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A Comparison of Two MIMO Relaying Protocols in Nakagami-m Fading

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Abstract

Transmit antenna selection with receive maximal-ratio combining (TAS/MRC) and transmit antenna selection with receive selection combining (TAS/SC) are two attractive multiple-input multiple-output (MIMO) protocols. In this paper, we present a framework for the comparative analysis of TAS/MRC and TAS/SC in a two-hop amplify-and-forward relay network. In doing so, we derive exact and asymptotic expressions for the symbol error rate (SER) in Nakagami-m fading. Using the asymptotic expressions, the SNR gap between the two protocols is quantified. Given that the two protocols maintain the same diversity order, we show that the SNR gap is entirely dependent on the array gain. Motivated by this, we derive the SNR gap as a simple ratio of the respective array gains of the two protocols. This ratio explicitly takes into account the impact of the number of antennas and the fading severity parameter m. In addition, we address the fundamental question of "How to allocate the total transmit power between the source and the relay in such a way that the SER is minimized?". Our answer is given in the form of new compact expressions for the power allocation factor – a practical design tool that optimally distributes the total transmit power in the network.

Index Terms

MIMO, Nakagami, relays.

I. INTRODUCTION

Wireless relaying is pivotal to extending network coverage and providing connectivity in hard-to-reach areas [1, 2]. The question is how to deliver high data rates over large areas at relatively low overhead cost. To meet the demand for high data rate transmission, multiple-input multiple-output (MIMO) processing is fundamental [3, 4]. MIMO relaying – where the source, relay, and destination have multiple antennas – has garnered much research interest. From a capacity perspective, the information theoretic analysis of MIMO relaying has been reported in [5–7]. More recently, from an error rate performance perspective, the design and analysis of such networks has been presented in [8–10]. The advantages of MIMO relaying, however, come at a cost of complex transceivers and power intensive signal processing modules. In particular, the high power consumption required to operate multiple radio-frequency (RF) chains presents a major challenge to the direct implementation of MIMO relaying in battery-operated wireless ad hoc and sensor networks.

An attractive solution to the aforementioned challenge is to apply MIMO selection protocols in MIMO relaying. Among them, transmit antenna selection with receive maximal-ratio combining (TAS/MRC) in MIMO relaying was proposed in [11]. In TAS/MRC relaying, a single antenna is selected at the transmitters and all the antennas at the receivers are MRC combined. To further reduce the number of RF chains in the network, transmit antenna selection with receive selection combining (TAS/SC) in MIMO relaying was considered in [12]. In TAS/SC relaying, a single antenna at the transmitters and a single antenna at the receivers are jointly selected. Several transmit antenna selection strategies for MIMO relaying were introduced in [13]. While the aforementioned works stand on their own merits, the purpose of this paper is to present a unified framework for the comparative analysis of TAS/MRC and TAS/SC in a two-hop amplify-and-forward relay network.

We begin with the question: *"What performance improvement does TAS/MRC relaying offer relative to TAS/SC relaying?"*. In tackling this, we derive exact and asymptotic expressions for the symbol error rate (SER) in Nakagami-m fading where m_1 and m_2 are the fading parameters in the source-to-relay link and the relay-to-destination link, respectively. Our asymptotic expressions demonstrate that the two protocols offer the same diversity order of $N_{\mathrm{R}} \times \min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})$, where N_{S}, N_{R}, and N_{D} are the number of antennas at the source, relay, and destination, respectively. This leads to the conclusion that the SNR gap between the two protocols is explicitly dependent on the array gain. Motivated by this, we pursue the question: *"Which distinct parameters determine this improvement in Nakagami-m fading?"*. In doing so, we formulate the SNR gap as a simple ratio of the array gains of the two protocols. This ratio captures the impact of the number of antennas and the impact of the m fading parameters on the SNR gap. To gain further insights, we examine the important question: *"What is the optimal transmit power allocation that minimizes the SER in each MIMO relaying protocol?"*. In addressing this, we derive easy-to-calculate expressions for the power allocation factor to optimally allocate the total transmit power between the source and the relay. The power allocation factor is a simple yet powerful design tool that maximizes the network performance without expending additional resources.

II. PROTOCOL DESCRIPTION

Consider the distributed MIMO relay network illustrated in Fig. 1, where the source, relay, and destination are equipped with $N_{\rm S}$, $N_{\rm R}$, and $N_{\rm D}$ antennas, respectively. The source has no direct link to the destination. We detail two distributed MIMO relaying protocols. In each protocol, the transmission from the source to the destination via the relay is facilitated by a two-step process. In the following, $\mathcal{E}_{\rm S}$ is the source transmit power, $\mathcal{E}_{\rm R}$ is the relay transmit power, $\mathcal{N}_0$ is the variance of the additive white Gaussian noise (AWGN), $\| \cdot \|$ is the Euclidean norm, $| \cdot |$ is the absolute value, $(\cdot)^\dagger$ is the conjugate transpose, and $\mathbf{I}_N$ is the $N \times N$ identity matrix.

A. TAS/MRC Relaying

Step 1: In the source-to-relay link, a single antenna from $N_{\rm S}$ antennas at the source is selected to maximize the received SNR at the relay. In doing so, all the $N_{\rm R}$ antennas at the relay are MRC combined. The received signal vector at the relay is given by

$$\mathbf{y}_{\rm SR} = \sqrt{\mathcal{E}_{\rm S}}\mathbf{h}_1^{\max} x + \mathbf{n}_{\rm R}, \tag{1}$$

where x is the transmit symbol, and $\mathbf{n}_{\rm R}$ is the $N_{\rm R} \times 1$ AWGN vector satisfying $\mathbf{E}[\mathbf{n}_{\rm R}\mathbf{n}_{\rm R}^\dagger] = \mathbf{I}_{N_{\rm R}}\mathcal{N}_0$. In (1), we define $\| \mathbf{h}_1^{\max} \| = \max_{1 \le i \le N_{\rm S}} \left\{ \| \mathbf{h}_1^i \| \right\}$ where $\mathbf{h}_1^i$ is the $N_{\rm R} \times 1$ complex channel vector between the i-th transmit antenna at the source and the $N_{\rm R}$ receive antennas at the relay, with independent and identically distributed (i.i.d.) Nakagami-m fading entries.

Step 2: In the relay-to-destination link, a single antenna from $N_{\rm R}$ antennas at the relay is selected to maximize the received SNR at the destination. In doing so, all the $N_{\rm D}$ antennas at the destination are MRC combined. Prior to forwarding, a variable gain of $G = 1/\sqrt{\mathcal{E}_{\rm S} \| \mathbf{h}_1^{\max} \|^4 + \tau\mathcal{N}_0 \| \mathbf{h}_1^{\max} \|^2}$ is applied to counteract the fading in the source-to-relay link. We consider two variable gain protocols: channel-noise-based (CNB) relaying that takes into account the noise variance with $\tau = 1$, and channel-based (CB) relaying that does not account for the noise variance with $\tau = 0$.

The received signal vector at the destination is given by

$$\mathbf{y}_{\rm RD} = \sqrt{\mathcal{E}_{\rm R}}\mathbf{h}_2^{\max} G y'_{\rm SR} + \mathbf{n}_{\rm D}, \tag{2}$$

where $y'_{\rm SR} = (\mathbf{h}_1^{\max})^{\dagger}\mathbf{y}_{\rm SR}$, and $\mathbf{n}_{\rm D}$ is the $N_{\rm D} \times 1$ AWGN vector satisfying $\mathbf{E}[\mathbf{n}_{\rm D}\mathbf{n}_{\rm D}^{\dagger}] = \mathbf{I}_{N_{\rm D}}\mathcal{N}_0$. In (2), the channel vector norm in the relay-to-destination link is $\| \mathbf{h}_2^{\max} \| = \max_{1\leq i\leq N_{\rm R}} \left\{\| \mathbf{h}_2^i \|\right\}$ where $\mathbf{h}_2^i$ is the $N_{\rm D} \times 1$ complex channel vector between the i-th transmit antenna at the relay and the $N_{\rm D}$ receive antennas at the destination, with i.i.d. Nakagami-m fading entries.

