Simulation and Identification Techniques for Floating Structures

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Abstract:
The dynamic behaviour of floating structures is studied in this work. These types of structures are normally simplified into spring-mass-damper systems where frequency dependent mass and damping are used to model the hydrodynamics. A method based on using digital filters to simulate the time response is tested in this work. The problem to identify added mass and added damping coefficients from measurement data is also examined. This is done by using the simulation model to generate time data. The predicted added mass and added damping can then be compared with the true coefficients and the identification method can be evaluated. Finally, an experimental system is studied and compared with simulation results.

Keywords:
Floating Structures, Dynamic Behaviour, Added Mass, Added Damping, System Identification
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Chen Yousheng
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Notations

\( \rho \) Density
\( g \) Gravitational Acceleration
\( L_s \) Draft
\( F \) Excitation Force
\( f_R \) Radiation Force
\( t \) Time
\( f \) Frequency
\( f_s \) Sampling Frequency
\( \omega \) Angular Frequency
\( r \) Radius
\( M_B \) Buoy Mass
\( C_B \) Viscous Damping
\( K_B \) Buoyancy Stiffness
\( M_A \) Added Mass
\( C_A \) Added Damping
\( Z_B \) Buoy Displacement
\( H_A \) Transfer Function between incident wave and wave force
\( H_B \) Transfer Function between wave force and buoy displacement
\( x \) Input Signal
\( y \) Output Signal
\( n \) External Noise Signal
\( a \) Filter a-coefficients
\( b \) Filter b-coefficients
\( N_A \) Number of a-coefficients
\( N_B \) Number of b-coefficients
\( d \) Initial Displacement
$G_{vf}(f)$  Cross Spectral Density (CSD)
$G_{ff}(f)$  Power Spectral Density (PSD)

**Abbreviations**

SDOF  Single Degree of Freedom System
FIR  Finite Impulse Responses
IIR  Infinite Impulse Responses
CSD  Cross Spectral Density
PSD  Power Spectral Density
1 Introduction

The hydrodynamic behaviour is essential to consider when designing structures in a sea environment. An increased knowledge within this field can help engineers to, for example, improve the stability of ships or optimize the performance of wave energy devices.

So far, diverse methods have been applied to the motions of floating bodies. They may be classified into three types: analytical methods [1-2], numerical methods [3], and experimental methods [4].

A set of theoretical added mass and added damping coefficients for a floating circular cylinder in finite-depth water has been investigate by Yeung [5]. Mciver and Linton [6] obtained numerical results for the added mass of the bodies heaving at low frequency in water of finite depth. E. V. Ermanyuk [7] used impulse response functions for evaluation of added mass and added damping coefficient of a circular cylinder oscillating in linearly stratified fluid. Experimental investigation of added mass effects on a Francis turbine runner in still water by C.G. Rodriguez [4].

A simplified theoretical model of a floating structure will be developed in this report. Then a methodology for solving the time response using digital filters will be shown. The simulation model will then be used to calculate the time response in various situations. In the time domain, a convolution integral is conventionally used to represent the fluid dynamic radiation force, characterised by added mass and added damping in the frequency domain. Thus, the simulation of these devices in time domain proves to be a very useful tool for both design of these device and predict theirs behaviour.

The goal is to better understand the dynamic behaviour and identify suitable measurement techniques for this type of problems. With the aim of estimate hydrodynamic parameters from real measurement data it is crucial to have reliable and accurate measurement and analysis techniques. It is therefore suitable to first test the performance of these methods on simulated data where the disturbance from contaminating noise can be controlled. In order to verify the modelling, simulation, identification methods presented, an experimental test in an aquarium will be performed as well.

The theoretical model is derived in Chapter 2 and a simulation routine is shown in Chapter 3. Identification methods are then studied in Chapter 4 and Chapter 5, followed by experimental test in Chapter 6 and conclusions in Chapter 7.
2 Simplified modelling of wave-buoy interaction

One of the goals of this work is to find a method to predict the buoy motion. For this, a simulation model is needed. A theoretical model is illustrated in the following sections.

2.1 Overview of Modelling.

Linear water wave theory is a widely used technique for determining how a wave gets diffracted by a fixed or floating structure. The linear water wave theory assumes that the fluid layer has a uniform mean depth, and that the fluid flow is inviscid, incompressible and irrotational. The underlying assumption of the theory is that the amplitudes of any wave or body motion are small.

Based on the linear theory, the motion of a floating buoy can be subdivided into a diffraction problem and a radiation problem. The diffraction problem concerns the force acting on a fixed buoy caused by incident wave. From the solution to the diffraction problem we can identify the external forces acting on the structure from the incident wave. These forces depend on the geometry of the structure, the water depth, boundaries and the oscillation frequency. For the radiation problem we study the waves generated by the oscillating structure. These waves will create reaction forces on the structure which are also depending on the geometry, water depth, boundaries and oscillation frequency. The reaction forces from generated waves are generally interpreted as added mass or inertia and added damping.

In order to predict the motion of a floating body subjected to ocean waves, it is necessary to know the wave force coefficients (diffraction problem) and the added mass coefficients and the added damping coefficients (radiation problem). All of these hydrodynamic parameters are frequency dependent. It is also necessary to know the mass of the structure and the buoyancy stiffness. All of these are summarized in Figure 2.1 where the two frequency dependent transfer functions are used to model the system.
The geometry of hydrodynamic structures can be idealized, for simplicity, to circular cylinders or rectangular floating bodies. It is then possible to find the hydrodynamic parameters from either analytical or numerical methods. In the following simulations we will assume that the structure is a vertical circular cylinder, as shown in Figure 2.2, in an infinite fluid domain with constant water depth. The added mass and added damping for heave mode of this type of geometry can be found in [8]. An example of added mass and added damping is shown in Figure 2.3, magnitude and phase of the wave force is also shown in Figure 2.4, for $r=0.5$ meter, draft $L_s=1.88\cdot r$ meter and water depth $=15\cdot r$ meter.

**Figure 2.1. System response of a floating buoy subjected to ocean wave.**

**Figure 2.2. A buoy with total height $L$, draft $L_s$, and three degrees of freedom (heave, surge, pitch).**
Figure 2.3. Added mass and added damping for the heave mode of a floating vertical cylinder with $r=0.5$, draft $1.88 \cdot r$ and water depth $15 \cdot r$.

