LETTER

Exact Outage Probability of Cognitive Underlay DF Relay Networks with Best Relay Selection

Vo Nguyen Quoc BAO†† and Trung Q. DUONG††, Members

SUMMARY In this letter, we address the performance analysis of underlay selective decode-and-forward (DF) relay networks in Rayleigh fading channels with non-necessarily identical fading parameters. Recently, many research works have investigated the performance of cognitive relaying networks under interference constraint (see, e.g. [1–4]). In particular, the authors of [1] derived the exact closed-form expression outage probability (OP) for two hop amplify-and-forward (AF) relaying networks. Assuming no direct link between source and destination, closed-form solutions for outage probability has been obtained for dual hop DF relaying networks under Nakagami-m fading channels [2]. By enabling only secondary relays, which satisfy the interference constraint, in the forwarding phase, Hussain et al. proposed a new best relay selection scheme for a cognitive network operating near a primary user [3]. Taking into account both the transmit power limit and the peak interference power constraint, Zhi Yan et al. provided the outage performance analysis of relay assisted hybrid overlay/underlay cognitive radio system over Rayleigh fading channels [4]. However, the performance study on the cognitive underlay DF relay networks with best relay selection has been limited so far. In particular, the exact expression for the end-to-end outage probability has not been explicitly given yet in Rayleigh fading channels. As stated in [5], the exact outage probability of such networks is generally very difficult to derive when conventional derivation approaches, i.e., employing the independence among channels, can not be used due to the interdependence of relaying links. To address this concern, several bounds for the outage probability have been provided in [5–7]. In particular, in [6], Guo et al. first derived the outage performance of underlay relay networks in Rayleigh fading channels. Subsequently, Lee et al. in [7] extended the work of Guo by considering the maximum transmission power of each node. However, both papers did not take into account the interdependence among first-hop links inflicted by the transmit power constraint. Very recently, Luo et al. [8] considered the dependence among relaying links and then obtained an lower bound on the outage probability. However, this lower bound is valid only for independent and identically distributed (i.i.d.) Rayleigh fading channels and more importantly it becomes very loose when the number of relays is large.

To the best of our knowledge, no exact closed-form expression for outage probability of DF networks under interference constraints has been reported in the literature. At the destination, the receiver can employ a variety of diversity combining techniques, e.g. maximal ratio combining (MRC) [8], equal-gain combining (EGC) [9], selection combining (SC) [10], to obtain spatial diversity from signal replicas, which are sent by relays and the source. Although optimum performance is highly desirable, practical wireless systems often sacrifice some performance in order to reduce their complexity. Furthermore, underlay spectrum sharing systems must operate in the low signal-to-noise ratio (SNR) regime to avoid causing any harmful interference on the primary networks [11] resulting in the fact that the advantage of including the direct link with maximal ratio combining is reduced by channel estimation errors, prevalent at low signal levels. As such, a selection diversity approach, which includes the direct link in the selection set, can have advantages and is a reasonable choice.

In this letter, by treating the dual-hop link as an virtually equivalent link accounting for both the possible outage of the first-hop link and the fading on the second-hop link, we for the first time derive an exact closed-form expression of outage probability for cognitive DF relay networks under interference constraints. This expression is applicable to all operating SNRs, valid for independent but not identically distributed (i.n.d.) Rayleigh fading channels and includes the i.i.d. channels as a special case.

1. Introduction

Recently, many research works have investigated the performance of cognitive relaying networks under interference constraint (see, e.g. [1–4]). In particular, the authors of [1] derived the exact closed-form expression outage probability (OP) for two hop amplify-and-forward (AF) relaying networks. Assum-
primary receiver. The data communication in this system consists of two phases: broadcasting phase and forwarding phase. In the broadcasting phase, the source transmits its signals to all the secondary relays and the secondary destination. Under DF, the best relay among successfully decoded relays, which has the best channel towards the destination, is selected in the forwarding phase. The other relays, which unsuccessfully decode the source message, will keep idle. Equipped with SC, the destination only selects the best signal out of two replicas for further processing and neglects the remaining one. This reduces the computational costs and may even lead to a better performance than MRC. Let $C$ be the set of successfully decoded relays, the instantaneous SNR at the destination is given by

