Trapping of water drops by defects on superhydrophobic surfaces

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ABSTRACT

In this work the effect of line-shaped defects on the motion of water drops on superhydrophobic surfaces have been investigated using high-speed video. The defects were introduced on superhydrophobic wax surfaces by a simple scratching method. It is shown that the motion of the drop in the vicinity of the defect can be approximated by a damped harmonic oscillator. Whether a drop gets trapped or not while traversing the defect is determined by the incident speed and the characteristics of the oscillator, more specifically by the damping ratio ζ and the nondimensional forcing constant a. We also show that it is possible to predict the trapping speed as well as the exit speed using a simple work-energy consideration in systems with negligible viscous dissipation.
1. INTRODUCTION

Recently, surfaces with extreme water-shedding properties, i.e. superhydrophobic surfaces, have attracted considerable scientific as well as industrial interest.\(^1\)\(^2\) Potential industrial application include water-proofing of textiles,\(^3\) self-cleaning windows,\(^4\) heat exchangers,\(^5\) water harvesting from humid air,\(^6\)\(^7\),\(^8\) and prevention of biofouling\(^9\) to name a few. The criteria for superhydrophobic behavior are a high water contact angle (\(>150^\circ\)), a low contact angle hysteresis (CAH) and a low roll-off angle.\(^10\) These surfaces are also referred to as self-cleaning,\(^11\) or as possessing the lotus effect\(^12\) and are usually composed of a low surface energy material and the surface topography includes protrusions in the nano- and microscale.\(^13\) These protrusions enable the formation of air pockets beneath the drop that can reduce the adhesion between the drop and surface, promoting higher droplet mobility.\(^14\) The friction losses in for example micro fluidics can subsequently be greatly reduced.\(^15\)

Trapping of water drops is interesting in for example lab-on-a-chip applications. In this work the aim was to determine the effect of line shaped topographical defects on the movement, and more specifically, the trapping of water drops on superhydrophobic coatings. We seek the system specific parameters governing the trapping conditions or in the case of nontrapping, the effect on the exit speed of drops. Trapping of drops have previously been achieved on electrically tunable wetting defects and was shown to be controlled by the trapping strength as well as the ratio between the viscous and the inertial time scale.\(^16\) Electrostatic actuation of drops on hydrophobic polymer surfaces have been implemented.\(^17\) Drop transportation has also been realized using gradients in wettability\(^18\) as well as roughness.\(^19\) By etching holes and grooves drops have been trapped and guided along microchannels, the anchoring strength has been shown to depend on the etched geometries and droplet size.\(^20\) Surface topography in the form of undercut edges have also been used to inhibit and control liquid spreading.\(^21\) By using sharp transitions in CAH drops have been deflected on tilted superhydrophobic surfaces.\(^22\)
In this work, the effects of droplet motion on tilted superhydrophobic alkylketene dimer (AKD) surfaces have been studied using high-speed video. The interactions between moving drops and the wetting defect have been investigated using a harmonic oscillator model as well as a less complicated work-energy consideration.

2. MATERIALS AND METHODS

2.1 Materials

A hydrophobic wax, alkyl ketene dimer (AKD), was supplied by EKA Chemicals (Bohus, Sweden) in the form of pellets with a mean particle diameter of approximately 4 mm. This wax contains unknown additives and was subsequently purified in ethanol prior to use. In brief, the AKD pellets were dissolved in warm ethanol. The solution contained a yellow emulsion that was removed by filtration. The AKD was recrystallized at room temperature and the crystals were filtered and washed with ethanol. This cleaning step was repeated twice and the resulting white crystals were dried in vacuum overnight. Ultrapure carbon dioxide (≥99.995 %) purchased from Strandmöllen AB (Ljungby, Sweden) was used as received. Microscope glass slides (VWR International, Radnor, USA) were cleaned in MilliQ water, ethanol and acetone prior to use.

2.2 Equipment and experimental procedures

2.2.1 Coating procedure

Cleaned glass slides were dip coated in AKD heated to 90 °C. The substrates were then placed horizontally and allowed to solidify at room temperature. The AKD coating was subsequently polished using a sequence of sandpapers with the following grit designations (CAMI): 240, 400, 800 and finally by a gloved (latex) finger.

