Abstract—In this work techniques for heating the fusion reactor ITER to thermonuclear temperatures, over 100 million kelvin, is investigated. The temperature is numerically computed for different heating configurations. The heat leakage is modeled to occur only via diffusion. The diffusion is assumed to be a combination of Bohm and gyro-Bohm diffusion. Basic conditions for a fusion reactor has been studied. The power needed for the different heat sources for the plasma to ignite is computed. Plots of the temperature profiles are included in the results together with plots showing the Q-value dependency on the power and the major radius.

I. INTRODUCTION

THE International Energy Outlook 2013 predicts that the world energy consumption will grow by 56 percent between 2010 and 2040[1]. The increase in the electric consumption will have to be produced somehow and as coal and oil will be increasingly expensive and is widely considered to be bad for the environment, an alternative is needed. That could be weather dependent renewables, gas, fission and hopefully fusion.

Gas releases carbon dioxide, many renewable sources are weather dependent and works intermittently and fission produces long lived highly radioactive waste. As an alternative research is being done on fusion with the goal to create a way to produce clean and sustainable electricity.

Fusion of hydrogen isotopes releases energy due to the difference in mass between the fused particles and the particles going in to the reaction. The reaction that is the easiest to achieve is the one between deuterium and tritium which produces an alpha particle and a neutron. The neutron will have a high speed. The neutrons from the fusion reactions gets absorbed by the lithium blanket in the reactor wall which in turn then heats water in a secondary cooling circuit. The temperature difference between the hot water and the cooling water creates an energy flow, in this case in the form of steam. The steam flows through a turbine and therefore turning it. The mechanical energy in the turbine is then converted to electrical energy in a generator connected to the turbine. The neutrons is captured by a section of the wall containing lithium which will create tritium that will later be used in the fusion reaction.

The first fusion reactor to produce net energy is called ITER and is being built in southern France. It is planned to be ready to run experiments in November 2020[2] and begin fusion in 2027. This reactor is build for the purpose of testing, it will not produce power to the grid. Some of the objectives with the projects are:

1) To create an ignited plasma
2) To get a $Q$, defined by \( Q = \frac{3nTe}{P_{\text{loss}}} \), value greater than 10.
3) To maintain fusion for eight minutes.

ITER is a tokamak type reactor which means that the plasma is magnetically confined in a torus. The plasma is confined in a way such that the high temperatures involved does not destroy the walls. The plasma is heated in several ways: injection of high energy deuterium, resistive heating, radio waves and finally, when the temperature is high enough: thermonuclear fusion.

II. IMPLEMENTATION

A tokamak is a type of fusion reactor where the vessel is shaped like a toroid. In order to confine the plasma and minimize the contact with the wall strong magnetic fields are applied. Note that the plasma temperature must be maintained and the walls must be kept cool to prevent meltdown of the reactor walls. The fuel is assumed to consist of an equal mix of completely ionized deuterium and tritium with the ion temperature being similar to the electron temperature. On a side-note, a completely ionized gas is the same thing as a plasma.

To achieve confinement, the orbits of the plasma particles must stay within the torus to avoid contact with the walls. This is achieved as the particles gyrate around the magnetic fields lines, making the trajectories helical. The poloidal magnetic field is a result of the toroidal movement of the charged particles in the plasma and the toroidal field is produced by coils surrounding the reactor vessel. The plasma current is induced via other coils.

Ignition is the state of the reactor when the heating from the fusion reactions is enough to maintain the temperature without auxiliary heating. The Lawson criteria is a simple criteria for estimating the conditions for ignition in a fusion reactor. The criterion is a product of the number density $n$, energy confinement time $\tau_E$, and the plasma temperature $T$ (here measured in eV). For fusion of deuterium and tritium the following version of the Lawson criterion holds.

\[
 n_e \tau_E T \geq 3 \cdot 10^{21} \tag{1}
\]

The Lawson criteria can be derived as follows: The confinement time $\tau_E$ is defined as the energy density in the plasma divided by the power loss $P_{\text{loss}}$

\[
 \tau_E = \frac{W}{P_{\text{loss}}} \Rightarrow P_{\text{loss}} = \frac{W}{\tau_E} \tag{2}
\]

