Input Filter Design to Prevent Line Oscillations in Buck Converter

Raghavendra Pappala
Raghavender Reddy Naredla

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Blekinge Institute of Technology
School of Engineering
Department of Electrical Engineering
Supervisor: Anders Hultgren
Examinar: Sven Johansson
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Contact Information:

Author:
1) Raghavendra Pappala
   Email: rag986p@gmail.com
2) Raghavender Reddy Naredla
   Email: raghavenderreddyn@gmail.com

Supervisor: Dr. Anders Hultgren

School of Engineering
Blekinge Institute of Technology
Web: www.bth.se/ing
37179 Karlskrona
Sweden
Phone: +46 455 385000
Abstract

Micropower AB is a company that design and produce battery chargers for heavy vehicles. Buck converters are used in some chargers as they provide a regulated output voltage. In buck converter, transistor acts as a switch which is controlled by Pulse Width Modulated signal. The input line voltage is applied to the buck converter through a small input filter capacitor. The charger can induce oscillations on the line current and the input filter capacitor voltage. These oscillations could be critical resulting, the buck converter not to work. This thesis addresses how to reduce or to limit these oscillations by adding a passive filter as an input to the buck converter. The systems are simulated in MATLAB.
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1. Introduction

Buck converters are widely used for their functionality of regulating voltage levels. In the buck converter the input voltage is bucked in amplitude and a lower amplitude voltage appears at the output. An input filter is often employed between the converter and the input as it attenuates the switching harmonics present in the converter input current. It has been noticed by Micropower that the charger in some situations can induce oscillations on the line current and the input filter capacitor voltage. Due to these oscillations, it has been observed that the input filter voltage can go below the desired output voltage causing the converter not to work. An attempt to reduce these oscillations has been done using different input passive filters. In Chapter 2, a simple buck converter has been designed and the function of the different components has been explained in short and the frequency diagram of the buck converter is plotted using Matlab. In Chapter 3, the Extra Element Theorem has been explained and the different impedances are defined accordingly which are used for the design of the input filter. In Chapter 4, Passive filters are explained in short and the design of the input filter is explained considering different impedances. In Chapter 5, Micropower Model is explained in brief and is implemented in Matlab and the output is plotted for different inductances. In Chapter 6, a short introduction to damp the filters is given and is further explained and Micropower model is damped with the best suited damping filter and the outputs are plotted for different values of inductances. In Chapters 7 & 8, an R-C filter is added in series with the inductor and damping the filter is shown. Also the capacitor and inductor are varied for different values and as a conclusion the best way recommended is explained.
2. Buck Converter

Buck converters are widely used in reducing the voltage of dc supply. Compared to the linear regulators, buck converters are efficient and don’t waste energy. The chapter explains the operation of the converter and the small signal model of the converter.

The block diagram of the buck converter is shown in Fig.2.1:

![Block Diagram of Buck Converter](image)

The converter uses a pair of switches, one controlled (Transistor, T) and one uncontrolled (Diode ‘D’) to achieve unidirectional power flow from input to output. The capacitor ‘C1’ and Inductor ‘L1’ store and transfer energy from input to output, and filter the output voltage. The inductor especially limit the current rush through the switch when circuit is ‘ON’ and the capacitor minimize the voltage overshoot and ripple present at the output of a step down converter. The transistor T is controlled by a Pulse Width Modulated signal. When the switch is ‘ON’, it conducts inductor current and the diode becomes reverse biased. When the switch is ‘OFF’, due to the inductive energy storage, inductor current continues to flow. This inductor current flows through the diode ‘D’ until the switch is turned ‘ON’ again.
2.1 Small signal model for the buck converter:

Consider the circuit shown in Fig. 2.2 and choose for the choice of best independent and dependent variables which in this case would be $i_2, v_1$ and $i_1, v_2$ respectively.
Plotting $i_2(t)$ and $v_2(t)$ versus time is shown in figure Fig.2.3 and Fig.2.4.

By time averaging, we have

$$< i_1 >_{T_s} = < i_2 >_{T_s} + < 0 > (1 - d) T_s$$

$$\Rightarrow < i_1 >_{T_s} = d < i_2 >_{T_s}$$

$$< Vg >_{T_s} = < 0 > (1 - d) T_s$$

$$\Rightarrow d < Vg >_{T_s}$$

Where $d = D + \hat{d}$

$\hat{d}$ = duty ratio

The above large signal models are linearized to obtain small signal model.

Starting point of large signal model for dependant I source in primary.
Multiply and neglecting second order terms, $\dot{d}i_2$ results

$$\Rightarrow D(i_2 + i_2) + i_2 \dot{d}$$

Fig.2.6. Current source in primary
Starting point of the large signal model for dependant V source in secondary

\[(D+\hat{d})(V_1+\hat{v}_1)\]

Fig. 2.7.

Multiplying and neglecting second order terms, we have

\[\Rightarrow D(V_1 + \hat{v}_1) + V_1\hat{d}\]

Fig. 2.8. Voltage source in secondary

By combining the small signal input/output circuits through DC transformer, we have
The Fig. 2.10 shows the frequency diagram of Buck converter shown in Fig. 2.2. with L1=18µH, C1= 45µf.
Fig. 2.10. Frequency diagram of the buck converter
3. Extra Element Theorem

Change of transfer function by the addition of impedance to the network is shown by the Extra Element Theorem of R. D. Middle Brook. The theorem leads to impedance inequalities which guarantee that an element does not substantially alter the transfer function. The theorem described below is based on R.W. Erickson[4].

