Safety Format for Non-linear Analysis of RC Structures Subjected to Multiple Failure Modes

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Doctoral Thesis in Structural Engineering and Bridges
Stockholm, Sweden 2015
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This dissertation is lovingly dedicated to the Holy Spirit who gave me enough strength to complete the present research with patience and confidence.

To Sara for being supportive, encouraging and loving. She stood by me during the most difficult times of this journey and consistently helped me keep perspective on what is important in life.

To Dad for instilling a hard work ethic, and to Mom for keeping her prayers and hope alive.
Preface

The research presented in this thesis was carried out at the Department of Civil and Architectural Engineering, KTH Royal Institute of Technology in Stockholm, Sweden, and at the Department of Structural, Geotechnical and Building Engineering, Polytechnic of Turin, Italy. It has been financed by both Universities.

I express my sincere gratitude to my Swedish supervisor, Dr. Johan Silfwerbrand, professor at the KTH Royal Institute of Technology in Stockholm, Sweden, who was always there when I needed advices and whom I owe many speeches of encouragement as I can hear him say “you can do this….don’t give up….you are almost there”. His thoughtful comments, suggestions, and continuous guidance were invaluable and led me down the correct path towards the final steps of accomplishing this personal and professional goal. My thankfulness also goes to my Italian supervisor, Dr. Giuseppe Mancini, professor at the Polytechnic of Turin, Italy, for support all along the way.

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Stockholm in March 2015

Filippo Sangiorgio
Abstract

This thesis treats the safety format for non-linear analysis of reinforced concrete (RC) structures subjected to multiple failure modes. The purpose is to identify questions which are poorly understood and ambiguous about the behaviour of structures that may fail due to a number of possible failure modes (e.g., bending, shear, buckling, crack propagation, fatigue) which might be used as a focus for the development of a more comprehensive approach to the evaluation of the structural safety level.

Nowadays non-linear analysis in concrete structures cannot be considered only as a research tool to improve the understanding of structural behaviour, but it is also a useful mean to design more and more enhanced structures and to estimate the actual safety level in existing structures. As a consequence, semi-probabilistic safety formats for the non-linear analysis of RC structures are of great practical interest for structural engineers.

Safety formats for non-linear analysis have mainly been tested on beams and columns subjected to normal forces and bending moments. Only recently there has been a noticeable effort in understanding whether available safety formats lead to the intended reliability when they are applied to structures that also may fail due to shear forces. However, the road ahead is still long and challenging.

The definition of a suitable safety format involves the clarification of (i) which values of geometric and material properties should be used in the non-linear analysis, considering that they all influence both the resistance and the ultimate behaviour of the whole system, (ii) when the incremental process of non-linear analysis should stop, and (iii) how to derive from the failure load the ultimate load that can be carried by the structure with the safety margins that are required by the semi-probabilistic approach. This thesis considers in some sense all three of these aspects.

The following major conclusions are based on the studies described in the appended papers: (1) the scatter of the shear capacity of RC slender members seems to be mainly due to the randomness of both tensile strength of concrete and shrinkage; (2) the structural behaviour at ultimate load of RC structures designed according to Eurocodes 2 is not unambiguous and may significantly vary depending on the structural system, load configuration, and capacity design; (3) the resistance of RC structures subjected to flexural and shear failure modes seems to be mainly influenced by the combination of mechanical properties of both longitudinal reinforcement and stirrups, and tensile strength of concrete; and (4) the resistance of RC structures subjected to multiple failure modes may have a general multimodal probability density function, in which each mode represents a specific failure mechanism.
Sammanfattning

Föreliggande avhandling behandlar säkerhetsformat för icke-linjär analys av armerade betongkonstruktioner som utsätts för flera brottmoder. Syftet är att identifiera frågor som är otillräckligt undersökt och hittills fått tvetydiga svar om beteendet hos konstruktioner eller konstruktionselement som kan gå till brott på grund av ett antal möjliga brottmoder (t.ex. böjning, skjuvning, knäckning, spricktillväxt, utmattnings). Nu kunskap kring problemen skulle kunna användas som grund för utvecklingen av en mer övergripande strategi för utvärderingen av strukturell säkerhetsnivå.


Säkerhetsformat för icke-linjär analys har främst studerats på balkar och pelare utsatta för normalkrafter och böjande moment. Först nyligen har det skett en märkbar ansträngning att förstå om tillgängliga säkerhetsformat leder till avsedd tillförlitlighet när de tillämpas på strukturer som också kan gå till brott p.g.a. stora tvärkrafter. Tyvärr är vägen fortfarande lång och utmanande.

Definitionen av ett lämpligt säkerhetsformat innebär förtydligande av (1) vilka värden på geometriska och materialegenskaper som bör användas i den icke-linjära analysen med tanke på att alla har inflytande på både motstånd och beteendet hos hela systemet, (2) när den inkrementella processen med icke-linjär analys bör avslutas och (3) hur man från det enklare brottet kan härleda den maximala belastning som kan bäras av strukturen med de säkerhetsmarginaler som krävs. Denna avhandling behandlar i någon mening alla dessa tre aspekter.

Följande viktiga slutsatser är baserade på studier som beskrivs i bifogade artiklar: (1) spridningen i skjuvkapacitet hos slanka armerade betongelement verkar främst bero på slumpmässighet hos betongens draghållfasthet och krympning; (2) betongens verkningssätt under maximal belastning är inte entydig i Eurocode 2 och kan variera avsevärt beroende på strukturens lastkonfiguration och kapacitet; (3) motståndet hos armerade betongkonstruktioner utsatta för brottmoder p.g.a. böj- och skjuvning verkar huvudsakligen påverkas av en kombination av mekaniska egenskaper hos både längsgående armering och bygglar samt draghållfasthet hos betong och (4) motståndet hos armerade betongkonstruktioner som utsätts för flera brottmoder kan ha en allmän probabilistisk tätfunktion, där varje mod representerar ett specifikt brott.
**Sommario**

Questa tesi tratta il safety format per l’analisi non lineare di strutture in cemento armato soggette a diversi modi di rottura (ad esempio, a flessione, per instabilità, a taglio, per formazione di cricche, a fatica, etc.). Lo scopo è quello di identificare le problematiche, ancora poco conosciute e ambigue, che influenzano il comportamento ultimo di questi tipi di strutture che possono essere utilizzate come punto di riferimento per lo sviluppo di un approccio più coerente per la valutazione del livello di sicurezza strutturale.

Oggi, l’analisi non lineare di strutture in cemento armato non può essere considerata solo uno strumento di ricerca volto ad una migliore comprensione del comportamento strutturale, ma deve essere un mezzo utile sia ad una progettazione strutturale sempre più avanzata che alla stima del livello effettivo di sicurezza di strutture esistenti. Di conseguenza, la definizione di un safety format a livello semi-probablistico per l’analisi non lineare di strutture in cemento armato è di grande interesse pratico per gli ingegneri strutturali.

Generalmente, i safety format per l’analisi non lineare sono stati testati su travi e pilastri sottoposti a sforzi normali e momenti flettenti. Solo recentemente c’è stato il tentativo di capire se i safety format attuali, applicati a strutture che si rompono anche a taglio, portano all’affidabilità prevista dagli standard. Tuttavia, la strada da percorrere è ancora lunga ed impegnativa.

La definizione di un adeguato safety format implica che vengano individuati: (i) i valori delle proprietà geometriche e dei materiali da essere utilizzati per l’analisi non lineare, visto che influenzano sia la resistenza strutturale che il relativo modo di rottura e che possono, in certe combinazioni, aumentare il rischio di rottura fragile dell’intero sistema; (ii) quando interrompere il processo incrementale dell’analisi non lineare; (iii) come derivare, dai risultati dell’analisi non lineare, il carico ultimo di rottura che può essere portato dalla struttura entro i margini di sicurezza richiesti dal metodo semi-probablistico. Questa ricerca vuole comprendere, in un certo qual modo, tutti e tre questi aspetti.

Le seguenti conclusioni generali sono basate sugli studi descritti negli articoli allegati: (1) lo scatter della resistenza a taglio di elementi snelli in c.a. sembra essere principalmente dovuto sia all’aleatorietà della resistenza a trazione del calcestruzzo, sia ad un elevato tasso di ritiro (shrinkage); (2) il comportamento ultimo di strutture in c.a. progettate secondo gli Eurocodici non è univoco, ma può variare notevolmente a seconda dello schema strutturale, della configurazione di carico, e della duttilità; (3) la resistenza di strutture in c.a. sottoposte a flessione e taglio sembra essere principalmente influenzata dalla combinazione delle proprietà meccaniche delle armature longitudinali e delle staffe, nonché dalla resistenza a trazione del calcestruzzo; e (4) la resistenza di strutture in c.a. sottoposte a molteplici modi di rottura, in generale, può avere una funzione di densità di probabilità multimodale in cui ciascun modo rappresenta uno specifico meccanismo di rottura.
List of Publications

Five journal papers form the basis of this Thesis.


**Paper II:** F. Sangiorgio, J. Silfwerbrand, and G. Mancini. "Scatter in the Shear Capacity of RC Slender Members without Web Reinforcement: Overview Study." Accepted for publication in the *Journal of Structural Concrete* (March 2015).


The planning of the papers, implementation of the numerical models, execution of the analyses, and writing has been performed by Sangiorgio. The co-authors have participated in the planning of the work and contributed to the papers with comments and suggestions for revisions.
Other Publications by the Author

Other publication by the author related to the topics of this thesis is as follows.


Moreover, Papers I and IV were also orally presented on two separate occasions:

**Paper I:** *ATINER 4th Annual International Conference on Civil Engineering, Structural Engineering and Mechanics*, 26-29 May 2014, Athens, Greece.

**Paper IV:** *ICBSE XII International Conference on Building Science and Engineering*, February 11-12 2015, Rio de Janeiro, Brazil.
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Paper V
1 Introduction

1.1 Background

Nowadays, non-linear analyses of concrete structures cannot be considered only as a research tool to improve the understanding of structural behaviour, but it is also an useful mean to design more enhanced structures and to estimate the actual safety level in existing structures. As a consequence, semi-probabilistic safety formats for non-linear analyses of reinforced concrete (RC) structures are of great practical interest for structural engineers.

Safety formats for non-linear analyses have mainly been tested on beams and columns subjected to normal forces and bending moments. Only recently, Schlune et al. [1 and 2] have made a noticeable effort in understanding whether available safety formats lead to the intended reliability when they are applied to structures that may fail due to shear forces. However, the road ahead is still long and challenging.

