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1 Introduction

The power plant short-term operation scheduling is a field of extensive and continuous research. Many methodologies and optimization techniques have been proposed that try to solve the problem of optimal unit commitment and power dispatch [1]. The deregulation of electricity markets in many countries has altered the objectives of power producers. While previously they were caring about how to follow a specific electrical load at the minimum cost, now the target is to maximize the profits by selling the electricity in the markets. In this economic environment the plant operators have to schedule their production under the uncertainty of electricity prices. Furthermore, the increased use of renewable energy sources in the grid results in higher volatility of electricity prices and higher demand for regulating power due to the intermittent power production. Therefore there is a need for new tools and techniques to be developed in order to handle all these new challenges.

Combined heat and power (CHP) plants can produce power and useful heat. Traditionally they find application in the industry or in residential district heating networks. The simultaneous power and heat production, however, makes the operation scheduling problem harder to solve compared to a conventional power plant as more restrictions are applied. A survey of the various methods proposed for the optimal CHP short-term operation is given in [2]. In this work, a model is proposed for the optimal short-term (24-hours ahead) operation planning of a CHP system under the uncertain parameters of electricity prices and heat demand. The uncertain parameters are incorporated into the model through a number of scenarios and stochastic programming framework is used for modeling the problem. To test the performance of the model, a case study is conducted.

2 Mathematical formulation of the problem

The formulation of the problem is done in three discrete steps: first the decision framework is considered, then the scenarios of the stochastic parameters are made and finally the mathematical model is formulated and solved. These steps are described in the following subsections.

2.1 The decision framework

In the stochastic programming framework the decisions are divided into decisions that have to be made before any stochastic parameter is realized (here-and-now) and decisions that are made with knowledge of the outcome (wait-and-see). This decision making process is formulated into a scenario tree with many stages where each stage represents a time point when decisions are made (fig.1). The stages on the left refer to earlier decisions. The proposed model derives the optimal power and heat production during the next day. The first stage decisions are made after the clearing of the spot market. At that time point the CHP producer knows exactly how much power has to produce next day (Day-1). This is equivalent to covering a specific load and the decision is about how to distribute the load among the units. Therefore the first stage variables are the unit commitment and power output of the units. The second stage variable is the heat output of the units during Day-1. This decision has not to be taken in advance but the producer can wait till the hour of the heat delivery to decide how much heat will be produced according to the heat demand. Finally, the third stage variables are the power offer in the spot market and the heat production the following
day (Day-2). This third stage is used in order to derive optimal unit commitment decisions for the final hour of Day-1, a method suggested in [3]. The scenario tree of the model is depicted in fig.1.

\[ \begin{align*}
\text{1st stage: Power dispatch} & \quad \text{Unit commitment} \\
\text{2nd stage: Heat dispatch} & \quad \text{Day-1 Heat demand: } Q_{D,t,\omega} \\
\text{3rd stage: Power Trading} & \quad \text{Heat dispatch: } Q_{D,t,\omega} \\
\end{align*} \]

Figure 1: Scenario tree of the proposed model

2.2 The scenarios

The scenario making is divided in two steps. In the first step a forecasting method is used to build a model that can predict future values of the stochastic parameters. There are many forecasting methods like time series analysis, neural networks, hybrid methods etc. In this work time series analysis is used to build a model that can predict future values of spot market prices and heat demand in a district heating network. A SARIMA model is used for the spot market prices and a SARIMAX for the heat demand where the external parameter is the outdoor temperature. In the second step Monte Carlo simulation is used to produce the scenarios using the previous models. Then the scenarios are combined to correspond to the scenario tree.

