The curricular importance of mathematics: a comparison of English and Hungarian teachers' espoused beliefs.

Paul Andrews, University of Cambridge

Abstract

This paper reports an interview study of 45 English and 10 Hungarian teachers of mathematics. The semi-structured interviews focused on teachers' professional life histories. Several questions, on which this paper is based, invited colleagues to discuss their beliefs about the necessary subject content for the teaching and learning of mathematics. The analyses indicated substantial differences between the two cohorts, which, it is argued, accord with well-defined national perspectives on education in general and mathematics education in particular and reflect, at national rather than individual levels, the expectations of the curricular frameworks within which teachers operate and the findings of studies of classroom practice undertaken in their countries. English teachers tended to view mathematics as applicable number and the means by which learners are prepared for a world beyond school. Hungarian teachers privileged mathematics as problem-solving and logical thinking.

Keywords: comparative education, curriculum, mathematics teachers, teacher role

Introduction


Evidence suggests also that children’s mathematical attainment varies according to the country in which they live and the form of measure used. This variation is particularly stark when the performance of English and Hungarian 13 and 14 year-olds is considered. The third international mathematics and science study (TIMSS) (Beaton et al 1996) and its repeats (Mullis et al 2000, 2004), which examined students’ performance on routine problems on topics representative of all curricula, found Hungarian students performing at significantly higher levels than their English counterparts. Alternatively, the Organisation for Economic Cooperation and Development’s programme of international student assessment (PISA) and its repeat, which examined mathematical literacy or the wider uses of mathematics in people's lives (OECD 2001, 2004), found Hungarian students performing at significantly lower levels than English. In the light of such findings, and drawing on the vocabulary of the second international mathematics study (Travers et al 1989), it is reasonable to infer such differences in the attained curriculum are due to differences in the ways in which intended curricula are implemented.

Teaching is ‘rooted in deeply held beliefs about the nature of the subject, the way students learn, and the role of the teacher’ (Stigler and Hiebert 1997: 19). Beliefs ‘play a significant role in shaping teachers’ characteristic patterns of instructional behavior’ (Thompson 1992: 130-131) due to the manner in which they inform the individual’s conception of and engagement with mathematics (Schoenfeld 1992).

Over recent years, participants' beliefs about mathematics and its teaching have been researched in a variety of contexts. These have included, for example, those of pre-service teachers in Spain (Carillo and Contreras 1994, Camacho et al 1998) and
Comparing beliefs

the United States (Frank 1990, Foss and Kleinsasser 1996, Cooney et al 1998), and serving teachers in Canada (Dionne 1988, Gattuso and Mailloux 1994, Chapman 1999), France (Nimier 1986), England (Andrews and Hatch 1999), New Zealand (Irwin and Britt 1994) and the United States (Thompson 1984, Bush et al 1990). In Canada the views of university mathematics teachers (Mura 1993) and mathematics educators (Mura 1995) have been explored as have those of mathematics undergraduates in Australia (Crawford et al 1994, 1998). However, and acknowledging the questionnaire study of Andrews and Hatch (2000), few comparative studies have been published in the field and even fewer drawing explicitly on the teachers’ voice. This paper, therefore, is unique in the field of teacher beliefs and mathematics education.

Beliefs are ‘understandings, premises or propositions about the world that are felt to be true’ (Richardson 1996: 103) and, in general, are products of individuals’ sense-making of the world around them (de Abreu et al 1997). They operate at two levels; at the lower level are single beliefs which are deeply personal, unaffected by persuasion, serendipitous in their formation and independent of social consensus or even internal consistency (Pajares 1992, Da Ponte 1994). At the higher level are belief systems - organised clusters of beliefs – which comprise both primary and derivative beliefs. Belief systems may be held in isolation of other belief systems; making the holding of conflicting beliefs a possibility (Green 1971). Snow et al (1996: 292) write that, humans ‘form and hold beliefs that serve their own needs, desires and goals’ with the consequence that they may ‘cause biases in perception and judgement’. Beliefs are filters through which experiences are interpreted (Pajares 1992, Gopnik and Meltzoff 1997) and are distinguishable from knowledge because they are non-consensual and disputable (Abelson 1979, Nespor, 1987). A ‘belief is in the first place an individual construct, while knowledge is essentially a social construct’ (Op’t Eynde et al 1999: 5).

Evidence indicates that teachers’ professional identities, their beliefs and sense of professional self and worth, ‘take shape as life unfolds and may ... solidify into a fixed identity ... or ... continue to grow and change’ (Connelly and Clandinin 1999: 95). Identities are informed by the individuals’ biographies (Hirsch 1993) which, for the secondary teacher, draw substantially on their relationship with their chosen subject (Grossman and Stodolsky 1995, Virta, 2000) and their own experiences of schooling (Feiman-Nemser and Buchmann 1986, Bush et al 1990, Woods 1993, Harel 1994, John 1996, Foss and Kleinsasser 1996). Identities, however, are unlikely to be single or immutable (Norquay 1990), nor are professional and personal identities likely to be distinct and unrelated (Thomas 1993). Teachers’ identities are multifaceted constructions derived from the stories they tell about themselves and the ways they interpret and represent significant episodes, both personal and social, in their lives (Connelly and Clandinin 1999).