Superhydrophobic coatings on the polished AKD were produced by spraying with the RESS technique. In brief, approximately 5 g of AKD was placed in the vessel and was heated to a temperature of
approximately 70 °C. The vessel was placed in an upright position so that the bottom of the vessel contained melted AKD and the outlet for the nozzle was placed at the top of the vessel. ScCO₂ with a pressure of 25 MPa and a temperature of 70 °C was subsequently delivered to the vessel and bubbled up through the melted AKD. All of the valves in the set-up were opened so that the CO₂ could pass directly through the entire system. The spray distance between the nozzle and the surface substrates was 3 cm.

2.2.2 Scratching procedure

In order to create scratches of specified widths and depths, custom made tips were used: Al₂O₃ half spheres with radii of 0.5 mm and 2.0 mm (Edmund Optics, Barrington, New Jersey, USA) were glued to the ends of steel cylinders with diameters of 2 mm and lengths of 7 mm. The tips were mounted in the ST-135 Cantilever of a nano scratch tester (CSM Instruments SA, Peseux, Switzerland). The samples were scratched at a rate of 0.1 mm/min with applied loads between 0.3 mN and 1.0 mN and the mean friction force \( F_t \) was monitored during the scratching procedure. The scratches were designated according to the tip radius \( r_{tip} \) and applied normal force \( F_N \) according to; scratch 1: \( r_{tip} = 2.0 \) mm and \( F_N = 0.3 \) mN, scratch 2: \( r_{tip} = 2.0 \) mm and \( F_N = 1.0 \) mN, scratch 3: \( r_{tip} = 0.5 \) mm and \( F_N = 0.3 \) mN, and scratch 4: \( r_{tip} = 0.5 \) mm and \( F_N = 1.0 \) mN. The designations of the different scratches are compiled in Table 1.

<table>
<thead>
<tr>
<th>Scratch no.</th>
<th>( r_{tip} ) [mm]</th>
<th>( F_N ) [mN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Table 1.** Summary of the scratch designations.

2.2.3 Scratch characterization

The width and depth of the scratches were characterized using a Microprof® 200 (FRT – Fries Research & Technology GmbH, Bergisch Gladbach, Germany) non-contact optical profilometer. Areas of
1000x1000 µm were analyzed with a sampling distance of 2 µm in the x and y directions in order to obtain topographic maps of the scratches. One measurement was performed for each sample.

2.2.4 Evaluation of drop motion

The superhydrophobic sliding resistance was measured on coated surfaces according to a previously developed method. In brief, water drops with volumes ranging from 3 µl to 40 µl were allowed to slide down coated surfaces with an inclination between 5° and 10°. The acceleration was measured using high speed video and the superhydrophobic sliding resistance, was calculated as a function of the square root of the Bond number according to:

\[
b_{sh} = \sqrt{Bo \left( \sin \beta - \frac{\dot{s}}{g} \right)}
\]

where β is the surface inclination, \(\dot{s}\) is the vector length of the drop acceleration, g is the gravitational acceleration constant and Bo is the dimensionless Bond number defined as:

\[
Bo = \frac{\rho g R^2}{\gamma}
\]

where ρ is the density of water, R is the nominal drop radius and γ is the surface tension of water. The drop speed \(\dot{s}\) was determined by fitting a second-degree polynomial to the position data and calculating the first derivative with respect to time. The final speed \(v_\infty\) on polished AKD, the incident speed \(v_i\) when the drop encountered the defect, as well as the exit speed \(v_1\) when it crossed the defect were calculated.

2.2.5 Contact angle measurements

Advancing (\(\theta_a\)) and receding (\(\theta_r\)) contact angles were measured on polished AKD using a CAM200 contact angle meter (KSV Instruments, Helsinki, Finland) with an automatic dispenser. A 20 µl water
drop was deposited on the surface and a stainless steel needle (external diameter 0.4 mm) was inserted into the drop. The drop volume was increased by injecting water at a rate of 0.5 µl/s. The injection was stopped and switched to suction when the drop volume had reached 50 µl. This suction, with a rate of 0.4 µl/s, resulted in a reduction of the droplet volume. The contact angles were calculated using Laplace fitting every second during both injection and suction. Contact angles were measured at three different positions on the polished AKD surface and the mean value was calculated.