2.3 The model

Objective function: The objective function of the problem (1) is to maximize the profits of the CHP producer which consist of the revenues from the power sold in the spot market during the 2nd day of the planning horizon (A) minus the variable production costs (B), the start-up costs (C) and the shut down costs (D) during the whole planning horizon.

\[
\text{Maximize :} \\
\sum_{\omega=1}^{N_\Omega} \pi_{\omega} \left( 2N_T \sum_{t=N_T+1}^{N_T} \sum_{g=1}^{N_G} \lambda_{t,g,\omega} P_{gt,\omega} - 2N_T \sum_{t=1}^{N_T} \sum_{g=1}^{N_G} \left( \lambda_{f,g} P_{\text{fuel},gt,\omega} + c_{\text{start},g} u_{gt,\omega} + c_{\text{stop},g} z_{gt,\omega} \right) \right) 
\]

Non-anticipativity constraints: These constraints (2-4) ensure that the structure of the scenario tree is applied into the problem. This means that the variables referred to common scenarios have to be assigned the same values.

\[
P_{gt,\omega} = P_{gt,\omega+1} \quad \forall g,t = 1,...,N_T,\omega = 1,...,N_\Omega - 1 \tag{2}
\]

\[
u_{gt,\omega} = u_{gt,\omega+1} \quad \forall g,t = 1,...,N_T,\omega = 1,...,N_\Omega - 1 \tag{3}
\]

\[
Q_{gt,\omega} = Q_{gt,\omega+1} \quad \forall g,t = 1,...,N_T,\omega = 1,...,N_\Omega - 1 : \text{ if } Q_{D,t,\omega} = Q_{D,t,\omega+1} \tag{4}
\]

Operational constraints: These constraints apply the operational limits of the CHP units. The most common type of steam turbine used in large CHP systems is the extraction condensing steam turbine. It is characterized for its flexibility. The feasible zone is described by (5-8). The fuel consumption is given by (9). In many CHP plants there are only heat producing boilers that are usually used during peak heat demand hours. The operational limits for these boilers are given by (10) and the fuel consumption by (11).
\[ \beta_{c,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} \leq \beta_{c,g} P_{max,g} u_{gt\omega} \quad \forall g, \forall t, \forall \omega \]  
\[ \beta_{c,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} \geq (\beta_{c,g} + \beta_{th,g}/r_{min,g}) P_{min,g} u_{gt\omega} \quad \forall g, \forall t, \forall \omega \]
\[ Q_{gt\omega} \leq Q_{max,g} \quad \forall g, \forall t, \forall \omega \]
\[ P_{gt\omega} \geq r_{min,g} Q_{gt\omega} \quad \forall g, \forall t, \forall \omega \]
\[ P_{fuel,gt\omega} = \beta_{c,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} + \beta_0, g u_{gt\omega} \quad \forall g, \forall t, \forall \omega \]
\[ Q_{min,g} u_{gt\omega} \leq Q_{gt\omega} \leq Q_{max,g} u_{gt\omega} \quad \forall g, \forall t, \forall \omega \]
\[ P_{fuel,gt\omega} = \frac{Q_{gt\omega}}{\eta_{heater,g}} \quad \forall g, \forall t, \forall \omega \]

**Heat balance constraints:** The heat load balance constraint ensures that the total heat production is equal to the total heat demand including the changes in the content of the heat storage tank (12-13). The capacity of the tank (14) limits the maximum heat content.

\[ V_{t+1\omega} = V_{t\omega} + \sum_{g=1}^{N_G} Q_{gt\omega} - Q_{D,t\omega} \quad t = 1, \ldots, 2N_T - 1, \forall \omega \]  
\[ V_{1\omega} = V_{2N_T} + \sum_{g=1}^{N_G} Q_{g2N_T\omega} - Q_{D,2N_T\omega} \quad \forall \omega \]
\[ V_{t\omega} \leq V_{max} \quad \forall t, \forall \omega \]  

**Power balance constraint:** The power balance constraint (15) is applied during the 1st day of the planning horizon according to the decision framework. It simply says that the total power output must satisfy the load.

\[ \sum_{g=1}^{N_G} P_{gt\omega} \geq P_{S,t} \quad t = 1, \ldots, N_T, \forall \omega \]  

**Unit commitment constraints:** These constraints (16-17) assign values to the binary variables \( u, y \) and \( z \) which keep the state of the units, operating, starting-up or shutting down respectively.

