals, may be written as [42],

\[
\hat{H}(f) = \sum_{r=1}^{N} \frac{Q_r \{\psi\}_r \{\psi\}_r^T}{j2\pi f - \lambda_r} + \frac{Q_r^* \{\psi\}_r^* \{\psi\}_r^{H}}{j2\pi f - \lambda_r^*}
\]

(6)

where \( r \) is the mode number, \( N \) is the number of modes used in the model, \( Q_r \) is the scaling factor of mode \( r \), \( \{\psi\}_r \) is the mode shape vector of mode \( r \), and \( \lambda_r \) is the pole belonging to mode \( r \).

Because two sources (references) were used during data acquisition, a method capable of handling multi-references is required. One such method is the polyreference least square complex exponential method developed by Vold [44, 45]. This method is defined for identification of MIMO-systems with the purpose of obtaining a global least-square estimate of the modal parameters. While this method was used in this work, the mode shapes were estimated using the frequency domain polyreference method [46]. The modal scaling method used was unity modal mass [43].

To assess the quality of the estimated parameters, the FRF’s were synthesized using the estimated parameters and overlayed with the estimated FRF’s. Furthermore, the Modal Assurance Criterion (MAC) [42] defined by

\[
MAC_{kl} = \frac{(\{\psi\}_k^H \{\psi\}_l^H)^2}{\{\psi\}_k^H \{\psi\}_k \{\psi\}_l^H \{\psi\}_l^H}
\]

(7)

was used as a measure of correlation between the mode shape \( \{\psi\}_k \) belonging to mode \( k \), and the mode shape \( \{\psi\}_l \) belonging to mode \( l \), where \( H \) is the Hermitian transpose operator.

### 2.5.1 Spectral Estimation Parameters

The estimation parameters used for the spectral density estimates, frequency response functions and operating deflection shapes are presented in Table 3, Table 4 and Table 5 respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Cutting process</td>
</tr>
<tr>
<td>Sampling frequency ( f_s )</td>
<td>51200 Hz</td>
</tr>
<tr>
<td>Block length ( N )</td>
<td>40960</td>
</tr>
<tr>
<td>Frequency resolution ( \Delta f )</td>
<td>1.25 Hz</td>
</tr>
<tr>
<td>Number of averages ( L )</td>
<td>20</td>
</tr>
<tr>
<td>Burst length</td>
<td>-</td>
</tr>
<tr>
<td>Window</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 3: Spectral density estimation parameters used in the production of the milling tool holder spectra during continuous machining.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Burst random</td>
</tr>
<tr>
<td>Sampling frequency $f_s$</td>
<td>51200 Hz</td>
</tr>
<tr>
<td>Block length $N$</td>
<td>40960</td>
</tr>
<tr>
<td>Frequency resolution $\Delta f$</td>
<td>1.25 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>200</td>
</tr>
<tr>
<td>Burst length</td>
<td>80%</td>
</tr>
<tr>
<td>Window</td>
<td>Rectangular</td>
</tr>
<tr>
<td>Overlap</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 4: Spectral density estimation parameters used in the production of the frequency response functions for the modal analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excitation signal</td>
<td>Cutting process</td>
</tr>
<tr>
<td>Sampling frequency $f_s$</td>
<td>51200 Hz</td>
</tr>
<tr>
<td>Block length $N$</td>
<td>40960</td>
</tr>
<tr>
<td>Frequency resolution $\Delta f$</td>
<td>1.25 Hz</td>
</tr>
<tr>
<td>Number of averages $L$</td>
<td>6</td>
</tr>
<tr>
<td>Burst length</td>
<td>-</td>
</tr>
<tr>
<td>Window</td>
<td>Hanning</td>
</tr>
<tr>
<td>Overlap</td>
<td>50%</td>
</tr>
</tbody>
</table>

Table 5: Spectral density estimation parameters used in the production of the operating deflection shapes.
2.6 Distributed Parameter Model of the Milling Tool Holder

The milling tool holder may be considered to be a beam with the cross section \( A(z) \) and the length \( l \). The Euler-Bernoulli beam theory may be utilized to approximately model a milling tool holder’s lower order bending modes \([40,47]\). The Euler-Bernoulli beam theory is generally considered for slender beams that have a diameter to length ratio exceeding 10 as this ratio allows the effects of shear deformation and rotary inertia to be ignored \([48]\). As a result, this theory tends to slightly overestimate the eigenfrequencies. This problem increases when dealing with the eigenfrequencies of higher modes \([48]\).

2.6.1 A Geometrical Model of the Clamped Milling Tool Holder

The milling tool holder has a complex structure and a cone interface is used for the particular clamping mechanism that attaches the tool holder to the spindle. Furthermore, the tool holder consists of a cylindrical shaft with a lip towards the spindle and in the center of the tool holder is cooling channel. In the model, spindle and the tool holder are assumed to be clamped rigidly. The geometry of the tool holder has also been simplified into a pipe in the analytical model. The cross-section of the milling tool holder and the corresponding analytical model are illustrated in Fig. 12 together with the assumed clamping.

![Diagram showing the cross-section of the milling tool holder and the simplified analytical model.](image)

Figure 12: a) The cross-section of the milling tool holder and in b) the simplified analytical model, where \( l=140.90 \text{ mm} \) is the length of the overhang, \( R_o= 24.00 \text{ mm} \) is the radius of the tool holder and \( R_i= 10.25 \text{ mm} \) is the radius of the coolant channel.

The cross-sectional properties of the simplified model are presented in Table 6.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>1.4795 \cdot 10^{-3}</td>
<td>m²</td>
</tr>
<tr>
<td>( I )</td>
<td>2.5191 \cdot 10^{-7}</td>
<td>m⁴</td>
</tr>
<tr>
<td>( J )</td>
<td>4.0838 \cdot 10^{-7}</td>
<td>m⁴</td>
</tr>
</tbody>
</table>

Table 6: The cross-sectional properties of the milling tool, where \( A \) is the area, \( I \) is the moment of inertia and \( J \) is the polar moment of inertia.
2.6.2 A Model of Transverse Vibrations

The Euler-Bernoulli differential equation describing the transversal motion of the milling tool holder in the y-direction may be written as [48]

\[ \rho A(z) \frac{d^2u(z, t)}{dt^2} + \frac{\partial^2}{\partial z^2} \left[ EI_x(z) \frac{\partial^2u(z, t)}{\partial z^2} \right] = f(z, t) \]  

(8)

where \( A(z) \) is the milling tool holder’s cross-sectional area, \( E \) is Young’s elastic modulus for the tool holder, \( I(z) \) is the cross-sectional area moment of inertia about the “x axis”, \( \rho \) is the density, \( t \) is the time, \( u(z, t) \) is the deflection in the y-direction and \( f(z, t) \) is the external force per unit length. It is assumed that both the cross-sectional area \( A(z) \) and the flexural stiffness \( EI(z) \) are constant along the milling tool holder. Eq.8 is often referred to as the Euler-Bernoulli beam equation. The model assumes that the following assumptions regarding the beam and its plane are true:

- The beam is uniform along its span-, or length-, and slender (diameter to length ratio\( <10 \)).
- The beam is composed of a linear, homogenous, isotropic elastic material without axial loads.
- The plane section remains plane.
- The plane of symmetry of the beam is also the plane of vibration so that rotation and translation are decoupled.
- Rotary inertia and shear deformation can be neglected.

To model the milling tool holder, a Fixed-Free Euler-Bernoulli beam model was applied. The beam has four boundary conditions, two at each end. One end is clamped and the other is free, see Fig. 13.

Figure 13: Model of a Fixed - Free beam, where \( \rho \) is the density, \( E \) is the elasticity modulus (Young’s coefficient), \( G \) is the shear modulus, \( A \) is the cross-sectional area, \( I \) is the moment of inertia, \( J \) is the polar moment of inertia and the length of the beam \( l = 140.9 \text{ mm} \).