The end-to-end SNR is written as $\gamma_{\rm TAS/MRC} = \Gamma_1\Gamma_2/(\Gamma_1 + \Gamma_2 + \tau)$, where $\Gamma_1 = \frac{\mathcal{E}_{\rm S}}{\mathcal{N}_0} \| \mathbf{h}_1^{\max} \|^2$ and $\Gamma_2 = \frac{\mathcal{E}_{\rm R}}{\mathcal{N}_0} \| \mathbf{h}_2^{\max} \|^2$ are the instantaneous SNRs in the source-to-relay link and the relay-to-destination link, respectively. In each link, we assume that channel state information (CSI) is estimated by the receiver using pilot symbols sent by the transmitter. In the source-to-relay link, the source transmits pilot symbols and the relay feeds back the index of the strongest transmit antenna to the source. In the relay-to-destination link, the relay broadcasts pilot symbols and the destination feeds back the index of the strongest transmit antenna to the relay.

B. TAS/SC Relaying

Step 1: In the source-to-relay link, a single antenna from $N_{\rm S}$ antennas at the source and a single antenna from $N_{\rm R}$ antennas at the relay are jointly selected to maximize the received SNR at the relay. The received signal at the relay is given by

$$y_{\rm SR} = \sqrt{\mathcal{E}_{\rm S}} h_1^{\max} x + n_{\rm R}, \tag{3}$$

where $n_{\rm R}$ is the AWGN with $\mathbf{E}[n_{\rm R}] = \mathcal{N}_0$. In (3), the magnitude of the channel coefficient in the source-to-relay link is $| h_1^{\max} | = \max_{1\leq i\leq N_{\rm S}, 1\leq j\leq N_{\rm R}} \left\{| h_1^{i,j} |\right\}$ where $h_1^{i,j}$ is the i.i.d. Nakagami-m fading channel coefficient between the i-th transmit antenna at the source and the j-th receive antenna at the relay.

Step 2: In the relay-to-destination link, a single antenna from $N_{\rm R}$ antennas at the relay and a single antenna from $N_{\rm D}$ antennas at the destination are jointly selected to maximize the received SNR at the destination. In this protocol, the relay applies a variable gain of $G = 1/\sqrt{\mathcal{E}_{\rm S} | h_1^{\max} |^2 + \tau\mathcal{N}_0}$. The received signal vector at the destination is given by

$$y_{\rm RD} = \sqrt{\mathcal{E}_{\rm R}} h_2^{\max} G y_{\rm SR} + n_{\rm D}, \tag{4}$$

where $n_{\rm D}$ is the AWGN with $\mathbf{E}[n_{\rm D}] = \mathcal{N}_0$. In (4), the magnitude of the channel coefficient in the relay-to-destination link is $|h_2^{\max}| = \max_{1\leq i\leq N_{\rm R}, 1\leq j\leq N_{\rm D}} \left\{|h_2^{i,j}|\right\}$ where $h_2^{i,j}$ is the i.i.d. Nakagami-m fading channel coefficient between the i-th transmit antenna at the relay and the j-th receive antenna at the destination.

The end-to-end SNR is expressed as $\gamma_{\mathrm{TAS/SC}} = \Upsilon_1 \Upsilon_2 / (\Upsilon_1 + \Upsilon_2 + \tau)$, where $\Upsilon_1 = \frac{\mathcal{E}_{\mathrm{S}}}{\mathcal{N}_0} \mid h_1^{\max} \mid^2$ and $\Upsilon_2 = \frac{\mathcal{E}_{\mathrm{R}}}{\mathcal{N}_0} \mid h_2^{\max} \mid^2$ are the instantaneous SNRs in the source-to-relay link and the relay-to-destination link, respectively. As in TAS/MRC relaying, CSI is estimated in each link by the receiver using pilot symbols sent by the transmitter.

III. Symbol Error Rates

In this section, we present new exact and asymptotic expressions for the SER of TAS/MRC relaying and TAS/SC relaying. In deriving the SER, we focus on CB relaying with $\tau = 0$. CB relaying is analytically tractable and its performance asymptotically approaches that of CNB relaying at high SNRs. We begin by presenting the exact cdf of the end-to-end SNRs, denoted by $F_{\gamma_{\mathrm{TAS/MRC}}}(\gamma)$ and $F_{\gamma_{\mathrm{TAS/SC}}}(\gamma)$. Based on these, the SER is directly evaluated according to

$$P_{\chi} = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^{\infty} \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_{\gamma_{\chi}}(\gamma) d\gamma, \tag{5}$$

where $\chi \in \{\mathrm{TAS/MRC}, \mathrm{TAS/SC}\}$. This applies to all general modulations with SER of the form $P = \mathbf{E}\left[aQ\left(\sqrt{2b\gamma}\right)\right]$, where a and b are constellation-specific constants [8].

A. TAS/MRC Relaying

1) Exact: The exact cdf of $\gamma_{\mathrm{TAS/MRC}}$ is evaluated by applying algebraic manipulation along similar lines to [14], according to

$$F_{\gamma_{\mathrm{TAS/MRC}}}(\gamma) = 1 - \int_0^{\infty} \tilde{F}_{\Gamma_1}\left(\gamma + \frac{\gamma^2}{\omega}\right) f_{\Gamma_2}(\omega + \gamma) d\omega, \tag{6}$$

where $\tilde{F}_{\Gamma_1}(\gamma) = 1 - F_{\Gamma_1}(\gamma)$ is the complementary cdf of Γ_1, $f_{\Gamma_2}(\cdot)$ is the pdf of Γ_2, and $\omega = \Gamma_2 - \gamma$.

In the source-to-relay link, the channel vector entries in $\mathbf{h}_1^i$ follow a Nakagami-m distribution with fading parameter m_1. As such, the cdf of Γ_1 is written as [4]

$$F_{\Gamma_1}(\gamma) = \left[1 - e^{-\gamma\frac{m_1}{\overline{\gamma}_1}} \sum_{k=0}^{m_1 N_{\mathrm{R}}-1} \frac{\left(\gamma\frac{m_1}{\overline{\gamma}_1}\right)^k}{k!}\right]^{N_{\mathrm{S}}} \tag{7}$$

where $\overline{\gamma}_1 = \frac{\mathcal{E}_{\mathrm{S}}}{\mathcal{N}_0}$ is the average SNR between each antenna pair in the source-to-relay link. In the relay-to-destination link, the channel vector entries in $\mathbf{h}_2^i$ follow a Nakagami-m distribution with fading parameter

m_2. Accordingly, the pdf of Γ_2 is written as

$$f_{\Gamma_2}(\gamma) = N_{\rm R}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2 N_{\rm D}} \frac{\gamma^{m_2 N_{\rm D}-1} e^{-\gamma\frac{m_2}{\overline{\gamma}_2}}}{\Gamma(m_2 N_{\rm D})}\left[1 - e^{-\gamma\frac{m_2}{\overline{\gamma}_2}} \sum_{l=0}^{m_2 N_{\rm D}-1} \frac{\left(\gamma\frac{m_2}{\overline{\gamma}_2}\right)^l}{l!}\right]^{N_{\rm R}-1} \tag{8}$$

where $\overline{\gamma}_2 = \frac{\mathcal{E}_{\rm R}}{N_0}$ is the average SNR between each antenna pair in the relay-to-destination link.

Substituting (7) and (8) into (6), then applying the identity [15, eq. 3.471.9], the cdf of $\gamma_{\rm TAS/MRC}$ is presented as

$$\begin{aligned} F_{\gamma_{\rm TAS/MRC}}(\gamma) = 1 &- \frac{2N_{\rm R}}{\Gamma(m_2 N_{\rm D})} \sum_{q_0=1}^{N_{\rm S}} \sum_{r_0=0}^{N_{\rm R}-1} \binom{N_{\rm S}}{q_0}\binom{N_{\rm R}-1}{r_0}(-1)^{q_0+r_0-1} \prod_{k=1}^{m_1 N_{\rm R}-1}\left[\sum_{q_k=0}^{q_{k-1}}\binom{q_{k-1}}{q_k}\left(\frac{1}{k!}\right)^{q_k - q_{k+1}}\right] \\ &\times \prod_{l=1}^{m_2 N_{\rm D}-1}\left[\sum_{r_l=0}^{r_{l-1}}\binom{r_{l-1}}{r_l}\left(\frac{1}{l!}\right)^{r_l-r_{l+1}}\right]\sum_{i=0}^{\sigma}\sum_{j=0}^{\varsigma+m_2N_{\rm D}-1}\binom{\sigma}{i}\binom{\varsigma+m_2N_{\rm D}-1}{j}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\varsigma+m_2N_{\rm D}-\frac{j-i+1}{2}} \\ &\times \left(\frac{m_1}{\overline{\gamma}_1}\right)^{\sigma+\frac{j-i+1}{2}}\left(\frac{q_0}{r_0+1}\right)^{\frac{j-i+1}{2}}\gamma^{\sigma+\varsigma+m_2N_{\rm D}} e^{-\gamma\left(\frac{q_0 m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)} K_{j-i+1}\left(2\gamma\sqrt{\frac{q_0(r_0+1)m_1m_2}{\overline{\gamma}_1\overline{\gamma}_2}}\right), \end{aligned} \tag{9}$$

where $K_\nu(\cdot)$ is the ν-th order modified Bessel function of the second kind [16, eq. 9.6.2]. In (9), we define $\sigma = q_1 + q_2 + \ldots + q_{m_1 N_{\rm R}-1}$ and $\varsigma = r_1 + r_2 + \ldots + r_{m_2 N_{\rm D}-1}$.