The energy is defined as

\[
 W = 3nTe \Rightarrow P_{\text{loss}} = \frac{3nTe}{\tau_E} \tag{3}
\]

The heating from the alpha particles in the fusion reactions has to exceed the power losses

\[
 fE_{\alpha} \geq P_{\text{loss}}
\]
Where $f = \frac{1}{4} n^2 \langle \sigma v \rangle$ is the number of fusion reactions occurring per volume and time and $E_\alpha$ is the kinetic energy of the alpha particle from the fusion. This gives us the inequality

$$\frac{1}{4} n^2 \langle \sigma v \rangle E_\alpha \geq \frac{3nT_e}{\tau_E}$$

(4)

By multiplying both sides with $T$ we get

$$\tau_E n T \geq \frac{12T_e^2 e}{E_\alpha \langle \sigma v \rangle}$$

(5)

By approximating $\langle \sigma v \rangle \propto T^2$ we get that the right hand side is a constant which according to [3, Eq. 1.5.5 p. 11] is $\tau_E n T > 3 \cdot 10^{21}$.

In order to achieve high core temperatures, the wall needs to be far away from the center for it to be able to withstand the temperatures. The temperature profile is independent of the minor radius of the containment vessel. The temperature decreases with the distance to the center. This means that the core temperature can be increased if the wall is further from the center.

ITER aims at getting the fusion energy gain factor $Q = 10$ where $Q$ is the fusion power divided by the auxiliary heating.

$$Q = \frac{P_{fusion}}{P_{auxiliary}}$$

(6)

This will require a temperatures in excess of 100 million Kelvin. This study looks at ITER, tokamaks and fusion in a very simplified way and give phenomenological explanations of some of the the requirements of the reactor.

A. Geometry

The geometry of the tokamak is a torus with major radius $R_0$ and a minor radius $a$. Figure 1 shows the geometry of the torus from above, the bright circle is a crosssection. It is somewhat complicated to use toroidal coordinates so from here on we will consider the toroid to be a periodic cylinder with a period of $2\pi R_0$ which is an equivalent problem.

Consider a torus shaped shell with a thickness $\delta r$ and a minor radius $r$ in the tube, we then get the circumference $O$ of the magnetic axis, the surface area $A$ of the shell and a small volume element $dV$.

$$O = 2\pi R_0, \quad A = \pi(r + \delta r)^2 - \pi r^2 = 2\pi r \delta r$$

$$dV = AO = 2\pi r \delta r O = 4\pi^2 R_0 \delta r$$

(7)

Since ITER will have a plasma that is toroidal with an elongation $\kappa$, the elongation will be used when integrating over the volume. The elongation $\kappa = \frac{b}{a}$ is defined as the major radius divided by the major axis of an ellipse, see Figure 2.

B. Derivation of the temperature equation

The temperature of the plasma can be calculated from the energy conservation [4, Eq 4.1 page 61]

$$\frac{3}{2} \left( \frac{\partial p}{\partial t} + \nabla \cdot p \nu \right) + p \nabla \cdot \nu + \nabla \cdot q = S$$

(8)

Where $p$ is the pressure, $t$ is the time, $\nu$ is the velocity, $q$ is the energy flux and $S$ is the power density from sources and sinks. The flows are in this case negligible, thus

$$\nabla \cdot p \nu = 0, \quad p \nabla \cdot \nu = 0$$

This simplifies equation (8) to

$$\frac{3 \partial p}{2 \partial t} + \nabla \cdot q = S$$

Stationarity gives

$$\nabla \cdot q = S$$

(9)

or

$$\iiint_\Omega \nabla \cdot q dV = \iiint_{\partial \Omega} q \cdot \hat{n} dA = \iiint_\Omega S dV$$

(10)
This tells us that the power flux through the surface, \( \partial \Omega \) is equal to the source inside the volume \( \Omega \). The divergence in cylindrical coordinates can be written as

\[
\nabla \cdot \mathbf{q} = \frac{1}{r} \frac{\partial}{\partial r} r q_r. \quad (11)
\]

