Project Joyride
The Ecofriendly Singleseater Aircraft of the Future

SA108X, Degree Project in Mechanical Engineering, First Level

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Abstract
This thesis presents an aircraft concept, The Joyride, supposed to satisfy the future demand for sustainable hobby aircrafts. The work was done through research on environmental friendly aircraft complimented by lectures on aircraft dynamics held by supervisor Arne Karlsson. A propulsion system was designed and two different future energy sources were examined. Sizing and performance analyses were executed in an iterative trial and error fashion to fulfil the specified aims for the aircraft. The final results present a zero-emissions, light weight, blended wing body aircraft.

The Joyride has got a maximum takeoff gross weight of less than 800 kg, wingspan of 16 m, wingarea of 24 m², cruise altitude of 6000 m and a maximum range of 850 km. It is equipped with an electric motor with a peak power of 120 kW~161 hp. This makes the Joyride a small and agile aircraft capable of reaching cruise altitude in less than 10 minutes and completing a 180° turn in just over 6 seconds, in other words perfectly suited for an aircraft enthusiast.

It is the authors’ belief that the Joyride is possible to realize within a timeframe of 50 years.
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Introduction
Today's increased awareness of environment- and energy issues have presented a new challenge for the aircraft industry, namely the development of aircraft with significantly smaller environmental impact. Project Joyride is a bachelor thesis at the Royal Institute of Technology in Stockholm, Sweden and it pertains to present a conceptual study of a future hobby aircraft of this kind. The concept is expected to be possible to realize within the authors' future working careers.

The aircraft was decided to be a single seater of blended wing body design with pusher configuration using a propeller connected to an electric motor for the propulsion. Aside from the environmental aims a lot of emphasis was put on flying qualities. The demand for a relatively high cruise altitude called for the integration of cockpit heating and a pilot oxygen system.

The decision to use a blended wing body design was due to the environmental benefits this specific airplane design gives. This aircraft design generates more lift and offers a reduction in drag compared to conventional design.

One aggravating circumstance was that it as of today, not considering military aircraft, only a couple of downsized experimental, unmanned aerial vehicles of this design has been produced. Because of this, some assumed values, normally acquired from previous aircraft of the same kind, instead had to be derived from research.
Method
Information was gathered through lectures held by supervisor Arne Karlsson, literature, papers and extensive research on the World Wide Web. Concepts and aircraft of similar design were studied and performance calculations were made with the numerical computation tool MATLAB, see Appendix C.

I. Sizing
The development of the Joyride began with our idea of how a future hobby aircraft should look and be able to perform and through those ideas a list of specific requirements which is seen in Appendix A was agreed upon.

I.I. Geometry & Design
It was early on decided that the aircraft would be of Blended Wing Body design and therefore initial sizing was done in part by looking at a previous aircraft of this kind, the Boeing X-48 experimental UAV but also through earlier research on this type of aircraft design.

Through known specifications of another BWB concept aircraft (Liebeck, 2004) a three-view image of a blended wing body design and geometric requirements, initial size values were attained. There is one constraint unique to the blended wing body configuration and that is that the whole aircraft is limited by a maximum thickness to chord ratio, i.e. the aircraft’s thickness, \( t \), and length, \( l \), are limited by one another. For the Joyride it was essential for this ratio to be as big as possible. That was because the Joyride is only meant for a single person and therefore shouldn’t be too oversized. The ratio cannot be too big either because then it hampers the aerodynamic properties of the design.

Figure 1: Three-view image of a BWB design
I.II. Weight

The initial estimation for the aircrafts takeoff gross weight, \( m_0 \), was set as the accepted maximum weight according to the specifications of requirement in appendix A. The empty weight was estimated by looking at the Boeing X-48B which has a wing area of 9.34 \( m^2 \) and a gross weight of 227 \( kg \), from this number 15 \( kg \) was subtracted because of motor weight and another 20 \( kg \) was subtracted as avionics and payload weight. The estimated empty weight of the X-48B was then adjusted by ratioing based on the difference in surface area between the X-48B and the Joyride (Raymer, 2012). The battery weight was then acquired from equation (2). Through other weight requirements concerning crew, payload and equipment together with an estimated empty weight an initial battery weight was also acquired.

\[
m_{\text{empty}} = \left( m_{X-48B\text{empty}} - 35 \right) \cdot \frac{s}{s_{X-48B}} \quad (1)
\]

\[
m_{\text{battery}} = m_0 - m_{\text{crew}} - m_{\text{payload}} - m_{\text{empty}} - m_{eq}. \quad (2)
\]

Later during the project when the energy needed for the whole flight mission had been calculated and the actual battery weight needed was known, equation (2) was revisited and a final accurate takeoff gross weight was acquired.

Another important geometric value needed is the mean aerodynamic chord, MAC. The MAC was calculated by splitting the wing area in two parts, inner and outer wing and then treating them as simple tapered wings. The respective MAC for these parts were then calculated with equation (3) and then the average value from these accepted as the Joyrides mean aerodynamic chord, equation (5).

\[
\bar{c} = \frac{1 + \lambda + \lambda^2}{1 + \lambda} \cdot \frac{2}{3} \cdot c_{\text{root}} \quad (3)
\]

\[
\lambda = \frac{c_{\text{tip}}}{c_{\text{root}}} \quad (4)
\]

Where \( c_{\text{tip}} \) and \( c_{\text{root}} \) is the chord length of the wingtip and wingroot, these were acquired from figure (1) for both wing parts.

\[
\bar{c} = \frac{\bar{c}_1 + \bar{c}_2}{2} \quad (5)
\]

I.II. Estimation of zero-lift drag coefficient, & drag-due-to-lift factor

The component buildup method is a method for establishing the zero-lift drag coefficient, \( C_{D0} \), with decent precision. This method sums the different parts of the aircraft and their properties instead of bundling them up as other methods such as the equivalent skin friction method do. The method accounts for skin friction, interference drag and geometry through the following variables.

Flat-plate skin friction coefficient \( C_F \)
Form factor \( F \)
Interference factor \( Q \)
Wetted surface \( S_{\text{wet}} \)

\[
C_{D0} = \frac{1}{2} \sum c \left[ C_{F,c} \cdot F_c \cdot Q_c \cdot S_{\text{wet},c} \right] + \Delta C_{D_{\text{misc}}} + \Delta C_{D_{\text{L&P}}} \quad (6)
\]

Where index \( c \) is for component number \( c \).
The joyride was divided in three components, fuselage, main wing and winglets, the terms inside the sum in equation (6) were evaluated for all of these components. The remaining right hand terms will only be taken into account on components of an aircraft who aren’t slender and not appropriate to be approximated as a flat plate. For the joyride these terms can be omitted.

The drag-due-to-lift factor, \( K \), was estimated from equation (7).

\[
K = \frac{1}{\pi AR e_0}
\]  

(7)

Where \( AR \) is the aircraft aspect ratio defined as the wing span squared divided by the wing area.

\[
AR = \frac{b^2}{S}
\]  

(8)

e_0 \) is the Oswald efficiency factor and was calculated from equation (9).

\[
e_0 = 4.61 \cdot (1 - 0.045 \cdot AR^{0.60}) \cdot (\cos \Lambda_{LE})^{0.15} - 3.1
\]  

(9)

Where \( \Lambda_{LE} \) is the leading edge sweep angle of the wing.

II. Performance

II.I. Steady and Level Flight

During steady and level flight the airplane is not accelerating in any direction, therefore the sum of all forces on the airplane must equal zero. Representing this with a free body diagram and a force balance, expressions for minimum thrust required, minimum power required and their corresponding velocities are derived. Other velocities of substantial interest such as stall-, cruise- and maximum velocity were also derived. Since the thrust during this condition is the thrust required to balance the drag it is referred to as \( T_r \), thrust required in the calculations below.

\[
L - m_0 \cdot g = 0
\]  

(10)

\[
T_r - D = 0
\]  

(11)

\[
L = m_0 \cdot g = \frac{1}{2} \cdot C_L \cdot \rho \cdot v^2 \cdot S \rightarrow C_L = \frac{m_0 \cdot g}{\frac{1}{2} \rho \cdot v^2 \cdot S}
\]  

(12)

Figure 2: Free Body Diagram, steady and level flight
\[ C_D = C_{D_0} + K \cdot C_L^2 = \frac{D}{\frac{1}{2} \rho v^2 S} \]  \hspace{1cm} (13)

\[ T_r = \frac{1}{2} \cdot C_D \cdot \rho \cdot v^2 \cdot S = \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot \left( C_{D_0} + K \cdot C_L^2 \right) \]  \hspace{1cm} (14)

\[ T_r = \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot C_{D_0} + \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot K \cdot C_L^2 \]  \hspace{1cm} (15)

\[ T_r = \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot C_{D_0} + \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot K \cdot \left( \frac{m_0 g}{\frac{1}{2} \rho v^2 S} \right)^2 \]  \hspace{1cm} (16)

To simplify this the dynamic pressure is represented by \( q \).

\[ q = \frac{1}{2} \cdot \rho \cdot v^2 \]  \hspace{1cm} (17)

\[ T_r = q \cdot S \cdot C_{D_0} + K \cdot \left( \frac{m_0 g}{\frac{1}{2} \rho v^2 S} \right)^2 \]  \hspace{1cm} (18)

The first term in equation (18) is called the zero-lift drag or parasite drag, \( D_0 \). The zero-lift drag is proportional to the velocity squared and consist mainly of skin friction drag. The second term, which is inversely proportional to the velocity squared, is the induced drag, \( D_i \) and is caused by the generation of lift.

This flight condition determines the aircrafts maximum- and cruise velocity. Since this aircraft aims to be environmentally friendly the cruise speed is chosen as the speed which minimizes the energy required per distance. The maximum velocity is given from the fact that it occurs when the maximum engine power is used and that once it is reached the aircraft is in a steady state where the thrust equals the drag. There are three more velocities of interest and that is the velocity which minimizes the thrust required, the velocity which minimizes the power required and the velocity at which the aircraft stalls.