Consider the linear circuit shown in Fig.3.1.

Network in the Fig.3.1 contains an input $V_{in}(s)$ and an output $V_{out}(s)$, also a port whose terminals are open circuited. Assume a transfer function from $V_{in}(s)$ to $V_{out}(s)$ be defined as

$$\frac{V_{out}(s)}{V_{in}(s)} = G(s)|_{Z(s)\rightarrow \infty} \quad (3.1)$$

The Extra Element theorem determines how the transfer function $G(s)$ is modified in the presence of an impedance $Z(s)$ connected between the terminals at the port.

Fig.3.2 shows the circuit with the addition of an element having impedance $Z(s)$ to the terminals at the port.
The result is

\[
\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = (G(s)|_{Z(s)\to\infty}) \left( \frac{1+\frac{Z_{\text{in}}(s)}{Z(s)}}{1+\frac{Z_{\text{out}}(s)}{Z(s)}} \right)
\]  

(3.2)

The right hand side terms involving \(Z(s)\) account for the influence of \(Z(s)\) on \(G(s)\), known as the “Correction Factor”.

Fig. 3.2. Addition of new element having impedance \(Z(s)\)
3.1 Extra Element Theorem for Dual form:

The theorem for the dual form where the extra element is replaced by short circuit is described below.

Consider the linear circuit shown in the Fig.3.3.

The transfer function in this form is initially known under the conditions that the port is short-circuited.
The short-circuit is replaced by the impedance $Z(s)$ as shown in the Fig. 3.4.

The addition of the impedance $Z(s)$ causes the transfer function to become

$$
\frac{V_{out}(s)}{V_{in}(s)} = \left( G(s) \bigg|_{Z(s) \to \infty} \right) \left( \frac{1 + \frac{Z(s)}{Z_N(s)}}{1 + \frac{Z(s)}{Z_D(s)}} \right)
$$

(3.3)
3.2 Relation between the Transfer functions:

The relation between the two \( G(s) \) transfer functions is described below.

By equating the equations (3.2) and (3.3)

\[
rac{G(s)|_{Z(s)=\infty}}{G(s)|_{Z(s)=0}} = \frac{Z_D(s)}{Z_N(s)}
\]

(3.4)

This is known as reciprocity relationship.

The quantities \( Z_N(s) \) and \( Z_D(s) \) can be found by measuring the impedances at the port.

The term \( Z_D(s) \) is known as the driving point impedance which is the Thevinin’s equivalent impedance seen looking into the port.

This impedance is found by setting the source \( V_{in}(s) \) to zero and measuring the impedance between the terminals of the port as shown in the Fig.3.5.

\[
Z_D(s) = \frac{V(s)}{i(s)}|_{V_{in}(s)=0}
\]

\[
Z_N(s) \text{ is determined as shown in Fig.3.6.}
\]
$Z_N(s)$ is found under the condition that the output $V_{out}(s)$ is nulled to zero. A current source $i(s)$ is connected to the terminals of the port which in the presence of input signal $V_{in}(s)$. The current $i(s)$ is adjusted so that the output $V_{out}(s)$ is nulled to zero. Under these conditions, the quantity $Z_N(s)$ is given by

$$Z_N(s) = \frac{V(s)}{i(s)} |V_{out(s)}_{null}\rangle$$
4. Input Filter Design

An input filter is often needed for the converter as it serves to prevent the converter switching current ripples from being reflected back into the source, into the line; also the input filter attenuates the switching harmonics from the line present in the converter input current. The input filter generally limits conducted electromagnetic interference (EMI). The input filter protects the converter and its load from transients that appear in the input voltage, thereby improving the system reliability.

4.1 Passive filters:

The combination of passive components is known as passive filters. Passive components doesn’t increase the power of the signal applied to them but can possibly cause power loss. They can be used to reduce power of signal, to select part of signal by its voltage, frequency or its time relationship to another signal to change the shape of a waveform or to pass a signal from one section of a circuit to another, but in every case the power of the signal are decreased, or unchanged but never increased.

Passive filters do not depend on an external power supply and they do not contain active components such as transistors. Passive components are Resistors, Capacitors and Inductors. Inductors block high frequency signals and conduct low frequency signals, while capacitors do the reverse.

It is because of the above stated properties of the passive components passive filter is chosen as the input filter for the buck converter.
4.2 Input filter design for the converter:

The Micropower converter is controlled by Current Programmed Controller. In the current programmed control the converter is controlled by the choice of peak transistor switch current peak \( i_s(t) \). The control input signal is a current \( i_c(t) \), and a simple control network switches the transistor on and off, such that the peak transistor current follows \( i_c(t) \). The transistor is still controlled by a PWM-signal, but duty cycle \( d(t) \) is not directly controlled. It depends on \( i_c(t) \) as well as on the converter inductor currents, capacitor voltages and power input voltage.