This study, drawing on professional life history interviews conducted with teachers of mathematics in England and Hungary, explores teachers’ beliefs about the necessary and appropriate curriculum content for the teaching and learning of mathematics. In so doing it has been important to acknowledge the educational traditions in which teachers operate and possible inconsistencies between a teacher’s espoused and enacted practice (Ernest 1989) or theory of action and theory in use (Argyris and Schön 1974).
Comparing beliefs

Method

Narrative research has been described as ‘probably the only authentic means of understanding how motives and practices reflect the intimate intersection of institutional and individual experience’ (Dhunpath 2000: 544). ‘Researchers looking at narrative (...) take seriously the idea that we live storied lives and that teachers and students tell stories to themselves and to others to make sense of the worlds they inhabit’ (Swidler 2000: 553).

With this in mind, forty-five teachers from England, and ten from Hungary, were interviewed in the months following a questionnaire study of their conceptions of mathematics and its teaching (Andrews and Hatch 2000). Hungarian informants worked in Budapest schools and the English in Manchester or Hampshire. In Budapest, colleagues were drawn from a variety of institutions including several eight grade primary schools (általános iskola) catering for the age range 6-14, a four-grade gimnázium catering for the age range 14-18, an eight-grade gimnázium catering for the age range 10-18, and a vocational school catering for the age range 14-17. Their locations included urban tenements, high rise estates, and mixed commercial and residential areas. In Manchester represented schools included single sex and coeducational, selective and comprehensive, denominational and non-denominational, urban and suburban schools. In Hampshire they included denominational and non-denominational coeducational comprehensives drawn from urban, suburban and semi rural areas. All the English schools catered for the 11-16 or 11-18 age ranges. Other indicators, like attainment on national tests or numbers of free school meals, showed that such schools reflected the diversity of all schools. However, despite confidence in the representativeness of the schools themselves, it is important to acknowledge that colleagues who volunteer for interview may have been atypical.

The interviews were semi-structured and invited colleagues to describe the manner in which their careers had developed and to discuss key episodes that had informed their professional lives. Interviews, which were conducted in colleagues’ schools by researchers from each teacher’s country, were tape-recorded and transcribed. Transcripts were posted to teachers to obtain their agreement as to content although not one was queried. Interviews lasted between thirty and forty minutes. Subsequent to their interviews, the Hungarian transcripts were translated into English by English-speaking Hungarian colleagues. The scale of the study meant that no funds were available for back translation. Confidence in the veracity of the translations was facilitated by a number of factors. The two teams had collaborated for several years prior to this project and had developed a shared vocabulary of mathematics classroom activity and an understanding of each other’s context. During this time I spent more than six weeks observing mathematics lessons in Budapest and was able to acquire sufficient linguistic sensitivity to be able to recognise, for example, the distinctive nature of the words gyakorlt (exercise), feladet (task) and probléma (problem). However, despite a belief that the translations were robust and that a shared vocabulary was, indeed, operational, substantial care was taken with regard to the analysis of the data and discussion of the results. It is important to note that there are few pronouns in Hungarian, so an explicit acknowledgement of the gender of a verb’s subject is rare. Consequently, Hungarians translating into English tend to report in the masculine. Therefore, unless the context made gender unambiguous, Hungarian utterances have been re-written to present a gender-neutral perspective.
Informants were invited to discuss their beliefs about the nature of mathematics, its content and processes, and the content necessary for learners to experience. It is on these beliefs that this paper focuses. In respect of analysis, transcripts were read, re-read, and categories of response noted. As new categories were identified, previously read transcripts were re-read to see if the new category applied also, in accordance with the constant comparison processes of grounded theory (Glaser and Strauss 1967, Strauss and Corbin 1998). An analysis of the categories led to the identification of several broad themes on which this paper is based.

Due to the different ways in which education is phased in the two countries, teachers differed in the accounts they gave of their professional identities. All the English teachers taught in secondary schools, that is, institutions catering for ages 11-16 or 11-18. In general, Hungarian secondary education begins at age 14, although recent legislative changes allow schools to admit younger students. Thus, of the ten interviewed teachers, six taught the upper primary age range (10-14) two the secondary age range (14-18) and two the extended secondary age range (10-18).

Results

As indicated above, there is no guarantee that either the schools or the informants drawn from them were representative of all possibilities. Consequently, caution has been exercised in respect of both the reporting and interpretation of colleagues’ utterances. The results are presented by teachers’ location, England followed by Hungary. All names are pseudonyms.

England

In respect of their beliefs about necessary curriculum mathematics, English teachers’ responses tended to fall into five broad curricular themes which, despite inevitable overlaps, were thought to be of sufficient robustness and interest to warrant acceptance. These can be seen in Table 1. However, as categories were identified, compared and refined, it became apparent that many were also indicative of teachers’ underlying educational orientations. Although not discussed in the same detail as the curricular themes these orientations are also presented in table 1.

<table>
<thead>
<tr>
<th>Curricular theme</th>
<th>Educational orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applicable number</td>
<td>Basics-oriented</td>
</tr>
<tr>
<td>Real-world preparation</td>
<td>Utility-oriented</td>
</tr>
<tr>
<td>Exploration</td>
<td>Autonomy-oriented</td>
</tr>
<tr>
<td>Curricular given</td>
<td>Conformity-oriented</td>
</tr>
<tr>
<td>Student characteristics</td>
<td>Differentially-oriented</td>
</tr>
</tbody>
</table>

Table 1: The five themes relating to English teachers’ beliefs about the curricular importance of mathematics and their corresponding educational objectives.
Comparing beliefs

All teachers touched on at least two themes and, by implication, aligned themselves with two educational orientations. Each theme with its associated orientation is discussed in turn.