$$
\gamma_{e2e} = \max(\gamma_0, \gamma_{2,k}),
$$

(1)

where $\gamma_0$ and $\gamma_{2,k} = \max_{k=1,...,K_0}$ denote the effective instantaneous SNRs of the direct link and from the best relay to the destination, respectively. Under the interference power constraint at the primary receiver, $I_p$, the transmit powers of the source and the $k$-th relay are limited at $P_s = I_p/|h_{sp}|^2$ and $P_k = I_p/|h_{sp}|^2$, respectively. Here, we adopt the general notation, $h_{uv}$, to denote the channel coefficients between node $u$ and node $v$ with $u \in \{s, r_k\}$ and $v \in \{s, d, p\}$. Under Rayleigh fading channels, the instantaneous channel gain, $|h_{uv}|^2$, is an exponential random variable with parameter $\lambda_{uv}$. Additionally, we can write the instantaneous SNRs of the links from the source to the destination, from the source to the $k$-th relay, and from the $k$-th relay to the destination as

$$
\gamma_0 = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}, \quad \gamma_{1,k} = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}, \quad \text{and} \quad \gamma_{2,k} = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2},
$$

respectively, where $N_0$ is the variance of the additive white Gaussian noise at all receivers.

3. Performance Analysis

The outage probability is one of the most commonly used performance metrics in wireless systems. Let $R$ be a predetermined requirement of data rate, the system outage probability is mathematically defined as

$$
\operatorname{OP} = \Pr\left(\frac{1}{2} \log_2 (1 + \gamma_{e2e}) < R\right) = \Pr(\gamma_{e2e} < \gamma_{th}),
$$

(2)

where $\gamma_{th} = 2^{2R} - 1$. To characterize the OP, we first need to derive the cumulative density function (CDF) of $\gamma_{e2e}$. With decode-and-forward relaying, to include all possible combinations of correct and erroneous decoding at the relays for which the end-to-end transmission is outage, in this letter we consider the system as effectively having $N + 1$ links between the source and destination [12]. With SC employed at the destination, this system can be thought of as a virtual SC scheme, where the input branches are the direct link and $N$ relaying links. To account for both the correctly and incorrectly decoded capability at the $k$-relay, let us denote $\gamma_k$ as the equivalent instantaneous SNR of the $k$-th cascaded link. Here, $\gamma_k$ implicitly includes both $\gamma_{1,k}$ and $\gamma_{2,k}$.

This type of analytical approach has been widely adopted in the performance analysis for DF relay (see, e.g., [12][14]), resulting in the probability density function (PDF) of $\gamma_k$ as [14]

$$
f_{\gamma_k}(\gamma|\gamma_{sp}) = \Theta_k \delta(0) + (1 - \Theta_k)f_{\gamma_{s,k}}(\gamma),
$$

(3)

where $\Theta_k$ represents the probability that the $k$-th relay is not included in the decoding set $C$ and $\delta(\cdot)$ is the dirac delta function. Mathematically, $\Theta_k = F_{\gamma_k}(\gamma_{sp})$ where $F_{\gamma_{s,k}}(\gamma|\gamma_{sp})$ is derived as follows:

$$
F_{\gamma_{s,k}}(\gamma|\gamma_{sp}) = \Pr\left(\frac{I_p |h_{sd}|^2}{N_0 |h_{sd}|^2} < \gamma\right) = \Pr\left(\frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2} < \gamma\right) = 1 - \exp\left(-\frac{\gamma}{\sigma_{\alpha_k}}\right),
$$

(4)

Furthermore, the PDF of $\gamma_{2,k}$ is $f_{\gamma_{2,k}}(\gamma) = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}$, $f_{\gamma_{2,k}}(\gamma)$, can be given by [15], p. 186

$$
f_{\gamma_{s,k}}(\gamma) = \int_0^\infty \frac{\gamma}{\sigma_{\alpha_k}} f_{\gamma_{s,k}}\left(\frac{\sigma_{\alpha_k}}{\gamma}\right) f_{\gamma_{sp}}(\gamma)d\gamma
$$

$$
= \frac{\alpha_{\gamma_k}}{(\gamma + \alpha_{\gamma_k})^2},
$$

(5)

where $\alpha_{\gamma_k} = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}$. The cumulative distribution function (CDF) of $\gamma_{2,k}$ is easily obtained by integrating $f_{\gamma_{2,k}}(\gamma)$ from 0 to $\gamma$

$$
F_{\gamma_{s,k}}(\gamma) = \int_0^\gamma f_{\gamma_{s,k}}(\gamma)d\gamma = \frac{\gamma}{\gamma + \alpha_{\gamma_k}}.
$$