\[ y_{gt\omega} \leq u_{gt\omega}, \quad y_{gt\omega} \leq 1 - u_{gt-1\omega}, \quad y_{gt\omega} \geq u_{gt\omega} - u_{gt-1\omega} \quad \forall g, \forall t, \forall \omega \]  
\[ z_{gt\omega} \leq u_{gt-1\omega}, \quad z_{gt\omega} \leq 1 - u_{gt\omega}, \quad z_{gt\omega} \geq u_{gt-1\omega} - u_{gt\omega} \quad \forall g, \forall t, \forall \omega \]

**Minimum up and down time constraints:** These constraints (18-23) are applied to avoid the frequent transitions from on state to off state and vice versa.

\[ \sum_{t=1}^{L_g} (1 - u_{gt\omega}) = 0 \quad \forall g, \forall \omega \]  
\[ \sum_{\tau=t}^{t+UT_g-1} u_{gt\omega} \geq UT_g y_{gt\omega} \quad \forall g, t = L_g + 1, \ldots, 2N_T - UT_g + 1, \forall \omega \]
\[ \sum_{\tau=t}^{2N_T} (u_{gt\omega} - y_{gt\omega}) \geq 0 \quad \forall g, t = 2N_T - UT_g + 2, \ldots, 2N_T, \forall \omega \]
\[ \sum_{t=1}^{F_g} u_{gt\omega} = 0 \quad \forall g, \forall \omega \]
\[ \sum_{\tau=t}^{t+DT_g-1} (1 - u_{gt\omega}) \geq DT_g z_{gt\omega} \quad \forall g, t = F_g + 1, \ldots, 2N_T - DT_g + 1, \forall \omega \]
\[ \sum_{\tau=t}^{2N_T} (1 - u_{gt\omega} - z_{gt\omega}) \geq 0 \quad \forall g, t = 2N_T - DT_g + 2, \ldots, 2N_T, \forall \omega \]

where \( L_g = \min \{2N_T, (UT_g - T^0_{up,g}) u^0_g\} \) and \( F_g = \min \{2N_T, (DT_g - T^0_{down,g}) (1 - u^0_g)\} \) are the hours in the beginning of the planning horizon that the unit is restricted to operate or to be offline respectively due to initial conditions.
3 Results

The system in the case study consists of two CHP units, one heat producing boiler and a heat storage tank. The parameters of the units are given in Appendix B. In fig.2 the scheduling of heat production is given. Because heat production is scenario dependent, this figure depicts one possible outcome. The existence of heat storage capacity increases the flexibility of the system. For example the use of the expensive heat boiler is avoided (e.g. hours 5-7) and the second unit can stop its operation when electricity prices are forecasted to be low (e.g. hours 41-43).

To estimate the value of heat storage, the model is solved with different amounts of storage capacity, running from 0 to 1000 MWh/h. The results show an increasing optimal value with decreasing change rate. It must be noted that the optimal value is negative as the objective function does not include the income from power sold in Day-1 and the income from heat sold. These incomes are fixed and do not affect the optimal solution.

<table>
<thead>
<tr>
<th>Capacity (MWh\textsubscript{th}/h)</th>
<th>0</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal value (€)</td>
<td>-94499</td>
<td>-91330</td>
<td>-90201</td>
<td>-89621</td>
<td>-89174</td>
<td>-88788</td>
</tr>
</tbody>
</table>

Table 1: Change of optimal value in accordance with the heat storage capacity

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References


Appendix A: Nomenclature

Indices and Numbers:
- \(g\) Index of units, running from 1 to \(N_G\)
- \(t\) Index of time periods in hourly resolution, running from 1 to \(2N_T\)
- \(\omega\) Index of scenarios, running from 1 to \(N_\Omega\)