Figure 2.4. Magnitude and phase of wave force for the heave mode of a floating vertical cylinder with $r=0.5$, draft $1.88 \cdot r$ and water depth $15 \cdot r$. 
2.2 Single-Degree-of-Freedom Model

The motion of a rigid body is characterized by six components corresponding to six degrees of freedoms. Assuming an appropriate coordinate system; surge, sway, and heave are translational motions in the x-, y-, and z-directions respectively. Roll, pitch, and yaw are corresponding to the rotational motions about x, y and z axes respectively. In this report, only heave mode is studied. The heave motion of the floating cylinder in an infinite fluid domain with constant water depth can be modelled as a single-degree-of-freedom system (SDOF) as shown in Figure 2.5.

\[
\begin{align*}
M_B + M_a(\omega) & \quad Z_B \\
K_B & \quad C_B + C_a(\omega)
\end{align*}
\]

Figure 2.5. \(M_B\) is the mass of the structure. \(M_a\) is the added mass. \(K_B\) is the buoyancy Stiffness. \(C_B\) is the viscous damping. \(C_a\) is the added damping. \(F\) is the wave force. \(Z_B\) is the heave motion.

In equilibrium, the following forces acts on the cylinder during heave motion:

- Hydrostatic force: This is due to the buoyancy stiffness. A restoring force is created which tries to return the buoy to the equilibrium position. Hence, the term buoyancy stiffness is used. For a cylinder with radius \(a\), the spring force can be written as
\[ F_s = K_B \cdot z = \rho g \pi r^2 \cdot z \]  \hspace{1cm} (2.1)

- **Excitation Force**: Excitation force due to incident waves.
- **Radiation Force** \((f_R)\): A reaction force from generated waves that the buoy produces.
- **Inertia Force**: A reaction force due to cylinders mass, \(M_B\).
- **Viscous Force**: Damping force due to the viscous damping, \(C_B\).

### 2.3 Added Mass and Added Damping

The differential equation for the system shown in Section 2.2 can be written as:

\[
M_B \cdot \ddot{z}(t) + C_B \cdot \dot{z}(t) + K_B z(t) + f_R = F(t) \hspace{1cm} (2.2)
\]

Or in frequency domain as

\[
\left(-w^2M_B + jwC_B + K_B\right) \cdot Z(w) + F_R(w) = F(w) \hspace{1cm} (2.3)
\]

Studying Eq. (2.2) and Eq. (2.3) it can be seen that the system is similar to the standard single-degree-of-freedom model. The only difference is the added force from radiation. In linear buoy theory it is common to assume the following form on \(F_R\):

\[
F_R(\omega) = Z_R(\omega) \cdot j\omega Z(\omega) \hspace{1cm} (2.4)
\]

\(Z_R\) is known as the radiation impedance and can be written as:

\[
Z_R(\omega) = C_A(\omega) + j\omega M_A(\omega) \hspace{1cm} (2.5)
\]

From Eq. (2.5) it can be seen that the real part of the radiation impedance is the added damping while the imaginary part is related to the added mass. As previously explained, both added damping and added mass depend on the geometry, water depth, boundaries and oscillation frequency.
2.4 Transfer Functions

As mentioned in previous section, two linear transfer functions can be defined for a floating buoy subjected to the ocean. They are the transfer function between incident wave and wave force and transfer function between wave force and buoy motion.

If incident waves make the buoy move, a linear transfer function between incident wave amplitude and wave force is

\[
H_A(\omega) = \frac{F(w)}{X_w(\omega)}
\]  

(2.6)

This transfer function represents the first box in Figure 2.1 and can be calculated when the wave force coefficients are known.

A typical transfer function between incident wave amplitude and applied force is shown in Figure 2.6 for \(r=0.5\) m, \(M_B=738\) Kg and \(C_A=100\) Ns/m. The wave force coefficients are shown in Figure 2.4.

![Figure 2.6](image)

*Figure 2.6. An example of a transfer function (force/wave) for a cylinder with \(r=0.5\) m, \(M_B=738\) Kg, \(C_A=100\) Ns/m.*
Assume wave force is known, the transfer function between the buoy motion and wave force is derived below.

Insert Eq. (2.4) into Eq. (2.3), gives

\[ Z_M(\omega) \cdot Z(\omega) + j\omega Z_R(\omega) \cdot Z(\omega) = F(\omega) \]  

(2.7)

Where, \( Z_M(\omega) = -w^2 M_B + iwC_B + K_B \)

Hence, the linear transfer function between wave force and resulting heave motion is

\[ H_B(\omega) = \frac{Z(\omega)}{F(\omega)} = \frac{1}{Z_M(\omega) + j\omega Z_R(\omega)} \]  

(2.8)

This can also be written as:

\[ H_B(\omega) = \frac{Z(\omega)}{F(\omega)} = \frac{1}{-\omega^2 (M_B + M_A(\omega)) + j\omega(C_B + C_A(\omega)) + K_B} \]  

(2.9)

A typical transfer function is shown in Figure 2.7 for \( r=0.5 \) m, \( M_B=738 \) Kg and \( C_A=100 \) Ns/m. The added mass and added damping used for this example is shown in Figure 2.3. As can be seen in Figure 2.7, a larger response is obtained when the buoy enters resonance. For this example, resonance occurs at approximately 0.45 Hz.
Figure 2.7. An example of a transfer function (motion/force) for a cylinder with $r=0.5$ m, $M_B=738$ Kg, $C_A=100$ Ns/m.

2.5 Conclusion

The dynamic behaviour of a floating buoy has been simplified to a SDOF-system and transfer functions for the system have been derived. Added mass is the imaginary part of the radiation impedance divide by angular frequency and the added damping is the real part of radiation impedance.
3 Simulation in the Time Domain

After transfer functions are obtained from the Chapter 2, digital filters are used to simulate the system which will be shown in this chapter. This methodology can lead to a very efficient simulation routine if stable and accurate filter coefficients can be found.

3.1 Digital Filters Properties

In order to simulate the buoy motion for a given incident wave, the transfer functions can be seen as two digital filters. A filter with input $x$ and output $y$ can be written in the following form:

$$
a_0 \cdot y_n = b_0 \cdot x_n + b_1 \cdot x_{n-1} + \ldots + b_{N_B} \cdot x_{n-N_B} - a_1 \cdot y_{n-1} - \ldots - a_{N_A} \cdot y_{n-N_A}$$

(3.1)

Eq. (3.1) is a standard difference equation. $N_A$ is the number of a-coefficients and $N_B$ is the number of b-coefficients. Eq. (3.1) can also be written in Z-domain as

$$
Y(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}
$$

(3.2)

The reason why filter is used to simulate the system is that the Eq. (2.2) is difficult to solve in time domain since the radiation force depends on the frequency. For the filter the buoy motion can be solved in frequency domain, then take inverse Fourier transform gives buoy motion in time domain. The processes using filters to simulate the time response is shown in Figure 3.1.
Figure 3.1. Incoming wave pass filter A gives wave force, wave force then pass filter B which produces the buoy motion.