In this chapter, the author will provide the objectives and motivation of the study and, afterwards, will discuss the research contribution and the organization of the thesis.

1.2 Aims and Scope

The overall aim of this thesis was to identify questions which are poorly understood and ambiguous regarding the behaviour of structures that may fail due to a number of possible failure modes (e.g., bending, shear, buckling, crack propagation, fatigue) which might be used as a focus for the development of a more comprehensive approach to the evaluation of the structural safety level. This purpose has been addressed in Paper V.

In order to achieve that goal, the fundamental step was a detailed understanding of the structural behaviour of RC structures before and at failure over a wide range of different parameters as the material properties, the vagueness associated with cross-section and transverse reinforcement geometry, and the model uncertainties, based on both sophisticated tools of structural analysis and probabilistic modelling; it has been invoked in Paper IV. The establishment of correlation between specific members’ damage conditions, environmental loadings, structural design, and changes in the structural response have also been pursued and approached in Paper III. The survey was limited to frame buildings subjected to axial, flexural and shear failure modes. Non-linear FE structural analyses have been used significantly.

The causes to the great shear failure scatter of RC slender members without web reinforcement have been studied preliminarily and addressed in Papers I and II. The study
was performed in a full probabilistic approach via Monte Carlo simulation and Data Mining techniques. Fig. 1.1 shows a schematic view of the structure of the research project.

Figure 1.1: Schematic view of the structure of the research project

Some specific objectives with this research project were:

1. The clarification of which values of geometric and material properties should be used in the non-linear analysis, considering that they all influence both the resistance and the ultimate behaviour of RC structures and may, in certain combination of property values, increase the risk of a brittle failure of the whole system.
2. The identification of when the incremental process of non-linear analysis should stop.
3. The explanation of how to derive, from the failure load, the ultimate load that can be carried by the structure with the safety margins that are required by the semi-probabilistic approach.

In this thesis the author has considered, in some sense, all three of those aspects.

Considering part (1), the uncertainties, a broad numerical investigation on different RC structures has been performed to evaluate the statistical significance and the probabilistic influences of geometric and material properties, and model uncertainty.

Point (2), when to stop the incremental process of the non-linear analysis, is treated through the use of both effective strain-based models, considering the maximum allowable strain of
1.3 Limitations

The author acknowledges the following limitations of this thesis.

- **Paper I**: Assessment of the ACI-DAfStb Database of Shear Tests on Slender Reinforced Concrete Beams without Stirrups for Investigations on the Shear Capacity Scatter.
  In the cluster analysis, the authors have assumed that the similarity between the different experimental tests could be due to only the geometrical parameters.

- **Paper II**: Scatter in the Shear Capacity of RC Slender Members without Web Reinforcement: Overview Study.
  In this study the authors have used an empirical approach to the problem. Throughout this process, results have not always conformed to the phenomenon itself and may be influenced by the particular methodology used. However, the obtained findings allow to speed up a more complicated method as a non-linear FEM analysis suitably calibrated and coupled with a probabilistic approach, giving an idea of which parameters may be convenient to be considered and which of them, if any, are less important or can even be neglected.

  In this study the authors have considered a multiple-resistance model describing respectively (1) both the axial and ultimate moment capacities, and (2) the shear strength of structural elements. To this end, the Eurocode 2 shear strength model for reinforced concrete members with web reinforcement has been used. Unfortunately, this is a codified shear strength method of design and does not lend itself well to be considered as predictive system of equations since it is intended to provide a conservative and safe lower bound to strength. Moreover, such multiple-resistance model involves the presence of two different model uncertainties, with different mean values and coefficient of variations, and this fact doubtless affects the interpretation of results.

- **Paper IV**: Resistance and Sub-Resistances of RC Beams Subjected to Multiple Failure Modes.
  Same as for Paper III.
  Same as for Paper III.

In addition, there are the following general limitations of the project: (1) housing structures; (2) no prestress; (3) no creep; (4) EC 2; and (5) no own lab tests.

Fig. 1.2 shows a schematic view of the limitations of the research project.

![Figure 1.2: Schematic view of the limitations of the research project](image-url)

1.4 Research Contribution

The broad numerical investigations on different RC structures presented in this thesis have resulted in the scientific contribution summarized below.

- The shear transfer mechanism of RC slender members without stirrups still presents significant uncertainties and the question has generated many controversies and debates since the beginning of the last century. Regrettably, until now the real causes of this problem are not yet clear to the scientific community and the issue is still important to investigate, especially nowadays that the minimizing of natural resources is of uppermost global interest. Due to the increased laboratory costs, actual studies are more and more often devoted to numerical simulations based on previous experiments. Unfortunately, it is difficult to find test results suitable for investigations on the shear capacity scatter in the available specialized literature. Therefore, thirteen groups of comparable experiments extracted from the ACI-DAfStb database of shear tests on slender reinforced concrete beams without stirrups are provided, each group containing a number of tests between 6 and 43, performed generally by different field
workers. These groups of reported test results will be of great importance both for the present research project and for researchers who investigate the causes of the shear failure scatter or develop improved shear design methods.

- All researchers that have tested the shear capacity of RC members without stirrups have observed a large scatter in the results. Hence, an overview study of the causes to the great shear failure scatter of RC beams without stirrups is conducted in a statistical and probabilistic way. The investigation highlights that both tensile strength of concrete and significant values of shrinkage may be useful to be considered for more in-depth studies of the phenomenon, whereas geometrical properties and concrete compressive strength seem to be less important or can even be neglected.

- A statistical and probabilistic investigation on the ultimate load behaviour of RC structures designed according to Eurocodes 2 and 8 and subjected to multiple failure modes is performed. Results show that the ultimate load behaviour is not unique and may vary from structure to structure depending on different factors: (a) the structural system, (b) the load configuration, and (c) the ductility class. The combination of those three factors may lead to a preferential performance of the structure; however this seems not to affect the main failure mechanism that has shown to be a ductile one. The performed investigation highlights that the European design specification is successful in preventing the formation of non-ductile failure mechanisms.

- The model uncertainty for the combination of both bending moment and buckling capacities for the material and geometric non-linear FE analysis of RC columns is assessed through a process of comparison with experimental results on representative examples.

- The model uncertainty for the shear capacity of RC structures with stirrups analyzed with the Eurocode 2 shear strength model is assessed through a process of comparison with experimental results on representative examples.

- Geometric and mechanical properties all influence the resistance of RC structures and may, in certain combination of property values, increase the risk of a brittle failure of the whole system. Therefore, a statistical and probabilistic investigation on the resistance of RC beams designed according to Eurocodes 2 and 8, and subjected to multiple failure modes, under both the natural variation of material properties and the uncertainty associated with cross-section and transverse reinforcement geometry is conducted. Results show that the ultimate load behaviour of RC structures subjected to flexural and shear failure modes seems to be mainly influenced by the combination of the mechanical properties of both longitudinal reinforcement and stirrups, and the tensile strength of concrete, of which the latter appears to affect the overall response of the system in a non-linear way. The model uncertainty of the resistance model used in the analysis plays undoubtedly an important role in interpreting results.

- Non-linear analysis is nowadays a useful mean to design more and more enhanced structures and to estimate the actual safety level in existing structures. As a consequence, semi-probabilistic safety formats for the non-linear analysis of RC structures are of great practical interest for structural engineers. Therefore, a study on the safety formats for non-linear analysis of RC structures, particularly focusing on the resistance Probability Density Function (PDF) of structures subjected to multiple failure modes, is conducted. A historical review of the topic is also given. The results highlight that the structural resistance may have, in general, a multimodal PDF in which each mode represents a specific failure mechanism.
1.5 Outline of the Thesis

This thesis is based on the work and results presented in the appended papers. The intention of the thesis is to set the different papers in the context of the research project. It also provides a space for additional explanations to the work presented. A summary on the safety formats for non-linear analysis of RC structures, including a historical review, is given in Chapter 2. Some underlying assumptions and conditions left out in the papers are also provided.

The research methods are described in Chapter 3.

Some considerations on the applicability of a safety format for non-linear analysis to RC structures subjected to multiple failure modes are given in Chapter 4.

General conclusions based on the research project are presented in Chapter 5. Proposals for further research are given in the same chapter. Specific conclusions regarding each study are found in the appended papers.
2 Safety Format for Non-linear Analysis of RC Structures

2.1 General Introduction

The resistance of reinforced concrete (RC) structures can be predicted by appropriate modelling of material properties, geometry variables and uncertainties associated with the applied model (Bertagnoli et al. [3]; Červenka [4 and 5]; and Sykora and Holicky [6]). In this regard, the effects of variability of both material properties and geometry have been extensively investigated in past years (see Paper II for instance) and are now relatively well understood for structures subjected to only one failure mode; however, at present, there is limited research on their influence on the ultimate load behaviour of RC structures subjected to multiple failure modes.

A recent study (Paper III), for instance, has tried to shed light on the ultimate load behaviour of RC structures designed according to Eurocodes 2 [7] and 8 [8] and subjected to axial, bending, and shear failure modes. Later, an extension of that study (Paper IV) has shown the important role played by both the combination of particular mechanical properties and model uncertainty.

2.2 Safety Formats before EN 1992-2:2005

As resumed by Macchi [9], the first approach to non-linear analysis may be found in some ancient CEB Bulletins like no. 21/30/34/52/53/97, published prior to 1977. However, only in Bulletins 101 and 105 a systematic approach to safety format for non-linear analysis can be found, as included within Model Code 1978 [10] and subsequent design examples (Macchi et al. [11]). In those early studies, the main problem encountered in the proposition of a safety format was the definition of both a reference constitutive law and strength values of material properties to be used in the non-linear analysis, in particular of concrete considering that a reduction of its strength, associated with the influence of the partial safety factor $\gamma_c$, also implies a consequent reduction of its Young modulus.

In this regard, as a first step, it was suggested in Model Code 1978 [10] to use the characteristic strength of concrete $f_{ck}$ and steel $f_{sk}$ for the overall analysis and to perform the local verification with the design strength values ($f_{cd}$ and $f_{yd}$, respectively).

