Study of Combustion Oscillations in Gas Fired Appliances

Palli Kishore Kumar
Dasari Vijay Kumar

Department of Mechanical Engineering
Blekinge Institute of Technology
Karlskrona, Sweden
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Supervisor: Kjell Ahlin, Professor Mech. Eng.
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Thesis submitted for completion of Master of Science in Mechanical Engineering with emphasis on Structural Mechanics at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden.

Abstract:
The thesis work discusses abnormal combustion noise in gas-fired appliances. An experimental model was made to provide insight into the causes of abnormal combustion noises. The experimental model was modelled mathematically considering it as a feedback loop system. Finally a stability plot was made to find out at what frequencies the system is stable.

Keywords:
Acoustic Impedance, feedback control system, combustion chamber, modal analysis.
Acknowledgements

This work was carried out at the Department of Mechanical Engineering, Blekinge Institute of Technology, Karlskrona, Sweden, under the supervision of Prof. Kjell Ahlin.

The thesis work was initiated in June 2005.

Finally, we want to thank Prof Kjell Ahlin for his guidance and support.

Karlskrona, January 2006

Palli Kishore Kumar

Dasari Vijay Kumar
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1 Notation

\begin{align*}
A & \quad \text{Area} \\
cc & \quad \text{Damping coefficient} \\
D & \quad \text{Damping ratio} \\
E & \quad \text{Young’s modulus} \\
e & \quad \text{Standardised error} \\
G & \quad \text{Shear modulus} \\
g & \quad \text{Acceleration of gravity} \\
J & \quad \text{Mass moment of inertia} \\
k & \quad \text{Spring coefficient} \\
m & \quad \text{Total mass of air} \\
r & \quad \text{Radius} \\
t & \quad \text{Time (sec)} \\
L & \quad \text{Length of the rod (m)} \\
C & \quad \text{speed of sound (m/s)} \\
\text{Rho} & \quad \text{Density of air (kg/m}^3\text{)} \\
Dl & \quad \text{Element length (m)} \\
Mi & \quad \text{Element mass (Kg)} \\
Ei & \quad \text{Equivalent young’s modulus} \\
Ki & \quad \text{Element to element spring constant} \\
Z & \quad \text{Acoustic impedance}
\end{align*}
2 Introduction

When a new gas-fired appliance is made there is a chance that the design is unacceptable for some operating conditions. When considering the same burner operating outside the combustion chamber with same operating conditions it will be quiet, it is also observed that noise will occur when the burner is inserted in the chamber. The choice of the fuel is also of prime importance; finding out the right combination and applying the changes for better design is expensive and time consuming.

The present thesis work focuses on “combustion oscillations” which belong to a class of self excited oscillations in which energy (flame) is modulated through a passive feedback loop (the combustion chamber and burner).

The experimental set up as shown in figure 2.1 is used to get insight into the study of abnormal combustion noise; the experimental setup consists of a cylindrical tube with length of 610 mm and outer diameter 30mm, propane gas cylinder with torch and extension tube of length 305 mm.
To get insight into the problem of abnormal combustion oscillation experiments can be performed by using the experimental set up as shown in fig 2.1. The gas flow is adjusted until the flame is about 10mm high and then inserted to 60mm into the combustion chamber; a loud noise is heard which is recorded with help of the microphone placed outside the chamber; the above experiment is repeated for various insertion depths of 15 mm and 30 mm. The results are discussed in chapter 3.