Deriving the maximum velocity.

\[ T_r \cdot v_{\text{max}} - D \cdot v_{\text{max}} = 0 \]  \hspace{1cm} (19)

\[ \eta_p \cdot P_{\text{eng max}} - D \cdot v_{\text{max}} = 0 \]  \hspace{1cm} (20)

\[ \eta_p \cdot P_{\text{eng max}} - v_{\text{max}} \cdot C_D \cdot S \cdot \frac{1}{2} \cdot \rho \cdot v_{\text{max}}^2 = 0 \]  \hspace{1cm} (21)

\[ \eta_p \cdot P_{\text{eng max}} - \left( C_{D_0} + K \cdot C_L^2 \right) \cdot \frac{1}{2} \cdot \rho \cdot v_{\text{max}}^3 \cdot S = 0 \]  \hspace{1cm} (22)

\[ C_L = \frac{m_0 g}{\frac{1}{2} \rho v_{\text{max}}^2 S} \]  \hspace{1cm} (23)

\[ \eta_p \cdot P_{\text{eng max}} - \left( C_{D_0} + K \cdot \left( \frac{m_0 g}{\frac{1}{2} \rho v_{\text{max}}^2 S} \right)^2 \right) \cdot \frac{1}{2} \cdot \rho \cdot v_{\text{max}}^3 \cdot S = 0 \]  \hspace{1cm} (24)

From this expression \( v_{\text{max}} \) was solved numerically.

Next the cruise speed is derived.

The energy required per distance is acquired by integrating the thrust over the distance.

\[ E_r = \int_{x_1}^{x_2} T_r \, dx = T_r \cdot (x_2 - x_1) \]  \hspace{1cm} (25)
By differentiating this expression with respect to velocity, the velocity which minimizes the energy required per distance is found, i.e. the cruise speed.

\[
\frac{dE}{dv} = \frac{d}{dv} \left[ \frac{1}{2} \rho \cdot v^2 \cdot S \cdot C_{D_0} + 2 \cdot K \cdot \frac{m_0^2 g^2}{\rho \cdot v^2 \cdot S} \right] \cdot (x_2 - x_1) = 0
\]  

(26)

\[
\rho \cdot S \cdot C_{D_0} \cdot v_{\text{cruise}} \cdot - \frac{4K \cdot m_0^2 g^2}{\rho \cdot S \cdot v_{\text{cruise}}^3} = 0
\]  

(27)

This gives the following expression for the sought velocity.

\[
v_{\text{cruise}} = \sqrt{\frac{2m_0 g}{\rho S} \cdot \frac{K}{C_{D_0}}} \]  

(28)

The third velocity, which minimizes the thrust required is derived by differentiating the thrust with respect to velocity. That is what we just did, except for the distance factor which got cancelled, so the velocity requiring minimum thrust is actually the same velocity which requires the minimum amount of energy per distance.

\[
v_{r_{\text{min}}} = v_{\text{cruise}} = \sqrt{\frac{2m_0 g}{\rho S} \cdot \frac{K}{C_{D_0}}} \]  

(29)

The velocity resulting in the minimum power required is now derived in a similar fashion.

\[
T_r \cdot v = D \cdot v
\]  

(30)

\[
P_r = \frac{1}{2} \rho \cdot v^3 \cdot S \cdot C_{D_0} + \frac{1}{2} \rho \cdot v^3 \cdot S \cdot K \cdot \left( \frac{m_0 g}{\rho \cdot v^2 \cdot S} \right)^2
\]  

(31)

\[
\frac{dP_r}{dv} = \frac{d}{dv} \left[ \frac{1}{2} \rho \cdot v^3 \cdot S \cdot C_{D_0} + 2 \cdot K \cdot \frac{m_0^2 g^2}{\rho \cdot v^2 \cdot S} \right] = 0
\]  

(32)

\[
\frac{3}{2} \rho \cdot v^2 \cdot P_{r_{\text{min}}} \cdot S \cdot C_{D_0} - 2 \cdot K \cdot \frac{m_0^2 g^2}{\rho \cdot S \cdot v_{r_{\text{min}}}^3} = 0
\]  

(33)

\[
v_{P_{r_{\text{min}}}} = \sqrt{\frac{2m_0 g}{\rho S} \cdot \frac{K}{3C_{D_0}}} \]  

(34)

Lastly, aircrafts must maintain a certain velocity to prevent the airflow from separating from the wing, causing a massive drop of lift. This lower limit for the velocity is the stall speed. The stall speed is an important factor in the aircrafts performance, especially during takeoff, climb and landing conditions. The aircrafts stall speed is acquired from the maximum lift coefficient requirement seen in appendix A.

\[
C_{L_{\text{max}}} = \frac{L}{\frac{1}{2} \rho v_{\text{stall}}^2 \cdot S}
\]  

(35)

Equation (32) then gives an expression for the stall speed.

\[
v_{\text{stall}} = \sqrt{\frac{m_0 g}{\frac{1}{2} \rho C_{L_{\text{max}}} \cdot S}}
\]  

(36)

By examining equation (36) it is seen that the stall speed is inversely proportional to the density of the surrounding air which in turn is inversely proportional to the altitude, hence the stall speed is increasing with increasing altitude.
The energy required during this part of the flight mission was calculated using equation (37).

\[ E_{\text{slf}} = \frac{P_{\text{eng}}}{} \cdot \eta_m \cdot v_{\text{cruise}} \cdot R \]  
\[ (37) \]

Where \( P_{\text{eng}} \) is the continuous motor power, \( \eta_m \) the motor efficiency and \( R \) the required flight mission range found in appendix A.

II.II. Steady Climb

During steady climb, the aircraft will travel with a velocity, \( v \) and climb angle, \( \gamma \), the forces acting on the airplane are the same as in steady and level flight. See figure (3).

\[ T - D - m_0 \cdot g \cdot \sin \gamma = 0 \]  
\[ (38) \]

\[ L - m_0 \cdot g \cdot \cos \gamma = 0 \]  
\[ (39) \]

This gives the following expressions for the climb angle and lift coefficient, with \( q \) still equalling the dynamic pressure.

\[ \sin \gamma = \frac{T-D}{m_0g} \]  
\[ (40) \]

\[ c_L = \frac{m_0g \cdot \cos \gamma}{q \cdot S} \]  
\[ (41) \]

The rate of climb, \( R/C \) is the vertical component of the velocity vector, \( v \).

\[ R/C = v \cdot \sin \gamma = v \cdot \frac{(T-D)}{m_0g} = \frac{P_{pr} - Dv}{m_0g} = \eta_{pr} \cdot \frac{P_{\text{eng}} - Dv}{m_0g} \]  
\[ (42) \]

In equation (42), \( T \cdot v \) is the power available, i.e. the propulsive power and \( D \cdot v \) is the power required to balance the drag. Assuming yet again that the aircraft has a parabolic drag polar, equation (13), a new expression can be derived where \( q \) is the dynamic pressure.

\[ q = \frac{1}{2} \cdot \rho \cdot v^2 \]  
\[ (43) \]

\[ D = q \cdot C_D \cdot S = q \cdot S \cdot \left(C_{D_0} + K \cdot C_L^2\right) \]  
\[ (44) \]

\[ D = q \cdot S \cdot \left(C_{D_0} + K \cdot \left(\frac{m_0g \cdot \cos \gamma}{q \cdot S}\right)^2\right) \]  
\[ (45) \]

\[ D = q \cdot S \cdot C_{D_0} + K \cdot \left(\frac{m_0g \cdot \cos \gamma}{q \cdot S}\right)^2 \]  
\[ (46) \]

Now substituting this equation (46) into equation (42) gives equation (47). According to (1. Karlsson, 2013) an acceptable approximation when solving this is to set \( \cos^2 \gamma = 1 \).
\[
R/C = \frac{n_P P_{\text{eng}}}{m_0 g} - \frac{q^4 C_{D_0} v^4}{m_0 g} - 2 \cdot K \cdot \frac{m_0 g^{-1}}{\rho v} \cdot \frac{1}{\rho v} \cdot \frac{1}{\rho v} = 0
\]  
(47)

The climb angle \( \gamma \) is acquired from equation (42).

\[
\sin \gamma = \frac{R/C}{v}
\]  
(48)

By differentiating equation (47) with respect to velocity and searching for what value this equals zero, a velocity which maximizes the rate of climb is found, \( v_{R/C_{\text{max}}} \).

Recall that \( q = \frac{1}{2} \cdot \rho \cdot v^2 \) from equation (17).

\[
\frac{d(R/C)}{dv} = -C_{D_0} \cdot \frac{3}{2} \cdot \rho \cdot v_{R/C_{\text{max}}}^2 \cdot \frac{s}{m_0 g} + 2 \cdot K \cdot \frac{m_0 g}{\rho v_{R/C_{\text{max}}}^2} = 0
\]  
(49)

\[
v_{R/C_{\text{max}}} = \left[ \frac{4}{3} \cdot \frac{K}{C_{D_0}} \cdot \left(\frac{m_0 g}{S \cdot \rho}\right)^{2/3} \right]^{1/2}
\]  
(50)

Notice that \( v_{R/C_{\text{max}}} \) is inversely proportional to the density which is decreasing with altitude, this gives that \( v_{R/C_{\text{max}}} \) is increasing with the altitude. Another important thing is that values for \( v_{R/C_{\text{max}}} \) cannot be too small because then there is danger of stalling the aircraft. Therefore a lower limit for \( v_{R/C_{\text{max}}} \) was needed.

\[
v_{R/C_{\text{max}}} \geq 1.2 \cdot v_{\text{stall}}
\]  
(51)

When substituting equation (50) into equation (47) \( R/C_{\text{max}} \) is acquired, see equation (55) and (56), once again recall that \( q = \frac{1}{2} \cdot \rho \cdot v^2 \).

\[
R/C_{\text{max}} = \frac{n_P P_{\text{eng max}}}{m_0 g} - \frac{1}{2} \cdot \rho \cdot C_{D_0} v_{R/C_{\text{max}}}^3 \cdot \frac{s}{m_0 g} - 2 \cdot K \cdot \frac{m_0 g}{\rho v_{R/C_{\text{max}}}^2} = 0
\]  
(52)

\[
R/C_{\text{max}} = \frac{n_P P_{\text{eng max}}}{m_0 g} - 4 \cdot \left(\frac{4}{27} \cdot K^3 \cdot C_{D_0}\right)^{1/2} \cdot \sqrt{R/C_{\text{D_0}}}
\]  
(53)