Mathematics as applicable number (basics-oriented education)

Twenty-nine (64%) English teachers indicated that the importance of mathematics lay in number and its applications. For some this was manifested in comments concerning the real-world while for others it was related to a belief that further study of the subject was impossible without a secure understanding of, and facility with, arithmetic.

In respect of the former, Brian, a male head of department of fifteen years' experience, said,

I think it’s more important they know about basic numeracy of numbers, being able to use numbers to help them in whatever they are going to do... I think the numeracy side is going to be most useful... I think for their everyday experiences of what they’re going to use Maths for, being able to work with the basic numbers is the most important.

More particular comments were made by Alison, a female teacher of five years’ experience. She said, tentatively, that

Their number work is extremely important and that’s just fundamental. ... If they can’t use numbers then how can they go on to use anything else? And how can they see how things happen? I think just putting everything into a practical sense so that they know they can measure dimensions. They know, if they’re going to put up wall paper or whatever, that they can estimate how much they’re going to need. Estimate, being able to estimate is important. How much is it going to cost me, roughly, that sort of thing.

Chris, a male head of department of twenty years’ experience, added a new dimension, discussed below, concerning student ability. He said,

I think the mental maths side of it, the basic arithmetic and a ... fluency with number are, perhaps, the most important life skills. ... The middle to less able students, if they’re to get anything, that would be what I would like them to be able to leave school with. However ... I’m not in favour of just teaching those elements of maths, I would be in favour of a broad curriculum. ... Those skills come up in all ... topics within maths.

In terms of number as the precursor to other aspects of mathematics, Dave, a male second in department of eight years experience who entered teaching after a career in industry, was clear that confidence with number was essential for successful study of the subject. He said,

I see numeracy skills as being absolutely paramount in all areas of maths. ... I’ve noticed that the one thing that seems to make the most difference is the numeracy - confidence with the numbers. Because whatever area of maths you do, there's almost certainly going to be some numbers involved somewhere and if they’re confident with the numbers and they can do the basic calculations without thinking then the rest seems to follow on much more easily. So I think if I was to pick out one thing that would be it.
Danny, a male teacher of just two years’ experience, referring to such skills as the building blocks of mathematics, said,

Everyone’s got to have the basics before you can progress. ... Because if you haven’t got those ... then you can’t progress in the way that you should. So I think they, therefore, must be important in the same way that, in English, you’ve got to be able to write and you’ve got to be able to know your alphabet before you can do anything creative. So I think it’s the building blocks aspect, that these things must come first and therefore are more important in the learning process.

Eleanor, a second in department of almost thirty years’ experience, presenting both perspectives, while alluding to notions of mathematics as determined by teachers’ perceptions of students’ ability, said,

If I’m talking about what is required outside then I think children need the numeracy skills, and I think if we don’t get youngsters being confident with their own numeracy then we’re doing them a disservice later on. So I think it’s important that we do emphasise those and perhaps they might take a greater priority than some .... topics. ... Once children’s numeracy skills are on a par then you would increase the amount of abstraction that you’ve used for algebra, for instance. So I wouldn’t have said there are things that are more important but I think there are things that you need ... to be sure that the youngsters are sound with before you ... do huge amounts of other work, although I’m not saying that because a child cannot add up, subtract, multiply or divide, you never do anything else. That is a diet of sheer boredom. But without the understanding of how the numbers work, I think you’re very limited as to what else you can usefully expect a child to understand.

In summary, almost two-thirds of the English informants indicated that the curricular significance of mathematics lay in numerical competence and confidence, which was manifested in two ways. For some, like Alison, Brian and Chris, mathematics was applicable number. Brian’s use of expressions like ‘basic numeracy’ and ‘everyday experience’, Alison’s belief that number work is ‘fundamental’, coupled with her desire to put ‘everything into a practical sense’ and Chris’ suggestion that ‘basic arithmetic’ and ‘fluency with number’ are the ‘most important life skills’ all suggest a worldview in which education focuses on preparing for a life beyond school; a view in which mathematics draws its curricular authority from a notion of functional arithmetic. For others, like Dave and Danny, arithmetical competence was an essential mathematical pre-requisite. Dave’s comment that the skills of numeracy are ‘absolutely paramount in all areas of maths’, and Danny’s view that one has to ‘have the basics before you can progress’ indicate a desire to secure their learners’ access to higher levels of mathematical understanding. Eleanor’s comments emphasised both the functional and the pre-requisite nature of number facility.

Such perspectives, focused on the development of the numerical skills necessary for either further study of mathematics or a successful engagement with a world beyond school, reflect a particular view of mathematics located in a belief system privileging education as the acquisition of basic skills. These teachers, it could be argued, could be described as basics-oriented.

Mathematics as preparation for the real-world (utility-oriented education)
Fifteen teachers (33%) described mathematics as a tool to facilitate learners’ understanding of, or participation in, the real-world. Several, as indicated above, focused on the utilitarian aspects of number. Others made more general statements while some focussed on particular aspects of the curriculum.

Annie, a female teacher of twenty years’ experience, said,

I think the relevance of maths is to everyday life. It's important because a lot of them think that maths is maths and that it gets shut away in the cupboard at the end of the day and never gets used again. And I think the fact that you can try and get them to relate it to other things, I feel is important.

Bryony, a second in department of fifteen years’ experience, while resonating with Annie’s generality, offered a more particular perspective. She said,

Things that they use in everyday life are very important because they’re useful and they’re actually going to go on and use them afterwards, but I definitely think it’s worth them experiencing all different aspects. ... It’s not just adding, it’s not just algebra, ... not just solving equations and things. ... I suppose things that they’re going to see when they get out of school are the most important, you know, like handling data, ... bar charts and the sort of things they might see in the papers. And percentages and simple number stuff that they’re going to have to use afterwards. It depends on the ability range as well again.