This procedure implies the adoption, in the analysis, of an idealized tri-linear moment-rotation diagram $(M, \theta)$ in which the localized plastic rotation is introduced at two different values of bending moment, $M_{yd}$ and $M_{yk}$, design and characteristic yielding moments, respectively (see
Chapter 2. Safety Format for Non-linear Analysis of RC Structures

Fig. 2.1). In practice, the structural section or region, in which the steel plasticization is reached first, is described by the \((M,\theta)\) design relationship, the other ones remaining with the characteristic \((M,\theta)\) behaviour. That means in general a high level of redistribution of internal actions in the critical section or region up to the fully exploitation of available plastic rotation which, in agreement with Červenka [4 and 5], may (a) be physically unrealistic since design material values are extremely low and do not represent a real material and (b) also change the failure mode. Furthermore, since in non-linear analysis material criteria are satisfied implicitly within constitutive laws, it seems illogical to perform local verifications.

![Figure 2.1: Idealized trilinear moment-rotation relationship](image)

As a second step, about ten years later this procedure was slightly modified in both Eurocode 2 Part 1-1 (ENV 1992-1-1:1991) [12] and Model Code 1990 [13] by substitution of characteristic value of concrete strength with the mean value \(f_{cm}\), without a substantial changing of safety format. However, in presence of second order effects, the mean strength value for concrete was reduced by the application of a coefficient \(\gamma_c\), being \(\gamma_c = 1.2\) in both Model Code 1978 [10] and Model Code 1990 [13] or \(\gamma_c = 1.35\) in Eurocode 2 Part 1-1 [12]. By the way, several studies have been proposed in the literature between 1972 and 1988 (Levi [14]; Leporati and Levi [15 and 16]; Mancini [17]; Carbone et al. [18]; and Levi et al. [19]) having the task to consider model uncertainties in the limitation of redistribution of internal actions and to reduce the over-proportional effects induced by \(\gamma_c > 1\), adopted in structures sensitive to second order effects.

The remarks to this proposed safety format for non-linear analysis can be summarized into:

- The region in which steel plasticization is reached first is established by convention, whereas it changes, in general, in every load case.
- Impossibility to apply this procedure for non-linear FEM analysis, where the combination of internal actions cannot be described by \((M,\theta)\) diagrams.

More research work was carried out in this direction and, afterwards, a more generalized approach for safety format in non-linear analysis was proposed in 1995 by König et al. [20] based on the consideration that in hyperstatic structures, assumed that the scattering of material properties and direct/indirect actions is known and evaluable by means of their stochastic distribution functions, only the sensitivity of the overall structural behaviour to the scattering of those variables remains to be investigated. By remaining within the field of semi-probabilistic approach, a new safety coefficient related to the overall structural strength was defined (the so called global safety factor \(\gamma_{Gl}\)), which may be interpreted as a structural strength reserve due to redundancy. This safety factor should cover the probability that all
along the structure, for the assigned actions distribution, the strength values of material properties fall down to the design values. The safety format was expressed as follows:

$$\gamma_G G + \gamma_Q Q \leq \frac{F_m}{\gamma_{Gl}}$$

(2.1)

where the left hand part of Equation (2.1) denotes the external action \((G\) being the dead load, \(Q\) the live load, and \(\gamma_G, \gamma_Q\) their corresponding partial safety factors) whereas \(F_m\) represents the maximum value reached by direct/indirect action in the non-linear analysis. Here non-linear analysis is suggested to be performed with “realistic” (most probable) materials constitutive laws and strength mean values for both concrete and steel \(f_{cm} \) and \(f_{ym}\), respectively. However, being \(\gamma_{Gl}\), defined by two different values depending on whether the collapse is dictated by a brittle failure (concrete crushing) or a ductile failure (steel yielding), it was successively proposed by König et al. [21] to use a reduced concrete strength \(f_c = 0.85 \cdot f_{ck}\) so that to have a common value of \(\gamma_{Gl}\) for both material failures.

Three main remarks have been attributed to this proposal by Mancini [22]: (1) the global safety factor is applied in the actions domain omitting the consideration of internal actions path and, therefore, is not able to distinguish the different structural behaviour in regions in which the limit strains for materials are reached (linear, over-proportional, under-proportional); (2) this safety format, being applied in the actions domain, is not consistent with the semi-probabilistic approach, in which acting external and resisting internal actions are compared; and (3) it does not take into account the model uncertainties on both acting and resisting side, despite their importance is fundamental in non-linear processes.

Maintaining the approach of a global safety coefficient, two extensions of this method have been proposed in the following years (Six [23] and Henriques et al. [24]), both based on strength mean values for materials properties:

$$S(\gamma_G G + \gamma_Q Q) \leq \frac{R(f_{ym}, f_{cm})}{\gamma_R \left( e_{sl} + \rho \frac{\rho_2}{\rho_1} \right)}$$

[23]

and

$$S(\gamma_G G + \gamma_Q Q) \leq \frac{R(f_{ym}, f_{cm})}{\gamma_R \left( \frac{x}{d} \right)}$$

[24]

Both the proposals are transferred in the external and resisting internal actions domain \(S\) and \(R\), respectively, with \(G, Q, \gamma_G, \gamma_Q\) as previously described for Eq. (2.1).

In Equation (2.2), the resistance safety factor \(\gamma_R\) depends on the reinforcement steel strain in the tensile reinforcement (or the less compressed reinforcement layer) \(e_{sl}\), the reinforcement ratio \(\rho_{tot}\), and the ratio between the reinforcement amount of the most compressed layer and the least compressed layer \(\rho_2/\rho_1\); whereas, in Equation (2.3), it is based on the relative position of the neutral axis \(x/d\), where \(x\) is the compressive zone depth and \(d\) is the effective depth of the cross-section.
Moreover, the safety factor is entirely defined by the resistance of the structure and, therefore, not anymore dependent on the material failure. Regrettably, model uncertainties are still not taken into account.

In order to find a technique that considers also model uncertainties and overcomes all those remarks, the following safety format proposed in 2002 by Mancini [22] needs to be examined. In this proposal, the approach of a global safety coefficient is maintained. Again, the safety verifications are applied in the external and resisting internal actions domain $S$ and $R$, respectively:

$$ S(\gamma_G G + \gamma_Q Q) \leq \frac{R(q_u)}{\gamma_{Gl}} = \frac{R(q_u)}{\gamma_{Rd} \gamma_{Gl}} $$  

(2.4)

where $q_u$ is the maximum level of direct/indirect actions reached in nonlinear analysis, performed with reduced concrete strength $f_c = 0.85 f_{ck}$ and mean value of steel strength $f_{ym}$, and $G$, $Q$, $\gamma_G$, $\gamma_Q$ as previously described for Eq. (2.1).

Model uncertainties have here been explicitly taken into account being $\gamma_{Gl} = \gamma_{Rd} \gamma_{Gl}$, with $\gamma_{Rd} = 1.08$ model uncertainty factor and $\gamma_{Gl} = 1.2$ global resistance factor. Unfortunately, their effects were not differentiated, whereas, as clarified in other studies (e.g., JCSS Probabilistic Model Code [25] and Drogue et al. [26]), they arise from uncertainties and errors associated with the structure of the model (load and resistance), stemming from abstractions, assumptions, and approximations; therefore, their scatter depends on both the type of analysed structure (frames, plates, shells, solids, etc.) and failure mode.

Meanwhile, the appearance on the European standards scenario of both Eurocode 0 (EN 1990:2002) [27] and the new Eurocode 2 Part 1-1 (EN 1992-1-1:2004) [7] clearly established the fundamental parameters of non-linear analysis (focusing on the identification of the stress-strain relation that should be used and on the importance of model uncertainties) so giving a guidance to define completely a safety format in agreement with the established issues of research in this field during the last 50 years (see Bertagnoli et al. [3] for more details).

### 2.3 Safety Formats from EN 1992-2:2005 to fib Model Code 2010

The long cognitive process explained in Section 2.2 has led to the adoption in the recent Eurocode 2 Part 2 (EN 1992-2:2005) [28] of the previously mentioned safety format proposed by Mancini [22], slightly modified, as recommended safety format for non-linear analysis of RC structures. However, the comments expressed for Mancini [22] apply, in the same manner, here. Moreover, this safety format for non-linear analysis is not applicable to beams which fail in shear tension failure (see again Schlune et al. [1 and 2]).

In the following years, in agreement with Eurocode 0 [27] and Eurocode 2 Part 1-1 [7], a new method of studying the safety of RC structures in combination with non-linear analysis was developed both on the basis of probabilistic investigation and on the assumption that the random distribution of resistance is according to lognormal distribution. The idea that a lognormal distribution may be typical for structural resistance is even today generally accepted by the scientific community. Nevertheless, this is a significant hypothesis that was
already challenged in 2002 by Henriques et al. [24], where it was demonstrated that, even considering a simple probabilistic model and structures that may have only one failure mode (flexural bending), the resistance distribution has mono or bi-modal shape, depending on the amount of reinforcement, with, in the latter, a higher scatter.

A first procedure for the definition of global resistance factors based on probabilistic analysis of beams, slabs and columns, has been studied by different authors (Holicky and Sykora [29] and Allaix et al. [30 and 31]). Here, the global resistance factor $\gamma_G$ is defined by Holicky and Sykora [29] as the ratio between the mean value $R_m$ and the design value $R_d$ of the distribution of the structural resistance:

$$\gamma_G = \frac{R_m}{R_d} \approx \exp(\alpha_R \beta V_R) \quad (2.5)$$

The mean value of the resistance $R_m$ is determined by performing a non-linear analysis with the mean values of material properties and nominal values of geometrical parameters, whereas the design value $R_d$ is identified with the probabilistic relationship recommended in Eurocode 0 [27]:

$$R_d = R_m \exp(-\alpha_R \beta V_R) \quad (2.6)$$

where $\alpha_R$ is the resistance sensitivity factor and $\beta$ is the reliability index. For the Ultimate Limit States, the values $\alpha_R = 0.8$ and $\beta = 3.8$ for an intended life of fifty years are recommended. The coefficient of variation $V_R$ of the distribution of the structural resistance $R$ is obtained by simulations via the Monte Carlo method. Model uncertainties are explicitly taken into account and may be differentiated depending on both the type of analysed structure and failure mode.

The following remarks can be made regarding this approach: (1) the global resistance factor $\gamma_G$ is not unique; (2) the probabilistic simulations may be time-consuming; and (3) the lognormal distribution, as previously discussed, may not correctly represent the structural resistance.