2.2.6 Drop dimension measurements

In order to determine the radius of the contacting disc $r$ between the drop and the surface and the nominal radius $R$ as functions of the projected radius $R_p$, a CAM200 contact angle meter (KSV Instruments, Helsinki, Finland) was used. Drops with volumes varying between 3 µl and 50 µl were placed on the superhydrophobic surface and photographs were taken parallel to the surface. The nominal radius $R$ was related to the drop volume $V$ according to

$$R = \left(\frac{3V}{4\pi}\right)^{1/3}. \quad (3)$$

The projected radius and the radius of the contacting disc were measured using image analysis software (National Instruments Vision Assistant 2012, National Instruments, Austin TX, USA).

3. RESULTS

Topographical maps of the 4 different scratches can be seen in Fig. 1. The friction forces are 14.3 ± 3.4 mN, 38.5 ± 5.7 mN, 8.7 ± 2.6 mN and 35.3 ± 4.4 mN for scratch no. 1, 2, 3 and 4, respectively. The scratch width was larger for the larger tip radii with 300 µm for both scratch 1 and 2; the width is 200 µm and 250 µm for scratches 3 and 4, respectively. The scratch depth increased with increasing load and decreasing tip radius, with the following values: 5 µm, 8 µm, 6 µm and 10 µm for scratch 1, 2, 3 and 4,
respectively. This behavior was expected since a smaller tip will penetrate deeper into a material at a constant load than a larger tip. The dimensions of the scratches are compiled in Table 2.

![Topographical maps of scratches](image)

**Figure 1.** Topographical maps of a) scratch 1, b) scratch 2, c) scratch 3, and d) scratch 4.

The measured values of the advancing contact angle $\theta_a$ and the receding contact angle $\theta_r$ for a polished AKD surface were $134^\circ$ and $66^\circ$, respectively. We consequently approximated $\theta_0 = (\theta_a + \theta_r)/2 \approx 100^\circ$.

Fig. 2 illustrates how the nominal radius $R$, the contacting disc radius $r$ and the projected radius $R_p$ were measured. In Fig. 3 the measured values of $R$ and $r$ are plotted as functions of $R_p$ using second degree polynomials. In the case of a 21 $\mu$l drop the nominal radius $R$, the contacting disc radius $r$ and the projected radius $R_p$ were 1.71 mm, 1.03 mm and 1.81 mm, respectively.
Figure 2. Photograph of a drop resting on a superhydrophobic AKD surface. The projected radius $R_p$ and the radius of the contacting disc $r$ are denoted by dashed arrows.

<table>
<thead>
<tr>
<th>Scratch no.</th>
<th>$F_r$ [mN]</th>
<th>Scratch width, $2d$ [$\mu$m]</th>
<th>Scratch depth [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.3 ± 3.4</td>
<td>300</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>38.5 ± 5.7</td>
<td>300</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>8.7 ± 2.6</td>
<td>200</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>35.3 ± 4.4</td>
<td>250</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 2. Summary of scratch width, scratch depth and friction force. The errors represent one standard deviation.
4. DISCUSSION

We consider a drop that moves rectilinearly across a macroscopically flat surface. A Cartesian coordinate system with the $xy$ plane aligned with this surface is introduced, so that $x$ is oriented in the direction of motion of the drop. When a water drop is placed on the surface, a contacting disc with radius $r$ is formed in the region

$$\Omega(s) = \{(x, y): (x - s)^2 + y^2 < r^2\}, \quad (4)$$

where $s$ is the position coordinate of the centroid $C$ of the contacting disc. We consider two different model surfaces: First a flat, polished AKD surface, which we refer to by the index 0. Secondly, a macroscopically flat AKD surface covered by a superhydrophobic layer of AKD applied using the RESS process. This superhydrophobic surface is referred to by the index 1.
4.1 Drop-defect interactions

Consider a superhydrophobic surface with a defect of width $2d$ centered on the line $x = 0$ perpendicular to the path of the drop. This defect covers the region

$$\Omega_0 = \{(x, y): x < d\}, \quad (5)$$

where we use the index 0 to indicate that we consider the defect region to be an area of type-0 surface. We assume that $d \ll r$ so that the shape of the drop, including its contacting disc $\Omega$, is not significantly affected by the defect. This geometry of the contacting disc and defect is illustrated in Fig. 4.

![Figure 4](image)

**Figure 4.** Assumed geometry of a contacting disc $\Omega$ of radius $r$ interacting with a defect $\Omega_0$ of half-width $d$.

