Very high cycle fatigue of duplex stainless steels and stress intensity calculations

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Faculty of Health, Science and Technology
Materials Engineering

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Abstract

Very high cycle fatigue (VHCF) is generally considered as the domain of fatigue lifetime beyond 10 million ($10^7$) cycles. Structural components which are subjected to $10^7$-$10^9$ load cycles during their service life are e.g. engine parts, turbine disks, railway axles and load carrying parts of automobiles. Therefore, the safe and reliable operation of these components depends on the knowledge of their fatigue strength and the prevalent damage/failure mechanisms. Moreover, the fatigue life of materials in the VHCF regime is controlled by the fatigue crack initiation and early growth stage of short cracks.

This study focused on the evaluation of fatigue properties of duplex stainless steels in the VHCF regime using ultrasonic fatigue testing equipment. Tests were conducted on cold rolled and hot rolled duplex stainless steel grades. Considerable attention was devoted to the evaluation of fatigue crack initiation and growth mechanisms using high resolution scanning electron microscopy. The fatigue crack initiation was found to be a surface initiated phenomena for all tested grades, albeit with some differences in initiation mechanisms.

The second part of this thesis work was the development of a distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half plane. Cracks with dimensions much smaller than the overall size of the domain were analysed. The main goal of the development of this technique was the evaluation of the stress intensity factor at the crack tip. The results were compared to results obtained using the Finite Element Method (FEM) software Abaqus and a difference of less than 1% was achieved for Jacobi polynomial expansion of sixth order in the dipole density representation.
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Special thanks to my family and friends for their love and support throughout my life.
List of Papers:

Literature review
Review of VHCF studies, short crack models and theory of vibrations
Muhammad Waqas Tofique

Paper I
Fatigue initiation and strength of duplex stainless steel strip specimens in the very high cycle fatigue regime
Muhammad Waqas Tofique, Jens Bergström, Nils Hallbäck, Christer Burman
Proceedings of the 6th International conference on very high cycle fatigue, Oct 2014, China.

Paper II
Fatigue strength, crack initiation and localized plastic fatigue damage in VHCF of duplex stainless steels
Muhammad Waqas Tofique, Jens Bergström, Nils Hallbäck, Anders Gåård, Christer Burman
To be submitted.

Paper III
Development of a distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half plane
N. Hallbäck, M.W. Tofique
International Journal of Solids and Structures, Published April, 2014.
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1 Introduction

Fatigue is a type of damage mechanism which results in crack initiation and propagation under repeating loads. Mechanical fatigue, which is the focus of this study, involves cyclic mechanical loads. The very high cycle fatigue (VHCF) regime corresponds to a fatigue life greater than 10 million loading cycles ($>10^7$ cycles) and has attracted a lot of attention in the recent years [1-7]. It has been shown that most engineering materials can fail due to fatigue at stress levels lower than the fatigue limit at $>10^9$ load cycles. Many engineering components are designed for fatigue life in the VHCF range; therefore, the choice of fatigue limit at $10^6$ cycles is not safe. The interest in VHCF research has grown in recent years due to ever increasing demands on higher speeds, productivity, reliability and efficiency of engineering components.

The concept of a fatigue limit is a consequence of practical limitation. With conventional equipment for fatigue testing, operating at 10-100 Hz, the testing times required to establish the fatigue strength of engineering materials in the VHCF regime are very long. However, the ultrasonic fatigue testing equipment, operating at a typical frequency of 20 kHz, can generate S-N (stress-fatigue life) data for fatigue life greater than $10^9$ cycles for engineering materials in a reasonable amount of time. For instance, the time required to generate data for $10^7$ cycles is 9 minutes, while conventional testing takes around 12 days. Meanwhile, the test time required to complete $10^9$ cycles is 14 hours with the ultrasonic equipment compared to three years required to test a single specimen at 100 Hz with conventional equipment.

Studies have shown that the VHCF regime is associated with characteristic crack initiation mechanisms. The importance of the crack initiation stage in the VHCF regime is emphasized by the fact that more than 90% of the total fatigue life is spent in this stage [2]. In the initiation stage, the crack is small compared to other dimensions of the body. In order to establish a better understanding of the stress state around the short cracks, different theoretical modelling techniques can be used. One such method is the distributed dislocation dipole technique, which makes use of dislocation pairs (dipoles). The dislocation dipoles are theoretical tools, which induce displacements along the crack lines in an otherwise uncracked body. This technique is based on the Bueckner's principle, described in the later sections. It can be used to calculate the stress intensity factor at the crack tip which governs the propagation of the crack.
2 Aims of research

The main aims of this research work can be summarized as below:

1. Testing of hot rolled and cold rolled stainless steel grades in the VHCF regime.
2. Investigation of fatigue crack initiation and propagation mechanisms active in the selected stainless steel grades in the VHCF regime.

