Forecasting accuracy for ARCH models and GARCH (1,1) family

Which model does best capture the volatility of the Swedish stock market?
Abstract

In the recent years more research has been focusing on the forecasting accuracy of heteroscedastic models. The aim of the study is to examine which autoregressive conditional heteroskedasticity model that has the best-forecast accuracy using stocks from the Swedish stock market. It can be seen that stocks with higher kurtosis were better predicted by using GARCH with student’s t innovations and stocks with lower kurtosis were forecasted better by using EGARCH. The results were estimated by using MSE (Mean Squared Error) as a measure of the difference between the predicted volatility and the stocks squared continuously compounded rate of return. It should be mentioned that there was a small difference between the different models MSE.

**Keywords:** Volatility, forecast, ARCH, ARCH with Student’s t distribution, GARCH, GARCH with Student’s t distribution, IGARCH, EGARCH, GARCH-M, TGARCH, Ericsson, H&M, Investor, Swedish stock.
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1. Introduction

1.1 Background and aim

It has become more important for financial institute after the financial crises to capture the movements of a financial asset. The movements are usually measured by the volatility (conditional standard deviation of the underlying assets return), and can also be seen as the risk of the asset – the more the asset moves, the more likely its value will decrease.

One of the biggest problem with modelling the volatility is one of its features, it has periods with low movements and then suddenly periods with high movements. The first model that assumed that the volatility is not constant is ARCH (autoregressive conditional heteroskedasticity model) by Engle in 1982. Because of his findings he got, The Sveriges Riksbank (Swedish Central bank) Prize in Economic Sciences in Memory of Alfred Nobel in 2003. Engel’s basic model has been transformed and developed too more sophisticated models, such as GARCH, IGARCH, TGARCH, EGARCH and GARCH-M. Even though the models should make the forecast accuracy of the original ARCH model better, are the new models really an improvement of the original ARCH?

Many of the different models have different features, which makes the forecast accuracy better. TGARCH is created to capture the negative movements of the volatility that usually is bigger than the positive movements. Another example is the EGARCH that should allow for the unequal changes of the volatility. But there is one question that is reasonable to ask: do the more sophisticated models capture the volatility better?

The first thought is that if they capture the volatility better it would lead to a more efficient forecast accuracy. However sometimes the simpler ARCH model maybe has a good forecast accuracy. So the question is: Which model has the best forecast accuracy?

There are a couple of studies that has been focusing on the forecast accuracy of models, which are assumed to capture the volatility movements.
1.2 Previous studies

Laurent, Rombouts and Violante (2012) examined which multivariate GARCH models performed best for 10 stocks. They compared 125 different GARCH models forecast precision under a 10 years period with forecasts of 1-, 5- and 20 days ahead. They found that under the unstable markets the multivariate GARCH models perform poorly.

Ramasamy and Munisamy (2012) did a study where they examined the prognostic correctness of GARCH, GJR (Glosten, Jagannathan and Runkle GARCH) and EGARCH models on the daily exchange rate for four currencies: Australian dollar, Thailand bhat, Singapore dollar and Philippine peso. The errors were estimated by using the predicted values and compared with the actual values for 2011. They found that the GARCH models are efficient to predict volatility exchange rates and that the improvement that the leverage does in EGARCH and GJR does not improve the forecasts a great deal.

Marcucci (2005) found that the GARCH model performs much better than the complex models such as MRS-GARCH (Markov Regime-Switching GARCH) in long horizons. But the MRS-GARCH model outperforms the GARCH model in a short time horizon. Maccucci means that due to the properties of the GARCH model it “implies to smooth and too high volatility forecasts”.

Cheong Vee, Nunkoo Gonpot and Sookia (2011) examine the forecasts for the US dollar and Mauritian Rupee using GARCH (1,1) with Generalized Error Distribution (GED) and with Student’s –t distribution. They used daily data and the difference between the daily data and the predicted forecast estimate MAE (Mean Absolute Error) and RMSE (Root Mean Squared Error). The result showed that GARCH (1,1) with GED had a better forecast than GARCH (1,1) with Student’s-t distribution.

Ederington and Guan (2004) found that the financial market has longer “memory” than explained by the GARCH (1,1) model but it has little impact on the ability to predict the further volatility. They found out that GARCH (1,1) had a better forecast then EWMA (Exponentially Weighted Moving Average) but between GARCH and EGARCH they couldn’t choose a favourite.
Awartani and Corradi (2005) study the forecasting ability of different GARCH (1,1) against asymmetric GARCH models and they found that the asymmetric models perform better than the GARCH (1,1) model on one step ahead forecast and on longer time horizon. They found out that GARCH (1,1) was only defeated by models that allows for asymmetry. For other models GARCH (1,1) performed well.