The clamped side of the beam will be fixated. Thus the displacement and the slope of the displacement in this point \( z = 0 \) will equal zero and the two first boundary conditions become

\[ u(z, t)|_{z=0} = 0, \quad \left. \frac{\partial u(z, t)}{\partial z} \right|_{z=0} = 0 \]  

(9)

The other end is free, so that no bending moment or shear force constrains the beam at the coordinate \( z = l \) when the beam vibrates. This yields two other boundary conditions that can be described as

\[ EI \left. \frac{\partial^2u(z, t)}{\partial z^2} \right|_{z=l} = 0, \quad EI \left. \frac{\partial^3u(z, t)}{\partial z^3} \right|_{z=l} = 0. \]  

(10)
2.6.3 A Model of Torsional Vibrations

In the same way as for the transverse vibration model, a vibration model for the torsional vibrations may be derived by considering the equation of motion for an infinitesimal element of the beam [48]. The differential equation describing the torsional motion for the milling tool holder around and along the z axis may be written as [48]

\[ \frac{\partial^2 \theta(z,t)}{\partial t^2} - \left( \frac{G}{\rho} \right) \frac{\partial^2 \theta(z,t)}{\partial z^2} = \tau(z,t) \]  (11)

where \( \theta(z,t) \) is the angular deflection, \( G \) is the shear modulus, \( \rho \) is the density and \( \tau(z,t) \) is the externally applied torque load per unit length. The clamped boundary condition is applied to where the milling tool holder is connected to the spindle and yields zero deflection. At the other end there is no torque in the case of the free vibration model. Thus, the boundary conditions for a milling tool holder with a coolant channel modeled as a hollowed shaft becomes

\[ \theta_z(z,t)\big|_{z=0} = 0 \]  (12)

\[ \frac{\pi}{2} (R_o^4 - R_i^4) G \theta_z(z,t)\big|_{z=l} = 0 \]  (13)

where \( R_o \) is the outer radius of the milling tool holder and \( R_i \) is the radius of the coolant channel. The relation between the shear modulus \( G \) and the elasticity modulus \( E \) is given by [49]

\[ G = \frac{E}{2(1+\nu)} \]  (14)

where \( \nu \) is Poisson’s ratio.

2.7 A Finite Element Model of the Milling Tool Holder

The milling tool holder was modeled in a CAD program and a finite element analysis was conducted to estimate the natural frequencies and mode shapes of the tool holder. The finite element mesh of the milling tool holder, consisted of 73470 nodes and 42728 elements, is presented in Fig. 14 where the white lines represent the borders of the elements connected at the nodes.

In the FE analysis the clamping surface on the back of the holder, i.e. behind the lip to the left in Fig. 14, was constrained to be fixed for all degrees of freedoms.

3 Results

This experimental investigation resulted in a large amount of vibration data that was collected from both experimental setups from both milling machines. However, the results presented in this report only constitute a small part of the investigation, but they represent the essence of the results. The results from the experimental examination are presented in terms of measured acceleration signals as a function of time and power spectral densities of the acceleration signals. Operating deflection shapes were estimated for one of the milling machines and are presented. Also, results from experimental modal analysis of the tool holder mounted in the milling machine are given. Finally, resonance frequencies and mode shapes calculated based
on distributed-parameter system models of the milling tool holder were generated as well as the corresponding results from the finite element analysis of the milling tool holder.

### 3.1 Spatial Measurements of Vibration

In order to get an overview of the measured acceleration signals during machining, the acceleration of the workpiece moving in the feed direction \( (+x_4) \) on the DMU 80FD Duoblock milling machine during machining is presented in the time domain in Fig. 15.

![Figure 15](image)

Figure 15: a) Acceleration of the workpiece in the feed direction \( (+x_4) \) during a milling operation performed in the DMU 80FD Duoblock milling machine. The radial depth \( a_e \) was 23 mm, the axial depth \( a_p \) was 2 mm, the feed speed \( v_f \) was 1401 mm/min and the spindle speed \( n \) was 1401 rev/min b) and the corresponding acceleration record zoomed in.

Results in terms power spectral density estimates of workpiece vibration in the feed direction \( (+x_4) \) are presented in Fig. 16 a) and b) for five different axial depths \( (a_p = 1, 2, 3, 4, 5 \text{ mm}) \) in the DMU 80FD Duoblock milling machine. The radial...
depth was \( a_e = 26.0, 25.6, 25.2, 24.2, 24.0 \) mm, the feed speed \( v_f \) was mm/min and the spindle speed \( n \) was 1401 r.p.m. The periodic components found in the power spectral density estimates in Fig. 16 a) are related to the spindle speed \( n \). Furthermore, a broadband response to an underlying structure may be observed in Fig. 16 a). Power spectral density estimates workpiece vibration, zoomed in frequency to the interval of the dominating resonance peak at approx. 770 Hz for, for the five different axial depths are presented in Fig. 16 b).

![Image of power spectral densities](image)

Figure 16: Power spectral densities of workpiece vibration in the feed direction (+x_4) during milling for different axial depths \( a_p \) in the DMU 80FD Duoblock milling machine. In a) (for the gray solid line \( a_p = 1 \) mm and for the black solid line \( a_p = 5 \) mm) and the radial depth \( a_e \) was 26 mm and 24 mm respectively, the feed speed \( v_f \) was mm/min and the spindle speed \( n \) was 1401 r.p.m. b) Zoomed in frequency to the interval of the dominating resonance peak for the five different axial depths \( (a_p = 1, 2, 3, 4, 5 \) mm) and the corresponding radial depth was \( a_e = 26.0, 25.6, 25.2, 24.2, 24.0 \) mm.

Results in terms power spectral density estimates of workpiece vibration in the feed direction (+x_4) when changing the feed speed \( v_f \) are presented in Fig. 17 for five different feed speeds \( (v_f = 1401, 1501, 1601, 1701, 1801 \) mm/min) in the DMU 80FD Duoblock milling machine. The radial depth was \( a_e = 24.6, 24.2, 23.8, 23.4, 23.0 \) mm, the axial depth \( a_p \) was 2 mm and the spindle speed \( n \) was 1401 r.p.m. In Fig. 17 a) no particular changes can be observed, but when zooming in on the peaks as illustrated in Fig. 17 b) a small difference in magnitude is observable.

Results in terms power spectral density estimates of workpiece vibration in the feed direction (+x_4) when changing the spindle speed \( n \) are presented in Fig. 18 a), for five different spindle speeds \( (n = 1401, 1465, 1528, 1592, 1656 \) r.p.m.) in the DMU 80FD Duoblock milling machine. The radial depth was \( a_e = 26.0, 25.8, 25.4, 25.2, 25.0 \) mm, the axial depth \( a_p \) was 2 mm and the feed speed \( v_f \) was 1401 mm/min. In Fig. 18 a), it is observable how the frequency of the harmonics changes with the change of spindle speed. In Fig. 18 b), typical power spectral density estimates of workpiece vibration in the feed direction (+x_4) during machining conducted in the Hurco BMC-50 milling machine is presented. The radial depth \( a_e \) was 26 mm, the axial depth \( a_p \) was 1 mm, the feed speed \( v_f \) was 1401 mm/min and the spindle speed \( n \) was 1401 r.p.m. Also, when carrying out the machining in the Hurco BMC-50 milling machine both a large number of narrow-banded peaks and a broadband response of
3.1.1 Operating Deflection Shapes

To obtain information on how the spindle frame vibrates relative to the workpiece, spatial measurements of the acceleration of these structural parts were carried out. Accelerometer positions and measurement directions on the spindle frame and the workpiece are illustrated in Fig. 19 a). To facilitate illustration of the operating deflection shapes, the spindle frame and the workpiece are simplified into a skeleton structure where the measurement positions are illustrated by black circles, defined as nodes, as also shown in Fig. 19 a). In Fig. 19 b) the simplified skeleton structure of the spindle frame and the workpiece is shown in the y-z plane and in Fig. 19 c) it is shown in the x-z plane.

Observe that the fourth node in the skeleton structure is fixed on the workpiece and thus moving away from the other nodes, along the x-axis, as the tool is cutting the workpiece. The spatial motion of this structure has a complex behavior and changes with time. However, an operating deflection shape at one of the dominant peaks in the spectral density previously presented (see Fig. 16), i.e. at 780 Hz, was estimated during a short time sequence and is presented in Fig. 20 a) and b). The deformation shape is presented with arrows in the figure. Observe that the size of the arrows does not represent the absolute magnitudes of the four positions deflection; their magnitudes are displayed in an enlarged scale to make them observable.

In order to show the complex spatial behavior of the measurement positions on the spindle frame and the workpiece, a trajectory for node two is presented during a time sequence of 15.625 ms, see Fig. 21 a) and b). The trajectory was produced by filtering the acceleration signals with a band-pass filter having a center frequency at 780 Hz. The frequency response function for the band-pass filter is presented in Fig. 22.