3 Experimental

In the experimental portion of this thesis work, duplex stainless steel grades were tested for their fatigue strength in the VHCF regime using ultrasonic fatigue testing equipment. Efforts were made to develop a fixture to conduct ultrasonic fatigue testing on duplex stainless steel sheets. Furthermore, considerable attention was devoted to the investigation of prevalent fatigue crack initiation and propagation mechanisms in all the tested grades.

3.1 Materials

In this study, two different types of duplex stainless steels were investigated:

- Cold rolled strip duplex stainless steel
- Hot rolled plate duplex stainless steels

3.1.1 Cold rolled duplex stainless strip steel

The properties of cold rolled strip steels are particular due to the fact that cold rolling induces texture and a fine microstructure. The strip steel material tested in this study has a nominal chemical composition of 25% chromium, 7% nickel and 4% molybdenum and 60-40% austenitic-ferritic duplex microstructure, shown in Figure 1. The grain size was in the range of 1-3 µm. The strip thickness was ~1 mm, and the specimens were extracted from the full thickness of the rolled material. It is worth noting that the tested grade is not produced to the final thickness of 1 mm, therefore, the rolling process was interrupted and some material was drawn for manufacturing the test specimens used in this study. The tested material was annealed and further cold rolled to the tensile
strength of 1150 MPa. The mechanical properties of the tested grade are presented in Table I.

Table I. Mechanical properties of the cold rolled duplex stainless strip steel

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>0.2% Proof strength (MPa)</th>
<th>Tensile strength (MPa)</th>
<th>Elongation A50 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.971</td>
<td>1030</td>
<td>1150</td>
<td>10</td>
</tr>
</tbody>
</table>

Figure 1. The austenite-ferrite microstructure of an etched specimen in the short transverse to rolling direction of the cold rolled duplex stainless strip steel grade. The brighter phase is austenite and the darker phase is ferrite.

3.1.2 Hot rolled plate duplex stainless steel

In Paper II, the studied materials were the two duplex stainless steel grades 2304 SRG and LDX 2101®. The chemical composition of both steel grades is shown in Table II.

Table II. Chemical composition (Weight %) of 2304 SRG and LDX 2101® grades

<table>
<thead>
<tr>
<th>Grade</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>N</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>2304 SRG</td>
<td>0.02</td>
<td>0.80</td>
<td>1.45</td>
<td>0.02</td>
<td>0.02</td>
<td>22.70</td>
<td>4.65</td>
<td>0.30</td>
<td>0.09</td>
<td>0.2</td>
</tr>
<tr>
<td>LDX 2101®</td>
<td>0.03</td>
<td>0.70</td>
<td>5.00</td>
<td>-</td>
<td>-</td>
<td>21.50</td>
<td>1.50</td>
<td>0.30</td>
<td>0.22</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The main alloying elements of the two tested grades are chromium, nickel and manganese. Chromium is a ferrite stabilizer while nickel and manganese are
added to give a phase balance with 51-49% austenite-ferrite microstructure to both steel grades. The lean duplex stainless steel LDX 2101® has a lower content of nickel, but manganese is added in order to obtain the phase balance. Both grades are hot rolled to a thickness of 40-84 mm. The specimens were extracted from the Short Transverse (ST) compared to the rolling direction of the billets. The 2304 SRG grade was annealed to a temperature of 1020 °C followed by slow cooling in air. The LDX 2101® grade was annealed to 1050 °C followed by slow cooling in air. Furthermore, the 2304 SRG grade has undergone a special melting shop treatment that controls the amount, size and distribution of non-metallic inclusions, primarily oxides and sulphides, in order to enhance machinability. The LDX 2101® grade has good machinability and, hence, does not require this sort of treatment. The microstructure of the two tested grades is shown in Figure 2 in which the grains of the two distinct phases are elongated along the rolling direction. The mechanical properties of the two grades are shown in Table III.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Proof strength $R_{p0.2}$ (MPa)</th>
<th>Ultimate Tensile Strength (MPa)</th>
<th>Elongation $A_s$ [%]</th>
<th>Hardness (HV)</th>
<th>Elastic Modulus (GPa)</th>
<th>Density (kg/m$^3$)</th>
<th>Ferrite [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2304 SRG</td>
<td>450</td>
<td>670</td>
<td>35</td>
<td>224</td>
<td>210</td>
<td>7750</td>
<td>49</td>
</tr>
<tr>
<td>LDX 2101®</td>
<td>460</td>
<td>675</td>
<td>33</td>
<td>224</td>
<td>210</td>
<td>7720</td>
<td>49</td>
</tr>
</tbody>
</table>
Figure 2. Optical microscope images showing the duplex microstructure in the short transverse to the rolling direction of (a) 2304 SRG (b) LDX 2101® grades
3.2 Equipment and testing method

A detailed description of the ultrasonic fatigue test equipment is included in the literature review done as a part of this study.

