Analysis of Active Gyro Based Roll-Stabilization of Slender Boat Hulls

TAO ZHANG

Master of Science Thesis
Stockholm, Sweden 2014
Analysis of Active Gyro Based Roll-Stabilization of Slender Boat Hulls

Tao Zhang
taozhang@kth.se

Academic Supervisor: Ivan Stenius
Examiner: Jakob Kuttenkeuler
Date: March 4, 2014

KTH, School of Engineering Sciences (SCI), Aeronautical and Vehicle Engineering, Naval Systems
Stockholm, Sweden
Currently, traffic congestion often happens in big cities every day. People demand a new conceptual vehicle which has a slender shape to reduce space, lightweight structure to decrease the fuel consumption and innovative technology to adapt for multiple transportation conditions. NEWT is such a conceptual amphibious vehicle that satisfies people’s requirements. However, everything has two sides. Slender shape and high centre of gravity will result in instability. When NEWT runs in low speed, it easily gets rolled over. In order to make up for its drawback, gyro-stabilizer has been applied to the vehicle. By tilting the rotational gyro, it generates a counter torque counteracting the roll motion to make the vehicle recover to an upright position.

Therefore this master thesis analyses the original stability of the vehicle and the possible improvement by adding the gyro system for both land and water-conditions. The model can handle the problem that the vehicle meets periodic disturbance forces, such as wave excitation force and wind force.
First I really appreciate my supervisor Ivan Stenius for his help. His constant feedback, comments, discussion, and encouragement always enlighten me and show me the right way.

I would also like to acknowledge my teachers: Jakob Kuttenkeuler, Anders Rosén, Karl Garme, and Stefan Hallström. They have educated me with their immense knowledge during the two years study in KTH.

Furthermore, I would like to express my thankfulness to all my classmates, you give me an unforgettable memory. I would cherish the time studying with you.

Finally I am grateful to my family for their endless love and support. They are my drives!
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>FOREWORD</td>
<td>3</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>4</td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td>6</td>
</tr>
<tr>
<td>1.1 Background</td>
<td>6</td>
</tr>
<tr>
<td>1.2 Method</td>
<td>8</td>
</tr>
<tr>
<td>1.3 Limitations</td>
<td>8</td>
</tr>
<tr>
<td>2 LITERATURE REVIEW</td>
<td>9</td>
</tr>
<tr>
<td>3 THE STABILITY THEORY</td>
<td>10</td>
</tr>
<tr>
<td>3.1 Hydrostatic stability</td>
<td>10</td>
</tr>
<tr>
<td>3.2 Gyroscopic stability</td>
<td>12</td>
</tr>
<tr>
<td>4 INITIAL STABILITY ANALYSIS</td>
<td>14</td>
</tr>
<tr>
<td>4.1 Digitizing the model</td>
<td>14</td>
</tr>
<tr>
<td>4.2 Original Stability Curves</td>
<td>16</td>
</tr>
<tr>
<td>4.3 Potential GZ Improvement with Gyro Stabilizer</td>
<td>20</td>
</tr>
<tr>
<td>4.4 Conclusion and discussion</td>
<td>23</td>
</tr>
<tr>
<td>5 MODELLING OF ROLL MOTION</td>
<td>24</td>
</tr>
<tr>
<td>5.1 Implementation of the model in land mode</td>
<td>24</td>
</tr>
<tr>
<td>5.2 Implementation of the model in water mode</td>
<td>25</td>
</tr>
<tr>
<td>5.3 Enhanced model with disturbing forces</td>
<td>31</td>
</tr>
<tr>
<td>5.4 Validation of the model</td>
<td>36</td>
</tr>
<tr>
<td>6 RESULTS</td>
<td>39</td>
</tr>
<tr>
<td>7 CONCLUSIONS</td>
<td>40</td>
</tr>
<tr>
<td>8 FUTURE WORK</td>
<td>41</td>
</tr>
</tbody>
</table>
REFERENCES ........................................................................................................................................ 42

7.1 Literature .................................................................................................................................... 42
7.2 Internet resource .......................................................................................................................... 42

APPENDIX A: BRITFAIR FILE QUOTATION .................................................................................. 43
APPENDIX B: STATE SPACE FORM .................................................................................................. 44
APPENDIX C: WAVE MEASUREMENTS .............................................................................................. 46
APPENDIX D: MATLAB SCRIPTS ....................................................................................................... 48
1 INTRODUCTION

1.1 Background

In Stockholm area there are nearly 1 million commuters. [23] The primary commuting routes running along the major arterial roads, rail networks and metro networks are becoming more and more congested. Therefore since 2007, the government had to implement the Stockholm congestion tax (Swedish: Trängselskatt i Stockholm) to reduce the traffic congestion and improve the environmental situation in central Stockholm. Traffic congestion can be found as well in other European countries. Heading the list from last year was Belgium where drivers wasted 55 hours in traffic. Holland was the next-worst country for jams, followed by France, statistics from traffic Information Company INRIX showed. [24]

Let’s take a glance at Asian city, Hong Kong. Like Stockholm, Hong Kong has a big population and it is also an archipelago city. The citizens suffer from traffic jam every day. According to government data [25], average vehicle speed on Lung Cheung Road (an expressway) has declined by 40 per cent from 2005 to 2010, and on Waterloo Road (a commuter artery) by 22 percent. Furthermore, bus journey times have increased, 99 per cent of its routes experiencing increased journey times, with the rise an average of 16 percent. Worsening congestion means more delays for commuters and a higher toll on the environment. The delays and variability of road transport naturally push commuters to use the subways, which is environmentally friendly. But railway crowding and the rising wealth of the middle class are pushing people back to road transport (private cars), which leads a vicious cycle.

As a result, people demand a new type of vehicles to change this situation, NEWT is such a new conceptual commuter vehicle that can run both on water and land safely and efficiently. On land it is designed as a two-wheeled motorcycle with a narrowly enclosed shell (See Figure 1.1.1). In principle narrow motorcycle results in the reduction in space and air drag. And on water two wheels can be retracted into body, NEWT becomes a conventional boat. NEWT can be widely used in coastal and archipelagic cities which have complex and various transportation systems. People can easily drive it across river and inner sea without transferring from autos to ships. Presumably, NEWT could help ease the traffic congestion in the future.

NEWT has two different modes in water, one is planing hull mode. NEWT can rise from the water with the help of two hydrofoils at high speed. Another one is displacement mode, it runs like a normal boat at lower speed. The problem is that it might lack of stability in displacement mode at low speed because of its slender hull shape and relatively high CG position. Especially in wavy condition, roll motion might make it wobble and even flip over.

The way that was investigated to improve its stability is to use gyroscopic stabilizer. So far, gyroscopic theory has been applied in various fields. It can be used in navigation as a sensor, and
also can be an actuator here generates righting moment counteracting the disturbing moment from gravity, wave, wind, etc.

The aim of the paper is to analyse the improvements of the ship stability with implementing the gyro-model in computer simulation and find out if it can be utilized in real sea water condition in Stockholm.

![Figure 1.1.1 Draft NEWT](image)
1.2 Method

To start with, a simple analysis is carried out to see how much the gyro-stabilizer can help the stability theoretically. First the ship hull is digitized and imported to a computer program to calculate its GZ-curve. After that, the maximum counter torque caused by gyro precession is computed theoretically and considered as an imaginarily added GZ lever arm. The new GZ-curve is compared with the stability criterion to see the improvements and potential in an overall perspective of using active gyro stabilization.

A control system model is developed and simulated by a MATLAB program. The angle and angular velocity of ship and gyro are the state vectors, tilting moment is the input, and the angle of ship and gyro are defined as output vectors. At each time step, calculate the error, and decide how much input torque is needed in next time step iteratively. This model can simulate the boat’s performance both in land and water modes respectively.

In the end, by adding more disturbances like wind force and wave excitation force to the system, the model can be more realistic and accurate. Eventually the behaviours of the ship with the aid of gyroscope can be anticipated in realistic conditions in Stockholm.

