Constructing Bidding Curves for a CHP Producer in Day-ahead Electricity Markets

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***Abstract*—The operation of Combined Heat and Power (CHP) systems in liberalized electricity markets depends both on uncertain electricity prices and uncertain heat demand. In the future, uncertainty is going to increase due to the increased intermittent power induced by renewable energy sources. Therefore, the need for improved planning and bidding tools is highly important for CHP producers. This paper applies an optimal bidding model under the uncertainties of day-ahead market prices and the heat demand. The problem is formulated in a stochastic programming framework where future scenarios of the random variables are considered in order to handle the uncertainties. A case study is performed and conclusions are derived about the CHP operation and the need for heat storage.**

***Index Terms*—Combined Heat and Power, CHP, bidding curves, stochastic programming, operation planning**

I. Introduction

Combined Heat and Power (CHP) systems are widely used in cases where there is a need for electric power and heat at the same time. Traditionally such systems find application in the industry and in district heating networks. More recently small CHP systems, called micro CHP, are being installed in commercial and residential buildings. Compared to conventional power plants, CHP systems achieve a higher efficiency, resulting in reduced fuel consumption and exhaust gas emissions. This is the reason why they are actively promoted in Europe [1] and other countries. As the use of CHP spreads, the optimal operation of such systems becomes more important in achieving lower production costs and better utilization of the produced energy. The interdependence, however, between heat and power generation increases the complexity of the production planning.

Various models have been proposed for the CHP production planning. Some of the initial related works are [2] and [3]. The realistic modeling of the CHP operation, results in the most cases in a non-linear mixed integer optimization problem with constraints. Therefore, there isn't a simple method used to solve the problem. Instead many methods, from priority list and Lagrange relaxation [4], to genetic algorithms and other bio-inspired techniques [5] have been used so far.

The electricity market liberalization has introduced many uncertain factors in the production scheduling of power systems. To minimize the operational risk uncertainties must be handled properly. For that reason tools like stochastic programming are used to incorporate all these uncertainties. In the stochastic programming framework uncertain parameters are treated as stochastic variables and are usually represented in scenarios. The solution provided is optimal for all the scenarios weighted by their probability of occurrence. Although there is extensive literature utilizing stochastic programming in conventional, hydro and res power systems, this is not the case for the CHP systems. This is probably due to the fact, that CHP plants till recently were operating with fixed heat load and the power was considered as a byproduct sold at a fixed price. However, the deregulation of power markets, allows CHP systems to take advantage from their inherent flexibility and schedule the power production in a way to achieve higher profits. That makes stochastic programming an important tool in CHP operation planning. Some relevant works incorporating stochastic programming in CHP planning can be found in [6] and [7]. In a recent work [8], authors use dynamic programming to handle the uncertainties of trading on multiple power markets. The reader is also referred to [9] and [10] for further reading on short-term operation planning of CHP systems.

The aim of this paper is to provide a model used by CHP producers for constructing bidding curves in day-ahead electricity markets. Stochastic programming is used to incorporate the uncertainties. The stochastic parameters include the day-ahead electricity prices and the heat demand which are modeled through a number of scenarios. The proposed model is formulated as a two stage mixed integer linear programming problem.

The following sections are organized as follows: Section II describes how uncertain parameters are modeled in a stochastic programming framework. This is specifically done for a) day-ahead electricity prices and b) heat demand in a district heating network. Section III provides the mathematical formulation of the model. In section IV a case study is presented based on realistic data and finally a concluding discussion is provided in section V. For reference, the nomenclature used throughout this paper is provided in the appendix.

II. Modeling CHP uncertainties

Within a stochastic programming modeling framework stochastic processes are represented by scenario trees [11].

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A scenario tree can be seen as a set of nodes and arcs (Fig. 1). Nodes constitute the time points where decisions are taken and the arcs represent the possible realizations of the random variables. The first node on the left is called root node and represents the point at the beginning of the planning horizon where a decision is made without any of the uncertain parameters having been realized. This is the first-stage decision. The nodes on the right are called leaves and represent the points where second stage decisions are made with full information about the value of the uncertain parameters. If the decision framework consists of more stages then the decision tree is formulated in a suitable way (multistage stochastic programming problem). Each path that connects the root node with a leaf is named scenario. The construction of the scenarios and the formulation of the decision tree is an important and difficult task within the stochastic programming framework. There are many techniques used to construct scenarios of random variables. In this paper a model of each uncertain parameter is formulated based on time series analysis of historical data. Then a Monte Carlo simulation is used to produce the scenarios.