In the current programmed control, the latch input is a clock pulse which initiates the switching period causing the latch output to be high and turning on the transistor. The transistor conducts and its current \( i_s(t) \) is equal to the inductor current \( i_L(t) \). This current increases depending on the value of inductance and converter voltages. Eventually the switch current \( i_s(t) \) becomes equal to the control signal \( i_c(t) \) resulting the controller to turn the transistor switch off and the inductor current decreases for the remaining PWM period \( T_s \).

But adding an input filter, the dynamics of the converter are modified. It modifies the transient response of the system which can lead the system to become unstable. Also the output impedance becomes large over some frequencies exhibiting resonances. An L-C filter is added to buck converter as in Fig. 4.2, the small-signal equivalent circuit model is modified as in Fig. 4.3. The input filter elements affect all transfer functions of the converter, including the control-to-output transfer function \( G_{vd}(s) \) and the converter output impedance resulting in possible...
destabilizing the system. By introducing damping into the input filter and by designing the input filter such that its output impedance is sufficiently small, the effect of the input filter on the control-to-output transfer function can be reduced.

The control-to-output transfer function is defined as:

\[ G_{vd}(s) = \frac{V_o(s)}{\hat{d}(s)} \]  

(4.1)

To determine the control-to-output transfer function in presence of input filter, set \( V_i(s) = 0 \) and solve for \( V_o(s)/d(s) \). The input filter can then be represented by its output impedance \( Z_0(s) \). The input filter thus is treated as an extra element having impedance \( Z_0(s) \). The modified transfer function in addition of the input filter using “Middle brook’s Extra Element” theorem is expressed as

\[ G_{vd}(s) = \left( G_{vd}(s)_{|Z_0(s)=0} \right) \frac{1+\frac{Z_0(s)}{Z_0(s)}}{1+\frac{Z_0(s)}{Z_0(s)}} \]  

(4.2)

Where, \( (G_{vd}(s)_{|Z_0(s)=0}) \) is the control-to-output transfer function with no input filter.

The Fig.4.2. shows the buck converter with an L-C input filter,

![Fig.4.2. buck converter with an L-C input filter](image-url)
The small-signal model for the above buck converter with input filter is shown in Fig.4.3.

Fig.4.3. Small-signal model for the above buck converter with input filter

\[ Z_D(S) \] is equal to the converter input impedance under the condition, \( i_c \dot{d}(s) = 0 \).

\[ Z_D(s) = Z_i(s)|_{\dot{d}(s)=0} \quad (4.3) \]
Upon setting $\hat{d}(s) = 0$, $Z_D(s) = Z_i(s)$, the above figure minimizes to,

![Circuit Diagram]

Fig 4.4 determination of $Z_D(s)$

Therefore,

$$Z_D(s) = \left(sL1 + \frac{\frac{1}{sC1}}{\frac{1}{R} + \frac{1}{sC1}}\right) \frac{1}{D^2}$$

(4.4)

$$= \frac{1}{D^2} \left(sL1 + \frac{R}{sRC1 + 1}\right)$$

$$= \frac{1}{D^2} \left(\frac{Rs^2L1C1 + R}{sRC1 + 1} + sL1\right)$$

$$= \frac{R}{D^2} \left(1 + \frac{sL1}{R} + s^2L1C1\right)$$

$$= \frac{R}{D^2} \left(\frac{1 + s\frac{L1}{R} + s^2L1C1}{sRC1 + 1}\right)$$

(4.5)
$Z_N(s)$ is equal to the converter input impedance under the condition, the controller varies $d(s)$ to maintain $\bar{v}_o(s)$ equal to zero.

$$Z_N(s) = Z_i(s)|_{\bar{v}_o(s)=0} \quad (4.6)$$

The input filter design criteria for Buck converter is given by,

$$Z_N(s) = -\frac{R}{D^2} \quad (4.7)$$

$$Z_D(s) = \frac{R}{D^2} \frac{1+\frac{sl_1}{R}+s^2C_1}{1+sRC_1}$$

$$Z_e = \frac{sl_1}{D^2}$$

From equation, it is noted that the addition of input filter causes the control to output transfer function to be modified by the factor,

$$\frac{1 + \frac{Z_0(s)}{Z_N(s)}}{1 + \frac{Z_0(s)}{Z_D(s)}} \quad (4.8)$$
called the correction factor.

When the following inequalities are satisfied

\[ \|Z_0\| \ll \|Z_N\|, \|Z_0\| \ll \|Z_D\| \]  

(4.9)

the correction factor has a magnitude of approximately unity. Also the input filter does not alter the control-to-output transfer function. These inequalities limit the output impedance of the input filter.

When,

\[ \|Z_0\| \ll \|Z_e\|, \|Z_0\| \ll \|Z_D\| \]

the converter output impedance is not affected by the input filter.

\(Z_D(s)\) is taken from the input filter design criteria of the buck converter.

\(Z_e(s)\) is equal to the converter input impedance under the conditions that the converter output is shorted.

\[ Z_e = Z_{l|\delta_0=0} \]

Consider,

\[ G_{\text{vout}}(s) = (G_{\text{vout}}(s)|_{Z_0(s) = 0}) \frac{1 + \frac{Z_0(s)}{Z_N(s)}}{1 + \frac{Z_0(s)}{Z_D(s)}} \]  

(4.10)

To find how the input filter modifies the control-to-output transfer function of the converter.