By combining the trajectory plots for each of the four measurement positions
Figure 18: a) Power spectral densities of workpiece vibration in the feed direction (+\(x_4\)) during milling for five different spindle speeds \((n = 1401, 1465, 1528, 1592, 1656\) r.p.m.) in the DMU 80FD Duoblock milling machine. The radial depth was \(a_e = 26.0, 25.8, 25.4, 25.2, 25.0\) mm, the axial depth \(a_p\) was 2 mm and the feed speed \(v_f\) was 1401 mm/min. b) Power spectral density of workpiece vibration in the feed direction (+\(x_4\)) performed in the Hurco BMC-50 milling machine. The radial depth \(a_e\) was 26 mm, the axial depth \(a_p\) was 1 mm, the feed speed \(v_f\) was 1401 mm/min and the spindle speed \(n\) was 1401 r.p.m.

Figure 19: a) presents a 3d-view of the spindle frame, tool holder and the milling table with the workpiece. The measurement positions shown as black circles connected by straight black lines forming a skeleton structure, b) present the skeleton structure in the y-z plane and c) presents the skeleton structure in the x-z plane.
Figure 20: Operating deflection shape for the spindle frame and the milling table with the workpiece at the frequency 780 Hz, estimated during machining. The radial depth $a_e$ was 24.4 mm, the axial depth $a_p$ was 4 mm, the feed speed $v_f$ was 1401 mm/min and the spindle speed $n$ was 1401 r.p.m. In a) the shape is presented in the y-z plane and in b) the shape is presented in the x-z plane.

Figure 21: Trajectory plot of the measured acceleration signals in node two at 780 Hz during machining. The radial depth $a_e$ was 24.4 mm, the axial depth $a_p$ was 4 mm, the feed speed $v_f$ was 1401 mm/min and the spindle speed $n$ was 1401 rev/min. a) and b) viewed from two different perspectives.
3.1.2 Angular Vibrations

The angular vibrations of the milling tool holder were measured at three different positions, two on the tool holder and one at the spindle close to the clamping of the tool holder. In Fig. 24, the angular vibrations of the three positions versus the number of revolutions of the tool holder are shown in the same diagram. In this figure, the angular vibrations during approximately the first 35 revolutions are measured prior to engagement of the tool in the workpiece. Note how, during the first revolutions, when no machining is carried out, the angular vibration of the tool holder and the spindle are still observable.

All three sensors show a good agreement on the angular vibrations when there is no cutting, see Fig. 25 a). However, during machining a discrepancy between the angular vibrations measured by the sensor on the spindle and the angular vibrations measured by the sensors on the tool holder is observable, see Fig. 25 b).

By plotting a waterfall diagram of the order spectra of the angular tool holder vibrations closest to the tool, it is obvious that the main angular vibration is directly related to the first order, see Fig. 26. The radial depth $a_r$ was 24.0 mm, the axial depth $a_p$ was 5 mm, the feed speed $v_f$ was 1401 mm/min and the spindle speed $n$ was 1401 rev/min. To facilitate observability of the peaks of the higher orders in the order spectra the first order was excluded from them and they were again plotted in a waterfall diagram as illustrated in Fig. 26 b). Also, in this figure an underlying broadband dynamic angular response of the tool holder may be observed. The seventh, 14:th and 21:th order of the spindle speed are slight higher than the direct neighboring orders, see Fig. 26 b). These orders are also the first, second and third order of the tooth-passing frequency.
Investigation of the Dynamic Properties of a Milling Tool Holder

Figure 23: Spatial motion of the spindle frame and the workpiece for a short time interval, based on band pass filtered acceleration signal measured during cutting. The radial depth $a_e$ was 24.4 mm, the axial depth $a_p$ was 4 mm, the feed speed $v_f$ was 1401 mm/min and the spindle speed $n$ was 1401 r.p.m. The ellipses represent the motion of the measured nodes for the frequency 780 Hz. The circle on each ellipse represents a synchronization point for all the nodes (measurement positions) at a certain time instant and is followed by a solid ellipse line which indicates the direction of the motion. In a) the motion is presented in the y-z plane and in b) the motion is presented in the x-z plane.

Figure 24: Angular vibrations of tool holder and the spindle, measured at two positions on the tool holder and at one position on the spindle. The first 35 revolutions are measured prior to engagement of the tool in the workpiece directly followed by the engagement phase. After approx. 40 revolutions the material removal process is carried out according to the selected cutting data. The radial depth $a_e$ was 24.2 mm, the axial depth $a_p$ was 2 mm, the feed speed $v_f$ was 1501 mm/min and the spindle speed $n$ was 1401 rev/min.
Figure 25: The angular vibrations of the milling tool holder and the spindle during machining when the radial depth $a_e$ was 24.2 mm, the axial depth $a_p$ was 2 mm, the feed speed $v_f$ was 1501 mm/min and the spindle speed $n$ was 1401 rev/min. a) Shows the angular vibrations prior to cutting and b) shows the angular vibrations during cutting.
Figure 26: Waterfall plot of the order spectra of the angular vibration during 30 s of machining. The radial depth $a_e$ was 24.0 mm, the axial depth $a_p$ was 5 mm, the feed speed $v_f$ was 1401 mm/min and the spindle speed $n$ was 1401 rev/min. a) presents the order spectra with the first order included in the plot and b) presents the order spectra when the first order has been removed.
3.2 Modal Analysis Results

Results in terms of accelerance function estimates and coherence function estimates from the experimental modal analysis (EMA) carried out on two milling machines are presented first in this section next to each other. In other words, the accelerance function estimates from the Hurco BMC-50 milling machine are presented in Fig. 27 a), Fig. 28 a) and Fig. 29 a) and the accelerance function estimates from the DMU 80FD Duoblock milling machine are presented in Fig. 27 b), Fig. 28 b) and Fig. 29 b). The two EMA setups differed slightly between the two machines, see section 2.1.5. A significant peak is noticeable around 650 Hz in the accelerance functions produced from the EMA carried out in Hurco BMC-50 milling machine and a peak around 750 Hz is noticeable in the accelerance functions produced from the EMA carried out in the DMU 80FD Duoblock milling machine.

![Figure 27: Accelerance magnitude function estimates between the force input in x-direction and the acceleration responses in x-direction based on the experimental modal analysis measurements, a) in the Hurco BMC-50 milling machine and b) in the DMU 80FD Duoblock milling machine.](image)

Typical multiple coherence function estimates obtained during the experimental modal analysis are illustrated in Fig. 30. The coherence function estimates presented in Fig. 30 shows values above 0.9 for most frequencies between 350 Hz and 1800 Hz. This indicates that the level of forces and accelerations was fairly good in the region of interest, that is, between 350 Hz up to 1400 Hz. Some dips may be observed around 800 Hz in estimates done from both EMA setups and one larger around 1500 Hz in the coherence function estimate from the EMA carried out in the DMU 80FD Duoblock milling machine.

3.2.1 Mode Shapes

Based on the accelerance functions estimated from the modal analysis setups carried out in the Hurco BMC-50 milling machine and in the DMU 80FD Duoblock milling machine, a number of resonance frequencies were estimated. The estimated resonance frequencies and their relative damping are presented in Table 7.

Furthermore, for each resonance frequency a corresponding mode shape of the spindle house - tool holder system was estimated. A figure (Fig. 31) defining the positions of the sensors together with two tables (Table 8 and Table 9) presents
Investigation of the Dynamic Properties of a Milling Tool Holder

Figure 28: Accelerance magnitude function estimates between the force input in y-direction and the acceleration responses in y-direction based on the experimental modal analysis measurements, a) in the Hurco BMC-50 milling machine and b) in the DMU 80FD Duoblock milling machine.