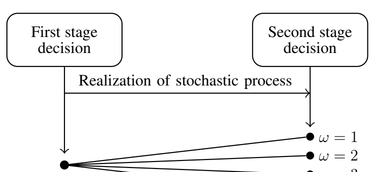

Figure 1. Scenario tree for two-stage decision making problems

A. Electricity prices

Electricity prices in day-ahead electricity markets (or spot markets) are characterized by volatility and periodicity. Fig. 2 depicts on the left side the time series of electricity prices in Elspot day-ahead electricity market [12] for a week's (168 hours) period. Prices range from approx. 20 to 40 €$/MWh$ and the changes are very frequent showing their volatile nature. A diurnal pattern can also be seen meaning that there is a periodicity of 24 hours. This is clearer in the autocorrelation function (ACF) on the right side of fig. 2. This function shows a strong correlation between the current price value and the values of the prices 1-3 hours ago and 24 hours ago (lags 1-3 and 24 respectively). Apart from the diurnal pattern, day-ahead electricity prices usually present weekly and seasonal patterns.

To make electricity price scenarios a forecasting model needs to be built. Many techniques have been used to forecast electricity prices. Among them we can find artificial neural networks [13], time series models [13][14], and hybrid models [15][16]. In this paper we build a SARIMA time series model based on historical data of the Elspot day-ahead electricity market. SARIMA models are suitable for fitting non stationary data with some degree of seasonality. A SARIMA model can be formulated in the following form:

$$\varphi_p(B)\Phi(B)(1-B)^D(1-B^s)y_t = \theta_q(B)\Theta(B)\varepsilon_t \qquad (1)$$

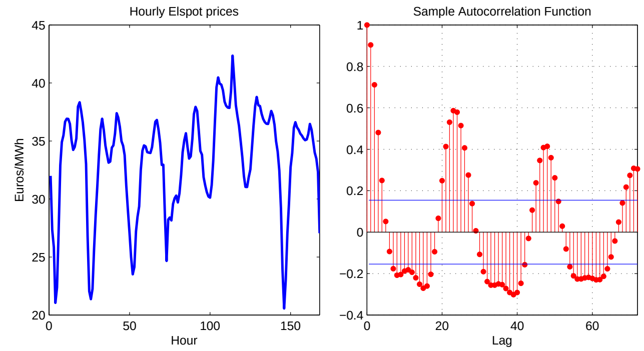

Figure 2. Spot market prices *(left)* and autocorrelation function *(right)*

where y_t is the electricity price at time t, $\varphi_p(B)$ and $\theta_q(B)$ are the autoregressive (order p) and moving average (order q) functions of the backshift operator $B : B^l y_t = y_{t-l}$ respectively, $\Phi(B)$ and $\Theta(B)$ the seasonal autoregressive and moving average functions respectively, D the nonseasonal integration degree, s the order of seasonality and ε_t the error term.

Box-Jenkins methodology is used for building a qualitative forecasting model [17]. This is a trial and error process consisting of four steps:

1) Model identification: After applying the logarithmic transformation and integrating the time series as many times needed to become stationary, the autocorrelation (ACF) and partial autocorrelation functions (PACF) are used to select the orders of p, q and s.
2) Model estimation: Given the orders of the model from the previous step, a least squares or maximum likelihood method is used to estimate the parameters of the model.
3) Diagnostic checking: In this step the residuals error is checked whether it is a white noise process. There are also some tests that can be used in order to check the fitness of the model. These are the Bayesian information criterion (BIC) and the Akaike information criterion (AIC). If the tests fail then the modeler goes back to the first step and modifies the initial model.
4) Forecasting: After the model has been fitted to the historical data, it can be used to forecast future values of the time series.

We apply the previous steps on historical data from Elspot day-ahead electricity market. The SARIMA forecasting model which is derived has the following form:

$$\begin{aligned}&\left(1-\varphi_1 B^1-\varphi_2 B^2\right)\left(1-\varphi_{23} B^{23}-\varphi_{24} B^{24}\right)\left(1-B^1\right)\\ &\times(1-B^{24})log\left(y_t\right)=\left(1-\theta_1 B^1-\theta_2 B^2-\theta_3 B^3-\theta_4 B^4\right.\\ &\left.-\theta_5 B^5\right)\times\left(1-\theta_{23} B^{23}-\theta_{24} B^{24}-\theta_{25} B^{25}-\theta_{168} B^{168}\right)\varepsilon_t\end{aligned} \qquad (2)$$

The prediction performance of the model can be measured in different ways. One is to compare the model with other competing models that are known to work in a reliable manner. The other way is the use of some measures of performance. The most common of them are the mean square error (MSE) and the mean absolute percentage error (MAPE) which are calculated by:

$$MSE = \frac{1}{N} \sum_{t=1}^{N} \left(X_t - \hat{X}_t \right)^2 \tag{3}$$

$$MAPE = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{X_t - \hat{X}_t}{X_t} \right| \tag{4}$$

where X_t is the actual value and $\hat{X}_t$ the forecasted value. The proposed model achieves an average daily MAPE score of 4.04%.