\(Z_N(s)\) and \(Z_D(s)\) is taken by considering the buck converter.

Setting \(d(s) = 0\), \(Z_D(s)\) is equal to the input impedance of the R-L-C filter, divided by the square of the turns ratio.

\[ Z_D(s) = \frac{1}{D^2} \left( sL_1 + R \right) \frac{1}{sC_1} \]  

(4.11)

Series resonance occurs at output filter frequency,

\[ f_0 = \frac{1}{2\pi \sqrt{L_1C_1}} \]

The value of asymptotes at resonant frequency \(f_0\) given by characteristic impedance \(R_0\), transformer primary:
The Q-factor is given by,

\[ Q = \frac{R}{R_0} = R \sqrt{\frac{C_1}{L_1}} \]  \hspace{1cm} (4.12)

The value of \( \|Z_D(j\omega)\| \) is,

\[ \|Z_D(j\omega)\| = \frac{R_0}{D^2} / Q \]  \hspace{1cm} (4.13)

\( Z_N(s) \) is taken by,

\[ Z_N(s) = Z_l(s) \big|_{v_o(s) \to 0} \]  \hspace{1cm} (4.14)

This impedance is equal to the converter input impedance \( Z_l(s) \), under the condition that \( d(s) \) is varied to maintain the output voltage \( v_o(s) \) at zero.

Since the voltage \( v_o(s) \) is zero, the current through the capacitor and load impedances are also zero. The inductor current \( i(s) \) and transformer winding currents are zero and hence the voltage across the inductor is also zero.

A test current \( i_{test}(s) \) is injected at the converter input port. The impedance \( Z_N(s) \) can be viewed as the transfer function from \( i_{test} \) to \( v_{test}(s) \):

\[ Z_N(s) = \frac{v_{test}(s)}{i_{test}(s)} \big|_{v_o \to 0} \]  \hspace{1cm} (4.15)

Since the currents in windings of the transformer model are zero, the current \( i_{test}(s) \) is equal to the independent source current \( d(s) \):

\[ i_{test}(s) = L \hat{d}(s) \]  \hspace{1cm} (4.16)

The voltage \( v_i(s) \), equal to the output voltage plus the inductor voltage, is also zero.

As \( v_i(s) \) is equal to zero, the voltage applied to the secondary of the transformer model is equal to the independent source voltage. Upon dividing by the turns ratio \( D \), \( v_{test}(s) \) is:

\[ v_{test}(s) = - \frac{v_i(s)}{D} \]  \hspace{1cm} (4.17)

From \( i_{test}(s) \) and \( v_{test}(s) \),

\[ Z_N(s) = \left( \frac{-v_i(s)}{D \hat{d}(s)} \right) = - \frac{R}{D^2} \]  \hspace{1cm} (4.18)
When the independent source voltage \( v_i(s) \) is set to zero the network reduces to the figure below.

![Fig 4.6 determination of the filter output impedance \( Z_0(s) \)](image)

\( Z_0(s) \) is given by the parallel combination of the inductor \( L_2 \) and \( C_2 \):

\[
Z_0(s) = sL2\left|| \frac{1}{sc_2} \right|
\]

Here, the magnitude \( \|Z_0(j\omega)\| \) is dominated by the inductor impedance at low frequency, and by the capacitor impedance at high frequency. The inductor and the capacitor asymptotes intersect at the filter resonant frequency:

\[
f_f = \frac{1}{2\pi\sqrt{L_2c_2}}
\]

The filter has the characteristic impedance

\[
R_{0f} = \sqrt{\frac{L_2}{c_2}}
\]

Since the input filter is not damped, its Q-factor is ideally infinite. From the bode plot of the filter output impedance we can determine whether the impedance inequalities in the equation (4.9) are satisfied.

Fig.4.7. is generated considering the input filter used in the Micropower converter shown in Fig.5.1. which shows the output impedance of the filter with different values of line inductance \( L_1 \) (\( L_2=2\mu H, L_2=20\mu H, L_2=200\mu H \)) in comparison with \( Z_N \) and \( Z_D \).
Fig. 4.7. Output impedance of the filter with different values of L2 in comparison with $Z_N$ and $Z_D$

It can be seen from the Fig. 4.7. that the output impedance of the input filter is larger than $Z_N$ and $Z_D$, which does not satisfy the condition in equation (4.9).
5. Micropower Model

The chapter explains the Micropower company’s design and is tested in Matlab and the outputs are shown.

The micro power model of the buck converter is shown in Fig.5.1

![Micropower converter model](image)

The company Micropower uses a full bridge converter in one of their battery chargers that can be modeled as buck converter with L1 inductor and C1 capacitor. An L-C filter is added as an input to the buck converter with inductor L2 and capacitor C2. The input filter is given an input 125volts through the voltage source V1 and the output is assumed to be 85-95volts at V2. The converter used is a current programmed controlled.
The circuit is modeled as a differential system using Kirchhoff’s laws and graph theory.