### 3.2 Digital Filters Design

In order to predict the buoy motion according to the incident wave, two digital filters shown in Figure 3.1 will be designed next. Digital filters can basically be classified into FIR (finite impulse responses) and IIR (infinite impulse responses) filter [10].

<table>
<thead>
<tr>
<th>FIR</th>
<th>IIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non feedback</td>
<td>Feedback</td>
</tr>
<tr>
<td>Always stable</td>
<td>May be unstable</td>
</tr>
<tr>
<td>Can be linear phase</td>
<td>Difficult to control phase</td>
</tr>
</tbody>
</table>

If computational cost is important, low-complexity IIR filter is recommended to use. If phase response is important, FIR filter is suitable to use. In this problem, the phase response is not linear, then we care about the cost and IIR filter will be used.

As mentioned in [8], the impulse response for the excitation force is non-causal. A system that has some dependence on input values from the future (in addition to possible dependence on past or current input values) is termed as a non-causal system. The impulse response functions related to the excitation forces is non-causal, because the wave may hit a part of the body and exert a force, before the arrival of the wave at the origin, and the latter is used as reference. Hence, an excitation force can be created before the reference wave is observed.
The impulse response related to excitation force as shown in Figure 3.2 for the system shown in Figure 2.2.

![Impulse response related to heave excitation force for a vertical cylinder buoy.](image)

*Figure 3.2. Impulse response related to heave excitation force for a vertical cylinder buoy.*

In this work, the following steps are followed in order to simulate the wave forces

1. The inverse Fourier transform is calculated to find the impulse response.
2. The impulse response is shifted so that the impulse response only exist for positive time (causal).
3. Take the Fourier Transform of the new impulse response, which gives a new transfer function.
4. The filter the coefficients are found for the new system.
5. The Filter coefficients are used to simulate the wave force to an arbitrary wave signal.
6. The wave force signal is phase-shifted in order to compensate for the delay introduced in step 2.

The inverse Fourier transform of the heave excitation force given in Figure 3.2 for a cylinder with \( r=0.5 \text{ m}, M_B=738 \text{ Kg}, C_A=100 \text{ Ns/m} \).

To make the impulse response causal, the impulse response is delayed and illustrated in Figure 3.3. Taking the Fourier transfer of the new impulse response, which gives the result shown in Figure 3.4. As expected, the magnitude is not changed but the delay can clearly be seen when studying the phase information.

![Figure 3.3 A delayed version of the impulse response shown in Figure 3.2.](image)
MATLAB [9] Command “invfreqz” can be used to find a discrete-time transfer function that corresponds to a given complex frequency response. The a-coefficients and b-coefficients can be found by using “invfreqz” to produce a stable IIR filter A, which has the same magnitude and phase as the transfer function which shows in Figure 3.3.

In order to know whether the filter A can be represent the transfer function $H_n(\omega)$, a command called ‘freqz’ can be used to get the transfer function of the filter B. Magnitude and phase of the true transfer function $H_n(\omega)$ is compared with the filter response in Figure 3.5. In this case a stable and accurate filter has been found which uses 2 a-coefficients and 2000 b-coefficients.

*Figure 3.4. Filter transfer function*
Next, the filter coefficients are used to simulate the excitation force to an arbitrary wave signal. After phase correcting the output we get the excitation signal which can represent the wave force associated with applied incident wave. This approach is verified with a simulation. The transfer function is calculated from (phase-corrected) time data and then compared with the desired transfer function between incident wave and excitation force. The result is shown in Figure 3.6. The curves are close to each other which implies that the simulation is correct. Only a smaller difference can be seen at higher frequencies since the filter response is not identical to the transfer function at these frequencies.

Figure 3.5. Filter magnitude and phase response is plotted in red and real transfer function $H_n(\omega)$ is plotted in black.
Figure 3.6. Transfer function for the box 1 in Figure 3.1 is plotted in red and the transfer function obtained from the simulation is plotted in black.

The transfer function $H_B(\omega)$ between external force and buoy motion as shown in Figure 2.7 is studied next. In contrast, the impulse responses corresponding to $H_B(\omega)$, are casual because their inputs are the actual cause of their response. It also can be seen from the impulse response function associated with $H_B(\omega)$ shown in Figure 3.6.
Figure 3.7. Impulse response corresponding to $H_B(\omega)$.

Taking the inverse Fourier transform of $H_B(\omega)$ gives impulse response in Figure 3.8. The a-coefficients and b-coefficients for $H_B(\omega)$ can be found by using a MATLAB function called “invfreqz” to produce a stable IIR filter $B$. Magnitude and phase of the true transfer function $H(\omega)$ is compared with the filter response in Figure 3.8.
As can be seen in Figure 3.8, the filter response is very close to actual transfer function \( H_B(\omega) \). Thus filter B can be used to simulate the transfer function \( H_B(\omega) \). A simple example is shown below to verify that the filter can be used to simulate the transfer function \( H_B(\omega) \). Assume that the periodic input signal is

\[
f_w(t) = 5 \sin(2\pi ft) + 3 \cos(2\pi ft) = (-5i + 3)e^{jwt}
\]

where, \( f=0.7 \).

Let \( f_w(t) \) pass the filter B, we get the buoy motion \( z_B(t) \). However, the steady-state solution can be found directly in the frequency domain.

When \( f=0.7 \), added mass and added damping are 232 Kg, 14.4 Ns/m respectively (see Figure 2.3). Insert these into Eq.(2.9) gives,
\[ H_B(2\pi f) = -9.0131e-005 - 4.0976e-006i \]

The steady-state response can then be calculated as

\[ X_B(w) = H_B(w) \cdot F_w(w) \]

The system is linear which gives that

\[ x_B(t) = X_B(w) \cdot e^{i\omega t} = (-2.9088e-004 + 4.3836e-004i)e^{i2\pi 0.7t} \]

The simulation result and exact solution are plotted in Figure 3.9.

![Buoy motion](image)

**Figure 3.9.** \( z_B(t) \) obtained from filter \( B \) is plotted in blue and the steady-state solution \( x_B(t) \) is plotted in red.

The amplitude of \( z_B(t) \) is bigger than \( x_B(t) \) at the beginning, after 20 seconds \( z_B(t) \) and \( x_B(t) \) match each other. As expected, \( z_B(t) \) shows some
transient response in the beginning. After approximately 40 seconds the response settles to the steady-state response which shows that the filter response is correct.

### 3.3 Simulation of Buoy Motion

To predict the buoy motion caused by the incident wave, two digital filters can be used to simulate the buoy motion. An incident wave passing two filters which have been designed in Section 3.3 gives the buoy motion.