Ding and Meade (2010) examine which model of GARCH, EWMA and SV model (Stochastic Volatility model) that is to prefer under different volatility scenarios. They did a simulation experiment when they simulated volatility that is in the range from high to low. They found that SV had a better forecast when it comes to scenarios with high volatility while EWMA was to prefer under medium volatility and it converges faster than any GARCH models.

Hansen and Lunde (2005) compared 330 different ARCH (GARCH) models to test if any of these models outperformed the GARCH (1,1). The models were tested on the Deutsche Mark (DM) –USD dollar ($) exchange rate and IBM return data (realized variance). On the exchange data GARCH (1,1) was not outperformed by any of the more sophisticated models. But on the IBM return data the GARCH (1,1) has a poor performance compared to the models with leverage effect.

Köksal (2009) compared over 1000 volatility models (not only ARCH/ GARCH) in how well they could capture the movements of the historical ISE-100 Index data and they out-of-sample adjusting. After comparing it with more than 1000 models it was EGARCH (1,1) with Student’s t distribution that performed best. Köksal claims that it is because of the “t-distribution seems to characterize the distribution of the heavy tailed returns better than the Gaussian distribution or the generalized error distribution”. Köksal also found that the models with leverage effect will have a better performance than other models.

Goyal (2000) used CRSP value weighted returns (stocks) and examine the performance for some GARCH models. The result indicated that the GARCH-M has a bad forecast and simple ARMA specification performed better in the out-of-sample test.
So it is obvious that all these studies gives different results based on the data but also on the choice of included models. So the aim of this thesis is not to examine all the transformations of the GARCH and all the lags of ARCH and GARCH. Rather it is to test some of the most common models against each other for real data. Three Swedish stocks have been chosen to test the models on. Then there is the problem with the number of lags. To make it easy the number of lags is chosen to (1,1) for the GARCH and for ARCH the number of lags are the significant lags for the partial autocorrelation for the squared rate of return (see part 4.1, 4.2 and 5).

1.3 Disposition

The next chapter is an introduction to what volatility is and how it is compounded from the stocks closing price. The third section demonstrates and explains the models, which will be investigated. The fourth section is the method, which explains how to choose the ARCH order and how the result is done. The fifth part states the data and the distributions of the data. Part six is the results. The final part is discussion and conclusion and here the result is discussed and compared to the earlier studies and finally a recommendation for continuous research is done.
2. Important terminology

2.1 Volatility

In financial trading, one of the central parts is to try to capture the movements of the underlying asset, which is usually known as volatility. The volatility is the conditional standard deviation of the underlying assets return and denoted by $\sigma_t$. The volatility has some important features. One of the most important is that the volatility changes over time and that it is not directly visible in daily data since there is only one observation each trading day. The volatility depends on the trading in each day and between the days (the over night volatility) (Tsay 2005, p.97f).

The volatility is calculated by the assumption that it has a geometric Brownian motion. The geometric Brownian motion is derived by the Black-Scholes formula and it explains to how the price of an asset moves. According to the model the price is a stochastic process with a log normal distribution (Moles and Terry, 2005).

There are more characteristics of the volatility that should be mentioned and the first is that the volatility appears in clusters. The second is that the volatility changes over time and that jumps in the volatility are unusual. The third is that the volatility does not grow to infinity; it rather stays within some spans. The forth characteristics is that the volatility reacts different on a drop in the prices of the underlying asset then it does for an increase in the price of the underlying asset. When the price of the underlying asset drops the volatility increases more which is known as leverage (see TGARCH) (Tsay 2005, p.97f).

2.1 Continuously compounded rate of return

To be able to assess the forecast accuracy of the ARCH and GARCH models without using autoregressive and moving average terms the series has to be stationary. One way to make financial time series stationary is to use continuously compounded rate of return. Let’s denote the logarithmic rate of return as $r_t$. To calculate the logarithmic rate of return, let $P_t$ be the closing price on an asset at time $t$. Assume that the asset is hold under $t-k$ to $t$ periods then it will give the simple gross return on the asset for $k$ periods.
$1 + R_t(k) = \frac{P_t}{P_{t-k}} = \frac{P_t}{P_{t-1}} * \frac{P_{t-1}}{P_{t-2}} * \ldots * \frac{P_{t-k+1}}{P_{t-k}}$

$= (1 + R_t)(1 + R_{t-1}) \ldots (1 + R_{t-k+1})$

$= \prod_{j=0}^{k-1} (1 + R_{t-j})$

(2.1)

Take the logarithm of the simple gross return, which yields the rate of return ($r_t$) (Tsay 2005, p. 3f).