The current voltage equations for the Micropower model are:

**Case 1: When the switch is closed**

\[
\frac{du_{C2}}{dt} = \frac{i_{L2}}{C2} - \frac{i_{L1}}{C2}
\]

\[
\frac{du_{C1}}{dt} = -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1}
\]

\[
\frac{dl_{L2}}{dt} = -\frac{u_{C2}}{L1} + \frac{V_1}{L2}
\]

\[
\frac{dl_{L1}}{dt} = \frac{u_{C2}}{L1} - \frac{u_{C1}}{L1}
\]

**Current-Voltage equations**

**Case 2: When the switch is open**

\[
\frac{du_{C2}}{dt} = \frac{i_{L2}}{C2}
\]

\[
\frac{du_{C1}}{dt} = -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1}
\]

\[
\frac{dl_{L2}}{dt} = -\frac{u_{C2}}{L1} + \frac{V_1}{L2}
\]

\[
\frac{dl_{L1}}{dt} = -\frac{u_{C1}}{L1}
\]

**Current-Voltage equations**
With the aid of given specifications and the above equations, the circuit is modeled and simulated in MATLAB and the output is shown in the figure below.

Fig. 5.2. Output of Micropower buck converter model

X-axis: time in seconds.

Y-axis: voltage in volts.

Here from the Fig. 5.2. It is observed the oscillations \((iL_2)\) are induced by the charger into the line current and the input filter capacitor voltage. The Micropower model is then varied with three different cases of line inductance with \(L_2=2\text{uh}, 20\text{uh}, 200\text{uh}\) and is simulated in Matlab and the results are shown in Fig. 5.3, Fig. 5.4, Fig. 5.5.
The line inductance is modeled as three different cases:

Micropower Model with L2 = 2µh (the case of stiff line)

Fig. 5.3. Output of Micropower buck converter model with L2 = 2µh
Micropower model with L2= 20µh (case of intermediate stiff line)

Fig. 5.4. Output of Micropower buck converter model with L2= 20µh
Micropower Model with L2 = 200µh (case of weak line):

![Micropower Model with L2 = 200µh](image)

The different L2’s will imply that the input filter bandwidth will depend on the line inductance.

Fig.5.5. Output of Micropower buck converter model with L2 = 200µh

The Fig.5.6. below shows the comparison of the different input filter frequency diagrams with buck converter frequency diagram.
Fig. 5.6. Comparison of the different input filter frequency diagrams with buck converter frequency diagram
6. Damping the Micropower Model

The chapter explains the need for the input filter to be damped and methods to damp the input filter and the best method chosen to damp the filter is explained and simulated in Matlab.

Adding an input filter to the current programmed Micropower converter causes the controller to oscillate in some cases.

The input filter changes the dynamics of the converter. Adding the filter also modifies the transient response, resulting the control system to become unstable. The output impedance is large over some frequency range exhibiting resonance. The input filter elements affect all the transfer function of the converter.

However, this instability problem can be mitigated by introducing adequate damping into the input filter and can also be avoided by decreasing the bandwidth of the controller.

6.1 Resistor in parallel with the Capacitor ‘C2’:

The filter could be damped by adding a resistor R3 in parallel with capacitor C2 as shown in the Figure 8.1.

![Diagram](image-url)

Fig.6.1. Resistor in parallel with C1

It results in power dissipation problem. The dc voltage V1 is applied across R3 and therefore R3 dissipates power \( \frac{V1^2}{R3} \) which is greater than the load power.
6.2 Resistor in parallel with Inductor ‘L2’

The dc voltage across L2 is zero, so there is no power loss in resistor R3. But this degrades the high frequency asymptote and there is no attenuation provided by L2.

![](diagram1.png)

Fig.6.2. Resistor R3 in parallel with L2

In this case it is not recommended to damp the inductor L2 as it is a line inductance and cannot be modified.

6.3 R3-C3 in parallel with C2

A dc blocking capacitor C3 is added in series with R3. Power loss is eliminated as the dc current cannot flow through R1. The value of C3 should be large, so that the output impedance of the network reduces to output impedance of the filter and the impedance of R4-C3 branch should be less than R4. This provides adequate damping.

![](diagram2.png)

Fig.6.3. R3-C3 in parallel with C1

There are some other approaches to damping the input filter which are R3-L3 parallel damping across L2 and R3-L3 series damping the L2, they are not recommended as the inductor L2 is line inductance and cannot be damped.
So by considering R3-C3 parallel damping across the capacitor C2, the circuit is modeled as shown in the Fig.6.4. below.

\[ V_1 = 125V, \ V_2 = 90V, \ L_2 = 0.2\mu H, \ L_1 = 18\mu H, \ C_2 = 225\mu f, \ C_1 = 45\mu f, \ C_3 = 1.3615 \times 10^{-5}, \ R = 0.0251\Omega, \ R_3 = 0.5075\Omega \]

![Fig.6.4. Considering R3-C3 parallel damping across the capacitor C2](image)

The current-voltage equations have been defined accordingly using Kirchhoff’s laws.

