Acoustic mixing in microfluidic chip using a Langevin transducer

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Abstract

This thesis aims to develop a new ultrasonic mixer for mixing in plastic microfluidic chips. The mixer is intended to be used for a biological application in the RAPP-ID project, where magnetic beads are mixed in a plastic micro-chamber to facilitate a DNA capture process. The requirements from the RAPP-ID project is the mixer should not permanently integrated with the chip and does not need additional preparation of the chip.

The new mixer consists of a bolt-clamped Langevin-type transducer. The transducer is placed on top of the chip, and has direct contact with the chip without any coupling medium in between. A high power source is used to drive the transducer. We found that the streaming is dependent on the level of contact between the horn and the chip. The streaming is slow, but with an adequate amount of contact, the magnetic beads are dispersed uniformly over the whole chip area within 3 minutes.

Two different designs of the Langevin transducer were evaluated, where the main difference is the profile of the horn. Initially, a transducer with a stepped horn was used. Later, a new transducer with exponential horn was made and used in the experiments. Finite element analysis was used to simulate the Langevin transducer and find its resonance frequencies. The dimensions of the transducer with exponential horn was adjusted until the fundamental resonance frequency matched the output signal from a driver board used in the experiments.

The main problem with the initial experimental setup is high heat generation in the transducer with the stepped horn. Another problem is that the transducer fixture is not rigid enough to hold the transducer. A new exponential horn and a new fixture for the transducer were designed to rectify these problems. Experiments with the new design showed that the heat generation in the transducer with exponential horn was reduced compared to the heat generated in the transducer with the stepped horn. The temperature of the exponential horn tip reached 27 °C after a 1 min drive, while the stepped horn tip had a temperature of 50 °C.
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1. Introduction

This thesis covers the design and application of a Langevin transducer in microfluidic mixing. The work was carried out as a Master’s Thesis in the Biomedical and X-ray Physics group at The Royal Institute of Technology.

1.1 Background and Motivation

The process of fluid mixing has a wide variety of applications in chemistry and biology, on both a large scale and a small scale. Large scale mixing is often used in industries, while small scale mixing is often used for analysis or research and can be achieved by mixing fluids in microfluidic devices. The dimensions of the flow channels in microfluidic systems typically have a large surface to volume ratio, which leads to a low specific Reynolds number. In such low Reynolds number regimes, mixing of the species is largely determined by diffusion, which is an inherently slow process. Microfluidic mixing schemes can be divided into two categories: passive and active. Passive mixers are designed to enhance the diffusion effect between the species, by increasing the contact area and/or the contact time of the multiple species. They achieve this by designing the microfluidic device itself in such a way. Examples of microfluidic devices where the species are mixed passively are: Luer system or a serpentine channel. Active mixers apply an external force in order to perturb the fluid. Among active mixers, there are some techniques which uses a magnetic rod to stir the fluid sample. The rod is in contact with the species. There are also non-contact methods where the device applying the external force is not physically in contact with the fluid.

Acoustic and ultrasonic techniques have been used in the development of active mixers. Ultrasound is known to generate streaming effects in fluids, and it can be applied to a microfluidic device for the purpose of mixing. One mixer design presented by Yaralioglu et al. [1] uses a simple microfluidic channel with embedded piezoelectric transducers which operate at 450 MHz. The channel uses continuous flow, where the fluids start to mix through diffusion without any external forces. Then, ultrasound is applied to increase the mixing speed. Another mixer design presented by Ahmed et al. [2] uses ultrasound to excite a trapped air bubble, which then generates microstreaming in a local area inside a microfluidic channel.
1.2 Goal and Objectives

The aim of this project is to develop a new ultrasonic mixer for plastic microfluidic chips. We propose a new setup which does not require any permanent integration with the plastic chips, since they are for single-use and should be easily disposable. We will try to induce streaming in microchips where the fluid volume is stationary, and there is no mixing effect from a continuous laminar flow.

The experiments has been done in collaboration with the RAPP-ID project, which aims to design a lab-on-a-chip system for circle-to-circle amplification (C2CA). The project imposes some requirements on the mixer design, such as mixing under a stationary flow and easy integration with a microfluidic device.

1.3 Outline

This thesis is divided into 5 chapters. Chapter 1 describes previous work and gives motivation for the work performed in this thesis. Chapter 2 presents the theory and design of a Langevin transducer. Chapter 3 describes the simulations of the Langevin transducer. Chapter 4 describes the experimental setup and presents the experimental results. Finally, Chapter 5 provides some discussion, summarizes the main conclusions of this thesis and presents an outlook for future work.
2. Theory and design of Langevin transducers

Langevin (bolt-clamped) transducers were invented originally for underwater (sonar) applications. A Langevin transducer is a composite resonator, and consists of a stack of piezoelectric elements sandwiched between two metal blocks. This is why it is also called sandwich transducer. We will refer to the front metal block as the horn, and the back metal block as the reflector.

The cross-section of the piezoelectric stack is typically ring-shaped. This allows the piezoelectric stack to be clamped together between the metal blocks by using a compression bolt. The inner radius of a piezoelectric ring is slightly larger than the nominal diameter of the compression bolt. This allows a piece of insulator, typically a plastic sheet, to be inserted into the space between the piezoelectric stack and the bolt. It is important that there is an insulator, which prevents the electrodes from a short-circuit. The electrode between the two piezoelectric rings is connected to the positive potential, while the two other electrodes is connected to the ground. Figure 2.1 is a schematic of a conventional Langevin transducer.

![Figure 2.1: Schematic of conventional Langevin transducer](image)

The total length of the transducer should be exactly $\lambda/2$, or a whole coefficient of $\lambda/2$, such that it can operate in resonance. The front horn is usually made of aluminum or titanium,
for durability, small material losses and acoustic impedance matching purposes. The reflector and the compression bolt is typically made of stainless steel. The main rule of thumb is that the acoustic impedance of the materials on the back end of the piezoelectric stack should be much higher than the acoustic impedances on the front end of the piezoelectric stack, so that the majority of the acoustic energy is directed towards the front, and guided through the horn. The electrodes are usually made of a copper alloy.

The pre-stress applied on the compression bolt (see Figure 2.1) has to have a certain torque in order to achieve a high electromechanical coupling. Normally, the application of an electrical field on a dielectric material causes electric dipole moments to align in the solid, which creates a polarization. However, piezo ceramics have a compression load limit, which means that if the applied torque is too high, then the piezo ceramics will be damaged and depolarization may occur. Depolarization means that the piezoelectric effect will be weaker. A typical piezo ceramic material can withstand pressures up to 250 MPa.

In the technical data sheets provided by Tohnichi Mfg. Co., Ltd. [3], the relationship between the torque and tension of a bolt is given by

\[ T = F_f \cdot d \cdot K \]  \hspace{1cm} (2.1)

where \( T \) is the torque [Nm], \( F_f \) is the axial force [N] from the bolt (preload), \( d \) is the nominal diameter [mm] and \( K \) is the torque coefficient. The formula for torque coefficient can also be found in Tohnichi Mfg. Co., Ltd. [3]. A typical value for the torque coefficient varies from 0.15 to 0.3.

### 2.1 Basic piezoelectric model

Piezoelectricity is the combined effect of the electrical and mechanical properties (Hooke’s law) of the material. Thus, the constitutive equations for a piezoelectric element are given by

\[ D_i = \varepsilon_{ij} \cdot E_j, \hspace{0.5cm} i, j = 1, 2, 3 \]  \hspace{1cm} (2.2)

where \( D \) is the electric charge density displacement, \( \varepsilon \) is permittivity and \( E \) is electric field strength and

\[ S_{ij} = s_{ijkl} \cdot T_{kl}, \hspace{0.5cm} i, j, k, l = 1, 2, 3 \]  \hspace{1cm} (2.3)

where \( S \) is strain, \( s \) is compliance and \( T \) is stress.