4.1.1 Wetting energy

We proceed to investigate the wetting energy of the drop when it moves across a superhydrophobic surface with a defect. We assume that never-wet and dewetted surface have the same surface energy. According to the Young–Dupré equation, the work of adhesion per unit area between the drop and the defect is $\gamma(1+\cos\theta_0)$, with $\theta_0$ the equilibrium contact angle between a water drop and the polished AKD
surface. Let $\theta_{a0}$ and $\theta_{r0}$ represent the advancing and the receding contact angle, respectively, of this polished surface. Then $\theta_0$ is estimated by their average: $\theta_0 \approx (\theta_{a0} + \theta_{r0})/2$. The work of adhesion between the drop and the superhydrophobic surface is assumed to be negligible since it is essentially an air–liquid interface. The total wetting energy can then be written in the form of a potential:

$$W_o(s) = -\gamma (1 + \cos \theta_0) \int_{\Omega_o \cap \Omega(s)} \mathrm{d}x \mathrm{d}y \cos \theta_0,$$  

(6)

where $\cap$ denotes the section of the two regions. The integral of Eq. (6) is the area of the shaded region in Fig. 4. This area is analytically available from simple geometry (not shown). The wetting potential for a water drop with contacting disc radius $r = 0.5$ mm on a RESS-AKD surface having a $2d = 200$ μm wide defect with wetting angle $\theta_0 = 100^\circ$ is illustrated in Fig. 5 (solid line).

\[W_d = -W_o(0) = \gamma (1 + \cos \theta_0) r^2 \left( \pi - 2 \arccos \frac{d}{r} + \frac{2d}{r} \sqrt{1 - \frac{d^2}{r^2}} \right). \quad (7)\]

**Figure 5.** Wetting potential experienced by the contacting disc of a water drop on a superhydrophobic surface with a defect (solid black line). The parameters are $r = 0.5$ mm, $\beta = 10^\circ$, $d = 100$ μm, and $\theta_0 = 100^\circ$. The wetting potential is also approximated by a harmonic potential (dashed red line).
Thus, the wetting potential can be approximated by a truncated harmonic potential:

$$W_0^* = W_d \left[ \frac{s^2}{(r + d)^2} - 1 \right], \quad |s| < r + d . \quad (8)$$

This approximation is illustrated in Fig. 5 (dashed red line).

### 4.1.2 Dissipation at the defect

When a water drop travels across a polished, type-0 surface, the lateral traction acting on its contacting disc can be represented by a viscous force component $F_{v0} = -2r\lambda_0 \dot{s}$ and a pinning force component $F_{p0} = -2r\gamma (\cos \theta_{t0} - \cos \theta_{a0})$. Here, however, we introduce a constant $\sigma_0 = \gamma (\cos \theta_{t0} - \cos \theta_{a0})$. Then, the constants $\lambda_0$ and $\sigma_0$ are to be determined experimentally.

When a water drop travels across a superhydrophobic, type-1 surface, previous experiments show that the viscous and pinning force components become $F_{v1} = 0$ and $F_{p1} = -b_{sh} \text{Bo}^{-1/2} mg$, respectively. Here, $b_{sh}$ is the superhydrophobic sliding resistance, which is determined on a defect free surface using Eq. (1).

When a drop moves across a defect in the superhydrophobic surface, it will inevitably be in a mixed state of wetting with different physics governing dissipation in the different states of wetting. This results in a complicated mix of type-0 and type-1 lateral traction.

As a first approximation for the lateral traction on a drop interacting with a defect, $|s| < r + d$, we apply a mixing rule which takes the respective force contributions to be in proportion to the amount of surface of each type in the interaction region:

$F_v = \frac{d}{r + d} F_{v0} + \frac{r}{r + d} \frac{2rd\lambda_0}{r + d} \dot{s} \quad (9a)$

$F_v = \frac{d}{r + d} F_{p0} + \frac{r}{r + d} F_{p1} \quad (9b)$
4.2 Equations of motion

Consider a water drop sliding down an inclined superhydrophobic surface while interacting with a defect, $|s| < r + d$. Let $\beta$ be the inclination angle. Newton’s second law for rectilinear particle motion then gives

$$\frac{dW_0}{ds} + F_v + F_p + mg \sin \beta = m\ddot{s}, \quad (10)$$

where we used the harmonic approximation for the wetting potential. This Eq. (10) has the form of a damped harmonic oscillator:

$$\ddot{s} + 2\zeta \omega \dot{s} + \omega^2 s = a, \quad |s| < r + d \quad (11)$$

with

$$\omega = \sqrt{\frac{2W_d}{m(r+d)^2}}, \quad \zeta = \frac{rd\lambda_0}{\sqrt{2mW_d}}, \quad a = g \sin \beta + \frac{F_p}{m} \quad (12)$$

the natural angular frequency, the damping ratio and the constant forcing function, respectively.