For the joyride the maximum rate of climb, the corresponding velocity and then also a required engine power was acquired through an iterative process but first an expression for the climb time was needed. The minimum climb time to cruise level is acquired from the maximum rate of climb and cruise altitude, \( \gamma_c \), by integrating the inverse of the maximum rate of climb over the height difference from sea level to cruise altitude. As can be seen in appendix A, specific requirements for the cruise altitude and minimum climb time exist.

\[
t_{\text{min}} = \int_0^{\gamma_c} 1/(R/C_{\text{max}}) \, dh
\]  
(54)

This minimum climb time requires a certain engine power, \( P_{\text{eng}} \), which was solved for by calculating the maximum rate of climb for a sequence of engine powers until the minimum climb time requirement was satisfied. With the required engine power known it was also possible to calculate the energy required during climb from equation (55).

\[
E_{\text{climb}} = \frac{P_{\text{eng}}}{\eta_m \cdot t_{\text{min}}}
\]  
(55)

It was also of interest to analyse the maximum climb angle and the climb performance corresponding to this condition. This was done through combining equation (47) and (48) and then following the same steps used for maximum rate of climb.
\[
\sin \gamma = \frac{\eta_P P_{\text{eng}}}{m_0 g v} - q S C_D_0 - 2 \cdot K \cdot \frac{m_0 g}{\rho v^2 S} \tag{56}
\]

\[
\frac{d(sin \gamma)}{dv} = - \frac{\eta_P P_{\text{eng}}}{m_0 g v^2} - \frac{C_D_0 \rho v S}{m_0 g} + \frac{4 K m_0 g}{\rho S v^3} = 0 \tag{57}
\]

\[
\rightarrow \frac{C_D_0 \rho S}{m_0 g} \cdot v_{\gamma_{\text{max}}}^4 + \frac{\eta_P P_{\text{eng}}}{m_0 g} \cdot v - \frac{4 K m_0 g}{\rho S} = 0 \tag{58}
\]

This fourth degree equation was solved numerically with Newton Raphson’s method and so \( v_{\gamma_{\text{max}}}, \) was acquired.

Equation (56) together with the known \( v_{\gamma_{\text{max}}}, \) then provides \( \gamma_{\text{max}} \), see equation (59).

\[
\sin \gamma_{\text{max}} = \frac{\eta_P P_{\text{eng}}}{m_0 g v_{\gamma_{\text{max}}}^2} - q S C_D_0 - 2 \cdot K \cdot \frac{m_0 g}{\rho v_{\gamma_{\text{max}}}^2 S} \tag{59}
\]

The maximum rate of climb corresponding to the maximum climb angle is acquired from equation (60).

\[
\left( \frac{R/C_{\text{max}}}{\gamma_{\text{max}}} \right) = \frac{\eta_P P_{\text{eng}}}{m_0 g} \cdot \frac{1}{2 \rho S C_D_0 v_{\gamma_{\text{max}}}^3} - 2 \cdot K \cdot \frac{m_0 g}{\rho v_{\gamma_{\text{max}}} S} \tag{60}
\]

II.III. Level Turning Flight

During this condition the aircraft is tilted creating a horizontal component of the lift which causes the aircraft to turn, see the free body diagram in figure (4). The total lift on the aircraft, i.e. the vertical component plus the horizontal component of the lift, now equals a factor times the aircraft weight. This factor is called load factor and it is represented by \( n \). This load factor is essential when determining the turn radius and the turn rate, i.e. the angular velocity of the aircraft. Level turning flight can be divided in two subcategories due to turn rate requirements, the first is instantaneous turn rate the other is sustained turn rate. During an instantaneous turn the tangential velocity of the aircraft is allowed to decrease but during a sustained turn it is not. The minimum speed allowed is still 1.2 times the stall speed. The turn radius and turn rate are derived from the free body diagram in figure (4), force balance and basic mechanics.

The turn radius, \( r, \) is dependent on the radial acceleration \( a_r \).

\[
r = \frac{v^2}{a_r} \tag{61}
\]

Force balance in the radial direction gives.
\[ m_0 \cdot g \cdot \tan \theta = m_0 \cdot a_r \quad (62) \]

\[ a_r = g \cdot \tan \theta \quad (63) \]

\[ a_r = g \cdot \sqrt{n^2 - 1} \quad (64) \]

Combining equation (61) and (64) gives equation (65).

\[ r = \frac{v^2}{g \cdot \sqrt{n^2 - 1}} \quad (65) \]

From equation (65) it is seen that the turn radius is minimized for the maximum load factor and minimum velocity.

The turn rate, \( \omega \), is calculated through equation (66) - (67).

\[ \omega = \frac{v}{r} \quad (66) \]

\[ \omega = \frac{g}{v} \cdot \sqrt{n^2 - 1} \quad (67) \]

From equation (67) we can see that the turn rate is maximized for the maximum load factor and minimum velocity.

It is apparent that the maximum load factor is needed for both the maximum turn rate and minimum turn radius.

\[ T = D \quad (68) \]

\[ L = L_x + L_y = m_0 \cdot g \cdot n \quad (69) \]

From the definition of the lift coefficient we also have equation (70).

\[ L = \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot C_L \quad (70) \]

Combining equation (69) and (70) the first expression for the load factor is acquired in equation (71).

\[ n = \frac{\frac{1}{2} \rho v^2 \cdot S \cdot C_L}{m_0 g} \quad (71) \]

From equation (71) it is obvious that the load factor is maximized by the maximum lift coefficient.

Next a neat expression for the bank angle is constructed through equation (72) - (76).

\[ L^2 = (m_0 \cdot g)^2 + (m_0 \cdot g \cdot \tan \theta)^2 \quad (72) \]

\[ n^2 = \frac{L^2}{(m_0 g)^2} = 1 + \tan^2 \theta \quad (73) \]

\[ n = \sqrt{1 + \tan^2 \theta} \quad (74) \]

\[ L \cdot \cos \theta = m_0 \cdot g \quad (75) \]

Combining equation (75) and (69) gives equation (76).

\[ \cos \theta = \frac{1}{n} \quad (76) \]

Which due to the nature of the cosine function shows that the maximum bank angle is acquired from the maximum load factor.
Now equation (17), (68) and (13) is used to derive the second equation for the load factor.

\[ T = \frac{1}{2} C_D \cdot \rho \cdot v^2 \cdot S = \frac{1}{2} \cdot \rho \cdot v^2 \cdot S \cdot (C_{D_0} + K \cdot C_L^2) \]  
(77)

\[ T = \left\{ q = \frac{1}{2} \cdot \rho \cdot v^2 \right\} = q \cdot S \cdot C_{D_0} + q \cdot \left( \frac{n \cdot m_0 \cdot g}{q \cdot S} \right)^2 \]  
(78)

\[ T = q \cdot S \cdot C_{D_0} + \frac{(n \cdot m_0 \cdot g)^2}{q \cdot S} \]  
(79)

\[ n = \sqrt{\left( T - q \cdot S \cdot C_{D_0} \right) \cdot \frac{q \cdot S}{K} \cdot \frac{1}{m_0 \cdot g}} \]  
(80)

\[ n = \sqrt{\left( T \cdot v - q \cdot S \cdot C_{D_0} \cdot v \right) \cdot \frac{1}{K} \cdot \frac{\rho \cdot v \cdot S}{m_0 \cdot g}} \]  
(81)

Recall that \( T \cdot v \) equals the propulsive power that is the engine power times the propeller efficiency, \( P_{\text{eng}} \cdot \eta_p \). The final expression for the load factor is seen in equation (82).

\[ n = \sqrt{\left( \eta_p \cdot P_{\text{eng}} - q \cdot S \cdot C_{D_0} \cdot v \right) \cdot \frac{\rho \cdot v \cdot S}{2 \cdot K \cdot m_0^2 \cdot g^2}} \]  
(82)

It is now apparent that the load factor is maximized for a high engine power.

With two equations that limits the maximum load factor, equation (71), (82) and their intersection point will yield the final maximum load factor and corresponding minimum velocity. These limitations were combined with equation (65) and (67) to find the intersection point yielding minimum turn radius and maximum turn rate as well.

There is a third limit to the load factor and that is the structural limit. This limit, although ignored due to the time limit of this project, is recognized.

II.IV. Gliding Flight

Gliding flight data is necessary to calculate in case of engine failure or other circumstances leading to the aircraft being unable to generate thrust. In this scenario it is important to know how far the aircraft can glide as this can mean the difference between an emergency landing on or off an airfield. The calculations below show the minimal negative vertical speed the aircraft can achieve, allowing maximum distance to be covered without the aid of the motors.