1.3 Limitations

- The vehicle in modelling analysis is simplified to be a box-shaped vehicle.
- The forward speed of the vehicle is zero, so there is no track curvature when heels.
- Only roll motion is considered in the analysis.
- The control system has its limitation of the initial heeling angle. It cannot handle the situation out of its capacity.
- Disturbance forces are assumed to be cosine functions
2 LITERATURE REVIEW

Ship’s intact stability is the prerequisite when doing initial design whatever the types of ship. It’s usually represented as GZ-curve. The theory and principle has explained by Anders Rosen [1]. And the criterion of GZ-curve for small boat is described by The Marine Safety Agency, UK [10].

Date back to last century, the history of the gyroscope is concluded by Ljiljana Veljović [3]. A variety of stabilizers used in ships for different functions that are introduced by Samoilescu, G and Radu, S [2]. Apart from ships, it has been used in two-wheeled vehicle [4]. A lot of research has achieved for developing a model to simulate the behavior of the vehicle [5][6][7][8][9].

In order to fit the model to NEWT boat, some changes are made such as redefining the equations of motion (EOM) and state space form. Additionally some disturbance forces are added to the model. For instance wave excitation moment is introduce to the model, the theory is from [1].
3 THE STABILITY THEORY

In this Chapter, the theory of hydrostatic stability and gyroscopic stability is described. Section 3.1 explains the basis of ship hydrostatic stability and the equations of righting lever arm as well as righting moment. In Section 3.2, the way by which the gyroscopic stabilizer works is presented and the equation of righting moment from the gyro is given as well. Therefore, the total righting moment is supposed to be the summary of hydrostatic moment and gyroscopic moment.

3.1 Hydrostatic Stability

In principle, a vessel in calm water and not moving is mainly subjected to two forces, gravitational force and buoyancy force. And when it floats, these two forces are equal in magnitude. More than floatation, people expect ships can always go upright, which is related to ship’s stability.

When a ship is forced to a heeling angle, the centre of buoyancy moves as a result, and the gravitational force and buoyancy force hereby together creates a righting moment to restore the ship to upright position. The lever arm GZ is the horizontal distance between the centre of gravity and the centre of buoyancy, and is commonly used to quantify the ship’s stability. More specifically, the positive GZ indicates that the ship has the ability of recovering from heeling, and the negative GZ, which generate a torque with the same direction as the roll motion, will speed up the heeling.

As can be seen in Figure 3.1.1, GZ can be expressed as:

\[ GZ = GM \sin(\rho), \]  
(3.1)

where GM is the metacentric height which is the distance between the centre of gravity of a ship and its metacentre. \( \rho \) is the roll angle. GM can be expressed as:

\[ GM = KB + BM - KG, \]  
(3.2)

where KB and KG are the vertical positions of ship’s centre of buoyancy and gravity, BM can be expressed as:

\[ BM = \frac{I_{WAX}}{V}, \]  
(3.3)

where \( I_{WAX} = \int_{WL} \frac{b(x)^3}{12} \, dx \)  
(3.4)

\( I_{WAX} \) is the moment of inertia, b is the width of ship, V is the displacement. GZ is affected by KG, mass and width of ship hull. The righting moment (RM) can be calculated as:

\[ RM = GM \times F_G, \]  
(3.5)

where \( F_G \) is the displacement of the vessel.
Figure 3.1.1 Midship section

Figure 3.1.2 Example of GZ curve
3.2 Gyroscopic Stability

A spinning top is a toy in childhood designed to be spun rapidly on the ground. The way it works is described by Newton's Laws of Rotation. While this can get pretty complicated in detail, there are some circumstances where the object will spin in a very simple manner. The object’s spin about the rotation axis gives it an angular momentum, which will remain constant until some outside torque works on it.

The gyroscopic stabilizer is utilized to eliminate ship roll in modern ships which has the similar principle as the spinning top, can keep its stability at sufficient spinning speed. As shown in Figure 3.2.1, a flywheel is rotating around Z axis. Following the right hand rule, the spin vector which can represent the spinning motion is going upward. Tilting the rotating flywheel around Y axis is called as precession, its vector direction goes in leftward direction. A counter torque (righting torque) is generated around X axis, which points outwards. These three vectors $\hat{x} \hat{y} \hat{z}$ are mutually perpendicular. The forefinger points in the direction of the spin vector $\hat{z}$, middle finger points in the direction of the precession vector $\hat{y}$, and thumb points in the direction of the counter torque vector $\hat{x}$.

NEWT vehicle system here comprises a ship hull, a gyro stabilizer mounted in a single-axis gimbal frame, a controller, and a motor. Spin axis of the gyro is vertical, precession axis Y is orthogonal to the spin axis Z. The gimbal frame is attached to the ship hull so that when the ship is upright, the keel line is perpendicular to both the precession axis and the gyro spin axis. When the ship heels, the controller will command the motor tilts the gyro fore or aft ward about the precession axis, creating a righting moment which is orthogonal to both Z axis and Y axis. The righting moment exerts on the ship hull. When properly controlled, the righting moment can counteract the heeling moment and turn the ship back to upright position.

Theoretically the counter-torque (righting moment from the gyro) $\tau$ is calculated by the equation [3.6]:

$$\tau = I_G \cdot \Omega \cdot \dot{\theta} \cdot \cos \theta,$$

where $\Omega$ is the rotational velocity of the flywheel, $\dot{\theta}$ is the precession velocity, $\theta$ is the precession angle of gyro, and the moment of inertia of flywheel ($I_G$) is considered as a cylinder and the expression is given by:

$$I_G = \frac{1}{2} \cdot m \cdot r^2,$$

where $m$ is the mass of flywheel, $r$ is the radius.
Figure 3.2.1 Back view of the gimbal and flywheel
4 INITIAL STABILITY ANALYSIS

The initial stability analysis has been studied in this Chapter 4. In Section 4.1, the NEWT draft geometry has reformatted to Britfair and imported to the stability tool to calculate the original GZ curve. The original GZ curves are plotted with various ship parameters in Section 4.2. And in Section 4.3, the possible GZ improvement with active gyro-stabilization is calculated in theory. The result is compared with the criteria, although the result seems promising, further work is still needed to optimize the system.

4.1 Digitizing the Model

A three-dimensioned CAD model of left side profile of NEWT amphibious vehicle, which agrees with the definition of the global right-handed Cartezian coordinate system, has been created already and shown in Figure 4.1.1. X-coordinate has direction from stern to bow, y from starboard to port and z from keel upwards. The real model is not exactly the same as the CAD model, since there would be some gaps for hydrofoil and wheels, but there will be no big differences in the initial stability analysis.

A Rhino plug-in program SectionTools is used in order to easily get its lines plan. As shown in Figure 4.1.2, section lines are automatically created along x and z directions. Multiple sets of points are applied at every intersection and corner of the section line. The CAD file, in order to be recognized by the specific hydrostatic tool, needs to be reformatted manually as a britfair file (See Appendix A for detail), which is shown in Figure 4.1.3.
Figure 4.1.2 Section lines

Figure 4.1.3 Britfair format
4.2 Original Stability Curves

_Hydrostatics_ is a Matlab program developed by KTH Centre for Naval Architecture which can load and modify britfair file and calculate the GZ-curve with the given ship's parameters (See Figure 4.2.1).

The britfair file NEWT.bri created in last Section, is here used with the ship parameters in Table 4.2.1. To analysis the stability of NEWT, Gz- curves are plotted with different mass, width and KG according to the value in Table 4.2.2.

Based on the stability theory introduced in Section 3.1 of Chapter 3, the larger the value of mass and breadth is, the better the stability is, and KG is on the contrary. Negative GZ means the ship has lost its ability of returning upright. Figure 4.2.2, 4.2.3 and 4.2.4 indicate that the total mass doesn't play an important role in this case, when it goes up from 350 kg to 450 kg, GZ will not change too much with the same width B and KG. But breadth and KG affect the GZ considerably. For example, when B=0.8 m, KG=0.6m, M=350kg, it has a large negative GZ value from 0 to 90 degress. If increase width B to 1.4m or decrease KG to 0.3m, GZ value will become positive. In addition, as the graph in the lower right corner of Figure 4.2.2 shown, if there is only one driver in the boat, LCG will approximately move forwards from 1.25 to 1.6 meters, which decreases the stability.