In summary, a third of informants saw the importance of mathematics in its applicability to a world beyond school. Expressions like ‘everyday life’ permeated several accounts, as did an emphasis on the applicable skills of statistical awareness. However, such comments were frequently mediated by other beliefs about the subject or its relationship to the learner; Bryony’s indication, for example, that they should be exposed to a breadth of mathematical ideas according to their ability. There was also a sense in the comments of both teachers that the real world provided both the context and the justification for the teaching of mathematics.

These teachers could be described as utility-oriented. Their perspectives on education seemed located in a belief that their responsibility was to prepare learners for everyday life. This disposition was represented in the ways in which they privileged those topics perceived as having real-world applicability; the concrete rather than the abstract aspects of mathematics.

Mathematics as an exploratory activity (autonomy-oriented education)

Nine teachers (20%) indicated that the importance of mathematics lay in its being an exploratory activity. For some this manifested in terms related to problem-solving while for others it was more explicitly investigative. Of this group, only one privileged mathematics as applicable number or real-world preparation. Grace, a teacher of more than twenty-five years’ experience, said

I think they have to have confidence to not immediately see an answer to any sort of problem, but to say, ‘Well, if all the facts are here and if I’ve been taught appropriate skills, then it’s worthwhile that I try and put together the information and try and get some solution’. So even if it's just arithmetic and it’s a knowledge of place value or if it's trigonometry and a knowledge of the rules and they say, 'Well, how can I put these rules with this situation to find out what I need?' I think they’ve got to be encouraged and taught how to solve a problem they might not actually have seen before but that you say to them,
‘You do have the skills for this, see if you can’ ... with guidance, ‘put two and
two together. ... I think that’s important because we can never prepare them
for everything they might see, which is going back to the understanding part
of it. If they can understand as well, they can meet unfamiliar situations and
be able to cope because they know, ‘Well, it’s got to be something to do with
this, what do I know about this and that?’ This is the ideal I’m talking about.
Sometimes it is just slog and I think if we’re not aiming for that then you’ve
got to aim for the ultimate really which is producing people who can
eventually come up with their own ideas.

Helen, a woman of five years’ experience who entered teaching from a career in the
pharmaceutical industry, offered a more focused perspective and implicit criticism of
the curriculum and its expectations;

They should all be able to do the investigative side. ... Children like finding
things out for themselves, but just to give them something and say, Go and
find out is not enough. Children, I think, need an awful lot of guiding. ... But
we could be so much further on if they already had the basic ideas of what it is
they are going to do, how to set it out, how to explain to other people. ... Their
language is very limited mathematically. ... There doesn’t seem to be much
mathematical talk. ... Investigative skills, where they learn to go and find other
things that are similar and relate to it, they know where to go and look
something up in the library or they know to go the computer and be able to
set out a diagram. It helps them in everything else they do in school. And it
makes their lives so much easier.

A different perspective was offered by Hazel, a female head of department of
twenty-five years' experience. She said, in addition to comments about problem-
solving activity, that ‘It’s the experiencing. It’s the experiencing that’s going to lead
them on to sorting out whatever it is they’ve got to sort out. .... I think the
experience of doing maths is far more important. What else is there to do? The
content is almost irrelevant.’ She went on to say, arguing for learning as
participation, that ‘it doesn’t matter what I do. If they don’t take part ... if they don’t
do what I ask them to do, then it’s all just a waste of time.

In summary, a fifth of English informants privileged the exploratory qualities of
mathematics. For some, like Grace, this lay in an emphasis on the heuristics of
problem-solving while for others, like Helen and Hazel, it was in the experience.
Both Helen and Grace acknowledged the importance of teacher support, while all
three implied that mathematical problem-solving provides transferable skills while
acknowledging that its attainment is not unproblematic.

The comments of these teachers indicate an educational orientation focused on the
development of learner autonomy derived from the skills of investigation and
problem-solving. Their comments suggested that they were less interested in a
conception of mathematics as problem-solving than on the transferable skills
acquired through problem-solving. Indeed, Hazel’s argument that learners should
experience mathematics seems to support such a conjecture. Such teachers could be
said to be autonomy-oriented.

Mathematics as an intended curricular given (conformity-oriented education)

Fourteen teachers (31%) indicated that the mathematics curriculum was a given and,
basically, beyond negotiation. For some this was mediated by an
acknowledgement that while individuals have preferred topics, the pursuance of such preferences would be detrimental to their students’ education.

Of those who seemed unconditionally accepting of the curriculum as given, Edward, a head of department of thirty years’ experience said, ‘I don’t think I’ve got any particular problem about the broad content of the National Curriculum, ... There’s plenty there. There’s nothing there I’d say is in itself more important than anything else.’ While George, a head of department of twenty years experience commented that, ‘I feel quite comfortable really with the given curriculum.’

In respect of those who acknowledged preferences, John, a former head of department and now an assistant head teacher of twenty-five years’ experience, said, I have my own personal preferences but I recognise the value of the broad base and know that other people have other preferences and I’d be compromising the education of children if I put more favour on one section than another. Obviously all that would come out in my presentation, in my sheer enthusiasm in certain areas and lack lustre in others. I successfully put most of our good mathematicians off statistics for life, unintentionally, and they’ve all done mechanics. ... I think all areas have equal validity and equal importance and should be presented as such to the children.