(6)

From (3), the conditional CDF of $\gamma_k$ is derived as

$$
F_{\gamma_k}(\gamma|\gamma_{sp}) = \Theta_k + (1 - \Theta_k)F_{\gamma_{2,k}}(\gamma)
$$

$$
= 1 - \frac{\alpha_{\gamma_k}}{\gamma + \alpha_{\gamma_k}} \exp\left(-\frac{\gamma_{sp}}{\sigma_{\alpha_k}}\right),
$$

(7)

It is important to note that although $h_{kd}$, is independent, $\gamma_k$ random variables, for $k = 1, 2, \ldots, [C]$, are dependent, i.e. they have the common term of $\gamma_{sp}$ as can be seen from (7). Therefore, using the law of conditional probability, the CDF of $\gamma_{e2e}$ can be written as (8) as shown in the top of the next page. With the current form of (8), it is very difficult to proceed further. To get around this difficulty, we use the mathematical relationship as in (9) (see Appendix A) thus (9) can be rewritten as (10). Finally, carrying out the integrations and evaluating the result at $\gamma_{sp}$ produces the desired result, which is given by (11), where $\alpha_{\gamma_{1,k}} = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}$ and $\alpha_0 = \frac{I_p |h_{sp}|^2}{N_0 |h_{sp}|^2}.

For i.i.d. case, i.e., $\alpha_0 = \alpha_{1,k} = \alpha_{2,k} = \alpha$, the end-to-end
from one to four, the outage performance improves and the  

\[
F_{\gamma_{c2}}(\gamma) = \Pr (\gamma_{c2} < \gamma) = \int_0^\infty \left[ 1 - \exp \left( -\frac{\gamma \gamma_{dp}}{\gamma_{sp1} \lambda_{dp}} \right) \right] \prod_{k=1}^{N} \left( 1 - \frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) \right) \frac{1}{\lambda_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) d\gamma_{dp}
\]

(8)

\[
\prod_{k=1}^{N} \left( 1 - \frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) \right) = 1 - \sum_{k=1}^{N} (-1)^{k-1} \sum_{n_1 \leq \cdot \cdot \cdot n_k < n_0} \prod_{p=1}^{k} \frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) \frac{1}{\lambda_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) d\gamma_{dp}
\]

(9)

\[
F_{\gamma_{c2}}(\gamma) = \int_0^\infty \left[ 1 - \exp \left( -\frac{\gamma_{dp}}{\gamma_{sp1} \lambda_{dp}} \right) \right] \prod_{k=1}^{N} \left( 1 - \frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) \right) \frac{1}{\lambda_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) d\gamma_{dp} + \int_0^\infty \frac{1}{\lambda_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma_{sp1} \lambda_{dp}} \right) d\gamma_{dp}
\]

(10)

\[
\text{OP}_{\text{in.d}} = \frac{\gamma_{th}}{\gamma_{th} + \alpha_0} + \sum_{k=1}^{N} (-1)^{k-1} \sum_{n_1 \leq \cdot \cdot \cdot n_k < n_0} \left[ \frac{\gamma_{th} \gamma_{dp}}{\gamma + \gamma_{dp}} \exp \left( -\frac{\gamma_{dp}}{\gamma + \gamma_{dp}} \right) \right] \left( \frac{1}{1 + \gamma_{th}^2 \alpha_{dp} \lambda_{dp}} \right)
\]

(11)

\[
\text{OP}_{\text{in.d}} = \frac{\gamma_{th}}{\gamma_{th} + \alpha} + \sum_{k=1}^{N} (-1)^{k-1} \left[ \frac{\alpha}{\gamma_{th} + \alpha} \right] \left( \frac{\gamma_{th}}{\gamma_{th} + \alpha} \right)^k
\]

(12)

outage probability is simplified as eq. (12).