It can be seen that NEWT has a strict limitation about its breadth and KG with respect to its stability, though the original intention of the design indicates its slender shape (small breadth) and lightweight structure (high KG mostly dominated by pilot). A stabilizer has to be applied to NEWT to resolve the conflict.

<table>
<thead>
<tr>
<th>Table 4.2.1 Main parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOA</td>
</tr>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>Height</td>
</tr>
<tr>
<td>Mass</td>
</tr>
<tr>
<td>LCG</td>
</tr>
<tr>
<td>KG</td>
</tr>
<tr>
<td>Density</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4.2.2 Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 4.2.2</td>
</tr>
<tr>
<td>Figure 4.2.3</td>
</tr>
<tr>
<td>Figure 4.2.4</td>
</tr>
</tbody>
</table>
Figure 4.2.2 GZ curves with M=350kg, KG=0.3~0.6 m, B=0.8~1.4 m
Figure 4.2.3 GZ curves with $M=400\text{kg}$, $K_G=0.3\sim0.6\text{ m}$, $B=0.8\sim1.4\text{ m}$
Figure 4.2.4 GZ curves with $M=450$ kg, $KG=0.3$~0.6 m, $B=0.8$~1.4 m
4.3 Potential GZ Improvement with Gyro Stabilizer

When the ship heels at a small angle, the righting moment helps the ship to recover upright. But from the last section 4.2, NEWT boat with slender hull and high centre of gravity does not have a good stability. At some heeling angles, GZ is even negative, thus the ship will not get righting moment, but instead, the gravitational force and buoyancy will make the ship flip over even more quickly. Here, the gyroscope stabilizer is implemented to analyse how this could improve the stability.

The principle of gyro stabilizer has been explained in Section 3.2, according to equation (3.6) and (3.7), the gyro will generally lose its effectiveness when the precession angle goes to 90 degree. So supposedly, the maximum righting moment from gyro \( \tau \) occurs at very small precession angle (\( \theta=0 \)):

\[
\tau = I_G \Omega \theta
\]

\[
I_G = \frac{1}{2} m r^2
\]

The total righting moment can be expressed as:

\[
MR = GZ FG + \tau, \quad (4.2)
\]

and

\[
MR = (GZ + GZ_{added}) FG \quad (4.3)
\]

So the theoretical increase of GZ can be derived as:

\[
GZ_{added} = \tau / FG \quad (4.4)
\]

Where FG is the weight of the boat, see Figure 4.3.1.

New GZ is thus found as:

\[
GZ_{new} = GZ + GZ_{added} \quad (4.5)
\]

For the safety reason, GZ curve has to fulfil the criteria, which is quoted from [10] “THE SAFETY OF SMALL WORKBOATS & PILOT BOATS”, Chapter 11.1.3 Intact stability of new vessels of less than 15 metres in length which carry cargo or a combination of passengers and cargo weighing not more than 1000kg and are not fitted with lifting devices, The Marine Safety Agency, UK. It states the constraint for the GZ curve:

- The area should be not less than 0.055 metre*Radians up to 30° heel angle
- The area should be not less than 0.09 metre*Radians up to 40° heel angle
- The area between 30° and 40° should be not less than 0.03 metre*Radians.
- GZ should be at least 0.20 metres at an angle of heel equal to or greater than 30°
- The maximum GZ should occur at an angle of heel of not less than 25°.

Taking a GZ plot in Figure 4.2.4 with B=1m and KG=0.4m, to make it satisfy all the criteria mentioned above in Table 4.3.1, GZ_{added} should be at least 0.1374 metres. See the improvement in Figure 4.3.2.

In order to see how light and how slow the flywheel can be and still meet the criteria, two sets of parameters are calculated, as illustrated in Table 4.3.2. Note that the radius of the flywheel and
precession velocity are defined as constants here.

![Figure 4.3.1 Midsection ship](image)

<table>
<thead>
<tr>
<th>Table 4.3.1 Check the GZ curve with criteria</th>
<th>Old GZ</th>
<th>New GZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>The area should be not less than 0.055 metre*radians up to 30° heel angle</td>
<td>0.0073 m*rad</td>
<td>0.0793 m*rad ✓</td>
</tr>
<tr>
<td>The area should be not less than 0.09 metre*radians up to 40° heel angle</td>
<td>0.0143 m*rad</td>
<td>0.1102 m*rad ✓</td>
</tr>
<tr>
<td>The area between 30° and 40° should be not less than 0.03 metre*radians.</td>
<td>0.006 m*rad ✗</td>
<td>0.03 m*rad ✓</td>
</tr>
<tr>
<td>GZ should be at least 0.20 metres at an angle of heel equal to or greater than 30°</td>
<td>0.32 m at 114° ✓</td>
<td>0.46 m at 114° ✓</td>
</tr>
<tr>
<td>The maximum GZ should occur at an angle of heel of not less than 25°.</td>
<td>114° ✓</td>
<td>114° ✓</td>
</tr>
</tbody>
</table>
Figure 4.3.2 The GZ curve with and without gyro theoretically.

Table 4.3.2 Assumed parameters of gyro

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type1</th>
<th>Type2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of flywheel</td>
<td>20 kg</td>
<td>2.9 kg</td>
</tr>
<tr>
<td>Radius</td>
<td>0.2 m</td>
<td>0.2 m</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>1447 RPM (151.5 rad/s)</td>
<td>10000 RPM (1047 rad/s)</td>
</tr>
<tr>
<td>( \dot{\theta} )</td>
<td>10 rad/s</td>
<td>10 rad/s</td>
</tr>
<tr>
<td>( \tau )</td>
<td>606 N*m</td>
<td>606 N*m</td>
</tr>
<tr>
<td>GZ_{added}</td>
<td>0.1374 m</td>
<td>0.1374 m</td>
</tr>
</tbody>
</table>
4.4 Conclusion and discussion

The initial analysis shows that the gyro-stabilizer is able to significantly improve the ship stability. Which makes the old GZ curve raise substantially and fulfil all the criteria.

However, the result is not fully convincing although it is promising. In reality, the gyro will lose its effectiveness gradually and the righting counter torque will decrease to zero when the gyro tilts to 90° where the axis of rotating flywheel parallels with the axis of roll motion. Besides that, the maximum counter torque cannot always be applied, otherwise the boat will overreact. Only the necessary counter torque should be properly controlled and applied to the boat. At last, periodic disturbance force should be considered in real water area. In the following sections, a more advanced model is derived to analyse the problem in more detail.

In next Chapter:

- the problem is solved in the time domain to analyze the applicable stabilizer period,
- a controller design of a gyro-based stability system for this type of vehicle, and
- the ability of the system to handle disturbance forces is also examined.
5 MODELLING OF ROLL MOTION

As discussed in last Chapter, counter torque from gyro depends on the rotational velocity of the flywheel, precession velocity and the fly-wheel inertia. Keeping the rotational velocity and physical parameters constant, the angular velocity of tilting gyroscope (precession velocity) becomes the input value which can determine how much righting moment needs to be generated to balance the vehicle. In order to get the desired output value (zero angle of vehicle and gyro), a control system has to be implemented and well designed.

Ideally, an active gyro-control system consists of 5 elements: vehicle body, gyroscope, sensor, motor and controller. Firstly the roll angle of body is detected by the sensor and fed to the controller. The controller calculates the necessary input value (precession velocity of the gyroscope), and commands the motor to tilt the gimbal aft or forward with the input speed. The gyro then generates the counter torque counteracting the instability. For each time step, the output value (roll angle and angle of gyro) is detected by the sensor again and compared with the expected value (zero angle), the controller calculates its error (difference between current output and desired output value) and adjusts the precession speed of gyroscope with the help of motor. Finally the ship and gyro return to upright position.