After having built the price model, the next step is to make the price scenarios. Monte Carlo simulation is applied for that purpose. The Monte Carlo method simulates sample paths by taking random error terms ε_t from a Gaussian distribution with zero mean. This procedure, applied for 24 sequential steps, gives a scenario of 24 day-ahead hourly prices. The simulation is applied many times until the desired number of scenarios is reached [11]. In fig. 3 five day-ahead price scenarios are shown. The actual price series is also plotted.

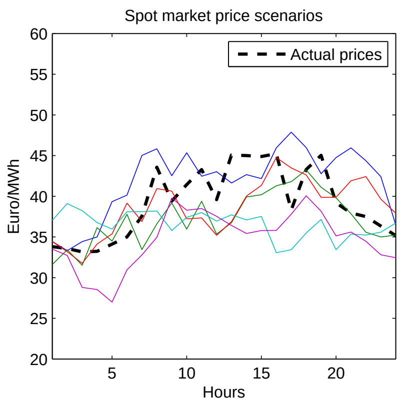

Figure 3. Five spot market price scenarios produced using a Monte Carlo simulation method. Actual prices are plotted with a dashed line

Functions from Matlab Econometrics Toolbox [18] are used to apply the previous steps for price modeling and scenario making.

B. Heat demand

The heat demand in a district heating network constitutes an uncertain parameter that the CHP operator has to take into account in order to make optimal operational decisions. It is characterized by lower volatility compared to electricity prices. This is apparent in fig. 5 where the hourly heat demand for a specific day is plotted with a dashed line. It is also apparent in the ACF in fig. 4 where the current heat demand is strongly correlated to the heat demand of previous hours which means that there aren't any steep changes. In the ACF a 24 hour periodicity can also be seen. This diurnal pattern is caused by people's social behavior. However, the most characteristic property of heat demand is its correlation with the outdoor temperature. In fig. 4 we plot the average heat demand for the Stockholm area during a month's period and the average daily outdoor temperature during the same period. It is clear that there is a strong negative cross correlation between these two parameters. That means the prediction of the outdoor temperature can be used as an external variable to the heat demand prediction, practice that is usually applied in various forecasting techniques. Among them, we can find from simple forecasting models like in [19], to artificial neural networks [20] and time series models [21].

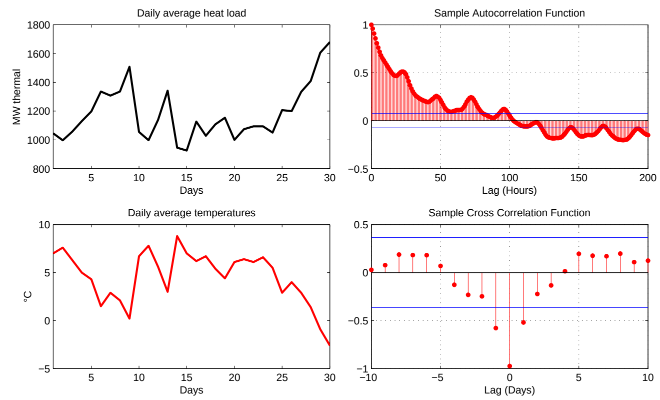

Figure 4. Average daily heat demand *(upper left)*, ACF *(upper right)*, average daily outdoor temperature *(lower left)* and cross correlation function *(lower right)*

In this paper we build a SARIMAX model to forecast next day's hourly heat demand. The procedure followed is the same as in the electricity price modeling with the difference that the outdoor temperature is included in the model as an external variable. The general form of the SARIMAX model is:

$$\varphi_p(B)\Phi(B)(1-B)^D(1-B^s)y_t = \beta x_t + \theta_q(B)\Theta(B)\varepsilon_t \tag{5}$$

where y_t here is the heat demand and x_t is the outdoor temperature. Because of their strong cross correlation at lag 0 (fig. 4) only the temperature at current hour is taken into account and not at previous hours. The rest terms are exactly the same as in (1). Because of the unavailability of hourly outdoor temperatures, daily average values are used instead, which are considered fixed throughout the whole day. That means the accuracy of the model is not as high as if hourly temperatures were used. Furthermore, unlike in reality, actual and not predicted values of outdoor temperatures are used. However, the purpose of the model is to make some heat demand scenarios for the formulation of the problem and the accuracy is not the most important factor in this paper.

The proposed heat demand forecasting model is given by:

$$\begin{aligned} &\left(1-\varphi_1 B^1-\varphi_2 B^2\right)\left(1-\varphi_{24} B^{24}\right)\left(1-B^1\right)\left(1-B^{24}\right) y_t \\ &= \beta x_t+\left(1-\theta_1 B^1-\theta_2 B^2-\theta_3 B^3\right) \\ &\times\left(1-\theta_{21} B^{21}-\theta_{24} B^{24}-\theta_{45} B^{45}-\theta_{48} B^{48}-\theta_{69} B^{69}\right) \varepsilon_t \end{aligned} \tag{6}$$

The average daily MAPE score of the model is 3.12%. To produce the heat demand scenarios a Monte Carlo simulation is applied, too. Five heat demand scenarios for the next day are depicted in fig. 5. The actual heat demand is also depicted with a dashed line.