Danny, whose thoughts about mathematics as applicable number were discussed above, offered a similar comment, There’re certain things that I like, that I think are relevant and should be taught, and there are things that I don’t. But that’s just personal preference and probably stems from my own weaknesses and strengths and what I enjoyed when I was at school .... I wouldn’t like to argue particular cases there, or I think this is more important.

In summary, almost a third of English teachers described the intended curriculum as beyond negotiation. Some framed their responses with reference to the English National Curriculum, as in Edward’s comments and John’s allusion ‘to the value of the broad base’. Others asserted that they were employed to deliver not to challenge the curriculum, as in the comments of Michael, a second in department of six years’ experience, who said, ‘it is not my job to question the National Curriculum, it is my job to teach it to the best of my ability’. For some, the curricular given was mediated by an acknowledgement of personal preference such John’s dislike of statistics.

These teachers, seemingly content with given frameworks, were unprepared to critique the intended curriculum. They appeared, despite particular preferences, comfortable with what was expected of them and reluctant to consider alternatives. These teachers appear professionally conformist in that their role is to encourage conformity rather than challenge. The beliefs of these teachers, it is argued, are rooted in a conformist-orientation.

Mathematics as determined by student characteristics (differentially-oriented education)

Eleven teachers (24%) indicated that their perspectives on appropriate mathematics varied according to their perceptions of the characteristics of their students. For most this was related to student ability while for a few it was linked to cognitive development. Typical comments in respect of ability were made by Naomi, a second in department of six year’s experience,
I think it depends what type of kids we're talking about. If we're talking about low ability ones then ... I'm coming round now to thinking that we have to teach them social maths. ... I think for the weaker students there should be something other than trigonometry and Pythagoras' theorem. ... It should be a lot more relevant. But ... I quite like the broader subject for the able and the ... middle students.

Maureen, a female head of department of more than thirty years' experience, offered a related perspective focused, initially, on the able student. She said,

I think it depends which group you're talking about. ... If you've got your top group you want to cover as much algebra as you can ... to get them into that kind of thing. They're the ones who you want to do A level and the more fluent they are with that, and the more understanding, then the better. ... If you're talking about other groups then ... algebra all the time is a loser because ... they can't see where it's going or what it's for. So you've got to home in much more on the maths that's relevant, I would have said, like, the statistics section. ... They need ... to be able to interpret what's going on.

In respect of the second group, Pete, a second in department of eighteen years' experience, said,

It depends what level you're working on. I can see at certain levels that number ... would be important to a lot of students. ... If I was talking to a top set year ten or year eleven, in some ways I would suggest that algebra is very important because I know that if they're going to go on and study maths then a lot of it is going to be algebra. ... So I can see from a ... functional point of view, that number's more important for some students and if they go on further algebra's more important. But from a theoretical point of view, you know, I think geometry's just as important, or ... data criticism. So I can see how all the elements, theoretically I wouldn't say that one's more important. But functionally, at different stages of the child's development I can see how some are considered more important.

In summary, and despite caveats implicit in 'it depends', almost a quarter of informants linked beliefs about curriculum mathematics to perceptions of their students. Some, like Naomi and Chris, suggested that the less able should be taught social mathematics and not abstractions like algebra. Others, such as Maureen and Pete, argued that algebra is appropriate only for the able while utilitarian topics like critical data handling should be offered the rest. In respect of students at each end of the ability spectrum these teachers’ comments seemed unequivocal; the able study mathematics in all its abstractions while the weak study applicable number and functional topics like data handling. There was ambiguity, however, in respect of the average student. The mathematics appropriate for the average student was presented as either a diluted form of the former or a rigorous version of the latter according to some arbitrary predisposition on the part of individual teachers.

These teachers’ comments tend to indicate an inconsistent or variable underlying educational objective. In the explicit manner in which these teachers differentiated their beliefs according to perceptions of the characteristics, usually ability, of their students, they could be described as being differentially-oriented.

Hungary
Comparing beliefs

The categories derived from the ten Hungarian teachers merged into just two robust themes which can be seen in table 2. Additional analyses led to the identification of two further educational orientations which can also be seen in table 2.

<table>
<thead>
<tr>
<th>Curricular theme</th>
<th>Educational orientation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-world facilitator</td>
<td>Empowerment-oriented</td>
</tr>
<tr>
<td>Problem-solving</td>
<td>Intellectually-oriented</td>
</tr>
</tbody>
</table>

Table 2: the two themes relating to Hungarian teachers’ beliefs about the curricular importance of mathematics and their corresponding educational objectives.

Mathematics as real-world facilitator (empowerment-oriented education)

Four Hungarian teachers commented, in varying degrees of importance, on the teaching of mathematics and its relationship to some sense of real-world. Anna, a mentor teacher – nominated by the university to support the school-based work of teacher training students – of ten years’ experience, said:

I think we teach mathematics to help children find their way in life more confidently. Whatever they become, a cleaning lady or a banker or a doctor..., mathematics is a logical skill; facts and things thought over in a logical way will help them to make their way more confidently in society.

While Eva, a teacher of nine years’ experience, said, ‘I think they have to see that mathematics is about life. They have to see that in mathematics you have to think logically’. On a related theme, Johanna, a teacher of twelve years’ experience, commented that,

I am an engineer. ... I keep in mind the practicality of things, and that’s the first thing for me, ... children have to learn the basic things. Knowing how to count and measure is not mathematics, but in the first four years they have to be taught them so that by the time they enter the fifth year, they can do them quite well. ... I like it if they can think logically. So, being inducted into intelligent thinking is how it should be. ... They should learn these skills so that by the time they enter the fifth year, they can do real mathematics.