This type of control system for unmanned vehicle has previously been used in e.g. [7][8]. Here, a similar procedure is followed with some modifications. Section 5.1 explains the principle of the model and implements it to the vehicle in land mode. In the end it derives the plot of the motion of the vehicle and gyro. Section 5.2 and 5.3 introduce the implementation of the model in water mode without and with disturbance forces and all the motions are plotted subsequently. The last Section 5.4 validates the correctness of the model in the perspective of power.

5.1 Implementation of the model in land mode

The vehicle model is simplified as a box shape with forward speed equalling zero. A gimbal is attached to the body of the vehicle, so the vehicle and gyroscope are considered as two separate rigid bodies that are kinematically constrained. Additionally, only consider the roll motion in YZ plane and the rotating speed of flywheel is constant, see Figure 5.1.1.
The terms and notations are defined below:

- \( B, G \) refer to the body of ship and gyroscope respectively.
- \( m \) and \( I \) with subscripts \( B \) or \( G \) represent the mass and the moment of inertia of corresponding body.
- \( \theta \) is precession angle of the gyro.
- \( \rho \) is the roll angle of the vehicle body.
- \( d_G \) and \( d_B \) are the distance from CG of gyro and vehicle to the pivot point.
- \( \Omega \) is the rotational speed of the gyro.
- \( M_\mu \) is the input precession torque.
- \( L_{WH} \) is the diameter of the wheel.
- \( K_G \) is the centre of gravity of the vehicle.
- \( g \) is gravity constant.

The equation of motion in roll is derived as two coupled nonlinear 4th order differential equations:

\[
(l_{B11} + m_B d_B^2 + l_{G11} \cos^2 \theta + m_G d_G^2 + l_{G33} \sin^2 \theta)\ddot{\rho} = -2\cos\theta\sin\theta(l_{G33} - l_{G11})\hat{\rho} - \\
\Omega \cos\theta l_{G33} \ddot{\theta} + (m_B d_B + m_G d_G)gsin \rho
\]

\[
l_{G22} \ddot{\theta} = M_\mu + \ddot{\rho}^2 \cos\theta\sin\theta(l_{G33} - l_{G11}) + \Omega \cos\theta l_{G33} \dot{\rho}
\]

The first equation (5.1) states all the torques on the vehicle about \( \rho \) axis. The left side is an inertial term due to the masses of the bicycle and gyro, which is the angular acceleration times the moment of inertia. The first term on the right is Coriolis term since it involves the product of the two rates \( \dot{\rho} \) and \( \dot{\theta} \). These terms arise when dealing with systems having multiple rotating reference frames. Second term is righting torque generated by gyro (see section 3.2 for more details) and the last term represents the heeling moment due to gravity.

The second equation (5.2) sums all the torque about \( \theta \). The left hand side of (5.2) is also inertial term. On the right side, they are input precession torque generated by the motor, centripetal term affected by the angular velocity of the vehicle, and counter torque produced by the angular
velocity of the vehicle.

A control system model is developed and simulated by MATLAB. First the two equations are converted to state-space form and linearized at upright position, which means \( \theta = \dot{\theta} = \rho = \dot{\rho} = 0 \) (See Appendix B).

\[
\dot{x} = Ax + Bu \\
\dot{y} = Cx + Du
\]  

(5.3)  

(5.4)

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\rho}
\end{bmatrix} = 
\begin{bmatrix}
0 \\
0
\end{bmatrix} \mu = M_\mu
\]  

(5.5)

\[
A = 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & a2 & a3 & 0
\end{bmatrix} \\
B = 
\begin{bmatrix}
b1 \\
0 \\
0
\end{bmatrix} \\
C = [1 \ 0 \ 1] \\
D = [0],
\]  

(5.6)

\[
a1 = \Omega^* l_{G33}/l_{G22},
\]  

(5.7)

\[
a2 = -\Omega^* l_{G33}/(l_{G11}+m_B d_B^2+m_G d_G^2+l_{G11}),
\]  

(5.8)

\[
a3 = (m_B d_B+m_G d_G)^* g/ (l_{G11}+m_B d_B^2+m_G d_G^2+l_{G11}),
\]  

(5.9)

\[
b1 = 1/l_{G22},
\]  

(5.10)

where \( \dot{x} \) is state vector, \( y \) is output vector, \( \mu \) is input vector, \( A \) is state matrix, \( B \) is input matrix, \( C \) is output matrix, \( D \) is feedthrough matrix.

After defining all the physical parameters and response speed, with the help of state-space form, the control gains \( K \) are calculated from the linear approximation model by using Ackermann’s formula (in MATLAB the function name is Acker) which is a solution for pole assignment method. Pole assignment is a method to adjust the poles of a given system by adjusting the feedback gains from state variables.

The gain factor \( K \) is a proportional controller to calculate the input precession torque \( M_\mu \):

\[
M_\mu = -K \cdot \dot{x}
\]  

(5.11)

The controller gain \( K \) guarantees the motor properly generate necessary torque to tilt the gyro and thus keep the vehicle stable dynamically.

The two differential equations (5.1)(5.2) can be solved by time domain integration with the aid of MATLAB function ode45 (it can solve non-stiff differential equations) with a defined time step 0.01 second and initial conditions 5 degree of roll. Based on the state condition \( \dot{x} \) and the controller gains \( K \), the differential equations are iteratively solved until the vehicle stay vertical. And all the parameters are shown in Table 5.1.1.

Finally the angle of the gyro, the angular velocity of gyro, the angle of vehicle, the angular velocity of vehicle, the precession torque and the righting moment are plotted respectively in Figure 5.1.2. According to the figure, the vehicle can recover from 5 degrees maximum roll angle to upright position in about 1 second. More than 5 degrees, the vehicle cannot get recovered in this model, but by tuning the controller gain \( K \), it can slightly increase the maximum roll angle with slower response time.
Table 5.1.1 Parameters in land mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b$</td>
<td>430 kg</td>
</tr>
<tr>
<td>$m_g$</td>
<td>20 kg</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>1047 rad/s (10000 RPM)</td>
</tr>
<tr>
<td>$h_b$ (height of ship)</td>
<td>1.3 m</td>
</tr>
<tr>
<td>$W_b$ (width of ship)</td>
<td>1 m</td>
</tr>
<tr>
<td>$R_g$ (radius of gyro)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>$T_g$ (thickness of gyro)</td>
<td>0.03 m</td>
</tr>
<tr>
<td>Density of gyro</td>
<td>7874 kg/m$^3$ (Iron)</td>
</tr>
</tbody>
</table>

Initial condition $[\theta \dot{\theta} \rho \dot{\rho}] = [0 0 5 0]$ degree

$K = [-207 10 -845 132]$

*Note that the flywheel is made of iron with density 7874 kg/m$^3$, to reduce the mass of the gyro, some gaps are drilled on the flywheel.

Figure 5.1.2 The motion of vehicle and gyro with 5 degrees of roll in land mode.
5.2 Implementation of the model in water mode

When the vehicle goes to the water, buoyancy force will help the boat recover stability. So in principle, the gyro system should have better performance than in land mode.

Here the boat is simplified as box shape and with zero forward speed. The equations of motion have derived similarly as in previous section as follows (Figure 5.2.1).

\begin{align}
(l_{B11} + m_Bd_B^2 + l_{G11} \cos^2 \theta + m_Gd_G^2 + l_{G33} \sin^2 \theta)\ddot{p} &= -2\cos \theta \sin \theta (l_{G33} - l_{G11}) \dot{\theta} - \Omega \cos \theta l_{G33} \dot{\theta} - m_{ALL} \sin \theta \\
l_{G22} \ddot{\theta} &= M_\mu + \rho^2 \cos \theta \sin \theta (l_{G33} - l_{G11}) + \Omega \cos \theta l_{G33} \dot{\theta}
\end{align}

\[(5.12)\]
\[(5.13)\]

![Figure 5.2.1 Equations of motion in water mode](image)

The only difference here compared to equations (5.1) and (5.2) is the last term in equation (5.12) which represents the righting moment from the vehicle itself in water mode (see section 3.1 for more details). Equations (5.12) and (5.13) are solved as in previous section. Consequently, the proportional controller gains $K$ are changed as the system has changed. Input all the parameters in Table 5.2.1. New plot has obtained in Figure 5.2.2. Even at 20 degrees roll angle, the boat can still return stable in 3.5 seconds. Additionally, by increasing the reaction time, the boat can recover from larger roll angle.