This Eq. (11) is nondimensionalized by introducing the nondimensional drop position, $\hat{s} \equiv s/(r + d)$, and the nondimensional time, $\hat{t} \equiv \omega t$, giving

$$\frac{d^2\hat{s}}{d\hat{t}^2} + 2\zeta \frac{d\hat{s}}{d\hat{t}} + \hat{s} = \hat{a}, \quad |\hat{s}| < 1 \quad (13)$$

with the nondimensional forcing constant

$$\hat{a} = \frac{1}{2W_d} \left[(r+d)mg \sin \beta + dF_{p0} + rF_{p1}\right] \quad (14)$$

representing the balance between gravity and pinning forces. Because $\hat{a}$ also represents the equilibrium position of the oscillator—the stationary solution is $\hat{s} = \hat{a}$—trapping is only be possible when $\hat{a} < 1$. That is, there will be no trapping if the tilt angle is too great. An illustration of water drops interacting with defects (scratch 1) at a surface inclination of 15° can be seen in Fig. 6, $t = 0$ is defined when the drop reaches the defect. The top sequence $a)$ illustrates a larger drop passing the defect without trapping and the bottom sequence $b)$ illustrates a smaller drop trapped by the defect.
Figure 6. Illustration of water drops interacting with defects (scratch 1) at a surface inclination of 15°, \( t = 0 \) is defined when the drop reaches the defect. Sequence \( a) \) illustrates a drop passing the defect without trapping and sequence \( b) \) illustrates a drop trapped by the defect.

Our procedure deviates slightly from that of Mannetje et al.\(^{15}\) in that we choose the inverse natural angular frequency instead of the viscous relaxation time as the reference time. This ensures that the familiar damping ratio \( \zeta \) becomes one of the parameters of the system.
4.3 Trapping criterion and exit speed

A drop that starts interacting with a defect at time \( t = 0 \), approaching from the region of negative \( x \) at a speed \( v \), corresponds to an initial value problem represented by Eq. (13) and the initial conditions

\[
\ddot{s}(0) = -1, \quad \frac{ds}{dt}{|_{t=0}} = \hat{v} = \sqrt{\frac{mv^2}{2W_d}}, \tag{15}
\]

where \( \hat{v} \) is the nondimensional incident speed. Trapping is defined to occur if \( \ddot{s} < 1 \) until the time when \( \ddot{s}d \) approaches zero.\(^{16}\)

Considering the above, only three parameters determine the fate of the drop: \( \hat{v}, \hat{a} \) and \( \zeta \). Particularly, we seek a trapping incident speed, \( \hat{v}_{\text{trap}}(\hat{a}, \zeta) \), below which the drop becomes trapped to formulate our trapping criterion. Although the solutions to the initial value problem are analytically available (Appendix B), \( \hat{v}_{\text{trap}}(\hat{a}, \zeta) \) needs to be evaluated numerically. If the drop is not trapped, we seek the exit speed \( \hat{v}_{\text{exit}}(\hat{v}, \hat{a}, \zeta) \), which is the speed of the drop when \( \ddot{s} = 1 \). This nondimensional velocity \( \hat{v}_{\text{exit}} \) is also available from the solutions to the initial value problem (Appendix B).

The physical parameters have been determined experimentally: The constant \( \lambda_0 \) was determined by varying the drop size and the inclination angle of a polished AKD surface, a straight line appeared when the group \( mg \sin \beta / (2r) \) was plotted against \( v_e \) (see Appendix A and Fig. 7). The slope of the linear fit was \( \lambda_0 \). In this case, the value of \( \lambda_0 \) was found to be 0.035. The pinning forces of the polished AKD surface depend on the roughness and chemistry of this surface. These properties were unknown for the defect area, meaning that \( \sigma_0 \) was effectively unknown. Hence, we used \( \sigma_0 \) as a fitting parameter. Particularly, we fit the measured, normalized exit speed \( \hat{v}_1 \) to the predicted, normalized exit speed \( \hat{v}_{\text{exit}} \).