Ultrasonic fatigue testing of thin strip specimens is problematic due to gripping issues. Hence, data in the VHCF regime is scarce [8-10]. In this study, several gripping concepts were developed and tested. However, finally, among the tried concepts the horseshoe type of fixture was found to be the best concept in terms of workability. Tests were conducted by designing complete load train to work in resonance at 20 kHz under fully reversed tension-compression loading \( R = -1 \). The load train comprises the ultrasonic horn, the horseshoe fixture and the specimen. The main idea of fixturing concept was that a design of amplification horn with a natural frequency of approximately 20 kHz was selected. The specimen along with the fixture was designed as one unit separately with a natural frequency of 20 kHz. Therefore, when the two units were put together the whole assembly also possessed a natural frequency of approximately 20 kHz. A detailed view of the specimen and fixture assembly mounted at the end of an amplification horn is shown in Figure 3a. In Figure 3b, the design of the horseshoe fixture made of titanium Ti6Al4V grade is presented. A detailed description of the whole designing procedure of the load train, along with the specimen design used in this study, is described in the Paper I.
The ultrasonic fatigue testing of strip specimens was conducted, using the above explained assembly, under fully reversed tension-compression loading mode $R=-1$. Compressed air cooling was used to maintain the temperature of the specimens near room temperature. A high resolution scanning electron microscope LEO 1530 – FEG SEM was used to examine the fracture surfaces and to measure and identify the fatigue fracture initiation defects. Estimates of the fatigue crack growth rate were achieved by fatigue striation measurements in Paper I.

Ultrasonic fatigue testing of cylindrical bar specimens was done on the hot rolled plate material of duplex stainless steel grades 2304 SRG and LDX 2101®, presented in Paper II. The ultrasonic fatigue testing technique is well established for the case of cylindrical bar shaped specimens. Examples of previous studies can be found in the literature [11-14]. The Finite Element Method (FEM) was used to determine the resonance length and design of the hourglass shaped specimens. Subsequently, a dynamic stress analysis was conducted on the designed specimens by applying a pulsating displacement with 20 kHz frequency at the end of specimens. The applied displacement excited the natural mode of the specimen at 20 kHz and a simulation of 50 cycles was done in order to determine the stresses in the specimen. The stress-

Figure 3. (a) A model of the horseshoe fixture and specimen connected to the bottom end of an amplification horn (b) detailed view of the horseshoe fixture
displacement factor of 29.70 MPa/µm was obtained i.e. for each 1 µm of applied displacement the stress level of 29.70 MPa at the surface of smallest section of the specimen was reached. The dimensions of the designed specimen are shown in Figure 4.

![Figure 4. The design of hourglass shaped cylindrical specimens used in Paper III.](image)

Testing was carried out at a load ratio of R=-1. To avoid excessive heating in the specimens, testing was conducted by dipping specimens in circulating water mixed with anti-corrosion additive in the ratio of 1:33. Furthermore, the fatigue testing was conducted using the staircase testing method. A fatigue life of $10^8$ cycles was selected where the fatigue strength needs to be determined. If the specimen failed before $10^8$ cycles then the next specimen was tested at 10 MPa lower stress level and vice versa.

4 Summary of papers

Three research papers constitute the main part of the research work done in this thesis. Two papers are based on the fatigue test results while the third paper is based on theoretical modelling of short cracks. A brief summary of all three papers is presented in the following sections.

4.1 Ultrasonic fatigue testing of duplex stainless steels (Papers I, II)

The results of fatigue testing conducted on the cold rolled duplex stainless strip steel grade are presented in Figure 5. The fatigue failures occurred in the 400-500 MPa range. A specimen run-out life length of $10^9$ cycles was chosen,
therefore, the data points with white markers were considered the run-outs and in fact have not failed. The fatigue stress levels showed very little variation with the fatigue life length of each failed specimen. However, no evidence of fatigue limit was observed for the tested duplex stainless strip steel grade.

![Fatigue testing results presented in the form of SN data for the cold rolled duplex stainless strip steel grade.](image)

**Figure 5.** Fatigue testing results presented in the form of SN data for the cold rolled duplex stainless strip steel grade.