Finally, we derive the exact SER of TAS/MRC relaying shown in (10) by substituting (9) into (5) and applying identities [15, eq. 3.326.2] and [17, eq. (2.16.6.3)] to solve the resulting integrals after some algebraic manipulations.

$$\begin{aligned} P_{\rm TAS/MRC} = \frac{a}{2} &- \frac{a\sqrt{b}N_{\rm R}}{\Gamma(m_2 N_{\rm D})} \sum_{q_0=1}^{N_{\rm S}} \sum_{r_0=0}^{N_{\rm R}-1} \binom{N_{\rm S}}{q_0}\binom{N_{\rm R}-1}{r_0}(-1)^{q_0+r_0-1} \prod_{k=1}^{m_1 N_{\rm R}-1}\left[\sum_{q_k=0}^{q_{k-1}}\binom{q_{k-1}}{q_k}\left(\frac{1}{k!}\right)^{q_k - q_{k+1}}\right] \\ &\times \prod_{l=1}^{m_2 N_{\rm D}-1}\left[\sum_{r_l=0}^{r_{l-1}}\binom{r_{l-1}}{r_l}\left(\frac{1}{l!}\right)^{r_l-r_{l+1}}\right]\sum_{i=0}^{\sigma}\sum_{j=0}^{\varsigma+m_2N_{\rm D}-1}\binom{\sigma}{i}\binom{\varsigma+m_2N_{\rm D}-1}{j}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{\sigma}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\varsigma+m_2N_{\rm D}-j+i-1} \\ &\times \left(b+\frac{q_0 m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)^{j-i-\sigma-\varsigma-m_2N_{\rm D}+\frac{1}{2}} (r_0+1)^{i-j-1}\, 2^{i-j-\sigma-\varsigma-m_2N_{\rm D}-\frac{3}{2}} \\ &\times \frac{\Gamma(\sigma+\varsigma+m_2N_{\rm D}-\frac{1}{2}-j+i)\Gamma(\sigma+\varsigma+m_2N_{\rm D}+\frac{3}{2}+j-i)}{\Gamma(\sigma+\varsigma+m_2N_{\rm D}+1)} \\ &\times {}_2F_1\left[\frac{\sigma+\varsigma+m_2N_{\rm D}-\frac{1}{2}-j+i}{2}, \frac{\sigma+\varsigma+m_2N_{\rm D}+\frac{1}{2}-j+i}{2};\right. \\ &\left.\sigma+\varsigma+m_2N_{\rm D}+1; 1-4\left(\frac{q_0(r_0+1)m_1m_2}{\overline{\gamma}_1\overline{\gamma}_2}\right)\left(b+\frac{q_0m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)^{-2}\right]. \end{aligned} \tag{10}$$

It is important to note that (10) consists of finite summations of the hypergeometric function, ${}_2F_1[a,b,c,z]$ [16, eq. 15.1.1], which is available in commonly used numerical software packages such as Matlab and Mathematica.

2) Asymptotic: To evaluate the asymptotic SER, we begin by deriving the first order expansion of our cdf expression in (9). A key observation is that the first order expansion of the cdf must be evaluated for three separable cases reflecting the fading severities in the source-to-relay link and the relay-to-destination link. In each case, we denote $\overline{\gamma}_2 = \rho\overline{\gamma}_1$ and apply the Maclaurin series expansion for $K_\nu(z)$ from [15, eq. 8.446] as $\overline{\gamma}_1 \to \infty$. Our first order expansion of the cdf is given by

$$F^{\infty}_{\gamma_{\mathrm{TAS/MRC}}}(\gamma) = \Psi\left(\frac{\gamma}{\overline{\gamma}_1}\right)^{N_{\mathrm{R}}\times\min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})} + o\left(\overline{\gamma}_1^{-N_{\mathrm{R}}\times\min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})}\right) \tag{11}$$

where Ψ captures the three cases expressed as

$$\Psi = \begin{cases} \frac{m_1^{m_1 N_{\mathrm{S}} N_{\mathrm{R}}}}{(m_1 N_{\mathrm{R}})!^{N_{\mathrm{S}}}}, & \textit{Case 1: } m_1 N_{\mathrm{S}} < m_2 N_{\mathrm{D}}, \\ \frac{m_2^{m_2 N_{\mathrm{R}} N_{\mathrm{D}}}}{\rho^{m_2 N_{\mathrm{R}} N_{\mathrm{D}}}(m_2 N_{\mathrm{D}})!^{N_{\mathrm{R}}}}, & \textit{Case 2: } m_1 N_{\mathrm{S}} > m_2 N_{\mathrm{D}}, \\ \frac{m_1^{m_1 N_{\mathrm{S}} N_{\mathrm{R}}}}{(m_1 N_{\mathrm{R}})!^{N_{\mathrm{S}}}} + \frac{m_2^{m_2 N_{\mathrm{R}} N_{\mathrm{D}}}}{\rho^{m_2 N_{\mathrm{R}} N_{\mathrm{D}}}(m_2 N_{\mathrm{D}})!^{N_{\mathrm{R}}}}, & \textit{Case 3: } m_1 N_{\mathrm{S}} = m_2 N_{\mathrm{D}}, \end{cases} \tag{12}$$

and $o(\cdot)$ represents the higher order terms (i.e., we write $f(x) = o[g(x)]$ as $x \to x_0$ if $\lim_{x\to x_0}\frac{f(x)}{g(x)} = 0$).

The asymptotic SER of TAS/MRC relaying is obtained in closed-form by substituting (11) into (5) and solving the resultant integral using [15, eq. 3.326.2], which results in

$$P^{\infty}_{\mathrm{TAS/MRC}} = \frac{a\Psi\Gamma\left(\frac{1}{2}+q\right)}{2\sqrt{\pi}}(b\overline{\gamma}_1)^{-q} + o\left(\overline{\gamma}_1^{-q}\right). \tag{13}$$

where $q = N_{\mathrm{R}} \times \min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})$. From (13), we find that TAS/MRC achieves a diversity order of $G_d = N_{\mathrm{R}} \times \min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})$, which represents the slope of the asymptotic SER curve defined as $G_d \triangleq \lim_{\overline{\gamma}_1\to\infty}\frac{-\log P_{\mathrm{TAS/MRC}}}{\log\overline{\gamma}_1}$ [18]. We further derive the array gain from (13) as

$$G_{a,\mathrm{TAS/MRC}} = b\left(\frac{2\sqrt{\pi}}{a\Psi\Gamma\left(\frac{1}{2}+q\right)}\right)^{\frac{1}{N_{\mathrm{R}}\times\min(m_1 N_{\mathrm{S}}, m_2 N_{\mathrm{D}})}}, \tag{14}$$

in which the SNR advantage of the asymptotic SER curve relative to a reference curve of $\overline{\gamma}_1^{-G_d}$ [18] is presented.

B. TAS/SC Relaying

1) Exact: The exact cdf of $\gamma_{\mathrm{TAS/SC}}$ is evaluated according to

$$F_{\gamma_{\mathrm{TAS/SC}}}(\gamma) = 1 - \int_0^\infty \tilde{F}_{\Upsilon_1}\left(\gamma + \frac{\gamma^2}{\omega}\right) f_{\Upsilon_2}(\omega+\gamma)d\omega, \tag{15}$$

where $\tilde{F}_{\Upsilon_1}(\gamma) = 1 - F_{\Upsilon_1}(\gamma)$ is the complementary cdf of Υ_1, $f_{\Upsilon_2}(\cdot)$ is the pdf of Υ_2, and $\omega = \Upsilon_2 - \gamma$.