$r_t(k) = ln\left(1 + R_t(k)\right) = ln\left[(1 + R_t)(1 + R_{t-1}) \ldots (1 + R_{t-k+1})\right]$

$= ln(1 + R_t) + ln(1 + R_{t-1}) + \ldots + ln(1 + R_{t-k+1})$

$= r_t + r_{t-1} + \ldots + r_{t-k+1}$

(2.2)
3. Heteroskedasticity Models

3.1 Autoregressive conditional heteroskedasticity model (ARCH)

The Autoregressive Conditional Heteroskedastic models are used to capture the return of an asset. Engel created the first heteroskedastic model in 1982 to capture the movements of the inflation rate in UK (Bollerslev, 2009). Because of the characteristics of the volatility for any financial time series the ARCH model is built on two assumptions. The first assumption is that high volatility appears in clusters and therefore the movement of the assets return is dependent on the previous values but for the whole time series it’s uncorrelated. The second assumption is that the distribution of the asset returns \( a_t \) can because of its dependence with the previous values be explained by a quadratic function of the former lagged values. The model is built on the information set that exists at time \( t-1 \). The conditional variance depends on the previous m lagged innovations:

\[
\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2
\]  

(3.1)

In the equation it can be seen that large values of the innovation of asset returns has a bigger impact on the conditional variance because they are squared, which means that a large shock have tendency to follow the other large shock and that is the same behaviour as the clusters of the volatility.

The ARCH (m) model becomes:

\[
a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2
\]

(3.2)

where \( \epsilon_t \sim N(0,1) \) iid, \( \omega > 0 \) and \( \alpha_i \geq 0 \) for \( i > 0 \). An assumption in the model is that \( \epsilon_t \) are assumed to follow a standard normal, student t or generalized error distribution (Tsay 2005, p. 102f).
3.2 ARCH with student t distribution (ARCH with t innovation)

Recall Equation (3.2) that is an ARCH (m) with Gaussian distribution, the difference between them is the first part \( a_t = \sigma_t \epsilon_t \) where \( \epsilon_t \sim N(0,1) \ iid \). In ARCH with student t distribution \( a_t = \sigma_t y_t \) where \( y_t \sim t(d) \) and \( d \) is unknown degrees of freedom (Bollerslev, 2009).

3.3 Generalized autoregressive conditional heteroskedasticity model (GARCH)

In the ARCH model there is several restrictions that has to be fulfilled so that the model can sufficiently estimate the volatility, which can be a problem. Therefore Bollerslev (1986) recommend a transformation of the ARCH model, to a generalized ARCH model (GARCH). To be able to explain the GARCH model, lets start with the continuously compounded log return series \( r_t \). Before \( a_t \) was the innovations of the asset returns but now let the innovation at time \( t \) (\( a_t \)) be \( a_t = r_t - \mu_t \). Now we can rewrite \( a_t \) in a GARCH (m,s) model:

\[
a_t = \sigma_t \epsilon_t,
\]
\[
\sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2,
\]

(3.3)

where \( \epsilon_t \sim N(0,1) \ iid \), the parameter \( \alpha_i \) is the ARCH parameter and \( \beta_i \) is the GARCH parameter and \( \omega > 0, \alpha_i \geq 0, \beta_j \geq 0 \) and \( \sum_{i=1}^{\text{max}(m,s)} (\alpha_i + \beta_i) < 1 \).

The restriction on the ARCH and GARCH parameters (\( \alpha_i, \beta_i \)) suggests that the volatility (\( a_t \)) is finite and that the conditional standard deviation increases (\( \sigma_t \)). If \( s = 0 \) then the model GARCH parameter (\( \beta_i \)) becomes extinct and then the left over is an ARCH (m) model.
The GARCH model can be rewritten in an ARMA (max \{m, s\}, s) expression.

\[
a_t^2 = \omega + \sum_{i=1}^{\text{max}(m,s)} (\alpha_i + \beta_i)a_{t-i}^2 + \eta_t - \sum_{j=1}^{s} \beta_j \eta_{t-j},
\]

(3.4)

where \(\eta_t = a_t^2 - \sigma_t^2\), which gives that \(\sigma_t^2 = a_t^2 - \eta_t\) (Tsay 2005, p. 113f).