After the a-coefficients and b-coefficients are known, a MATLAB command called “filter” can be used to rapidly simulate the response from a given input. In order to avoid the aliasing, the maximum frequency in the input signal should be smaller than 0.5 fs (sampling frequency). The process to simulate the buoy motion from an arbitrary incident wave is summarized below.

First, the incident wave pass the low pass filter which gives $x_L(t)$ (without frequencies higher 0.5 fs). Then $x_L(t)$ pass through filter A obtains $ff(t)$ which has the right amplitude but wrong phase for the wave force. The phase is corrected in $ff(t)$ which gives $f(t)$ (true wave force according to the applied incident wave). Finally, wave force pass filter B which gives the buoy motion. The whole process can also be seen from Figure 3.10.

![Diagram of the simulation process](image)

**Figure 3.10.** The incident wave should be filtered through a low pass filter before it passes through the filter A to avoid aliasing.

An example of a time response is shown in Figure 3.8. Here the incident wave is shown together with the simulated buoy displacement as a function of time. In this example, the incident wave contains some higher frequencies. However, these are not seen when studying the buoy motion.
This is because the system as a whole has a low transfer at higher frequencies as can be seen from Figure 3.8 and Figure 3.6.

Figure 3.11. Incident wave is shown together with the simulated buoy displacement, as a function of time.

3.4 Conclusion

Two digital filters are used to simulate the dynamic behaviour of a buoy subjected to ocean waves. The incident waves pass through two filters which gives buoy displacement in time domain. It solves the problem that Eq. (2.2) is difficult to solve in the time domain.

The transfer function between the incident wave and wave force are non-causal. Instead of designing a non-causal filter, the filter coefficients are found for a delayed impulse response function. This introduces some phase errors which can easily be compensated for. The whole system (from incident wave to buoy motion) can then be simulated with two digital filters in series.
4 System Identification from Time Responses

In chapter 2, the heave motion of the floating cylinder was simplified as a single degree of freedom (SDOF) system, as shown in Figure 2.5. The expression for the transfer function is shown in Eq. (2.9). Identification of the parameters in Eq. (2.9) from given time responses, will be studied in this chapter. The problem is simplified by assuming that the excitation force is known i.e. only the transfer function between applied force and resulting buoy motion is considered (box 2 in Figure 2.1).

Added mass and added damping can be obtained from Eq. (2.9), if applied force and buoy motion can be measured from an experiment. In the measurement, there is no hope to measure simply input signal and output signal without at least one of these signals being contaminated by external noise. To understand how the noise would affect the result, extraneous noise will be discussed in Section 4.1. Coherence function will be investigated in Section 4.2 to make sure the measurement is reliable. Identification for added mass and added damping is illustrated in Section 4.3.

4.1 Extraneous Noise

As mention the input signal and output signal will be contaminated by external noise. In this case, ‘noise’ is everything that a linear model cannot explain. However, it is common to assume that the noise is uncorrelated with either the input or output signal. Here we assume that the input signal has no disturbance but the output contains noise. An illustration of this is shown in Figure 4.1. In this Figure, \( v_B(t) \) is the true output from the linear system and \( z_B(t) \) is the measured output.
Figure 4.1 The simulation of linear response system were the output cannot be measured without disturbance.

If the transfer function $H_B(\omega)$ is calculated by following expression:

$$\hat{H}_B(\omega) = \frac{Z_B(\omega)}{F(\omega)}$$  \hfill (4.1)\]

It is impossible to acquire a good result. External noise would destroy the measurement result, since this estimator has a large variance as is shown in Figure 4.2.
**Figure 4.2** Extraneous noises destroy the estimated transfer function between applied force and buoy motion.

In order to obtain the reliable transfer function between applied force and buoy motion $H_1$-estimator or $H_2$-estimator [11] should be used.

From Figure 4.1, the output has noise disturbance, thus $H_1$-estimator is suitable to compute the frequency response. (If the input signal is disturbed by noise, $H_2$-estimator has to be used.) $H_1$-estimator for the transfer function $H_B(\omega)$ can be written as

$$
\hat{H}_B(\omega) = \frac{\hat{G}_{zf}(\omega)}{\hat{G}_{ff}(\omega)}
$$

Where, $G_{zf}(\omega)$ is cross spectral density between input force and output buoy motion, and $G_{ff}(\omega)$ is power spectral density of the applied force. The symbol ^ (hat) denote that we are dealing with estimated functions. These quantities can be calculated using Welch’s Method [12].

The true transfer function is shown in Figure 4.3 together with the estimated transfer function using Eq. (4.2).
Figure 4.3 shows that a reliable transfer function may be obtained by using Eq. (4.2). However, for a real case the true transfer function between the applied force and the buoy motion is unknown. In order to know if the result is reliable, it is necessary to study the coherence function.

4.2 Coherence Function

The coherence function is defined as the ratio between the $H_1$-estimate and $H_2$-estimate, that is

$$
\hat{\gamma}^2_{zf}(f) = \frac{\hat{H}_1(f)}{\hat{H}_2(f)} = \frac{|\hat{G}_{zf}(f)|^2}{\hat{G}_{zz}(f)\hat{G}_{ff}(f)} \tag{4.3}
$$

Where, $0 \leq \hat{\gamma}^2_{zf}(f) \leq 1$
If \( \gamma_f^2(f) = 1 \) then \( \hat{H}_1 = \hat{H}_2 \) which implies that we have no extraneous noise, and moreover that the measured output derives solely from the measured input. The coherence functions are used to understand the relative importance of the various contributions to the response of the system being analyzed. The reason for the coherence function to deviate from 1 can be summarized as:

1. The noise cannot be ignored.
2. The truncation effect due to the measurement time being too short.
3. The system is non-linear or not time invariant.
4. Bias error due to a time delay between the input and output signals.

A coherence function is shown in Figure 4.4, for the analysed data in Figure 4.3.

![Coherence Function](image)

Figure 4.4. Coherence function for the transfer function shown in Figure 4.3
The coherence deviates from 1 around the resonance frequency due to leakage effects. Overall the coherence function indicates that the estimated transfer function is reliable.

### 4.3 System Identification

Added mass and added damping can be obtained from Eq. (2.9), if input signal and output signal can be measured from an experiment. In this section, the identification of hydrodynamic parameters from periodic data will be discussed in Sub-section 4.3.1. The identification of hydrodynamic parameters from random data will be discussed in Sub-section 4.3.2. Finally, identification from transient data will be studied in Sub-section 4.3.3 and identification from initial values will show in Sub-section 4.3.4.

#### 4.3.1 Identification from Periodic Data.

Identifying added mass and added damping can be relatively simple if the system is excited with a single frequency input.

After the transfer function for a certain frequency is computed, it can be inserted into Eq. (2.9). The added mass and added damping can be calculated as

\[
\begin{align*}
\hat{M}_a(\omega_0) &= \text{Re}\left(\frac{1}{\hat{H}_B(\omega_0) - K_B}\right) - M_B \\
\hat{C}_a(\omega_0) + C_B &= \text{Im}\left(\frac{1}{\hat{H}_B(\omega_0) - K_B}\right) / \omega_0
\end{align*}
\]

#### 4.3.2 Identification from Random Data

In this section we assume to the input force signal is a normally distributed random signal. Random signals have continuous spectra which contain all
frequencies. It implies that added mass and added damping for all frequencies can be estimated.

When the input data and output data are known, by using Eq. (4.2), the estimated transfer function can be computed. In order to calculate the cross spectral density between input force and output buoy motion, and spectral density of the applied force, Welch’s method is used.

Substituting estimated transfer function into Eq. (2.9) gives,

\[
\hat{M}_a(\omega) = \frac{\text{Re}\left(\frac{1}{\hat{H}_B(\omega)} - K_B\right)}{-\omega^2} - M_B
\]  
(4.6)

\[
\hat{C}_a(\omega) + C_B = \frac{\text{Im}\left(\frac{1}{\hat{H}_B(\omega)} - K_B\right)}{\omega}
\]  
(4.7)

Where \(\text{Re}(.)\) means the real part and \(\text{Im}(.)\) means the imaginary part.

Eq. (4.6) and (4.7) give a way to identify the added mass and added damping if \(z_B(t)\) and \(f(t)\) are known from a measurement. \(K_B\) is the buoyancy stiffness. \(M_A\) is the mass of the buoy. When \(H_B(\omega), K_B, \omega,\) and \(M_A\) are known, added mass can be calculated. The value on the viscous damping (\(C_B\)) can be difficult to find. Instead, with Eq. (4.7) we estimate the total damping in the system.

### 4.3.3 Identification from Transient Data

Transient data like random signals have continuous spectra. However, as opposed to random signals, transient signals do not continue infinitely. One example can be a sudden hit applied to the buoy (impulse testing).

The theory shown in previous section can be used for transient signals as well. The only difference is how the spectral densities are calculated. For random signals, Welch’s method is used while the following formula can be used to calculate the spectral densities for impulse testing:

\[
\hat{G}_{ff}(\omega) = \frac{1}{N} \cdot \sum_{m=1}^{N} F_m(\omega) \cdot F_m^*(\omega)
\]  
(4.8)
In Eq. (4.8) and Eq. (4.9), \( N \) is the number of averages (number of hits if impulse testing is used). \( F_m(\omega) \) is the Fourier transform of the measured force and \( Z_m(\omega) \) is the Fourier transform of the measured buoy response. Eq. (4.2), Eq. (4.6), and Eq. (4.7) can then be used to calculate the added mass and added damping.

### 4.3.4 Identification using Initial Values

In some cases it is not possible to apply an external force (random or transient) to the buoy structure. In this case it can still be possible to identify the system by changing the initial values. For example, if the buoy is pressed down a small distance and then released, it will start to move until the equilibrium position is found. If this free response is measured and the initial offset from equilibrium is known, it can be possible to find the added mass and added damping.

For initial value \( \dot{z}(0) = 0 \), \( z(0) = d \) and external force \( f(t) = 0 \), the governing equation for heave motion of a floating cylinder can be written as:

\[
(M_A(\omega) + M_B) \cdot \ddot{z}(t) + (C_A(\omega) + C_B) \cdot \dot{z}(t) + K_B z(t) = 0
\]  

(4.10)

Take Laplace transform of Eq. (4.10) gives

\[
(M_A(\omega) + M_B) \cdot (s^2 Z(s) - sz(0) - \dot{z}(0)) + (C_A(\omega) + C_B) \cdot (sZ(s) - z(0)) + K_B Z(s) = 0
\]

(4.11)

Substituting the initial value into Eq. (4.11) produces

\[
Z(s) = \frac{[s(M_A(\omega) + M_B) + (C_A(\omega) + C_B)]d}{[s^2(M_A(\omega) + M_B) + (C_A(\omega) + C_B)s + K_B]}
\]

(4.12)

Eq. (4.12) can also be written as:
\[
\frac{K_B}{d} \frac{d}{Z(s)} - s = s(M_A(\omega) + M_B) + (C_A(\omega) + C_B) \quad (4.13)
\]

Inserting \( s= \omega \), into Eq. (4.13) gives

\[
\frac{K_B}{d} \frac{d}{Z(j\omega) - j\omega} = \omega(M_A(\omega) + M_B) + (C_A(\omega) + C_B) \quad (4.14)
\]

And the added mass and added damping can be identified as

\[
M_A(\omega) = \frac{1}{\omega} \text{Im} \left\{ \frac{K_B}{d} \frac{d}{Z(j\omega)} - j\omega \right\} - M_B \quad (4.15)
\]

\[
C_A(\omega) + C_B = \text{Re} \left\{ \frac{K_B}{d} \frac{d}{Z(j\omega) - j\omega} \right\} \quad (4.16)
\]

### 4.4 Conclusion

In this chapter identification methods of the added mass and added damping are derived for the periodical signal, random signal, transient signal, and initial value (for the input signal cannot be measured).

Identification from random signal and transient signal, they share the same formulas. The different part is that they used different methods to calculate transfer function between the excitation force and buoy displacement. Random signal used Eq. (4.2) to calculate the transfer function. Welch’s method is used to estimate the cross spectral density and spectral density. Transient data share the same equation with random signal to estimate the transfer function, however the method for transient data to estimate the cross spectral density and spectral densities are different from Welch’s method.
For the initial value, different equations are derived to estimate the added mass and added damping.
5 Simulation Verification

The identification methods shown in Chapter 4 will be tested on simulated data in this chapter. In this chapter, it is assumed that the buoy is in the calm water (without incident wave), instead of wave force, applied force is acting on the buoy. A digital filter is used to simulate the system, with different input signals, for instance, periodical signal, random signal or transient signal. Simulation data is then used to identify the added mass and added damping.

All simulations done in this chapter are based on the simple example with a floating vertical circular cylinder the following parameters: radius $r=0.5\text{ m}$ and a draft $L_s=1.88a\text{ m}$ on water depth $h=15a$ is used. The mass of the cylinder is $m=738\text{ kg}$ and the buoyancy stiffness is $7704\text{ N/m}$.

Identification results are followed in section 5.1, followed by conclusions in Section 5.2.

5.1 Identification Result

In order to know if the equations presented in chapter 4 are suitable, identification results for added mass and added damping for the different input signal are shown in this section. Simultaneously, identification results for added mass and added damping for initial value problem, which cannot measure the input signal, are also shown in this section.

5.1.1 Estimate Hydrodynamic Parameters from Periodic Data.

Assume that periodic input signal is,

$$f_w(t) = 5 \sin(2\pi ft) + 3 \cos(2\pi ft) = (-5i + 3)e^{i\omega t} \quad \text{Where, } f=0.7.$$  

Letting $f_w(t)$ pass the filter $B$, which gives buoy motion $z_B(t)$. Transfer function at $f=0.7$ is estimated from input signal and simulated output signal. Then Eq. (4.4) and Eq. (4.5) are used to estimate the added mass and added damping. The estimated added mass and added damping for the periodic signal when $f=0.7$ are 232.3 and 14.2 respectively. The estimated added
mass and added damping is compared with the true system added mass and added damping below. Identified added mass and added damping is plotted in red ‘o’ maker, and the true added mass and added damping is plotted in black.

![Graphs showing comparison between identified and true added mass and added damping](image)

*Figure 5.1. Comparison between identified added mass and added damping and system added mass and added damping.*

Identification results are very close to the true data as can be seen in Figure 5.1. It means the equations for identifying the added mass and added damping for the periodical signal are correct.

### 5.1.2 Estimate Hydrodynamic Parameters from Random Data.

The MATLAB command ‘randn’ is used to create a random force signal \( f(t) \). This force is then low-pass filtered around 1 Hz. The buoy response is calculated by using the methods in Chapter 4. In order to make the simulation closer to reality, a small amount of noise is added to the output signal before the analysis. A short segment of the applied force, \( f(t) \), and the buoy motion \( z_B(t) \) can be seen in Figure 2.5.
The spectral densities are calculated with Welch’s method where averaging and windows are used to decrease the random error. According to Welch’s method, the number of averages is increased by using overlaps [11]. In the all random tests, 500 averages, hanning window, and 50% overlap have been used to estimate the spectral densities.

The true added mass and added damping are compared with the estimated added mass and added damping from no noise in input or output signal is illustrated in Figure 5.2 and coherence function for the input and output is plotted in Figure 5.3 as well. These estimated added mass and added damping were calculated using Eq. (4.6) and Eq. (4.7).

Figure 5.2. A short segment of applied force and buoy motion plotted as functions of time.
Figure 5.3. Comparing true added mass and added damping with estimated added mass and added damping without disturbance either in output or input signal.

Figure 5.4. Coherence function for the input and output without noise.
Figure 5.5. Comparing true added mass and added damping with estimated added mass and added damping with disturbance in output signal.

Figure 5.6  Coherence function for the estimated data shown in Figure 5.4
Figure 5.3 shows that added mass and added damping can be found for all the frequency, when there is no disturbance in input or output signal. When there is noise in the output signal, added mass and added damping can also be found by using averaging and overlapping to analyze the data, as shown in Figure 5.5. The coherence function shows that the data is reliable.

5.1.3 Estimate Hydrodynamic Parameters from Transient Data.

In order to remove the random errors, averaging is used. For the transient signal, several simulations (or experiments) should be performed so that an average can be calculated.

MATLAB functions are used to create transient data. An example of a transient force and resulting heave motion of the buoy motion is shown in Figure 5.6. In all the transient tests, 10 averages have been used to estimate spectral densities.

![Graph](image)

*Figure 5.7. Transient force and the buoy motion for the heave mode*
Figure 5.8. True added mass and added damping together with estimated added mass and added damping without disturbance neither in output or input signal.

Figure 5.9. Coherence function for estimated data in Figure 5.8.
Figure 5.10. True added mass and added damping and estimated added mass and added damping with disturbance in output signal.

Figure 5.11. Coherence function for estimated data in Figure 5.10
The true added mass and added damping are compared with the estimated values without noise and with noise in output signal in Figure 5.8 and Figure 5.10 respectively. In order to know, if the estimated data are reliable, coherence functions are plotted in Figure 5.9 and 5.11. The noise level can be seen when studying the coherence function in Figure 5.11.

5.1.4 Estimate Hydrodynamic Parameters using Initial Value.

To simulate the initial value problem, a constant force is applied to the cylinder for a certain time until the cylinder stop moving. The force is kept for a while, and then suddenly released. An example of force and resulting response is shown in Figure 5.12

![Figure 5.12. Simulation of the initial value problem.](image)
Figure 5.12 shows the cylinder motion after releasing the force which can be seen as the initial value problem. Here $\dot{z}(0)=0$, $z(0)=0.013$ meter

\begin{figure}[h]
\centering
\includegraphics[width=0.6\textwidth]{image1.png}
\caption{The heave motion with initial value $\dot{z}(0)=0$, $z(0)=0.013$ m}
\end{figure}

For the initial value problem Eq. (4.15) and Eq. (4.16) can be used to estimate added mass and added damping. The true added mass and added damping and estimated added mass and added damping are shown in Figure 5.14 without disturbance at output and with disturbance at output in Figure 5.15.
Figure 5.14. True added mass and added damping estimated added mass and added damping without disturbance either in output or input signal.

Figure 5.15. True added mass and added damping and estimated added mass and added damping with disturbance in output signal.

Simulated added mass and added damping and the true added mass and added damping are in good agreement except the low frequencies and high frequencies. The reason for they are not fit together is that the expressions
to get added mass and added damping are divided by angular frequency which is close to zero when frequency is close to zero. In Figure 5.14 and 5.15, added damping coefficients are not as good at higher frequencies. The reason for this relatively large error is not clear. However, when the buoy is instantly released it is excited with very high frequencies. It is hard to simulate the high frequency response, hence it is possible that the simulation error makes it hard to estimate the added damping coefficient.

5.2 Conclusion

In this chapter, simulation and identification of the added mass and added damping from the simulation data for a simple example have been studied. The result shows that estimated added mass and added damping are in good agreement with the true added mass and added damping. It implies that those equations derived in chapter 4 to identify the added mass and added damping from different input signals are correct.

In section 5.2 identification results have been illustrated for periodic signal, random signal, transient signal, and initial value problem.

To begin with, identification for the periodic signal is very accurate. It is especially useful at some frequency which added mass and added damping can not be estimated by random data or transient data. The disadvantage for identification from the periodic signal, it is that only added mass and added damping at the specific frequency can be found. Hence, it can be very time consuming.

Identification results from the random signal and transient data indicates that added mass and added damping of a floating buoy can be found for the all the frequencies and the expressions derived in chapter 4 may be suitable for a real measurement (when the input contains a certain level of noise). Identification from the random data may be simpler because only one measurement is needed. In order to reduce the random error, the measurement data can be subdivided into several segments, and then averaged. In contrast, several measurement are needed for transient signals to reduce the random errors.

Finally, identification results from the initial value problem shows that added mass can be found for all the frequencies and the equations derived in chapter 4 are suitable for real measurement. However added damping can only be found for low frequencies.
6 Experimental Test

In this chapter experiments will be carried out to verify modelling, identification and simulation methods presented in previous chapters. A floating circular cylinder in an aquarium will be investigated below as shown in Figure 6.1. The result from this experiment cannot be compared directly with the theoretical system studied previously. In Chapter 1 to Chapter 5 a cylinder in infinite sea was used as an example but for the experimental system the boundaries in the aquarium will affect the result. The added mass and added damping in the experimental system will therefore be heavily influenced by the boundaries in the aquarium (finite sea) [13]. However, the aim with this experimental study is to investigate the dynamic response of a system influenced by hydrodynamic interaction.

The studied system and the experimental setup are shown in Section 6.1 and Section 6.2. Experimental data will be analyzed to check the linearity for the structure and a simulation model will then be created and compared with experimental data. Digital filters are used to simulate the system as shown in Chapter 3. In the end the measured force signal from the experiment is used as input to the simulation model and then compared with the output measured in the experiment. If the simulation data shows good agreement with the measured data, it means that modelling, identification and simulation methods presented in previous chapters are suitable for this type of system.

6.1 Structure under Test

The structure that will be tested in the experiment is shown in Figure 6.1. A floating cylinder (radius 9.6 cm and height 20 cm) is fixed with a beam in an aquarium. The beam is free to rotate around point 1 as shown in Figure 6.1.

The reason why the cylinder is fixed with a beam is to force the cylinder to only move in the heave mode. The floating cylinder and beam can be seen as a rigid body. Heave motion and the applied heave force are measured from this structure.
6.2 Measurement Setup

A list of the equipment used is shown in Table 6.1. The SignalCalc Mobilyzer and Amplifier are shown in Figure 6.2.

Table 6.1. Equipments for experiment.

<table>
<thead>
<tr>
<th>Signal generation and data acquisition</th>
<th>SignalCalc Mobilyzer, LDS PA 100 E Power Amplifier, Computer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaker</td>
<td>LDS Shaker</td>
</tr>
<tr>
<td>Testing structure</td>
<td>Beam, cylinder and aquarium,</td>
</tr>
<tr>
<td>Sensors</td>
<td>Accelerometer (1000 mv/g), Force Transducer (112.410 mv/N)</td>
</tr>
<tr>
<td>Software</td>
<td>MATLAB &amp; SignalCalc Mobilyzer</td>
</tr>
</tbody>
</table>
The SignalCalc Mobilyzer and the amplifier are used to generate a voltage signal to the shaker. The experimental setup for the shaker, testing structure, and signal measurement equipment (force transducer and accelerometer) are shown in Figure 6.3.
The force transducer is used to measure the force acting on the structure, and the accelerometer is used to measure the acceleration response from the system.

A typical segment of force and acceleration measured from this structure is plotted in Figure 6.4 and Figure 6.5.

![Figure 6.4 A segment of measured force as a function of time.](image1)

![Figure 6.5 A segment of measured acceleration as a function of time.](image2)
The measurement data selected for further analysis are shown in table 6.2. Random signals are used in Dataset 1-4 and a chirp signal is used in Dataset 5. The input spectrum was modified for Dataset 2-3 by adding more energy at lower frequencies.

### Table 6.2. List of measurements

<table>
<thead>
<tr>
<th>Dataset</th>
<th>Measurement Time</th>
<th>Frequency range</th>
<th>Input spectrum</th>
<th>RMS of force</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30 min</td>
<td>Random signal (0-100) Hz</td>
<td>Flat</td>
<td>0.89 N</td>
</tr>
<tr>
<td>2</td>
<td>110 min</td>
<td>Bandpassed signal (0.5-7Hz)</td>
<td>Modified</td>
<td>0.67 N</td>
</tr>
<tr>
<td>3</td>
<td>65 min</td>
<td>Bandpassed signal (0.5-7Hz)</td>
<td>Modified</td>
<td>5.21 N</td>
</tr>
<tr>
<td>4</td>
<td>65 min</td>
<td>Bandpassed signal (1.2-2.2Hz)</td>
<td>Flat</td>
<td>0.06 N</td>
</tr>
<tr>
<td>5</td>
<td>110 min</td>
<td>Chirp signal (1-2.5Hz)</td>
<td>-</td>
<td>0.01 N</td>
</tr>
</tbody>
</table>

### 6.3 Analysis of Measurement Data

In this section, the measurement data shown in Table 6.2 will be used to analyse the system. The transfer function and coherence function obtained from analysing Dataset 1 is shown in Figure 6.6 and Figure 6.7. This transfer function was estimated by using Welch’s method with windowing and averaging and using the H1-estimator.
Figure 6.6. Estimated transfer function using dataset 1 (see Table 6.2)

Figure 6.7. Estimated coherence function using Dataset 1 (see Table 6.2)
The most interesting frequency range for this system is from 0 to 3 Hz as can be seen from the transfer function. Several resonances occur between 1.5-2 Hz. Coherence is bad at low frequencies (0 – 1 Hz). It indicates that the measurement data in this range is not reliable.

The coherence is bad at the low frequencies which could depend on, for example, equipment limitation, nonlinear system response, or too low response at low frequencies.

The transfer functions calculated for different force levels (using Dataset2 and Dataset3) are shown in Figure 6.8. The result is here zoomed in between 1-3 Hz.

![Transfer functions](image.png)

**Figure 6.8.** *Transfer functions for two different force levels between 1-3 Hz.*

Transfer functions for different force levels are fairly close to each other. However, a small nonlinearity can be seen in Figure 6.8.
The amplitude probability density function (APDF) for the acceleration measured with low force is shown in Figure 6.9. The APDF with a high force level is shown in Figure 6.10.

Figure 6.9. APDF comparison for the measured data with a low force (black) compared with a normal distribution (red).

The APDF is close to a normal distribution when the applied force is low. However, the APDF for the measured data deviates from a normal distribution when the applied force is high. Based on this analysis, a linear assumption is good when the applied force is low.
Figure 6.10. *APDF comparison for the measured data with a high force (plot in black dots) and normal distribution (plot in red).*

An attempt was made to improve the coherence around 1-2 Hz. Since the acceleration is very low at the low frequencies, it is better to put more power at the lower frequencies. With a higher response at low frequencies, it is easier to measure the force and acceleration.