Results from the ultrasonic fatigue testing of hot rolled duplex stainless steel grades 2304 SRG and LDX 2101® are presented in Figure 6. Again, no evidence of fatigue limit was observed from the fatigue test data. Instead, a decreasing trend of fatigue strength levels with fatigue life length was observed. However, the fatigue stress levels were much lower compared to the cold rolled duplex stainless strip steel grade due to the difference in the rolling condition and mechanical properties. The cold rolled steel grade has a much finer microstructure and is strain hardened to a higher tensile strength compared to the hot rolled steel grades.
4.1.1 Fracture surface morphology

A typical example of fracture surface morphology of the failed cold rolled duplex stainless steel strip steel specimen is shown in Figure 7a. Depending on fracture surface morphology and crack growth rates, the fracture surface was divided into four different regions. Region 1 consists of the fatigue crack initiating surface defect and the immediately surrounding region which has a fine grained (FGA) appearance. This has been observed previously by researchers in the surrounding region of crack initiation occurring from non-metallic inclusions [2, 15]. In the case of cold rolled strip steel specimens, the fatigue crack initiation invariably occurred at the surface defects which were leftover from the cold rolling process. An example of a typical crack initiating surface defect is shown in Figure 7b. No evidence of internal fatigue crack initiation with fish-eye type of morphology was observed in the tested grade.
Figure 7. (a) Typical fracture appearance of a failed specimen \((2.67 \times 10^8 \text{ cycles, } 450 \text{ MPa})\) from the cold rolled duplex stainless strip steel grade (b) Fine Granular Area (FGA) surrounding the surface crack initiation defect.

A typical fracture surface chosen to represent the case of hot rolled duplex stainless steel grades 2304 SRG and LDX 2101® is shown in Figure 8. In Figure 8a, an overview of the fracture surface is shown which is further divided into four different regions. Region 1 consists of a surface crack initiation site and the surrounding region in which the growth of a short crack occurred along crystallographic planes, shown in detail in Figure 8b. It should be noted that in the case of the hot rolled duplex stainless grades no evidence of fatigue crack initiation from any kind of surface rolling or machining defects was found. The grains in Region 1 appear to have smooth faceted appearance and the crack path angle changes upon crossing the grain boundaries. In Region 2, the fracture surface had a rather smooth morphology and the crack growth occurred perpendicular to the applied load direction. The distinct morphology of the austenitic-ferritic phases can be observed in Region 2, shown in Figure 8c, d. In Region 3, the crack growth rate increase evidenced by widely spaced striations was observed before the specimen failed due to single overload in Region 4, which has ductile failure morphology.
4.1.2 Fatigue crack initiation mechanism

In the case of the cold rolled duplex stainless strip steel grade, the crack initiation always occurred at surface defects left by the cold rolling process. However, it should be mentioned that since the tested grade is not produced to the final thickness of 1 mm, the surface conditions presented here are not the ones found in the actual product. A typical example of a fatigue crack initiating surface defect is shown in Figure 9.
Figure 9. A typical example of a cold rolling induced surface defect found on the longitudinal surface of specimen (2.67x10^8 cycles, 450 MPa) that resulted in fatigue crack initiation.

Similarly, for the case of hot rolled duplex stainless steel grades, an analysis of the crack initiation mechanism was done by investigating the external surface of the specimens. For both tested grades, 2304 SRG and LDX 2101®, the fatigue crack initiation occurred due to accumulation of plastic strain at the external surfaces of the specimens. However, there were some fundamental differences in the underlying mechanism of the accumulation of the plastic damage between the two grades. As shown in Figure 10a, for 2304 SRG the plastic fatigue damage was observed to occur mainly along the austenite-ferrite phase boundaries. In the LDX 2101® grade, the plastic fatigue damage was observed to be concentrated in the ferrite phase and the austenite grains appeared to be completely damage free as shown in Figure 10b.
4.1.3 Fatigue crack propagation

For the cold rolled duplex stainless strip steel grade, the fatigue crack growth rate was estimated by measuring the striation spacing on three specimen running up to 35, 46 and 214 million cycles to failure. It was possible to detect the striations as close as around 20 µm away from the crack initiation site. The crack growth rate is plotted in Figure 11 against the distance from the crack initiation point. One can observe that the lowest crack growth rate estimated was around 15-60 nm/cycle. The growth rate stayed almost constant over a certain distance before it started to rise considerably and resulted in final failure.
For the hot rolled steel grades 2304 SRG and LDX 2101®, the crack growth rate was again estimated from measurements of striation spacing at the fracture surface using SEM. Two specimens were chosen from each grade to represent the typical cases of short fatigue life and gigacycle fatigue life. The estimated crack growth rate was plotted against the distance from the fatigue crack initiation site, as shown in Figure 12. The results showed a similar trend for all four specimens, where, initially the growth rate was low around 10 – 40nm/cycle and stayed constant over a certain distance on the fracture surface. After a certain distance, which varied for each specimen, the crack growth rate started to rise considerably before reaching the final failure. However, one can notice a similarity in the crack growth rate trend for the cold rolled duplex stainless strip steel grade and the hot rolled grades.
5 Theoretical modelling of short cracks