4. Numerical results and Discussion

In this section, we present some representative numerical examples and simulation results for the outage probability of the cooperative spectrum sharing with DF relays in Rayleigh fading channels.

Fig. 1 shows the outage probability of DF relay networks with different numbers of relays. The channel parameters are set as \( \lambda_{id} = 3 \), \( \lambda_{sp} = \lambda_{th} = 3 \), and \( \lambda_{dp} = 3 \) for all \( k \). It is seen that by increasing the number of relays from one to four, the outage performance improves and the DF relay network always outperforms direct transmission. Clearly, the analytical results match very well with simulations, which verifies the correctness of our proposed analysis.

In Fig. 2 we examine the effect of interference level and the relative average channel gains between primary and secondary users on the system performance. In particular, for the cognitive networks, we assume that all fading parameters are equal to some specific value \( \lambda_{c} \), i.e., \( \lambda_{id} = \lambda_{sp} = \lambda_{th} = \lambda_{dp} \). Similarly, all fading parameters for primary networks are considered to be equal to a certain values \( \lambda_{p} \), i.e., \( \lambda_{id} = \lambda_{sp} = \lambda_{dp} \). Then, the outage probability performance is plotted versus the fraction of these two parameters, i.e., \( \frac{\lambda_{id}}{\lambda_{dp}} \).
increases with the increase of the ratio of expected. The result also shows that the outage probability rate of interference level results in lower outage performance as shown in Fig. 2. We can see that higher maximum tolerate interference level results in lower outage performance as shown in Fig. 2. We can see that higher maximum tolerate interference level results in lower outage performance.

Fig. 2  Effect of the maximum tolerate interference level and average channel power form secondary nodes to the primary node, $N = 3, \gamma_{th} = 3$.

Fig. 3  Effect of i.i.d. and i.n.d. channels, $N = 3, \gamma_{th} = 3$.

5. Conclusion

In this paper, we have derived the outage probability of DF relay networks under underlay interference constraints. The exact outage probability expression is simple and exact, and requires no special function evaluations. Numerical results illustrate that under the constraint of interference level, DF relay systems still hold advantages over the direct transmission.

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Appendix A: Proof for (9)

The purpose of this appendix is to derive (9). By first expanding the product and then grouping together like terms, (9) can be derived as follows:

$$\prod_{k=1}^{N} \left[ 1 - \frac{\alpha_{2,k} \gamma}{\gamma + \alpha_{2,k}} \right] = 1 - \sum_{n_1=1}^{N} (-1)^{n_1} \frac{\alpha_{2,n_1}}{\gamma + \alpha_{2,n_1}} - \sum_{n_1=1, n_2=1}^{N} (-1)^{n_1+n_2} \frac{\alpha_{2,n_1}}{\gamma + \alpha_{2,n_1}} \frac{1}{\gamma + \alpha_{2,n_2}} - \sum_{n_1=1, n_2=1, n_3=1}^{N} (-1)^{n_1+n_2+n_3} \frac{\alpha_{2,n_1}}{\gamma + \alpha_{2,n_1}} \frac{1}{\gamma + \alpha_{2,n_2}} \frac{1}{\gamma + \alpha_{2,n_3}} - \ldots$$

$$= 1 - \sum_{n_1=1}^{N} (-1)^{n_1} \frac{\alpha_{2,n_1}}{\gamma + \alpha_{2,n_1}} \prod_{p=1}^{N} \frac{1}{\gamma + \alpha_{2,n_p}}$$

as shown in Fig. 2. We can see that higher maximum tolerate interference level results in lower outage performance as expected. The result also shows that the outage probability increases with the increase of the ratio of $\lambda_s / \lambda_d$.

In Fig. 3, we study the impact of different fading channel conditions by generating the fading parameters from uniformly distributed random values $0 \to 2$. For the primary network, all average channel gains are fixed to be one, i.e., $\lambda_{sp} = \lambda_{sd} = 1$. As a baseline, we also plot the OP of the i.i.d. case, i.e., $\lambda_{sd} = \lambda_{sp} = \lambda_{d} = \lambda_{s} = 1$. As can be observed from Fig. 3, the simulation results are in excellent agreement with the analytical results and the i.i.d. network provides better performance than the i.n.d. case.
References