In this work we assume that the energy flux \( q_r \) given by Fick’s first law

\[
q_r = -D_{gb} \nabla U \quad (12)
\]

\( D_{gb} \) is given by [16] and \( U \) is the energy density.

\[
q_r = -D_{gb} \frac{\partial U}{\partial r}
\]

where

\[
U = \frac{3}{2} \left( n_e T_e e + n_i T_i e \right) \quad [4] \text{ p. 61}
\]

With the density \( n_c = n_i \) and the temperature(in eV) \( T_i = T_e = T \) we get

\[
q_r = -\frac{3}{2} n_c e D_{gb} \frac{\partial T}{\partial r}
\]

(13)


\[
\Rightarrow \frac{1}{r} \frac{\partial}{\partial r} \left( r D_{gb} \frac{\partial T}{\partial r} \right) = -\frac{S(r, T)}{3 n_c e}
\]

(14)

The term \( S \) is a sum of all the heat sources and sinks:

\[
S = S_\alpha + S_B + S_{NB1} + S_{ECHRH} + S_{ICRH} + S_O \quad (15)
\]

Those terms are explained in their own subsections. \( S_\alpha \) is the fusion source, the energy from the alpha particle, \( S_B \) is the sink from Bremsstrahlung.

C. Bohm diffusion

In this work we use a combined Bohm/gyro-Bohm diffusion coefficient[5]

\[
D_{gb} = a_1 \chi_{\text{Bohm}} + a_2 \chi_{\text{Gyro-Bohm}} \quad (16)
\]

Where we use \( a_1 = \frac{1.6 \times 10^{-7} + 8.5 \times 10^{-5}}{2} \), \( a_2 = \frac{1.75 \times 10^{-7} + 3.5 \times 10^{-2}}{2} \)

and

\[
\chi_{\text{Bohm}} = \left( \frac{2 \pi a^2 B_{\text{tor}}}{\mu_0 R_0 I_{\text{plasma}}} \right)^2 \frac{T}{B_{\text{tor}}} \quad (17)
\]

and

\[
\chi_{\text{Gyro-Bohm}} = \frac{T \rho}{a B_{\text{tor}}} \quad (18)
\]

\( B_{\text{tor}} = 5.3 \) T[6] and the gyro radius \( \rho \) is

\[
\rho = \frac{v_{th}}{\omega_g} \quad (19)
\]

Where \( \omega_g = \frac{e B_{\text{tor}}}{m} \) is the cyclotron frequency and \( v_{th} = \sqrt{\frac{2kT}{m}} \) is the thermal velocity. Since \( \rho \propto \sqrt{m} \) and the plasma is a combination of deuterium and tritium we construct an average mass, \( m \), as

\[
\sqrt{m} = \frac{\sqrt{m_d} + \sqrt{m_t}}{2}
\]

\[
\Rightarrow m = \frac{\left( \sqrt{m_d} + \sqrt{m_t} \right)^2}{4} \quad (20)
\]

After some algebra we get that

\[
\rho = \sqrt{\frac{T}{2eB_{\text{tor}}^2}} \left( \sqrt{m_d} + \sqrt{m_t} \right) \quad (21)
\]

\( m_d \) and \( m_t \) is the mass of deuterium respectively tritium.

D. Ohmic heating

Ohmic heating is caused by the collisions of electrons and ions in the plasma. The ramping up of the externally applied magnetic field induces a current in the plasma. The resistivity of the plasma creates friction, creating a force balance with the electric field from the change in the magnetic field.

The power density per unit volume of Ohmic heating \( S_O \) is

\[
S_O = \eta j^2 \quad (22)
\]

Where \( j \) is the current density and \( \eta \) is the resistivity

\[
\eta = \frac{(2m_e)^{1/2} e^2 \ln(\Lambda)}{12e_0^2 (\pi T_e)^{3/2}} \quad [7] \text{ Eq. 11.30}
\]

(23)

Where \( m_e \) is the electron mass, \( e \) is the elementary charge, \( e_0 \) is the vacuum permittivity and \( T_e \) is the electron temperature in eV. The numerical value is

\[
\eta = \frac{5 \cdot 10^{-5} \ln(\Lambda)}{T_e^{3/2}} \quad (24)
\]

\[
\ln(\Lambda) \approx 20[\text{Eq. 37}] [4]
\]

(22) can be derived as follows. Consider an infinitesimal work on a charged particle \( dW = Fdr \). The power on \( N \) particles becomes

\[
P = N \frac{dW}{dt} = NF \frac{dr}{dt} = NFv \quad (25)
\]

The Coulomb force \( F = qE \) and the electric field is \( E = \eta j \). This gives

\[
P = qv \eta j N \quad (26)
\]

Where \( v \) is the difference is velocity between the ions and the electrons. From the definition of power density we get

\[
S_O = \frac{P}{V} = \frac{Nqv \eta j}{V} \quad (27)
\]

With \( \frac{N}{V} = n \) we get

\[
S_O = \frac{P}{V} = nqv \eta j \quad (28)
\]

and with the current density \( nqv = j \) we end up with the desired

\[
S_O = \eta j^2
\]
E. Neutral beam injection

Neutral Beam Injection (NBI) is a heating process involving injection of fast neutral particles, in this case deuterium, into the plasma chamber. Since neutral particles are not affected by magnetic fields they are able to penetrate into the plasma. Inside the neutrals undergo charge-exchange in which they lose an electron and thus becomes confined by the magnetic field.

In order to calculate the NBI power source \( S_{NBI} \) we assume \( N_0 \) particles each having a mass \( m \) is inserted each second. The total power is \( P \)

\[
P = N_0 \frac{mv^2}{2} \Rightarrow v = \sqrt{\frac{2P}{N_0m}} \tag{29}
\]

Every second there is a probability of ionization via charge-exchange. Charge-exchange is a collision between a charged and a neutral particle where charge is transferred between the particles in the collision while the energy of the particles stays the same.

The probability of ionization per second will be denoted as \( p \). Assuming that the particles have the same velocities, \( v \), before and after the collisions we define the penetration depth, \( \lambda_{NBI} \), as

\[
\lambda_{NBI} = \frac{v}{p} \tag{30}
\]

The penetration depth is the distance into the plasma where the number of particles flowing through have decreased to \( N_0e^{-1} \). We assume that the fast neutrons will be inserted orthogonal to the torus, towards the symmetry axes shown in Figure 3. Call this direction \( \hat{x} \).

The difference in the number of non-ionized deuterium particles \( N \) in \( dx \) is

\[
\frac{dN_{Neu}}{dx} = -\frac{1}{\lambda_{NBI}} N \Rightarrow N_{Neu}(x) = N_0e^{-\frac{x}{\lambda_{NBI}}} \tag{31}
\]

The change rate of ions correlated to the change in numbers of neutral particles:

\[
dN_{Ion} = -\frac{dN_{Neu}}{dr} \delta r
\]

Each inserted neutral deuterium particle has an energy of \( E_{In} \)

\[
P_{Beam} = E_{In} dN_{Ion} \tag{32}
\]

\[
S_{NBI} = \frac{dP_{Beam}}{dV} \%
\]

With \( \lambda_{NBI} \) we get

\[
S_{NBI} = \frac{E_{In} - \frac{dN_{Neu}}{dr} \delta r}{4\pi^2 R_0^2 \delta \bar{r}} = -\frac{E_{In}}{4\pi^2 R_0^2} \frac{dN_{Neu}}{dr} \tag{33}
\]

\[
S_{NBI}(r) = -\frac{E_{In}}{4\pi^2 R_0^2} \left( \frac{dN_{Neu}}{dx} \bigg|_{x<a} + \frac{dN_{Neu}}{dx} \bigg|_{x>a} \right) \tag{34}
\]

\[
\frac{dN_{Neu}^{<a}}{dx} = -\frac{N_0}{\lambda_{NBI}} e^{-\frac{x-a}{\lambda_{NBI}}}
\]

\[
\frac{dN_{Neu}^{>a}}{dx} = -\frac{N_0}{\lambda_{NBI}} e^{-\frac{x+a}{\lambda_{NBI}}}
\]

To account for the finite width of the beam the source is corrected by a factor \( \sqrt{\frac{r}{\epsilon^2 + r^2}} \), where \( \epsilon \) is the width of the beam, which also prevents the diverging behavior in the formula for small \( r \)

\[
S_{NBI}(r) = \frac{E_{In} N_0}{4\pi^2 R_0 \sqrt{\epsilon^2 + r^2}} e^{-\frac{x-a}{\lambda_{NBI}}} + e^{-\frac{x+a}{\lambda_{NBI}}} \tag{35}
\]

As NBI sends through highly energetic particles with an exponentially decaying profile, some energy will be deposited into the opposite side of the beam injector, in this case the center. Too much of this energy will damage the reactor and so the maximum shine trough allowed for NBI is set to five percent of the power.

F. Fusion reactions

The kind of nuclear fusion that will be used in ITER is \( ^2\text{D} + ^3\text{T} \rightarrow ^3\text{He}(3.5\text{MeV}) + ^1\text{H}(14.1\text{MeV}) \) \[4, Eq. 2.17\].

Because the neutron is neutral, it will not become a part of the plasma and instead go straight to the reactor wall. This means that only the alpha particles will contribute to the plasma heating.

According to \[4, Eq. 4.5 p. 63\] the power density \( S_\alpha \) is given by

\[
S_\alpha = \frac{E_{\alpha} n^2 \langle \sigma v \rangle}{4} \tag{36}
\]

where \( \langle \sigma v \rangle \) is given by

\[
\langle \sigma v \rangle = C1 \cdot \theta \sqrt{\frac{\xi}{m_\alpha c^2 T^3}} e^{-3\xi} \tag{8, Eq 12}
\]

\[
\theta = \frac{T}{1+T(C2+C4+C6)} \tag{8, Eq 13}
\]

\[
\xi = \left( \frac{B_0^2}{4\delta} \right)^{1/3} \tag{8, Eq 14}
\]

The constants are given in Table I.
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>B_{G}(\sqrt{keV})</th>
<th>m_{e}c^{2}(keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.17302 · 10^{-2}</td>
<td>1124572</td>
</tr>
<tr>
<td>C2</td>
<td>1.53861 · 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>7.51886 · 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>4.60643 · 10^{-3}</td>
<td></td>
</tr>
<tr>
<td>C5</td>
<td>1.35000 · 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>C6</td>
<td>−1.06750 · 10^{-4}</td>
<td></td>
</tr>
<tr>
<td>C7</td>
<td>1.36600 · 10^{-5}</td>
<td></td>
</tr>
<tr>
<td>T_{i}, range keV</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta(\sigma v))_{max}(</td>
<td>%)</td>
<td>0.2 − 100</td>
</tr>
</tbody>
</table>

Table I

**IMPROVED FORMULAS FOR FUSION CROSS-SECTIONS AND THERMAL REACTIVITIES**

G. Radio-frequency heating

There are two types of radio frequency heating involved in this project. Ion Cyclotron Resonance Heating (ICRH) and Electron Resonance Heating (ECRH). The waves get absorbed when the wave frequency is the same as the cyclotron frequency \( \omega_{g} \) of the ions and electrons respectively. Since \( B_{\text{tor}} \) depends on the distance from the toroidal axis the cyclotron frequency will also depend on the distance. This means that there will be an area where the cyclotron frequency is the same as source frequency at which the energy of the wave will be absorbed. Since the wave frequency can be controlled, the absorption area can be chosen fairly freely. From [9] we identify that the absorption curve will take on a Gaussian profile. In our equations we chose that the ECRH will be centered at a radius 0.3a and with a width 0.08a, while the ICRH will be centered at the magnetic axis with a width of 0.2a.

H. Bremsstrahlung

Bremsstrahlung is the radiation process when a charged particle is accelerated by another charged particle through Coulomb collision. The power density radiated from bremsstrahlung is

\[
S_{B} = 5.35 \cdot 10^{3} Z_{\text{eff}} n_{20}^{2} (T)^{1/2} \tag{37}
\]

Where \( Z_{\text{eff}} \approx 1 \) is the effective number of protons, \( n_{20} \) is the number density is units of \( \frac{10^{20}}{m^{3}} \) and \( T \) is the temperature in keV.

I. The temperature equation on integral form

Equation (34) is rewritten here for numerical analysis. Let’s first define

\[
y(r) = r D_{gb} \frac{\partial T}{\partial r}, \quad f(r) = -\frac{r S(r, T)}{3n_{e}c} \tag{38}
\]

Integrating (34) with respect to \( r \) gives the equation

\[
\int_{r_{1}}^{r'} \frac{dy}{dr} dr = \int_{r_{1}}^{r'} f(r) dr
\]

\[
y(r') - y(r_{1}) = \int_{r_{1}}^{r'} f(r) dr \tag{38}
\]

With \( T \frac{\partial T}{\partial r} = \frac{1}{2} \frac{\partial T^{2}}{\partial r} \), we get

\[
y(r') = T K \frac{dT^{2}}{2} dr = \frac{dT^{2}(r)}{dr} \frac{2y(r')}{r'K} \tag{39}
\]

Where

\[
K = \frac{1.6 \cdot 10^{-4} + 8 \cdot 10^{-5}}{2} \left( \frac{2\pi a^{2} B_{\text{tor}}}{\mu_{0} R_{\text{plasma}}} \right)^{2} \frac{1}{B_{\text{tor}}} +
\]

\[
\frac{1.75 \cdot 10^{-2} + 3.5 \cdot 10^{-2}}{2} \frac{\rho}{a B_{\text{tor}}} \tag{39}
\]

Integration gives

\[
T^{2}(r) - T^{2}(r_{0}) = \int_{r_{0}}^{r} 2y(r') r'K dr' \tag{40}
\]

Inserting the expression for \( y(r') \) from (38)

\[
\Rightarrow T^{2}(r) = T^{2}(r_{0}) + \int_{r_{0}}^{r} \frac{2}{r'K} \left( \frac{r_{1} K dT^{2}}{2} \right) r_{1} - \int_{r_{1}}^{r} \frac{r'' S(r'', T)}{3n_{e}c} dr'' \tag{40}
\]

With \( r_{0} = a \) and \( r_{1} = 0 \) we get that \( \frac{dT^{2}}{dr} \bigg|_{0}^{a} = 0 \) and to handle the divergent behavior when \( r \) approaches 0 we use \( r = \sqrt{r^{2} + (0.01 \cdot a)^{2}} \) in the denominator.

\[
T^{2}(r) = T_{a}^{2} - \int_{a}^{r} \frac{2}{\sqrt{r^{2} + (0.01 \cdot a)^{2}} K} \left( \int_{r_{1}}^{r} \frac{r'' S(r'', T)}{3n_{e}c} dr'' \right) dr' \tag{41}
\]

is the equation to be solved numerically, see section III.

### III. METHODS

**The way this problem will be solved**

1. Set the boundary temperature \( T_{a} \). Assume \( T_{a} \) to be the temperature everywhere and save it to \( T_{\text{list}} \).
2. Create a list of \( r \) of equal length starting at \( \text{minor radius length} \times T_{\text{list}} \) and ending at minor radius.
3. Interpolate \( T_{\text{list}} \) to \( T \).
4. Integrate the inner integral using quad.
5. Integrate the outer integral using quad.
6. Save \( T_{\text{list}}(r) \).
7. Estimate the error in \( T_{k} \).
8. If the absolute error is too large: go to II.

The relative error in \( T \) is

\[
\epsilon_{rel} = \frac{||T_{k+1} - T_{k}||}{||T_{k}||} \tag{42}
\]

Where the index denotes the iteration number.

The norm used is the \( L_{2} \) norm on a domain \( Q \)

\[
||f||_{L_{2}} = \sqrt{\int_{Q} f(x)^{2} dV} \tag{42}
\]

\[
T^{2}(r) = T_{a}^{2} - \int_{a}^{r} \frac{2}{K \sqrt{r^{2} + (0.01 \cdot a)^{2}}} \cdot y dr' \tag{42}
\]

inner integral

\[
y = \int_{0}^{r} \frac{r'' S(r'', T)}{3n_{e}c} dr'' \tag{42}
\]

outer integral
C3. TOKAMAK HEATING

IV. RESULTS

The simulations were based on the parameters shown in Table II.

<table>
<thead>
<tr>
<th>Name</th>
<th>Denotation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major radius</td>
<td>$R_0$</td>
<td>6.2 m</td>
</tr>
<tr>
<td>Minor radius</td>
<td>$\alpha$</td>
<td>2 m</td>
</tr>
<tr>
<td>Elongation</td>
<td>$\kappa$</td>
<td>$\approx$ 1.7</td>
</tr>
<tr>
<td>Plasma current</td>
<td>$I_{\text{Plasma}}$</td>
<td>15 MA</td>
</tr>
<tr>
<td>Tritium mass</td>
<td>$m_t$</td>
<td>$5.016 \cdot 10^{-27}$ kg</td>
</tr>
<tr>
<td>Deuterium mass</td>
<td>$m_d$</td>
<td>$3.344 \cdot 10^{-27}$ kg</td>
</tr>
<tr>
<td>ln($\Lambda$)</td>
<td>$\ln \Lambda$</td>
<td>20</td>
</tr>
<tr>
<td>Plasma volume</td>
<td>$\text{volume}$</td>
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</tr>
<tr>
<td>Plasma density</td>
<td>$n$</td>
<td>$10^{20}$ m$^{-3}$</td>
</tr>
<tr>
<td>Boundary temperature</td>
<td>$T_0$</td>
<td>$4 \cdot 10^4$ keV</td>
</tr>
<tr>
<td>Toroidal magnetic field</td>
<td>$B_{\text{tor}}$</td>
<td>5.3 T</td>
</tr>
<tr>
<td>Energy of the $\alpha$-particle</td>
<td>$E_\alpha$</td>
<td>3.5 MeV</td>
</tr>
<tr>
<td>Energy of the neutron</td>
<td>$E_{\text{neutron}}$</td>
<td>14.1 MeV</td>
</tr>
<tr>
<td>Penetration depth</td>
<td>$\lambda_{\text{NBI}}$</td>
<td>1.3 m</td>
</tr>
</tbody>
</table>

Table II

The calculated temperature profiles and the iterations of the numerical model is shown in Figure 4. The temperature is the highest in the center and the lowest at the boundary, as expected. The diffusion coefficient of the last iteration is then shown in Figure 5. The diffusion coefficient is decreasing with increased radius, as expected. A greater temperature difference is supposed to induce a larger diffusive flow.

The core temperature is in excess of 100 million Kelvin, as required for efficient fusion. With the auxiliary power at 50 MW the core temperature is supposed to be around 25 keV. The simulated value is a tad bit high, although still somewhat acceptable. While the fusion energy gain factor is too large to be realistic, the curve is supposed to be linear, as seen in Figure 6.
V. DISCUSSION

The gyro-Bohm coefficients have parameters from an experiment at JET, which might give an order of magnitude wrong compared to ITER. Because the system is in steady state, the power at radius \( r \) due to the diffusion will be equal to the integral over the sources from zero to \( r \).

In Figure 7 it is shown that Equation 10 is fulfilled, except for at the boundary, where the derivative routine does not work.

The working mechanisms of the Bohm diffusion has not been studied as it is far beyond the scope of a bachelor thesis.

VI. CONCLUSIONS

The trajectories of the plasma particles in a tokamak reactor are complex to model, the scientific community has been working on models since the 50’s and is still unable to explain some of the phenomena occurring. One such phenomena is a greater temperature gradient close to the plasma boundary.

In this work the heat flux is assumed to only occur via Bohm and gyro-Bohm diffusion, which is a simplification of the actual process. The coefficients may vary as much as by several orders of magnitude, meaning that the simulation is a rough approximation.

Because of the high temperatures, ignition is achieved even without external power, meaning that when we increase the external power the Q-value decreases as shown in Figure 8. In reality ignition is not achieved unless sufficient external power is supplied, however when ignition is achieved most of the external power will be unnecessary.

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