(2.2) and (2.3) can be combined into coupled equations containing \( D, E, S \) and \( T \). (2.4) and (2.5) represents the strain-charge form, given by Sherman and Butler [4, p. 35]. Another way to write the coupled equations is the stress-charge form (see section A.1).

\[ S = s^E \cdot T + d^e \cdot E \Leftrightarrow S_{ij} = s^i_{jkl} T_{kl} + d_{kij} E_k, \hspace{0.5cm} i, j, k, l = 1, 2, 3 \]  \hspace{1cm} (2.4)

\[ D = d \cdot T + \varepsilon^T \cdot E \Leftrightarrow D_i = d_{ijk} T_{jk} + \varepsilon^T_{ij} E_j, \hspace{0.5cm} i, j, k = 1, 2, 3 \]  \hspace{1cm} (2.5)
\( \mathbf{d} \) is the coupling matrix in the strain-charge form and its transpose is \( \mathbf{d}^T \). \( \mathbf{s}^E \) is the compliance under a constant (or zero) electric field, and \( \varepsilon^T \) is permittivity under a constant (or zero) stress field.

A three-dimensional tensor of rank four, such as \( s^E_{ijkl} \) in (2.4), has 81 components. If the piezoelectric material used is transversely isotropic, then the tensors describing the material constants will be symmetric. In this case there will only be 36 distinct components, which means the tensor can be reduced to a 6x6 matrix. Using Voigt notation, equations (2.4) and (2.5) can be reduced to (2.6) and (2.7) by re-labeling the subscripts as shown in Table 2.1.

\[
\begin{array}{cccccc}
ij & 11 & 22 & 33 & 23, 32 & 13, 31 & 12, 21 \\
m, \nu & 1 & 2 & 3 & 4 & 5 & 6 \\
\end{array}
\]

Table 2.1: Voigt notation

\[
S_m = s^E_{mn} T_{m} + d_{km} E_k, \quad m, \nu = 1,2, \ldots , 6 \quad k = 1,2,3 \\
D_i = d_{im} T_{m} + \varepsilon^T_{ij} E_j, \quad m = 1,2, \ldots , 6 \quad i, j = 1,2,3
\]

Furthermore, for permanently polarized electrostrictive materials, many of the coefficients in (2.6) and (2.7) are zero (Sherman and Butler [4, p.36]). The plane of isotropy is defined here as the 12-plane (xy-plane). It follows that the poling axis of the material is the 3-axis (z-axis).

If we were to expand (2.6) and (2.7) for a material such as PZT, we would get the following:

\[
\begin{bmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{bmatrix} = \begin{bmatrix}
s^E_{11} & s^E_{12} & s^E_{13} & 0 & 0 & 0 \\
s^E_{21} & s^E_{22} & s^E_{23} & 0 & 0 & 0 \\
s^E_{31} & s^E_{32} & s^E_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & s^E_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & s^E_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & s^E_{66}
\end{bmatrix} \begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix} + \begin{bmatrix}
0 & 0 & d_{31} \\
0 & 0 & d_{32} \\
0 & 0 & d_{33} \\
0 & d_{24} & 0 \\
d_{15} & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_1 \\
D_2 \\
D_3
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 0 & 0 & d_{15} & 0 \\
0 & 0 & 0 & d_{24} & 0 & 0 \\
d_{31} & d_{32} & d_{33} & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_5 \\
T_6
\end{bmatrix} + \begin{bmatrix}
\varepsilon^T_{11} & 0 & 0 \\
0 & \varepsilon^T_{22} & 0 \\
0 & 0 & \varepsilon^T_{33}
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2 \\
E_3
\end{bmatrix}
\]

The subscripts 4,5 and 6 represent engineering shear strains around the 1-axis, 2-axis and 3-axis respectively (or x, y and z axes). Due to symmetry, we have

\[
s_{ij} = s_{ji} \forall i,j
\]
Further, because of transverse isotropy, we have

\[ s_{11} = s_{22} \]
\[ s_{13} = s_{23} \]
\[ s_{44} = s_{55} \]
\[ d_{31} = d_{32} \]
\[ d_{24} = d_{15} \]
\[ \varepsilon_{11} = \varepsilon_{22} \]

2.2 Design choices

Typically, a Langevin transducer is designed to have a certain resonance frequency. Normally, a Langevin transducer has a low slenderness ratio, i.e. its lateral dimensions is much smaller than its longitudinal dimensions, as this ensures that it will be operating in a longitudinal mode. In this case, one-dimensional theory for a long, thin bar can be used to accurately predict the resonance frequency. However, if the lateral dimensions exceed a quarter of the longitudinal wavelength, then one-dimensional theory will not be accurate in predicting the resonance frequency. In this case, the vibration of the transducer is a coupled one of longitudinal and lateral vibrations [5].

We will be using a piezoelectric element from a high-power Langevin transducer, which usually has a large cross-section. Additionally, the higher the driving frequency is, the smaller the wavelength will be, which means the lateral dimension has to be smaller to not produce lateral modes of vibrations.

Initially, a transducer with a stepped horn was used. Experiments were done with the stepped horn to evaluate the transducer, the driving board, the effects of streaming in microfluidic chips and the rest of the experimental setup. Then, a new design was proposed to try and improve the setup and rectify any potential issues. Part of the new design consists of using a horn with an exponential profile instead of the stepped profile. CAD drawings of the stepped horn can be found in section A.3. The new design including the exponential horn is shown in sections A.4 to A.8.

The transducer will be operated with a driver board with an output of 28 kHz. The new design will make changes mainly to the horn, and the rest of the transducer will remain the same. The stepped horn has a resonance frequency near 24 kHz, which means the new horn will have to be made a little shorter to obtain a resonance frequency at 28 kHz. The length of the horn is determined by finite-element analysis in chapter 3.

The material of the horn was changed from titanium (old design) to aluminum (new design), because of better acoustic impedance matching. In the new design, we have also chosen to add a supporting flange to the horn, which allows the transducer to be mounted to a fixture.
2.2.1 Piezoelectric stack

The piezoelectric rings used in the Langevin transducer should be polarized in the thickness direction. [6] provides some relationships used in transducer design to determine the axial dimensions of the piezoelectric stack, horn and reflector.

The piezoelectric element used in our transducer was obtained from an existing Langevin transducer. The materials used to build the original transducer is unknown. However, we do know that the transducer is used for ultrasonic cleaning applications since the accompanying driver board has a high power output. Ultrasonic cleaning uses high intensity, which is why we have assumed that the type of piezoelectric material is for high voltage applications.

These types of piezoelectric materials have a high mechanical quality factor and low dissipation. Later for the simulations in chapter 3, we use high-voltage piezoelectric material called Pz26. Some theoretical calculations will use material properties provided by the manufacturer of Pz26, to illustrate the relations between the different components, which could be considered in future designs where a specific piezoelectric material is chosen to build the transducer.

The piezoelectric rings used in our Langevin transducer has an outer diameter of 45 mm, an inner diameter of 15 mm and a height of 5.25 mm. There are two piezoelectric rings in the stack so the total height is 10.5 mm.

2.2.2 Horn

In high-intensity applications, the main purpose of the horn is to provide a high amplitude vibration at the output with a small vibration as input. The horn acts as a waveguide, and by using a tapered structure it is possible to concentrate the waves and amplify the vibration at the tip of the horn.

A transducer is always vibrating in the longitudinal mode at its fundamental (first) resonance frequency. However, the first resonance mode may not generate the largest vibration amplitude. The transducer designed by Li et al. [11] has its largest vibration amplitude at the second resonance mode.

Stepped profile

A common shape of the front horn is the stepped profile. The horn consists of two sections: the first section has the same cross-section as the piezoelectric stack, while the second section (positioned towards the load) has a smaller cross-section.

The stress concentration is very high at the step discontinuity. If the horn is not driven at its resonance frequency, then there will be an increase in thermal losses, due to the stress-strain relationship. It is possible to add a fillet to the step in order to reduce the stress around the step. This will also reduce the amount of thermal losses.
The stepped horn used in our transducer has a length of 43 mm and an input diameter of 45 mm (the diameter of the first section, next to the piezoelectric stack). It is made of titanium (Grade 2). The velocity of longitudinal wave in titanium is 4825 m/s (Table 2.2), and this means a quarter wavelength at a frequency of 28 kHz is 43 mm, which is approximately equal to the input diameter of the stepped horn at 45 mm. If a large part of the horn has a cross-section larger than a quarter wavelength, then there may be lateral vibrations in the transducers.