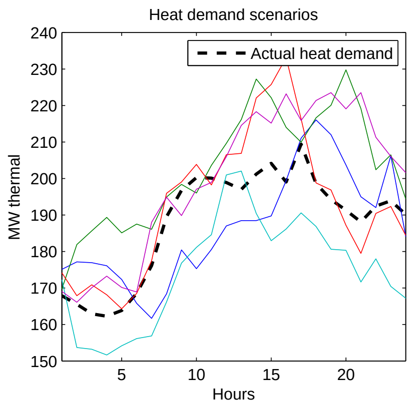

Figure 5. Five heat demand scenarios for the next day produced using a Monte Carlo simulation method. Actual prices are plotted with a dashed line

III. Problem formulation

As stated in the introduction, the bidding decision framework of a CHP producer in day-ahead electricity markets is formulated as a two stage stochastic mixed integer linear programming problem. The stochastic parameters are the day-ahead electricity prices and the hourly heat demand during the next day. The first stage decision variable is the power volume which is going to be offered at a specific price to the day-ahead market. This decision is taken by the producer before noon the day before the power delivery. The second stage decisions are the unit commitment and heat dispatch decisions which are taken before the actual production. The scenario tree of the model is depicted in fig. 6. Two things have to be commented here. The first is that the model would be more realistic if it was formulated as a three stage problem where the second stage decision would be the unit commitment and the third stage decision would be the heat dispatch. This model would follow the exact decision framework where unit commitment is decided some hours before the power dispatch because of the start-up process and the actual heat demand is not known to the operator. But this formulation applied here, apart from providing a simpler model to solve, it is also not too far from reality because the second stage decision is taken very close to power dispatch and the heat demand can be considered fixed in that moment. The second comment is related with the structure of the scenario tree. First stage decisions are taken before any stochastic parameters have been realized and are not related to them. That's why there is usually one root node in the scenario tree as in fig. 1. Here a methodology described in [11] is followed in order to obtain the bidding curves in day-ahead electricity markets. According to this, the uniqueness of the root node is relaxed as seen in fig. 6. That means the power volume that is offered to the spot market is not unique but it depends on the electricity price scenarios. This is done because it is not adequate for a producer to determine a single quantity of power to be traded in the market, but instead a different quantity is offered for every possible price realization. This technique seems to split the initial scenario tree (fig. 6, dashed lines) into many different sub-trees resulting in many sub-problems which can be solved independently. This is not true because the requirement for ascending bidding curves imposes a constraint that doesn't allow the independent solution of the sub-problems.

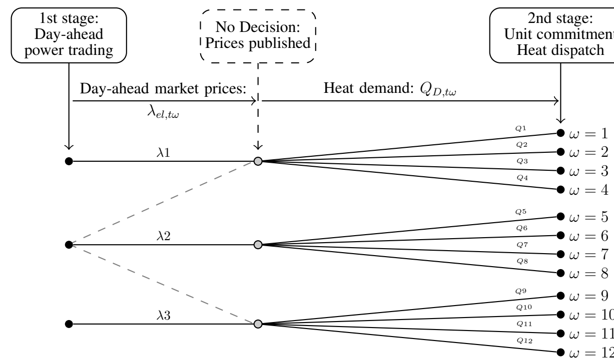

Figure 6. Scenario tree of the proposed model

The objective function of the problem is to maximize the net profit of the CHP producer for all possible scenarios (7). Every scenario's profit is equal to the revenue obtained from trading in the day-ahead market minus the production cost for the scenario. The production cost consists of the operation cost and the cost of starting-up the units. The revenue from selling the heat produced is not included in the objective function as it is fixed and doesn't change the optimal solution. The net profit is the summation of these scenario dependent profits multiplied with their own probability of occurrence.

$$Maximize:$$

$$\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_T} \sum_{g=1}^{N_G} \left(\lambda_{el,t\omega} P_{gt\omega} - (\lambda_{f,g} P_{fuel,gt\omega} + c_{start,g} y_{gt\omega})\right) \tag{7}$$

The constraints of the problem are related to the structure of the scenario tree as explained previously, the operational limits of the units, the heat balance and the on/off status and minimum up/down times of the units. The non-decreasing constraint (8) ensures the ascending order of power bids and the non-anticipativity constraint (9) ensures common power offers for the same price scenario.

$$P_{gt\omega} \leqslant P_{gt\omega'} \qquad \forall g, \forall t, \forall \omega, \omega' : if\ \lambda_{el,t\omega} \leqslant \lambda_{el,t\omega'} \tag{8}$$

$$P_{gt\omega} = P_{gt\omega+1} \qquad \forall g, \forall t, \omega = 1, ..., N_\Omega - 1 : \quad if\ \lambda_{el,t\omega} = \lambda_{el,t\omega+1} \tag{9}$$