Here, \( \hat{v}_1 = \sqrt{mv_1^2 / (2W_d)} \), where \( v_1 \) was the measured exit speed. The optimal fit (Fig. 8) was found for \( \sigma_0 = 0.119 \text{ N/m} \).
Figure 7. Plot of $(mg \sin \beta)/2r$ against the final speed $v_\infty$ of the drop. The solid line represents the linear fit: $\lambda_0 v_x + s_0$, the slope indicates $\lambda_0 = 0.035$ (See Appendix A).

Figure 8. Measured, normalized exit speed $\hat{v}_1$ of untrapped drops plotted against the predicted, normalized exit speed $\hat{v}_{\text{exit}}$. The solid line represents $\hat{v}_1 = \hat{v}_{\text{exit}}$. 
With this value for $\sigma_0$ from the fit, we plot the measured, normalized incident speed $\hat{v}$ against the predicted, normalized trapping speed $\hat{v}_{\text{trap}}$ in Fig. 9. The data corresponding to trapped drops are represented by $\times$-symbols while those of untrapped drops are denoted by circles. In most cases, when the incident speed fell below the predicted trapping speed, the drop was indeed trapped and \textit{vice versa.}

![Figure 9](image.png)

\textbf{Figure 9.} Measured, normalized incident speed $\hat{v}$ of untrapped drops (black circles) and trapped drops (red $\times$-symbols) plotted against the predicted, normalized trapping speed $\hat{v}_{\text{trap}}$. The solid line represents $\hat{v} = \hat{v}_{\text{trap}}$.

4.4 Simplified model

In cases when viscous forces $F_v$ are negligible as compared to pinning forces $F_p$, the dynamics of the drop is captured by the work–energy theorem of mechanics. When traversing the defect, the drop loses gravitational potential, $\Delta W_g = -2(r + d)mg \sin \beta$, but there is no net change in wetting potential. The work exerted on the drop by the pinning forces is $U = 2(r+d)F_p$. 
The drop will become trapped when the kinetic energy of the incident drop and the contribution from the gravitational potential are too small to overcome the counteracting work of the pinning forces. This condition reads

\[ \frac{1}{2}m v^2 - \Delta W_g < -U , \quad (16) \]

and in the limiting case we have

\[ v = v_{\text{trap}} = \sqrt{- \frac{4}{m} \left[ (r + d) mg \sin \beta + d F_{p0} + r F_{p1} \right]}. \quad (17) \]

In cases when the expression for trapping speed becomes imaginary, the drop will always pass the defect. The incident speed is plotted against the trapping speed in Fig. 10 using the simplified trapping prediction of Eq. (17), and, again, \( \sigma_0 = 0.119 \) N/m. The data corresponding to trapped drops are represented by red \( \times \)-symbols while those of untrapped drops are denoted by black circles. As for the harmonic oscillator model, in most cases, trapping and nontrapping was accurately predicted with this simplified model.

**Figure 10.** Measured incident speed \( v \) of untrapped drops (black circles) and trapped drops (red \( \times \)-symbols) plotted against the predicted trapping speed \( v_{\text{trap}} \) using the simplified expression, Eq. (17). The solid line represents \( v = v_{\text{trap}} \).
When, \( v > v_{\text{trap}} \) or when \( v_{\text{trap}} \) does not exist, the drop will pass the defect with an exit speed \( v_{\text{exit}} \). The work-energy theorem of mechanics then yields

\[
U = \Delta W_g + \frac{1}{2} m (v_{\text{exit}}^2 - v^2) \Leftrightarrow v_{\text{exit}} = \sqrt{v^2 + \frac{4}{m} \left[ (r + d) mg \sin \beta + d F_{\text{pl}} + r F_{\text{pl}} \right]}. \tag{18}
\]

The observed exit speed \( v_1 \) of untrapped drops is plotted against the predicted exit speed in Fig. 11. When comparing Fig. 11 to Fig. 8 it becomes clear that neglecting the viscous forces slightly impairs the prediction of the exit speed; the data points show greater scatter in the simplified model.