The output from the ultrasonic driver board is typically not very clean, i.e. there are multiple frequencies. This is also the case for the driver board used in our experiments (see section 4.5). There are frequencies up to 80 kHz in the output. In this case it might be better to have a thinner profile for the front horn, even if the amplification factor may not be large at the fundamental resonance frequency. However, if there are lateral vibrations coupled with longitudinal vibrations at a given frequency, this would mean that the amplification factor (or output at the tip of the horn) would be smaller.

For the reasons given above, a choice was made to design a new horn with an exponentially tapered profile.

**Exponentially tapered profile**

The profile of an exponential horn can be described as

\[ S_1 = S_0 \cdot e^{-\gamma x} \quad (2.8) \]

where \( S_0 \) is the cross-sectional area of the surface at \( x = 0 \), \( S_1 \) is the cross-sectional area of the surface at \( x = L \) and \( \gamma \) is the taper factor (or flare constant) of the exponential profile.

Inserting \( x = L \) into Equation 2.8, we get

\[ \gamma = \frac{\ln \frac{S_0}{S_1}}{L} \]

The specific form of the wave equation for a horn, given by Ensminger and Bond [12, p.141], is

\[ \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} - \frac{1}{S} \frac{\partial S}{\partial x} \frac{\partial \xi}{\partial x} - \frac{\partial^2 \xi}{\partial x^2} = 0 \]

where \( \xi \) is the particle displacement. Using Equation 2.8 and assuming a harmonic motion \( \frac{\partial \xi}{\partial t} = v = j\xi\omega \), we get the differential equation for the exponential horn:

\[ \frac{\partial^2 v}{\partial x^2} - \gamma \frac{\partial v}{\partial x} + \frac{\omega^2}{c^2} v = 0 \]

where \( v \) is the particle velocity. The solution to the differential equation [12, p.145] is

\[ v = e^{\gamma x/2} k_1 \cos \sqrt{\frac{\omega^2}{c^2} - \frac{\gamma^2}{4}} x + k_2 \sin \sqrt{\frac{\omega^2}{c^2} - \frac{\gamma^2}{4}} x \]
To obtain a wave motion, the frequency has to satisfy the following relation:

\[
\frac{\omega^2}{c^2} - \frac{\gamma^2}{4} > 0
\]

This gives us a cut-off frequency of

\[f_c > \frac{\gamma c}{4\pi}\]

where \(c\) is the longitudinal velocity of the horn.

If the input frequency is below the cut-off frequency, the exponential horn will transmit nothing as its impedance is purely reactive, and the acoustic waves will be reflected [13]. It is important that the input frequency is higher than the cut-off frequency. This imposes a limit on how small the tip of the horn can be. The smaller the output area is compared to the input area, the higher the value of gamma will be, which means the cut-off frequency will be higher. We have chosen to keep the same output area as the stepped horn, since the effect of the streaming may change depending on the coupling area between the tip and the chip.

The horn is connected to the rest of the transducer by inserting the steel bolt to a mounting hole in the horn. In this case, any horn needs to have a straight section initially before tapering off. This is to avoid the waves reflecting against the steel bolt in the center when the profile starts to taper off.

In our design of the exponential horn, starting from the interface between the piezoelectric element and the front horn, the horn has a straight section of 13 mm, then a 65 mm long exponential taper from a radius of 22.5 mm to 3.5 mm, and finally another straight section of 5 mm. This gives us a gamma of

\[
\gamma = \frac{\ln \frac{0.0225^2}{0.065}}{0.065} = 57.25
\]

We will see later in the frequency sweep that the transducer with our exponential horn has a resonance frequency close to 28 kHz. We will now check the cut-off frequency of the horn to make sure that we can drive the transducer at its fundamental resonance frequency.

The manufactured exponential horn is made of aluminium 6082. For this material, we have a modulus of elasticity of 70 GPa, and a density of 2700 kg/m³ [9]. This gives us a longitudinal speed of

\[c = 5091.8 \text{ m/s}\]

and a cut-off frequency of

\[f_c = 23.2 \text{ kHz}\]

The cut-off frequency is below 28 kHz, which means that we will be able to drive the horn at its fundamental resonance frequency.
2.2.3 Reflector

The reflector should be made of a material with high density and high elasticity. Such a material will have a high acoustic impedance which causes most of the acoustic waves to be directed towards the front horn. The most common shape is a cylinder with the same diameter as the piezoelectric stack.

The reflector can also be designed with some kind of tapered shape. The reason for doing this is to make most of the acoustic energy transmit to the horn. The law of mass conservation implies that by reducing the mass of the tail, less acoustic energy will be used to create the mechanical vibrations in the reflector.

2.2.4 Supporting flange

A flange can be a plate or rim which extends outwards from an object, and is commonly found at the end of a pipe and used to fasten two pipe sections together. In our case, a flange was added to the exponential horn so that the transducer can be fixated to a supporting structure. The flange also helps to align the surfaces between the horn and the microfluidic chip such that there are as few air gaps as possible, which would otherwise cause reflection of the ultrasound.

A drawing of the exponential horn can be found in section A.5 and the assembled transducer in section A.4. The flange is first fastened to a hollow cylinder, which is a casing and encapsulates the piezoelectric stack and the reflector. Then, the cylinder is fixated to a larger aluminum plate, which makes the transducer more rigid during operation. The flange is manufactured as an integrated piece of the exponential horn, as opposed to a separate disc inserted between the horn and the piezoelectric rings. A fewer number of separate surfaces in the transducer should result in less coupling loss.

The cylinder is made of plastic so that the rest of the setup is electrically isolated from the transducer. A hollow plastic cylinder needs to have a certain thickness to prevent it from breaking, especially when mounting screw holes are drilled into the cylinder. This is why the cylinder has a large thickness of 15 mm. The diameter of the flange is 110 cm, which is much larger than the piezoelectric stack with a diameter of 45 mm. The extra space on the perimeter is used to accommodate the plastic cylinder. Also, the transducer we used had fairly large electrode connectors sticking out from the side, so there needs to be extra space between the piezoelectric stack and the casing.

The lateral dimensions of the transducer parts are normally less than a quarter of wavelength to avoid coupling between the longitudinal and lateral modes. If a circular flange is added, there may be radial resonances. Simulations show that there is a radial resonance at 19 kHz (see ??). One way to reduce the lateral vibrations at higher frequencies is to add perforated holes around the flange. The flange used in the manufactured exponential horn does not have any perforated holes.
2.2.5 Material

The acoustic impedance of the different parts of the Langevin transducer has to be considered to provide an efficient transmission of acoustic energy. Ideally, the following equation [6] should be satisfied:

\[ Z_{\text{piezo}} = \sqrt{Z_{\text{horn}} \cdot Z_{\text{refl}}} \]  (2.9)

where \( Z_{\text{piezo}} \), \( Z_{\text{horn}} \) and \( Z_{\text{refl}} \) are the specific acoustic impedance of the piezoelectric material, horn and reflector respectively. The specific acoustic impedance of a material is calculated as

\[ Z = \rho \cdot c \]

where \( \rho \) is the density and \( c \) is the acoustic velocity.

Table 2.2 shows some properties for the materials used in the transducer [7, 8, 9, 10]. The velocity of longitudinal waves have been calculated for a thin long rod, \( c = \sqrt{Y / \rho} \), where \( Y \) is the modulus of elasticity and \( \rho \) is the density.

We will begin by estimating the acoustic velocity of the piezoelectric material used in our transducer. The datasheet for Pz26 has the following values of modulus of elasticity [7].

\[
\begin{align*}
Y_{11}^E & = 76.93 \text{ GPa} \\
Y_{33}^E & = 50.92 \text{ GPa} \\
Y_{11}^D & = 86.16 \text{ GPa} \\
Y_{33}^D & = 95.61 \text{ GPa}
\end{align*}
\]

The superscript E means that \( Y \) is measured under a constant electric field, or in other words, it is measured while the piezoelectric element is short-circuited (closed circuit). The superscript D means that \( Y \) is measured while the piezoelectric element is in an open circuit. Naturally, a open circuit means that there is additional strain of the piezoelectric element due to the generated electric field, which is why the modulus of elasticity is higher. The subscript 33 represents the axis of polarization of the piezoelectric material, which in our case is in the same direction as the longitudinal mode of the transducer.