Now, go back to the Figure 4.2.4, the plot in the top right corner. The green curve is the GZ-curve with $M=450kg$, $KG=0.5m$, $B=1.0m$ and $GM_\theta=-0.1m$. It clearly shows GZ-curve is negative from 0 to 70 degrees roll angle which represents the boat has an unqualified stability. Import the parameters with initial roll angle of 12 degrees maximum (See Table 5.2.2) and plot in Figure 5.2.3. It shows that the boat can recover to vertical position in 2.5 seconds at 12 degrees roll angle. The stability has improved significantly.

However this case is not exactly the same as NEWT vehicle will suffer. Wave excitation force and some disturbing force (e.g. wind force) are not considered here. Thus, the model should be
modified to fit the real case.

Table 5.2.1 Parameters in water mode with 20 degrees of roll with positive GM0

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b$ (mass)</td>
<td>430 kg</td>
</tr>
<tr>
<td>$m_g$ (mass)</td>
<td>20 kg</td>
</tr>
<tr>
<td>$\Omega$ (angular velocity)</td>
<td>1047 rad/s (10000 RPM)</td>
</tr>
<tr>
<td>$h_b$ (height of ship)</td>
<td>1.3 m</td>
</tr>
<tr>
<td>$W_b$ (width of ship)</td>
<td>1 m</td>
</tr>
<tr>
<td>$R_g$ (radius of gyro)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>$T_g$ (thickness of gyro)</td>
<td>0.03 m</td>
</tr>
<tr>
<td>$Zwl$ (thickness of gyro)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Density of gyro</td>
<td>7874 kg/m$^3$ (Iron)</td>
</tr>
</tbody>
</table>

Initial condition $[\dot{\theta} \ \dot{\rho} \ \dot{\dot{\rho}}] = [0 \ 0 \ 20 \ 0]$ degree

$K=\begin{bmatrix} 32.3 & 6.85 & -31.4 & 411.9 \end{bmatrix}$

Figure 5.2.2 The motion of vehicle and gyro with 20 degrees of roll in water mode with positive GM0.
Table 5.2.2 Parameters in water mode with 12 degrees of roll with negative GM0

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b$ (mass of ship)</td>
<td>430 kg</td>
</tr>
<tr>
<td>$m_g$ (mass of gyro)</td>
<td>20 kg</td>
</tr>
<tr>
<td>$\Omega$ (angular velocity)</td>
<td>1047 rad/s (10000 RPM)</td>
</tr>
<tr>
<td>$h_b$ (height of ship)</td>
<td>1.3 m</td>
</tr>
<tr>
<td>$W_b$ (width of ship)</td>
<td>1 m</td>
</tr>
<tr>
<td>$R_g$ (radius of gyro)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>$T_g$ (thickness of gyro)</td>
<td>0.03 m</td>
</tr>
<tr>
<td>$Z_{wl}$ (depth of waterline)</td>
<td>0.2 m</td>
</tr>
<tr>
<td>Density of gyro</td>
<td>7874 kg/m$^3$ (Iron)</td>
</tr>
</tbody>
</table>

Initial condition $[\theta \dot{\theta} \rho \dot{\rho}] = [0 0 12 0]$ degree

$K = [-38.3 6.85 -52.9 394.4]$

Figure 5.2.3 The motion of vehicle and gyro with 12 degrees of roll in water mode with negative GM0.
5.3 Enhanced model with disturbing forces

In reality, wave motion force and wind force often act on the boat in any sea conditions, to make
the model deal with more realistic problems, an enhanced model has been created. Start from
equations of motion, extra disturbance moment \( F_w \) and \( F_0^{wm} \) can be added to the equations:

\[
(I_{B11} + m_B d_B^2 + I_{G11} \cos^2 \theta + m_G d_G^2 + I_{G33} \sin^2 \theta) \ddot{\theta} = -2 \cos \theta \sin \theta (I_{G33} - I_{G11}) \dot{\theta} \dot{\rho} - \\
\Omega \cos \theta I_{G33} \dot{\theta} - m_{A1L2} G M 0 \sin \rho + F_w d_w \cos \rho + F_0^{wm}
\]

\[
I_{G22} \ddot{\rho} = M_\mu + \dot{\rho}^2 \cos \theta \sin \theta (I_{G33} - I_{G11}) + \Omega \cos \theta I_{G33} \dot{\rho},
\]

where

\[
F_0^{wm} = \tau \rho^* g \frac{H_s}{2} L * e^{- kT} \left( \frac{b}{k} \cos \left( \frac{k b}{2} \right) + \frac{2}{k^2} \sin \left( \frac{k b}{2} \right) \right),
\]

and

\[
k = \frac{4 \pi^2}{g \cdot T_z^2} \quad (5.17)
\]

\[
F_w = 0.5 \cdot \rho_{air} \cdot v^2 \cdot A. \quad (5.18)
\]

Where \( F_w \) is constant wind force horizontally acting on the side of boat, \( \rho_{air} \) is the density of air, \( v \) is the wind speed, \( A \) is the side surface of the boat. \( F_0^{wm} \) is the maximum wave excitation
moment. \( H_s \) is significant wave height, \( L \) is the LOA, \( k \) is wave number, \( T_z \) is the average wave
period, \( b \) is width of ship, \( T \) is the draught. See Figure 5.3.1.

\[
\text{Wind speed is assumed to be 5m/s (gentle breeze), and pick the value from the wave}
\text{measurement in Stockholm inner city water ways (See Appendix C). The motion is plotted in}
\text{Figure 5.3.2 based on the parameters in Table 5.3.1.}
\]

Since the disturbing forces are defined as constant forces here, the gyrooscope will keep tilting
until it loses the efficiency. More specifically, when the gyro has rotated to 90 degrees, the
righting moment from the gyro has no effect on counteracting the roll motion (See equation
(3.6)). Instead, it makes the boat yaw (rotate around B3 axis). Furthermore, when the gyro turns
over 90 degrees, the counter torque will speed up the roll motion.

As shown in Figure 5.3.2, the gyro can approximately last 1.5 seconds effectively with the maximum initial roll angle of 8 degrees. Compared with the sea state measured in inner city water ways in Stockholm, the effective duration of the gyro stabilizer (1.5 second) is less than the average wave period. The result is presented in Table 5.3.2. But keep in mind that the wave excitation force is assumed as constant force here. In reality, it is a periodic force. So wave excitation moment is redefined as a cosine function as below:

\[ F_{wm}^{\text{wm}} = F_{0}^{\text{wm}} \cos (\omega \cdot t) \] (5.19)

\[ \omega = \frac{2\pi}{T_w} \] (5.20)

Where \( \omega \) is the frequency of wave, \( t \) is the time

Combine the equations from (5.14) to (5.20) to the model, and the result is plotted in Figure 5.3.3.

As shows in Figure 5.3.3, the boat can even keep upright about 8 seconds, which considerably exceeds the average wave period in each area of Stockholm inner city water ways. See Table 5.3.3.

Furthermore, change the constant wind force to the periodic force as well, assume the wind period \( T_w \) is 5 seconds, increase the wind speed \( v \) to 15 m/s (moderate gale) the result has shown in Figure 5.3.4 and Table 5.3.4. The gyro system can work effectively more than 10 seconds which is much larger than average wave period in Stockholm inner city water ways.