Starting with the free body diagram in figure (5) and a force balance an expression for the minimal negative vertical speed is derived.
\[
D = m_0 \cdot g \cdot \sin \gamma
\]
\[
L = m_0 \cdot g \cdot \cos \gamma
\]
\[
\frac{L}{D} = \frac{m_0 g \cdot \cos \gamma}{m_0 g \cdot \sin \gamma}
\]
\[
\frac{L}{D} = \frac{1}{\tan \gamma}
\]

From equation (86) it is obvious that the sink angle, \(\gamma\), is inversely proportional to the lift to drag ratio and therefore the minimum sink angle is achieved for the maximum lift to drag ratio therefore an expression for the maximum lift to drag ratio needs to be derived. This is done by assuming yet again the simple parabolic drag polar and then solving for which value of the lift coefficient the lift to drag ratio is maximized.

\[
\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D0} + K \cdot C_L^2}
\]
\[
\frac{d}{dC_L} \left( \frac{C_L}{C_D} \right) = \frac{C_{D0} + K \cdot C_L - 2 \cdot K \cdot C_L^2}{(C_{D0} + K \cdot C_L^2)^2} = 0
\]
\[
C_{D0} + K \cdot C_L^2 - 2 \cdot K \cdot C_L^2 = 0
\]
\[
C_L = \sqrt{\frac{C_{D0}}{K}}
\]

Equation (87) and equation (90) now gives the maximum lift to drag ratio and as a consequence the minimum sink angle and corresponding rate of descent, \((R/D)_{\gamma_{\text{min}}}\), is acquired.

\[
\left( \frac{L}{D} \right)_{\text{max}} = \frac{1}{2 \sqrt{C_{D0} \cdot K}}
\]
\[
\gamma_{\text{min}} = \tan^{-1} \left( 2 \cdot \sqrt{C_{D0} \cdot K} \right)
\]
\[
(R/D)_{\gamma_{\text{min}}} = \sin \gamma_{\text{min}} \cdot \sqrt{\frac{W}{S} \cdot \frac{2 \cdot \cos \gamma_{\text{min}}}{\rho \cdot C_L}}
\]

Having these variables solved we can also find the maximum glide range of the aircraft, the horizontal velocity and the time remaining until impact. See equation (94), (95) and (96).

\[
t = \frac{y_{\text{in}}}{(R/D)_{\gamma_{\text{min}}}}
\]
\[
v_H = \frac{(R/D)_{\gamma_{\text{min}}}}{\tan \gamma}
\]
\[
z_{\text{glide}} = v_H \cdot t = \frac{(R/D)_{\gamma_{\text{min}}}}{\tan \gamma} \cdot \frac{y_{\text{in}}}{(R/D)_{\gamma_{\text{min}}}} = \frac{y_{\text{in}}}{\tan \gamma}
\]
II.V. Takeoff Analysis

The takeoff analysis is an important part of the design process, it is necessary for the aircraft to be able to lift off of conventional runways. There are four parts that need to be analysed. The grounded-, the roll-, the lift- and the stabilized climb part. The takeoff turns into flight once the aircraft has reached the minimum vertical obstacle clearance height.

The total ground roll is equal to $S_G + S_R$ where $S_R$ is the distance the aircraft travels during rotation. The rotation time is mostly dependent on the pilot but for small aircraft the time is in the magnitude of one second. The acceleration during rotation is negligible because the small time value. This means $S_R = V_{TO} = 1.1 \cdot V_{Stall}$.

\[
F = m_0 \cdot a
\]

\[
a = \frac{g}{m_{0g}} \cdot [T - D - \mu \cdot (m_0 \cdot g - L)]
\]

\[
g = \left[ \frac{\left( \frac{T}{m_{0g}} - \mu \right) + \frac{\rho}{2 \cdot m_{0g} \cdot g} \cdot (C_{D_0} - K \cdot C_L^2 + \mu \cdot C_L) \cdot v^2} \right]
\]

\[
K_T = \left( \frac{T}{m_{0g}} \right) - \mu
\]

\[
K_A = \frac{\rho}{2 \cdot m_{0g} \cdot g} \cdot (\mu \cdot C_L - C_{D_0} - K \cdot C_L^2)
\]

\[
S_G = \int_{V_i}^{V_f} \frac{V}{a} \, dv = \frac{1}{2} \int_{V_i}^{V_f} \frac{1}{a} \, d(v^2) = \frac{1}{2} g \int_{V_i}^{V_f} \frac{d(v^2)}{K_T + K_A v^2}
\]

\[
S_G = \left( \frac{1}{2 \cdot g \cdot K_A} \right) \cdot \ln \left( \frac{K_T + K_A v_f^2}{K_T + K_A v_i^2} \right)
\]

When transitioning from takeoff to a stabilized climb angle, the aircraft follows a circular path, i.e. the aircraft is starting a vertical turn. The transition turns into steady flight once the aircraft stops moving in this way and instead climbs at a constant speed. During the transition the aircraft
accelerates from the takeoff speed $1.1 \cdot v_{\text{Stall}}$ to the climb speed $1.2 \cdot v_{\text{Stall}}$, assuming the acceleration is constant the speed during the transition can be written as $1.15 \cdot v_{\text{Stall}}$. We assume the average lift coefficient is about 90% of the maximum lift. First the load factor $n$ is calculated to assure that its maximum value is not exceeded.

$$n = \frac{L}{m_0 g} = \frac{\frac{1}{2} \rho 0.9 C_{L_{\text{max}}}^{-1} 1.15 v_{\text{Stall}}^2}{\frac{1}{2} \rho S C_{L_{\text{max}}} v_{\text{Stall}}^2} = 1.15^2 \cdot 0.9 = 1.2$$  \hspace{1cm} (104)$$

$$R = \frac{v_T^2}{0.2g}$$  \hspace{1cm} (105)$$

Assuming the aircraft travels in a circular path we derive the following, where $\gamma_{\text{climb}}$ is the central angle of the arc length illustrated in figure (6).

$$T = \frac{\eta_p P_{\text{eng}}}{v_{\text{to}}}$$  \hspace{1cm} (106)$$

$$\sin \gamma_{\text{climb}} = \frac{T-D}{m_0 g} \approx \frac{T}{m_0 g} - \frac{1}{(\frac{L}{D})}$$  \hspace{1cm} (107)$$

$$S_{TR} = R \cdot \sin \gamma_{\text{climb}} = R \cdot \left(\frac{T-D}{m_0 g}\right) \approx R \cdot \left(\frac{T}{m_0 g} - \frac{1}{(\frac{L}{D})}\right)$$  \hspace{1cm} (108)$$

$$h_{TR} = R(1 - \cos \gamma_{\text{climb}})$$  \hspace{1cm} (109)$$

As we finally reach the climb segment of takeoff, we need to know how far we travel horizontally before we clear the minimum vertical obstacle height required. The minimum vertical obstacle clearance height is usually 15.24 meters for small civil aircraft.

$$S_C = \frac{h_{\text{obstacle}}-h_{TR}}{\tan \gamma_{\text{climb}}}$$  \hspace{1cm} (110)$$

In some cases however the obstacle is cleared before reaching climb, when $h_{TR} \geq 15.84 \text{ m}$. In this case $S_C = 0$. If this is the case the horizontal distance traveled is calculated by

$$S_{TR} = \sqrt{R^2 - (R - h_{\text{obstacle}})^2}$$  \hspace{1cm} (111)$$

II.VI. Static Pitch Stability

![Free Body Diagram, Pitch Stability](image)
The pitch stability of the Joyride was evaluated through formulating a stability criteria and then analysing whether it was possible to satisfy or not. This was done by determining how the Joyride would respond to a slight disturbance in its angle of attack. First it was recognized that the lift force and pitching moment are both dependant on the angle of attack and it is naturally perceived that an increase in angle of attack means an increase in lift. The relation between pitching moment and angle of attack is a little harder to visualize but by definition a body is statically stable if it after a disturbance always returns to the latest equilibrium point. So a disturbance which increase the angle of attack turns the Joyrides nose upwards and in order to return to the last equilibrium point this disturbance should induce a nose down pitching moment from the aircraft. This means that we have the following expressions.

\[ \frac{dL}{d\alpha} > 0 \]  \hspace{1cm} (112)

\[ \frac{dM}{d\alpha} < 0 \]  \hspace{1cm} (113)

\[ \Rightarrow \frac{dM}{dL} = \frac{dM}{d\alpha} \frac{d\alpha}{dL} < 0 \]  \hspace{1cm} (114)

This knowledge was then applied on the moment around the centre of gravity derived from the free body diagram in figure (7). The moment around the aerodynamic centre, \( M_{a.c.} \), is due to the aircrafts asymmetric profile.

\[ M_{c.g.} = M_{a.c.} + L \cdot l_{c.g.} \]  \hspace{1cm} (115)

\[ \frac{dM_{c.g.}}{dl} = l_{c.g.} < 0 \]  \hspace{1cm} (116)

What this means is that in order for the Joyride to be statically stable in pitch, the aerodynamic centre of the aircraft needs to be located further aft than the centre of gravity.

This was done in three steps, first the position of the structures centre of gravity was calculated. Then the position of the aerodynamic centre calculated and lastly the remaining weights were positioned inside the aircraft accordingly in order to fulfil the stability criteria. When calculating the structures centre of gravity it was divided into four unique geometric parts and it was assumed that these parts all had a homogeneous mass distribution. Notice that there are doublets of parts 3 and 4. The position of the geometric centres for these parts were then calculated from the nose of the Joyride and the previous assumption makes these points “local” centres of gravity and having them known was essential when calculating the structure’s “global” centre of gravity. These parts were then geometrically evaluated and estimated values for each part’s total volume fraction acquired.

![Figure 8: Overview of the Joyride and it's structural parts.](image-url)
The structure's centre of gravity was then acquired.

\[
\begin{align*}
    r_{c.g.} &= r_1 \cdot \frac{m_1}{m_{\text{empty}}} + r_2 \cdot \frac{m_2}{m_{\text{empty}}} + r_3 \cdot \frac{m_3}{m_{\text{empty}}} + r_4 \cdot \frac{m_4}{m_{\text{empty}}} \\
    r_{c.g.} &= r_1 \cdot \frac{v_1}{v_{\text{tot}}} + r_2 \cdot \frac{v_2}{v_{\text{tot}}} + r_3 \cdot \frac{v_3}{v_{\text{tot}}} + r_4 \cdot \frac{v_4}{v_{\text{tot}}}
\end{align*}
\]  

(117)  

(118)

In figure (9) the positioning of the different equipment, crew and payload is highlighted. At a distance of \( r_b \) from the nose of the Joyride the batteries and payload was placed. At a distance of \( r_c \) from the nose of the aircraft the pilot, laptop, and oxygen cylinder was placed. At a distance of \( r_m \) from the nose of the aircraft the motor, and motor controller was placed.

The Joyrides centre of gravity was acquired from equation (119).

\[
    r_{c.g.} = r_{c.g.} + \left( \frac{(m_{\text{battery}} + m_{\text{payload}}) r_b + (m_{\text{laptop}} + m_{\text{cylinder}} + m_{\text{crew}}) r_c + (m_{\text{motor}} + m_{\text{m.c.}})}{m_o} \right)
\]

(119)

Next the roll wise position of the aerodynamic centre was needed. According to (4. Karlsson, 2013) the aerodynamic centre is located at the 25% mark of the mean aerodynamic chord. The mean aerodynamic chord was calculated in chapter I.I. Geometry and Design.