For a piezoelectric element, we should use \( Y_{33}^E \) to calculate the acoustic longitudinal velocity, otherwise there will be an additional bias as mentioned above. With \( Y_{33}^E = 50.92 \text{ GPa} \) and \( \rho = 7700 \text{ kg/m}^3 \), we get a longitudinal speed of

\[ c_{33,pz26} = 2572 \text{ m/s} \]

The specific acoustic impedance for the different materials is also shown in Table 2.2.
The same piezoelectric material was used in the two transducer designs. The left-hand side of (2.9) is equal to 19.8 MRayls. For the transducer with stepped horn, the right-hand side is equal to
\[ \sqrt{Z_{\text{titanium}} \cdot Z_{\text{steel}}} = 29.6 \text{ MRayls} \]

The corresponding value for the exponential horn is
\[ \sqrt{Z_{\text{aluminum}} \cdot Z_{\text{steel}}} = 23.4 \text{ MRayls} \]

The right-hand side value for the new transducer design is closer to the left-hand side value compared to the transducer with stepped horn. This means that a transducer with a horn made of aluminum has a higher acoustic transmission efficiency than a titanium horn, all other things being equal.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density [kg/m³]</th>
<th>Velocity of longitudinal waves [m/s]</th>
<th>Specific acoustic impedance [MRayls]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel AISI 4340</td>
<td>7850</td>
<td>5110</td>
<td>40.1</td>
</tr>
<tr>
<td>Pz26</td>
<td>7700</td>
<td>2572</td>
<td>19.8</td>
</tr>
<tr>
<td>Titanium Grade 2</td>
<td>4510</td>
<td>4825</td>
<td>21.8</td>
</tr>
<tr>
<td>Aluminum 6082</td>
<td>2700</td>
<td>5092</td>
<td>13.7</td>
</tr>
</tbody>
</table>

Table 2.2: Material data for different parts of the transducer
3. Simulations

Both equivalent circuit and finite element modeling are widely used in the transducer design and analysis. In this project, we focused on finite element analysis using the software package COMSOL. The software version used is COMSOL 4.3b (build 4.3.2.189). The Langevin transducer is simulated in an isolated environment, in order to find a design which has a resonance frequency matching that of the driver board and efficient transmission of acoustic waves. Ultimately we are interested in observing the streaming patterns in the chip, so in this regard it would be interesting to couple the output of the transducer to a fluid dynamics model of the chip. However, because of time and computation limitations, no fluid dynamic simulations were made. Also, the surrounding mechanical structure would have to be modeled due to fluid solid interaction [14].

3.1 Finite-element model

The geometry was drawn in a 2D axisymmetric plane (Figure 3.1a) and then revolved 360 degrees around the z-axis to generate a 3D model. It would have been possible to solve the FEM with only a 2D model and utilizing the symmetry around the z-axis, in order to reduce the amount of calculations. However, a 2D model would not capture any bending modes of the transducer.

We are interested in finding the resonance frequencies of the transducer. The resonance frequencies can be determined by calculating the electrical impedance over a given frequency range. The electrical impedance $Z$ of a piezoelectric ring can be formulated by the general Ohm law:

$$Z = \frac{V}{I}$$

$$I = \int_0^r j_z(r) 2\pi r \, dr$$

$$Y = \frac{1}{Z}$$

where $V$ is the voltage across the 2 surfaces of the piezoelectric ring, $I$ is the current flowing across the surface and $j_z$ is the current density component along the z-axis and $Y$ is the admittance.

To model the Langevin transducer, we use the Physics module Acoustic-Piezoelectric Interaction, Frequency Domain (acpz) (found under Acoustics → Acoustic-Structure Interaction). The
frequency sweep is performed using the Study Type called *Frequency Domain*. The Physics module contains the built-in variable `acpz.Y11`, which gives the admittance of the transducer (`acpz.Y11` may not exist in earlier COMSOL versions than the version used for this project). In order to simulate an impedance analysis and then compare with measurements, we define two variables with the expressions `real(acpz.Y11)` and `imag(acpz.Y11)`, which gives us the real part (conductance) and the imaginary part (susceptance) of the admittance. Furthermore, a damping property needs to be added to obtain a non-zero conductance over the frequency domain.

### 3.1.1 Geometry of stepped horn

The stepped horn was made of an unalloyed titanium (Grade 2). Table 3.1 shows the materials used in COMSOL for the respective domains. Figure 3.1a shows the half cross-section for $x > 0$, while Figure 3.1b shows a 3D model of the transducer with a stepped horn.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Transducer part</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reflector</td>
<td>Steel AISI 4340</td>
</tr>
<tr>
<td>2</td>
<td>Bolt</td>
<td>Steel AISI 4340</td>
</tr>
<tr>
<td>3</td>
<td>Upper piezo ring</td>
<td>Pz26</td>
</tr>
<tr>
<td>4</td>
<td>Lower piezo ring</td>
<td>Pz26</td>
</tr>
<tr>
<td>5</td>
<td>Stepped horn</td>
<td>Ti Grade 2 (UNS R50400) [solid,oxidized]</td>
</tr>
</tbody>
</table>

Table 3.1: FEM domains of stepped horn
3.1.2 Geometry of exponential horn

The material used to manufacture the exponential horn is the aluminum alloy 6082. The exact same aluminum alloy was not found in COMSOL. Instead, aluminum alloy 6063-T83 was used, which should be very similar to the alloy used in the real horn. Table 3.2 shows the material used for the respective domains. Figure 3.2a shows the half cross-section for x > 0, while Figure 3.2b shows a 3D model of the transducer with the exponential horn, which has a flange.

![Half cross-section](image)

![3D model](image)

Figure 3.2: FEM model of exponential horn with flange

<table>
<thead>
<tr>
<th>Domain</th>
<th>Transducer part</th>
<th>Property</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Reflector</td>
<td>Linear Elastic Material</td>
<td>Steel AISI 4340</td>
</tr>
<tr>
<td>2</td>
<td>Bolt</td>
<td>Linear Elastic Material</td>
<td>Steel AISI 4340</td>
</tr>
<tr>
<td>3</td>
<td>Upper piezo ring</td>
<td>Piezoelectric Material</td>
<td>Pz26</td>
</tr>
<tr>
<td>4</td>
<td>Lower piezo ring</td>
<td>Piezoelectric Material</td>
<td>Pz26</td>
</tr>
<tr>
<td>5</td>
<td>Exponential horn with flange</td>
<td>Linear Elastic Material</td>
<td>Aluminum 6063-T83</td>
</tr>
</tbody>
</table>

Table 3.2: FEM domains of exponential horn

3.1.3 Boundary conditions

The same boundary conditions were used for both models containing the stepped and exponential horn respectively. Table 3.3 shows the defined boundary conditions. The electrodes should have the electric potential as shown in Figure 2.1.
The pre-stress load was not applied as any boundary condition. This means that we assume the mechanical coupling is perfect between the horn and the piezoelectric stack.

One thing to note is that since the chosen physics is in the Frequency Domain, this means that the boundary values are harmonic (periodic) by default. For example, if the input voltage is defined as 5 V, then the actual value will be $5 \sin \omega t$. There is no need to explicitly define the voltage as a trigonometric function.

3.1.4 Mesh
The default tetrahedral element was used for meshing the whole model. A rule of thumb is to use an element size which gives 5-10 elements per wavelength in each material, in order for the FEM solution to converge. When performing the impedance analysis in COMSOL, we will be solving for a range of frequencies. If a Langevin transducer needs to be designed for a very high resonance frequency in the megahertz range, then the number of elements will also be very large. In this case it may be a good idea to use other element types where suitable to reduce the number of elements. However, in our case, we are performing calculations in the kHz range so it is not necessary to change the default.