To study the vibration behavior of the structure experimentally, modal analysis techniques (EMA) can be used from which the mode shape of the pressure of the open air column inside the hollow cylindrical tube (combustion chamber) can be calculated.
The experimental model was modeled mathematically considering the 
abnormal combustion noise in combustion chamber as a feedback loop 
system; elements of the feedback system is defined by the transfer function 
of the combustion chamber (cylindrical tube) and flame, transfer function 
of the chamber is the complex ratio of pressure to volume velocity which is 
“acoustic impedance”. The acoustic impedance can be calculated 
mathematically by two methods. Matlab codes were written to plot the 
Acoustic impedance of open air columns in hollow cylindrical tubes. One 
method was by building up the M and K matrices for an open air column in 
an hollow cylindrical tube and Matlab codes were made to plot the 
Frequency Response Function from which the acoustic impedance was 
calculated. The second method was by deriving the impulse response of the 
open air column in the cylindrical tube which is derived by considering a 
Dirac pulse propagating along a longitudinal section of a metal rod when 
excited by a horizontal impulse force. Instead of the flame transfer function 
a low pass Butterworth filter can be used as a feedback element, finally 
stability of the feedback system can be checked for different cut off 
frequencies and gain of the feedback.
3 Problem Description

In general it is very difficult to observe the build up of flame oscillations in the combustion chamber; it is a lot easier to record the pressure oscillation that is formed in the combustion in connection with the oscillation of flame.

In the design of the combustion chamber it is important to understand the difference between resonance and self exited oscillation because the change in design may solve the resonance problem but it does not solve the problem of combustion oscillation.

Thus an experimental setup was made in the lab to get insight into the causes of abnormal combustion oscillation.
Fig. 3.1. Diagram showing the combustion process in a hollow tube [1]

Several measurements were taken with the burner at various insertion depths inside the cylindrical combustion chamber. The curve in Fig. 3.1 shows a plot of sound frequencies obtained with the burner placed at 15 mm inside the hollow cylindrical tube. A spectrum until 1200 Hz frequency range was measured and it has three peaks at 354, 712 and 1075 Hz. Fig. 3.2 shows a plot of sound frequencies obtained with the burner placed at 30 mm inside the hollow cylindrical tube. A spectrum until 1200 Hz frequency range was measured and it has three peaks at 346, 705 and 1068 Hz. Fig. 3.3
shows a plot of sound frequencies obtained with the burner placed at 60 mm inside the hollow cylindrical tube. A spectrum until 1200 Hz frequency range was measured and it has three peaks at 335, 695 and 1065 Hz.

**Fig. 3.2.** Sound frequencies measured when the burner is on and placed at 15 mm inside the hollow cylindrical tube.

**Fig. 3.3** Sound frequencies measured when the burner is on and placed at 30 mm inside the hollow cylindrical tube.
The most important information to be gleaned from a comparison of the curves of Figs. 3.2, 3.3, 3.4, is that the frequency of the peaks changes with the burner insertion depth which clearly shows that the resulting sound frequencies are a result of self excited oscillations, if it was a resonance problem it could be solved by changing the natural frequency. By changing the insertion depths of burner natural frequency of the tube will change, since natural frequency is inversely proportional to wave length (natural frequency=speed of sound/wavelength). For this reason it is usually impossible to prevent combustion oscillations by changing the natural frequency of the combustion chamber. Except in rare cases, such a change would merely change the pitch of the noise without reducing the intensity. This can be demonstrated by changing the length of the tube forming the combustion chamber. Such a change in length changes the natural frequencies of the combustion chamber in an inversely proportional manner. The result is that the frequency of the noise changes with the natural frequency of the combustion chamber but the intensity of the noise is not diminished. This illustrates why it is important to distinguish between the self-excited oscillations and true resonances. If the sound were caused by a true resonance, then a change in the natural frequency of the combustion chamber would be a sure way for reducing the intensity of the
noise. This proves that the phenomenon is not a true resonance but a self-excited oscillation.
4 Build up of Combustion oscillations

The relation between the cause of pressure oscillation and flame oscillation and their interdependence is very difficult to analyze and is a known fact since the oscillation of the flame is accompanied by the pressure oscillation in combustion chamber

As more than one mechanism is involved it is difficult to analyze. It stands to reason that, however, that if there is any way at all for the pressure oscillations to modulate the burning of the flame, then there is a possibility of a vicious circle in which the pressure oscillations, caused by flame oscillations, will cause even bigger oscillations of the flame. This vicious circle is called the “feedback loop”.