III. Equipment, Energy & Propulsion

III.I. Propulsion System

The propulsion system designed for the joyride can be divided into four steps and the process is seen in the chart in figure (10). The batteries are connected to the motor controller which in turn powers the motor. In order to maximize the overall efficiency there typically is a gearbox between the motor and propeller to adjust the rpm of the motor to something more in line with the propellers preferences. This is not needed on the joyride because the motor rpm is relatively low and there are propellers available for this speed. Electric motors do not need gearboxes in the same way combustion engines do, that is because of the difference in which rpm they reach maximum torque. A combustion engine normally needs to reach 3000-5000 rpm in order to hit maximum torque while the electric motor in the joyride has full torque much earlier, between 200-300 rpm. The efficiency of an electric motor varies quite a lot so this had to be analysed. The motor chosen for the joyride has a
peak efficiency of 95% but this can, particularly at low speeds fall as low as 50%. That is why some electric vehicles use multi speed transmission to maximize the efficiency at all rpm’s. This was not seen to be as important for an aircraft because the rpm is not expected to change too much during flight and the multi speed transmission would basically only matter during takeoff, increasing acceleration which was regarded unnecessary.

III.I Batteries
The single biggest reason why aircrafts today still rely purely on gasoline as their source of energy is the fact that electrical energy is difficult to store. Available batteries on the market today are insufficient for an airplane due to their low energy density. This is however due to change with the development of future lithium air batteries. Lithium air batteries use oxygen from the air instead of an internal oxidizer. This gives them a very high theoretical energy density, even comparable to gasoline, (12 kWh/kg for Li-Air batteries compared to 13 kWh/kg for gasoline). The high energy density would provide enough energy to make this type of aircraft more than feasible. Because there aren’t any on the market today, shape and size was chosen to best fit the aircrafts intended weight distribution.

Another source of energy examined was a structural battery. A structural battery is simply put a battery within the aircrafts casing. The density of this type of material is 1750 kg/m³ and it has a theoretical energy storing maximum of 372 mAh/g. Structural batteries are being researched, although in small scale. If used, structural batteries could make up the majority of the aircrafts casing, giving the aircraft ~0.8 · m_{empty} kg of structural battery power. This translates into 372 · 0.8 · m_{empty} Ah of energy. The 0.8 factor is an estimation that structure corresponding to 80 % of the aircrafts empty weight can be used as a structural power source. This comes from the problem that the structural batteries are not strong/resilient enough to be used everywhere. Especially not on parts of the aircraft that are subject to a lot of stress, such as the inner part of the wing.

III.III. Electric AC Motor & Controller
The joyride is equipped with an electric motor because of the environmental benefits compared to conventional combustion engines. There are absolutely zero emissions from a pure electric vehicle and charging the batteries with electric power generated from renewable resources, a significant reduction in environmental impact is achieved. The motor chosen for the Joyride is the YASA-750, it has a peak power of up to 120 kW and a peak efficiency of 95%. The complete specifications together with a system efficiency graph can be seen in Appendix B.

The chosen controller is the Sevcon Gen4 Size 8 controller which is developed by Sevcon in cooperation with YASA motors and optimized to work well with the YASA 750 electric motor. What the motor controller essentially does is that it takes the DC voltage from the batteries and transforming it to 3-phase AC power for the motor. Additionally the controller also tracks the pilots throttle setting in order to generate the requested power.
III.IV. Propeller
The propeller chosen was made of carbon fibre due to the low weight and excellent durability. The diameter of the propeller had one requirement. It couldn’t be so long that the edges would travel at speeds over Mach 1 which would create shock waves greatly decreasing the propellers efficiency and increased noise. Where the latter would not cope well with the environmental profile of the Joyride.

With these constraints the diameter of the propeller was calculated as follows where \( a \) is the speed of sound at cruise altitude.

\[
r \cdot v_{prop} \leq r \cdot 0.9 \cdot a = \omega = \frac{2 \pi n}{60}
\]

\[
r \leq \frac{2 \pi n}{60 \cdot 0.9 \cdot a}
\]

\[
d_{prop} \leq \frac{\pi n}{54 \cdot a}
\]

III.V. Flight Instruments
In accordance with instrumental flight rules which are a set of regulations to be followed when aircraft are mainly operated by the aid of flight instruments these items are needed.

- An airspeed indicator
- An artificial horizon
- An altitude indicator
- Turn and slip indicator
- Directional gyro i.e. horizontal situation indicator
- Rate of climb/descent indicator
- Global Positioning System
- Two-way communications link

Besides the navigation instruments listed above the Joyride will need some engine instrumentation as well.

- Engine RPM
- Engine Temperature
- Throttle

Lastly information regarding energy left and an estimated range indicator is desired.

It is perceived that within the time limit of this concept, all of these items will be available on a computer program and therefore a laptop will be sufficient for the flight instrumentation. The weight of the laptop and accessories is estimated to be 3 kg.

III.VI. Pilot Oxygen System
Because of the high operation altitudes of the aircraft an oxygen system is needed for the pilot. Instead of pressurizing the whole cockpit it was decided to save weight and instead use a portable aviation oxygen system like the one offered by Skyox. With a duration of 15.27 \( h \) possible to maximize to 46.21 \( h \) if used with an oxymizer. The cylinder has a volume of 17 \( l \), corresponding to 14 16 \( l \) of uncompressed oxygen and the cylinder weighs 16.5 \( kg \) while being attachable to back of the pilots’ seat.
III.VII. Cockpit Heating

The standard atmospheric temperature at 6000 m is $-22.09\,^\circ C$ which is not considered a very pleasant environment for the pilot. Because of the overheating nature of the Li-Air batteries it was decided that this excessive heat was enough to satisfy the needs. No additional weight.

Results

Sizing

Maximum Thickness to Chord Ratio of 17%

It was decided that $t = 1.5\,m$ would be sufficient space for the pilot.

The following are initial geometry values settled on.

- Aircraft maximum thickness $t = 1.5\,[m]$
- Aircraft length $l = 8.82\,[m]$
- Wingspan $b = 15.86\,[m]$
- Wing area $S = 23.60\,[m^2]$
- Aspect ratio $AR = 12.78$
- Drag due to lift factor $K = 0.12$
- Zero-lift drag coefficient $C_{D0} = 0.0077$
- Maximum lift coefficient $C_{L_{max}} = 1.7$
- Leading edge sweep angle $\Lambda_{LE} = 40^\circ$
- Empty weight $m_{empty} = 487.68\,[kg]$
- Crew weight $m_{crew} = 100\,[kg]$
- Payload weight $m_{payload} = 100\,[kg]$
- Battery weight $m_{battery} = 28.31\,[kg]$
- Motor weight $m_{motor} = 25\,[kg]$
- Controller weight $m_{controller} = 10\,[kg]$
- Oxygen cylinder weight $m_{cyl} = 16.5\,[kg]$
Laptop weight  \( m_l = 3 \ [kg] \)
Takeoff gross weight  \( m_0 = 770.49 \ [kg] \)
The mean aerodynamic chord  \( \bar{c} = 4.46 \ [m] \)

**Performance**

Total energy required during flight mission  \( E_{tot} = 220.8 \ [kWh] \)
Cruise speed  \( v_{cruise} = 62.23 \ [m/s] \)
Maximum speed  \( v_{max} = 118.59 \ [m/s] \)
Speed yielding minimum power required, steady level flight  \( v_{Pr_{min}} = 47.29 \ [m/s] \)
Minimum power required, steady level flight  \( P_{min} = 25.60 \ [kW] \)
Energy required, steady level flight  \( E_{slf} = 199.68 \ [kWh] \)

In figure (11) two plots are highlighting the thrust and power required as a function of velocity during steady level flight.

The climb results were as follows.

Minimum climb time to cruise altitude  \( t_{min} = 8.8 \ [min] \)
Required maximum climb time  \( t = 15 \ [min] \)
Required engine power  \( P_{eng_r} = 80.6 \ [kW] \)
Energy required during climb part of mission  \( E_{climb} = 21.12 \ [kWh] \)
In figure (12) the aircrafts final climbing properties are highlighted, it consists of three plots presenting velocity, rate of climb and climb angle plotted vs. altitude. The blue curves are for maximum climb angle conditions and the green curve for maximum rate of climb conditions. The red curve in the velocity plot is the stall speed. Notice that the velocity corresponding to maximum climb angle is smaller than the stall limit for the whole interval and hence this theoretical maximum climb angle is not reachable.

![Velocity vs. Altitude](image)

![Rate of Climb vs. Altitude](image)

![Climb Angle vs. Altitude](image)

*Figure 12: Tangential velocity, rate of climb & climb angle vs. altitude.*

The turn results:

The maximum load factor, minimum turn radius and maximum turn rate can be seen in figure (13), (14) and (15).

- Maximum load factor \( n_{\text{max}} = 2.02 \)
- Minimum turn radius \( r_{\text{min}} = 66.97 \text{ [m]} \)
- Maximum turn rate \( \omega_{\text{max}} = 29.09 \text{ [°/s]} \)
- Maximum bank angle \( \theta = 60.4 \text{ [°]} \)

This result indicate that a 180° turn is made in 6.2 seconds which is considered sufficient in order to satisfy the demand of good flying qualities.
Figure 13: Load factor vs. velocity

Figure 14: Turn radius vs. velocity
The following numbers are specifically for an unexpected loss of thrust during steady level flight at cruise altitude except for the minimum sink angle which was a constant.

Minimum sink angle \( \gamma_{\text{min}} = 3.53^\circ \)

Time until impact \( t = 25.86 \text{ [min]} \)

Maximum glide range \( z_{\text{glide}} = 95.39 \text{ [km]} \)

Impact velocity \( v_{\text{imp}} = 45.10 \text{ [m/s]} \)

More general gliding flight results are shown in figure (16) where the minimum rate of descent, horizontal velocity, time remaining until impact and glide range are presented for altitudes ranging from sea level to cruise altitude.
The takeoff analysis yielded the following results:

Ground distance \( S_G = 81.89 \, [m] \)

Roll distance \( S_R = 19.27 \, [m] \)

Transition to flight distance \( S_{TR} = 65.71 \, [m] \)

Total take-off distance \( S_{tot} = 166.88 \, [m] \)

Height after total ground roll and transition to climb \( h_{TR} = 68.12 \, [m] \)

Because of \( h_{TR} \geq 15.84 \, [m] \) it was no need to calculate the climb part of the takeoff.