3.2 Piezoelectric element
In our simulations, we have used material constants from a lead zirconate titanate (PZT) material called Pz26. This material is suitable for high voltage applications [16]. The complete set of material constants for Pz26 can be found in a spreadsheet provided by Ferroperm Piezoceramics A/S [7].

3.2.1 Material constants
In the version of COMSOL used for this project, the piezoelectric material constants could be defined in either the strain-charge form or the stress-charge form. The ordering of the variables in COMSOL follows the Voigt notation. If another ordering was used, we would have to rearrange the material constants below.
We have used the strain-charge form (2.6) and (2.7), which requires the compliance matrix \(s\) (the inverse of the elastic stiffness matrix), the coupling matrix \(d\) and the relative permittivity constants under constant stress. The compliance constants for PZ26 are shown below.

\[
\begin{bmatrix}
    s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\
    s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\
    s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\
    0 & 0 & 0 & s_{44}^E & 0 & 0 \\
    0 & 0 & 0 & s_{55}^E & 0 & 0 \\
    0 & 0 & 0 & 0 & s_{66}^E & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
    13.0 & -4.35 & -7.05 & 0 & 0 & 0 \\
    -4.35 & 13.0 & -7.05 & 0 & 0 & 0 \\
    -7.05 & -7.05 & 19.6 & 0 & 0 & 0 \\
    0 & 0 & 0 & 33.2 & 0 & 0 \\
    0 & 0 & 0 & 0 & 33.2 & 0 \\
    0 & 0 & 0 & 0 & 0 & 34.7 \\
\end{bmatrix} \cdot 10^{-12} \left[ \text{m}^2/\text{N} \right]
\]

The values for the coupling matrix \(d\) is shown below.

\[
\begin{bmatrix}
    0 & 0 & 0 & 0 & d_{15} & 0 \\
    0 & 0 & 0 & d_{24} & 0 & 0 \\
    d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \\
\end{bmatrix}
= 
\begin{bmatrix}
    0 & 0 & 0 & 0 & 3.27 & 0 \\
    0 & 0 & 0 & 3.27 & 0 & 0 \\
    -1.28 & -1.28 & 3.28 & 0 & 0 & 0 \\
\end{bmatrix} \cdot 10^{-10} \left[ \text{C}/\text{N} \right]
\]

The relative permittivity constants under constant stress are

\[
\varepsilon_{11,r}^T = 1193 \\
\varepsilon_{33,r}^T = 1326
\]

The polarization direction of the piezoelectric material as given by the matrices above is set along the z-axis. In the FEM model, the upper and lower piezoelectric rings are defined as 2 separate entities, where the upper piezoelectric ring (domain 3) has a rotated coordinate system with the z-axis pointing in the opposite direction of the global coordinate system. This is because the material constant matrix defined in subsection 3.2.1 is for a piezoelectric element polarized along the z-axis.

### 3.2.2 Material losses

Losses for the piezoelectric material are included in the FEM model by adding Damping and Loss under the entities Piezoelectric Material. Structural damping can be described by the Rayleigh equation, where the damping coefficient is

\[
c = \alpha m + \beta k
\]

where \(\alpha\) is the mass multiplication factor and \(\beta\) is the stiffness multiplication factor. Normally, viscous damping occurs in piezoelectric transducer vibration at low kHz range [15]. For viscous damping we have \(\alpha = 0\) and \(\beta = \frac{1}{\omega Q_m}\).

With a value of \(Q_m = 100\) (see Figure 3.3), the damping value at 28 kHz is \(\beta = 5.7 \cdot 10^{-8}\).
Without structural damping, the conductance of the transducer cannot be measured. The electrical analog of structural damping is resistance, and if there is no resistance in the circuit then the conductance is zero.

Ferroperm’s datasheet also gives the dielectric dissipation factor, which is presented as

\[ \tan \delta^E = 0.003 \]

The loss factor represents the ratio of conductance to susceptance of a parallel equivalent circuit of the ceramic element [16]. \( \delta \) refers to the angle between the resistive (lossy) component of an electromagnetic field and its reactive (lossless) component in a phasor diagram. \( \tan \delta \) is also the reciprocal to the quality factor, which means that the quality factor is larger than 3000. A quality factor above 1000 is considered to be very good, where only a small fraction of the applied energy is lost. However, keep in mind that if the applied power is very high, then the total energy loss and generated heat may still be significant even if the dielectric loss factor is small. Even though that the loss factor is only given in the 3-axis direction, we can assume that the loss is uniform. Thus, the electrical damping for Pz26 was set to an isotropic loss factor of 0.003.

### 3.3 Simulation results

The plots of the simulated electrical admittance curves in the frequency domain can be found in subsection 3.3.1, where they are compared to real measurements. Furthermore, in subsection 3.3.2 and subsection 3.3.3 we look at the total displacement of the transducer.

#### 3.3.1 Electrical admittance measurements

A transducer should be operated at one of its resonance frequencies, in order to maximize its vibration amplitude and minimize the heat generated due to energy losses. An admittance analyzer (Z-Check 16777 k, SinePhase, Austria) was used to identify the resonance peaks. The admittance curves are measured under an output load of air medium. If the transducer is oper-
ated in water, its resonance will decrease because of the increased acoustic impedance. However, the acoustic impedance of a plastic microfluidic chip can be considered to be negligible, so the measurements should match the resonance of the transducer during operation.

Initially, there was a decision to be made regarding the amount of pre-stress to be applied (see section 4.1). Figure 3.4 shows the conductance (real part of admittance) from 1 kHz to 100 kHz for three different values of torque: 30, 50 and 70 Nm. We can see that the fundamental resonance frequency increases with the amount of torque, as well as the conductance for each resonance peak. A torque of 70 Nm gives a good coupling between the piezoelectric element and the horn.

![Figure 3.4: Measured conductance of stepped horn with different pre-stress values](image)

We also compare simulated values of the conductance with measured values for the two horn profiles in Figure 3.5. The frequency range is from 22 kHz to 65 kHz, and a stepsize of 100 Hz is used for the measurements, while a stepsize of 25 Hz is used for the simulations. The resonance frequencies are largely determined by the geometries of the transducers, which is why it is important to simulate the whole transducer.
Figure 3.5: Admittance curves for the two different horns used in the experiments

The first resonance peak for both horns are quite accurate when comparing the measurements.
and simulations. The measured fundamental resonance frequency for the stepped horn is 24000 Hz, while the simulated frequency is 23850 Hz. As for the exponential horn, the measured fundamental resonance frequency is 28100 Hz, while the simulated frequency is 28150 Hz.

The stepped horn was manufactured before simulating an impedance analysis. However, in order to use the driver board, a resonance frequency of 28 kHz was desired. The design of the new exponential horn was first made in Comsol. The length of the exponential horn was adjusted until the fundamental resonance frequency became close to 28 kHz.

Different piezo material constants may give slightly different resonance frequencies of the transducer. This is one possible reason why the results deviates from the measurements, since the piezoelectric material used in the measurements is unknown. Also, the simulations do not account for any coupling loss between the different interfaces, which means that the magnitude of a resonance peak is likely to be higher in the simulations compared to the measurements. This may be why some higher order resonance peaks show up in the simulations but not in the measurements.

3.3.2 Displacement of transducer with stepped horn

The figures below show the total displacement of the transducer with stepped horn at different resonances, when it is driven by an input voltage of 1 V\textsubscript{rms}. The total displacement includes both longitudinal and lateral vibrations. The deformation due to the displacement is visible in the 3D model. The resonance frequency is shown at the top of each figure.

![Figure 3.6: Total displacement of 1st resonance](image)
Figure 3.7: Total displacement of 2nd resonance

Figure 3.8: Total displacement of 3rd resonance
Figure 3.6 shows the first resonance of the transducer, and we can see that it is operating in the longitudinal mode. The total displacement at the tip of the transducer is on the order of $10^{-6}$ m. Figure 3.7 shows the second resonance, and here the order of magnitude of the tip displacement is $10^{-8}$ m. The displacement at the tip is less compared with the first resonance, even though the conductance is higher. In Figure 3.8 and Figure 3.9, we can see the lateral vibrations clearly. There is almost no displacement at the tip for these higher order resonances.