Regarding the operational limits of the units, these are depended on the type of the units operating in the CHP system. *Back-pressure steam turbines* are used when there is a need for simultaneous power and heat production, since these two quantities can only be produced in a constant power-to-heat ratio (11a). The operation cost is proportional to the fuel consumption. Since power and heat are strongly connected, the fuel consumption depends only on the produced power (10). The fixed term $\beta_{0,g}$ expresses an additional cost at minimum power production and is the result of the linearization of the function. The upper and lower limits are described by (12). The same equations are also valid for *gas turbines* which are usually used in smaller CHP systems. Heat produced by gas turbines is extracted from the exhaust gases by means of a heat exchanger. Alternatively heat in exhaust gases can be directly released to the environment through some auxiliary cooling system. If such an auxiliary cooling system is present then the relation between power and heat production is given by (11b).

$$P_{fuel,gt\omega} = \beta_{el,g} P_{gt\omega} + \beta_{0,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{10}$$

$$P_{gt\omega} = r_g Q_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{11a}$$

$$P_{gt\omega} \geq r_g Q_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{11b}$$

$$P_{min,g} u_{gt\omega} \leqslant P_{gt\omega} \leqslant P_{max,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{12}$$

Extraction condensing steam turbines are characterized for their flexibility. They offer the possibility to extract steam from different stages of the turbine. That means heat and power production are not strongly connected by a constant power-to-heat ratio as in the case of back-pressure steam turbines. Instead, the turbine can operate inside a feasible operation zone which is described by (14-17) and depicted in fig. 7 (Lines 1-4 respectively). This is the reason why this kind of turbines is frequently used in CHP systems. The fuel consumption (13) depends both on the power and heat produced.

$$P_{fuel,gt\omega} = \beta_{el,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} + \beta_{0,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{13}$$

$$\beta_{el,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} \leqslant \beta_{el,g} P_{max,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{14}$$

$$\beta_{el,g} P_{gt\omega} + \beta_{th,g} Q_{gt\omega} \geqslant \left(\beta_{el,g} + {\beta_{th,g}}/{r_{min,g}}\right) P_{min,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{15}$$

$$Q_{gt\omega} \leqslant Q_{max,g} \quad \forall g, \forall t, \forall \omega \tag{16}$$

$$P_{gt\omega} \geqslant r_{min,g} Q_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{17}$$

Heat producing boilers are often included in CHP systems to cover peak heat demand. Boiler's efficiency η relates the fuel consumption with the produced heat (18). The unit limits are also applied (19).

$$P_{fuel,gt\omega} = \frac{Q_{gt\omega}}{\eta_g} \qquad \forall g, \forall t, \forall \omega \tag{18}$$

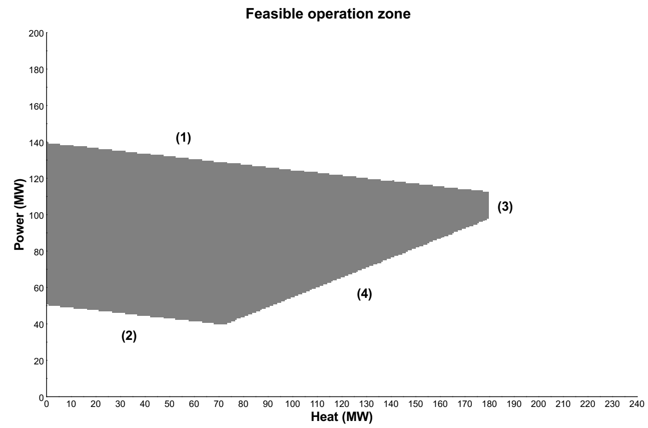

Figure 7. Feasible operation zone of an extraction condensing turbine: 1. Line of maximum injection of fuel, 2. Line of minimum injection of fuel, 3. Line of maximum heat flow and 4. Line of minimum power-to-heat ratio

$$Q_{min,g} u_{gt\omega} \leqslant Q_{gt\omega} \leqslant Q_{max,g} u_{gt\omega} \qquad \forall g, \forall t, \forall \omega \tag{19}$$

There is also the possibility for the heat to be stored in *heat storage tanks*. This unit increases the flexibility of the CHP system as heat can be stored for later use. The only limitation here is the tank's capacity (20).

$$V_{t\omega} \leqslant V_{max} \quad \forall t, \forall \omega \tag{20}$$

The heat balance constraint is necessary to determine how much heat is going to be produced during every time period. This is described by (21a) where in every time period and scenario the heat content in the storage tank plus the heat produced by every heating production unit minus the heat demand equals the tank's heat content in the beginning of next time period. If there is no heat storage tank, then the heat production is restricted to be equal to the heat demand. In some cases there might be special equipment that enables *heat cooling*. Although it makes power production more flexible, heat cooling is not always allowed because it decreases the overall efficiency of the plant. When heat cooling exists, equation (21b) is applied. To determine the heat production during the last time period, the heat content of the storage tank at the end of the last period (i.e. the beginning of the following planning horizon) is defined to be equal to the heat content in the first period (22). In case the heat is retained for the following planning horizon, the equality is then replaced by a lower inequality.