Remember equation (3.3) the GARCH (1,1) (where 1,1 stands for first lags) can be written as

\[
a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 + \beta \sigma_{t-1}^2
\]

(3.5)

where \(0 \leq \alpha, \beta \leq 1, (\alpha + \beta) < 1\).

From this model we can see that big values for \(a_{t-1}^2\) and \(\sigma_{t-1}^2\) tend to give big values on \(\sigma_t^2\) that is happening when the volatility exists in clusters (Tsay 2005, p. 113f).

3.4 GARCH with student t distribution (GARCH with t innovation)

In equation (3.5) the first part \(a_t = \sigma_t \varepsilon_t\) where \(\varepsilon_t \sim N(0,1)\) iid. In GARCH with student t distribution (t innovation) the first part is written as \(a_t = \sigma_t \gamma_t\) where \(\gamma_t \sim t(d)\) and \(d\) is unknown degrees of freedom (Bollerslev, 2009).
3.5 Integrated GARCH (IGARCH)

Consider the GARCH model in ARMA format equation (3.4)

\[(1 - \alpha(L) - \beta(L))a_t^2 = \omega + (1 - \beta(L))\eta_t\]  \hspace{1cm} (3.6)

where \(\alpha(L)\) and \(\beta(L)\) is approximate defined lag polynomials. Then enforce a unit root process on the AR part \((1 - \alpha(L) - \beta(L)) = \varphi(L)(1 - L)\) and rewrite the model to:

\[\varphi(L)(1 - L)a_t^2 = \omega + (1 - \beta(L))\eta_t.\]  \hspace{1cm} (3.7)

The IGARCH (1,1) is written as:

\[a_t = \sigma_t \epsilon_t, \hspace{0.5cm} \sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + (1 - \beta)a_{t-1}^2\]  \hspace{1cm} (3.8)

where \(\epsilon_t \sim N(0,1)\) iid and \(0 < \beta_t < 1\) (Tsay) in Stata \((1 - \beta)\) is denoted by \(\alpha\). The IGARCH model is not covariance stationary (Bollerslev, 2009).

3.6 GARCH in the Mean (GARCH-M)

The M in GARCH-M stands for “in the mean”. The GARCH (1,1)-M is written as:

\[r_t = \mu + c\sigma_t^2 + a_t,\]
\[a_t = \sigma_t \epsilon_t, \]
\[\sigma_t^2 = \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2\]  \hspace{1cm} (3.9)

The equation for \(a_t\) and \(\sigma_t^2\) is equal the GARCH (1,1); the only part that differ is the equation for \(r_t\), where \(\mu\) and \(c\) are constants. The constant \(c\) is the risk premium parameter and a positive value shows that the return \((r_t)\) has a positive relation to its own volatility.

The formula for \(r_t\) (rate of return) indicates that there is serial correlation that can depend on the risk premium (Tsay 2005, p. 123).
3.7 Exponential GARCH model (EGARCH)

Even if GARCH is an improvement of ARCH, the GARCH model still has some issues with managing financial time series. Therefore Nelson suggested a new model in 1991, the exponential GARCH model (EGARCH) (Nelson, 1991). The change he proposed was that there should be a weighed invention to the model that should allow for the unequal changes of the volatility in the return of the asset.

Let $\alpha_t$ still be the innovation of the asset return at time $t$, then the EGARCH $(m,s)$ model can be written as:

$$
\alpha_t = \sigma_t \epsilon_t,
$$

$$
\ln(\sigma_t^2) = \omega + \sum_{i=1}^{s} \alpha_i \left| \alpha_{t-i} \right| + \theta_i \alpha_{t-i} \sigma_{t-i} + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-1}^2)
$$

(3.10)

Then EGARCH $(1,1)$ is written as

$$
\alpha_t = \sigma_t \epsilon_t,
$$

$$
\ln(\sigma_t^2) = \omega + \alpha(\left| \alpha_{t-1} \right| - E(\left| \alpha_{t-1} \right|)) + \theta \alpha_{t-1} + \beta \ln(\sigma_{t-1}^2)
$$

(3.11)

where $\epsilon_t$ and $|\alpha_{t-1}| - E(|\alpha_{t-1}|)$ are iid and have mean zero. When the EGARCH has an Gaussian distribution of the error term then $E(|\epsilon_t|) = \sqrt{2/\pi}$, which gives:

$$
\ln(\sigma_t^2) = \omega + \alpha(\left| \alpha_{t-1} \right| - \sqrt{2/\pi}) + \theta \alpha_{t-1} + \beta \ln(\sigma_{t-1}^2)
$$

(3.12)

There is one property that should be mentioned and this is that negative shocks of the volatility tend to have a bigger impact and therefore $\theta$ is often assumed to be negative (Tsay 2005, p. 124f). Because the model uses logarithms it causes difficulties when it comes to estimate an unbiased forecast (Bollerslev, 2009).
3.8 Threshold GARCH model (TGARCH)

The last model is the Threshold GARCH model (TGARCH) created by Glosten, Jagannathan and Runkle in 1993 and Zakoian in 1994. The idea behind TGARCH is that it should be better to capture the movements of the negative shocks, due to the fact that they have a bigger effect on the volatility than the positive shocks have (Tsay 2005, p. 130). To be able to capture the movements the model lets the conditional standard deviation be determined by the sign of the earlier lagged values (Bollerslev, 2009).

Remember that \( a_t = r_t - \mu_t \) and let \( N_{t-i} \) be an indicator variable if the previous lagged values of \( a_t \) is negative. Then

\[
N_{t-i} = \begin{cases} 
1 & \text{if } a_{t-i} < 0 \\
0 & \text{if } a_{t-i} \geq 0 
\end{cases}
\]

(3.13)

Mentioned earlier the formula for GARCH \((m,s)\) model is

\[
\sigma_t^2 = \omega + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2
\]

(3.14)

in the ARCH parameter of the GARCH \((m,s)\) model add the indicator variable with a constant \((\gamma_i)\) and get

\[
\sigma_t^2 = \omega + \sum_{i=1}^{m} (\alpha_i + \gamma_i N_{t-i}) a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2
\]

(3.15)

which is the TGARCH \((m,s)\) model, where \( \alpha_i, \gamma_i \) and \( \beta_j \) are positive parameters which has nearly the same properties as the GARCH models parameters. The difference between the GARCH and TGARCH is that in the TGARCH because of \( \gamma_i > 0 \) and if \( a_{t-i} \) is negative then \( a_{t-i}^2 \) has a bigger impact on \( \sigma_t^2 \) (Tsay 2005, p. 130).

TGARCH \((1,1)\) is written as:

\[
\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2
\]

(3.16)
4. Data and Method

4.1 Data material

The data that has been used is daily closing prices for the stocks Ericsson, H&M and Investor during a ten year period from 31st of March 2004 to 31st of March 2014, which gives 2519 observations for each stock. The data was collected using Nasdaq OMX Nordic database for historical share prices.

Graph (4.1) Daily closing prices for Ericsson, H&M and Investor from 31st of March 2004 to 31st of March 2014

![Graph showing daily closing prices](image)

*Notation: The stocks closing price is in SEK (Swedish krona).*

To capture the volatility without using ARMA extensions on the ARCH and GARCH models, the series has to be stationary and as mentioned in section 2.2 one way to make the data stationary is to use continuously compounded rate of return. The continuously compounded rate of return is calculated by using the equation (2.2).

*Graph (4.2) is the continuously compounded rate of return for Ericsson, Graph (4.5) is the continuously compounded rate of return for H&M and finally Graph (4.8) is the continuously*
compounded rate of return for Investor. And it can be seen in these graphs that all of them have more movements around the financial crises in the end of 2007 under 2008 and 2009 and that the volatility exists in clusters, there are periods with high volatility and periods with low volatility.

The *Graph (4.2)* is the rate of return for Ericsson. The graph shows that there are some extreme values, especially in the end of 2007. The Ericsson has volatility clustering, which indicates that there is an ARCH effect, which means some stationary parts and some more changeable parts.

*Graph (4.2) Continuously compounded rate of return for Ericsson’s stock*

It seems like there is an ARCH effect for the rate or return for Ericsson. An LM test for autoregressive conditional heteroskedasticity (ARCH) effect was done (see *Appendix (1)*). On the first lag of the test the p-value is 0.0131, which is smaller than 0.05, which means that the null hypothesis is rejected and there is ARCH effect.
Graph (4.3) Autocorrelation Function for continuously compounded rate of return for Ericsson

The Graph (4.3) is the sample autocorrelation function. The grey area is the 95 per cent confidence interval. The lags are the blue lines.

Graph (4.4) Continuously compounded rate of return for H&M’s stock
The *Graph (4.4)* demonstrates the continuously compounded rate of return for the H&M. And it can be seen from the graph it appears that there exists an ARCH effect (due to volatility clustering), so an LM test for autoregressive conditional heteroskedasticity (ARCH) effect was done for H&M (*Appendix (2)*). All the lags in the test have p-value 0.0000 and the null hypothesis is rejected, so there exists ARCH effect.

*Graph (4.5) Autocorrelation Function for continuously compounded rate of return for H&M*

*Graph (4.5)* is the sample autocorrelation for H&M’s rate of return.
Graph (4.6) shows the continuously compounded rate of return for Investor’s stock. It gives the impression that there occurs ARCH effect (due to volatility clustering). And as for the previous series, LM test for autoregressive conditional heteroskedasticity (ARCH) effect was carried out. The p-value for all lags was 0.0000 and the null hypothesis was rejected – the series has an ARCH effect (see Appendix (3)).
**Graph (4.7) Autocorrelation Function for continuously compounded rate of return for Investor**

Graph (4.7) is the sample autocorrelation function for the Investor’s continuously compounded rate of return.

**Table (4.1) Descriptive statistic over the continuously compounded rate of return for Ericsson, H&M and Investor**

<table>
<thead>
<tr>
<th>Stock</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ericsson</td>
<td>-.0000765</td>
<td>-.2718863</td>
<td>.153216</td>
<td>.0219925</td>
<td>-1.01495</td>
<td>17.92541</td>
</tr>
<tr>
<td>H&amp;M</td>
<td>.0003973</td>
<td>-.1024785</td>
<td>.0954885</td>
<td>.0158312</td>
<td>.0358333</td>
<td>7.066792</td>
</tr>
<tr>
<td>Investor</td>
<td>.0004496</td>
<td>-.1039605</td>
<td>.1364117</td>
<td>.0167374</td>
<td>.0982346</td>
<td>8.011327</td>
</tr>
</tbody>
</table>

In Table (4.1) is the descriptive statistics for the rate of return. It can be observed that Ericsson’s rate of return has higher skewness and kurtosis than H&M and Investor. Either of the series seems to be normally distributed according the skewness and kurtosis.
4.2 Building an ARCH-model (ARCH specifications)

In order to properly assess the effect of a model on dataset it is important to decide which model should be used (Javed and Mantalos, 2013), therefore the right ARCH model that should be estimated has to be identified. Recall that the ARCH (m) model is

\[ a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2. \]  

(3.2, 4.1)

In the model \( a_t^2 \) is an unbiased estimator of \( \sigma_t^2 \) and because of the unbiased there should be a linear relationship between \( \sigma_t^2 \) and \( a_{t-1}^2, \ldots, a_{t-m}^2 \) like there is in an autoregressive model. Therefore one can use the PACF (Partial Autocorrelation Function) of \( a_t^2 \) to identify the order of the ARCH model.

Because the stock market is only trading during the week and on a normal week there is five days except the weekends – therefore the first five lags of the partial autocorrelation function for the squared rate of returns is used (Craioveanu and Hillebrand, 2009). Out of the first five lags only those lags were used which were significant at five per cent level.

*Graph (4.8) Partial Autocorrelation Function for the squared continuously compounded rate of return for Ericsson*
As can be observed from *Graph (4.8)* lag one, two and three are significant for the first five lags, which specifies the ARCH (1,2,3) for Ericsson’s rate of return.

*Graph (4.9) Partial Autocorrelation Function for the squared continuously compounded rate of return for H&M*

From *Graph (4.9)* it is obvious that all the first five lags are significant, so for H&M’s continuously compounded rate of return the ARCH model are ARCH (1,2,3,4,5).
From *Graph (4.10)* it can be observed that all first five lags are significant and the specification of the ARCH model according to Craioveanu and Hillebrand (2009) should be ARCH (1,2,3,4,5).

### 4.3 Why use GARCH models (1,1)

According to Javed and Mantalos (2013) numerous studies that investigate model selection for the GARCH models find that the “performance of the GARCH (1,1) model is satisfactory”. Javed and Mantalos (2013) claim that the first lag is sufficient to capture the movements of the volatility. To be able to compare the results, I will for the other GARCH models also use (1,1).

### 4.4 Why use Students t distribution only for GARCH (1,1)

In the GARCH model $a_t = \sigma_t \epsilon_t$ are assumed to follow a normal distribution but it has been showed that $a_t = \sigma_t \epsilon_t$ sometimes seems to have thicker tails compared to the normal therefore Bollerslev created the GARCH-t (GARCH with student t distribution) model in 1987 as a specific transformation of the GARCH. For the other transformed GARCH models they still assumes to follow a normal distribution (Bollerslev, 1987).
4.5 Method

Following the steps below has been used to try to evaluate the model with the best-forecast accuracy. The following procedures has been estimated for the three stocks (Ericsson, H&M and Investor) and for all the models mentioned above (ARCH, ARCH with Student’s t distribution, GARCH, GARCH with Student’s t distribution, IGARCH, EGARCH, GARCH-M and TGARCH) using Stata.

i. First estimated the coefficients for the first 2419 observations using log likelihood.

ii. Then estimated the conditional variance

iii. Then generated the error as each stocks squared rate of return minus the conditional estimated variance, for the last 100 observations

iv. And then finally calculated the MSE (mean squared error) for each model and stock

The formula for the MSE is

\[
\frac{1}{100} \sum_{i=2419}^{2519} (r(k)_{it}^2 - \hat{a}_{it}^2)^2
\]

where \( r(k)_{it}^2 \) is the squared continuously compounded rate of return for each one of the three stocks (Ericsson, H&M and Investor) at time t and \( \hat{a}_{it}^2 \) is the conditional estimated variance (Hansen and Lunde, 2005).
5. Analysis and Results

As declared in section 4.2 the model specification for Ericsson gives ARCH (1,2,3), for H&M gives ARCH (1,2,3,4,5) and for Investor (1,2,3,4,5).

Table (5.1) Estimated coefficients for the ARCH models and the respective MSE

<table>
<thead>
<tr>
<th></th>
<th>Constant ((\omega))</th>
<th>Lag1 ((\alpha_1))</th>
<th>Lag2 ((\alpha_2))</th>
<th>Lag3 ((\alpha_3))</th>
<th>Lag4 ((\alpha_4))</th>
<th>Lag5 ((\alpha_5))</th>
<th>MSE</th>
<th>Degrees of freedom</th>
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<tr>
<td>Ericsson</td>
<td>-0.0000912 (-0.21)</td>
<td>0.1129968 (5.17)</td>
<td>0.1274772 (8.75)</td>
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<td>-</td>
<td>9.937e-08</td>
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<td></td>
<td>.0002641 (12.65)</td>
<td>0.205244 (4.33)</td>
<td>0.1751373 (4.29)</td>
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<td>-</td>
<td>8.054e-08</td>
<td>3.988018</td>
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<tr>
<td>H&amp;M</td>
<td>.0001308 (32.00)</td>
<td>0.1394124 (6.98)</td>
<td>0.073913 (3.89)</td>
<td>0.1118382</td>
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<tr>
<td></td>
<td>.000121 (11.93)</td>
<td>0.1374968 (3.89)</td>
<td>0.1020548 (2.98)</td>
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<td>0.0657648</td>
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<tr>
<td>Investor</td>
<td>.0000937 (16.82)</td>
<td>0.0994687 (4.81)</td>
<td>0.1449146 (6.85)</td>
<td>0.1679876</td>
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<td></td>
<td>.0000943 (11.16)</td>
<td>0.0972777 (3.25)</td>
<td>0.1670039 (4.83)</td>
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</table>

Notation: The t-values within the parentheses

For the Ericsson’s continuously compounded rate of return it can be seen that the estimated coefficients are significant for all the three lags but not significant for the constant. The ARCH model with the lowest MSE is ARCH (1,2,3) with student t distribution.

For H&M’s rate of return the ARCH (1,2,3,4,5) models are significant and ARCH (1,2,3,4,5) with Gaussian distribution had the lowest MSE.

And finally for Investor both ARCH (1,2,3,4,5) with Gaussian and Student t distribution were significant, however ARCH (1,2,3,4,5) with Student t distribution has the lowest MSE.
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<th>Model</th>
<th>Constant</th>
<th>ARCH Lag1 (α)</th>
<th>GARCH Lag1 (β)</th>
<th>EGARCH (θ)</th>
<th>TGARCH (γ)</th>
<th>ARCH-M (r)</th>
<th>MSE</th>
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</thead>
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<td>Investor GARCH</td>
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Notation: The t-values within the parentheses
The GARCH models that were estimated for the three stocks were GARCH (1,1), GARCH (1,1) with student t distribution, IGARCH (1,1), EGARCH (1,1), TGARCH (1,1) and GARCH-M (1,1) (see section 4.3 for model specification).

As can be observed from Table (5.2) two of the models for Ericsson’s continuously compounded rate of return did not have significant coefficients: TGARCH (1,1) and GARCH-M (1,1). The GARCH (1,1) model with the lowest MSE for Ericsson’s rate of return is GARCH with Student t-distribution.

Table (5.2) shows that one of the models that were used on H&M’s continuously compounded rate of return had an insignificant coefficient, the GARCH-M (1,1). The model with the lowest MSE had EGARCH (1,1).

For Investor’s rate of return the same models as mentioned before has been used and as for the two other data sets the GARCH-M (1,1) had an insignificant coefficient. The model with the lowest MSE was EGARCH (1,1).

If both the ARCH- and GARCH models are compared than the model with the lowest MSE for Ericsson’s continuously compounded rate of return was GARCH (1,1) with Student t-distribution. For H&M’s and Investor’s continuously compounded rate of return it was EGARCH (1,1).
6. Discussion and conclusion

This paper has examined the forecast accuracy of different GARCH models. To assess which model that has the best-forecast precision the MSE (Mean squared error) was used. Previous research has shown that the volatility many times is higher than the estimated volatility using GARCH models. The investigation of MSE has shown that there is a difference between the forecasting accuracy for the different models. There was a difference in the MSE between the stocks.

Ericsson’s continuously compounded rate of return series had more movement, weaker ARCH effect (than the other two) and it had the lowest number of partial autocorrelation lags that were significant for the squared rate of return (three lags).

H&M’s rate of return had more constant movements and stronger ARCH effect (than Ericsson). The model that had the lowest MSE was EGARCH (1,1).

Finally, Investor’s rate of return had stable fractions and parts with more volatility, strong ARCH effect and as well as for H&M five significant lags for the partial autocorrelation function. EGARCH (1,1) was the model with the lowest MSE.

The results of this study indicate that there is a difference when it comes to the models. It seems like when the continuously compounded rate of return is more unpleasant then the GARCH (1,1) with student t innovation did a more accurately forecast model than any of the other ARCH- and GARCH models. When the rate of return is more nicely behaving then it gives the impression that EGARCH (1,1) is better than the rest of the models.

According to Hansen and Lunde (2005) models with leverage effects (like EGARCH) should have a better-forecast accuracy than GARCH on return series. But according to Ramasamy and Munisamy (2012) the leverage effect should not improve the forecast much. As can be seen from Table (5.2) the difference between GARCH and a GARCH with leverage effect, EGARCH and TGARCH are small, approximately $0.480e^{-08}$ and $0.021e^{-08}$, respectively. Ederington and Guan (2004) could not decide a favourite between GARCH and EGARCH because the difference was small which is consistent with the result of the study.

---

1 The values is the mean difference between the GARCH and EGARCH / TGARCH for all three stocks.
One other finding showed that GARCH-M (1,1) was insignificant for all rates of return and therefore it should not be used to analyse rate of return series. One explanation why the GARCH-M (1,1) didn’t work can have something to do with how the model is built. Considering equation (3.9) it has two constants μ and c that affect the GARCH-M (1,1) (r_t). When a log return series is used then the mean and the intercept is almost zero, which can affect the model to be insignificant. Goyal (2000) discovered the GARCH-M badly performance on return series, that implies that GARCH-M is not to prefer.

Cheong Vee, Nunkoo Gonpot and Sookia (2011) found that GARCH with GED (Generalized error distribution) performed better than GARCH with Student’s –t distribution. In this study the GED was not used but there was a difference between the GARCH (1,1) with Gaussian distribution and GARCH (1,1) with Student’s t distribution. GARCH (1,1) with Student’s t distribution performed better than GARCH (1,1) with Gaussian distribution, the mean difference for all the three stocks were $0.223 e^{-08}$.

One of the biggest problems when it comes to forecasting is that volatility is difficult to observe, which makes it difficult to estimate the actual performance of the models (Tsay). Therefore all the results are biased. To be able to draw some conclusion more data should be analysed or using some bias correction methods – like bootstrap. More data could also improve the study because more precisely conclusions could be drawn. So, further research should be done to include data over several stocks or have a difference for “calm” periods with low volatility or “intense” periods with high volatility.
References


Appendix

Appendix (1) LM test for autoregressive conditional heteroskedasticity (ARCH) effect for Ericsson

. estat archlm, lag(1/30)
LM test for autoregressive conditional heteroskedasticity (ARCH)

<table>
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H0: no ARCH effects vs. H1: ARCH(p) disturbance
**Appendix (2) LM test for autoregressive conditional heteroskedasticity (ARCH) effect for H&M**

```
. estat archlm, lag(1/30)
LM test for autoregressive conditional heteroskedasticity (ARCH)

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H0: no ARCH effects  vs.  H1: ARCH(p) disturbance
### Appendix (3) LM test for autoregressive conditional heteroskedasticity (ARCH) effect for Investor

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LM test for autoregressive conditional heteroskedasticity (ARCH)
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H0: no ARCH effects  vs.  H1: ARCH(p) disturbance