The stability analysis proved the aircraft to be neither unstable nor stable but neutral, i.e. \( l_{c.g.} = 0 \). It could easily be made statically stable by moving weight further forward but because the static margin would still not be very large a more spacious area around the pilot was prioritized.

The results from the geometric evaluation of the four parts are shown in table (1) together with calculated positions for the structures local centres of gravity and the weight distribution.
Aerodynamic centre position \( r_{a.c.} = 4.5 \text{ [m]} \)

Centre of gravity position \( r_{c.g.} = 4.5 \text{ [m]} \)

These values for \( r_{a.c.} \) and \( r_{c.g.} \) were measured from the nose of the Joyride.

The propeller diameter could not exceed 133 cm without the tips reaching a speed of Mach 1 hence the propeller diameter had to be smaller than this.

\( d_{\text{prop}} = 120 \text{ [cm]} \)

**Discussion**

**Pusher Configuration**

The major reason pusher configuration was chosen for the Joyride is the design of the blended wing body. Because the wing is swept, it is difficult to mount an engine in a good way, therefore the best option is to have the propeller in the back of the aircraft.

**Batteries**

There are quite a few problems with Lithium Air batteries which will need to be solved by 2060 when this theoretical aircraft will be used. The first problem you will face is that these batteries are extremely hard to recharge once discharged. If they were to be used today the user would have to buy new batteries after each flight since they currently also degrade a lot from the recharging.

Another big problem with these batteries is the overheating factor, the chemical reaction required to produce electricity also produces a lot of heat. This combined with the fact that these batteries are extremely flammable causes a huge problem. This is partially solved as air is needed to power the battery and the air at flight altitude is very cold.

The third and perhaps biggest problem is the batteries sensitivity towards water. If any water touches the battery it quickly degrades, so you need a filter which lets air but not water through. This isn’t feasible today and would make flight in an area with high air moisture impossible.

The energy density of this battery seems too high to be true, but this is because we calculated the energy density excluding the weight of oxygen needed as this would be provided by the outside air for free and does not need to be stored.

Although it was originally planned to use structural batteries this idea was scrapped as it wasn’t reasonable to expect them to be operational by 2060. The research is still in its infancy and few people are working on it. In the case of Lithium-Air batteries several leading companies such as IBM are working on these, making it far more plausible that these will be readily available by 2060. We did however calculate how much energy could potentially be stored. Using the method described in part III.II. **Batteries** the energy stored in the structural battery is anywhere from 100 to 464 kWh of

<table>
<thead>
<tr>
<th>Part / Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Batteries &amp; Payload</th>
<th>Laptop, crew and cylinder</th>
<th>Motor &amp; controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{V_{\text{part}}}{V_{\text{tot}}}, [%] )</td>
<td>13.6</td>
<td>30.6</td>
<td>2 \cdot 14.4</td>
<td>2 \cdot 13.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{\text{part/item}}, [m] )</td>
<td>2.53</td>
<td>6.10</td>
<td>6.10</td>
<td>7.29</td>
<td>0.3</td>
<td>2.21</td>
<td>8.5</td>
</tr>
</tbody>
</table>

*Table 1: Volumetric fractions and position of the centre of gravity for each part of the structure*
energy, depending on voltage, meaning structural batteries alone could power the Joyride. As stated earlier though, it wasn’t an option because the aircraft needs to be possible by 2060.

**Blended Wing Body**

When doing research for this bachelor thesis the blended wing body design was eventually discovered and found very interesting. As the thesis is based around building an environmentally friendly aircraft, the BWB design seemed great. The fuselage is part of the wing which generates a high lift to drag ratio enabling increased fuel economy and range.

The choice did however come with some problems. BWB is a very unusual design which made finding information difficult. We first used the X-48 design by Boeing and NASA to extrapolate data such as wing area/weight ratio for a blended wing design. The X-48 scaled upwards was however not a good design for us as it was too large and heavy for a single person aircraft powered by batteries. Instead we chose to look at another concept for inspiration (Liebeck, 2004).

There are a few reasons this design hasn’t hit home in the aircraft passenger industry. It is difficult to pressurize correctly as a tube is easier to pressurize then an oval shape. Windows are very far apart and in some BWB designs not even used. The design pushes passengers and cargo off the center line of the aircraft which causes the vertical motion felt increase when the aircraft rolls.

**Takeoff**

The reason for making a take-off analysis is to know if the aircraft can lift off of a standard runway. A standard runway today is anywhere from 1800 meters to 2400 meters at higher altitudes. The 0.9 factor in determining the load factor is a general assumption and are due to the use of take-off flaps. It is however debatable if BWB aircrafts can reach such a high value. Because they lack a tail to trim the resulting pitching moments the trailing edge control surfaces cannot be used as flaps for takeoff. We chose to calculate it with this value anyway to see that the maximum load factor would not be possible to exceed. The Joyride required a runway of 166.8 meters making it able to take off from almost any runway in the world, (even aircraft carriers). The span of our aircraft is also small enough to work on all runways. This was assumed beforehand as the aircraft is light and has great aerodynamic properties. Overall the takeoff properties are great.

**Gliding Flight**

The result of 95 kilometers glide range means that you can glide to an airport in Sweden almost no matter where the engine failure occurs. The only airports with a distance greater than 190 kilometers between them are Luleå and Kiruna. This means that should engine failure occur right in between these, you would not be able to do an emergency landing on an airport. This shows that the aircraft is not well suited for this route and perhaps it shouldn’t be flown. All other airports in Sweden have less than 190 kilometers between them making this an excellent aircraft for traveling inside Sweden.
Static Pitch Stability
With the current weight distribution the Joyride is pretty much neither stable nor unstable but neutral because the centre of gravity and aerodynamic centre are at pretty much equal distance from the nose of the aircraft. Because the blended wing design is tailless, it has to have its aerodynamic centre behind its centre of gravity. This is for us possible but the safety margin is just not going to be as large as we would like. This however can be corrected through manipulations of the wing, making different parts of the wing have different aerofoils. This would allow more lift to be generated at the outer wing parts who are further aft and hence create more nose down pitch moment. Another possibility is to twist the wing which would move the aerodynamic centre further back. If decided to allow the safety margin to be negative the aircraft could still be flyable by integrating an advanced control system that activates the aircrafts control surfaces according to disturbances and situation. Regarding the position of the aerodynamic centre there is some uncertainty in the calculations. That is because of the assumption that the aerodynamic centre should lie at the 25% mark of the mean aerodynamic chord. This holds true for thin aerofoils and whether the Joyride can be considered thin or not is controversial.

Absolute Ceiling
The absolute ceiling occurs when the rate of climb eventually reaches zero. For the Joyride the absolute ceiling is at 28500 meters. The reason for it being so high is that electrical motors don’t lose power due to the air density decreasing at higher altitudes. In reality the Joyride would never get close to this altitude due to structural limits.

Conclusion
The concept has united environmental consciousness and aircraft design, the final results are in line with the initial requirements and present a sustainable aircraft for the future.
Division of Labour

The table below visualizes respective authors’ areas of responsibility and their contribution to the work, some of the topics can be seen in several columns and that is intended. As editor, Mårten was responsible for the report.

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<th>Gustav</th>
<th>Together</th>
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<td>Estimation of $C_D$</td>
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<td>Performance</td>
<td>Steady and level flight</td>
<td>Gliding Flight</td>
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Table 2: Division of labour

Acknowledgment

The authors’ would like to thank:

1. Our supervisor associate professor Arne Karlsson who’ve helped us throughout this thesis.
2. Professor Dan Zenkert, who on short notice met with us and discussed the possibilities and future of structural power.
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Papers


Literature
Appendices

Appendix A. Specifications of Requirements
Project Joyride shall be an eco-friendly single seater aircraft of pusher configuration and must in combination with good flying qualities also be operable on quite high altitude.

Design Takeoff Gross Weight

\[ m_0 \leq 800 \text{ [}kg\text{]} \]

Payload Weight

\[ m_{\text{payload}} \leq 100 \text{ [}kg\text{]} \]

Crew Weight

\[ m_{\text{crew}} \leq 100 \text{ [}kg\text{]} \]

Range

\[ R \leq 850 \text{ [}km\text{]} \]

Maximum Lift Coefficient

\[ C_{l_{\text{max}}} = 1.7 \]

Maximum climb time to cruise altitude

\[ t_{\text{min}} = 15 \text{ [}min\text{]} \]

Cruise altitude

\[ y_{\text{cruise}} = 6000 \text{ [}m\text{]} \]

Maximum velocity

\[ v_{\text{max}} \geq 100 \text{ [}m/s\text{]} \]
Appendix B. YASA 750 Electric Motor Data

<table>
<thead>
<tr>
<th>Motor Name</th>
<th>YASA 750</th>
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</thead>
<tbody>
<tr>
<td>Peak torque @ 360A</td>
<td>750Nm</td>
</tr>
<tr>
<td>Continuous torque</td>
<td>400Nm</td>
</tr>
<tr>
<td>Peak power @ 400V</td>
<td>120kW</td>
</tr>
<tr>
<td>Continuous power</td>
<td>&gt;50kW</td>
</tr>
<tr>
<td>Peak efficiency</td>
<td>95%</td>
</tr>
<tr>
<td>Total volume</td>
<td>7 litres</td>
</tr>
<tr>
<td>Total weight</td>
<td>25kg</td>
</tr>
</tbody>
</table>

![Motor performance chart](image)
Appendix C. Matlab Code
%2013-03-07
%Kandidatexamensarbete Flygteknik SA108X VT2013

%Project Joyride

%Författare: Mårten Kring med små instick av Gustav Jufors
%Handledare: Arne Karlsson

clear all; close all; clc;

%%% Konstanter
%-------------------------------------------------------------
%---------------------------------------------------------------------------
%--------------------------------------------------------------
%g               = 9.807;
% [m/s^2]               Gravitations konstant
R               = 850000;
% [m]               Req. Range
n_b            = 0.65;
% []               Battery efficiency
e               = n_b*43.2*10^6;
% [J/kg]               Batteriernas energidensitet
etap            = 0.9;
% []                Propeller verkningsgrad
etam            = 0.95;
% []               Motor verkningsgrad
y_c            = 6000;
% [m]               Reg. Cruise altitude
b            = 15.86;
% [m]               Wing span
S            = 23.62;
% [m^2]               Wing area
AR             = 1.2*b^2/S;
% []                Aspect Ratio
t            = 1.5;
% [m]               Maximal tjocklek av planet
l            = 8.82;
% [m]               Flygplanets långd
CL_max            = 1.7;
% []               Reg. max lift coefficient
P_eng_max            = 120000;
% [W]               Maximal motor effekt
P_eng_kont            = 50000;
% [W]               Kontinuerlig motor effekt
V            = [15:0.9406:110];
% [m/s]               Hastighetsvektor till minimum thrust/power
Height                = [0:60:6000];
% [m]               Höjdvektor
[T_H p_H rho_H a_H]              = Atmosphere(Height);
% [K],[Pa],[Kg/m^3],[m/s]               Atmosfärsdata mellan 0m och 60000m
m_0                = 770.4877;
% [kg]               Design Takeoff Gross Weight (med start värde)
m_c                = 100;
% [kg]               Weight Crew
m_p                = 100;
% [kg]               Weight Payload
m_cyl                = 16.5;
% [kg]               Weight oxygen cylinder
m_lt                = 3;
% [kg]               Weight laptop
m_motor = 25;
% [kg] Weight motor
m_motorkont = 10;
% [kg] Weight motorcontroller
m_e = 487.68;
% [kg] Empty Weight
m_b = m_0 - m_c - m_p - m_cyl - m_motor - m_motorkont;
% [kg] Initialt värde på batteri vikten
m_b_in = m_b;
V_cruise_guess = 60.1694;
% [m/s] Uppskattat värde för V_cruise till sizing.
V1 = [65];
% [m/s] Initial gissning för max hast.
P_eng = 50000;
% [W] Emotorns kontinuerliga effekt
Height2 = [0:500:50000];
% [m] Höjdvektor 2 - Till absolut ceiling
[T_H p_H rho_H2 a_H] = Atmosphere(Height2);
% [K], [Pa], [kg/m^3], [m/s] Atmosfärdata mellan 0m och 300000m
V3 = [0:1.1:110];
% [m/s] Hastighetsvektor. Till level turning flight.
my = 1.789*10^-5;
% [Ns/m^2] Dynamiska viskositeten på cruise altitude
Ret = 5*10^5;
% [] Transition reynolds number
tc_max = 0.17;
% [] Max thickness to chord ratio
xc_max = 0.25;
% [] Chordwise position of maximum thickness
A = 1700;
% [] Konstant till flat-plate skin friction coefficient
theta_m = 36;
% [º] Wing sweep utefter linjen för xc_max
S_wet = 2.5*S;
% [m^2] Wetted area
A_max = 3.39;
% [m^2] Maximum cross section area för "fuselage"
theta_m_W = 50;
% [º] Winglet wing sweep
xc_max_W = 0.25;
% [] Chordwise position of maximum thickness för winglet
tc_max_W = 0.1;
% [] Max thickness to chord ratio för winglet
l_W = 0.015;
% [m] Winglet chordan
theta_LE = 40;
% [º] Leading edge sweep
c_(1) = 4.46;
% [m] Mean aerodynamic chord
%% Zero lift drag coefficent & drag due to lift factor ------------------------------

% Flat-plate skin friction coefficient CF
Re(1) = rho_H(1)*V_cruise_guess*c_(1)/my;
%Reynolds tal vingen
Re(2) = rho_H(1)*V_cruise_guess*1/my;
%Reynolds tal fuselage
Re(3) = rho_H(1)*V_cruise_guess*1_W/my;
%Reynolds tal winglet
for i = 1:3;
   if Re(i) > Ret
      CF(i) = 0.523/((log(0.06*Re(i)))^2);
   end
   if Re(i) < Ret
      CF(i) = 1.328/sqrt(Re(i));
   end
   if (0.9*Ret<=Re(i)&& Re(i)<=1.1*Ret)
      CF(i)=0.455/((log10(Re(i))^2.58)-A/Re(i));
   end
end
% Form factor F
Ma = V_cruise_guess/a_H(end);
f  = l/sqrt(4/(pi*A_max));
F(1) = (1+0.6/xc_max*tc_max+100*tc_max^4)*(1.34*Ma^0.18*cosd(theta_m)^0.28);
F(2) = 1+60/f^3+f/400;
F(3) = (1+0.6/xc_max*tc_max+100*tc_max^4)*(1.34*Ma^0.18*cosd(theta_m_W)^0.28);
% Interference factor Q
Q = [1 1 1.05];
% Wetted area
Swet(1) = S_wet*0.31;
Swet(2) = S_wet*0.65;
Swet(3) = S_wet*0.04;
CD0 = 1/S*sum(F.*CF.*Q.*Swet);
% Drag due to lift factor
e0 = 4.61*(1-0.045*AR^0.68)*cosd(theta_LE)^0.15 -3.1;
K = 1/(pi*AR*e0);

%%% Steady level flight
-----------------------------------------------------------------------------------------------------------------------------------------------
Vtr_min  = sqrt(2/rho_H(end)*sqrt(K/CD0)*m_0*g/S)
V_cruise = Vtr_min
Vpr_min  = sqrt(2*m_0*g/(rho_H(end)*S)*sqrt(K/(3*CD0)))

Tr   = (CD0+K*(m_0*g)^2/((1/2*rho_H(end)*Vtr_min^2)^2*S^2))*(1/2*rho_H(end)*Vtr_min^2)*S;
Tr_vekt = (CD0+K*(m_0*g)^2/((1/2*rho_H(end).*V.^2).^2*S^2)).*(1/2*rho_H(end).*V.^2)*S;
Dl   = (K*(m_0*g)^2/((1/2*rho_H(end).*V.^2).^2*S^2)).*(1/2*rho_H(end).*V.^2)*S;
D0   = CD0*(1/2*rho_H(end).*V.^2)*S;

figure(1);
% Plottera thrust req. och power req. som funktion av hastigheten vid service ceiling
subplot(1,2,1);
plot(V,D0,'r',V,Dl,'g',V,Tr_vekt,'b',V,Tr,'k-'); grid on;
legend({'D0', 'Dl', 'Tr', 'Tr_m_i_n','Location', 'North'});
xlabel('Velocity [m/s]');
ylabel('Thrust, [N]');
Pr  = CD0*1/2*rho_H(end)*Vpr_min^3*S+K*2*(m_0*g)^2/(rho_H(end)*Vpr_min*S)
Pr_vekt = CD0*1/2*rho_H(end).*V.^3+S*K*2*(m_0*g)^2./(rho_H(end).*V.*S);

subplot(1,2,2);
plot(V,Pr_vekt,V,Pr,'k-'); grid on;
legend('Pr','Pr_m_i_n','Location','North');
xlabel('Velocity [m/s]');
ylabel('Power, [W]');

%Maximum velocity
i = 1;
c = 1;
while c > 0
    koll(i) = etap*P_eng_max-V1(i)*(1/2*rho_H(end)*V1(i)^2*S*CD0 + 2*K*m_0^2*g^2/(rho_H(end)*V1(i)^2*S));
    if koll(i) < 0
        c = -1;
    end
    i=i+1;
    V1(i)=V1(i-1)+0.01;
end
V_max = V1(end-1);
V_stall_vekt = sqrt(m_0*g./(1/2*rho_H*CL_max*S));

%%% Steady Climb

V2 = [V_stall_vekt(1):0.9154:V_max];
r = 1;
while r<=length(Height)
    ROC(r,:)=P_eng*etap/(m_0*g)-CD0*1/2*rho_H(r)*V2.^3.*S./(m_0*g)-K*2*(m_0*g^2*S)/(rho_H(r).*V2);
    S_gamma(r,:)=(ROC(r,:)./V2);
    gamma(r,:)=asind(S_gamma(r,:));
    r=r+1;
end

figure(2);
% Plottar stigvinkel gamma som funktion av hastigheten vid olika höjder
for i = 1:10;
    subplot(5,2,i);
    plot(V2,gamma(i*10,:)); grid on;
title(['Climb angle vs. Velocity at altitude ~= ' num2str(round(Height(i*10))) ' [m]']);
xlabel('Velocity [m/s]');
ylabel('\gamma [º]');
end

figure(3);
% Plottar stigvinkel gamma som funktion av höjden vid olika hastigheter
for i = 1:5;
    subplot(3,2,i);
    plot(Height,gamma(:,i*4));grid on;
title(['Climb angle vs. Altitude at velocity ~= ' num2str(round(V2(i*4))) ' [m/s]']);
xlabel('y [m]');
ylabel('\gamma [º]');
end
figure(4);
% Plottar rate of climb som funktion av hastigheten vid olika höjder
for i = 1:10;
    subplot(5,2,i);
    plot(V2,ROC(i*4,:)); grid on;
    title(['Rate of climb vs.Velocity at altitude ~= ' num2str(round(Height(i*4))) ' [m]']);
    xlabel('Velocity [m/s]');
    ylabel('(R/C) [m/s]');
end

figure(5);
% Plottar rate of climb som funktion av höjden vid olika hastigheter
for i = 1:5;
    subplot(3,2,i);
    plot(Height,ROC(:,i*4)); grid on;
    title(['Rate of climb vs. Altitude at velocity ~= ' num2str(round(V2(i*4))) ' [m/s]']);
    xlabel('y [m]');
    ylabel('(R/C) [m/s]');
end

% The maximum rate of climb, Velocity giving maximum rate of climb
% and climb angle at maximum rate of climb all plotted vs. altitude

P_eng_guess = 50000;
t_min=16*60;
while t_min > 8.8*60;
    % Andra till 8.8*60 så fås p_eng_req = p_eng_max
    for r=1:length(Height)
        Vroc_max(r) = (4/3*K/CD0*(m_0*g/(S*rho_H(r)))^2)^(1/4);
        if Vroc_max(r) < 1.2*V_stall_vekt(r)
            Vroc_max(r)=1.2*V_stall_vekt(r);
        end
        roc_max(r) = etap*P_eng_guess/(m_0*g)
                    - Vroc_max(r)*4/sqrt(3)*sqrt(K*CD0);
        gamma_roc_max(r) = asind(etap*P_eng_guess/(m_0*g*Vroc_max(r))
                                   - 4/sqrt(3)*sqrt(K*CD0));
    end
    t_min = trapz(Height,1./roc_max);
    % Minsta stigtid i minuter upp till 6000 meters höjd med full motoreffekt
    % görs på 8.8 minuter
    E_climb = P_eng_guess/etam*t_min;
    E_slf   = P_eng/(etam*V_cruise)*R;
    E_tot   = E_slf+E_climb;
    m_bat_korr = E_tot/(e);
    P_eng_guess = P_eng_guess + 200;
end

m_0_korr = m_e + m_lt + m_p + m_c + m_cyl + m_motor + m_motorkont +
           m_bat_korr;
t_min = t_min / 60
P_eng_req = P_eng_guess

figure(6);
% Vroc max & roc_max plotted vs. altitude
subplot(3,1,1)
plot(Height, Vroc_max); grid on;
title('Velocity giving maximum rate of climb vs. Altitude');
xlabel('y [m]');

ylabel('V_{(R/C)}_{max}');

subplot(3,1,2);
plot(Height, roc_max); grid on;
title('Maximum rate of climb vs. altitude');
xlabel('y [m]');
ylabel('(R/C)_{max} [m/s]');

subplot(3,1,3);
plot(Height, gamma_roc_max); grid on;
title('Climb angle at (R/C)_{max} vs. Altitude');
xlabel('y [m]');
ylabel('$\gamma_{(R/C)}_{max} [^\circ]$');

% The maximum climb angle - using newton raphsons method to solve
% 4th-degree equation

C=1;
while C<=length(Height)
    for H=1:length(Height)
        x=ones(size(Height));
        i=1;
        zprim=[];
        Q=[];
        while x(H)>1e-10 && i<15;
            z=CD0.*rho_H/(m_0*g/S).*x.^4+etap*P_eng_max/(m_0*g).*x-
            4*K*(m_0*g/S)./rho_H;
            zprim=4*CD0.*rho_H/(m_0*g/S).*x.^3+etap*P_eng_max/(m_0*g);
            Q=x-(z./zprim);
            x=Q;
            i=i+1;
        end
        Q(i-1);
        V_gamma_max=x;
        kontroll(H)=V_gamma_max(H)^4+(etap*P_eng_max/(m_0*g))*((m_0*g)/S)/(rho_H(H)*CD0)*V_gamma_max(H)-4*((m_0*g)/S)^2*K/(rho_H(H)^2*CD0); % = 0 om x är en rot
        S_gamma_max(H) = etap*P_eng_max/(m_0*g*V_gamma_max(H))-
            CD0*1/2*rho_H(H)*V_gamma_max(H)^2*(S/(m_0*g))-
            K*2*(m_0*g/(S*rho_H(H)*V_gamma_max(H))^2));
        gamma_max(H) = asind(S_gamma_max(H));
        if H==length(Height)
            C=C+1;
            H=1;
        end
    end
end
ROC_gamma_max = V_gamma_max.*S_gamma_max;

figure(7);
subplot(3,1,1);
plot(Height,V_gamma_max, Height, Vroc_max, Height, V_stall_vekt); grid on;
title('Velocity vs. Altitude');
xlabel('y [m]');
ylabel('V [m/s]');
legend(['\gamma_{max}', '(R/C)_{max}', 'Stall limit']);

subplot(3,1,2);
plot(Height, ROC_gamma_max, Height, roc_max); grid on;
title('(R/C) vs. Altitude');
xlabel('y [m]');
ylabel('(R / C) [m/s]');

subplot(3,1,3);
plot(Height,gamma_max, Height, gamma_roc_max); grid on;
title('Climb angle vs. altitude');
xlabel('y [m]');
ylabel('\gamma [°]');

%% CEILINGS -----------------------------------------------

% Beräknar absolute ceiling samt service ceiling

% Beräknar densiteten då RoC_max = 0 m/s -> abs ceiling
i=1;
c=0;
while i<length(Height2) && c==0
    x(i)=P_eng*etap/(m_0_korr*g)-
        4*(4/27*K^3*CD0)^1/4*sqrt((m_0_korr/S)./rho_H2(i));
    if x(i)<0;
        rho_abs_c=rho_H2(i-1);
        c=i-1;
    end
    i=i+1;
end

abs_C = Height2(c) % Absolute ceiling

% Beräknar densiteten för RoC = 0.508 -> service ceiling

% Level Turning Flight -----------------------------------

n_cl = 1/2*rho_H(end).*V3.^2*CL_max/(m_0_korr*g/S);
n_pmax = sqrt((etap*P_eng_max-
    1/2*rho_H(end).*V3.^3*CD0)*rho_H(end)*S.*V3./(2*K*m_0_korr^2*V3^2));
figure(8);
plot(V3,n_cl,V3,n_pmax); grid on;
title('Load factor vs. Velocity');
xlabel('V [m/s]');
ylabel('n');
legend('n_C_L_m_a_x', 'n_P_e_n_g_m_a_x', 'Location', 'North');

i = 1;
c = 1;
V4 = [0];
while c > 0
```matlab
koll(i) = 1/2*rho_H(end)*V4(i)^2*CL_max/(m_0_korr*g/S) -
sqrt((etap*P_eng_max -
1/2*rho_H(end)*V4(i)^3*S*CD0)*rho_H(end)*S.*V4(i)/(2*K*m_0_korr^2*g^2));
if koll(i) > 0
    c = -1;
    Vskarn = V4(i);
end
i=i+1;
V4(i)=V4(i-1)+0.5;
end
n_max = 1/2*rho_H(end).*Vskarn^2*CL_max/(m_0_korr*g/S);
b_angle = acosd(1/n_max);

% Turn radius
r_cl = V3.^2./(g*sqrt(n_cl.^2-1)); r_pmax = V3.^2./(g*sqrt(n_pmax.^2-1));
figure(9);
plot(V3,r_cl,V3,r_pmax); grid on;
title('Turn radius vs. Velocity');
xlabel('V [m/s]');
ylabel('r [m]');
legend('r_{C_L,m_a,x}', 'r_{P_e,n_g,m_a,x}', 'Location', 'North');
r_min = Vskarn^2/(g*sqrt(n_max.^2-1));

% Turn rate
w_cl = 180/pi*g./V3.*sqrt(n_cl.^2-1); w_pmax = 180/pi*g./V3.*sqrt(n_pmax.^2-1);
figure(10);
plot(V3,w_cl,V3,w_pmax); grid on;
title('Turn rate vs. Velocity');
xlabel('V [m/s]');
ylabel('\omega [º/s]');
legend('\omega_{C_L,m_a,x}', '\omega_{P_e,n_g,m_a,x}', 'Location', 'North');
w_max = (g./Vskarn*sqrt(n_max.^2-1));
disp(['Maximum load factor: ' num2str(n_max) ' - Minimum turn radius: ' num2str(r_min) ' - Maximum turn rate: ' num2str(180/pi*w_max)]);

%%% Gliding Flight ----------------------------------------------

gamma_min = atand(2*sqrt(CD0*K));
RoD = sind(gamma_min)*(sqrt(m_0_korr*g/S*2*cosd(gamma_min)./(rho_H.*sqrt(CD0/K))));
t = 1/60*Height./RoD;
t_max = t(end);
Vh = RoD/tand(gamma_min);
z = Vh*60.*t;
z_max = z(end);
figure(11);
subplot(2,2,1);
plot(Height, RoD); grid on;
title('Minimum rate of descent vs. Altitude');
xlabel('y [m]');
ylabel('(R / D)_{m_i,n} [m/s]');
subplot(2,2,2);
plot(Height, Vh); grid on;
title('Horizontal velocity vs. Altitude');
xlabel('y [m]');
ylabel('V_H [m/s]');
subplot(2,2,3);
plot(Height, t); grid on;
```
title('Time remaining until impact vs. Altitude');
xlabel('y [m]');
ylabel('t [min]');
subplot(2,2,4);
plot(Height, z); grid on;
title('Glide range vs. Altitude');
xlabel('y [m]');
ylabel('x_g_l_i_d_e [m]');

%% Takeoff Analysis  
-----------------------------------------------------

n_to = 1.15^2;
% < n tillåten från turning flight, ok.
Vto = V_stall_vekt(1)*1.1;
Vtr = V_stall_vekt(1)*1.15;
T = P_eng_max*etap/Vto;
W = m_0_korr*g;
my = 0.03;
CL = 2*W/(rho_H(1)*1.1*V_stall_vekt(1)^2*S);
Kt = (T/W)-my;
Ka = rho_H(1)*S/(2*W)*(my*CL-CD0-K*CL^2);
SG = (1/(2*g*Ka))*log(Kt+Ka*Vto^2/Kt)
SR = Vto;
Rv = Vtr^2/(0.2*g);
D = CD0+K*CL^2/(rho_H(1)*Vtr^2*S);
gamma = asind((T-D)/W);
STRx = Rv*sind(gamma)
STR = sqrt(Rv^2-(Rv-10.7)^2)
HTR = Rv*1-cosd(gamma));
Stot = STR+SR+SG

%% Pitch Stability  
--------------------------------------------------------

m_vekt = [m_bat_korr+m_p m_l+m_c+m_cyl m_motor+m_motorkont m_e*0.136
m_e*0.306 m_e*0.288 m_e*0.27];
% Avstånd till delarnas geometriska centrum från planets framkant.
l_vekt = [3.37 5.45 5.45 1.76 1.92];
r_vekt = [0.3 0.25*1 0.5];
r_vekt(4) = 3/4*l_vekt(1);
r_vekt(5) = l_vekt(1)+l_vekt(2)/2;
r_vekt(6) = l_vekt(1)+l_vekt(2)/2;
r_vekt(7) = l_vekt(3)+(l_vekt(4)+l_vekt(5))/2;
r_cg = sum(m_vekt.*r_vekt)/m_0_korr;
ac = 4.5;