Figure 29: Accelerance magnitude function estimates between the force input in y-direction and the acceleration responses in x-direction based on the experimental modal analysis measurements, a) in the Hurco BMC-50 milling machine and b) in the DMU 80FD Duoblock milling machine.
Figure 30: Typical coherence functions between the force inputs (the asterisk denotes all inputs) and the acceleration responses $+6Y,-5Y$ (dashed line) and $+6X,+5X$ (solid line) from the experimental modal analysis measurements, a) in the Hurco BMC-50 milling machine and b) in the DMU 80FD Duoblock milling machine.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq. [Hz]</th>
<th>Damping [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>615.4</td>
<td>3.91</td>
</tr>
<tr>
<td>2</td>
<td>650.4</td>
<td>2.58</td>
</tr>
<tr>
<td>3</td>
<td>783.0</td>
<td>0.60</td>
</tr>
<tr>
<td>4</td>
<td>920.6</td>
<td>1.49</td>
</tr>
<tr>
<td>5</td>
<td>985.5</td>
<td>1.23</td>
</tr>
<tr>
<td>6</td>
<td>1141.1</td>
<td>1.75</td>
</tr>
<tr>
<td>7</td>
<td>1305.3</td>
<td>1.96</td>
</tr>
<tr>
<td>8</td>
<td>1349.9</td>
<td>1.68</td>
</tr>
<tr>
<td>9</td>
<td>1518.0</td>
<td>4.11</td>
</tr>
<tr>
<td>10</td>
<td>1611.3</td>
<td>0.38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq. [Hz]</th>
<th>Damping [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>744.1</td>
<td>2.68</td>
</tr>
<tr>
<td>2</td>
<td>755.4</td>
<td>2.61</td>
</tr>
<tr>
<td>3</td>
<td>809.6</td>
<td>2.38</td>
</tr>
<tr>
<td>4</td>
<td>925.8</td>
<td>1.24</td>
</tr>
<tr>
<td>5</td>
<td>992.8</td>
<td>2.03</td>
</tr>
<tr>
<td>6</td>
<td>1064.0</td>
<td>1.34</td>
</tr>
<tr>
<td>7</td>
<td>1128.2</td>
<td>1.50</td>
</tr>
<tr>
<td>8</td>
<td>1509.5</td>
<td>1.17</td>
</tr>
<tr>
<td>9</td>
<td>1692.8</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 7: Estimated resonance frequencies and their relative damping coefficients from the modal analysis setups carried out in the Hurco BMC-50 milling machine and in the DMU 80FD Duoblock milling machine.
the two first mode shapes for the respective milling machine setup in the form of magnitude and angle.

![Image of tool holder and spindle house](image)

Figure 31: Cross-section view of the spindle house and the tool holder illustrating the mode shape for the first bending mode. In a) for the Hurco milling machine and in b) for the DMU 80 milling machine.

<table>
<thead>
<tr>
<th>Position</th>
<th>Norm. mag</th>
<th>Angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hurco milling machine mode at 615.4 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>65.4</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.36</td>
<td>50.4</td>
</tr>
<tr>
<td>4</td>
<td>0.58</td>
<td>47.9</td>
</tr>
<tr>
<td>5</td>
<td>0.75</td>
<td>44.7</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>45.3</td>
</tr>
<tr>
<td>DMU 80FD milling machine mode at 744.1 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>47.5</td>
</tr>
<tr>
<td>2</td>
<td>0.01</td>
<td>48.3</td>
</tr>
<tr>
<td>3</td>
<td>0.28</td>
<td>54.2</td>
</tr>
<tr>
<td>4</td>
<td>0.69</td>
<td>56.2</td>
</tr>
<tr>
<td>5</td>
<td>1.00</td>
<td>57.1</td>
</tr>
</tbody>
</table>

Table 8: Mode shape table presenting magnitude values for each measurement position and the angle relative to the x-axis, for the first mode estimated for respective milling machine setup.

In order to be able to evaluate the quality of the estimated modal parameters, a synthesized accelerance function are produced and overlaid on top of the corresponding estimated accelerance function. In Fig. 32 and Fig. 33, the driving point accelerance functions and the transfer accelerance functions between x and y direction in the driving point position, are presented together with their synthesized functions. The synthesized functions show good agreement with the estimated accelerance functions. In order to check the quality of the estimated mode shapes, a MAC matrix was produced. These matrixes are presented in Fig. 30, and as can be seen the orthogonality between the two first modes are excellent, while a high correlation exists for higher order modes.

### 3.3 Distributed Parameter Model Results

The results from the distributed parameter models of a clamped milling toolholder, in terms of natural frequency estimates, for both bending and torsional modes, are presented in Table 10.

The mode shapes, based on the distributed parameter models of a clamped milling tool holder, for the first and second bending mode as well as for the first torsional mode is presented in Fig. 35. Note that the first and second bending mode shapes are
Table 9: Mode shape table presenting the magnitude values for each measurement position and angle relative to the x-axis, for the second mode estimated for respective milling machine setup.

<table>
<thead>
<tr>
<th>Position</th>
<th>Norm. mag</th>
<th>Angle [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03</td>
<td>-24.2</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>-39.3</td>
</tr>
<tr>
<td>4</td>
<td>0.55</td>
<td>-42.1</td>
</tr>
<tr>
<td>5</td>
<td>0.72</td>
<td>-45.1</td>
</tr>
<tr>
<td>6</td>
<td>1.00</td>
<td>-44.5</td>
</tr>
</tbody>
</table>

Figure 32: The magnitude of the synthesized and the measured accelerance functions for the milling tool holder when clamped in the Hurco BMC-50 milling machine, a) between the force signal and the acceleration signal from location 5X and b) between the force signal at location 5Y and the acceleration signal from location 5X.

Figure 33: The magnitude of the synthesized and the measured accelerance functions for the milling tool holder when clamped in the DMU 80FD Duoblock milling machine, a) between the force signal and the acceleration signal from location 4X and b) between the force signal at location 4Y and the acceleration signal from location 4X.
Figure 34: a) MAC matrix presenting the correlation between the estimated mode shapes. In a) the matrix based on the mode shapes estimated from the Hurco BMC-50 milling machine setup, while in b) the matrix is based on the mode shapes estimated from the DMU 80FD Duoblock milling machine setup.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [Hz]</th>
<th>Type of mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1902.34</td>
<td>First bending</td>
</tr>
<tr>
<td>2</td>
<td>11921.74</td>
<td>Second bending</td>
</tr>
<tr>
<td>3</td>
<td>6321.53</td>
<td>First torsional</td>
</tr>
</tbody>
</table>

Table 10: Natural frequency estimates based on the distributed parameter model of a clamped milling tool holder.
in the transverse direction, while the first torsional mode shape represents a rotation deformation of the structure around and along its own centerline.

![Normalized mode shape](image)

Figure 35: The first and second bending modes together with the first torsional mode. Calculated based on the distributed-parameter system models of a clamped milling tool holder.

### 3.4 Finite Element Model

The first six natural frequencies estimated based on the milling tool holder FE-model are presented in Table 11. The corresponding mode shapes are illustrated in Fig. 36 to Fig. 38.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [Hz]</th>
<th>Type of mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1437.7</td>
<td>First bending</td>
</tr>
<tr>
<td>2</td>
<td>1440.4</td>
<td>First bending</td>
</tr>
<tr>
<td>3</td>
<td>5417.6</td>
<td>First torsional</td>
</tr>
<tr>
<td>6</td>
<td>6728.7</td>
<td>Second bending</td>
</tr>
<tr>
<td>7</td>
<td>6744.2</td>
<td>Second bending</td>
</tr>
<tr>
<td>8</td>
<td>7789.2</td>
<td>First longitudinal</td>
</tr>
</tbody>
</table>

Table 11: Natural frequency estimates based on FE analysis of a clamped milling tool holder.

### 4 Summary and Conclusion

A number of different machining measurements have been conducted which show good agreement with the expectations grounded in cutting theory. A pronounced periodicity is present in acceleration measurements carried out during milling operations. This periodicity is directly related to the spindle speed \( n \) and also to the harmonics observable in Fig. 16 a) and b). Furthermore, when increasing the cutting depth \( a_p \), the vibration level increases as can be seen in Fig. 16 b). This also agrees with the theory [1], showing that increasing cutting depth results in increasing cutting forces, and thus also in greater excitation levels. When changing the feed speed (feed rate), no significant changes are observed although a small increase in vibration level with
Figure 36: a) The milling toolholder mode shape belonging to the natural frequency at 1437.7 Hz, and b) the mode shape belonging to the natural frequency at 1440.4 Hz, estimated with a FE model. The solid part represents the shape of the mode and the transparent part drawn in thin lines is the un-deformed part.

Figure 37: a) The milling toolholder mode shape belonging to the natural frequency at 5417.6 Hz, and b) the mode shape belonging to the natural frequency at 6728.7 Hz, estimated with a FE model. The solid part represents the shape of the mode and the transparent part drawn in thin lines is the un-deformed part.