The VHCF life is controlled by the behaviour of the crack when it is short in comparison to other dimensions of the body. Knowledge of the crack tip stress intensity factor in this regime is of high importance. A distributed dislocation dipole technique developed as a part of this work can be used to compute the stress intensity factor for cracks which are short compared to the dimensions of the whole body.
5.1 Stress intensity factor [16]

A crack tip in an arbitrary body is schematically shown in Figure 13. For a linear elastic and isotropic material, the stresses around the crack tip, in terms of polar coordinates, are given by:

$$\sigma_{ij} = \left( \frac{k}{\sqrt{r}} \right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^m g_{ij}^{(m)}(\theta)$$

(1)

where \(\sigma_{ij}\) is the stress tensor, \(r\) is the distance to the point of interest, \(\theta\) the angle to the point of interest, \(k\) is a constant and \(f_{ij}\) is a dimensionless function of \(\theta\). The higher order terms after the addition sign depend on the geometry of the body and are influenced by the remote boundary conditions.

The stress field close to the crack tip, shown in figure 13, can be represented as:

$$\lim_{r \to 0} \sigma_{ij} = \left( \frac{k}{\sqrt{r}} \right) f_{ij}(\theta) + \sum_{m=0}^{\infty} A_m r^m g_{ij}^{(m)}(\theta) = \left( \frac{k}{\sqrt{r}} \right) f_{ij}(\theta)$$

(2)

When \(r\) approaches 0 the leading term in equation (2) goes to infinity whereas the higher order terms remain finite or go to zero. Thus, the governing term for the stress field is \(1/\sqrt{r}\) close to the crack tip, i.e. a stress singularity exists in the vicinity of crack tip in an elastic material.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} = \frac{k}{\sqrt{2\pi r}} \begin{bmatrix} 1 - \sin \left( \frac{\theta}{2} \right) \sin \left( \frac{3\theta}{2} \right) \\ 1 + \sin \left( \frac{\theta}{2} \right) \sin \left( \frac{3\theta}{2} \right) \\ \sin \left( \frac{\theta}{2} \right) \cos \left( \frac{3\theta}{2} \right) \end{bmatrix}$$

(3)

Each component of the stress is proportional to a single constant, \(K\), called the stress intensity factor. If this constant is calculated, the entire stress distribution at the crack can be computed by equation (3). The stress intensity factor fully characterizes the stress distribution around the crack tip because it represents the magnitude of the dominant singular term in a series expansion of the stress state. For small \(r\), the leading singular term dominates the behaviour of the stresses. If it is assumed that the fracture occurs locally at some critical combination of stress and strain, then it follows that fracture must occur at a critical stress intensity factor, \(K_C\). Therefore, \(K_C\) is a measure of fracture toughness.
The stress intensity factor in the generalized form can be written as:

\[ K_I = \sigma_a Y \sqrt{\pi a} \]  

(4)

where \( \sigma_a \) is applied stress and \( Y \) is a geometrical factor, which depends on the geometry of the body and the crack.

In a linear elastic material, the fatigue crack growth is purely a function of the stress intensity factor. The most frequently used expression, obtained from correlation of experimental data, is the empirical Paris law [17]:

\[ \frac{da}{dN} = A(\Delta K)^n \]  

(5)

Where \( A \), \( n \) are the constants which can be found in data books for many materials. From the above discussion, we have seen that the stress intensity factor is the controlling parameter for the growth of cracks in a body not only under monotonic but also under cyclic loading conditions.
5.2 Distributed dislocation dipole technique – brief introduction

5.2.1 Dislocation dipole

In a distributed dislocation dipole technique, the dislocation dipoles act as strain nuclei which are distributed along the crack lines in an otherwise perfect body. An infinitesimal dipole consists of a pair of dislocations of opposite signs separated by an infinitesimal distance from each other. It should be clarified that, although the dislocation dipoles, employed in this technique, are composed of pairs of edge dislocations existing as lattice defects; the presence of lattice defects is _not_ implied. The dislocation dipoles are only the mathematical tools to introduce a controlled, self-consistent state of stress in the body.

In Figure 14, an example of two dislocation dipoles, \( b_{11} \) and \( b_{21} \), is shown which correspond to opening and sliding across the \( x_2 \) axis. The subscripts in the notation \( b_{ij} \) indicate whether the dipole corresponds to opening or sliding displacement. The first subscript \( i \) indicates the direction of burgers vector whereas the second subscript \( j \) indicates the direction normal to the distance between the dislocation pair.