Table 5.3.1 Parameter in water mode with more disturbance with initial 8 degrees of roll

<table>
<thead>
<tr>
<th>Vehicle parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_b ) (mass of boat)</td>
<td>430 kg</td>
<td>( \Omega )</td>
<td>1047 rad/s (10000 RPM)</td>
</tr>
<tr>
<td>( m_g ) (mass of gyro)</td>
<td>20 kg</td>
<td>KG</td>
<td>0.4 m</td>
</tr>
<tr>
<td>( h_b ) (height of ship)</td>
<td>1.3 m</td>
<td>g</td>
<td>9.81</td>
</tr>
<tr>
<td>( W_b ) (width of ship)</td>
<td>1 m</td>
<td>( d_0 )</td>
<td>0.3 m</td>
</tr>
<tr>
<td>( R_g ) (radius of gyro)</td>
<td>0.2 m</td>
<td>( d_g )</td>
<td>0.2 m</td>
</tr>
<tr>
<td>( H_g ) (thickness of gyro)</td>
<td>0.03 m</td>
<td>GM0</td>
<td>0.1 m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disturbance environmental parameters</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_s ) (significant wave height)</td>
<td>0.2 m</td>
<td>( T_z )</td>
<td>1.6 s</td>
</tr>
<tr>
<td>( Zwl ) (waterline)</td>
<td>0.2 m</td>
<td>( d_W )</td>
<td>0.8 m</td>
</tr>
<tr>
<td>( v ) (wind speed)</td>
<td>5 m/s</td>
<td>( \rho_{air} )</td>
<td>1.225 kg/m(^3)</td>
</tr>
<tr>
<td>( A ) (cross-sectional area)</td>
<td>3.25 m(^2)</td>
<td>FW</td>
<td>50 N</td>
</tr>
</tbody>
</table>

Initial condition \([\theta \dot{\theta} \dot{\rho}]= [0 \ 0 \ 8 \ 0] \) degree

\[ K=\begin{bmatrix} -23 & 16 & -2902 & 283.8 \end{bmatrix} \]
Figure 5.3.2 The motion of vehicle and gyro with 8 degrees of roll in water mode with constant disturbance forces.

| Table 5.3.2 Effective duration of gyro against average wave period (constant disturbance) |
|-----------------------------------------------|-------------------------------|-------------------|-------------------|
| Average wave period Tz (s)                   | Lidingbron                  | NackaStrand      | Slussen          |
| 1.6                                           | 1.9                          | 2.0              |                  |
| Endurance (1.5 s)                            | ×                            | ×                | ×                |
Figure 5.3.3 The motion of vehicle and gyro with 8 degrees of roll in water mode with constant wind forces and periodic wave excitation force.

Table 5.3.3 Effective duration of gyro against average wave period (periodic wave excitation force and constant wind force)

<table>
<thead>
<tr>
<th></th>
<th>Lidingbron</th>
<th>NackaStrand</th>
<th>Slussen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average wave period $T_z$ (s)</td>
<td>1.6</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Endurance (8 s)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
Figure 5.3.4 The motion of vehicle and gyro with 8 degrees of roll in water mode with periodic wave excitation force and wind force.

Table 5.3.4 Effective duration of gyro against average wave period (periodic wave excitation force and wind force)

<table>
<thead>
<tr>
<th></th>
<th>Lidingbron</th>
<th>NackaStrand</th>
<th>Slussen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average wave period</td>
<td>1.6</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>Tz (s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Endurance (more than 10 s)</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>
5.4 Validation of the model

Since the gyro-system is just a device that transfers energy from motor torque to roll direction, the input power is supposed to equal with outputs regardless of drag. The expression for power is derived as follow:

\[
\begin{align*}
    P_{\text{input}} &= P_{\text{output}} \\
    P_{\text{input}} &= M_\mu \cdot \dot{\theta} \\
    P_{\text{output}} &= \Omega \cos \theta I_{G33} \dot{\theta} \cdot \dot{\rho} \\
    M_\mu \cdot \dot{\theta} &= \Omega \cos \theta I_{G33} \dot{\theta} \cdot \dot{\rho}
\end{align*}
\]

In order to check the validity of the model, the input power and output power in land mode, water mode, water mode with periodic wind and wave forces are plotted as a function of time respectively, see Figure 5.4.1, Figure 5.4.2 and Figure 5.4.3:

According to the Figures 5.4.1 to 5.4.3, the input power approximately equals to outputs. Therefore, it turns out the model runs correctly.

![Figure 5.4.1 Input and output power of the system in land mode.](image)
Figure 5.4.2 Input and output power of the system in water mode without disturbance forces.
Figure 5.4.3 Input and output power of the system in water mode with periodic wind and wave forces.
6 RESULTS

According to the simulation, with the predetermined parameters of gyroscope and vehicle, and zero forward speed of vehicle, gyro-stabilizer can easily handle about 5 degree roll angle in land mode, 20 degree in water mode and 8 degree in water mode with disturbing wave and wind force, see Table 6.1 for the results. The first column is the maximum initial roll angle, the second is the determined controller gains K, the third is the max righting moment from the gyro, the last column is the time for vehicle to recover upright. In addition, the gyro can maintain stability for more than 10 seconds which is much longer that average wave period in Stockholm inner city water ways.

By changing the controller gains and physical parameters, the performance will change subsequently. But the limitation of the total weight and the power of engine should be considered as well.

It is hard to give the exact maximum roll angle that the gyro can tackle with. Because of the different controller design, the behaviour may vary significantly.

Table 6.1 Performance of the gyro system in four conditions

<table>
<thead>
<tr>
<th></th>
<th>Initial roll angle (deg)</th>
<th>Controller gains K</th>
<th>Max righting moment (N m)</th>
<th>Time for recovery (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land mode</td>
<td>5</td>
<td>[-207 10 -845 132]</td>
<td>2966</td>
<td>0.8</td>
</tr>
<tr>
<td>Water mode A</td>
<td>20</td>
<td>[32.3 6.85 -31.4 411.9]</td>
<td>452</td>
<td>3.5</td>
</tr>
<tr>
<td>Water mode B</td>
<td>12</td>
<td>[-38 6.85 -52.9 394.4]</td>
<td>626.6</td>
<td>2.5</td>
</tr>
<tr>
<td>(negative GZ)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water mode with wave and wind force</td>
<td>8</td>
<td>[-23 16 -2902 283.8]</td>
<td>7227</td>
<td>0.38</td>
</tr>
</tbody>
</table>
7 CONCLUSIONS

From the beginning, the problems have defined as:

- Does NEWT have a good stability?
  No, Chapter 4 Initial Stability Analysis Section 4.1 and 4.2 have proved that NEWT has a bad initial stability with high CG and slender hull. With the standard configuration \( M=450 \text{kg}, B=1 \text{m} \) and \( \text{KG}=0.4 \), the GZ curve does not fulfil the stability criteria.

- Can gyroscopic stabilizer improve its stability in principle?
  Yes, Section 4.3 calculated the potential improvements with the gyro-stabilizer, it significantly increase the GZ and therefore fulfils the stability criteria.

- What conditions can the gyro stabilizer handle?
  Chapter 5 Modelling of Roll Motion elaborates the motion of vehicle and gyro in the modes of land, water and water with disturbance. The detailed results are presented in Chapter 6. Although the gyro-stabilizer has its limitation, it has been shown in this thesis that the stability can be improved significantly.
8 FUTURE WORK

In this work, forward speed of the vehicle is assumed to be zero, but in reality it is not, so the trajectory of the vehicle should be considered as well. And in principle, with the high forward speed, the original stability might be better. Therefore if continue to work with this and introduce the forward speed to equations of motion, the result will be more precise.

Another thing is the controller, the controller here is designed as a simply proportional controller. Instead of P controller, A PI or PID controller may give the gyro system better performance. Because the vehicle will gradually recover from one side to upright position instead swaying from one side to the other side repeatedly until vertical.