Figure 38: a) The milling toolholder mode shape belonging to the natural frequency at 6744.2 Hz, and b) the mode shape belonging to the natural frequency at 7789.2 Hz, estimated with a FE model. The solid part represents the shape of the mode and the transparent part drawn in thin lines is the un-deformed part.
increasing speed may be noticed, see Fig. 17. The dependence of the harmonics on the spindle speed is observable in Fig. 18 where the spindle speed was changed. In all the acceleration records, an underlying dynamic response may be observed. This is confirmed from the results obtained from the modal analysis carried out in both milling machines; compare the acceleration spectra in Fig. 18 b) with the accelerance function in Fig. 27 a) where a peak near 620 Hz is present in both figures. The results from the modal analysis, presented in Table 8 and Table 9, suggest that this is the first bending mode of the tool holder when the tool holder is clamped in the Hurco BMC-50 milling machine. This can also be seen when comparing the acceleration spectra in Fig. 16 with the accelerance function in Fig. 28 b), where a peak near 790 Hz is present in both figures. The results from the modal analysis, presented in Table 8 and Table 9, from the setup in the DMU 80FD Duoblock milling machine suggest this is the first bending mode of the tool holder. The difference in frequency of the first bending modes of the tool holder in the different machines indicates a more rigid clamping in the DMU 80FD Duoblock milling machine. Also the fact that the natural resonance frequencies estimated for both EMA setups in the same frequency range, presented in Table 7, are in general higher in the case of the DMU 80 Duoblock milling machine compare to the case of the Hurco BMC-50 milling machine, supports this conclusion.

The operating deflection shape analysis of the spindle frame-workpiece of the milling machine provided information concerning their spatial motion during machining. The deflection shape of the spindle frame-workpiece, can also be connected to the first bending mode, and the complex behavior of the shape may be explained by the fact that the milling tool holder is rotating while it is assumed to have a motion that is in itself related to a first bending mode. This assumption is however, not confirmed since no accelerometer measured the bending vibration of the milling tool holder during cutting.

To move on, angular vibration measurements of the milling tool holder showed the significance of the various orders. For example, the first order of the spindle speed had a major impact on the angular vibration level which suggests a significant unbalance of spindle - tool holder system. An unbalance will generally introduce transverse vibration directly related to the first order of the spindle speed. However, due to the sensor setup, the discs mounted on the milling tool holder, will because of the unbalance, be displaced from the rotation center and thus influence the angular velocity measured by the sensors. The first order of the spindle speed was already present in the angular vibration even before cutting took place, see the angular vibration before cutting in Fig. 25 a). It may also be noticed that the largest vibrations for all measurements in all nodes occur at a frequency, where one of the orders of the spindle speed coincides with a fundamental bending mode of the milling tool holder. The main purpose with using rotation sensors was to discover any angular motion relating to torsional modes of the milling tool holder. One important conclusion from the measurement was that no such dynamic behavior could be observed or demonstrated.

The analytical models together with the finite element model provided rough estimates of where in frequency the natural frequencies of the various modes may be expected to be found and how the mode shapes will look like. This is important when setting up the measurement and selecting sensor configuration. Both the analytical model and the finite element model were configured in order to overestimate the natural frequencies. In the real setups, the milling tool holder was configured with a tool that made the structure longer and at the same time adding mass to the end of the
structure. The clamping of the milling tool holder was furthermore assumed to be infinitely rigid for the model, which is not the case in reality.

One of the most interesting results, can be found in the accelerance function in Fig. 29, where significant peaks near 800, 1000 and 1600 Hz were found in the HurcoBMC-50 milling machine setup, while peaks near 925 was found in the DMU 80FD Duoblock milling machine setup. These peaks were not found in the accelerance functions in Fig. 27 and Fig. 28. The presence of the peaks in the accelerance function in Fig. 29 suggests that the structure is sensitive to forces applied in the orthogonal direction with respect to the direction of the response. If this result is a property of a milling tool holder clamped in a spindle and supported by bearings, or if it is something else, needs further investigations. However, it should be noted that the similar results (concerning the orthogonal sensitivity) were obtained for two different milling machines.

Acknowledgments

The present project is sponsored by the company Acticut International AB in Sweden which has multiple approved patents covering active control technology.

References


Part V

Preliminary Investigation of Active Control of a Milling Tool Holder
This part is based on the publication:

Preliminary Investigation of Active Control of a Milling Tool Holder

Henrik Åkesson$^{1,2}$, Tatiana Smirnova, Lars Håkansson and Ingvar Claesson
$^1$Blekinge Institute of Technology, Department of Signal Processing, 372 25 Ronneby, Sweden
$^2$Acticut International AB, Gjuterivägen 6, 311 32 Falkenberg, Sweden

Abstract

In milling, vibration is a common problem. During cutting, the cutting process excites the tool and the tool holder continuously. Having a milling tool holder with a long overhang will easily result in high vibration levels. The vibration level depends on many different parameters such as material type, dimensions of the workpiece and tool holder, cutting data, etc. High vibration levels result in reduced tool life, poor surface finish and disturbing sound. There are different methods to reduce the vibration levels, e.g. trial and error by an operator, introducing a passive tuned damper or applying active vibration control. This paper presents an active vibration control solution for a milling tool holder and preliminary results from the active control. The major challenge of transferring electrical power while maintaining signal quality to and from a rotating object is discussed and the proposed solution to this challenge is presented.

1 Introduction

Vibration problems during metal cutting occur frequently in the manufacturing industry. The vibration level depends on many different parameters such as material type, the dimensions of the workpiece, the rigidity of the tooling structure and the cutting data. The milling process is typically characterized by the usage of a tool with one or several teeth that rotates about a fixed axis [1], (there are several types of milling machines today that have several axes and that can perform complex milling operations about a non-fixed axis). The cyclic variation of forces that arise due to the rotating tool in the machining process might excite any of the natural modes of the milling machine structure. One part of the machine tool structure is the milling tool holder, and using a long overhang for this part will most likely result in high vibration levels.
Several methods have been developed to find cutting parameters that may reduce instabilities during machining [2,3]. One alternative method is to change some of the parameters during the milling operation [2], i.e. changing the spindle speed or the feed rate of the workpiece. The goal with changing one of the cutting parameters is to reduce the feedback and thereby avoid instability. Active solutions have also been proposed, for example by J.L. Dohner et al. [4], where an active structural control system was developed to alter the system’s dynamics.

In order to determine which method should be used and how, the dynamic properties of the system have to be thoroughly analyzed. One study concerning the dynamic properties of one type of milling tool holder and the dynamic properties of two milling machines have been reported in [5]. This paper claims that the vibrations display a high correlation with the two fundamental bending modes of the milling tool holder.

This paper discusses the implementation and properties of an active milling tool holder with vibration sensors and an embedded piezoceramic actuator. It furthermore presents a solution for transferring the vibration sensor signal and the electrical actuator power signal between a rotating active milling tool holder and a non-rotating control unit placed, for example, outside the milling machine. Results from the measurement of the power and sensor signals are presented for a number of different rotation speeds. Finally, results from initial experiments concerning the active control of a milling tool holder vibrations are also presented.

2 Materials and Methods

The milling machine used in the experiments was a Hurco BMC-50 vertical CNC machining centre. The spindle was of the ATC type, i.e. the speed can be varied between 10-3000 RPM in steps of 20 RPM and the maximum torque is 428 Nm, see Fig. 1.

![Figure 1: The Hurco BMC-50 milling machine.](image-url)
2.1 Active Tool Holder, Tool and Inserts

The milling tool holder is the interface between the spindle and the tool which holds all the inserts. The milling tool holder used in the experiments was a modified tool holder based on a standard tool holder of the type E3471 5525 16160 which has an overhang of 160 mm and a diameter of 38 mm, see Fig. 2 a).

An actuator was embedded a small distance from the center line in order to be able to produce a bending moment. The milled space for the actuator can be observed in Fig. 2 a). On the milling tool holder, a tool of the type R220.69-0040-12-5A was mounted and is illustrated in Fig. 2 b). This tool has a cutting diameter of $D_c = 40$ mm and a five teeth, $z_n = 5$ (inserts) configuration. The insert used in the tool configuration was XOMX120408TR-M12 T250M.

![Active milling tool holder](image1.png) ![Milling tool](image2.png)

Figure 2: a) The modified tool holder and b) the tool with the five teeth mounted.