As a second step, maintaining the approach of probabilistic investigation, a new technique was introduced by Červenka [4 and 5] so that to save the time that would be required to perform the Monte Carlo simulations and, therefore, to speed up the procedure of the safety assessment. This method is called the “Estimate of Coefficient of Variation” method (ECOV) and is based on the idea that the random distribution of resistance, which is again described by the coefficient of variation $V_R$, can be estimated from mean $R_m$ and characteristic values $R_k$. The underlying assumption is again that random distribution of structural resistance is according to lognormal distribution. In this case, it is possible to express the coefficient of variation as:

$$V_R \leq \frac{1}{1.65} \ln \left( \frac{R_m}{R_k} \right) \quad (2.7)$$

The definition of the global resistance factor $\gamma_G$, Equation (2.5), remains unchanged. The key steps in this method are to determine the mean and characteristic values $R_m$, $R_k$. It was proposed to estimate them through two separate non-linear analyses using mean and characteristic values of input material parameters, respectively.
However, some comments should be expressed: (1) observations expressed for the
distribution of resistance still apply here; (2) it introduces some approximations with respect
to the more generic probabilistic analysis; and (3) effects of model uncertainties should be
treated separately.

With the arrival of the new fib Model Code 2010 [32 and 33], a different perspective was
placed on non-linear analysis and safety assessment. It represents, in fact, the state-of-the art
for design of RC structures and first introduced non-linear analysis in the design process. The
design condition to be used in the safety format for non-linear analysis is written in the
external and resisting internal actions domain:

\[
F_d \leq R_d
\]  \hspace{1cm} (2.8)

where \( F_d \) is the design value of the actions and \( R_d \) the design resistance. Three different
approaches are proposed to evaluate the design resistance \( R_d \) (depending on various levels of
implementation of probabilistic theory): (1) the probabilistic method; (2) the global resistance
methods; and (3) the partial factor method.

The probabilistic assessment of the structural resistance is based on two models. A non-linear
finite element model is suggested to represent the response of the structure and a probabilistic
model is required to account for the uncertainty of the model parameters (e.g., material
properties, geometrical dimensions, boundary conditions). The design value of resistance \( R_d \)
can be evaluated by probabilistic analysis. In this approach the resistance \( R \) is represented by
non-linear structural analysis. The safety can be evaluated with the help of the reliability
index \( \beta \), or alternatively by the failure probability \( P_f \) taking into account all uncertainties due
to random variation of material properties, dimensions, and possibly other random effects.

Global design resistance \( R_d \) shall be obtained as:

\[
R_d = \frac{R(\alpha \beta)}{\gamma_{Rd}}
\]  \hspace{1cm} (2.9)

where \( \gamma_{Rd} > 0 \) is the model uncertainty factor and \( R(\alpha \beta) \) is the resistance corresponding to
reliability index \( \beta \), which is reduced by factor \( \alpha < 1 \) in order to account for a separate safety
assessment of resistance. This safety format is complex and may be used for structures to be
designed and for existing structures in cases where an increased effort is economically
justified.

In the global resistance format, the resistance is considered on a global structural level. It is
especially suitable for design based on non-linear analysis, where verification of limit states is
performed by numerical simulations. The aim of the global resistance methods is to estimate
the design resistance \( R_d \) by dividing the resistance computed with properly chosen
representative values \( f_{rep} \) for the material resistance and both the global resistance factor \( \gamma_R \)
and the model uncertainty factor \( \gamma_{Rd} \):

\[
R_d = \frac{R(f_{rep})}{\gamma_R \gamma_{Rd}}
\]  \hspace{1cm} (2.10)

Two alternative methods are mentioned in fib Model Code 2010 [33] for the derivation of \( R_d \):
(1) the global resistance factor method (which is adopted from Eurocode 2 Part 2 [28],
slightly modified); and (2) the ECOV method (Červenka [4 and 5]).
The partial safety factor (PSF) format is the usual way of verifying structural design. It is a simplified verification concept, which is based on past experience and calibrated in such a way that the general reliability requirements are satisfied with a sufficient margin during a defined period of time. According to this method, the design resistance $R_d$ is estimated by means of a non-linear analysis with the design values of the material resistances.

$$ R_d = R(f_d) \quad (2.11) $$

However, as mentioned in Section 2.2, the selection of the design values may be physically unrealistic and unconservative.

### 2.4 Safety Formats after fib Model Code 2010

After the new fib Model Code 2010 [32 and 33], although the topic is still controversial, only two contributions were found in the literature.

The first contribution was presented to the scientific community between 2011 and 2012 by Schlune et al. [1 and 2]. Safety formats for non-linear analysis have mainly been tested in past years on structures subjected to normal forces and bending moments. Therefore, a new safety format was proposed with the intention of including the safety of structures that may also fail due to shear loading. This safety format follows the approach of the global resistance methods and uses the mean yield strength of the reinforcement steel $f_{ym}$, the mean “in situ” concrete compressive strength $f_{cm,is}$, and the nominal values of the geometric parameters $a_{nom}$ as input quantities for the non-linear analysis. The design resistance $R_d$ is then derived by division of the obtained resistance by the global resistance factor $\gamma_R$:

$$ R_d = \frac{R(f_{ym}, f_{cm,is}, a_{nom})}{\gamma_R} \quad (2.12) $$

Based on the assumption of a lognormal distributed resistance, $\gamma_R$ is calculated from probabilistic considerations:

$$ \gamma_R = \frac{\exp(\alpha_R V_R)}{\theta_m} \quad (2.13) $$

with $\alpha_R$ and $\beta$ according to Eurocode 0 [27] (as previously described in Section 2.3).

Model uncertainties are explicitly taken into account through the use of the bias factor $\theta_m$, which is defined as the mean ratio of experimental to predicted resistance. Its value varies between 0.7 and 1.2 for failure in compression, bending and shear. It is important to note the distinction between $\theta_m$ and the model uncertainty factor $\gamma_{RD}$ (fib Model Code 2010 [32 and 33]), although both take model uncertainties into account.

The coefficient of variation of the structural resistance $V_R$ is written as follows:

$$ V_R = \sqrt{\psi^2 + \theta^2 + \psi^2} \quad (2.14) $$
where $V_g$, $V_m$, and $V_f$ are the coefficients of variation of the geometrical, model, and material uncertainties, respectively. Suggestions for $V_g$ and $V_m$ are also proposed.

When the main material parameters are the concrete compressive strength and the yield stress of the steel, the coefficient of variation $V_f$ can be estimated by means of:

$$V_f = \frac{\sqrt{\left(\frac{R_m - R_{cm}}{\Delta f_c}\right)^2 \sigma_f^2 + \left(\frac{R_m - R_{ym}}{\Delta f_y}\right)^2 \sigma_f^2}}{R_m}$$  \hspace{1cm} (2.15)$$

where

$\sigma_{fc}$, $\sigma_{fy}$ are the standard deviations of the concrete compressive strength and the yield stress of the steel, respectively.

$\Delta f_c$, $\Delta f_y$ are finite variations of the material resistances.

$R_{\Delta f_c}$, $R_{\Delta f_y}$ are results of non-linear analyses performed using the values $f_{cm} - \Delta f_c$ for the concrete compressive strength and $f_{ym} - \Delta f_y$ for the yield stress.

The coefficient of variation $V_R$ can be estimated by means of three non-linear analyses: one performed with the mean values of the material resistances, the other two with the values $f_{cm} - \Delta f_c$ and $f_{ym} - \Delta f_y$, respectively. In this sense, this technique could be considered as an improved ECOV method because, overcoming the time-consuming concerns usually associated with probabilistic simulations, also considers model uncertainties.

Shortly thereafter, the second contribution (Allaix et al. [34]) was also published. This safety format is based both on the approach of the global resistance methods and on the assumption that the random distribution of resistance is according to lognormal distribution. The non-linear analysis is here performed using the mean values of the material resistances $f_m$ and the nominal values of the geometrical dimensions $a_{nom}$. The design resistance $R_d$ is derived by division of the obtained resistance by both the global resistance factor $\gamma_R$ and the model uncertainty factor $\gamma_{Rd}$:

$$R_d = \frac{R(f_m, a_{nom})}{\gamma_R \gamma_{Rd}}$$  \hspace{1cm} (2.16)$$

In this case, the global resistance factor $\gamma_R$ is derived from the coefficient of variation of the structural resistance $V_R$, estimated in turn via probabilistic simulation using the Monte Carlo method (a non-linear analysis up to failure is performed using an FE model for each sample of the random variables in order to obtain the distribution of the structural resistance $R$):

$$\gamma_R = \exp(\alpha_R \beta V_R)$$  \hspace{1cm} (2.17)$$

with $\alpha_R$ and $\beta$ according to Eurocode 0 [27] (as previously described in Section 2.3).

The results of the investigation depend on the assumptions underlying the models used in the non-linear analysis. Clearly, the approach is meaningful if the structural model covers all relevant failure mechanisms. The probabilistic model used for the uncertainty analysis of the structural response should cover the material resistances and the model uncertainties. A probabilistic characterization of the geometrical dimensions of the cross-section and the positions of the reinforcing bars could also be considered.
The model uncertainty factor \( \gamma_{Rd} \) takes into account the difference between the real behaviour of a structure and the results of a numerical model suitable for the specific structure. From these features, it is suggested to be derived from the comparison of experimental tests and numerical calculations, but through probabilistic considerations. Given, in fact, the distribution of the resistance model uncertainty \( \vartheta_R \), the model uncertainty factor \( \gamma_{Rd} \) can be derived using the following expression (König and Hosser [35]):

\[
\gamma_{Rd} = \frac{1}{\exp(-\tilde{\alpha}_R \beta V_{\vartheta R})} = \exp(\tilde{\alpha}_R \beta V_{\vartheta R})
\]  

(2.18)

where \( \tilde{\alpha}_R = 0.4 \alpha_R \) is the sensitivity factor for the resistance model uncertainty and \( V_{\vartheta R} \) is the coefficient of variation of the resistance model uncertainty \( \vartheta_R \).

The advantage of this approach is that it is simple if global resistance factors have been firstly estimated for classes of structures, for example according to the simplified level II approach (König and Hosser [35]).

### 2.5 Examples of Application

The performances of some of the presented safety formats have been tested by Červenka [4 and 5] on different examples ranging from a simple statically determinate structure with bending failure mode up to statically indeterminate structures with shear failure modes. Those safety formats are: (a) the global approach proposed by Eurocode 2 Part 2 [28]; (b) the ECOV method (Červenka [4 and 5]), (c) the probabilistic assessment; and (d) the Partial Safety Factor method (PSF).