In summary, four teachers indicted that the importance of mathematics lay in its facilitating learners’ preparation for some sort of real-world. This was evidenced in Anna’s helping learners ‘find their way in life more confidently’, Eva’s assertion that ‘mathematics is about life’ or Johanna’s keeping ‘in mind the practicality of things’. However, despite this brief but explicit link to a real world, the emphasis of all these teachers lay in the development of, or induction into, logical or intelligent thinking. That is, even when focusing on real-world contexts, the induction of learners into higher order logical thinking appeared the more important objective.

This desire to prepare learners for a successful engagement with a world beyond school, which was mediated by an explicit desire to induct learners into intelligent or logical thinking, indicated an educational objective related to the empowerment of learners. Empowerment-oriented teachers see education as the means of providing learners with the critical, transferable, skills necessary for a successful engagement with the real-world.
Comparing beliefs

**Mathematics as problem-solving (intellectually-oriented education)**

Nine of these ten teachers spoke about mathematics as a problem-solving discipline and suggested that their role was one of presenting non-routine problems. Typical of comments made by others were those of Ferenc, a teacher of fifteen years’ experience. He said:

> I think it is very important to make children realise that mathematics is actually solving problems. ... So I find it really important to help them collect what they know in connection with the problem, what conditions they know, what knowledge they have and how they can order it to create a solution. ... You can't be sure they always get to the solution, but it's still good if they can find a way where, or how, they can look for a solution. The other important thing is that ... children have the right to make mistakes. They have to learn that, when solving a problem, if they make an error, it doesn't matter. It's a part of solving the problem ... and goes with any kind of problem-solving, particularly in mathematics. And if they get this experience in mathematics, it helps them get through problems in life too. ... It's very important for children to understand that a problem is solved when they have ... checked it and are sure that there are no other solutions; and if there are other solutions then they have found them all.

A variation on the theme was provided by Klára, a teacher of eight years’ experience. She said:

> One of them is the joy of exploration; ... children should be brave and able to get close to an unknown problem. ... It's also important that they understand that things are built on each other, not just in mathematics but in other subjects ... and everyday life too. And if one part is left out ... then you should replace it in time so that you can keep on building the building, ... I find it important for them to experience the joy of cooperation. Often in mathematics it happens that some children cannot get to the end of a problem when a child, one who wasn't talking until that point, suddenly puts everything together, ... says something completely new and is the one who offers the solution. And I think one of the most important parts of the modern age ... is that you have to be able to work in a group and you have to be happy that you're one who gave something to that exploration. I don't think the greatest merit is that you do something alone and you solve the problem yourself, it's the thinking together and working in group.

Both Ferenc and Klára implied that mathematics is synonymous with problem-solving. Moreover, Ferenc’s comments about multiple-solution problems which, according to Klara, are solved collaboratively, confirm that the word used, *probléma*, and the manner of its contextualisation could not have been interpreted as *gjakorlít* (exercise) or *feladet* (task). Indeed, Klára asserted that the greatest merit lies in solutions obtained collectively while Ferenc argued that learners should feel confident with failure. Importantly, the skills of mathematical problem-solving are seen as transferable to a world beyond school.

Klára also alluded to mathematics as a building constructed systematically and logically from one stage unto the next. This reflected the comments of several, including Erika, a teacher of almost thirty years' experience, who said that, ‘this building up is very important ... and often the building up is at least as important as the taking apart’. Interestingly, Erika highlighted, not only mathematics as a
construction, but also, as discussed by Ferenc, the importance within the problem-solving process, of acknowledging and then managing the conditions embedded within the problem. She said that when solving problems we ‘start from nothing and, step by step, the conditions will narrow our way’

The construction metaphor seemed important to Hungarian teachers. Not only did it offer a didactic perspective on the subject but also a philosophy of mathematics. Erika added,

> Often we discuss in our classes what mathematics is; ... if there were no human beings, there would be no prime numbers, because they’re not in the world itself, they were created by humans, ... many, many ... years ago. And we think about it, in connection with, say, a definition. We define something, that it should be like this, or could be defined like this ... and we can show that the alternative definition would have been stupidity.

In respect of teachers’ perceptions of student ability, an issue of some importance to many English teachers, Klára was the only one to raise it, saying

> Lessons should be set up in such a way that everybody should have a problem to solve. ... We must make sure that the weaker doesn't fall behind the average because then they cannot take it anymore. ... The gifted child should be given good, colourful, interesting and exciting problems so that they are not bored in their lessons, while those in the middle level shouldn’t be forgotten. It's very important that you should adapt yourself for the class you're teaching. And of course you have to look around. ... to freshen up your collection of problems.

Such comments suggest that beliefs about student ability as a curriculum determinant are mediated by a continuing desire to pose appropriate problems. That is, despite an apparent emphasis on ability, mathematics as problem-solving remains the dominant perspective.

In summary, the nine teachers of this group indicated that problem-solving should dominate learners’ mathematical experiences. Even when discussing a world beyond school they emphasised the transferability of the skills of problem solving. Their comments, as indicated by the frequency of the building metaphor and a sense of the collaborative, are rooted in practice and a sense of how students’ mathematical thinking might be developed. There is also a sense, inferred from the comments of Ferenc, Erika and Klára, that they do not expect the problems they pose to be simple but accessible, non-routine and multi-step.