2.2 Slip Ring Device

In order to provide electrical transfer paths for power and sensor signals to a rotating milling tool holder, the slip-ring method was used. A prototype able to transfer 15 ampere, 200 volt at a rotating speed of 8000 RPM (for a tool holder with a diameter of max 50 mm), or a velocity of 60 m/s, at the contact surfaces, was designed. Two poles are required for transferring power to one actuator and two poles are required to connect to one sensor, i.e. if wires are used. An alternative is to transfer power to two actuators and a wireless sensor signal transmitter. The sliding contacts are spring mounted and have two contact surfaces each towards the slip rings so that in case of large vibrations the risk of losing electrical contact is reduced. The slip ring device prototype enables the use of four sliding contacts per slip ring, thus providing eight contact surfaces if needed. From each slip ring, a conducting wire is accessible from the bottom side of the slip ring device for the connection of actuators and/or sensors. A sketch of the prototype is presented in Fig. 3 a) and a photo is presented in Fig. 3 b).
2.3 Experimental Setup

Four different experiments were conducted.

Firstly, the quality of the signals over the slip ring device was tested at the maximum rotation speed of the spindle, i.e. $n = 3000$ RPM. This was done by connecting the four conductors in the rotating part in pairs - resulting in two input poles and two output poles - and using broad band noise, 0-50000 Hz, as the input signal measuring the input and the output signals simultaneously.

Secondly, the actuator and the sensors were connected to the slip ring device. The actuator was excited in the frequency range of 0-5kHz and a channel estimation/control path - the transfer function between the voltage over the actuator and the response of the vibration sensor - was performed using either the piezo-film or the accelerometer for a number of different RPM, from 100 RPM up to 1000 RPM in steps of 100.

Furthermore, in order to determine the dynamic properties of the clamped milling tool holder by means of experimental modal analysis, an impulse hammer was used. The milling tool holder was excited close to the milling tool tip with an impulse hammer at opposite sides to the accelerometer attached to the milling tool holder, see Fig. 4.

The fourth experiment involved cutting with and without active control. As the control algorithm, the filtered-x algorithm was used, and the controller was set to a frequency range of 0 to 4000 Hz. A block diagram of the main structure of all of the setups is presented in Fig. 4.

2.4 Cutting Parameters

In order to test active control of the milling tool holder vibrations, we cut a workpiece consisting of the material SS1312. The workpiece had approximately the dimensions 70x50x500 mm ($y$, $z$, $x$), see Fig. 5. The workpiece was clamped to the milling table, and the milling table moved in the x-direction, resulting in a continuous cutting process along the workpiece, see Fig. 5. The cutting data used in the active control of the milling vibration were: feed speed $v_f$ was set to 1000 mm/min, the spindle speed $n = 700$ RPM, cutting a radial depth $a_e = 10$ mm and an axial depth $a_p$ equal to 2mm. During the cutting process, the active control algorithm was turned on and off.
Figure 4: The main structure of the experimental setups. Note that only one sensor is connected to the slip ring device at a time. The slip ring device is represented as a block in the figure in order to present the actuator and the piezo-film configurations, otherwise these components are covered by the slip ring device.

Figure 5: The side milling configuration of the cutting setup during the milling measurements.
3 Results

Results from the preliminary examination of the slip ring path and the control path involving the slip ring device are presented in terms of magnitude of frequency response estimates together with the quality measure in terms of coherence [6]. Then, the results from the active control setup are presented in terms of power spectral density estimates [6].

3.1 Setup One, Slip Ring Test

The first experimental setup shows a good transfer of the signal with a signal loss of less than 0.05 dB over a wide frequency range and a coherence close to one, see Fig. 6.

![Graph](image)

Figure 6: a) The frequency response function estimate between the input noise and the output noise passed through the slip ring device at 3000 RPM, with the conductors connected in pairs. And b) the corresponding coherence function estimate.

3.2 Setup Two, Control Path Estimation

The second experimental setup involves both the actuator and the sensors. First, a control path estimate is conducted with one sensor at a time when the spindle is not rotating, see Fig. 7. It is observable that the accelerometer sensor has better signal quality compared to the piezo film in the frequency range of 200 Hz to 1000 Hz, see the coherence in Fig. 7 b). It should be noted that the accelerometer has a known sensitivity of 100mV/g while the piezo film’s sensitivity is unknown. Also, the voltage/force relation for the actuator is specific for the particular configuration of the milling tool holder and has not been calibrated. Thus, the transfer function estimates are presented as sensor voltage over actuator voltage.

Since the accelerometer provided the best performance, it was used for the control path estimates for the various spindle speeds. Results of the control path estimates from 100 RPM up to 1000 RPM in steps of 100 RPM are presented in Fig. 8. In Fig. 8 a) only small changes are observable in the control path frequency response function estimates for different RPM. However, in the corresponding coherence functions in Fig. 8 b) it is observable that the quality of the coherence is reduced in the frequency range of 50 to 800 Hz for increasing RPM.
Figure 7: a) The frequency response function estimate between the actuator and the vibration sensor, when the spindle is not rotating. b) The corresponding coherence function estimate.

Figure 8: a) Control path estimates between the actuator and the accelerometer, when the spindle rotates from 100 RPM up to 1000 RPM in steps of 100 RPM. b) The corresponding coherence function.
3.3 Setup Three, Impact Test

In the third setup, an experimental modal analysis was conducted using an impulse hammer. Both sensors were used at the same time, since the spindle did not rotate. The frequency response function estimate is presented as voltage over force from the sensor signals. From the magnitude function in Fig. 9 a), the difference between strain and acceleration with respect to frequency is obvious. It should also be observed that the piezo-film is a capacitive sensor. With the help of the coherence function in Fig. 9 b), the signal quality of the sensors may be compared. The quality of the signal from the accelerometer seems to be better when compared to the signal from the piezo-film. The piezo-film signal quality is degraded by a 50 Hz disturbance and related harmonics, and this results in periodic dips in the coherence function.

![Figure 9: a) The frequency response function estimate between the impulse hammer and the vibration sensors. b) The corresponding coherence function estimate.](image)

3.4 Setup Four, Active Control

The results from the cutting experiment are presented as power spectral densities of the acceleration signal from the milling tool holder, using two different frequency resolutions. Also, a wide frequency range is presented together with zoomed-in power spectral density estimates. Firstly, a resolution of \( \delta f = 12500/4096 \approx 3 \) Hz was used giving a fairly high resolution of the power spectral density estimates. As can be seen in Fig. 10, the power spectral density estimates have several harmonics with a distance of \( 700/60 \approx 11.7 \) Hz which is related to the spindle speed. In the power spectral density estimates in Fig. 10 a), an underlying dynamic system is observable. In Fig. 10 b), the corresponding power spectral density estimates for the frequency range of 590 Hz to 640 Hz are shown.

Secondly, a spectral resolution of \( 12500/256 \approx 49 \) Hz was used in the power spectral density estimates of the milling tool holder acceleration and these spectra are presented in Fig. 11.
Figure 10: a) Spectral densities estimates of the acceleration signal from the milling tool holder without active control (solid line) and with active control (dashed line). b) The corresponding power spectral density estimates zoomed in.

Figure 11: a) Spectral densities estimates of the acceleration signal from the milling tool holder without active control (solid line) and with active control (dashed line). b) The corresponding spectra zoomed in.
4 Summary and Conclusions

Preliminary results from experiments using an active milling tool holder have been described in this paper. A method to transfer electrical signals and electrical power to a rotating tool holder has been presented. The slip ring device seems to be a successful way of transferring signals and power without the loss of any significant signal quality. This is shown in Fig. 6, where the conductors have been connected in pairs at the rotating side, while rotating with the maximum rotation speed of this milling machine. Results concerning the control path estimates showed a slight loss of signal quality. This may, however, be explained by the fact that the sensors pick up other vibrations than those originating from the actuator. Thus, uncorrelated noise affects the control path estimate when the spindle is rotating. In addition to this, the impact test confirms the quality of the frequency response function estimates during the measurements performed during rotation. During the experiments, the bending of the milling tool holder, the acceleration close to the tool tips and the excitation of the tool holder with the actuator were successfully sensed by the setup. When it comes to the active control feasibility study, some attenuation of the milling tool holder vibration has been introduced. However, this may well be rectified when more work has been carried out to find the optimal performance of the suggested setup.

Acknowledgments

The present project is sponsored by the company Acticut International AB in Sweden which has multiple approved patents covering active control technology.