![Figure 14. Dislocation dipoles corresponding to opening (a) and sliding displacement (b) across \( x_2 \) axis][18]

The dislocation dipoles can be interpreted as sources of strain within the body. Therefore, stresses due to an arbitrary infinitesimal dislocation dipole are proportional to the displacement induced by it, as expressed by the following equation [19]:

\[
d\sigma_{ij} = \frac{2\mu}{\pi(x+1)}db_{kl}L_{ij}^{kl} \tag{6}
\]
where $\mu$ is the shear modulus and $\kappa = 3 - 4\nu$ for plane strain or $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress. The factors $L_{ij}^{kl}$ are dipole influence functions, depending on the geometry of the body, which can be found in Hills et al. [19].

Figure 15. General dislocation dipole in an infinite half plane [18]

### 5.2.2 Bueckner’s theorem

Bueckner’s principle is illustrated in Figure 16. Figure 16a shows the cracked body subjected to far-field loading in which the stress intensity factors at tips of cracks 1 and 2 needs to be computed. The problem of a cracked body subjected to far-field traction in Figure 16a is equivalent to superposition of the solution obtained by solving for the stress state $\sigma_{22}^{(i)}, \sigma_{12}^{(i)}$ at the imaginary crack lines in the externally loaded but uncracked body in Figure 16b and applying the equal but opposite stresses $-\sigma_{22}^{(i)}, -\sigma_{12}^{(i)}$ at the crack faces in the cracked body but devoid of external loads in Figure 16c. The superscript $i$ denotes the crack segment. In response to the applied stresses the crack faces will exhibit opening and sliding displacements. In the distributed dislocation dipole technique, dislocation dipole densities represent the displacement of crack faces which are the unknowns of the analysis.
5.2.3 Method

The problem is reduced to the determination of unknown dislocation dipole densities in an uncracked body, which generate equal and opposite stresses to those generated by the far-field stress. This makes the crack faces to be traction-free. Once the dislocation dipole density is determined, the stress intensity factor, and the stress state at any point in the body, can be computed. In *Paper III*, this technique has been used for the analysis of different geometries of cracks in an elastic half plane.

6 Summary of Paper III

In Paper III, a distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half plane was developed. The main goal of this work was the evaluation of stress intensity factors at each crack tip. Furthermore, the evaluation of the stress state at an arbitrary position in the body was also described. Contact between the crack faces was ignored and the developed method applies only for fully open cracks.
The numerical procedure was based on the Bueckner’s theorem [20], described already in section 4.2.2. The opening and sliding displacements of the crack surfaces, in reaction to the applied stresses, were represented by unknown distributions of dislocation dipole densities along each crack segment. A brief description of a dislocation dipole, and the stresses induced by it, has already been presented in section 4.

The dipole density distribution was expressed by aid of a weight function $w^{(q)}(s)$ multiplied by a regular function $\varphi^{(q)}_{m2}(s)$. The weight function $w^{(q)}(s) = (1 - s)^{\gamma^+(q)} (1 + s)^{\gamma^-(q)}$ captured the singularity at the crack ends and the regular function $\varphi^{(q)}_{m2}(s) = \sum_{n=0}^{Np} c^{(q)}_{m2,n} p_n^{(q)}(s)$ was expressed as a series of $Np+1$ Jacobi polynomials with coefficients $c^{(q)}_{m2,n}$, where $n=0,1,...,Np$ and $m=1,2$. The expression for the dipole density distribution is given by equation (7):

$$B_{m2(q)}(s) = (1 - s)^{\gamma^+(q)} (1 + s)^{\gamma^-(q)} \sum_{n=0}^{Np} c^{(q)}_{m2,n} p_n^{(q)}(s) + B_{m2,+1}^{(q)} \Delta(\gamma^+(q)) \frac{1}{2^{\gamma^+(q)}} (1 + s)^{\gamma^-(q)} + B_{m2,-1}^{(q)} \Delta(\gamma^-(q)) \frac{1}{2^{\gamma^-(q)}} (1 - s)^{\gamma^+(q)}$$

(7)

The first and second factors of the weight function correspond to behaviour of the dipole density at the positive ($s=+1$) and the negative ($s=-1$) endpoints, respectively. The second and third terms are the conditional terms that represent the opening and sliding displacements at the crack segment endpoints joined to a kink or a branch. Williams [21] gave an expression for the stress state in the vicinity of corner in an elastic body. For a corner in which the elastic material occupies an angle $\Delta\theta>\pi$ the stresses become infinite as the distance $r$ from the tip of corner approaches zero. The singularity associated with the symmetric part of stress state is stronger than the singularity associated with the asymmetric part. Therefore, in this paper, the asymptotic behaviour of the stresses at crack kinking or branching was assumed to behave according to the most singular part, i.e. $\sigma_{ij} \sim r^{\lambda_1^{(1)}-1}$. Here, the term $\lambda_1^{(1)}$ is the leading term of the series of eigenvalues of the symmetric part in the Williams’ series expansion.
The stress conditions were enforced at collocation points along each crack segment, which resulted in two coupled integral equations for each component of stress at each collocation point. The integral equation was obtained by inserting the expression for the dipole density distribution into the equation for the stresses due to a dislocation dipole equation (6), and integrating it over the crack length of each crack segment \( q \). This resulted in the following integral equation.