The mentioned examples were analysed with program ATENA for non-linear analysis of concrete structures. The numerical analysis was based on finite element method and non-linear material models for concrete, reinforcement and their interaction. Tensile behaviour of concrete was described by smeared cracks, crack band and fracture energy; compressive behaviour of concrete was defined by damage model with hardening and softening. The reinforcement was modelled by truss elements embedded in two-dimensional isoparametric concrete elements. Non-linear solution was performed incrementally with equilibrium iterations in each load step.

**Example 1**: Simply supported beam in bending uniformly loaded as shown in Fig. 2.2. The beam has a span of 6 m, rectangular cross-section of \( h = 0.3 \) m and \( b = 1 \) m. It is reinforced with \( 5 \phi 14 \) along the bottom surface. The concrete type is C30/37 and reinforcement has yield strength of 500 MPa. The failure occurs due to bending with reinforcement yielding.

![Figure 2.2: Beam geometry with distributed design load for example 1 (from Červenka [5])](image-url)
**Example 2**: Continuous deep shear beam with two spans (shown in Figs. 2.3 and 2.4). It corresponds to one of the beams tested at Delft University of Technology by Asin [36]. It is a statically indeterminate structure with a brittle shear failure.

**Figure 2.3**: Continuous deep beam in Example 2 (from Červenka [5])

**Figure 2.4**: Laboratory test of the continuous deep beam (from Červenka [5])

**Example 3**: Bridge pier. This example is chosen in order to verify the performances of the various safety formats in the case of a problem with second order effect (i.e., geometric non-linearity). It is adopted from a practical bridge design in Italy that was published by Bertagnoli et al. [3]. It is a bridge pier loaded by normal force and moment in the top, Fig. 2.5.
Example 4: Railway bridge frame structure in Sweden, shown in Fig. 2.6. It fails by a combined action of bending and shear. It is an existing bridge that was subjected to a field test up to failure by a single load in the middle of the left span.
Chapter 2. Safety Format for Non-linear Analysis of RC Structures

For each example, a full probabilistic analysis was also performed. Each probabilistic analysis consisted of several (at least 32 to 64) non-linear analyses with randomly chosen material properties. The design resistance $R_d$ was then obtained by a probabilistic analysis of randomly generated resistances.

Calculated design resistances $R_d$ for all examples and the various methods are compared in Table 1. The design resistances $R_d$ are normalized with respect to the values obtained for partial factors method (PSF) to simplify the comparison. Consequently, values below unity indicate a lower safety level.

Table 1: Comparison of various safety formats expressed as relative resistance (from Červenka [5])

<table>
<thead>
<tr>
<th>Example</th>
<th>PSF</th>
<th>ĖCOV</th>
<th>ĖN 1992-2</th>
<th>Probabilistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1</td>
<td>1.0</td>
<td>1.0</td>
<td>0.95</td>
<td>0.96</td>
</tr>
<tr>
<td>Bending</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example 2</td>
<td>1.0</td>
<td>1.02</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>shear beam</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example 3</td>
<td>1.0</td>
<td>1.06</td>
<td>0.98</td>
<td>1.02</td>
</tr>
<tr>
<td>bridge pier</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example 4</td>
<td>1.0</td>
<td>0.97</td>
<td>0.93</td>
<td>1.01</td>
</tr>
<tr>
<td>bridge frame</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>1.0</td>
<td>1.01</td>
<td>0.96</td>
<td>0.99</td>
</tr>
</tbody>
</table>
According to Červenka [5], for the investigated range of problems, the following conclusions can be drawn:

- The differences between all methods are not significant.
- Eurocode 2 Part 2 [28] method, using a fixed global safety factor $\gamma_{Gl} = 1.27$ evaluated on a one-dimensional model, gives more conservative results comparing to other methods when used with a two-dimensional model (the model uncertainty for the two models are different).
- The probabilistic assessment is sensitive to the type of random distribution assumed for input variables.

However, different results have been obtained by other studies [1 and 2], underlining that for different FE models (two- or three-dimensional) the safety format according to Eurocode 2 Part 2 [28] may not lead to the intended reliability level.
3 Research Methods

The research methods basically consist of the following five steps: (1) design of the case studies; (2) full probabilistic model; (3) resistant model; (4) assessment of the model uncertainty; and (5) data analysis.

3.1 Design of the Case Studies

3.1.1 Scatter in the Shear Capacity of RC Slender Members without Web Reinforcement

A total number of 13 sets of shear tests on comparable slender RC beams without stirrups extracted from the ACI-DAfStb evaluation database (Reineck et al. [37]) and grouped in Paper I were identified and considered. In order to extract valid statistical information, the amount of data available from each set of comparable tests was increased with numerical experiments.

3.1.2 Resistance and Ultimate Load Behaviour of RC Structures subjected to Multiple Failure Modes

Six sets of beams (from a simply supported beam to various unsymmetrical continuous beams), and a set of isolate short columns (idealizing the inner column of a sway asymmetric frame) were properly designed on the basis of linear elastic analysis for both serviceability and ultimate limit states according to Eurocodes 2 [7] and 8 [8] for various ductility classes (DCL for low, DCM for medium, and DCH for high). Similarly, a further set of slender columns was also designed, this time on the basis of non-linear analysis; for reasons of simplicity, long term effects of shrinkage and creep were not considered in the design process.

The length of the beams varies between 5 m and 20 m. The isolate short columns have a height equal to 3 m whereas the isolate slender columns have 6 m height. Details are given in Table 2.

The examples studied are all connected to buildings (and not civil engineering structures) with concrete C25/30, steel S500, and exposure class X0. The design loads are close to the maximum probable loads to occur during approximately a 50-year period of time and multiplied by load factors. Each member of the designed structures has constant cross-section dimensions (e.g. constant width, depth, and cover) along its length; conversely, longitudinal reinforcements and stirrups are variable and defined depending on the specific needs. Beams
have a rectangular shape, with fixed width \((b = 300 \text{ mm})\) and depth \((h\) value from design), and were supposed to carry a typical domestic floor \((l = 4.00 \text{ m total length})\); columns have a square cross-section with constant side length \(h\) (value from design). An example is reported in Fig. 3.1. A total number of forty-nine different structures were considered.

Table 2: Design samples– Part A

<table>
<thead>
<tr>
<th>Exp. Set</th>
<th>Structural System Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>(G_d = 31.4 \text{ kN/m} \quad Q_d = 12.0 \text{ kN/m} )</td>
</tr>
<tr>
<td></td>
<td>(G_d+Q_d)</td>
</tr>
<tr>
<td></td>
<td>B-30x45-1S-500-DCL</td>
</tr>
<tr>
<td></td>
<td>-DCL</td>
</tr>
<tr>
<td></td>
<td>-DCM</td>
</tr>
<tr>
<td></td>
<td>-DCH</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>(G_d = 31.4 \text{ kN/m} \quad Q_d = 12.0 \text{ kN/m} )</td>
</tr>
<tr>
<td></td>
<td>L1) (G_d+Q_d)</td>
</tr>
<tr>
<td></td>
<td>B-30x45-1S-500-DCL</td>
</tr>
<tr>
<td></td>
<td>-DCL</td>
</tr>
<tr>
<td></td>
<td>-DCM</td>
</tr>
<tr>
<td></td>
<td>-DCH</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>(G_d = 32.9 \text{ kN/m} \quad Q_d = 12.0 \text{ kN/m} )</td>
</tr>
<tr>
<td></td>
<td>L1) (G_d+Q_d)</td>
</tr>
<tr>
<td></td>
<td>B-30x45-2S-500-500</td>
</tr>
<tr>
<td></td>
<td>-DCL</td>
</tr>
<tr>
<td></td>
<td>-DCM</td>
</tr>
<tr>
<td></td>
<td>-DCH</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>(G_d = 34.4 \text{ kN/m} \quad Q_d = 12.0 \text{ kN/m} )</td>
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<td></td>
<td>L1) (G_d+Q_d)</td>
</tr>
<tr>
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<td>B-30x60-2S-750-500</td>
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<td>-DCL</td>
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<tr>
<td></td>
<td>-DCM</td>
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<tr>
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<td>-DCH</td>
</tr>
<tr>
<td><strong>E</strong></td>
<td>(G_d = 31.4 \text{ kN/m} \quad Q_d = 12.0 \text{ kN/m} )</td>
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<td>L1) (G_d+Q_d+Q_d)</td>
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<td>B-30x75-2S-1000-500</td>
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<td>-DCL</td>
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<td>L2) (G_d+Q_d)</td>
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<td>B-30x45-3S-500-500-500</td>
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### 3.1. Design of the Case Studies

Table 2: Design samples – Part B

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<td></td>
<td>$Q_d = 12.0 \text{ kN/m}$</td>
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<td>$G_d + Q_d$</td>
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</tr>
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<td></td>
<td>$G_d + Q_d$</td>
</tr>
<tr>
<td></td>
<td>-DCL</td>
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<tr>
<td></td>
<td>-DCM</td>
</tr>
<tr>
<td></td>
<td>-DCH</td>
</tr>
<tr>
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<td>$G_d$</td>
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<td>$G_d + Q_d$</td>
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<td>-DCL</td>
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<td>-DCM</td>
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<tr>
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<td>-DCH</td>
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<td>-DCL</td>
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<td>-DCM</td>
</tr>
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<td>-DCH</td>
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<td>B-30x60-3S-500-1000-500</td>
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<table>
<thead>
<tr>
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<th>Structural System Description</th>
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<tr>
<td></td>
<td>$H_d = 75.4 \text{ kN}$</td>
</tr>
<tr>
<td></td>
<td>$M_d = 4.8 \text{ kNm}$</td>
</tr>
</tbody>
</table>

**Diagram:**

- G
  - $N_d$
  - $H_d$
  - $M_d$

**Labels:**

- C-55-300
- SC-55-600
3.2 Full Probabilistic Model

A full probabilistic model based on JCSS Probabilistic Model Code [25] able to describe both the mechanical properties of concrete and reinforcement steel, the reinforcement area, the geometrical properties of the cross-section, the out of plumbness, and the resistance model uncertainty was defined. Effects of additional variations on the concrete compressive strength
due to the special placing, curing and hardening conditions, concrete age, and on the tensile strength due to the gravel type and size, composition of cement, and climatical conditions were also taken into account. The model uncertainty $\theta_k$ is modelled in terms of uncertain perturbations by introducing three lognormal distributed stochastic variable ($\theta_{k,B}$ for axial-bending, $\theta_{k,S}$ for shear with stirrups, and $\theta_{k,SC}$ for shear without stirrups) with both expected mean values and standard deviations estimated on the basis of the model assessment procedures described in Section 3.5. The model uncertainty $\theta_k$ is then multiplied by the outcomes of the predictive model ($\theta_{k,B}$ if the failure is due to axial-bending, $\theta_{k,S}$ if it is due to shear with stirrups, and $\theta_{k,SC}$ if it is due to shear without stirrups).