### 3.3.3 Displacement of transducer with exponential horn

The figures below show the total displacement of the transducer with exponential horn at different resonances, when it is driven by an input voltage of $1 \text{ V}_{\text{rms}}$. The total displacement includes both longitudinal and lateral vibrations. The deformation due to the displacement is visible in the 3D model. The resonance frequency is shown at the top of each figure.

If the horn has a supporting flange, then it is extremely important that, along the axial dimension of the transducer, there is a displacement node located exactly at the flange [17]. If the axial displacement at the flange is large, then the ultrasonic waves will propagate out toward the flange and into the whole system. This could create undesired resonance frequencies, which would then interfere or change the fundamental resonance frequency of the transducer itself.
Figure 3.10: Total displacement of 1st resonance

Figure 3.11: Total displacement of 2nd resonance
Figure 3.10 shows the first resonance of the transducer, and we can see that it is operating in the longitudinal mode. The total displacement at the tip of the transducer is on the order of $10^{-7}$ m. Figure 3.11 shows the second resonance, and here the order of magnitude of the displacement is also $10^{-7}$ m.

For the given dimensions, there seems to be less lateral vibrations in the transducer with the exponential horn compared with the stepped horn, especially for the higher order resonances.
in Figure 3.12 and Figure 3.13. However, at the second resonance, there are ultrasonic waves propagating along the flange.
4. Mixing experiments

One of the objectives of the RAPP-ID project is to design a lab-on-a-chip system for circle-to-circle amplification (C2CA). One of the steps in C2CA is DNA target capture, and this process will be integrated into the chip. A possible method for capturing DNA strands is to specifically hybridize the targets to magnetic beads (1 µm size) that have been pre-conjugated with capture probes through biotin-avidine chemistry. After target capture the unspecific and unbound DNA is washed away and the target amplified on the bead by rolling circle amplification (RCA) of ligated padlock probes.

The mixer consists of a conventional Langevin transducer. The transducer used in the initial experiments had a stepped horn. Then, more experiments were made with a new transducer design which include an exponential horn.

4.1 Assembly of the transducer

The transducer is assembled by threading the compression bolt through the reflector, piezo stack and finally screwing it into the horn. Then, the compression bolt has to be tightened with a high torque, in order to ensure a good mechanical coupling between the piezo element and the horn. The applied torque used when assembling the transducer is 70 Nm. This value was chosen based on testing 3 different torques (see subsection 3.3.1 and also looking at a reference table from Tohnicchi Mfg. Co., Ltd. [3]. The transducer used in the experiments has a compression bolt with a nominal diameter of 12 mm. For a standard high durability screw joint M12, it can handle a torque of 76 Nm. Using (2.1), and a conservative torque coefficient value of 0.1, the applied torque corresponds to an axial tension of 58.3 kN. The piezoelectric ring has an outer radius of 22.5 mm and an inner radius of 7.5 mm, which means the surface area of the piezo stack is 14.1 cm². This gives us a compressive stress of 41.3 MPa over the piezo stack. The transducer used by Ando and Kagawa [17] has a piezo stack with a height of 9 mm and a diameter of 55 mm. The authors used a compressive stress of 35.7 MPa. The dimensions of these two transducers are similar, which suggests that the compressive stress is on an adequate level.

A clicker type torque wrench was used to apply the torque. These are somewhat more accurate than a beam type torque wrench. However, the accuracy may still be ±20%. When assembling the bolt-clamped transducer, some safety measures will have to be taken. First, there is a limit on the compressive stress of the piezo element, so the applied torque may not be too large. Second, after applying the pre-stress, static charge will be built up in the piezo element.
4.2 Experimental methods

The experiments are conducted by placing a Langevin transducer on top of a plastic microfluidic chip. The chip is filled with a DNA solution, and magnetic beads are placed in the middle with a magnet. The transducer was driven by the provided driver board, at 28 kHz. The magnetic beads are placed in the middle of the chip manually with the help of a small bar magnet, hence forming a round clump initially.

The transducer was first lowered until the tip came into contact with the chip. Then, the transducer was lowered again, which causes the tip to press against the chip, thereby providing a better coupling. The transducer was lowered by a micrometer (positioned in the direction of the z-axis) until the level of contact was reached. The streaming effect is dependent on the level of contact (i.e. how much the tip is pressed against the chip). A larger value means a better contact, since the transducer was pressed down more, reducing the air gap. A source of measurement error is manually reading the level of contact from the micrometer.

4.3 Experimental setup using the stepped horn

Figure 4.1 and Figure 4.2 show the initial setup of the transducer with the stepped horn. It was held in place with a gripper.

Figure 4.1: Transducer with stepped horn held by a gripper
The fine position of the transducer can be adjusted with an xyz-stage. The xyz-stage is fixated on a height-adjustable stand, which has magnetic feet as a base. The xyz-stage can be lowered and raised along the stand together with the transducer. It is important that all of the magnetic feet are turned on during operation because there is a small difference in the positioning of the magnetic foot when it is turned off and on. If one of the magnetic foot is turned off while the other three are turned on, then the xyz-stage will not be in a perfectly horizontal position.

One problem with the setup is that the transducer cannot be fixated rigidly when the gripper is used. The transducer can rotate around the gripping axis as seen in Figure 4.1. When the transducer is pushed at a point far from the gripping axis, for example at the tip of the horn, then it is easy to cause a rotational movement. Another issue is that a rotation would cause the surface of the tip of the transducer to be misaligned (non-parallel) with the surface of the chip. In the case of non-parallel surfaces, small gaps of air will appear between the interfaces, which causes the transmission of ultrasound to decrease significantly due to an increased reflection and a reduced total contact area. The repeatability of the experiments is thus very bad when using the gripper to fixate a long transducer.

The coupling between the transducer and the plastic chip could be increased by adding ultrasonic coupling agent on the surface of the chip. However, since the plastic chips are for single use and should be easily disposable, one of the requirements of the project was that the chips should not require any additional preparation. Thus, no coupling agent was used in the experiments.

The microfluidic chip is placed in a conventional bright-field microscope. A DSLR camera is attached to the microscope lens, which enables us to take photos and record videos of the experiments.
4.4 Experimental setup using the exponential horn

Figure 4.3 shows the newly designed exponential horn together with the support structure. The horn is made of Aluminum 6082. The piezo stack, reflector and compression bolt are identical to the previous setup with the stepped horn.

Figure 4.3: Exponential horn together with support structure (new design)

4.4.1 Support structure

Figure 4.4 shows how the new parts are mounted to the xyz-stage on the height-adjustable stand. The supporting structure helps to make the transducer more rigid during operation. The hollow cylinder (seen in Figure 4.4 as the black cylinder), is made of a thermoplastic called polyoximetylen (POM) and used to insulate the transducer from the rest of the setup.

Figure 4.4: The exponential horn and support structure mounted on the xyz-stage
4.5 Driver board output

Figure 4.5 shows the driver board used to drive the transducer. It uses transformer-based circuitry to generate a high voltage output. The driver board is specified to have a frequency of 28 kHz and a power of 100 W, according to the vendor.

![Driver board for transducer](image)

Figure 4.5: Driver board for transducer

Figure 4.6 shows the high voltage probe (Tektronix P6015A) used to measure the signal from the driver board.

![Measuring the driver board signal with a high voltage probe](image)

Figure 4.6: Measuring the driver board signal with a high voltage probe

The oscilloscope (Tektronix TDS7104) shows the time-averaged signal from the driver board in Figure 4.7. We can see that the signal is not a pure sinus wave, and is heavily modulated. We might have guessed that this was the case since the purpose of the original transducer was for cavitation in liquid. The peak-to-peak voltage shown on the oscilloscope is 1.52 V. The probe
used for this measurement has an attenuation factor of 1000, which means that the actual voltage is 1520 V.

![Image](image1.png)

Figure 4.7: Time-averaged driver board signal

Furthermore, we let the oscilloscope perform a Fast Fourier Transform (FFT) on the signal. Figure 4.8 shows the result of the FFT and we can see that there are a wide range of frequencies. The two strongest peaks occur at 25 kHz and 85 kHz.