9.1 Literature


[3] Ljiljana Veljović, “History and Present of Gyroscope Models and Vector Rotators”, University of Kragujevac, Faculty of Mechanical Engineering


9.2 Internet Resource


SHORT INTRODUCTION TO THE BRITFAIR-FORMAT FOR HULL DEFINITION

The Britfair is a fairly powerful, yet straightforward hull geometry definition data format. This document briefly explains the most basic data format for the definition of sequential sections forming a hull. When you create your hull keep in mind that:

1. Data separation is usually done using blanks (mellanslag) and not tab.

2. The coordinate system is defined as $x$ being the length coordinate from stern to bow, $y$ is positive to port and $z$ from keel upwards, usually with the hull lower point as $z=0$. For sailboats, usually the lowest point of the canoe-body is used as $z=0$.

3. Define the sections in sequence starting with the aft-most section.

4. Define each section with the offset points sequentially starting at the keel and then in counter-clockwise direction.

5. You only need to define the right (starboard) side of the hull if the hull is symmetric.

6. You shall save your file as an ASCII textfile with extension .txt or .bri (not .doc or similar).

The following is an example of a Britfair file with explanations:

<table>
<thead>
<tr>
<th>File content</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Britfa1</td>
<td>Arbitrary hull name.</td>
</tr>
<tr>
<td>1</td>
<td>Start of section 1 definition... It has to be here, just accept that 0...</td>
</tr>
<tr>
<td>8 -4.42 -4.42</td>
<td>No of section offsets section x-coord. section y-coord. section z-coord.</td>
</tr>
<tr>
<td>0.00 0.00</td>
<td>$y$, $z$</td>
</tr>
<tr>
<td>0.04 0.00</td>
<td>etc...</td>
</tr>
<tr>
<td>0.04 9.20</td>
<td></td>
</tr>
<tr>
<td>5.52 9.88</td>
<td></td>
</tr>
<tr>
<td>6.28 10.68</td>
<td></td>
</tr>
<tr>
<td>8.84 12.08</td>
<td></td>
</tr>
<tr>
<td>10.32 13.76</td>
<td></td>
</tr>
<tr>
<td>11.12 15.24</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>End of section 1</td>
</tr>
<tr>
<td>11 0 0</td>
<td>No of section offsets section x-coord. section y-coord.</td>
</tr>
<tr>
<td>0.00 0.00</td>
<td>$y$, $z$</td>
</tr>
<tr>
<td>0.16 0.00</td>
<td>etc...</td>
</tr>
<tr>
<td>0.24 5.00</td>
<td></td>
</tr>
<tr>
<td>0.56 7.20</td>
<td></td>
</tr>
<tr>
<td>1.40 8.12</td>
<td></td>
</tr>
<tr>
<td>2.68 8.72</td>
<td></td>
</tr>
<tr>
<td>6.52 10.04</td>
<td></td>
</tr>
<tr>
<td>8.76 11.16</td>
<td></td>
</tr>
<tr>
<td>10.52 12.76</td>
<td></td>
</tr>
<tr>
<td>11.56 14.60</td>
<td></td>
</tr>
<tr>
<td>11.96 16.20</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>End of section 2</td>
</tr>
<tr>
<td>...</td>
<td>Section nr 3</td>
</tr>
<tr>
<td>...</td>
<td>etc...</td>
</tr>
<tr>
<td>...</td>
<td>etc...</td>
</tr>
<tr>
<td>0</td>
<td>End of section xx</td>
</tr>
<tr>
<td>8 162 162</td>
<td>No of section offsets section x-coord. section y-coord.</td>
</tr>
<tr>
<td>0.0 3.5</td>
<td>$y$, $z$</td>
</tr>
<tr>
<td>0.0 4.0</td>
<td>etc...</td>
</tr>
<tr>
<td>0.0 6.0</td>
<td></td>
</tr>
<tr>
<td>0.0 8.0</td>
<td></td>
</tr>
<tr>
<td>0.0 10.0</td>
<td></td>
</tr>
<tr>
<td>0.0 12.0</td>
<td></td>
</tr>
<tr>
<td>0.0 14.0</td>
<td></td>
</tr>
<tr>
<td>1.0 16.44</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>End of section</td>
</tr>
<tr>
<td>0 0 0</td>
<td>End of file indicator.</td>
</tr>
</tbody>
</table>
APPENDIX B: STATE SPACE FORM

The equations of motion have derived as follow:

\[
\begin{align*}
(l_{B11} + m_Bd_B^2 + l_{G11}\cos^2\theta + m_Gd_G^2 + l_{G33}\sin^2\theta)\ddot{\rho} &= -2\cos\theta\sin\theta(l_{G33} - l_{G11})\dot{\theta} - \\
\Omega\cos\theta l_{G33}\ddot{\theta} + (m_Bd_B + m_Gd_G)gsin\rho
\end{align*}
\]

(1)

\[
l_{G22}\ddot{\theta} = M_\mu + \dot{\rho}^2\cos\theta\sin\theta(l_{G33} - l_{G11}) + \Omega\cos\theta l_{G33}\dot{\rho}
\]

(2)

Where

- \(B, G\) refer to the body of ship and gyroscope respectively.
- \(m\) and \(I\) with subscripts \(B\) or \(G\) represent the mass and the moment of inertia of corresponding body.
- \(\theta\) is precession angle of the gyro
- \(\rho\) is the roll angle of the vehicle body
- \(d_B\) and \(d_G\) are the distance from CG of gyro and vehicle to the pivot point
- \(\Omega\) is the rotational speed of the gyro (the same as \(\omega_{dsk}\) described in previous Sections)
- \(M_\mu\) is the input precession torque.
- \(L_{WH}\) is the diameter of the wheel
- \(K_G\) is the centre of gravity of the vehicle
- \(g\) is gravity constant

Converting equations (1) (2) to state-space form as below:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &=Cx + Du
\end{align*}
\]

(3)

\[
x = \begin{bmatrix} \dot{\theta} \\ \dot{\rho} \\ \dot{\rho}\end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \end{bmatrix}
\]

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{2\cos\theta\sin\theta(l_{G33} - l_{G11})}{l_{G22}} & -\frac{\dot{\rho}^2\cos\theta\sin\theta(l_{G33} - l_{G11})}{l_{G22}} & -\frac{\Omega\cos\theta l_{G33}\dot{\rho}}{l_{G22}} \\ 0 & 0 & 1 \end{bmatrix}
\]

\[
B = \begin{bmatrix} -\frac{m_Bd_B + m_Gd_G}{l_{G22}} \\ \frac{l_{G22}}{l_{G22}} \\ 0 \end{bmatrix}
\]

(4)

(5)
\[
\dot{\mathbf{x}} = \begin{bmatrix}
\dot{\theta} \\
\dot{\rho} \\
\dot{\rho}
\end{bmatrix}
\]
(6)
\[
\mathbf{y} = \begin{bmatrix}
g_1(x) \\
g_2(x)
\end{bmatrix} = \begin{bmatrix}
\theta \\
\rho
\end{bmatrix}
\]
(7)
\[
\mu = M\mu
\]
(8)

Where \( \mathbf{x} \) is state vector, \( \mathbf{y} \) is output vector, \( \mu \) is input vector, \( A \) is state matrix, \( B \) is input matrix, \( C \) is output matrix, \( D \) is feedthrough matrix.