![Figure 11](image.png)

**Figure 11.** Measured exit speed \( v_1 \) of untrapped drops plotted against the predicted exit speed \( v_{\text{exit}} \) using the simplified model, Eq. (18). The solid line represents \( v_1 = v_{\text{exit}} \).

5. CONCLUSIONS

The interactions between a drop and a band-shaped defect on a superhydrophobic surface comprises a wetting potential and forces resulting from contact line motion. The motion of the drop in the vicinity of the defect can be approximated by a damped harmonic oscillator characterized by two nondimensional parameters \( \zeta \) and \( \hat{a} \). Here, \( \zeta \) is the damping ratio, and \( \hat{a} \) is a quantity that includes the sum of gravitational and pinning forces.
Whether a drop with a given incident speed becomes trapped or not when traversing the defect is determined by this incident speed and the characteristics \((\zeta, \dot{a})\) of the oscillator. A trapping speed can be computed for each oscillator, so that if the incident speed falls below this trapping speed, the drop becomes trapped. In systems with low dissipation, the damping becomes negligible, and the trapping speed as well as the exit speed can then be predicted by a simple work-energy consideration. We find that a fair prediction can be achieved with this simplified method for water drops on superhydrophobic AKD surfaces.

ASSOCIATED CONTENT

Supporting Information.

Appendix A: Calculation of contact line forces
Appendix B: Solutions of the damped harmonic oscillator

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Author Contributions

The manuscript was written through contributions of all authors. All authors have given approval to the final version of the manuscript.

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Supporting Information

Trapping of water drops by wetting defects

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Appendix A  Contact line forces

We consider a water drop travelling down an inclined, polished AKD surface with inclination angle $\beta$. When it reaches its final speed $v_\infty$, there is a force balance between the gravitational force and the integral of the surface traction:

$$mg \sin \beta + F_{v_0} + F_{p_0} = 0,$$

where the viscous and pinning forces are negative. This force balance is rewritten as

$$\frac{mg \sin \beta}{2r} = \lambda_0 v_\infty + \sigma_0.$$  \hspace{1cm} (20)

Hence, by varying the drop size and the inclination angle, a straight line appears when the group $mg \sin \beta/(2r)$ is plotted against $v_\infty$. The slope and intercept is $\lambda_0$ and $\sigma_0$, respectively.
Appendix B  Solutions of the damped harmonic oscillator

We consider the solutions to the initial value problem of Eqs. (13) and (15). The solution to this problem is analytically available\textsuperscript{25}

\[
\hat{s}(\hat{t}) = \begin{cases} 
A_1 e^{-\hat{t} \left( \zeta - \sqrt{\zeta^2 - 1} \right)} + B_1 e^{-\hat{t} \left( \zeta + \sqrt{\zeta^2 - 1} \right)} + \hat{a}, & \zeta > 1 \\
(A_2 + B_2 \hat{t}) e^{-\hat{t}} + \hat{a}, & \zeta = 1 \\
\left[ A_3 \cos\left( \hat{t} \sqrt{1 - \zeta^2} \right) + B_3 \sin\left( \hat{t} \sqrt{1 - \zeta^2} \right) \right] e^{-\zeta \hat{t}} + \hat{a}, & \zeta < 1,
\end{cases}
\]

Where the constants \( A_i \) and \( B_i \), \( i = 1, 2, 3 \), are determined from the initial conditions in each case:

\[
A_1 = \frac{\hat{v} - \left( \zeta + \sqrt{\zeta^2 - 1} \right) (1 + \hat{a})}{2 \sqrt{\zeta^2 - 1}} \tag{22a}
\]

\[
B_1 = \frac{\hat{v} - \left( \zeta - \sqrt{\zeta^2 - 1} \right) (1 + \hat{a})}{2 \sqrt{\zeta^2 - 1}} \tag{22b}
\]

\[
A_2 = -1 - \hat{a} \tag{22c}
\]

\[
B_2 = \hat{v} - 1 - \hat{a} \tag{22d}
\]

\[
A_3 = -1 - \hat{a} \tag{22e}
\]

\[
B_3 = \frac{\hat{v} - \zeta (1 + \hat{a})}{\sqrt{1 - \zeta^2}} \tag{22f}
\]
REFERENCES