In the source-to-relay link, the cdf of Υ_1 is given by

$$F_{\Upsilon_1}(\gamma) = 1 - \sum_{k=1}^{N_{\mathrm{S}} N_{\mathrm{R}}}\binom{N_{\mathrm{S}} N_{\mathrm{R}}}{k}(-1)^{k-1}\left(\frac{\Gamma\left(m_1, \gamma\frac{m_1}{\overline{\gamma}_1}\right)}{\Gamma(m_1)}\right)^k, \tag{16}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [16, eq. 6.5.3]. In the relay-to-destination link, the pdf of Υ_2 is given by [19]

$$f_{\Upsilon_2}(\gamma) = N_{\rm R}N_{\rm D}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{m_2}\frac{\gamma^{m_2-1}e^{-\gamma\frac{m_2}{\overline{\gamma}_2}}}{\Gamma\left(m_2\right)}\sum_{l=0}^{N_{\rm R}N_{\rm D}-1}\binom{N_{\rm R}N_{\rm D}-1}{l}(-1)^l\left(\frac{\Gamma\left(m_2,\gamma\frac{m_2}{\overline{\gamma}_2}\right)}{\Gamma\left(m_2\right)}\right)^l. \quad (17)$$

The cdf of $\gamma_{\rm TAS/SC}$ is found by substituting (16) and (17) into (15), invoking [15, eq. (3.324.1)] to expand the incomplete gamma function, and applying the identity in [15, eq. 3.471.9] to solve the resulting integrals which results in

$$\begin{aligned} F_{\gamma_{\rm TAS/SC}}(\gamma) &= 1-\frac{2N_{\rm R}N_{\rm D}}{\Gamma(m_2)}\sum_{q_0=1}^{N_{\rm S}N_{\rm R}}\sum_{r_0=0}^{N_{\rm R}N_{\rm D}-1}\binom{N_{\rm S}N_{\rm R}}{q_0}\binom{N_{\rm R}N_{\rm D}-1}{r_0}(-1)^{q_0+r_0-1}\\ &\times\prod_{k=1}^{m_1-1}\left[\sum_{q_k=0}^{q_{k-1}}\binom{q_{k-1}}{q_k}\left(\frac{1}{k!}\right)^{q_k-q_{k+1}}\right]\prod_{l=1}^{m_2-1}\left[\sum_{r_l=0}^{r_{l-1}}\binom{r_{l-1}}{r_l}\left(\frac{1}{l!}\right)^{r_l-r_{l+1}}\right]\\ &\times\sum_{i=0}^{\sigma}\sum_{j=0}^{\varsigma+m_2-1}\binom{\sigma}{i}\binom{\varsigma+m_2-1}{j}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{\sigma+\frac{j-i+1}{2}}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\varsigma+m_2-\frac{j-i+1}{2}}\\ &\times\left(\frac{q_0}{r_0+1}\right)^{\frac{j-i+1}{2}}\gamma^{\sigma+\varsigma+m_2}e^{-\gamma\left(\frac{q_0m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)}K_{j-i+1}\left(2\gamma\sqrt{\frac{q_0(r_0+1)m_1m_2}{\overline{\gamma}_1\overline{\gamma}_2}}\right). \end{aligned} \quad (18)$$

In (18), we define $\sigma = q_1+q_2+\ldots+q_{m_1-1}$ and $\varsigma = r_1+r_2+\ldots+r_{m_2-1}$.

We derive the exact SER in closed-form by substituting (18) into (5). After applying identities [15, eq. 3.326.2] and [17, eq. (2.16.6.3)] to simplify the integrals and performing some algebraic manipulation, we obtain the exact SER expression given by

$$\begin{aligned} P_{\rm TAS/SC} &= \frac{a}{2}-\frac{a\sqrt{b}N_{\rm R}N_{\rm D}}{\Gamma(m_2)}\sum_{q_0=1}^{N_{\rm S}N_{\rm R}}\sum_{r_0=0}^{N_{\rm R}N_{\rm D}-1}\binom{N_{\rm S}N_{\rm R}}{q_0}\binom{N_{\rm R}N_{\rm D}-1}{r_0}(-1)^{q_0+r_0-1}\\ &\times\prod_{k=1}^{m_1-1}\left[\sum_{q_k=0}^{q_{k-1}}\binom{q_{k-1}}{q_k}\left(\frac{1}{k!}\right)^{q_k-q_{k+1}}\right]\prod_{l=1}^{m_2-1}\left[\sum_{r_l=0}^{r_{l-1}}\binom{r_{l-1}}{r_l}\left(\frac{1}{l!}\right)^{r_l-r_{l+1}}\right]\\ &\times\sum_{i=0}^{\sigma}\sum_{j=0}^{\varsigma+m_2-1}\binom{\sigma}{i}\binom{\varsigma+m_2-1}{j}\left(\frac{m_1}{\overline{\gamma}_1}\right)^{\sigma}\left(\frac{m_2}{\overline{\gamma}_2}\right)^{\varsigma+m_2-j+i-1}\\ &\times\left(b+\frac{q_0m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)^{j-i-\sigma-\varsigma-m_2+\frac{1}{2}}(r_0+1)^{i-j-1}\,2^{i-j-\sigma-\varsigma-m_2-\frac{3}{2}}\\ &\times\frac{\Gamma(\sigma+\varsigma+m_2-\frac{1}{2}-j+i)\Gamma(\sigma+\varsigma+m_2+\frac{3}{2}+j-i)}{\Gamma(\sigma+\varsigma+m_2+1)}\\ &\times{}_2F_1\left[\frac{\sigma+\varsigma+m_2-\frac{1}{2}-j+i}{2},\frac{\sigma+\varsigma+m_2+\frac{1}{2}-j+i}{2};\right.\\ &\left.\sigma+\varsigma+m_2+1;1-4\left(\frac{q_0(r_0+1)m_1m_2}{\overline{\gamma}_1\overline{\gamma}_2}\right)\left(b+\frac{q_0m_1}{\overline{\gamma}_1}+\frac{(r_0+1)m_2}{\overline{\gamma}_2}\right)^{-2}\right]. \end{aligned} \quad (19)$$

2) Asymptotic: Following similar steps to the derivation of (11), we evaluate the first order expansion of the cdf in (18) as $\overline{\gamma}_1 \to \infty$ according to three separable cases. The result is

$$F^{\infty}_{\gamma_{\text{TAS/SC}}}(\gamma) = \Phi\left(\frac{\gamma}{\overline{\gamma}_1}\right)^{N_{\text{R}}\times\min(m_1 N_{\text{S}}, m_2 N_{\text{D}})} + o\left(\overline{\gamma}_1^{-N_{\text{R}}\times\min(m_1 N_{\text{S}}, m_2 N_{\text{D}})}\right). \tag{20}$$

where

$$\Phi = \begin{cases} \frac{m_1^{m_1 N_{\text{S}} N_{\text{R}}}}{(m_1!)^{N_{\text{S}} N_{\text{R}}}}, & \textit{Case 1: } m_1 N_{\text{S}} < m_2 N_{\text{D}}, \\ \frac{m_2^{m_2 N_{\text{R}} N_{\text{D}}}}{\rho^{m_2 N_{\text{R}} N_{\text{D}}}(m_2!)^{N_{\text{R}} N_{\text{D}}}}, & \textit{Case 2: } m_1 N_{\text{S}} > m_2 N_{\text{D}}, \\ \frac{m_1^{m_1 N_{\text{S}} N_{\text{R}}}}{(m_1!)^{N_{\text{S}} N_{\text{R}}}} + \frac{m_2^{m_2 N_{\text{R}} N_{\text{D}}}}{\rho^{m_2 N_{\text{R}} N_{\text{D}}}(m_2!)^{N_{\text{R}} N_{\text{D}}}}, & \textit{Case 3: } m_1 N_{\text{S}} = m_2 N_{\text{D}}. \end{cases} \tag{21}$$

Inserting (20) into (5) and using [15, eq. 3.326.2] to solve the resultant integral, we derive the asymptotic SER of TAS/SC relaying in closed-form as

$$P^{\infty}_{\text{TAS/SC}} = \frac{a\Phi\Gamma\left(\frac{1}{2}+q\right)}{2\sqrt{\pi}}(b\overline{\gamma}_1)^{-q} + o\left(\overline{\gamma}_1^{-q}\right) \tag{22}$$

where $q = N_{\text{R}} \times \min(m_1 N_{\text{S}}, m_2 N_{\text{D}})$. As indicated in (22), TAS/SC relaying retains a diversity order of $G_d = N_{\text{R}} \times \min(m_1 N_{\text{S}}, m_2 N_{\text{D}})$. This diversity order is the same as that of TAS/MRC relaying. From (22), the array gain of TAS/SC relaying is given by

$$G_{a,\text{TAS/SC}} = b\left(\frac{2\sqrt{\pi}}{a\Phi\Gamma\left(\frac{1}{2}+q\right)}\right)^{\frac{1}{N_{\text{R}}\times\min(m_1 N_{\text{S}}, m_2 N_{\text{D}})}}. \tag{23}$$

Comparing (14) and (23), it is clear that TAS/MRC relaying and TAS/SC relaying exhibit different array gains.