$$V_{t+1\omega} = V_{t\omega} + \sum_{g=1}^{N_G} Q_{gt\omega} - Q_{D,t\omega} \quad t = 1, ..., N_T - 1, \forall \omega \tag{21a}$$

$$V_{t+1\omega} \leq V_{t\omega} + \sum_{g=1}^{N_G} Q_{gt\omega} - Q_{D,t\omega} \quad t = 1, ..., N_T - 1, \forall \omega \tag{21b}$$

$$V_{1\omega} = V_{N_T\omega} + \sum_{g=1}^{N_G} Q_{gN_T\omega} - Q_{D,N_T\omega}$$
$$t = 1, ..., N_T - 1, \forall\omega \quad (22)$$

The following binary constraints are necessary to model the start-up and shut-down status of the units (23 and 24). That means the simultaneous commitment and decommitment of the units is avoided. In the way these constraints are formulated, only the on/off status variable u needs to be binary. The variables y and z referred to start-up and shutdown status respectively are automatically assigned to binary values.

$$y_{gt\omega} \leqslant u_{gt\omega}, \quad y \leqslant 1 - u_{gt-1\omega}, \quad y_{gt\omega} \geqslant u_{gt\omega} - u_{gt-1\omega}$$
$$\forall g, \forall t, \forall\omega \quad (23)$$

$$z_{gt\omega} \leqslant u_{gt-1\omega}, \quad z_{gt\omega} \leqslant 1 - u_{gt\omega}, \quad z_{gt\omega} \geqslant u_{gt-1\omega} - u_{gt\omega}$$
$$\forall g, \forall t, \forall\omega \quad (24)$$

Thermal power plants cannot switch from on to off status and vice versa very frequently because the risk of component failures is increased [6]. Therefore minimum operation and down times are defined to avoid these transitions. The following constraints (25-27) are modeling the minimum operation time:

$$\sum_{t=1}^{L_g} (1 - u_{gt\omega}) = 0 \quad \forall g, \forall\omega \quad (25)$$

$$\sum_{\tau=t}^{t+UT_g-1} u_{g\tau\omega} \geqslant UT_g y_{gt\omega}$$
$$\forall g, t = L_g + 1, ..., N_T - UT_g + 1, \forall\omega \quad (26)$$

$$\sum_{\tau=t}^{N_T} (u_{g\tau\omega} - y_{gt\omega}) \geqslant 0 \quad \forall g, t = N_T - UT_g + 2, ..., N_T, \forall\omega \quad (27)$$

where $L_g = min\left\{N_T, \left(UT_g - T^0_{up,g}\right) u^0_g\right\}$ is the time in the beginning of planning horizon that the unit is restricted to operate.

The same equations also apply for the minimum down time by changing $u_{gt\omega}$, $1 - u_{gt\omega}$, $y_{gt\omega}$, UT_g, $T^0_{up,g}$ with $1 - u_{gt\omega}$, $u_{gt\omega}$, $z_{gt\omega}$, DT_g, $T^0_{down,g}$ respectively.

IV. Case Study

In this section a case study is performed in order to verify the performance of the model. The system consists of an extraction condensing turbine and a heat boiler that covers the peak heat load. There is also a heat storage tank. The system parameters are taken from [6] and are shown in table I.

Data from Stockholm's district heating network and Elspot market are used to produce the heat demand and electricity price scenarios respectively. The scenario tree is constructed in the way it was explained in the previous section. The heat storage capacity for the base case is considered 50 MWh/h and the average heat demand is 195 MW thermal, close to the maximum heat output of the steam turbine. The solution of the problem provides the optimal power volume which will be produced at a specific hour and for every price scenario. Putting these volumes in ascending order, we take the bidding curves for every hour in day-ahead electricity market. In fig. 8 the bidding curves for four different hours are shown.

Table I
CHP parameters for the case study

Extraction condensing steam turbine	
Fuel	Gas
Fuel price, (€/MWh)	11
Min. power output, (MW)	35
Max. power output, (MW)	140
Max. heat output, (MW_{th})	200
Marginal fuel consumption for power production	2.4
Marginal fuel consumption for heat production	0.36
Fuel consumption at minimum output, (MW)	40
Minimum power-to-heat ratio	0.5
Start-up cost, (€)	15000
Minimum up time, (h)	6
Minimum down time, (h)	3
Heat boiler	
Fuel	Gas
Min. heat output, (MW_{th})	0
Max. heat output, (MW_{th})	80
Efficiency	0.9
Start-up cost, (€)	0
Heat storage tank	
Capacity, (MWh/h)	0 - 200