Now linearize the state-space form at upright position \( \mathbf{x}_0 = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \), \( \mu_0 = M\mu_0 = 0 \)

The matrix \( A, B, C, D \) can be derived by partial differential:

\[
A_{ij} = \left. \frac{\partial f_i}{\partial x_j} \right|_{\mathbf{x}_0 = \mu_0}
\]
(9)
\[
B_{ij} = \left. \frac{\partial f_i}{\partial \mu_j} \right|_{\mathbf{x}_0 = \mu_0}
\]
(10)
\[
C_{ij} = \left. \frac{\partial g_i}{\partial x_j} \right|_{\mathbf{x}_0 = \mu_0}
\]
(11)
\[
D_{ij} = \left. \frac{\partial g_i}{\partial \mu_j} \right|_{\mathbf{x}_0 = \mu_0}
\]
(12)

Finally the value of \( A, B, C, D \) is calculated:

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & a1 & 0 \\
0 & 0 & 0 & 1 \\
a2 & a3 & 0 & 0
\end{bmatrix},
B = \begin{bmatrix}
0 \\
b1 \\
0 \\
0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 1
\end{bmatrix},
D = [0]
\]
(13)

Where:
\[
a1 = \Omega^* I_{G33}/I_{G22}
\]
\[
a2 = -\Omega^* I_{G33}/(I_{B11} + m_b*d_b^2 + m_g*d_g^2 + I_{G11})
\]
\[
a3 = (m_b*d_b + m_g*d_g)*g/(I_{B11} + m_b*d_b^2 + m_g*d_g^2 + I_{G11})
\]
\[
b1 = 1/I_{G22}
\]
APPENDIX C: WAVE MEASUREMENTS

Wave height vs wave period

Heave distribution of samples

Wave heights sorted by magnitude

File: LG03.txt
Wave encounters = 843 at
T_z = 1.9 s : Average period of zero up-crossings
H_m = 0.14 m : Average wave height, crest through
H_max = 0.52 m : Maximum wave height, crest through
H(1/3) = 0.35 m : Average height of 33% highest waves
H(1/10) = 0.25 m : Average height of 10% highest waves

Wave height vs wave period

Heave distribution of samples

Wave heights sorted by magnitude

File: LG05.txt
Wave encounters = 460 at
T_z = 1.9 s : Average period of zero up-crossings
H_m = 0.14 m : Average wave height, crest through
H_max = 0.52 m : Maximum wave height, crest through
H(1/3) = 0.35 m : Average height of 33% highest waves
H(1/10) = 0.25 m : Average height of 10% highest waves
APPENDIX D: MATLAB SCRIPTS

imp3.m
clc, clear all, close all;

global u K

first_time1 = 0;

%% Physical Parameters
mB = 290;        % [kg] mass of ship body, exclude gyro, pilots
mG = 20;         % [kg] mass of gyro
mP = 70*2;    % [kg] mass of point load, pilots + engine, etc
mALL = mB+mG+mP;
rG = 0.2;        % [m] radius of gyro
tG = 0.03;       % [m] thickness of gyro, gyro is assumed to be cylinder.
wB = 1;          % [m] width of ship body
hB = 1.3;        % [m] height of ship
L = 2.5;         % [m] overall length of ship
Zwl = 0.2;          % [m]
dG = 0.5-Zwl;       % [m] distance between cg of gyro and rotational point
dB = 0.2;
GM0 = 0.1;
omega = 1047;    % [rad/s] rotational speed of gyro
g = 9.81;        % acceleration of gravity
ds=1025;         % [kg/m3] density of seawater

IG11 = mG*(3*rG^2+tG^2)/12;
IG22 = IG11;
IG33 = mG*rG^2/2;
IB11 = mB*(hB^2+wB^2)/12;

%% Linearized coefficients
a1 = omega*IG33/IG22;
a2 = -omega*IG33/(IB11+mB*dB^2+mG*dG^2+IG11);
a3 = -mALL*g*GM0/(IB11+mB*dB^2+mG*dG^2+IG11);
b1 = 1/IG22;

a1=1800;
a2=-10;
a3=1400;
if first_time1 == 0
% run once to get K
A=[0 1 0 0; 0 0 0 a1; 0 0 0 1; 0 a2 a3 0];
B=[0; b1; 0; 0];
C=[1 0 1 0];
D = 0;
SYS = ss(A,B,C,D);
G = tf(SYS);
P=[-20 -20 -20 -20];
K=acker(A,B,P);
first_time1 =1;
end

%% Simulation Parameters
deg2rad = 0.0174532925;
rad2deg = 57.2957795;
IC = [0*deg2rad 0*deg2rad 8*deg2rad 0*deg2rad];  % [angle of gyro, angular v of gyro, angle of ship, angular v of ship]
Tf = 2;
del = .01;
N = Tf/del;
DATA = [0 IC];
UDAT = [0];

%% Simulation
for ks = 1:N
%Get end values from last xx msec simulation
len = length(DATA(:,1));
t1 = DATA(len,1);
x(1) = DATA(len,2);
x(2) = DATA(len,3);
x(3) = DATA(len,4);
x(4) = DATA(len,5);
T_out(ks)=[omega*cos(x(1))*IG33*x(2)];
%Reset the Initial Conditions
IC = [x(1) x(2) x(3) x(4)];
%Do the simulation
ks*del
[T,Y] = ode45('imp3_func',[(ks-1)*del ks*del],IC);
%Record the data for the last time period
DATA = [DATA; T(length(T)) Y(length(Y),:)];
UDAT = [UDAT; u];
end;

T_out(ks+1)=omega*cos(DATA(len+1,2))*IG33*DATA(len+1,3);
T_out=T_out';

t=DATA(:,1);
theta=DATA(:,2)*rad2deg;
theta_dot=DATA(:,3)*rad2deg;
rho=DATA(:,4)*rad2deg;
rho_dot=DATA(:,5)*rad2deg;
uplot=UDAT(:,1);
max_turque=max(abs(T_out))

figure;
subplot(411)
plot(t, theta)
ylabel('angle of gyro [deg]')
grid on
subplot(412)
plot(t,rho)
ylabel('angle of vehicle [deg]')
grid on
subplot(413)
plot(t,uplot)
ylabel('Gimbal motor torque')
grid on
subplot(414)
plot(t,T_out)
ylabel('Precession torque')
xlabel('time [s]')
grid on
imp3_func.m
function dx = imp3_func(t,x)

global u K

%% Parameters
mB = 290;        % [kg] mass of ship body, exclude gyro,pilots
mG = 20;         % [kg] mass of gyro
mP = 70*2;    % [kg] mass of point load, pilots + engine, etc
mALL = mB+mG+mP;

rG = 0.2;        % [m] radius of gyro
tG = 0.03;       % [m] thickness of gyro, gyro is assumed to be cylinder.
wB = 1;          % [m] width of ship body
hB = 1.3;        % [m] height of ship
L = 2.5;         % [m] overall length of ship
Zwl = 0.2;          % [m]
dG = 0.5-Zwl;       % [m] distance between cg of gyro and rotational point
dB = 0.2;

IG11 = mG*(3*rG^2+tG^2)/12;
IG22 = IG11;
IG33 = mG*rG^2/2;
IB11 = mB*(hB^2+wB^2)/12;

%% Disturbance force

dW = 1-Zwl;          % [m] wind load
FW = 10;         % [N] wind force
Hs = 0.2;
Td = 0.166;            % [m] draught
Tz = 1.6;        % [s] average period
k = 4*pi^2/(g*Tz^2);

F_Max_WM = ds*g*(Hs/2)*L*exp(-k*Td)*(-wB*cos(k*wB/2)/k+2*sin(k*wB/2)/k^2); % [N*m]

wave roll excitation moment

%%

M_u = -K*x;
u=M_u;

%% Equations of motion

% dx(1) = x(2);
% dx(2) = (M_u+omega*cos(x(1))*IG33*x(4))/IG22;
% dx(3) = x(4);
% dx(4) =
(FW*dW*cos(x(3))-mALL*g*GM0*sin(x(3))+F_Max_WM-omega*cos(x(1))*x(2))/(IB11+mB*dB^2+mG*dG^2+(cos(x(1)))^2*IG11+(sin(x(1)))^2*IG33);

dx(1) = x(2);
dx(2) = (M_u+omega*cos(x(1))*IG33*x(4)+x(4)^2*cos(x(1))*sin(x(1))*(IG33-IG11))/IG22;
dx(3) = x(4);
dx(4) =
(FW*dW*cos(x(3))-mALL*g*GM0*sin(x(3))+F_Max_WM-2*cos(x(1))*sin(x(1))*(IG33-IG11)*x(2)*x(4)-omega*cos(x(1))*IG33*x(2))/(IB11+mB*dB^2+mG*dG^2+(cos(x(1)))^2*IG11+(sin(x(1)))^2*IG33);
dx=dx';
end