Out of the many known cases of self excited oscillations, combustion oscillations are a type of self excited oscillations. The acoustical oscillations of musical instruments such as flute’s and organ pipes are known examples of self excited oscillations. In all cases of self excited oscillations there must be a source of energy, the supply of which is being modulated by a feedback. In the case of the flute, the source of energy is a stream of air supplied by the player so as to be responsive to oscillations of the air inside the bore of the flute.

Figure 3. From reference [1] shows the shape and size of inner cone changes during the oscillation cycle. It is well established that the surface area of the inner cone of a Bunsen type flame is proportional to the amount of fuel burned per unit of time. Thus it can be seen that the flame burns at a faster rate during the part of the cycle represented by Frame “A” of the high speed movie than during any other part. During this part of the cycle the frame therefore consumes an extra amount of flue gases. Let the volume of the extra amount of fuel/air mixture be denoted by Q1 and that of the extra amount of flue gases be denoted by Q2. Since the density of the flue gases is about 1/7 of that of the fuel/air mixture Q2 is seven times as large as Q1 and the flame has, in effect, generated a small burst of excess volume. [1]

\[ Q_3 = Q_2 - Q_1 = G \times Q \]

Where G stands for gain in the volume.
Introducing a small burst of excess volume into the confinement of the combustion chamber causes a pressure rise. This pressure rise adds up for each successive cycle and forms a feedback loop resulting in combustion oscillations. We come to know that the spontaneous build-up of combustion oscillations depends critically on the timing of the pressure oscillation which is the effect of a flame oscillation relative to the pressure oscillation which was the cause of the pressure oscillation.[1]
5 Frequency Domain Evaluation

The differential equations for the time dependence of pressure and volume in the combustion chamber, for the flow of fuel/air mixture through the burner ports, and the combustion process were obtained from reference [1]. The differential equations were simplified by changing them into the frequency domain. The differential equations could be simplified further if the initial build-up does not involve non-linear phenomenon. This condition is mostly met in gas fired appliances, since the mean flows are small with respect to the speed of sound and pressure drops are small with respect to the atmospheric pressure. Under these conditions the frequency domain analysis techniques are employed for preventing self-excited oscillations in linear feedback control systems. The experimental evaluation of the elements of feedback loop can be done easily in the frequency domain, rather than in the time domain. [1]

Figure 5.1 shows the elements of the feedback loop in a form commonly used by control engineers. A symbolic representation of combustion process is shown if fig.5.1. Each element in the feedback loop is defined by an input and output which occur within the flame but not at separated points. The box Z in the feedback loop actually indicates the flame insertion in the combustion chamber. Similarly the input and output of the burner stand for the pressure oscillations in gas flow through the burner ports. The flame represents an amplifier because the volume of burnt gases is larger than the volume of fuel/air mixture entering the combustion process. The amplification is the main cause of self-excited oscillations.

The “transfer functions” describe the cause and effect relation for each element in the feedback loop, which are ratio of output by input for an element.
For the flame:

\[ G = \frac{q_3}{q_4} \] (5.1)

For the combustion chamber:

\[ Z = \frac{p}{q_3 + q_{ext}} \] (5.2)

For the burner:

\[ H = \frac{q_4}{p} \] (5.3)

Figure 5.1 Symbolic representation of Feedback Loop Involved in Self-Excited Combustion Oscillations

P = amplitude of pressure oscillation at burner ports
q_4 = amplitude of flow oscillation through burner ports
q_3 = amplitude of oscillations in the generation of excess flue gas volume
\( q_{\text{ext}} \) = amplitude of artificial excitation of combustion chamber.

Q3 denotes the excess volume generated by the combustion process.