![Image](image2.png)

Figure 4.8: FFT of driver board signal

Since the signal is heavily modulated, this may explain why there is so much heat generated in the stepped horn. The stepped horn only performs optimally for a single frequency, and has to be designed as such. However, an exponential horn has only a lower cutoff frequency, which allows more frequencies to be transmitted through the horn and thus generating less heat.

### 4.6 Plastic microfluidic chip

The plastic microfluidic chips are provided by ChipShop, and can be found in their product catalogue [18]. A typical chip used in the experiments has 2 inlets and 2 outlets, and is shown in Figure 4.9. A rhombic chamber chip (Product code: 12-0932-0131-05) is used for the experiments. It has a volume of 20 µL, depth of 400 µm, a lid thickness of 188 µm, is made of Zeonor (a type of polymer) and has a hydrophilized surface.
4.7 Mixing results

4.7.1 Results using stepped horn

Three different contact levels were tried for dispersing the magnetic beads: 15, 30, 50 µm. A higher contact level, such as 50 µm, means that the horn is pressed against the plastic chip more than a contact level of 15 µm.

The images in Figure 4.10 show the initial state (before driving the transducer) and the final state (after driving the transducer) for a duration of 30 seconds. A contact of 15 µm does not produce any dispersion, while a contact of 50 µm is needed for streaming to occur.

The magnetic beads have a brown color in the images. Figure 4.10a and Figure 4.10b show the initial and final state of the magnetic beads for a contact level of 15 µm, while Figure 4.10c and Figure 4.10d show the initial and final state of the magnetic beads for a contact level of 50 µm.
The longer the transducer is driven, the more likely it is for cavitation bubble generation of dissolved oxygen to occur. For a duration of 30 seconds, only streaming is observed from the ultrasonic pressure. However, when the duration is increased, up to 3 minutes, there are several cavitation bubbles appearing near the center of the placement of the ultrasonic horn. It is possible for the generated bubbles to move around in the liquid. Sometimes they remain at their original location, which is probably due to some kind of trapping, and sometimes they are pushed to the side. The cavitation bubbles occur randomly. Figure 4.11c and Figure 4.11d show the inlet and outlet of the chip after a drive of 3 minutes. We can see a lot of bubbles accumulated near the inlet and outlet.
We can see from the images in Figure 4.11 that a contact of 30 µm is still not enough to fully disperse the magnetic beads in a span of 3 minutes. However, if the contact is increased to 50 µm, the mixing effect is increased and the magnetic beads are spread out over a larger area for the same duration. Figure 4.12 shows a 3 minute drive with a contact level of 50 µm.
Figure 4.12: Duration: 3 min, Contact level: 50 µm, Transducer: Stepped horn

Figure 4.13 shows a time-lapse series for the run with contact level 50 µm and a duration of 3 minutes. The images are taken with an interval of 5 seconds. The mixing finishes after roughly 2 minutes with a contact level with 50 µm. The last two images in the series show the inlets of the chip. We can see that there are very few gas bubbles. The size of the bubbles are not large either, so this means that the majority of the streaming is from the boundary layer. The mixing time is considerably less when there are gas bubbles, since the localized streaming caused by the bubbles is much larger than the overall streaming in the chip.
Figure 4.13: Time-lapse series of streaming in chip, 5 second intervals, 50 μm contact level

4.7.2 Results using exponential horn

Experiments were also done with the new transducer design, which has a horn with an exponential profile. Due to limited samples of the magnetic beads, the only level of contact used was 30 μm. Another difference compared to the experiments with the stepped horn was that this time the transducer was not driven for a fixed time, but was driven until the sample was completely mixed.

A contact level of 30 μm was used for 3 different samples. The time it takes for the magnetic beads to be completely mixed varies a lot between the samples. The time to completion for the first sample is 110 seconds. Then, it took 70 seconds for the second sample and 10 seconds for the third sample. The large variation is because the mixing was completed by different types of streaming. Boundary layer streaming was seen in the first and second sample. In the first sample, the vortices were small so the mixing was slow. The mixing was only completed when cavitation bubbles appeared. They generate a turbulent flow in the whole chip, and the sample is mixed almost instantly. The turbulent flow generated by the cavitation bubbles seem to be larger compared to the results with the stepped horn. It could be that the bubbles collapse suddenly and generate an acoustic jet. The streaming vortices in the second sample were larger and covered the whole chip. After 70 seconds, the mixing was roughly 50% completed. Still, it
was the cavitation bubbles which generated a high streaming flow and finished the mixing. The third sample was mixed very quickly because the bubbles appeared immediately. Figure 4.14a and Figure 4.14b show the initial and final state of the first sample.

![Image of initial state](image1.png)  ![Image of final state](image2.png)

(a) $t = 0\ s$  (b) $t = 110\ s$

Figure 4.14: Duration: 110 s, Contact level: 30 µm, Transducer: Exponential horn
5. Discussion and conclusions

5.1 Repeatability problem caused by slide holder

Several problems were discovered during the experiments with the Langevin transducer. One such problem is flexing of the plastic chips. The plastic chips have the same size as an ordinary microscope slide and are placed in the ordinary microscope slide holder. The chip starts to bend when the transducer is pressed down on top of the chip. There are multiple chambers on a single chip, and if a chamber in the center of the chip is used, then the flexing is at its largest. On the other hand, if a chamber at the end of the chip is used, there is less flexing since the displacement from the transducer is closer to the slide holder. The flexing could be reduced by using a more solid microscope table.

5.2 Heat generation in transducer

Another important issue is the heat generated by the transducer. The stepped horn did not have a resonance frequency at 28 kHz, which meant that a lot of power was turned into heat. Also, higher frequencies from the output of the driver board generated lateral vibrations which caused less longitudinal vibration and also affected the stability of transducer. The exponential horn should be less sensitive to lateral vibrations compared to the stepped horn for the same input frequency, due to the slender profile of the exponential horn.

The temperature of the stepped horn was measured with a thermocouple, and during a drive of 1 minute the tip of the horn reached a temperature over 50 °C, starting initially from an ambient temperature of 22 °C. The temperature of the exponential horn was also measured with a thermocouple. The tip of the exponential horn reached a temperature of 27 °C during a drive of 1 minute under the same initial conditions.

Generally, the transducers should be operating at a temperature well below their Curie temperature, $T_c$, which is the temperature at which ceramics are completely depolarized and lose their piezoelectric properties. The piezoelectric material Pz26 has a Curie temperature of 330 °C. The temperature should be kept as close to ambient temperature as possible, when using the transducer for biological applications. For example, the DNA strands may be destroyed in the chip. If the horn is to be used for cell lysis, then the cells should be killed by the high pressure ultrasound and not by the increased heat. In this case, active cooling might be needed.

The piezoelectric material constants are temperature dependent. If the temperature of the
transducer increases during operation, this may lead to a shift in the resonance frequency of the transducer. Figure 5(b) in the paper by Ando and Kagawa [17] shows the ratio of resonant frequency change with the temperature for a similar transducer. Since the quality factor is very large, this means that the bandwidth of a resonance peak is narrow, so a small shift in the resonance frequency may greatly diminish the vibration of the transducer.

5.3 Cavitation bubble generation

When using high power ultrasound, cavitation can also occur. This happened quite often during the experiments, in particular when driving the transducer for a long duration. The fluid is heated up by two sources: the absorption of acoustic waves and thermal conduction between the tip of the transducer and the chip. The cavitation bubble generation of dissolved oxygen is likely caused by the fluid inside the chip heating up over time. Caupin and Herbert [19] show that the probability of cavitation increases with temperature.

![Figure 5.1: Cavitation threshold as a function of temperature [19]](image)

Makuta et al. [20] show that bubbles around 330 µm in diameter were generated through primary bubble generation using a hollow cylindrical ultrasonic horn. This is the resonance diameter of oxygen bubble in water under 19.5 kHz ultrasonic irradiation, according to the Rayleigh-Plesset equation. The height of the micro-chamber used in the experiments is 200 µm. If the bubbles generated in our experiments have a similar size, then they would be in contact between the ceiling and the floor of the micro-chamber. This means it would be difficult for the bubbles to be dissolved again, and they will remain in the chamber.