The randomness of shrinkage is considered by assuming that the total unrestrained drying shrinkage of concrete $\varepsilon_{c,d,0}$ is an uncorrelated random variable with known mean value, coefficient of variation of 30%, and lognormal probability distribution; no randomness is instead associated with autogenous shrinkage.

Details are given in Tables 3 and 4. Shown in the table are (a) all the basic random variables, (b) their symbols, (c) distribution types, (d) units, (e) mean values $\mu$, (f) standard deviations $\sigma$, (g) coefficients of variation C.o.V., and (h) coefficients of correlation $\rho_{ij}$.

Table 3: Probabilistic model for investigations on the scatter in the shear capacity of RC slender members without web reinforcement

<table>
<thead>
<tr>
<th>Basic Variable</th>
<th>Symbol/Equation</th>
<th>Dist.</th>
<th>Unit.</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>C.o.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>In situ concrete compressive strength</td>
<td>$f_c = \alpha(t,\tau)f_{c0}Y_1$</td>
<td></td>
<td>MPa</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- $\alpha(t,\tau) = 0.8[0.6 + 0.12 \ln(t)]$</td>
<td>D</td>
<td>day</td>
<td>1.42</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Age of concrete</td>
<td></td>
<td>D</td>
<td>day</td>
<td>28</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Concrete compressive strength</td>
<td>$f_{c0}$</td>
<td>LGN</td>
<td>MPa</td>
<td>$f_{c,m\text{group}}$</td>
<td>$\sigma(f_{c,m\text{group}})$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- $\lambda$</td>
<td>D</td>
<td></td>
<td>0.96</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- $Y_1$</td>
<td>LGN</td>
<td></td>
<td>1.00</td>
<td>-</td>
<td>0.06</td>
</tr>
<tr>
<td>Concrete tensile strength</td>
<td>$f_{ct} = 0.3f_{c}^{2/3}Y_2$</td>
<td></td>
<td>MPa</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>- $Y_2$</td>
<td>LGN</td>
<td></td>
<td>1.00</td>
<td>-</td>
<td>0.30</td>
</tr>
<tr>
<td>Bar area</td>
<td>$A_s$</td>
<td>N</td>
<td>mm$^2$</td>
<td>$A_{s,m\text{group}}$</td>
<td>-</td>
<td>0.02</td>
</tr>
<tr>
<td>Dimensions of cross-section</td>
<td>$h, b$</td>
<td>N</td>
<td>mm</td>
<td>$X_{m\text{group}}$</td>
<td>$4+0.006X_{m\text{group}}\leq 10$</td>
<td>-</td>
</tr>
<tr>
<td>Effective depth of cross-section</td>
<td>$d$</td>
<td>N</td>
<td>mm</td>
<td>$d_{m\text{group}}$</td>
<td>10.00</td>
<td>-</td>
</tr>
<tr>
<td>Model uncertainty</td>
<td>$\theta_{k,SC}$</td>
<td>LGN</td>
<td></td>
<td>1.10</td>
<td>-</td>
<td>0.10</td>
</tr>
<tr>
<td>Unrestrained drying shrinkage</td>
<td>$\varepsilon_{c,d,0}$</td>
<td>LGN</td>
<td>$\varepsilon_{c,d,0\text{m}}$</td>
<td>-</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Age of concrete at the beginning of drying shrinkage</td>
<td>$t_s$</td>
<td>D</td>
<td>day</td>
<td>7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ambient relative humidity</td>
<td>$RH$</td>
<td>D</td>
<td>%</td>
<td>60</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note. D = deterministic value; N = normal distribution; LGN = lognormal distribution
The correlation coefficients $\rho_{ij}$ (that in this thesis will be simply denoted as $r$) are also known as the product-moment coefficients of correlation or Pearson’s correlations (Snedecor and Cochran [38]). Correlations are interpreted by squaring the value of the correlation coefficients. The squared values represent the proportion of variance of one variable that can be predicted from the other variable. A rule of thumb for interpreting correlation coefficients has been established from experimental studies (Garcia [39]): (i) $0.0 \leq r < 0.2$, very weak; (ii) $0.2 \leq r < 0.4$, weak; (iii) $0.4 \leq r < 0.7$, moderate; (iv) $0.7 \leq r < 0.9$, strong; and (v) $0.9 \leq r \leq 1.0$, very strong.

The reference to JCSS Probabilistic Model Code [25] is made because, as reported in Vrouwenvelder [40], it gives guidance on the modelling of random variables in structural engineering.

For each simulation setting (comparable shear tests or designed structures), thousands of samples (150,000 for each shear test, 10,000 for the various types of beams; and 1,000 for the columns) were generated via Monte Carlo method through the use of Latin Hypercube Sampling (see McKay et al. [41]), from the mean values of their physical parameters, covering a wide range of mechanical and geometrical properties.

Table 4: Probabilistic model for investigations on the resistance and ultimate load behaviour of RC structures subjected to multiple failure modes

<table>
<thead>
<tr>
<th>Basic Variable</th>
<th>Symbol/Equation</th>
<th>Dist.</th>
<th>Unit.</th>
<th>$\mu$</th>
<th>$\sigma$</th>
<th>C.o.V.</th>
<th>$\rho_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>In situ concrete compressive strength</td>
<td>$f_c = \alpha(t,\tau) f_{c0}^{0.8} Y_1$</td>
<td>- MPa</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Age of concrete</td>
<td>$t$</td>
<td>D</td>
<td>year</td>
<td>1.42</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Concrete compressive strength</td>
<td>$f_{c0}$</td>
<td>LGN</td>
<td>MPa</td>
<td>0.96</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>- Concrete tensile strength</td>
<td>$f_{t} = 0.3 f_c^{2/3} Y_2$</td>
<td>- MPa</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Bar area</td>
<td>$A_s, A_s', A_{sw}$</td>
<td>N mm$^2$</td>
<td>$X_{design}$</td>
<td>0.02</td>
<td>1.00 0.50 0.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel yield stress</td>
<td>$f_y, f_{yw}$</td>
<td>N MPa</td>
<td>560</td>
<td>-</td>
<td>0.30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Steel ultimate strength</td>
<td>$f_u$</td>
<td>N MPa</td>
<td>1.15 $f_{y,nom}$</td>
<td>40</td>
<td>0.35 0.85 1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dimensions of cross-section</td>
<td>$h, b$</td>
<td>N mm</td>
<td>$X_{design}$</td>
<td>4+0.006$X_{design}$≤10</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Concrete cover to top steel</td>
<td>$c_s$</td>
<td>LGN</td>
<td>mm</td>
<td>$c_{s,design}$+10</td>
<td>10</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Effective depth of cross-section</td>
<td>$d$</td>
<td>N mm</td>
<td>$d_{design}$+10</td>
<td>10</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out of plumbness</td>
<td>$\Phi$</td>
<td>N rad</td>
<td>0.00</td>
<td>0.0015</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Model uncertainty for axial-bending</td>
<td>$\theta_{R,B}$</td>
<td>LGN</td>
<td>-</td>
<td>1.00</td>
<td>-</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>Model uncertainty for shear</td>
<td>$\theta_{R,S}$</td>
<td>LGN</td>
<td>-</td>
<td>1.60</td>
<td>-</td>
<td>0.15</td>
<td>-</td>
</tr>
</tbody>
</table>

Note. D = deterministic value; N = normal distribution; LGN = lognormal distribution
3.3 Resistant Model

3.3.1 Shear Capacity of RC Slender Members without Web Reinforcement

The presence of a high uncertainty in the shear transfer mechanism is doubtless reflected on the quality of the different predictive models implying that, at present, no advanced and sophisticated formulations exist that are able to predict the shear strength of reinforced concrete beams without web reinforcement with a high demand in precision and accuracy. In this regard, a painstaking research activity was carried out in past (see Reineck et al. [37]; Bresler and Scordelis [42]; Cho [43]; and Song and Kang [44]). The most promising way to investigate it is, undoubtedly, the use of a non-linear FEM analysis suitably calibrated; however, this procedure is also complicated and time-consuming.

Similarly, calculation of shrinkage is a difficult task, because shrinkage, degree of restraint, modulus of elasticity, Poisson’s ratio, creep, concrete age, and concrete quality all influence the stress. In accordance with Silfwerbrand [45], due to the large number of influencing parameters, an accurate prediction of shrinkage stresses is almost impossible in an arbitrary case and, if detailed knowledge of actual concrete properties and environmental effects is lacking, use of complicated models can hardly be motivated.

Therefore, since “the purpose of computing is insight, not numbers” (Hamming [46]), a simple formulation for both ultimate shear resistance and shrinkage strain such as the empirical one suggested by Eurocode 2 [7], which is, however, able to explain qualitatively this phenomenon, was considered sufficient for the aim of the study. Shear resistance and shrinkage strain equations were then combined together to take into account shrinkage effects.

Three different predictive models were considered and assessed through comparison with experimental results. From the simplest to the more complex, they are characterized as follows.

Model A: shear resistance model entirely according to Eurocode 2 [7]; shrinkage effects neglected.

\[ V_{R,c} = [C_{R,c}K(100\rho_l f_{c}(t))^{1/3}]b_wd \]  \hspace{1cm} (3.1)

Model B: same as model A considering the square root of the tensile strength of concrete multiplied by a corrective coefficient given by the ratio 1/0.3, instead of the cube root of the compressive strength (over many simulations this will not affect the mean value of the distribution of the shear capacity, but it will consider the greater scatter of the tensile strength of concrete respect the compressive strength).

\[ V_{R,c} = [C_{R,c}K(100\rho_l)^{1/3}(f_{ct}(t)/0.3)^{1/2}]b_wd \]  \hspace{1cm} (3.2)

Model C: same as model B including the shrinkage effects.

\[ V_{R,c} = [C_{R,c}K(100\rho_l)^{1/3}(f_{ct}(t)/0.3)^{1/2} + k_1\sigma_{cs}(t)]b_wd \]  \hspace{1cm} (3.3)
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The following assumptions were made for Model C: (i) age of concrete $t = 28$ days; (ii) age of concrete at the beginning of drying shrinkage $t_s = 7$ days; (iii) ambient relative humidity $RH = 60\%$; and (iv) $N$ strength class of cement.