Such perspectives indicate that these teachers are intellectually-oriented in the sense that mathematics is the means by which the mind of the learner is developed; through problem-solving the critical faculties of the learner are honed and through collaboration the skills of negotiation are practised.

Discussion

While refraining from making too strong assertions – the Hungarian sample was small, the representativeness of schools and teachers was difficult to guarantee and there remain possible ambiguities of translation – there is evidence to indicate that English and Hungarian teachers of mathematics differ significantly in respect of their espoused beliefs as to the importance of the subject they teach. Ernest (1989) cautioned against an over-reliance on teachers’ espoused beliefs as frequently there are mismatches between them and observations of practice. Nonetheless, as is shown below, the findings of this study resonate well with official curricular
documentation and externally conducted studies of classroom behaviour focused on
generalised descriptions of nationally distinctive practice. Consequently, such
concerns may, in the context of a study focused more at the national rather than the
teacher level, be misplaced. Also, as will be shown, teachers’ philosophical
perspectives resonate well with reports of systemic educational traditions and the
manner in which teachers’ identities are contextually and culturally located.

The evidence suggests that most teachers from each country are aligned with a
nationally-located perspective on mathematics which informs a culturally-defined
professional identity. There are exceptions, as with the English teachers who
privileged learner autonomy, but in general the evidence is indicative of generality.
The Hungarian perspective is dominated by mathematics as an intellectually-
challenging and problem-solving discipline and accords with an observational study
of Hungarian mathematics classrooms (Andrews 2003) and a comparative analysis of
national curricula (Sutherland 2000). Teaching is focused on learners’ transition to
adulthood through their acquisition of the skills of logical reasoning which are
developed through the solution of non-routine problems that allow for multiple
solution strategies and encourage higher level thinking (Hiebert and Wearne 1993,
Stein et al 1996). Hungarian teachers’ professional identities seemed rooted in
collaboration and the study of a subject with little explicit, but clearly-defined
implicit, real-world relevance. This resonates with an earlier observation that
Hungarian teachers appear ‘largely unconcerned with the applications or usefulness
of mathematics to the wider world’ (Andrews and Hatch 2000) and may help explain
Hungarian success on TIMSS.

The majority view of English teachers appeared concerned with functionality,
manifested in an emphasis on applicable number and others’ beliefs about real-
world preparation and differentiated curricula and accord with earlier findings
indicating that English teachers’ beliefs about mathematics and its teaching were
underpinned by notions of utility (Andrews and Hatch 2000). English teachers’
professional identities seemed rooted in a model of education in which learners’
attainment was frequently pre-determined, and therefore individualised, and
focused explicitly on real-world relevance. The acceptance, by many, of their
professional context indicated that English teachers see education as an induction
into conformity. Such perspectives resonate with studies of English teachers’
classroom practice which present mathematics as a technique-oriented subject
(Leung 1995, Kaiser 1999) premised on a model of content simplification (Haggarty
and Pepin 2000) and may explain English success on tests like PISA. This sense in
which teachers’ perspectives on mathematics informs their professional identities
accords with the findings of earlier studies (Grossman and Stodolsky 1995, Virta
2000).

The perceptual resonance between compatriot teachers seems to suggest that many
of the influences in the construction of their professional identities are implicit
rather than explicit. The stories that teachers tell, at least in respect of mathematics,
appear similar to those told by others from the same country and suggest not only
that they derive from similar experiences but also that these experiences have been
interpreted and represented similarly. Thus it is conjectured that not only are
cultural influences, such as a teacher’s own experiences of school (Harel 1994, John
1996, Foss and Kleinsasser 1996), significant but that their influences are stronger
than might have been expected. Also, the existence of such consistent and
culturally-located identities seems to suggest that elements of a teacher’s identity
Comparing beliefs

may be more stable than implied by, for example, Norquay (1990) and confirms Greer et al’s (2002: 285) observation that teachers’ ‘beliefs are shaped by both the immediate and wider instructional environments in which they work.’

In short, the espoused beliefs of the majority of teachers of both countries appear consistent with studies of their countries’ pedagogic traditions and tends to confirm the existence of each country’s characteristic pedagogical flow (Schmidt et al 1996), which are ‘the pedagogical strategies and approaches typical of a set of lessons’ which are ‘enacted repeatedly in a country’s classrooms’ and appear routine and almost ‘below the conscious level for most teachers’ (Cogan and Schmidt 1999: 71-72).

Of course, whether such beliefs and practices, indicative of the implemented curriculum, resonate with the intended curriculum is a moot point; there is research indicative of a substantial gap between espoused and observed practice (McRobbie and Tobin 1997, Leung et al 2001). In this regard, an analysis of each country’s curricular documentation should be helpful. Initially there are similarities. Both national curricula expect children to acquire the skills of logical thinking and problem-solving, which are presented as longitudinal themes permeating all subject content. However, the expected outcomes of these expectations differ significantly. The Hungarian curriculum (Ministry of Education 2000a, 2000b, 2000c) expects children, particularly from grade 5 onwards, to engage with the processes of proof including the construction of their own. The English (Department for Education and Employment/Qualifications and Curriculum Authority 1999) presents no such expectation; students, particularly older ones, are expected to understand proofs presented to them and be able to distinguish between a proof and a demonstration, but nowhere within the assessment criteria of the English document can be found the word proof. An interesting interview manifestation of these differences can be seen in informants’ use of the word problem (probléma). In 45 English interviews the word problem occurred 18 times while in ten Hungarian probléma occurred 32 times. That is, each English informant used the word 0.4 times compared with 3.2 utterances per Hungarian - a ratio of 1:8.