C. Comparison of TAS/MRC Relaying and TAS/SC Relaying

Noting that TAS/MRC relaying and TAS/SC relaying offer the same diversity order, we focus on their respective array gains. The SNR gap between TAS/MRC relaying and TAS/SC relaying is defined as the ratio of the array gains of TAS/MRC relaying in (14) and TAS/SC relaying in (23). Using (12) and (21) and performing some mathematical manipulations, the ratio is given as follows

$$\frac{G_{a,\text{TAS/MRC}}}{G_{a,\text{TAS/SC}}} = \begin{cases} \left(\frac{(m_1 N_{\text{R}})!^{N_{\text{S}}}}{(m_1!)^{N_{\text{S}} N_{\text{R}}}}\right)^{\frac{1}{m_1 N_{\text{S}} N_{\text{R}}}}, & \textit{Case 1: } m_1 N_{\text{S}} < m_2 N_{\text{D}} \\ \left(\frac{(m_2 N_{\text{D}})!^{N_{\text{R}}}}{(m_2!)^{N_{\text{D}} N_{\text{R}}}}\right)^{\frac{1}{m_2 N_{\text{D}} N_{\text{R}}}}, & \textit{Case 2: } m_1 N_{\text{S}} > m_2 N_{\text{D}} \\ \left(\frac{(m_1!)^{-N_{\text{S}} N_{\text{R}}} + (m_2!)^{-N_{\text{R}} N_{\text{D}}}(m_2/(m_1\rho))^{m_1 N_{\text{R}} N_{\text{S}}}}{((m_1 N_{\text{R}})!)^{-N_{\text{S}}} + ((m_1 N_{\text{S}})!)^{-N_{\text{R}}}(m_2/(m_1\rho))^{m_1 N_{\text{R}} N_{\text{S}}}}\right)^{\frac{1}{m_1 N_{\text{R}} N_{\text{S}}}}, & \textit{Case 3: } m_1 N_{\text{S}} = m_2 N_{\text{D}}. \end{cases} \tag{24}$$

IV. Optimal Power Allocation

Power allocation is an important design criterion which allows network service providers to maximize performance without squandering valuable resources. In this section, we apply our new asymptotic solutions to efficiently derive the optimal power allocation that minimizes the SER. To this end, we consider a general weight for the power allocation factor η such that $\mathcal{E}_{\mathrm{S}} = \eta\mathcal{E}_{\mathrm{T}}$ and $\mathcal{E}_{\mathrm{R}} = (1-\eta)\,\mathcal{E}_{\mathrm{T}}$, where $0 < \eta < 1$ and $\mathcal{E}_{\mathrm{T}}$ is the total power constraint.

A. TAS/MRC Relaying

To evaluate the optimal power allocation, we consider both $\mathcal{E}_{\mathrm{S}}$ and $\mathcal{E}_{\mathrm{R}}$ by taking into account the joint contributions of the source-to-relay link and the relay-to-destination link. The objective function for the optimal power allocation that minimizes the SER is derived based on (13). As such, it is given by

$$P^{\infty}_{\mathrm{TAS/MRC}} = \frac{a}{2\sqrt{\pi}}\left(\frac{(m_1\mathcal{N}_0)^{m_1N_{\mathrm{S}}N_{\mathrm{R}}}\,\Gamma\left(\frac{1}{2}+m_1N_{\mathrm{S}}N_{\mathrm{R}}\right)}{(\eta\mathcal{E}_{\mathrm{T}}b)^{m_1N_{\mathrm{S}}N_{\mathrm{R}}}\,(m_1N_{\mathrm{R}})!^{N_{\mathrm{S}}}} + \frac{(m_2\mathcal{N}_0)^{m_2N_{\mathrm{R}}N_{\mathrm{D}}}\,\Gamma\left(\frac{1}{2}+m_2N_{\mathrm{R}}N_{\mathrm{D}}\right)}{((1-\eta)\,\mathcal{E}_{\mathrm{T}}b)^{m_2N_{\mathrm{R}}N_{\mathrm{D}}}\,(m_2N_{\mathrm{D}})!^{N_{\mathrm{R}}}}\right), \quad (25)$$

which is formulated by examining all three cases of Ψ in (12). Indeed, (25) can be viewed as the first order expansion of the SER for *Case 3:* $m_1N_S = m_2N_D$, or the higher order expansion of the SER for *Case 1:* $m_1N_S < m_2N_D$ and *Case 2:* $m_1N_S > m_2N_D$.

The optimal power allocation is determined by solving $\partial P^{\infty}_{\mathrm{TAS/MRC}}/\partial\eta = 0$. After some mathematical manipulations, we find that the optimal power allocation is the value of η which satisfies

$$\frac{(1-\eta)^{m_2N_{\mathrm{R}}N_{\mathrm{D}}+1}}{\eta^{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}}\left(\frac{\mathcal{E}_{\mathrm{T}}b}{\mathcal{N}_0}\right)^{m_2N_{\mathrm{R}}N_{\mathrm{D}}-m_1N_{\mathrm{S}}N_{\mathrm{R}}} = \frac{m_2^{m_2N_{\mathrm{R}}N_{\mathrm{D}}+1}N_{\mathrm{D}}\Gamma\left(\frac{1}{2}+m_2N_{\mathrm{R}}N_{\mathrm{D}}\right)(m_1N_{\mathrm{R}})!^{N_{\mathrm{S}}}}{m_1^{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}N_{\mathrm{S}}\Gamma\left(\frac{1}{2}+m_1N_{\mathrm{S}}N_{\mathrm{R}}\right)(m_2N_{\mathrm{D}})!^{N_{\mathrm{R}}}}. \quad (26)$$

For the special case of $m_1N_{\mathrm{S}} = m_2N_{\mathrm{D}}$, (26) reduces to

$$\eta = \left(1 + \frac{m_2^{\frac{m_1N_{\mathrm{S}}N_{\mathrm{R}}}{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}}(m_1N_{\mathrm{R}})!^{\frac{N_{\mathrm{S}}}{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}}}{m_1^{\frac{m_1N_{\mathrm{S}}N_{\mathrm{R}}}{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}}(m_2N_{\mathrm{D}})!^{\frac{N_{\mathrm{R}}}{m_1N_{\mathrm{S}}N_{\mathrm{R}}+1}}}\right)^{-1}. \quad (27)$$

Considering Rayleigh fading (i.e., $m_1 = m_2 = 1$) and equal number of antennas at the source, relay and destination (i.e., $N_{\mathrm{S}} = N_{\mathrm{R}} = N_{\mathrm{D}}$), (27) reduces to $\eta = 0.5$.