According to Park and Paulay [47], shrinkage effects were modelled as a tensile restraining force, due to the bonded reinforcement, applied on the concrete at the level of the centroid of the steel (see Fig. 3.2). In the figure, consistent with Eurocode 2 [7], $\varepsilon_{cs}$ is the total unrestrained shrinkage of concrete (given by the combination of drying and autogenous components of shrinkage, $\varepsilon_{cd}$ and $\varepsilon_{ca}$ respectively). Relaxation of shrinkage strains due to creep of concrete is not taken into consideration.

Figure 3.2: Shrinkage effects

3.3.2 Structures Subjected to Multiple Failure Modes and NL-FEM Analysis

Two different numerical models based on the FEM incremental-iterative non-linear analysis of RC plane frames were implemented in MATLAB environment for the purpose of the present research project. The two resistance models consider the combination of failure modes of reinforced concrete structures and are based on Eurocode 2 [7] assumptions for members subjected to flexure, axial load, and shear. Such models are certainly more complicated than the standard approach, but also more realistic.

The first model, the fastest one, considers only the material non-linearities and is based on both the moment-curvature relation and the modified secant stiffness method (Kim and Lee [48]; Rasheed and Dinno [49 and 50]; Kwak and Kim [51]; and Valipour and Foster [52]). The moment-curvature relationship is numerically derived using the mathematical model of the full stress-strain curves of concrete and reinforcing steel, strain compatibility, and equilibrium equations for all the basic structural elements composing the structure. The second model considers both the material and geometric non-linearities and is based on the more sophisticated layer approach. The model uses the element type defined by Kang and Scordelis [53]. Unfortunately, this element type has shape functions that are not mutually consistent (it associates constant linear axial deformation to parabolic curvature) which may lead to an excessive rigidity, called eccentricity issue, that, however, can be mitigated reducing the size of the mesh (Ferretti et al. [54]). Both the proposed models use the load control approach and are able to analyse elements with variable geometry. The safety check is performed in the domain of the internal actions.

For each analysed structure, the number of elements was kept to the minimum required to adequately describe the system (e.g., depending on the variation of cross-section geometry,
3.3. RESISTANT MODEL

Longitudinal reinforcement and stirrups), whereas the FEM mesh was composed of one-dimensional elements and refined to reach internodal distances smaller than 100 mm.

Assumptions for the analysis were as follows: (i) Bernoulli-Navier beam theory; and (ii) load applied at the centroid of the element. Spurious sensitivity of results due to both load step and convergence criteria was reduced as follows: (a) loading was applied in a stepwise fashion with 1% increments; (b) convergence criteria were based on force and displacement, and the convergence tolerance limit was established for both calculations at 0.1%. Bond slip between steel and concrete, and long term loading effect were neglected.

The elevation and the cross section of an undeformed element which is assumed to be prismatic are shown in Fig. 3.3(a) and 3.3(b), respectively. The cross-section is assumed to have an axis of symmetry which coincides with the local y-axis. In order to account for the varied material properties within an element, the cross-section is divided into a discrete number of concrete (approximately one per cm) and reinforcing steel layers each of which is assumed to be in a state of uniaxial stress. The geometry of each layer is defined by its area and y coordinate.

![Diagram of cross-section and element](image-url)

Figure 3.3: Geometry of an undeformed FE
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The mathematical model for the stress-strain curve of concrete is shown in Fig. 3.4(a). The ascending part of the curve is a parabola as proposed by Sargin [55], which is also mentioned in Eurocode 2 [7], and represents concrete in compression, whereas concrete in tension is assumed to have a linear elastic behaviour up to the maximum tensile stress $f_{ct}$. After that point, tension stiffening effects are also considered according to Massicotte et al. [56]. Consistent with Eurocode 2 [7], a bilinear stress-strain curve is utilized for reinforcing steel as shown in Fig. 3.4(b). The meaning of symbols is as in the mentioned references.

![Figure 3.4: Modelling of material properties according to Eurocode 2 [7] and Massicotte et al. [56]](image)

### 3.4 Assessment of the Model Uncertainty

Models, descriptive or predictive, are the basic vehicles by which scientists reflect and express the understanding of some aspect of reality, a particular unknown of interest. As it is virtually impossible to grasp any situation in its entire complexity, models are always partial representations of reality. In other words, what it is known about the true nature of the unknown of interest is generally incomplete, resulting in a state of uncertainty. Accordingly, the uncertainties in model predictions arise from uncertainties in the values assumed by the model parameters, parameter uncertainty, and the uncertainties and errors associated with the structure of the model, model uncertainty, stemming from abstractions, assumptions, and approximations (Droguett and Mosleh [26]).

#### 3.4.1 Scatter in the Shear Capacity of RC Slender Members without Web Reinforcement

The assessment procedure for the model uncertainty $\theta_{R,SC}$ goes through a process of comparison with experimental results. Engineering judgment was also used. The ACI-DAfStb evaluation database (Reineck et al. [37]) was considered. The raw database was filtered so that all experiments carried out outside of the following range were excluded: (a) shear-to-span ratio $2.5 \leq \alpha_d \leq 7$; and (b) geometric percentage of longitudinal reinforcement $\rho_{sw} \leq 2\%$. Experiments lacking recorded test values for axial tensile strength of concrete $f_{ct,\text{test}}$ were neither taken into consideration. The shear tests results were direct compared with models A, B, and C predictions.

For each model, the model safety factor, defined herein as $\gamma_{mod} = V_{\text{test}}/V_{\text{mod}}$, was computed and results were visually evaluated.
3.4.2 Resistance and Ultimate Load Behaviour of RC Structures Subjected to Multiple Failure Modes

The assessment procedure of the model uncertainty goes through a process of comparison with both experimental results and the results of simulations performed with ADINA. Simple and continuous beams subjected only to bending failure were assessed through comparison with ADINA outputs. Columns subjected to buckling due to bending and axial compression were assessed through comparison with reported tests: (a) two cases studied by Kang and Scordelis [53] (Lin Beam B and Aroni Column A2 30c5), (b) the example 2.3, pp. 28-29, published in [57], and (c) the column II/4, loads of short duration, investigated by Gaede [58]. Results are summarized in Table 5. For the evaluation of the shear resistance model, the author refers to the study of Sykora et al. [59]. The statistical and probabilistic characteristics of the models uncertainties (θ_{R,B} for axial-bending, and θ_{R,S} for shear) were then evaluated and compared with suggestions of both JCSS Probabilistic Model Code [25] and Schlune et al. [1]. Engineering judgment was also used.

Table 5: Performance data for θ_{R,B} based on experimental results

<table>
<thead>
<tr>
<th>Experiment Notation</th>
<th>Experimental Ultimate Load (P_{exp}) [kN]</th>
<th>Ref.</th>
<th>FEM Ultimate Load (P_{u}) [kN]</th>
<th>P_{exp}/P_{u}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lin Beam B</td>
<td>178</td>
<td>[53]</td>
<td>200</td>
<td>0.89</td>
</tr>
<tr>
<td>Aroni Column A2 30c5</td>
<td>8.81</td>
<td>[53]</td>
<td>8.9</td>
<td>0.99</td>
</tr>
<tr>
<td>Column example 2.3</td>
<td>68.0</td>
<td>[57]</td>
<td>68.3</td>
<td>1.00</td>
</tr>
<tr>
<td>Column II/4</td>
<td>38.5</td>
<td>[58]</td>
<td>38.0</td>
<td>1.01</td>
</tr>
</tbody>
</table>

3.5 Data Analysis

The different designed case studies were then analysed up to complete loading and mechanical behaviour with the resistant model suitable for the specific investigation.

Results were then analyzed through both multivariate statistical and probabilistic methods and visually explored by box plots. Data mining techniques were also applied.
4 Considerations on RC Structural Systems

4.1 Reliability of Structural Systems

Any structural system has to satisfy more than one performance criterion. Even for a simple beam, the performance criterion could be strength related (e.g., bending moment or shear) or serviceability related (e.g., deflection or vibration). Thus, the beam can fail in more than one performance mode.

The concept to consider multiple failure modes and/or multiple component failures is known as system reliability evaluation (Haldar and Mahadevan [60]). A complete reliability analysis includes both component-level and system-level estimates.

In general, system reliability evaluation is quite complicated and depends on many factors. Some of the important factors are (1) the contribution of the component failure events to the system’s failure, (2) the redundancy in the system, (3) the post-failure behaviour of a component and the rest of the system, (4) the statistical correlation between failure events, and (5) the progressive failure of components. Considering the beam example, it is possible to estimate its reliability by calculating the probability of satisfying all the performance criteria.

An engineering system usually consists of multiple components, and system failure may occur when one or more components fail. This is still an active area of research and a detailed discussion of the argument is well beyond the scope of the present thesis (for different opinions see, for example, Haldar and Mahadevan [60] and Ditlevsen and Madsen [61]).

4.2 Distribution of Structural Resistance

The study conducted in Paper III has highlighted that, in case of structures subjected to multiple failure modes, the structural resistance $R$ may be described as combination of different structural sub-resistances $R_X, ..., R_Z$ where $X, ..., Z$ represent the diverse failure mechanisms of the structure (e.g., bending, shear, buckling, crack propagation, fatigue).

This hypothesis can be formulated as follows: “the sub-resistance $(R_X, R_Y, R_Z)$ of a structure subjected to multiple failure modes $(X, Y, Z)$ is defined as the fraction of the total resistance $(R)$ of the structure to the specific failure mechanism”.

By considering, for simplicity, a structure subjected to two different failure modes, e.g., (1) bending and (2) shear, the structural failure mechanism seems to depend on both the distance...
between the mean values of the two structural sub-resistance factors $\mu_1$ and $\mu_2$, respectively, and the magnitude of their standard deviations. Qualitatively speaking, if $\mu_1$ and $\mu_2$ are reasonably near and their standard deviations are such as to create a significant overlap between the two random variables, the failure would seem to be due to a combined effect of flexural strength and shear strength. This could imply that a clear distinction between a pure ductile failure (the flexural one) and pure brittle failure (the shear one) would not have been able to be made with certainty, and could lead to a mixed failure mode, the so-called flexural-shear failure. On the other hand, if $\mu_1$ and $\mu_2$ are enough far and their standard deviations are sufficiently small that the likely overlap of the two random variables is reduced to the minimum possible, the most probable structural failure mechanism is the one having the lowest strength.