Case 1: When the switch is closed

\[
\frac{du_{C2}}{dt} = -\frac{u_{C2}}{R3C2} + \frac{u_{C3}}{R3C2} + \frac{i_{L2}}{C2} - \frac{i_{L1}}{C2}
\]

\[
\frac{du_{C1}}{dt} = -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1}
\]

\[
\frac{du_{C3}}{dt} = \frac{u_{C2}}{R3C3} - \frac{u_{C3}}{R3C3}
\]

\[
\frac{dl_{L2}}{dt} = -\frac{u_{C2}}{L1} + \frac{V_1}{L2}
\]

\[
\frac{dl_{L1}}{dt} = \frac{u_{C1}}{L1} - \frac{u_{C2}}{L1}
\]
Case 2: When the switch is open

\[
\begin{aligned}
\frac{du_{C2}}{dt} &= -\frac{u_{C2}}{R3C2} + \frac{u_{C3}}{R3C2} + \frac{i_{L2}}{C2} \\
\frac{du_{C1}}{dt} &= -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1} \\
\frac{du_{C3}}{dt} &= \frac{u_{C2}}{R3C3} - \frac{u_{C3}}{R3C3} \\
\frac{dl_{L2}}{dt} &= -\frac{u_{C2}}{L1} + \frac{V_1}{L2} \\
\frac{dl_{L1}}{dt} &= -\frac{u_{C1}}{L1}
\end{aligned}
\]

Current-Voltage equations

6.4 Calculations to find \( R3 \) and \( C3 \):

The values of the inductor \( L1 \) and capacitor \( C1 \) used in the Micropower converter model are:

\( L2 = 0.2 \mu\text{h} \)

\( C2 = 225 \mu\text{f} \)

The output impedance of the filter is determined from Fig.4.5 by the equation below:

\[
z_0 = \frac{L2}{L2 + C2 + 1} \quad (6.1)
\]

\[
= 2 \times 10^{-7}
\]

Also, the characteristic impedance of the input filter is needed to find the values of \( L2 \) and \( C2 \) and is determined using the below equation.

The Characteristic impedance of the input filter is

\[
R_{of} = \sqrt{\frac{L2}{C2}} \quad (6.2)
\]

\[
= 0.02981\Omega
\]
The value of peak output impedance for the optimum design is chosen to be 1,

\[ ||Z_o||_{mm} = R_{of} \frac{\sqrt{2(2 + n)}}{n} = 1 \]  \hspace{1cm} (6.3)

Here, the quantity ‘n’ is defined as the ratio of the blocking capacitance C3 to the filter capacitance C2.

Further solving the above equation, the ratio of C3 to C2 is determined.

The ratio of the blocking capacitor C3 to the capacitor C2 is given by

\[ n = \frac{R_{of}^2}{||Z_o||_{mm}^2} \left(1 + \sqrt{1 + \frac{||Z_o||_{mm}^2}{R_{of}^2}} \right) \]  \hspace{1cm} (6.4)

\[ = 0.0605 \]

From the ratio of C3 to C2 the value of C3 can be defined.

Since C2 is given,

\[ n = \frac{C3}{C2} \]  \hspace{1cm} (6.5)

\[ C3 = n \times C2 = 1.3615 \times 10^{-5} f. \]

The value of damping resistance that leads to optimum damping is described by

\[ \frac{R3}{R_{of}} = \sqrt{\frac{(2+n)(4+3n)}{2n^2(4+n)}} \]  \hspace{1cm} (6.6)

The above expression is solved to determine the value of R3 and is shown below.

\[ R3 = R_{of} \sqrt{\frac{(2+n)(4+3n)}{2(n)^2(4+n)}} = 0.5075 \Omega \]
The model is then implemented in the MATLAB and the output is shown in the figure below.

![Permits current regulated buck converter](image)

Fig.6.5. Output of R3-C3 parallel damping across C2.

Also the inductor ‘L2’ is varied for different values in the circuit and accordingly the values of resistor R3 and the capacitor C3 are calculated and the circuit is simulated in the MATLAB and the outputs are shown in the below figures.
For $L_2 = 2\mu h$

$\begin{align*}
R_3 &= 0.5230\Omega \\
C_3 &= 4.4474 \times 10^{-5} f
\end{align*}$

![Permits current regulated buck converter graph](image)

**Fig. 6.6.** For $L_2 = 2\mu h$, X-axis: time in seconds, Y-axis: voltage in volts.

From the **Fig. 6.6.** it is noted that $iL_2$ oscillates at around 20V to 120V gradually decreases.
For $L_2 = 20\,\mu\text{H}$

$R_3 = 0.5690 \, \Omega$

$C_3 = 1.5565 \times 10^{-4} \, \text{f}$

Fig.6.7. For $L_2 = 20\,\mu\text{H}$, X-axis: time in seconds, Y-axis: voltage in volts.
For $L_2 = 200\mu h$

$R_3 = 0.6806 \Omega$

$C_3 = 6.6904 \times 10^{-4} f$

![Graph of a current regulated buck converter](image)

**Fig. 6.8.** For $L_2 = 200\mu h$, X-axis: time in seconds, Y-axis: voltage in volts.

From the figures 6.6 to 6.8 it is found that the line current $i_{L2}$ oscillates with increase in line inductance. This shows that the charger induces oscillations, which are critical for the buck converters performance.
Fig. 6.9. frequency diagram of damping the input filter with R3-C3 parallel damping with different values of L2

The Fig. 6.9. shows that even after the input filter is damped, the output impedance is still larger than $Z_N$ and $Z_D$.