Parameters
- \(\pi_\omega\) Probability of occurrence of scenario \(\omega\)
- \(\lambda_{el,t,\omega}\) Day-ahead market price in period \(t\) and scenario \(\omega\), (€/MWh)
- \(Q_{D,t,\omega}\) Heat demand in period \(t\) and scenario \(\omega\), (MWth)
- \(P_{S,t}\) Power load in period \(t\), (MW)
- \(\lambda_{f,g}\) Fuel price of unit \(g\), (€/MWh)
- \(c_{\text{start},g}\) Start-up cost of unit \(g\), (€)
- \(c_{\text{stop},g}\) Shut down cost of unit \(g\), (€)
- \(\beta_{el,g}\) Marginal fuel consumption for power production of unit \(g\)
- \(\beta_{th,g}\) Marginal fuel consumption for heat production of unit \(g\)
- \(\beta_{0,g}\) Fuel consumption at minimum output of unit \(g\), (MW)
- \(r_{\text{min},g}\) Minimum power-to-heat ratio of unit \(g\)
- \(\eta_{\text{boiler},g}\) Efficiency of boiler unit \(g\)
- \(P_{\text{min},g}, P_{\text{max},g}\) Power production limits of unit \(g\), (MW)
- \(Q_{\text{min},g}, Q_{\text{max},g}\) Heat production limits of unit \(g\), (MWth)
- \(V_{\text{max}}\) Heat storage capacity, (MWh/h)
- \(U_{T_g}\) Minimum up time of unit \(g\), (h)
- \(D_{T_g}\) Minimum down time of unit \(g\), (h)
- \(u_{g,t,\omega}, y_{g,t,\omega}, z_{g,t,\omega}\) Initial state of binary variables \(u_{g,t,\omega}, y_{g,t,\omega}\) and \(z_{g,t,\omega}\)
- \(T_{\text{up},g}\) Time periods of unit \(g\) has been on in the beginning of the planning horizon, (h)
- \(T_{\text{down},g}\) Time periods of unit \(g\) has been off in the beginning of the planning horizon, (h)

Variables
- \(P_{g,t,\omega}\) Power produced by unit \(g\) in period \(t\) and scenario \(\omega\), (MW)
- \(Q_{g,t,\omega}\) Heat produced by unit \(g\) in period \(t\) and scenario \(\omega\), (MWth)
- \(P_{f,el,g,t,\omega}\) Fuel consumption of unit \(g\) in period \(t\) and scenario \(\omega\), (MWh/h)
- \(V_{\omega}\) Heat storage content in period \(t\) and scenario \(\omega\), (MWh/h)
- \(u_{g,t,\omega}\) Binary variable for the on/off status of unit \(g\) in period \(t\) and scenario \(\omega\)
- \(y_{g,t,\omega}\) Binary variable for the start-up of unit \(g\) in period \(t\) and scenario \(\omega\)
- \(z_{g,t,\omega}\) Binary variable for the shut down of unit \(g\) in period \(t\) and scenario \(\omega\)

Appendix B: Case study parameters

Table 2: CHP parameters used in the case study

<table>
<thead>
<tr>
<th>CHP units</th>
<th>Unit1/Unit2</th>
<th>Heat boiler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel</td>
<td>Gas</td>
<td>Fuel</td>
</tr>
<tr>
<td>Fuel price, (€/MWh)</td>
<td>10</td>
<td>Fuel price, (€/MWh)</td>
</tr>
<tr>
<td>Min. power output, (MW)</td>
<td>35/30</td>
<td>Min. heat output, (MWth)</td>
</tr>
<tr>
<td>Max. power output, (MW)</td>
<td>140/120</td>
<td>Max. heat output, (MWth)</td>
</tr>
<tr>
<td>Max. heat output, (MWth)</td>
<td>200/180</td>
<td>Efficiency</td>
</tr>
<tr>
<td>Marg. fuel consumption for power prod.</td>
<td>2.4/2.5</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Marg. fuel consumption for heat prod.</td>
<td>0.36/0.37</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Fuel consumption at min output, (MW)</td>
<td>40/35</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Minimum power-to-heat ratio</td>
<td>0.5/0.5</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Start-up cost, (€)</td>
<td>10000</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Minimum up time, (h)</td>
<td>1/1</td>
<td>Capacity, (MWth)</td>
</tr>
<tr>
<td>Minimum down time, (h)</td>
<td>1/1</td>
<td>Capacity, (MWth)</td>
</tr>
</tbody>
</table>