This imply, as discussed in Paper V, that the distribution of the resistance of RC structures subjected to multiple failure modes may have, in general, a multimodal probability density function (PDF), resulting as a combination of the probability density functions (PDFs) of the structural sub-resistances. See Figs. 4.1 and 4.2 for instance. A similar result was reached by Henriques et al. [24] in their study on safety format for the design of concrete frames. The researchers have shown, in fact, that the resistance distribution for RC beams subjected only to flexural failure mode may be unimodal or bi-modal depending on the amounts of reinforcements.

As a consequence, the simple link between the mean (or, for analogy, characteristic) values of geometric and material properties and the mean (or characteristic) value of the structural resistance may not be always guaranteed. This fact may limit the applicability of actual safety formats for non-linear analyses of RC structures to generic cases in which the structural system may have multiple failure modes and/or multiple component failures. Unfortunately, nowadays, this is slightly taken into consideration being the probabilistic approach commonly suggested mostly based on the assumption that the structural resistance $R$ has a lognormal PDF.

Fig. 4.1 shows the PDF and cumulative density function (CDF) of both the flexural and shear sub-resistance factors $\lambda_X = R_X/L_d$, where $R_X$ is the sub-resistance of the beam related to failure mode $X$ and $L_d$ is the design load, for beam B-30x60-2S-750-500 (experimental set C, load case L1, and DCH), considered as representative of the case studies. The pdfs are not scaled by the probability of occurrence.

Fig. 4.2 displays the scatter plot, with PDFs and both the fitting of a least squares line and the correlation coefficients $r$, of the structural sub-resistance factors $\lambda_X$ of the different failure modes of beam B-30x45-2S-500-500 (experimental set B, load case L1, and DCM) with respect to the tensile strength of concrete $f_{ct}$. Here, data are grouped by failure modes: (1) flexural failure with crushing of concrete without yielding of tension reinforcements, (2) flexural failure with crushing of concrete with yielding of tension reinforcements, and (3) shear failure. The pdfs are not scaled by the probability of occurrence.
4.3 Material Uncertainty

Research carried out in Paper IV reveals the following main features.

- It is not always true that a decrease of material properties of a structure reduce the structural resistance. On the contrary, it has been shown that an increase of tensile strength of concrete $f_{ct}$ seems to induce a flexural brittle failure by crushing of the compressive concrete before the tension steel yields, which does not provide any warning before failure as the failure is instantaneous (apparent over-reinforcement of...
the beam), and, consequently, a reduction of the structural resistance. This phenomenon seems to affect only statically undetermined systems (e.g., continuous beams and, for analogy, frames) by preventing the redistribution of bending moments.

- It has been shown that both the structural resistance and the ultimate behaviour of RC structures subjected to multiple failure modes seem to be mainly influenced by the combination of the mechanical properties of both longitudinal reinforcement and stirrups. An increase of the steel yield stress $f_y$ of the longitudinal bars seems, in fact, to lead preferably to a shear failure and, consequently, to a brittle behaviour of the structure, whereas an increase of the steel yield stress $f_{yw}$ of stirrups seems to conduct to a preferential flexural failure and, accordingly, to an improvement of the structural ductility. This phenomenon may be, of course, of great importance for structural safety evaluations and, therefore, for the applicability of a safety format. However, more research is still needed before definite recommendations are reached.

- Compressive strength of concrete $f_c$ seems to be only of minor importance for the structural resistance of RC structures.

4.4 Geometrical Uncertainty

The geometrical uncertainty of RC structures subjected to multiple failure modes still remains quite small as for structures with only one failure mechanism (see, for instance, Schlune et al. [1 and 2] and JCSS Probabilistic Model Code [25]) and, therefore, continues to have a minor influence on the resistance.

Moreover, results of this research project show that their effects are quite small even on structures sensitive to geometrical imperfections as slender columns.

4.5 Model Uncertainty

The analyses conducted in Papers III and IV highlight that the model uncertainty of the resistance model plays the most important role in interpreting results and drawing conclusions.

Furthermore, numerical modelling of the structural behaviour of structures subjected to multiple failure modes may imply the use of different model uncertainties, depending on the accuracy of the model to each failure mechanism (e.g., bending and shear). This is a quite new problem poorly investigated in the past that should be better assessed in a safety format for non-linear analysis.
5 Conclusions

5.1 General Conclusions

The purpose of this thesis is enhancements to the safety format assessment procedure for non-linear analysis of reinforced concrete (RC) structures subjected to multiple failure modes. The focus lies on identifying questions which are poorly understood and ambiguous about the behaviour of structures that may fail due to a number of possible failure modes (e.g., bending, shear, buckling, crack propagation, fatigue) which might be used for the development of a more comprehensive approach to the evaluation of the structural safety level. The following conclusions are based on the studies described in the appended Papers.

- The scatter of the shear capacity of RC slender members seems to be mainly due to the randomness of the tensile strength of concrete which, in turn, is owed to the presence of flaws or micro-cracks that may vary in dimensions and orientation depending on different factors (e.g., gravel type and size, composition of cement, high rate of restrained shrinkage, and climatic and curing conditions). High values of shrinkage may also play an important role. Other parameters like (a) height of beam, (b) area of reinforcing steel, (c) width of web, (d) effective depth, and (e) compressive strength of concrete seem to be of only minor importance.

- The ultimate load behaviour of a RC structure designed according to Eurocodes 2 and 8 is not unequivocal and may vary from structure to structure depending on different factors: (a) the structural system, (b) the load configuration, and (c) the ductility class. The combination of those three factors may lead to a preferential performance of the structure; however this seems not to affect the main failure mode or, more realistically speaking, the main failure mechanism. In all the studied cases, in fact, the average shear strength exceeds the shear corresponding to the average flexural strength. That means that an overall ductile behaviour is expected. Therefore, it is possible to conclude that the European design specification is successful in preventing the formation of non-ductile failure mechanisms.

- The resistance of RC beams subjected to flexural and shear failure modes seems to be mainly influenced by the combination of the mechanical properties of both longitudinal reinforcement and stirrups. An increase of the steel yield stress $f_y$ of the longitudinal bars seems, in fact, to lead preferably to a shear failure and, consequently, to a brittle behaviour of the structure, whereas an increase of the steel yield stress $f_{yw}$ of stirrups seems to conduct to a preferential flexural failure and, accordingly, to an improvement of the structural ductility. Moreover, tensile strength of concrete $f_{ct}$ appears also to affect the overall response of the structure depending on both its intensity and the structural system. It has been shown, in fact, that moderate values of $f_{ct}$ have almost negligible effects on both the structural ultimate behaviour and resistance whereas, on the contrary, high values may reduce the ductility of the
structure inducing a brittle failure with crushing of concrete in compression before the tension steel yields (which does not provide any warning before failure). However, this reduction of ductility does not seem to affect the overall structural resistance of statically determined systems (as simply supported beams) while it may strongly reduce the resistance of statically undetermined systems (e.g., continuous beams and, for analogy, frames) by preventing the redistribution of bending moments. On the other hand, compressive strength of concrete $f_c$ seems to be of only minor importance. The uncertainty of the resistance model used in the analysis plays the most important role in interpreting results and drawing conclusions.

- The resistance of RC structures subjected to multiple failure modes may have, in general, a multimodal probability density function, in which each mode represents a specific failure mechanism. Moreover, it has been derived that the distribution of the structural resistance $R$ is a linear combination of the distributions of the structural sub-resistances $R_X$ multiplied by their corresponding probability of occurrence $\rho_X$.

Besides the conclusions stated above, this research project has given some practical contributions. In Paper I a collection of sets of comparable experiments extracted from the ACI-DAfStb evaluation database of shear tests on slender reinforced concrete beams without stirrups was established; these sets of comparable experiments are intended to be used by researchers who investigate the causes of the shear failure scatter or develop improved shear design methods.

A contribution of Paper III is the suggestion of two model uncertainties for (a) the combination of both bending moment and buckling capacities for the material and geometric non-linear FE analysis of RC structures, and (b) the shear capacity of RC structures with stirrups analyzed with the Eurocode 2 shear strength model both assessed through a process of comparison with experimental results on representative examples.

5.2 Further Research

5.2.1 Scatter in the Shear Capacity of RC Slender Members without Web Reinforcement

The scatter in shear is a complex problem subjected to randomness which can be better understood through the use of strain-based models, as a non-linear FE analysis suitably calibrated, coupled with a probabilistic approach. Together, these will provide a long-term perspective on the question.

Moreover, a more comprehensive study of the effects of (i) aggregates and their location in the cross-section and in relation to rebars, and (ii) microcracks and their random pattern is advisable.
5.2.2 Resistance and Ultimate Load Behaviour of RC Structures

An extension of the studies presented in Papers III and IV to more complex structural systems, and through the use of 2D or 3D non-linear FE analysis suitably calibrated, is desirable.

In addition, the following question is still unsolved:

- How to deal with uncertainties in existing structures?

5.2.3 Safety Format for Non-Linear Analysis of RC Structures Subjected to Multiple Failure Modes

Several factors are still not well assessed in actual safety formats for non-linear analysis of RC structures. Some of them are shown to be: (i) the possibility that the structural resistance may generally have a multimodal probability density function (PDF) whereas the probabilistic approach commonly suggested is based on the assumption that it has a lognormal PDF; (ii) the fact that a decrease of material properties of a structure may not always reduce the structural resistance whereas, on the contrary, an increase of tensile strength of concrete $f_{ct}$ seems to induce in continuous systems a flexural brittle failure by crushing of the compressive concrete before the tension steel yields and, consequently, a reduction of the resistance of the system; (iii) the risk that the link between the mean (or, for analogy, characteristic) values of geometric and material properties and the mean (or characteristic) value of the structural resistance may not always be guaranteed; and (iv) the circumstance that the model uncertainty of the resistance model used in the non-linear analysis may be not unambiguous.

This is still an active area of research and more studies are still necessary before any firm conclusion can be drawn.
Bibliography


König G, Pommerening D, Tue N. Safety Concept for the Application of Non-linear Analysis in the design of Concrete Structures - General Considerations. CEB Bulletin 229, 1995.


Appendix 0. Bibliography