The Hungarian documents make explicit reference to children being able to demonstrate different forms of mathematical thinking and understand the different ways in which knowledge is constructed. The English document makes only an implicit mention of the former and fails completely to consider the latter, preferring, it seems, to focus on specific skills rather than more generalised mathematical behaviours and understandings. The Hungarian documents, even when focused on vocational and trade schools, expect all learners to engage in a mathematical discourse, something not mentioned in the English (Ministry of Education and Culture 2000c).

These distinctions reflect the introductions to the two documents. The Hungarian documents offer a rationale for the teaching of mathematics, which the English does not. Indeed, the latter’s section about mathematics in the National Curriculum discusses only technical and structural details. It is as if, for the English, there is an assumed, and commonly held, agreement as to the nature of mathematics and mathematical learning which is thought unproblematic. For the Hungarians there is a clear expression of the unique contribution that mathematics makes to the intellectual development of all learners and a sense of its being a valuable cultural artefact. The nearest the English document gets to such a celebration is the inclusion of some banal sound bites from eminent mathematicians or users of mathematics.
In summary, the evidence of this and other research indicates, in each country, high levels of consistency between teachers' beliefs as to the importance of mathematics as a curriculum subject and the national curricula frameworks within which they operate. Reasons for this can be found in the historical and philosophical precedents that inform the educational traditions of each country. Author (3: 58) note the resonance of the Hungarian curricular tradition with rational encyclopaedism; a philosophy 'which privileges those subjects which encourage rational faculties, facilitates social change through rational knowledge and demands that all learners have access to the same curriculum'. This contrasts with the English humanist tradition, which emphasises utility and subordinates rationality to the wider aim of personal morality. The latter, as in English classrooms, results in mathematics as applicable number which children learn independently of their peers and which creates, implicitly, systematic denials of opportunity. The former, as in Hungarian classrooms, necessitates collaborative problem-solving and, explicitly, demands and provides equality of access to the curriculum. However, this sense of national resonance was not as secure for the English cohort as for the Hungarian. A fifth of English teachers, drawing on an autonomy-oriented philosophy, described mathematics as an exploratory activity which, in some sense, could be seen as resonating with 'Hungarian' perspectives. Indeed, like their Hungarian colleagues, they discussed mathematics in experiential terms, describing it as something that learners do rather than have done to them. However, unlike the Hungarians, they emphasised neither the logico-deductive nature of the subject nor the explicit sense of mathematics as problem-solving. Consequently, suggestions that these teachers reflected Hungarian perspectives would be naive. This dichotomy of English belief reflects the practices observed in the two schools of Boaler's (1999) study; a transmissive teaching involving much repetitive practice but with little transferability and a project-based approach leading to more positive attitudes and successful transfer.

Other issues of importance emerged. Hungarian children attend selective secondary schools. However, as curriculum documents indicate, all children are expected to experience an intellectually rigorous curriculum. That is, despite institutional differentiation, learners’ experiences of the curriculum appear consistent and challenging. In England, most learners attend comprehensive secondary schools. However, despite nominal entitlement to curricular equity, the beliefs of many English teachers lead to differentiated curricula, a systematic denial of opportunity and skills-based mathematics (Dowling 1996, Boaler et al 2000, Haggarty and Pepin 2002). The evidence implies, also, that few English teachers have any mathematical ambition for their learners. The utility- and basics-driven agendas reflect a view that mathematics has little intrinsic value. Unlike their Hungarian colleagues, for whom the study of mathematics develops and sustains high levels of intellectual activity, many English teachers seemed comfortable as providers of tool kits. The exception lay in the autonomy-oriented perspectives of a minority for whom mathematics may have implied intrinsic value and intellectual challenge beyond skills-acquisition.

In conclusion, it seems that teachers in both countries operate within long-held educational traditions rooted in societal beliefs about the purpose of education in general and mathematics education in particular and which create powerful unacknowledged influences on their professional identities. Moreover, most teachers’ beliefs about mathematics and its teaching reflect, at a general rather than particular level, the expectations of the curricular frameworks within which they
operate and the findings of studies of classroom practice undertaken in their
countries. That is, the beliefs of most teachers in respect of mathematics and its
curricular importance appear consistent with both the intended and the
implemented curricula of their respective countries.

In an era of international mobility, particularly within Europe, such culturally-
determined beliefs, expectations and classroom practices may make problematic the
transfer of teachers of mathematics from one educational system to another (Pepin
1998). Those responsible for the employment of foreign teachers should understand
that mathematics is not an unambiguous body of knowledge, nor does it carry with
it pedagogic or didactic uniformity. Employers need to understand that in each
educational system curricular mathematics is culturally-located and associated with
deeply held, but rarely articulated, beliefs and long-standing pedagogic traditions.

References

Abelson, R. (1979). Differences between belief systems and knowledge systems.


effectiveness (San Francisco: Jossey-Bass).

Mathematics in the middle school years: IEA's third international mathematics and
science study (Boston: Boston College).

Boaler, J. (1999) Participation, knowledge and beliefs: a community perspective on

disaffection, polarisation and the construction of failure. British Educational

mathematics: a study of three teachers from other disciplines. Focus on Learning
Problems in Mathematics, 12 (1), 41-60.

teachers' conceptions and attitudes towards mathematics. International Journal of
Mathematical Education in Science and Technology, 29 (3), 317-324.


Comparing beliefs

(eds.), Proceedings of the eighteenth international conference for the psychology of mathematics education (Lisbon: University of Lisbon).


Comparing beliefs


Comparing beliefs