\[
-\pi(\kappa+1)\frac{\sigma_{2g}^{(p)}}{2\mu} \mathcal{I}_n^{(p)}(t) = \\
\sum_{q=1,q\neq p}^{N_p} a_q \sum_{n=0}^{N_p} c_{m,2,n}^{(q)} \int_{-1}^{1} (1-s)\phi_{+}^{(q)} (1+s)\phi_{-}^{(q)} P_n^{(q)}(s) L_{2g}^{m2(q)}(t,s) ds \\
+ \frac{1}{2\gamma^2} B_{m,2,1}^{(q)} \Delta(\phi_{+}^{(q)}) \int_{-1}^{1} (1+s)\phi_{+}^{(q)} L_{2g}^{m2(q)}(t,s) ds \\
+ \frac{1}{2\gamma^2} B_{m,2,-1}^{(q)} \Delta(\phi_{+}^{(q)}) \int_{-1}^{1} (1-s)\phi_{+}^{(q)} L_{2g}^{m2(q)}(t,s) ds \\
+ \sum_{n=0}^{N_p} \left\{ \frac{c_{g,2,n}^{(p)}}{\alpha_p} \int_{-1}^{1} (1-s)\phi_{+}^{(p)} (1+s)\phi_{-}^{(p)} P_n^{(p)}(s) (s-t)^2 ds \\
+ a_p c_{m,2,n}^{(p)} \int_{-1}^{1} (1-s)\phi_{+}^{(p)} (1+s)\phi_{-}^{(p)} P_n^{(p)}(s) L_{2g}^{m2(p)}(t,s) ds \right\} \\
+ \frac{B_{m,2,1}^{(p)}}{2\gamma^2} \Delta(\phi_{+}^{(p)}) \int_{-1}^{1} (1+s)\phi_{+}^{(p)} ds + a_p \int_{-1}^{1} (1+s)\phi_{+}^{(p)} L_{2g}^{m2(p)}(t,s) ds \\
+ \frac{B_{m,2,-1}^{(p)}}{2\gamma^2} \Delta(\phi_{+}^{(p)}) \int_{-1}^{1} (1-s)\phi_{+}^{(p)} ds + a_p \int_{-1}^{1} (1-s)\phi_{+}^{(p)} L_{2g}^{m2(p)}(t,s) ds \right\}
\]

\( p = 1, \ldots N, g = 1,2 \)

In equation (8), for the case when \( q=p \) the hypersingular integrals are obtained i.e. when the collocation point is at the same crack segment where the dipole is located. These hypersingular integrals are inherent to the procedure. When \( s \rightarrow t \) the terms \( 1/(s-t)^2 \) in the integral equation (8) approach infinity in quadratic manner which means a stronger singularity. Therefore, in order to avoid singularity of solution the hypersingular integrals in equation (8) were solved by using the Hadamard principal value, which was further obtained by differentiating the Cauchy principal value [22]. The regular integrals in the integral equation were solved by the Gauss-Jacobi quadrature rule. The
equation (8) was repeatedly applied at each collocation point, and for each of the two stress components, resulting in a linear equation system.

In addition to the integral equation (8), there are certain continuity constraints that need to be enforced at crack kinking and/or branching. At crack kinking and/or branching the net opening and sliding displacements (i.e. the dislocation dipole densities) should cancel out. This condition could be expressed as:

\[ \sum_{q \in Q_i} \pm \alpha_{mk}^{(q)} B_{k2(q)}(s \to \pm 1) = 0 \]  

(9)

Taking the derivative of equation (9) ensures compatible rotations of crack segment end points. This condition, which serves to preserve the continuity of the dipole density at crack kinking or branching, is given by:

\[ \sum_{q \in Q_i} \pm \alpha_{mk}^{(q)} \frac{dB_{k2(q)}}{ds} (s \to \pm 1) = 0 \]  

(10)