References


Part VI

Noise Source Identification and Active Control in a Water Turbine Application
This part is based on the publication:

Noise Source Identification and Active Control in a Water Turbine Application

Henrik Åkesson\textsuperscript{1,2}, Andreas Sigfridsson and Thomas T. Lagö
\textsuperscript{1}Acticut International AB
Gjuterivägen 6, 311 32 Falkenberg, Sweden

Ingvar Andersson
Turab AB Förrådsgatan 2, 571 39 Nässjö, Sweden

Lars Håkansson
\textsuperscript{2}Blekinge Institute of Technology,
Department of Signal Processing,
372 25 Ronneby, Sweden

November 25, 2009

Abstract

When optimizing a water turbine application for high efficiency, it is not uncommon that disturbing vibrations occur. This may happen even when the generator is not used at maximum power. The challenge is to find the root cause of the high vibration levels. When the vibrations have started, more or less everything vibrates. This paper presents an approach that has been utilized in a real-life situation using multiple tools and a methodology to break down the global vibrations into cause and effect. The paper can act as an example of how modern vibration analysis and methodology can be utilized when vibration levels are too high and the cause of these vibrations is uncertain. This paper presents a vast range of data and shows multiple measurements that have been performed. The paper also shows how the results from these measurements can be used to locate the vibration source. In addition to this, a solution that may resolve this problem is presented together with the attenuation result. The successful attenuation of the structural vibration using the proposed solution confirms the conclusions with the aid of the method presented in the paper.

1 Introduction

In Sweden, power plants are commonly situated along water courses. Today, there are around 1900 hydroelectric power stations of various sizes, 700 producing a power of 1.5 MW or more and 1200 small-scale power plants producing a power less than 1.5 MW. Together they generate approximately half of the Swedish electrical power capacity. Hydroelectric power is a renewable energy resource and is thus considered to be a so-called a green source of energy. It is therefore likely to be beneficial to the
environment to use this sort of energy resource. By building dams, water is trapped and can be controlled to flow through tunnels in the dam, turning turbines and driving generators, see Fig. 1. However, when optimizing a water turbine application for high efficiency, it is not uncommon that disturbing vibrations occur. Occasionally, the vibrations cause damage to the power plant. Vibrations may be caused by various blade configurations even when the generator is not used at maximum power. These harmful vibrations have been experienced at several power plants and a solution to this problem has been pursued.

![Sketch of a hydroelectric power station.](image)

Figure 1: Sketch of a hydroelectric power station.

In order to find a suitable solution to the problem, a first step is to identify and understand the cause of the vibrations. The power plant station presented in this paper is principally configured according to Fig. 1 and the real power house is presented in Fig. 2.

## 2 Materials and Methods

The first step was to examine the vibration problem experienced in the hydroelectric power plant. In order to get an appropriate and detailed description of the actual problem, vibrations were measured and recorded during operation of the power plant and then used to perform operating deflection shape (ODS) analysis [1, 2]. The operating deflection shapes of a structure provide information on the structure’s spatial deformation, either for a certain time instance or for a certain frequency, depending on the analysis method. In order to examine whether the problem identified from the ODS analysis is related to the dynamic properties of the structure, an experimental modal analysis was performed. Also, coarse Finite Element Models (FEM) were used
to optimize sensor locations and get an indication of what motions to expect. Finally, after identifying the vibration problem, active control was implemented in order to attenuate the vibrations and also to confirm whether the previous analysis was correct or not.

The structures of interest are both the rotating turbine and its bearings along with the supporting frame. The main bearings are inside the blue frame illustrated in the model in Fig. 3 b).

### 2.1 Operating deflection shapes

To examine the behavior of the bearing frame during operation mode, the acceleration at a number of different spatial locations of the structure (was measured simultaneously. By considering the phase and amplitude of the response signals from the accelerometers on the operating structure, it is possible to produce an estimate of the ODS. The amplitude is estimated by either auto-power spectra or auto-power spectral density estimates depending on whether the signal is tonal or random [3,4]. The phase between each spatial position is then estimated from the cross-power spectrum. Thus, the expression of the ODS in terms of power spectra is given by [2]:

$$\{ODS(f)\}_{RMS} = \left\{ \sqrt{\hat{P}_{11}(f)} \sqrt{\hat{P}_{22}(f)e^{j\hat{\theta}_{21}(f)}} \ldots \sqrt{\hat{P}_{NN}(f)e^{j\hat{\theta}_{N1}(f)}} \right\}^T$$  \hspace{1cm} (1)

where $\hat{P}_{nn}(f)$ is the estimated power spectrum and $e^{j\hat{\theta}_{n1}(f)}$ is the phase function of the estimated cross-spectrum $\hat{P}_{n1}(f)$ for $n \in \{2, \ldots, N\}$.

For the ODS measurements, eleven different sensor locations were selected, six on the top of the generator and five on the bottom side of generator frame, see Fig. 4 and Fig. 5.
Figure 3: In a) the penstock is observable as the large dark tube going around the red beams which support the concrete structure. In the center, the generator shaft is located. Above this floor, the generator windings are located while below it the turbine blades are turned by the water flow. In b) an overview of the whole generator shaft (grey) and the main bearing frame (blue) is presented. The bottom side of the bearing frame is reachable from the room presented in a).

Figure 4: a) The top cover of turbine one. Below the floor plates, the generator windings are rotating. b) Six accelerometers were glued to two different rigid structures close to the shaft, measuring in the x- (left), y- (into picture) and z- (up) directions.
The acceleration occurring at the measurement positions was measured simultaneously during various operating conditions. In connection with this, the following conditions were examined:

1. Three power conditions: the turbine generating 2.6 MW, 1.5 MW and 3.1 MW.
2. Turbine standing still, (second turbine running in the background).
3. Turbine stopping, (shaft de accelerating).
4. Turbine starting, (shaft accelerating).

Figure 5: Five accelerometers measuring vibrations in the z-direction were glued on the bottom side of the bearing frame.

2.2 Modal Analysis

The primary goal of experimental modal analysis is to identify the dynamic properties by obtaining the modal parameters of the system under examination; i.e. to determine the natural frequencies, mode shapes and damping ratios from experimental vibration measurements [1]. The frequency transfer function \( \hat{H}(f_k) \) between the input signal \( x(n) \) and the output signal \( y(n) \) for a dynamic system may be estimated according to [4]:

\[
\hat{H}(f_k) = \frac{\hat{P}_{PSD}^{yx}(f_k)}{\hat{P}_{PSD}^{xx}(f_k)}, f_k = \frac{k}{N}F_s
\]

where \( 0 \leq k \leq N/2.56 \), \( F_s \) is the sampling frequency, \( \hat{P}_{PSD}^{yx}(f_k) \) is the cross-power spectral density between the input signal \( x(n) \) and the output signal \( y(n) \), and \( \hat{P}_{PSD}^{xx}(f_k) \) is the power spectral density for the input signal \( x(n) \).

In the previous ODS measurements, a number of accelerometers were used. However, in the experimental modal analysis, an excitation force has to be produced in order to be able to estimate the frequency response functions. This is normally done using an impulse hammer or a shaker. In this case, when the structure under investigation is large and relatively rigid, a shaker is preferable in order to be able to
inject sufficient vibration energy into the structure. The largest shaker available at
the time was used for experimental modal analysis and was mounted to one of the
beams of the bearing frame (position 3 in Fig. 5). The force applied to the structure
was measured simultaneously with the accelerometers previously described so that
the frequency response estimates could be produced.

![Figure 6: a) An overview of the shaft, the bearing frame (covered with plates) and a
shaker attached to one of the rigid beams. b) A close-up illustrating how the shaker
was attached to the bearing frame.](image)

### 2.3 Actuator Setup

The actuators used in the implementation of active control were inertia mass actuators
and are presented in Fig. 7 a). They can give a maximum force of 5200 N and have
been tuned to work in the range of 18 to 26 Hz, see Fig. 7 b). This results in a
resonance frequency of 22 Hz, a vibrating mass of 38.5 kg and a spring stiffness of
736 N/mm.

Two different positions were tested in order to evaluate the performance. Three
actuators were first mounted on the top of the generator structure, see Fig. 8 a), and
then secondly two actuators were positioned on the bottom side of the bearing frame,
see Fig. 8 b) and c). As a reference signal to the controller, an accelerometer placed
beside one of the actuators was used. During the active control measurements, two
different operating conditions were used, first when the generator produced 3.2 MW
and secondly when the generator produced 3.4 MW.

### 2.4 The Active Control Algorithm

As a digital controller, a single channel feedback filtered-x LMS algorithm imple-
mented in DSP was used. The algorithm is suitable in this application and gives a
robust solution [5]. The block diagram of the feedback filtered-x LMS algorithm is
shown in the Fig. 9 and is described by equations (3-7).