B. TAS/SC Relaying

Following the same procedure as outlined in the previous subsection, we optimize both $\mathcal{E}_{\mathrm{S}}$ and $\mathcal{E}_{\mathrm{R}}$ by jointly considering the source-to-relay link and the relay-to-destination link. The objective function for the optimal power allocation that minimizes the SER is derived based on (22). The objective function is given by

$$P^{\infty}_{\text{TAS/SC}} = \frac{a}{2\sqrt{\pi}} \left(\frac{(m_1\mathcal{N}_0)^{m_1 N_{\text{S}} N_{\text{R}}} \Gamma\left(\frac{1}{2} + m_1 N_{\text{S}} N_{\text{R}}\right)}{(\eta \mathcal{E}_{\text{T}} b)^{m_1 N_{\text{S}} N_{\text{R}}} (m_1)!^{N_{\text{S}} N_{\text{R}}}} + \frac{(m_2\mathcal{N}_0)^{m_2 N_{\text{R}} N_{\text{D}}} \Gamma\left(\frac{1}{2} + m_2 N_{\text{R}} N_{\text{D}}\right)}{((1-\eta)\,\mathcal{E}_{\text{T}} b)^{m_2 N_{\text{R}} N_{\text{D}}} (m_2)!^{N_{\text{R}} N_{\text{D}}}} \right), \quad (28)$$

which is formulated by examining all three cases of Φ in (21). Solving $\partial P^{\infty}_{\text{TAS/SC}}/\partial \eta = 0$, the optimal power allocation is the value of η which satisfies

$$\frac{(1-\eta)^{m_2 N_{\text{R}} N_{\text{D}}+1}}{\eta^{m_1 N_{\text{S}} N_{\text{R}}+1}} \left(\frac{\mathcal{E}_{\text{T}} b}{\mathcal{N}_0}\right)^{m_2 N_{\text{R}} N_{\text{D}} - m_1 N_{\text{S}} N_{\text{R}}} = \frac{m_2^{m_2 N_{\text{R}} N_{\text{D}}+1} N_{\text{D}} \Gamma\left(\frac{1}{2} + m_2 N_{\text{R}} N_{\text{D}}\right) (m_1)!^{N_{\text{S}} N_{\text{R}}}}{m_1^{m_1 N_{\text{S}} N_{\text{R}}+1} N_{\text{S}} \Gamma\left(\frac{1}{2} + m_1 N_{\text{S}} N_{\text{R}}\right) (m_2)!^{N_{\text{R}} N_{\text{D}}}} \quad (29)$$

For the special case of $m_1 N_{\text{S}} = m_2 N_{\text{D}}$, (29) reduces to

$$\eta = \left(1 + \frac{m_2^{\frac{m_2 N_{\text{R}} N_{\text{D}}}{m_1 N_{\text{S}} N_{\text{R}}+1}} (m_1)!^{\frac{N_{\text{S}} N_{\text{R}}}{m_1 N_{\text{S}} N_{\text{R}}+1}}}{m_1^{\frac{m_1 N_{\text{S}} N_{\text{R}}}{m_1 N_{\text{S}} N_{\text{R}}+1}} (m_2)!^{\frac{N_{\text{R}} N_{\text{D}}}{m_1 N_{\text{S}} N_{\text{R}}+1}}}\right)^{-1}. \quad (30)$$

For Rayleigh fading (i.e., $m_1 = m_2 = 1$) and equal number of antennas at the source and the destination (i.e., $N_{\text{S}} = N_{\text{D}}$), (30) reduces to $\eta = 0.5$, which is independent of N_{R}.

V. Results and Discussions

In this section, we highlight the impacts of the number of antennas and the fading parameters on the performance gap between TAS/MRC relaying and TAS/SC relaying. In the examples, '•' marks the Monte Carlo simulations for TAS/MRC relaying and '○' marks the Monte Carlo simulations for TAS/SC relaying. For both protocols, we consider CNB relaying ($\tau = 1$) in the Monte Carlo simulations. The solid curves represent the exact SER in (10) and (19). The dashed lines represent the asymptotic SER in (13) and (22). We see an excellent agreement between the simulations and the exact analytical curves in the medium to high SNR regime. Furthermore, the asymptotic lines accurately predict the diversity orders and array gains of both protocols.

Figs. 2, 3, and 4 consider BPSK modulation and plot the SER (which is equivalent to the BER for BPSK) of TAS/MRC relaying and TAS/SC relaying with equal average energies at the source and the relay (i.e., $\mathcal{E}_{\text{S}} = \mathcal{E}_{\text{R}}$). We address the following three cases based on $m_1 N_{\text{S}}$ and $m_2 N_{\text{D}}$.

1) Case 1: $m_1 N_S < m_2 N_D$. In this case, the relay-to-destination link is stronger than the source-to-relay link. In Fig. 2, we set $m_1 = 1$, $N_{\text{S}} = 1$, $m_2 = 2$, and $N_{\text{D}} = 3$. Given $G_d = N_{\text{R}} \times \min(m_1 N_{\text{S}}, m_2 N_{\text{D}})$, we identify that the diversity order, which is reflected in the slope of the asymptotes, is directly proportional to N_R. As such, we see that the slope of the asymptotes increases according to $G_d = 1$, 2, and 3 for $N_{\text{R}} = 1$, 2, and 3, respectively. We also see that for the same diversity order, the SNR gap between the two protocols increases with increasing N_R. Using (24), this SNR gap is conveniently calculated as $G_{a,\text{TAS/MRC}}/G_{a,\text{TAS/SC}} = 0$ dB, 1.5051 dB, and 2.5938 dB for $N_R = 1$, 2, and 3, respectively.

2) Case 2: $m_1 N_S > m_2 N_D$. In this case, the source-to-relay link is stronger than the relay-to-destination link. In Fig. 3, we set $m_1 = 2$, $N_{\text{S}} = 3$, $m_2 = 1$, and $N_{\text{D}} = 1$. We observe from the figure that for the same diversity order, there is no SNR gap between the two protocols at high SNRs. This observation is confirmed using (24).

3) Case 3: $m_1 N_S = m_2 N_D$. In this case, the source-to-relay link and the relay-to-destination link are in equilibrium. In Fig. 4, we set $m_1 = 1$, $N_{\text{S}} = 2$, $m_2 = 1$, and $N_{\text{D}} = 2$. Using (24), the SNR gap between the two protocols is $G_{a,\text{TAS/MRC}}/G_{a,\text{TAS/SC}} = 0.6247$ dB, 1.5051 dB, and 1.8616 dB for $N_R =$ 1, 2, and 3, respectively.

Fig. 5 plots the SNR gap between TAS/MRC relaying and TAS/SC relaying using (24). The figure highlights the impact of m_1 and m_2 on the SNR gap. The source-to-relay link and relay-to-destination link are in equilibrium with $N_S = N_R = N_D$ and $m_1 = m_2$. Setting $N_S = N_R = N_D = 2$ and $m_1 = m_2 = 1$, provides a SNR gap of $G_{a,\text{TAS/MRC}}/G_{a,\text{TAS/SC}} = 1.5051$ dB. We find that this gap increases with increasing m_1 and m_2. For example, setting $N_S = N_R = N_D = 2$ and $m_1 = m_2 = 3$, results in a larger SNR gap of $G_{a,\text{TAS/MRC}}/G_{a,\text{TAS/SC}} = 2.1684$ dB.

Fig. 6 plots the SER of TAS/MRC relaying and TAS/SC relaying with BPSK modulation. The figure examines the impact of η on the SER, where we set $\mathcal{E}_{\text{T}} = 20$ dB, $m_1 = 2$, $N_{\text{S}} = 2$, $N_{\text{R}} = 2$, $m_2 = 1$, and $N_{\text{D}} = 4$. Based on (27), the SER for TAS/MRC relaying is minimized at $\eta = 0.65$. As such, optimal power allocation is achieved by assigning 65% of the total transmit power to the source and 35% to the relay. Based on (30), the SER for TAS/SC relaying is minimized at $\eta = 0.58$. The values of η for optimal power allocation are confirmed in Fig. 6.

VI. Conclusion

This letter provides a SER performance comparison between TAS/MRC and TAS/SC relaying. To facilitate this, we derived new exact and asymptotic expressions for the SER in Nakagami-m fading. Based on the asymptotic expressions, we showed that both protocols offer the same diversity order of $N_{\text{R}} \times \min(m_1 N_{\text{S}}, m_2 N_{\text{D}})$. This means that the SNR gap between the two protocols is entirely dependent on the array gain. Motivated by this, we analyzed the SNR gap as a simple ratio of their respective array gains. We then examined the fundamental question of how to allocate the total transmit power between the source and the relay such that the SER is minimized. In addressing this question, we derived a simple and practical design rule for the power allocation factor of the two protocols.

REFERENCES

[1] D. Soldani and S. Dixit, "Wireless relays for broadband access," *IEEE Commun. Mag.*, pp. 58–66, Mar. 2008.

[2] S. Berger, M. Kuhn, and A. Wittneben, "Recent advances in amplify-and-forward two-hop relaying," *IEEE Commun. Mag.*, pp. 50–56, Jul. 2009.

[3] Q. Li, G. Li, W. Lee, M.-I. Lee, D. Mazzarese, B. Clerckx, and Z. Li, "MIMO techniques in WiMAX and LTE: A future overview," *IEEE Commun. Mag.*, pp. 86–92, May 2010.