\( q_{\text{ext}} \) represents any external excitation of the combustion chamber which may, for instance, be applied artificially in order to study the stability margin of the combustion system which does not oscillate spontaneously.
6 Conditions for Self-Excitation

Self-excited oscillations are oscillations in the generation of excess flue gas volume \( q_3 \) which either grow or; at least, maintain themselves without external excitation (i.e. when \( q_{ext} = 0 \)). The relationships between the three transfer functions that will give rise to self-excited oscillations can, therefore, be derived by first eliminating \( q_4 \) and \( p \). [1]

\[
Z \cdot H \cdot G = \frac{p}{(q_3 + q_{ext})} \cdot \frac{q_4}{p} \cdot \frac{q_3}{q_4} = \frac{q_3}{q_3 + q_{ext}} \quad (6.1)
\]

And then setting \( q_{ext} = 0; \)

\[
Z \cdot H \cdot G - 1 = 0 \quad (6.2)
\]

The transfer functions are all complex functions of frequency; their full description, therefore, requires plots of either their real or imaginary components as functions of frequency. \( Z, H \) and \( G \) are complex which means that self excitation can only occur at those frequencies at which the phase angles of \( Z, \) of \( H, \) and of \( G \) add up to zero or a multiple of 360 degrees. Equation 6 represents the border line condition under which self-excited oscillations maintain themselves but do not grow. Oscillations will only build up if the right hand side of Eq. (6.2) is larger than zero at a frequency at which the phase angle condition is met. The magnitude of the left hand side of Eq. (6.2) then is a measure of the build up rate. [1]

The objective of any design or modification for the prevention of self excited oscillations in gas-fired appliances is simply to get a negative rather than a positive build up rate. Stated mathematically, this requires that:

\[
|Z \cdot H \cdot G| < 1 \quad (6.3)
\]

At any frequency at which

\[
\angle Z \cdot H \cdot G = 0 \quad (6.4)
\]
These relations are somewhat easier to use if they are rewritten in the form:

\[ |Z| < \left| \frac{1}{H \times G} \right| \]  \hspace{1cm} (6.5)

\[ \angle Z = \angle \left( \frac{1}{H \times G} \right) \]  \hspace{1cm} (6.6)

This objective can be approached primarily in either of two ways:

1. Shift one or more of the phase angles so that Eq. (6.4) is satisfied only at frequencies where \( |Z| \ll \left| \frac{1}{H \times G} \right| \); this is usually controlled by burner design.

2. Reducing any peaks of \( |Z| \) so that Eq. (6.5) is satisfied at all frequencies where the phase angle of \( Z \) approaches that of \( \frac{1}{H \times G} \).
   This is usually controlled by the design of the combustion chamber.

When \( q_{\text{ext}} \neq 0 \);

\[ \frac{p}{q_{\text{ext}}} = \frac{Z}{1 - Z \times H \times G} \]  \hspace{1cm} (6.7)
Properties of Transfer Functions

Linear acoustic theory offers a basis from which we can predict the effect of frequency on the transfer functions of the combustion chamber and of the burner with considerable confidence and the effects of design parameters in at least a qualitative manner. Qualitative estimates allow us to draw general conclusions as to trends and to identify methods of attack which might be useful in special cases. A quantitative analysis of a specific design would require, of course, a detailed quantitative knowledge of the transfer functions for the particular design.
7.1 Transfer function of combustion chamber

The transfer function of the combustion chamber is the complex ratio of pressure to volume velocity, which is the “acoustic impedance”. For simple hollow cylindrical combustion chamber, the impedance as a function of frequency, dimensions of the cylinder, and flame insertion depth within the combustion chamber was calculated theoretically. The results obtained theoretically were compared with the results obtained from experiments. The impedance so calculated for a simplified combustion chamber has all the salient features of the impedance of the actual combustion chambers determined experimentally. The parameters like the number of spring mass systems considered for the theoretical model was modified from 200 to 62 i.e. the M and K matrices which was of the dimension 200*200 was reduced to a matrix with dimensions 62*62 so that the theoretical model fits the experimental model. We can observe from Fig. 7.1 that the magnitude of impedance as a function of frequency is dominated by a series of dips each representing the natural frequencies of the combustion chamber. The resonance frequency of the experimental setup was calculated by using sound excitation from a radio. The resonance frequencies obtained by this method were around 280 and 560Hz. We can see that the dips in the impedance plots were around 280 and 560 Hz by which we can say that the theoretical model fits the experimental model.
Figure 7.1 Calculated magnitude of Transfer function Z Of a cylindrical combustion chamber.

Figure 7.2 Resonance frequencies of the experimental setup obtained by random sound excitation from a radio
7.2 Transfer Function of the Burner and Flame

The burner and flame are so closely interrelated that they cannot be separately defined by different transfer functions. Eqs (6.5) and (6.6) use only the combined transfer functions $H \times G$ which indicate that they are closely interlinked. The transfer function of burners is the ratio of amplitude of oscillation of volume through burner ports to the ratio of pressure oscillations at burner ports and the angle of the flow oscillations relative to that of the pressure oscillations is the phase angle.

The part in the feedback loop which has very less information is the G and H which are the flame oscillations and the burner. The known thing about the flame is it has a gain which is due to increase in volume while combustion and has a phase lag which is due to the fact that a particle of gas doesn’t burn immediately upon leaving the burner port. Due to the various time delays the excess volume contributions from different parts of flame are not all in phase with each other. The result is that the total phase angle is not simply proportional to frequency and the total gain tends to diminish with increasing frequency. It is quite probable that the lower gains at high frequencies are one of the reasons why combustion oscillations occur usually only at the lowest natural frequency of the combustion chamber. So during the theoretical modelling, the transfer function of the burner and the flame is approximated by a low pass filter, since a low-pass filter passes relatively low frequency components in the signal but stops the high frequency components.
8 Modal Analysis of the combustion Chamber

The vibration behaviour of the structure should be studied experimentally in order to have a precise identification and know how of the dynamic characteristics, namely natural frequencies, mode shapes and damping. These are the characteristics which depend on the weight and stiffness of a structure and might affect the response due to excitation.

The phenomenon, Resonance, occurs when a structure excited at a frequency that coincides with the natural frequency of the structure. These are the frequencies that a structure will select if allowed vibrating freely without any excitation. For each of these natural frequencies there exists a specific pattern in which the structure deforms, so called the mode shape. At resonance the amplitude of the response increases dramatically, which can be restricted by the damping present in the structure. Continued vibration at resonance may result in fatigue.

The experimentation includes both data acquisition and its subsequent analysis giving the dynamic properties, which is termed modal testing and popularly known as Experimental Modal Analysis or EMA. Ultimately this gives the mathematical description of the structure.

In the present thesis Experimental modal analysis techniques were used to find out the mode shapes of the particle velocity of the open air column inside the hollow cylindrical tube (combustion chamber).
8.1 Solution for undamped force system

The equation of motion for a multiple degree of freedom system is given by

\[
[M] \ddot{x}(t) + [C] \dot{x}(t) + [K]x(t) = \{F(t)\}
\]  \hspace{1cm} (8.1)

In the above equation we have a mass matrix \([M]\), a diagonal matrix, and symmetric matrices\([C]\), damping matrix, and stiffness matrix \([K]\). Here we are interested in finding the mode shapes of the air column so \(M, K\) become

\[m = \frac{\rho AL}{N}\]
\[k = \frac{Nc^2 \rho A}{L}\]

\[K = \begin{bmatrix}
k & -k & 0 \\
-k & 2k & -k \\
-k & 2k & \ddots \\
\end{bmatrix}
\]

\[M = \begin{bmatrix}
\frac{\rho AL}{N} \\
\frac{\rho AL}{N} & \ddots \\
\ddots & \ddots \\
\end{bmatrix}
\]

The solution for a second order, linear, time invariant system begins often with an undamped system (\([C] = 0\)). The system of equations is then reduced as
\[ [M]\dddot{x} + [K]\dot{x} = \{F\} \]  

(8.2)

For brevity notation \( t \) is dropped. Now considering free solution (unforced), which means the forcing function is zero.

\[ \{F\} = 0 \]  

(8.3)

The general solution from the theory of calculus for equation (8.1) is known as

\[ \{x\} = \{X\}e^{\lambda t} \]  

(8.4)

Here \( \lambda \) is the complex valued frequency given by \( \lambda = \sigma + j\omega \). With equations 8.3 and 8.4 equation 8.1 becomes

\[ (\dot{\lambda}^2[M] + [K])\{X\} = 0 \]  

(8.5)

To obtain non trivial solution to the above equation, the determinant should be zero and on rewriting to decouple the system equation gives

\[ \left( \det[M]^{-1} + \lambda^2[I] \right)\{X\} = 0 \]  

(8.6)

The equation 8.6 is a polynomial equation whose roots give the Eigen frequencies (natural frequencies). The vector \( \{X\} \) that satisfies the equation (8.6) for the corresponding Eigen frequency is the Eigen vector or the mode shape vector \( \{\psi_r\} \).
8.2 Procedure

The mode shapes of the particle velocity in the open air column of the hollow cylindrical tube (combustion chamber) were measured by creating an impulse with the help of a pulse generator and sensing the response at the place of excitation with the help of a microphone. Ten points along the length of the open air column were chosen for excitation using a pulse generator and the response collected at each of the ten different positions. The microphone and the pulse generator were connected to the FFT analyzer which is connected to a computer to convert the data collected during experimentation from the time domain into the frequency domain. The data collected was later used in Matlab for modal analysis.

Figure 8.1 Experimental setup used for the study of abnormal combustion noise in gas fired appliances
Figure 8.2 Pulse generator used for producing impulse excitation in the hollow cylindrical tube

Figure 8.3 Hollow tubes used to find out the mode shapes of particle velocity in open air column
Figure 8.4 Circuit used to supply DC power to the microphone

Figure 8.5 Setup for data acquisition (computer + Signalcalc mobilyzer (dynamic signal analyzer)}
8.3 Results

The mode shapes of the combustion chamber at 1\textsuperscript{st} and 2\textsuperscript{nd} natural frequencies 280 Hz and 560 Hz respectively, calculated experimentally using experimental modal analysis techniques are shown in figure 8.6 and 8.8. Figure 8.7 and 8.9 shows the pressure and velocity pattern inside the cylindrical tube.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.6.png}
\caption{First mode shape obtained from modal analysis (280Hz)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.7.png}
\caption{Pressure (shown in black) and velocity (green) pattern inside cylindrical tube at 1\textsuperscript{st} natural frequency when both ends are open}
\end{figure}
Figure 8.8. Second mode shape obtained from modal analysis (560Hz)

Figure 8.9. Pressure (black) and velocity (green) pattern inside cylindrical tube at 2\textsuperscript{nd} natural frequency when both ends are open
9 Theoretical Derivation of Acoustical Impedance for open Air column

9.1 Variables considered

\[ Area = \pi \times \left( \frac{0.028}{2} \right)^2 \]  
Cross section of the tube [m²]

\[ L = 0.610; \]  
Length of the open air column [m]

\[ \rho = 1.21; \]  
Density of air [kg/m²]

\[ c = 343; \]  
Speed of sound in air [m/s]

\[ m = L \times area \times \rho; \]  
the total mass of the air [kg]

\[ dl = L/n; \]  
Element length [m]

\[ mi = m/n; \]  
Element mass [kg]

\[ E = \rho \times c \times c; \]  
Equivalent Yong's modulus [N/m²]

\[ ki = \frac{E \times area}{dl} \]  
Element-to-element spring constant [N/m]
9.2 Setting up of M and K matrices

The M and K matrices can be set up for an open column by replacing the variables m, k in the symmetric M and K matrices.

For an open air column

\[
\begin{align*}
  m &= \frac{\rho AL}{N} \\
  k &= \frac{Nc^2 \rho A}{L}
\end{align*}
\]

The size of the M and K matrices depends on the no of spring mass systems we want to choose to make the theoretical model fit to the experimental model. The number of spring mass systems was chosen to be around 62 in the theoretical model.

The M and K matrices for an open air column

\[
K = \begin{bmatrix} 
  k & -k & 0 \\
-2k & 2k & -k \\
 0 & -k & 2k & -k \\
  & -k & \ddots & \ddots & -k \\
  & & -k & k \\
\end{bmatrix}
\]

\[
M = \begin{bmatrix} 
  \frac{\rho AL}{N} & \cdot & \cdot \\
  & \frac{\rho AL}{N} & \cdot \\
  & & \cdot \\
\end{bmatrix}
\]

Matlab codes were written to plot the frequency response function for an M, C and K system. The frequency response obtained from M, C, and K system is mobility FRF which is the ratio of velocity by Force [(m/s)/N].
Since impedance $Z$ is the ratio of pressure/volume velocity [$\text{Pa/(m}^3\text{/s)}$] so the mobility FRF can be changed into the Impedance FRF by dividing the inverse of $H$ with $(\text{area})^2$ which results in the units of impedance. Result Shown in figure 9.2

$$Z = \frac{H}{(\text{area})^2}$$

The Acoustic Impedance was also calculated by deriving the impulse response of the tube. The impulse response is derived by considering a Dirac pulse moving in a tube just as a longitudinal wave passing in a cylindrical rod. When we excite the tube at some point along its length we produce a Dirac passing to the left and the other to the right of the tube. The time taken for the Dirac to go to the left end is taken as $T_1$ and the one to the right end as $T_2$. The Diracs oscillate along the length of the tube and based on the time intervals the Diracs interact and add up to each other the impulse response of the tube was derived. From the impulse response the impedance was calculated in the similar manner as above.

Figure.9.1.Example of a Dirac passing through a longitudinal bar

$$H\ (s)\ =\ (2\ +\ 1\ *\ e^{-sT_1}\ +\ e^{-sT_2})$$
For another $T$ it becomes

\[
= (2 + 1.e^{-sT_1} + e^{-sT_2}) \ast (2 + 1.e^{-sT_1} + e^{-sT_2} .... e^{-2nsT_1}) \\
= (2 + 1.e^{-sT_1} + e^{-sT_2}) \ast (1 - e^{-sT})^{-1} \\
= \frac{(2 + 1.e^{-sT_1} + e^{-sT_2})}{(1 - e^{-sT})}
\]

**Fig.9.2.** Green curve indicates the acoustic impedance from the impulse response of the tube and the black curve indicates the acoustic impedance derived from the $M$, $C$ and $K$ model.

Figure 9.2 shows the impedance derived from impulse response and $M$, $C$ and $K$ model, up to 900 Hz frequency of the both peaks are same after that black curve is not coherent with green curve. A plausible explanation for these phenomena is the system is being damped.
10 Feedback control system

The events in the burner and in the flame are so closely interrelated that any separation of the two is bound to be arbitrary. There are many types of burners for which such type of separation is unnecessary or impossible. This is why Equations (6.5) and (6.6) use only the combined transfer functions $H \times G$.

The poles and residues obtained from the Acoustic Impedance (Z) of the theoretical model were used to get the condensed forced time response filter coefficients for system; mobility of the system is defined by ratio of residues and poles. The theoretical model of the feedback control system with the burner and the combustion chamber was made in MATLAB, in which the transfer function of the burner and the flame are approximated by a low pass filter. A second order Butterworth filter was used to get the filter coefficients B, A, which were used to simulate the feedback system. Then a stability plot was made to find out at what frequencies the system is stable for the obtained theoretical results. The various plots obtained during the simulation in MATLAB are shown below.