The feedback filtered-x LMS algorithm with a leakage coefficient is defined by the
The following equations:

\[ y(n) = w^T(n)x(n) \]  \quad (3)

\[ e(n) = d(n) + y_C(n) \]  \quad (4)

\[ w(n + 1) = \gamma w(n) - \mu x_C(n)e(n) \]  \quad (5)

where \( \mu \) is the adaptation step size and

\[ x_C(n) = \left[ \sum_{i=0}^{I-1} \hat{c}_i x(n-i), \ldots, \sum_{i=0}^{I-1} \hat{c}_i x(n-M+1) \right]^T \]  \quad (6)

is the filtered reference signal vector, which usually is produced by filtering the reference signal \( x(n) \) with an FIR-filter estimate \( \hat{c}(i) \), \( i \in 0, 1, \ldots, I - 1 \) of the forward path. \( w(n) \) is the adaptive FIR filter coefficient vector, \( y(n) \) is the output signal from the adaptive FIR filter, \( e(n) \) is the error signal, measured by the accelerometer, \( y_C(n) \) is the secondary vibration, \( C \) is the estimate of the forward path, \( d(n) \) is the primary vibration, \( x_C(n) = [x(n), \ldots, x(n-M+1)] \) is the reference signal vector and \( x(n) \) is related to the delayed error signal as

\[ x(n) = e(n-1). \]  \quad (7)

This algorithm uses an estimate of the forward path (D/A converter, amplifier, actuator and structural path) to produce an adequate direct gradient estimate that enables a minimization of the mean square error in the control application. The error signal is produced by the sum of the primary vibration signal and the secondary vibration induced by the actuator and transferred through the forward path. The estimate of the forward path was made with the use of the LMS algorithm prior to the implementation of active control.
Figure 8: a) The actuators mounted from the top of the generator frame. b) The actuators mounted from the bottom side of the bearing frame where c) is the same configuration as in b) illustrating how the actuators are supposed to act on the bearing frame.
3 Results

The results from the ODS measurements are presented as linear power spectra since the responses contain significant tonal components. The operating deflection shape is presented for the "first mode" as well as for the finite element model analysis. For the experimental modal analysis result, a frequency response estimate is presented. For the active control setup, some representative results of vibration with and without control in form of power spectra are presented.

3.1 Vibration Spectrum and Operating Deflection Shape

Initially, vibration levels for three different power operating conditions of the power plant are presented, see Fig. 10. Two peaks that are more significant than the others may be observed here, one at 21 Hz and a second peak at approximately 800 Hz. Since the spectra has the unit acceleration and it is the level of displacement which is of importance, the first peak is larger and thus likely to be much more harmful in comparison to the second peak. This is the case because the acceleration is the second derivative of displacement and thus the magnitude of the displacement will decline with a factor of \((j2\pi f)^2\) with the frequency when compared with the acceleration. The frequency of 21 Hz is the same as the blade wheel frequency.

The power spectra from the accelerometers placed on the bottom side of the bearing frame generated when the turbine is running and when it has been stopped is presented in Fig. 11. It is clear that the vibrations have almost disappeared and the residual vibrations originate from the second turbine in the power plant station. It may also be observed that the peak at 800 Hz is almost unnoticed by the sensors located on the bottom side of the bearing frame.

The operating deflection shape is presented in Fig. 12 and illustrates clearly the larger motion towards the center as well as the fact that both sides operate in phase.

3.2 Finite Element Model

The first bending mode from the finite element model of the bearing frame shows the same behavior as measured during the operating modes, i.e. the largest motion is in the center of the frame, see Fig. 13.
Figure 10: a) Power spectrum from two positions on top of the generator for three different power operating modes of the power plant. b) Zoomed-in power spectrum.

Figure 11: a) Power spectrum from an accelerometer glued to the bottom side of the bearing frame when the turbine is running and when the turbine is off. b) Zoomed-in power spectrum.

Figure 12: The operating deflection shape at 21 Hz. The length of the arrows represents the magnitude of the displacement measured by the sensors positioned at the locations indicated by the red circles.
3.3 Modal Analysis

The frequency response function estimated from the signals obtained during the experimental modal analysis contained a lot of noise. This was due to the relatively rigid structure as compared to the force capacity of the shaker. However, it is possible to get an indication of the dynamics of the structure. Two peaks at 21 and 23 Hz are observable, see Fig. 14.

![Figure 13: A finite element model, a) the un-deformed body and in b) the deformed body together with the "wired" un-deformed body.](image)

Figure 13: A finite element model, a) the un-deformed body and in b) the deformed body together with the "wired" un-deformed body.

![Figure 14: The magnitude of the accelerance function estimate between the position where the shaker is attached (position 3) and the accelerometer close to the shaft (position 6), see Fig. 6.](image)

Figure 14: The magnitude of the accelerance function estimate between the position where the shaker is attached (position 3) and the accelerometer close to the shaft (position 6), see Fig. 6.

3.4 Active Control

An overview of the vibration spectra is presented in Fig. 15 to show the levels of vibration with and without attenuation. The vibrations are from an accelerometer mounted on the top of the generator denoted Z7+. It should be observed that the active control system does not handle the vibration peak at 800 Hz, but attenuates the intended vibration peak at 21 Hz by an order of six.
In Fig. 16, the vibration levels are presented with and without control for two different actuator configurations. The measured signal is from an accelerometer mounted on top of the generator denoted Z9+.

4 Summary and Conclusions

In this paper, an effective method that is able to identify vibration sources has been presented. An experiment was conducted where large vibrations were measured during various operating conditions, showing a stable tonal component at 21 Hz. The dynamics of the bearing structure indicate a dynamic weakness at the same frequency the shaft is excited with. Also, a solution of how to attenuate the vibrations has been proposed and a first implementation has demonstrated the successful attenuation of these vibrations. However, the performance of the active system may be enhanced further. For example, the number of actuators and the size of each actuator, the number of sensors used in the control algorithm as well other parameters in the digital controller, are still interesting to evaluate in order see what is possible to achieve while maintaining costs at a minimum.
Acknowledgments

The present project is sponsored by the company Acticut International AB in Sweden which has multiple approved patents covering active control technology. The project has been performed in collaboration with Turab AB in Sweden.

References


Vibration in metal cutting is a common problem in the manufacturing industry, especially when long and slender tool holders or boring bars are involved in the manufacturing process. Vibration has a detrimental effect on machining. In particular, the surface finish is likely to suffer, but tool life is also most likely to be reduced. Tool vibration also results in loud noise that may disturb the working environment.

The first part of this thesis describes the development of a robust and manually adjustable analog controller capable of actively controlling boring bar vibrations related to internal turning. This controller is compared with an adaptive digital feedback filtered-x LMS controller and it displays similar performance with a vibration attenuation of up to 50 dB.

A thorough experimental investigation of the influence of the clamping properties on the dynamic properties of clamped boring bars is also carried out in second part of the thesis. In relation to this, it is demonstrated that the number of clamping screws, the clamping screw diameter size, the screw tightening torque and the order the screws are tightened, have a significant influence on a clamped boring bar’s eigenfrequencies as well as on its mode shape orientation in the cutting speed - cutting depth plane. Also, an initial investigation of nonlinear dynamic properties of clamped boring bars was carried out.

Furthermore, vibration in milling has also been studied in relation to milling tool holders with a long overhang. A basic investigation concerning the spatial dynamic properties of the tool holders of milling machines, both when not cutting and during cutting, has been carried out. Also, active control of milling tool holder vibration has been investigated and a first prototype of an active milling tool holder was implemented and tested. The challenge of transferring electrical power while maintaining good signal quality to and from a rotating object is addressed and a solution to this is proposed.

Finally, vibration is also a problem for the hydroelectric power industry. In Sweden, hydroelectric power plants stand for approximately half of Sweden's electrical power production and are also considered to be a so-called green source of energy. When renovating water turbines in small-scale hydroelectric power plants and modifying them to optimize efficiency, it is not uncommon that disturbing vibrations occur in the power plant. These vibrations have a negative influence on the production capacity and will wear various components quickly. Occasionally, these vibrations may cause severe damage to the power plant. To identify this vibration problem, experimental modal analysis and operating deflection shape analysis were utilized. To reduce the vibration problem, active control using inertial mass actuators was investigated. Preliminary results indicate a significant attenuation of the vibrations.