The linear equation system, obtained by enforcing the stress condition at each collocation point and the continuity conditions, is solved by the Gaussian elimination method for the unknown coefficients of the dipole density distribution. Once the coefficients of the dipole density distribution are obtained, the stress intensity factors can be evaluated by the following relations:

\[ K_I^{(q)}(t = \pm 1) = \lim_{t \to \pm 1} \frac{2\mu}{(k+1)} \sqrt{\frac{\pi}{2a_q(1+t)}} B_{22(q)}(t) \]  

(11a)

\[ K_{II}^{(q)}(t = \pm 1) = \lim_{t \to \pm 1} \frac{2\mu}{(k+1)} \sqrt{\frac{\pi}{2a_q(1+t)}} B_{12(q)}(t) \]  

(11b)

Using the above explained numerical procedure, different cases of multiple straight, kinked and branched cracks were treated in order to assure that the theory and implementation worked. The solutions were compared with the multi-purpose Finite Element Method (FEM) software Abaqus. The results indicated that Jacobi polynomial expansions of order \( N_p = 6 \) gave an accuracy to within 1\% and that higher order polynomials improved the accuracy. The present procedure was compared to a simplified procedure suggested in the literature which ignores the stress singularities associated with corners at kinking/branching in the dipole density representation. From this comparison,
the workability of both methods was proved; however, the present method achieved the same accuracy using much lower order polynomial expansions.
7 Conclusions

Based on the experimental results and the analysis, the following conclusions were drawn:

1. The horseshoe type of fixture for thin strip testing presented in this study worked well for 20 kHz ultrasonic fatigue testing. Experimental data in the VHCF regime was obtained for the selected stainless steel grades.

2. No evidence of fatigue limit was observed for any of the tested grades.

3. In the case of the cold rolled duplex stainless strip steel grade, a stress intensity range threshold between 1-3 MPa√m needed to be overcome to initiate a crack from a surface defect, and another stress intensity range threshold in the range of 3-5 MPa√m was found at the crack transition out of the fine granular area.

4. In the case of the hot rolled duplex stainless steel, the fatigue crack initiation occurred due to the accumulation of plastic slip fatigue damage at the external surfaces of the specimens.

Based on the different cases of cracks analysed by the dislocation dipole distribution technique developed in this thesis work, the following conclusions were drawn:

5. The dislocation dipole density distribution technique attained an accuracy of less than 1% error using the sixth order Jacobi polynomials when compared with the results from FEM software Abaqus. The accuracy can be further improved by increasing the order of Jacobi polynomials.

6. The dislocation dipole density distribution technique showed faster convergence rate to attain the same level of accuracy when compared to the simplified procedure, which ignored the representation of a singularity at the crack kinking and/or branching.
8 Future work

Further investigation of the fatigue crack initiation mechanisms prevalent in both duplex stainless strip steel grades and the hot rolled grades is required. In the case of cold rolled duplex stainless steel, the Fine Granular Area (FGA) found around the fatigue crack initiating surface defects needs to be studied in further detail. In this regard, Focussed Ion Beam (FIB) cut-outs will be taken from the Fine Granular Area (FGA) and studied using Transmission Electron Microscope (TEM). Additionally, an investigation of the crystallographic nature of short crack growth in the Region 1 of the fracture surface in the hot rolled duplex stainless steel grades 2304 SRG and LDX 2101® is required. An Electron Backscattered Diffraction (EBSD) technique can be useful in this regard. Furthermore, the mechanism of inhomogeneous plastic fatigue damage accumulation among the two different crystallographic phases also needs to be further investigated.

The distributed dislocation dipole technique for the analysis of cracks needs to be further improved. The description of the singularity associated with the crack kinking and branching needs further improvement. The full description of the singularity including both the symmetric and asymmetric part of the Williams expression needs to be added in the dipole density representation. Another of the future goals is modelling of crack closure and contact between the crack faces.

9 References


Very high cycle fatigue of duplex stainless steels and stress intensity calculations

Very high cycle fatigue (VHCF) is considered as the domain of fatigue lifetime beyond $10^7$ load cycles. Structural components which are subjected to $10^7$-$10^9$ load cycles during their service life are engine parts, turbine disks and railway axles. The reliable operation of these components depends on the knowledge of the prevalent damage/failure mechanisms. This study focused on the evaluation of fatigue properties of duplex stainless steels in the VHCF regime. The ultrasonic fatigue tests were conducted on cold rolled duplex stainless strip steel and hot rolled duplex stainless steel grades. Considerable attention was devoted to the evaluation of fatigue crack initiation and growth mechanisms.

The second part of this thesis work was the development of a distributed dislocation dipole technique for the analysis of multiple straight, kinked and branched cracks in an elastic half plane. The main focus of this technique was the evaluation of the stress intensity factors at each of the multiple crack tips. Several test cases were analysed, and the results suggest that the accuracy of within 1% is achieved provided that Jacobi polynomial expansions up to at least the sixth order are used.