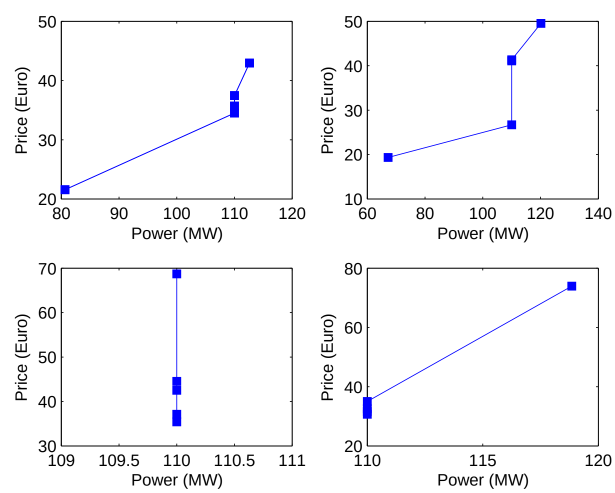

Figure 8. Bidding curves for four different hours

In fig. 9 the power/heat production of the steam turbine and the heat production of the boiler are shown for every scenario. The four plots correspond to the previous bidding curves. The conclusion here is that the steam turbine, as it was expected due to the high heat demand, operates in the area close to the maximum heat production. Especially for the bottom left case the heat demand is so high that the steam turbine provides the maximum heat flow for almost every scenario. That's why the corresponding bidding curve is a straight line. For the rest hours some scenarios allow the steam turbine to operate in partial heat load which gives the chance to provide more power. In general these plots show the interdependence between the power and heat production.

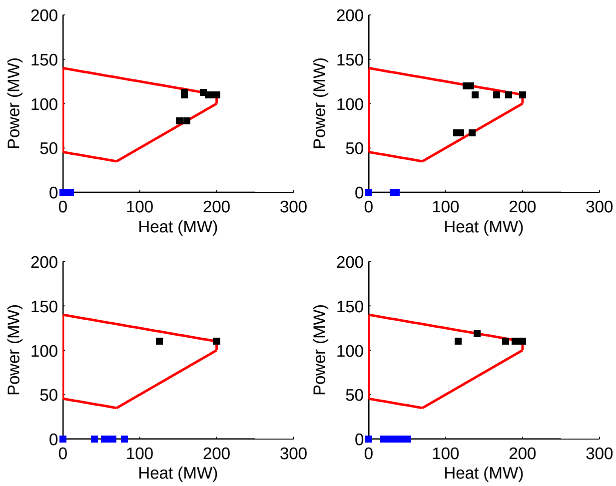

Figure 9. Scenario dependent production scheduling of the steam turbine (black squares) and the heat boiler (blue squares) for four different hours

For further evaluation of the model, three more cases are examined. In the first case the electricity prices are much higher compared to the base case, in the second case a storage tank with higher capacity is added and in the last case a partial heat load is considered. The energy production of the units for every scenario and at a specific hour is shown in fig. 10. When the prices are high the heat production of the boiler is generally bigger, allowing the steam turbine to provide more power. This is more obvious in the third case where the heat boiler in combination with the big storage tank allow the steam turbine to provide almost the maximum power for specific scenarios. For the case of partial heat load it might be profitable for few specific scenarios the operation of the heat boiler.

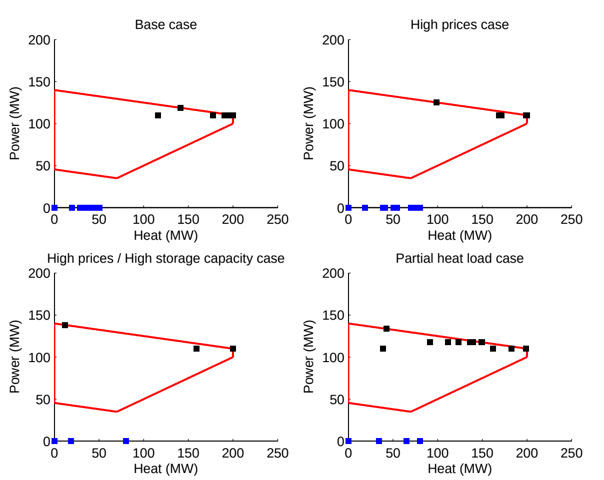

Figure 10. Scenario dependent production scheduling of the steam turbine (black squares) and the heat boiler (blue squares) for four different cases at a specific hour

The advantage of using a stochastic approach over a deterministic one can be measured with the *value of the stochastic solution (VSS)*. To obtain the VSS the initial stochastic programming problem is solved with the stochastic parameters replaced by their expected values. The solution to this deterministic problem provides optimal values for the first-stage variables. Then, the initial stochastic programming problem is solved again with the first-stage variables fixed to the values provided by the solution of the deterministic problem. For a maximization problem the VSS is the difference between the optimal value of the objective function of the stochastic programming problem and the optimal value of the objective function of the modified one. For a minimization problem the VSS is the opposite. For the base case the optimal value of the stochastic programming problem is 7353 and the optimal value of the modified problem is 6918. Therefore, VSS is 435.