For the impedance inequalities in equation (4.9) to be satisfied and for the filter to be properly damped, the capacitor is chosen and tested with different values which are larger than the company’s 225µf capacitor and the best value is chosen, provided the filter is damped by satisfying equation (4.9) i.e. output impedance of the input filter should be less than the impedances $Z_N$ and $Z_D$. 
Capacitor $C_2$ is chosen to be $2250 \mu f$ the output impedance of the filter is higher than $Z_N$ and $Z_D$ for $L_2 = 20 \mu h$ and $200 \mu h$

For $C_2 = 22500 \mu f$, it is noted that the output impedance of the filter is higher than $Z_D$ for $L_2 = 200 \mu h$.

For $C_2 = 2 \times 22500 \mu f$ $Z_O$ is equal but not less than $Z_D$ for $L_2 = 200 \mu h$.

For $C_2 = 3 \times 22500 \mu f$

![Input filter design comparison](image)

**Fig.6.10.** frequency diagram of damping the input filter with R3-C3 parallel damping with different values of L2 and for $C_2 = 3 \times 22500 \mu f$

From the Fig.6.10. it is noted that the filter is damped sufficiently for the input filter capacitor with $3 \times 22500 \mu f$, we see that the output impedance of the filter is lesser than the impedances $Z_N$ and $Z_D$ thus by satisfying the equation 4.9.

By considering $C_2 = 3 \times 22500 \mu f$ the output of the converter for three different inductances is plotted in the below figures.

For $L_2 = 2 \mu h$
R3 = 0.5014 Ω

C3 = 7.3685 \times 10^{-4} f

Fig. 6.11. For L2 = 2µh., X-axis: time in seconds, Y-axis: voltage in volts.

For L2 = 20µh
R3 = 0.5043Ω
C3 = 0.023f

Fig.6.12. For L2 = 20µh, X-axis: time in seconds, Y-axis: voltage in volts.

For L2 = 200µh
$R_3 = 0.5134\Omega$

$C_3 = 0.0076f$

Fig. 6.13. For $L_2 = 200\mu h.$, X-axis: time in seconds, Y-axis: voltage in volts.
7 Buck converter with R-C Input filter

The Buck converter with R-C Input filter with Resistor R3 connected in series with C1 is shown below with the values.

\[ V_1 = 125V, \quad V_2 = 90V, \quad L_2 = 0.2\mu H, \quad L_1 = 18\mu H, \quad C_2 = 225\mu f, \quad C_1 = 45\mu f, \quad R_1 = 0.5m\Omega, \quad R_2 = 5.44m\Omega, \quad R_3 = 3m\Omega \]

Fig.7.1. Buck converter with R-C Input filter with Resistor R3 connected in series with C2

A resistor R3 is connected in series with the capacitor C2. The value of R3 is taken to be the equivalent series resistance (ESR) of the capacitor ‘C2’ which is taken to be in between 2mΩ to 5mΩ.

The filter here is a low pass filter which attenuates the high frequency components. The oscillations produced are nothing but the high frequency components. The circuit is modeled as a differential algebraic system using Kirchhoff’s laws and graph theory, the equations for the closed switch and the open switch circuit is defined.
Case 1: When the switch is closed

\[
\frac{du_{C2}}{dt} = \frac{i_{L2}}{C2} - \frac{i_{L1}}{C2}
\]

\[
\frac{du_{C1}}{dt} = -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V2}{RC1}
\]

\[
\frac{di_{L2}}{dt} = -\frac{u_{C2}}{L1} - \frac{R3i_{L2}}{L2} + \frac{R3i_{L1}}{L2} + \frac{V1}{L2}
\]

\[
\frac{di_{L1}}{dt} = \frac{u_{C2}}{L1} - \frac{u_{C1}}{L1} + \frac{R3i_{L2}}{L1} - \frac{R3i_{L1}}{L1}
\]

Current-Voltage equations

Case 2: When the switch is open

\[
\frac{du_{C2}}{dt} = \frac{i_{L2}}{C2}
\]

\[
\frac{du_{C1}}{dt} = -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V2}{RC1}
\]

\[
\frac{di_{L2}}{dt} = -\frac{u_{C2}}{L1} - \frac{R3i_{L2}}{L2} + \frac{V1}{L2}
\]

\[
\frac{di_{L1}}{dt} = -\frac{u_{C1}}{L1}
\]

Current-Voltage equations
The circuit is simulated in MATLAB and the output is shown in the figure below.