The nyquist plot helps us to gain insight into the stability of the closed-loop system by analyzing the contour of the frequency response function in the complex plane. A nyquist plot was made to check the stability of the system at 432 Hz and the plots shows that the system is stable. Fig 10.2 shows the time response of the output obtained from feedback system which is modelled theoretically using the filter coefficients b, a, B, A. The coefficients b, a are the condensed forced time response filter coefficients for system, mobility of the system is defined by ratio of residues and poles which are obtained from the theoretical model of the Acoustic Impedance ‘Z’. Fig 10.4 shows the time response of the output obtained from feedback system which is modelled theoretically using the filter coefficients b, a, B, A where the filter coefficients B was multiplied by a factor 1.01. The PSD is plotted to check at what frequency in the frequency domain maximum power is concentrated, Fig 10.3 and Fig 10.5 which resulted in 432 Hz.
Fig. 10.1 Nyquist plot of the system

Fig. 10.2 Output obtained from a theoretically modelled feedback system using the coefficients b, a, B, A
Fig. 10.3 PSD of the feedback obtained from above system (432Hz)

Fig. 10.4 Output obtained from a theoretically modelled feedback system when the coefficient B is multiplied by a factor 1.01
Fig. 10.5. PSD of the output of the above feedback system with the coeff B multiplied by a factor 1.01 (432Hz)
11 Stability Diagram

The stability of the system is determined by the poles of the transfer function, the poles of transfer function decide whether the system is stable or unstable.

In s-plane, the correlation between dynamic behaviour and position of the poles and zeros are important in analysis and design of continuous system by the correlation $z = e^{Ts}$ these correlations can be used to derive analogous insights in $z$ plane, where $T$ is sampling interval. [9]

Pole and zeros in s and z plane are

$$s = -a \pm jb$$

$$z = e^{(-a \pm jb)T} = e^{-aT} e^{\pm jbT}$$

Stability condition: In the s-plane, the imaginary axis ($a=0$) is the boundary of the stable pole region, This axis maps into $z = e^{(\pm jbT)}$, a circle of unit radius about the origin ,called unit circle ,as b moves along imaginary axis. Stability requires that $a>0$, so $e^{(-aT)} < 1$.Hence follows the basic theorem for stability of control system

Stability theorem states that the all poles of the system z transfer function must lie inside the unit circle in the $z$-Plane.

Gain of the feedback defined as a function of how much output is feedback to the input,” If the loop gain too great the system may go in to oscillation”

Stability of the system is verified by calculating the poles of the system. The poles are obtained by calculating the roots of the denominator of the transfer function. If the poles of the system transfer function lie inside the unit circle then the system is stable. Stability of the system is verified for different gain values and cut off frequency of the second order butter worth low pass filter. By using cut off frequency, filter coefficients b, a of feedback loop element were calculated, which is assumed as the combined
burner and flame transfer function, the gain values were chosen in the range of \(1e^{-7}\) and \(10e^{-7}\) for the corresponding cut-off frequencies from 400 to 700 to check at what corresponding gain values of the feedback loop the system is stable or unstable. The stability of the system is plotted and is shown in fig below.

![Stability Diagram](image)

\(\text{Figure.11.1. Stability plot showing the stable and unstable regions of the feedback control system}\)
12 Conclusion

The aim of the project was to study the cause of feedback excitation oscillations. A model was made in the lab to check the cause of the feedback excitation oscillation experimentally. The experimental results obtained were useful in comparing the results from the mathematical model. The experimental model was modelled mathematically considering it as a feedback loop system. A mathematical model was also made to find out the stable regions of the system. The mathematical procedures were built into a modal tool box to be used in MATLAB. Finally we could find out at what gain and frequencies the system is stable or unstable. From the thesis work we could find an approach to make a mathematical model which can be later used to solve abnormal combustion noise problems. By modelling dampers mathematically to solve abnormal combustion noise problems and then making a prototype of the required damper saves a lot of money and time for the designer as it is a costly affair if the prototypes are manufactured and checked experimentally if they work or not.
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