Finally, to estimate the value of heat storage capacity, the problem is solved with various capacities ranging from 0 to 200 MWh/h. Fig. 11 depicts the change of the net profit in accordance with the installed capacity. The conclusion is that a heat storage tank makes the CHP system more flexible resulting in bigger profits. There is however a limit that above that there is no much gain to justify the investment of a bigger heat storage tank. To estimate this exact limit, an economic analysis needs to be done which will take into account the installation cost of the heat storage tank and the net present value of the future profits.

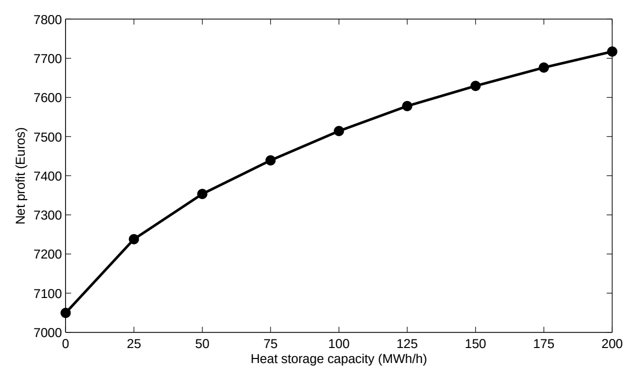

Figure 11. Profit change in accordance with the heat storage capacity

V. Conclusions

This work provides a tool for constructing the bidding curves in day-ahead electricity markets for a CHP producer. The problem is formulated using the stochastic programming framework to incorporate the uncertain parameters of the electricity prices and the heat demand. The resulting optimization problem is a two stage stochastic mixed integer programming problem. The first stage variable is the hourly power volume that will be offered to day-ahead market at a specific electricity price. The second stage comprises the unit commitment and power/heat dispatch variables. The model includes a heat storage tank to estimate the value of the increased flexibility offered by the use of the tank. The heat storage level at the end of the planning horizon is assumed to be set by a long-term planning. Further improvement of the model would be to use a multi-period approximation in order to deal with this assumption. The interdependence between the power and heat production for a CHP system is exemplified with some case studies. The value of the stochastic solution is also calculated and the value of the heat storage capacity is estimated.

Appendix: Nomenclature

Indices and Numbers:

g	Index of units, running from 1 to $\mathrm{N_G}$
t	Index of time periods in hourly resolution, running from 1 to $\mathrm{N_T}$
ω	Index of scenarios, running from 1 to $\mathrm{N_\Omega}$

Parameters

$\lambda_{el,t\omega}$	Day-ahead market price in period t and scenario ω, (€/MWh)
$Q_{D,t\omega}$	Heat demand in period t and scenario ω, ($\mathrm{MW_{th}}$)
π_ω	Probability of occurrence of scenario ω
$\lambda_{f,g}$	Fuel price of unit g, (€/MWh)
$c_{start,g}$	Start-up cost of unit g, (€)
$\beta_{el,g}$	Marginal fuel consumption for power production of unit g
$\beta_{th,g}$	Marginal fuel consumption for heat production of unit g
$\beta_{0,g}$	Fuel consumption at minimum output of unit g, (MW)
r_g	Power-to-heat ratio of unit g
$r_{min,g}$	Minimum power-to-heat ratio of unit g
η_g	Efficiency of boiler unit g
$P_{min,g}, P_{max,g}$	Power production limits of unit g, (MW)
$Q_{min,g}, Q_{max,g}$	Heat production limits of unit g, ($\mathrm{MW_{th}}$)
V_{max}	Heat storage capacity, (MWh/h)
UT_g	Minimum up time of unit g, (h)
DT_g	Minimum down time of unit g, (h)
u_g^0, y_g^0, z_g^0	Initial state of binary variables $u_{gt\omega}$, $y_{gt\omega}$ and $z_{gt\omega}$
$T_{up,g}^0$	Time periods of unit$g$ has been on in the beginning of the planning horizon, (h)
$T_{down,g}^0$	Time periods of unit$g$ has been off in the beginning of the planning horizon, (h)

Variables

$P_{gt\omega}$	Power produced by unit g in period t and scenario ω, (MW)
$Q_{gt\omega}$	Heat produced by unit g in period t and scenario ω, ($\mathrm{MW_{th}}$)
$P_{fuel,gt\omega}$	Fuel consumption of unit g in period t and scenario ω, (MWh/h)
$V_{t\omega}$	Heat storage content in period t and scenario ω, (MW)
$u_{gt\omega}$	Binary variable for the on/off status of unit g in period t and scenario ω
$y_{gt\omega}$	Binary variable for the start-up of unit g in period t and scenario ω
$z_{gt\omega}$	Binary variable for the shut down of unit g in period t and scenario ω

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