[4] Z. Chen, J. Yuan, and B. Vucetic, "Analysis of transmit antenna selection/maximal-ratio combining in Rayleigh fading channels," *IEEE Trans. Veh. Technol.*, vol. 54, pp. 1312–1321, Jul. 2005.

[5] B. Wang, J. Zhang, and A. H.-Madsen, "On the capacity of MIMO relay channels," vol. 51, pp. 29–43, Jan. 2005.

[6] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in MIMO relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 1433–1444, Jun. 2006.

[7] Y. Fan and J. Thompson, "MIMO configurations for relay channels: Theory and practice," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1774–1786, May 2007.

[8] H. A. Suraweera, P. J. Smith, A. Nallanathan, and J. S. Thompson, "Amplify-and-forward relaying with optimal and suboptimal transmit antenna selection," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 1874–1885, Jun. 2011.

[9] C.-K. Wen, K.-K. Wong, and C. T. K. Ng, "On the asymptotic properties of amplify-and-forward MIMO relay channels," *IEEE Trans. Commun.*, vol. 59, pp. 590–602, Feb. 2011.

[10] Y. Song, H. Shin, and E.-K. Hong, "A cooperative beamforming scheme in MIMO relay broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 940–947, Mar. 2011.

[11] P. L. Yeoh, M. Elkashlan, and I. B. Collings, "Exact and asymptotic SER of distributed TAS/MRC in MIMO relay networks," *IEEE Trans. Wireless Commun.*, vol. 10, pp. 751–756, Mar. 2011.

[12] H. A. Suraweera, G. K. Karagiannidis, Y. Li, H. K. Garg, A. Nallanathan, and B. Vucetic, "Amplify-and-forward relay transmission with end-to-end antenna selection," in *Proc. IEEE Wireless Commun. and Netw. Conf. (WCNC)*, Sydney, Australia, Apr. 2010, pp. 1–6.

[13] C. T. G. Amarasuriya and M. Ardakani, "Performance analysis framework for transmit antenna selection strategies of cooperative MIMO AF relay networks," *IEEE Trans. Veh. Technol.*, vol. 60, pp. 3030–3044, Sep. 2011.

[14] T. A. Tsiftsis, G. K. Karagiannidis, P. T. Mathiopoulos, and S. A. Kotsopoulos, "Nonregenerative dual-hop cooperative links with selection diversity," *EURASIP J. Wireless Commun. Netw.*, vol. 2006, article ID 17862, 8 pages, 2006.

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 6th ed. New York, NY, USA: Academic Press, 2000.

[16] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, 10th ed. New York, NY, USA: Dover, 1972.

[17] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series Volume 2: Special Functions*, 1st ed. New York, NY, USA: Gordon and Breach Science Publishers, 1986.

[18] Z. Wang and G. B. Giannakis, "A simple and general parameterization quantifying performance in fading channels," *IEEE Trans. Commun.*, vol. 51, pp. 1389–1398, Aug. 2003.

[19] T. Gucluoglu and T. M. Duman, "Performance analysis of transmit and receive antenna selection over flat fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, pp. 3056–3065, Aug. 2008.

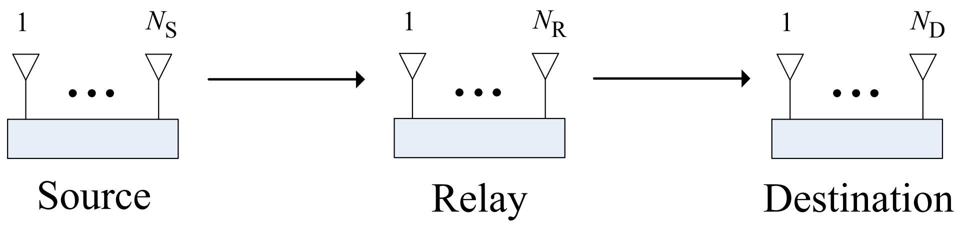

Fig. 1. MIMO relaying with N_{S}, N_{R}, and N_{D} antennas.

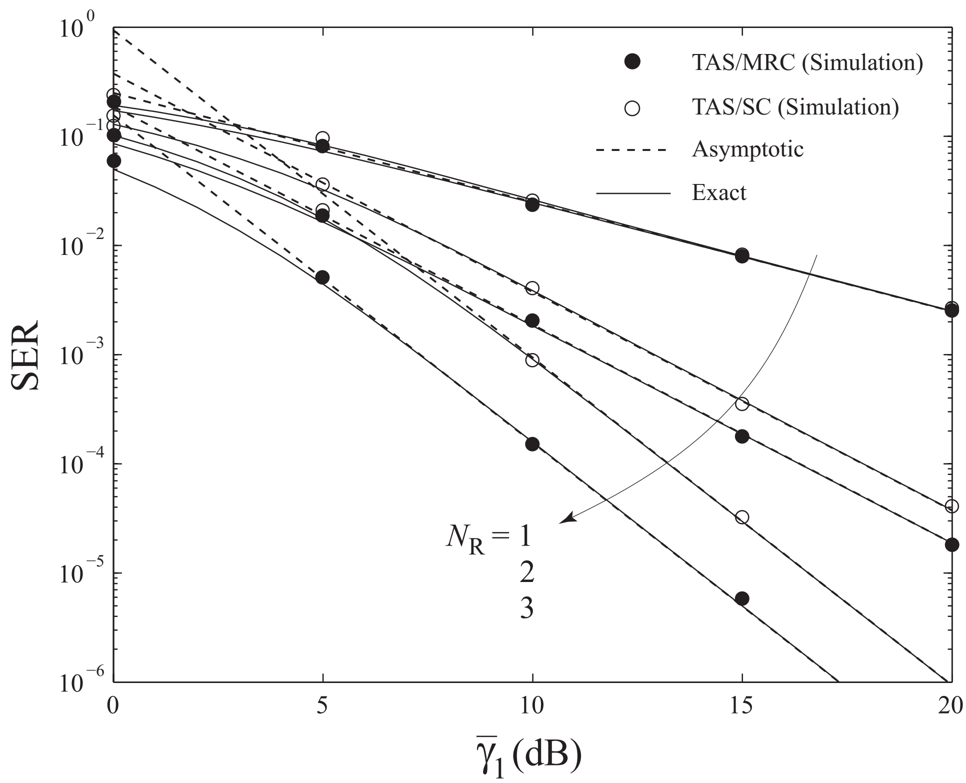

Fig. 2. SER of TAS/MRC relaying and TAS/SC relaying. *Case* 1 : Relay-to-destination is the stronger link. We set $m_1 = 1$, $N_{\mathrm{S}} = 1$, $m_2 = 2$, and $N_{\mathrm{D}} = 3$.

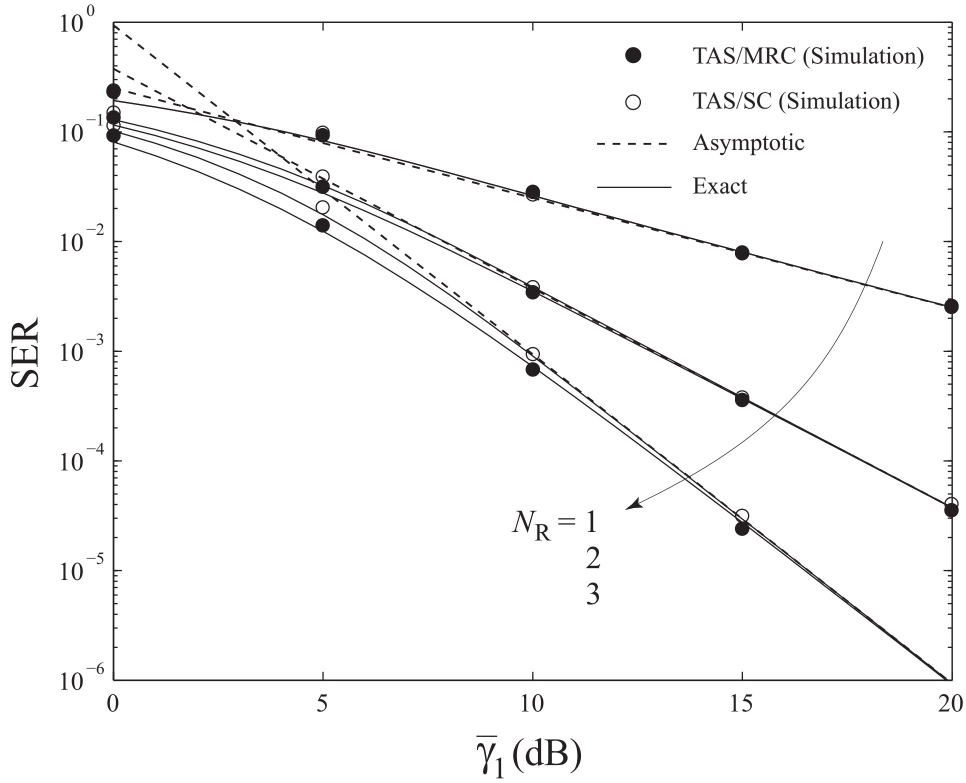

Fig. 3. SER of TAS/MRC relaying and TAS/SC relaying. $Case\ 2$: Source-to-relay is the stronger link. We set $m_1 = 2$, $N_S = 3$, $m_2 = 1$, and $N_D = 1$.