The transfer function and coherence for Dataset2 are shown in Figure 6.11 and Figure 6.12. Again, the transfer functions are calculated with Welch’s method and the $H_1$-estimator.
Figure 6.11. A segment of transfer function for Dataset2

Figure 6.12. A segment of coherence function for Dataset2
Coherence has been improved dramatically for frequency range 1-2 Hz, by putting more power at the low frequencies. However, coherence still deviates from 1 at some frequencies. To verify that the transfer function obtained from Dataset 2 is reliable, a chirp signal and another bandpassed (1.2-2.2 Hz) random signal have been used to test the system.

A comparison between the transfer function calculated from Dataset 2 and the estimate obtained from the chirp signal (Dataset 5) is shown in Figure 6.13. A comparison between Dataset 2 and Dataset 4 is shown in Figure 6.14.

Figure 6.13. The transfer function calculated with a chirp signal is compared with the transfer function calculated with random excitation.
Figure 6.14. The transfer function shown in figure 6.6 is compared with the transfer function calculated from Dataset 4 (bandpassed signal).

The transfer function from the chirp signal test shows good agreement with the transfer function calculated from random data. The transfer function from the bandpassed signal test shows good agreement with the transfer function calculated from random data. It means that the transfer function from Dataset2 is reliable and can be used to study the system.

6.4 Modelling and System Identification

A simulation model is created in this section based on the assumption that there is only heave motion and the motion is small. The dynamic behaviour for the heave mode in this system is modelled with an SDOF system with added mass and added damping (viscous damping is ignored).
Figure 6.15. $M_B$ is the mass of the structure. $M_A$ is the added mass. $K_B$ is the buoyancy Stiffness. $C_A$ is the added damping. $F$ is the applied force. $Z_B$ is the heave motion.

For this experiment, buoy mass and buoyancy stiffness are calculated as shown in Eq. (6.7) - Eq. (6.8). Here, $r$ is the radius of the cylinder and $L_S$ is the draft.

$$M_B = \rho \pi r^2 L_s = 1.45 \text{ (kg)} \quad (6.7)$$

$$K_B = \rho g \pi r^2 = 284 \text{ (N/m)} \quad (6.8)$$

Added mass and added damping can be obtained using the measured transfer function. The transfer function found from the Dataset2 in section 6.4 will be used to identify the added mass and added damping.
Figure 6.16. A segment of the estimated added mass from experimental data.

Figure 6.17. A segment of estimated added damping from experimental data.
Added mass and added damping estimated from the transfer function as shown in Figure 6.16 and Figure 6.17.

Added mass converges to 1.45 kg at the high frequencies as can be seen from Figure 6.16. This is the high-frequency limit added mass for the floating cylinder and it is approximately equal to the buoy mass. At some frequencies the estimated added mass is negative. One possible explanation for this is that the measurement errors are larger at these frequencies, especially at the antiresonances where the acceleration level is very small.

Added damping behaves as expected in Figure 6.17, it is high at the resonance, and then goes down to zero at higher frequencies. Added damping is close to zero at the high frequencies, however it converges to a small value because the viscous damping is ignored in the calculations.

Digital filter is used to simulate the transfer function. The MATLAB command “invfreqz” is used to find the filter coefficients. 1000 number of b-coefficients and 3 a-coefficients gave a good result. To verify the digital filter can be used to simulate the system, filter transfer function and system transfer function are plotted together in Figure 6.18.

![Figure 6.18. Comparison between system transfer function and filter transfer function.](image)
Figure 6.18 shows the digital filter can be used to represent the system. Filter transfer function has almost same magnitude and phase as the system transfer function has.

### 6.5 Verification of Simulation Model

In this section, the simulation model will be verified by sending the measured force signal to the digital filter found in section 6.4. The simulation output is then compared with the measured output. If they are in a good agreement, it implies that simulation model is correct.

A typical segment of the measurement data compared with the simulated result is shown in Figure 6.19.

![Comparison between simulated acceleration and measured acceleration.](image)

*Figure 6.19. Comparison between simulated acceleration and measured acceleration.*
Using the simulation model, a transfer function is then calculated using random data. This transfer function is compared with the transfer function obtained from the experiment in Figure 6.20.

Simulated data shows good agreement with the measured data as can be seen from Figure 6.19 and Figure 6.20. It means that the model and theoretical model presented in previous section are correct and suitable for the experimental system.

6.6 Conclusions from Experimental Test

The experimental system shown in this chapter can be assumed as a linear SDOF system when the applied force is low. Since the system response is
very low at the low frequencies, in order to better measure the response from system, more power has been added to the low frequencies for the input signal. The transfer function obtained from random data after compensating the spectrum gives a better coherence function but still not good enough at very low frequencies. Thus, it is difficult to find hydrodynamic parameters in the low frequency region.

Added mass and added damping were estimated from the transfer function. The result shows that there is a strong interaction in a region between 1-2 Hz. In this frequency range it was possible to see several vibrating modes in the aquarium. It is difficult to verify the estimated added mass and added damping since a similar setup cannot be found in the literature. However, at the high frequencies added mass is quite close to the result for a heaving cylinder in infinite sea. A simulation model based on digital filters was used to simulate the system response and a comparison between the simulation results with the measurement data shows good agreement.
7 Conclusion

The dynamic behaviour of floating structures has been studied in this work. The floating structures were simplified into an SDOF system where frequency dependent added mass and added damping are used to model the hydrodynamic interaction.

To predict the buoy motion, digital filters were used to simulate the time response from the floating structure. This simulation model has been tested by comparing digital filters transfer functions with the system transfer functions. The comparison results confirm that digital filters can be useful to simulate time response from hydrodynamic structures.

This work has also studied system identification methods to identify the added mass and added damping from experimental data. These methods have been evaluated by using the simulation model to predict the time response that corresponds to a given input. The predicted added mass and added damping from the simulation data were compared with the simulation system’s added mass and added damping. It shows that the identification method can be used to estimate the added mass and added damping from experimental data. However these methods may be sensitive to the noise disturbance.

Finally, an experimental system has been studied. The added mass and added damping for the tested structure has been estimated from the experimental data. The result shows large peaks in the added mass and added damping when the frequency is close to the mode shapes of the water in the aquarium. A simulation model based on digital filters was used to simulate the system response and a comparison between the simulation results with the measurement data shows good agreement. This indicates that the estimated added mass and added damping coefficients are reasonable. A suggestion for further work is to verify the obtained estimated added mass and added damping coefficients for the studied system with theoretical calculations.
8 References


12. Welch, PD; The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short,