If bubbles are present in the chamber, then the streaming pattern is greatly affected. Therefore, it is difficult to discern the streaming patterns. In the experiments with the magnetic beads, we can see that the time it takes to completely disperse the beads varies when repeating a run with the same contact. This is mainly due to the presence of the gas bubbles. Moreover, the gas
bubbles sometimes get pushed around by the pressure field inside the chip, similar to a chaotic system. In this case, the mixing speed is even faster. Even though gas bubbles may help in increasing the speed of the mixing, they are also undesired in lab-on-a-chip systems as they can accumulate in inlets and restrict the flow.

5.4 Driving frequency of transducer

In both experimental setups with the stepped horn and the exponential horn, the transducer is driven with a source which contains frequencies in the kHz range. The wavelength of the acoustic waves is larger than the dimensions of the chip, which means that the type of streaming most likely to occur is boundary layer streaming. However, if a higher input frequency in the MHz range is used, where the wavelength is shorter than the chamber dimension, then Eckart streaming will occur. The dimensions of the current Langevin transducer will have to be modified to obtain a higher resonance frequency. A miniature transducer designed by Li et al. [11] has resonance frequencies around 400 kHz.

5.5 Conclusions and further research

The contributions of this thesis work is three-fold. First, we have examined the feasibility of using a Langevin transducer for mixing in plastic microfluidic chips. We have shown that it is possible to obtain streaming in a microfluidic chip with stationary flow. The mixing effect depends highly on the coupling between the transducer and the chip. Initially, a transducer with a stepped horn was used. There were some issues with the experimental setup and also the transducer itself. If high input power is used, a lot of heat is generated in the transducer with stepped horn, which then causes cavitation bubble generation of dissolved gas in the fluid, which is not desired in a lab-on-a-chip system. The support structure is not rigid enough, which causes repeatability issues.

Second, a design of a new exponential horn was made in order to solve the above mentioned problems. Also, a new support structure was designed to hold the transducer. Finite element analysis with COMSOL was used to determine the dimensions (primarily the length) of the new transducer such that it matched the output frequency of the ultrasonic driver board. The fundamental resonance frequency found in the simulations are very close to the real measurements. Third, the stepped horn on the existing transducer was replaced with a new design, which includes an exponential horn and a support structure. The experiments were repeated, and we found that the heat generation in the transducer with exponential horn is much less than the transducer with stepped horn, when driven with the same ultrasonic driver board. The support structure of the setup with the exponential horn is more rigid than the holder used to fixate the stepped horn. The contact between the transducer tip and the chip was improved, since a smaller contact level at 30 µm is enough for the magnetic beads to be completely mixed in less than 3 minutes, while a contact level of at most 50 µm was needed when the stepped horn was used. A smaller contact level will also cause less flexing of the microfluidic chip.
There are three suggestions for future research. First, driving the transducer with a lower power and a frequency which is not modulated. The lower power may make it less likely for cavitation bubbles to appear.

Second, it would be interesting to try a higher driving frequency as discussed earlier. If a high enough frequency is used such that the streaming is of Eckart type, then the ultrasound should be sent in the largest direction. If the transducer tip surface is positioned in parallel to the chip, then the streaming is limited by the height of the chamber (200 µm). Some other chips have a larger height, up to 800 µm. The minimum frequency needed to obtain Eckart streaming in chips with a larger chamber height would be lower, making it easier to use a Langevin transducer as an external force.

Third, an alternative position of the transducer may be tried. If the transducer tip surface is positioned on the side of the chip, it is even easier to obtain Eckart streaming since the width of the chamber is close to 1 centimeter. However, the contact area on the side, equal to the height of the chip, is small and thus it is difficult to couple the transducer to the chip. Several components of the Langevin transducer has to be diminished in this case. The miniature transducer presented by Li et al. [11] may be a good fit for such a setup. An alternative method would be to direct the ultrasound towards the chip at an angle. This increases the propagation distance of a longitudinal wave, which makes it easier to generate acoustically induced Eckart streaming. A wedge adapter can be mounted on the tip of the horn (see Figure 5.2) in order to couple the transducer to the chip at an angle. It is important to design a suitable experiment setup so that good contacts are achieved between the horn and the wedge as well as between the chip and the wedge.

Figure 5.2: Plastic wedge adapter used to couple the transducer to the chip at an angle
Bibliography


A. Appendix

A.1 Coupled piezoelectric equations in the stress-charge form

The coupled equations can also be written in the stress-charge form which consists of (A.1) and (A.2) Sherman and Butler [4, p. 36].

\[
T = c^E \cdot S - e^l \cdot E \quad \Leftrightarrow \quad T_{ij} = c^E_{ijkl} S_{kl} - e^l_{kj} E_k, \quad i, j, k, l = 1, 2, 3 \quad (A.1)
\]

\[
D = e \cdot S + \varepsilon^S \cdot E \quad \Leftrightarrow \quad D_i = e_{ijk} S_{jk} + \varepsilon^S_{ij} E_j, \quad i, j, k = 1, 2, 3 \quad (A.2)
\]

The coupling matrix in the stress-charge form is \(e\) and its transpose is \(e^t\). \(c^E\) is the elasticity (stiffness) under a constant (or zero) electric field. \(\varepsilon^S\) is permittivity under a constant (or zero) strain field.

Transformation from the strain-charge to stress-charge form is given by (A.3), (A.4) and (A.5).

Sometimes piezo manufacturers only publish material data for \(d\), while finite element software packages require material data to be entered as \(e\).

\[
c^E = s^{E^{-1}} \quad (A.3)
\]

\[
e = d \cdot c^E \quad (A.4)
\]

\[
\varepsilon_S = \varepsilon_T - d \cdot c^E \cdot d^t \quad (A.5)
\]

A.2 Material constants for Pz26 in the stress-charge form

\[
\begin{bmatrix}
    c^E_{11} & c^E_{12} & c^E_{13} & 0 & 0 & 0 \\
    c^E_{21} & c^E_{22} & c^E_{23} & 0 & 0 & 0 \\
    c^E_{31} & c^E_{32} & c^E_{33} & 0 & 0 & 0 \\
    0 & 0 & 0 & c^E_{41} & 0 & 0 \\
    0 & 0 & 0 & c^E_{51} & 0 & 0 \\
    0 & 0 & 0 & 0 & c^E_{66} & 0 \\
\end{bmatrix}
\begin{bmatrix}
    16.8 & 11.0 & 9.99 & 0 & 0 & 0 \\
    11.0 & 16.8 & 9.99 & 0 & 0 & 0 \\
    9.99 & 9.99 & 12.3 & 0 & 0 & 0 \\
    0 & 0 & 0 & 3.01 & 0 & 0 \\
    0 & 0 & 0 & 0 & 3.01 & 0 \\
    0 & 0 & 0 & 0 & 0 & 2.88 \\
\end{bmatrix}
\times 10^{10} \text{[N/m}^2]\]

\[
\begin{bmatrix}
    0 & 0 & 0 & c_{15} & 0 \\
    0 & 0 & 0 & c_{24} & 0 \\
    0 & 0 & 0 & c_{31} & c_{32} \\
\end{bmatrix}
\begin{bmatrix}
    0 & 0 & 0 & 0 & 9.86 & 0 \\
    0 & 0 & 0 & 9.86 & 0 \\
    -2.80 & -2.80 & 14.7 & 0 & 0 \\
\end{bmatrix}
\text{[C/m}^2]\]
\[ \varepsilon_{11,r}^S = 828 \]
\[ \varepsilon_{33,r}^S = 700 \]
A.3 Drawing of stepped horn

(Unit: mm)
A.4 Drawing of assembled transducer with exponential horn
A.5 Drawing of exponential horn with flange

(Unit: mm)
A.6 CAD drawing of plate 1

(Unit: mm)
A.7 CAD drawing of plate 2

(Unit: mm)
A.8 CAD drawing of plastic cylinder

(Unit: mm)