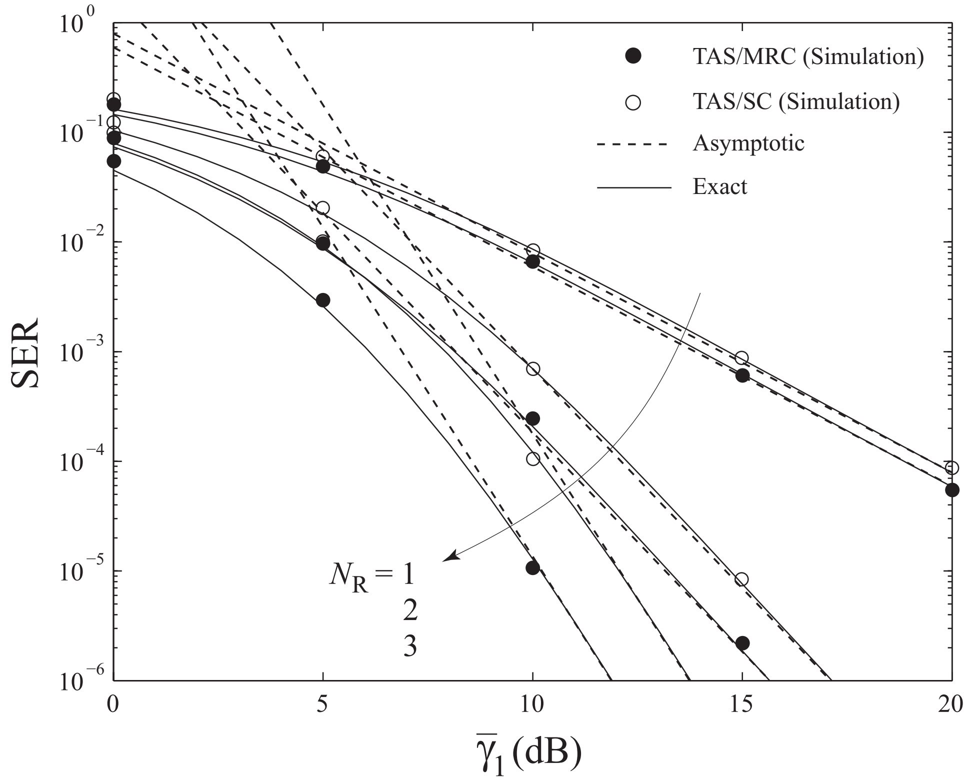

Fig. 4. SER of TAS/MRC relaying and TAS/SC relaying. $Case\ 3$: Source-to-relay link and relay-to-destination link are in equilibrium. We set $m_1 = 1$, $N_\mathrm{S} = 2$, $m_2 = 1$, and $N_\mathrm{D} = 2$.

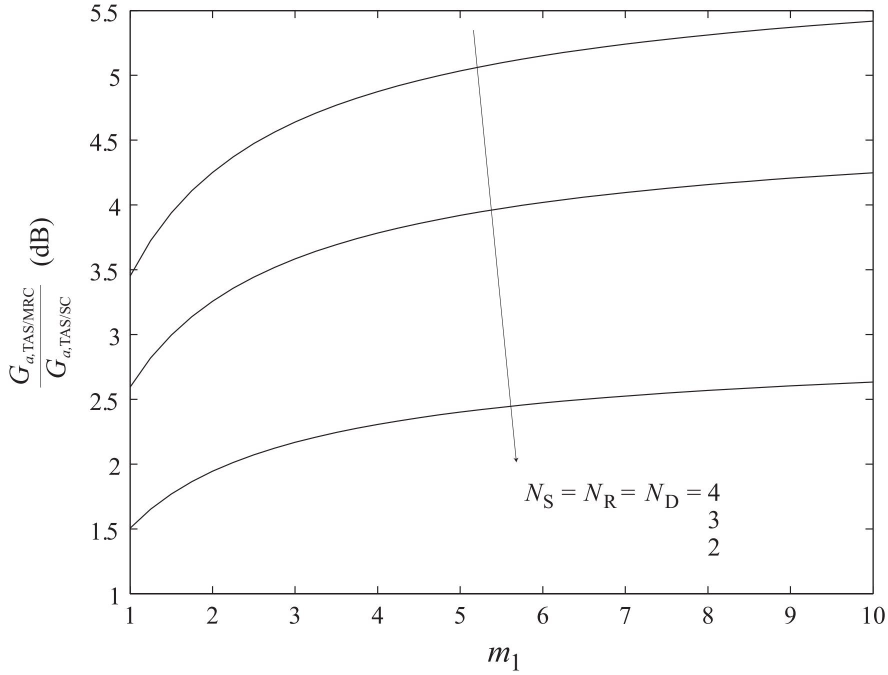

Fig. 5. Ratio of array gains of TAS/MRC relaying and TAS/SC relaying. The source-to-relay link and relay-to-destination link are in equilibrium with $N_S = N_R = N_D$ and $m_1 = m_2$.

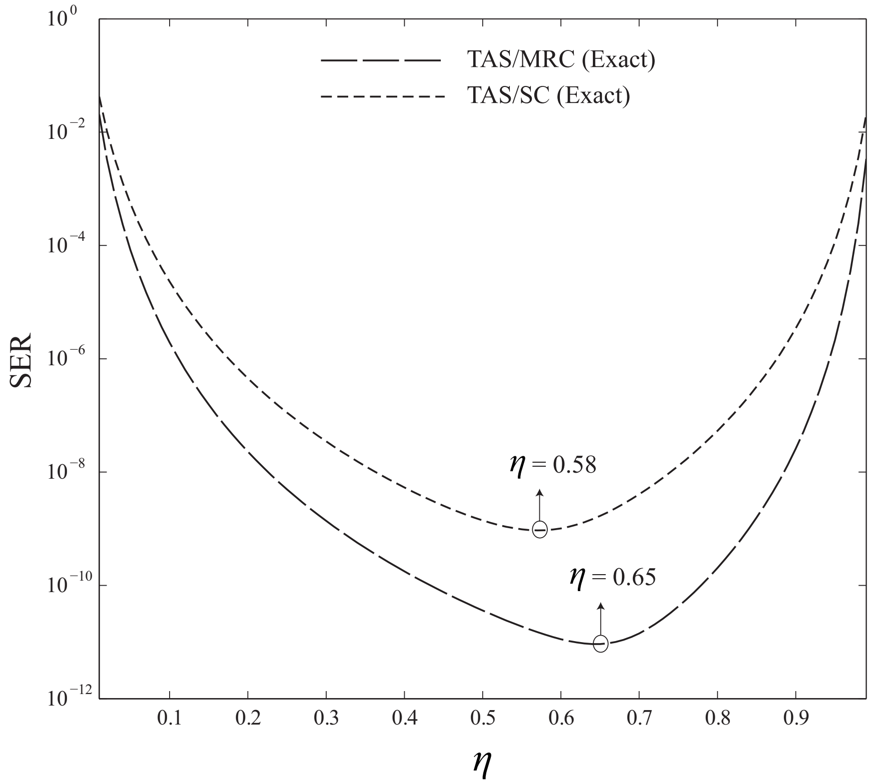

Fig. 6. SER of TAS/MRC relaying and TAS/SC relaying. The source-to-relay link and relay-to-destination link are in equilibrium with $\mathcal{E}_{\rm T} = 20$ dB, $m_1 = 2$, $N_{\rm S} = 2$, $N_{\rm R} = 2$, $m_2 = 1$, and $N_{\rm D} = 4$.