Fig. 7.2 Output of buck converter with input R-C filter, X-axis: time in seconds, Y-axis: voltage in volts.
The inductor $L_2$ has been varied for three different cases:

$L_2 = 2\mu h$ (stiff line case):

![Graph](image)

Fig. 7.3. For $L_2 = 2\mu h$, X-axis: time in seconds, Y-axis: voltage in volts.
L2=20µh (intermediate stiff line case)

Fig.7.4. For L2 = 20µh., X-axis: time in seconds, Y-axis: voltage in volts.
L2 = 200µh (weak line case):

Fig. 7.5. For L2 = 200µh, X-axis: time in seconds, Y-axis: voltage in volts.
Damping the R-C input filter and varying L2 and C2 in Micropower Model

Damping of the input filter with R-C can be shown with the values

\[ V_1 = 125\text{V}, \ V_2 = 90\text{V}, \ L_2 = 0.2\mu\text{H}, \ L_1 = 18\mu\text{H}, \ C_2 = 225\mu\text{f}, \ C_1 = 45\mu\text{f}, \ C_3 = 1.9406 \times 10^{-5}\text{f}, \]
\[ R_1 = 0.5\text{m}\Omega, \ R_2 = 5.44\text{m}\Omega, \ R_3 = 3\text{m}\Omega, \ R_4 = 0.3604\Omega \]

As the inductor L2 is the line inductance, the option left to damp the filter is the R4-C3 parallel damping the input filter. The closed and the open switch circuit current voltage equations are defined accordingly.
Case 1: When the switch is closed

\[
\begin{align*}
\frac{du_{C2}}{dt} &= -\frac{u_{C1}}{R4C2} + \frac{u_{C3}}{R4C2} + \frac{i_{L2}}{C2} + \frac{i_{L1}}{C2} \\
\frac{du_{C1}}{dt} &= -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1} \\
\frac{du_{C3}}{dt} &= \frac{u_{C2}}{R4C3} - \frac{u_{C3}}{R4C3} \\
\frac{di_{L2}}{dt} &= -\frac{u_{C2}}{L1} - \frac{R3i_{L2}}{L2} + \frac{V_1}{L2} \\
\frac{di_{L1}}{dt} &= \frac{u_{C2}}{L1} - \frac{u_{C1}}{L1} - \frac{R3i_{L1}}{L1}
\end{align*}
\]

Current-Voltage equations

Case 2: When the switch is open

\[
\begin{align*}
\frac{du_{C2}}{dt} &= -\frac{u_{C2}}{R4C2} + \frac{u_{C3}}{R4C2} + \frac{i_{L2}}{C2} \\
\frac{du_{C1}}{dt} &= -\frac{u_{C1}}{RC1} + \frac{i_{L1}}{C1} + \frac{V_2}{RC1} \\
\frac{du_{C3}}{dt} &= \frac{u_{C2}}{R4C3} - \frac{u_{C3}}{R4C3} \\
\frac{di_{L2}}{dt} &= -\frac{u_{C2}}{L1} - \frac{R3i_{L2}}{L2} + \frac{V_1}{L2} \\
\frac{di_{L1}}{dt} &= \frac{u_{C1}}{L1}
\end{align*}
\]

Current-Voltage equations

The values of R4 and C3 are calculated from the equations (6.1) to (6.6).
R4 = 0.3604Ω  
C3 = 1.9406×10^{-5} μF

The circuit is simulated in MATLAB and the output is shown in the figure below.

Fig. 8.2: Damping R-C input filter, X-axis: Time in seconds, Y-axis: Voltage in volts.
Varying $L_2$ and $C_2$ in the Micropower Model:

By varying the values of $C_2$ in the Micropower Buck converter model the circuit has been simulated in MATLAB. With the aid of same equations defined in the Micropower Buck converter model and by changing the value of $C_2$ to a smaller value compared to the Micropower converter model, the circuit has been simulated in the MATLAB and the outputs with different $L_2$’s are shown below.

For the case of stiff line ($L_1=2\mu H$), $C_1=10\mu F$:

![Diagram showing output of Micropower Buck converter model with $L_1=2\mu H$, $C_1=10\mu F$, X-axis: Time in seconds, Y-axis: Voltage in volts.]

Fig.8.3: Output of Micropower buck converter model with $L_1=2\mu H$, $C_1=10\mu F$, X-axis: Time in seconds, Y-axis: Voltage in volts.
For the case of intermediate stiff line (L1=20µH), C1=10µf:

![Graph](image)

**Fig. 8.4.** Output of Micropower buck converter model with L1 = 20µH, C1 = 10µf. X-axis: Time in seconds, Y-axis: Voltage in volts.

For the case of weak line (L1=200µH), C1=10µf:
Fig. 8.5. Output of Micropower buck converter model with $L_1 = 200\mu\text{H}$, $C_1 = 10\mu\text{f}$. X-axis: Time in seconds, Y-axis: Voltage in volts.
9. Conclusion

In this thesis, attempts were made to reduce or to limit the oscillations induced by the line current in Micropower converter model using a passive input filter. A small signal model for the buck converter is developed and discussed. With the aid of Extra Element theorem, the input filter design has been implemented using the control-to-output transfer functions and by considering impedance inequalities.

The line inductance in the Micropower model has been varied for three different cases found the charger to be inducing oscillations. The input filter in the Micropower model is then damped to reduce the oscillations. The input filter capacitor is varied for a better value and the filter is damped. A resistor is then added to the input filter in the Micropower model and outputs were shown accordingly. The input filter capacitor has been chosen to be smaller one compared to the Micropower model to reduce the cost effectiveness and the circuit has been simulated in the Matlab and found to be minimizing the oscillations on the line current.
References:


