Multivariate approaches for Value-At-Risk and Expected Shortfall on electricity forwards

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Multivariate approaches for Value-at-Risk and Expected Shortfall on electricity forwards

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Abstract
This study applies a group of multivariate volatility models to forecast 1-day ahead electricity forward returns with data supplied by Swedish energy company Skellefteå Kraft AB. From these we estimate the risk measures Value-at-Risk and Expected Shortfall. We find that there is seasonality in volatility which results in four distinct portfolios for analysis. Four multivariate volatility models are applied to these portfolios which are then subject to minimum variance optimization. The forecasted risk measures of the portfolios is then treated to various backtesting procedures to assess which model is most suitable. The models are able to predict well but we find no strong results that favors any particular model in the backtests, although some indications lead us to recommend the industry standard Riskmetrics™ model.

Key Words – Forwards, Electricity, Value-at-Risk, BEKK, DCC, Backtesting
JEL classification – G11, G17

1 Introduction
The deregulations of the global electricity markets have created an increased financial risk awareness amongst electricity companies, partly in how they distribute their financial positions to achieve satisfactory dividends and partly to alert executives as to what a financial crisis may have in store for the company. Another feature of the electricity market is its tendency to be highly volatile which calls for sound risk management (Liu and Wu, 2007). Participants on the market today have the ability to trade electricity in numerous ways for which futures, forwards, options and hourly delivery are to name a few. The risk partaken by electricity producers on the financial markets can be reduced by portfolio allocation. This is of course withholding that a well diversified portfolio is supported by carefully chosen financial contracts, preferably relying on risk management backed by theory. This study is devoted to the multivariate modelling of portfolio
electricity forward returns and the corresponding volatilities on the electricity market with the forward series being supplied by Skellefteå Kraft AB, one of the largest energy companies in Sweden.¹ A range of models will be utilized and compared to each other through backtesting for an assessment of which model is best suited for the task at hand in terms of the risk measures Value-at-Risk (VaR) and Expected Shortfall (ES).

The price fluctuations in the underlying asset electricity occur depending on a variety of factors like marginal pricing, climate changes as well as production. Accounting for the most important factors one should note first and foremost that electricity prices exhibit large spikes due to inelastic demand combined with an exponentially increasing curve of marginal costs. These large price changes could take place due to such circumstances as changing weather conditions. However, after these rapid changes in the price a mean reversion process is likely to take place. One should also take into account that seasonality has a strong presence in electricity price time series, usually over days, weeks or years. (Benth et al., 2007)

To achieve a meaningful assessment of the risk taken by an institution the risk measure VaR was conceived, initially only secluded to professionals within firms it quickly became a popular measure for practitioners. This was certainly achieved by JP Morgan in the publishing of RiskMetrics¹™ Technical Document by Guaranty (1994) which was later on recognized by the Basel Committee for regulatory purposes. In Thinking Coherently by Artzner et al. (1997) as well as in Coherent Measures of Risk also by Artzner et al. (1999) the point was proven that VaR cannot be considered a coherent risk measure as it does not fulfill the axioms set by these articles. In particular VaR violates the axiom of sub-additivity in which a portfolio will, at most, amount to a risk that is the sum of the risk in its sub-portfolios. The alternative risk measure ES, on the other hand, does fulfill the requirements to be called a coherent risk measure in that context. Even so, VaR remains more widely used in risk management.

When Engle (1982) introduced the Autoregressive Conditional Heteroscedasticity model it quickly paved the way for an extensive selection of models for usage in volatility modelling circumstances. Although this breakthrough made volatility modelling of individual assets possible it did not take into account the nature of co-movements in assets, which could have great implications on, for example, portfolio allocation. The proposed remedy for this shortcoming came in the advent of the multivariate GARCH models. As with the univariate counterpart a variety of models soon spawned, but to a certain extent there has been some struggle to justify its existence compared to the univariate approach; see for example Bauwens et al. (2006), Berkowitz and O’Brien (2002), Brooks and Persand (2003), Christoffersen (2009) and Caporin and McAleer (2009). Other issues also occur within this framework due to the need of the covari-

¹for more information visit their website at http://www.skekraft.se/default.aspx?di=3624.
ance matrix being positive definite and the “curse of dimensionality” being ever present with inherent restrictions on the number of assets reasonable to model. So what is the point in using these models? A defining feature of the more developed multivariate GARCH models like the diagonal conditional correlation (DCC) or the BEKK model is that they add the element of time-varying correlations to the mix while pertaining traits like a positive-definite matrix or parsimony, and if proven more suitable for the task at hand than the univariate models such a model should be employed to adhere to the multivariate nature of the data set, should no other contending model be present.

The VaR measure itself holds great importance but in many cases overconfidence in this measure or lack of knowledge may lead to model choices without any criticism. Even though a model may provide accurate point estimations and decent fit, as time progresses and the nature of the data changes the model may be less favorable to other options without the knowledge of the user. The natural step to engage such a problem would be to apply statistics, specifically statistical tests to ascertain whether a model is estimating accurately or to decide which model to use from competing models.

This paper is organized as follows apart from this introduction. Section 2 describes the fundamentals of forwards, VaR and ES, while Section 3 accounts for the models being employed as well as previous literature. Section 4 explains the data set, gives some descriptive statistics and sets out the method while Section 5 displays the results. Section 6 concludes with a discussion along with suggestions for further research.

2 Forwards, VaR and ES

2.1 Forwards

Different ways of financially hedging against risk might be the use of options, forwards, futures and other various financial derivatives. The applied risk management in this study will be based on a portfolio of selected electricity forward contracts with various maturities traded in Euro. The currency exchange risk from SEK to Euro will not be taking into account in this study but is also a factor that should be considered when studying the financial risk a participant in the market may be exposed to.

A forward is a financial contract in which an agreement between two parties is struck today for the price of a certain amount of an underlying asset that will be traded at some future date for a then agreed upon price. The features of a forward contract are similar to that of the future contract, with the main difference being that the forward contract is traded directly between two parties without a clearing house. This in turn means that the forward contracts have a much higher credit risk than the futures since the clearing house greatly reduces the credit risk by making the two parties pay a margin deposit or an initial margin before entering a future contract.
The daily closing price of each electricity forward contract prices considered will be transformed to a rate of return $r_{t,T}$ in the following manner for each month:

$$r_{t,T} = \frac{F_{t,T} - F_{t-1,T}}{F_{t-1,T}}$$

where $F_{t,T}$ is the closing price for the contract at day $t$ and $F_{t-1,T}$ is the corresponding closing price at the previous day $t-1$ for a forward contract maturing at month $T$. In this study we will use the log returns as opposed to the simple returns for our electricity forward price series. (Gorton et al., 2013)

Forward price volatility, ceteris paribus, tend to increase with their maturity, this is referred to as the “Samuelson effect” (see Samuelson (1965)) and it can partly be explained by the fact that news affecting a certain asset will have stronger impact on short-term forwards while long-term contracts remain unchanged due to it not being immediately relevant. This is an often observable phenomena when studying energy contracts such as electricity contracts, the volatility tend to increases during the last six months of the contract. With reference to the Samuelson effect it may be wise to model electricity forwards with long time to maturity contra electricity forwards with short time to maturity separately. (Geman, 2009)

2.2 Value-at-Risk

The VaR measure at the confidence level $\alpha \in (0, 1)$ of a portfolio can be defined as the smallest number $l$ such that the probability that the loss $L$ exceeds $l$ is no larger than $(1 - \alpha)$, which in turn can be formally written as

$$\text{VaR}_\alpha = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}$$

VaR is usually reported at $\alpha = 95\%$ and/or $99\%$ regarding market and credit risk while operational risk is preferred to be reported on $\alpha = 99.9\%$. (McNeil et al., 2010)

2.3 Expected shortfall

VaR is a useful measure but it could underestimate the actual loss, instead the expected loss may be more informative, this introduces us to ES which is defined as “the expected loss given that the VaR is exceeded”. For a normally distributed loss function the conditional distribution given that VaR is exceeded is truncated normal distributed, for a given upper tail probability $p$ let $\text{VaR}_q$ be the associated VaR to the $q$:th quantile of $X$ if $X$ is standard normal distributed $X \sim N(0, 1)$. 


Then given that $X > \text{VaR}$ the expectation of $X$ is

$$E(X|X > \text{VaR}_q) = \frac{f(\text{VaR}_q)}{p}$$  \hspace{1cm} (3)

and

$$f(x) = \left(\frac{1}{\sqrt{2\pi}}\right) \exp\left(-\frac{x^2}{2}\right)$$  \hspace{1cm} (4)

is the probability distribution function of $X$. Generally, the ES for a log return $r_t$ with a conditional distribution $N(\mu_t, \sigma_t^2)$ is expressed as (Tsay, 2005)

$$ES_q = \mu_t + \frac{f(\text{VaR}_q)}{p} \sigma_t$$  \hspace{1cm} (5)

### 3 Modelling approaches and previous literature

The aim of this section is to summarize the theory of the models which shall form the basis for our empirical research. We describe various univariate as well as multivariate approaches for modelling volatility and finally conclude with some portfolio allocation theory and means of backtesting VaR and ES.

#### 3.1 Generalized autoregressive conditional heteroscedasticity models

The volatility, which can be measured through $\sigma$ (the standard deviation), is a highly important factor to estimate when modelling financial data and especially when estimating VaR and ES, otherwise the risk of a financial portfolio may be underestimated. Another property that is often observed when studying price series is heteroscedasticity, this is seen when financial data exhibit volatility periods (volatility clusters), large price fluctuations seems to be followed by additional periods of fluctuations. The generalized autoregressive conditional heteroscedasticity (GARCH) models deal with the heteroscedasticity of time series and by the use of historical volatility makes it possible to better predict time series.

For a log return series $r_t$ let $a_t = r_t - \mu_t$ where $a_t$ is the innovation at time $t$. The $a_t$ follows a GARCH($m$, $s$) if

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{m} \theta_i a_{t-i}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2$$  \hspace{1cm} (6)

where $\epsilon_t$ is an i.i.d. sequence with mean 0 and variance 1. The GARCH model captures the volatility clusters in a way such as if there is a large $\sigma^2$ or a large $a^2$ in the previous time period, it will cause a large $\sigma^2$ in the next period, generating this well known feature of financial time series. (Jorion, 2007)
3.2 Modelling Multivariate Volatility

The univariate volatility model considered in the previous section can be extended to the multivariate case through modelling of the unconditional or conditional covariance matrix for the set of electricity forward contracts in question. This section will therefore extend the scope of our study to these models as well.

Santos et al. (2013) studies the importance of modelling portfolio market risk by the use of univariate and multivariate GARCH models and determine that the multivariate GARCH models are a more appropriate choice when forecasting one-step-ahead VaR. The evaluation procedure, backed by backtesting, shows that these two different methods, the univariate and multivariate GARCH, differ significantly in favor of the multivariate approach in an out-of-sample period. This result holds both when considering real-market data as well as applied to data produced by Monte Carlo simulations.

Another area where the multivariate approach may prove a more appropriate choice than the univariate case is in portfolio optimization problems. The portfolio optimization according to Markowitz is studied in Pojarliev and Polasek (2003). Fitted multivariate GARCH models seem to predict one of the key factors discussed above, the covariance matrix, at more precise levels than the univariate counterparts. An adequate prediction of the covariance matrix is key in the application of the Markowitz theory for portfolio optimization since global minimum variance (GMV) portfolios are found to be very sensitive to inputs such as the covariance matrix.

3.2.1 EWMA estimation

Given innovations \( \{a_1, a_2, \ldots, a_{t-1}\} \) a simplified estimator for the unconditional covariance matrix at \( t \) is

\[
\hat{\Sigma}_t = \frac{1}{t-1} \sum_{j=1}^{t-1} a_j a_j'
\]

which thereby applies equal weights. But it is in many applications reasonable to expect that equal weights is not the case and often one could assume that past innovations has less impact than more present innovations. Applying exponential smoothing

\[
\hat{\Sigma}_t = \frac{1 - \lambda}{1 - \lambda^{t-1}} \sum_{j=1}^{t-1} \lambda^{j-1} a_j a_j'
\]

the covariance matrix now varies with time in that regard. With an sufficiently large \( t \) we have that \( \lambda \approx 0 \) in which the equation above can be expressed as

\[
\hat{\Sigma}_t = (1 - \lambda) a_{t-1} a_{t-1}' + \lambda \hat{\Sigma}_{t-1}
\]

which is the Exponentially Weighted Moving Average (EWMA) estimate. (Tsay, 2005)
The parameter \( \lambda \) can vary from 0 to 1 and the value which it assumes will affect the relevance of past innovations in which a lower value will have an adverse effect and vice versa. Throughout this study we will assign \( \lambda = 0.94 \) as consistent with the Riskmetrics\textsuperscript{TM} framework with daily observations (Guaranty, 1994). The EWMA clearly resembles the GARCH model besides not being subject to quasi maximum likelihood parameter estimation as pointed out by Engle and Mezrich (1995).

### 3.2.2 Multivariate GARCH

Analogous to the volatility clusters in the univariate GARCH case the multivariate case extends this to having correlation clustering where assets may move together through time in response to news in the market, these correlation clusterings may be captured by a multivariate GARCH model. In specifying a multivariate GARCH model one first specifies univariate GARCH models for the time-varying conditional variance by also taking into account the time varying conditional covariance of each pair of assets. (Alexander, 2008)

To generalize the EWMA approach to modelling multivariate volatility Bollerslev et al. (1988) suggests the Diagonal Vectorization (VEC) model as a straightforward extension of the univariate GARCH modelling approach. The model is

\[
\Sigma_t = A_0 + \sum_{i=1}^{m} A_i \odot (a_{t-i}a_{t-i}') + \sum_{j=1}^{s} B_j \odot \Sigma_{t-j}
\]

where \( m \) and \( s \) are nonnegative integers, \( A_i \) and \( B_j \) are symmetric matrices and \( \odot \) denotes the Hadamard product. As seen in the formula the variances and covariances included in \( \Sigma_i \) are still represented as univariate GARCH\((m, s)\) models. Although greatly simplified, the model comes at the cost of neither reassuring the user that a positive-definite covariance matrix is achieved nor being able to allow for dynamic dependence structure between volatility series. (Tsay, 2005)

### 3.2.3 BEKK model

To assure a positive definite covariance matrix in estimating a low dimensional multivariate GARCH model Baba, Engle, Kraft and Kroner (BEKK) proposed the general BEKK model

\[
\Sigma_t = CC' + \sum_{i=1}^{m} A_i(a_{t-i}a_{t-i}')A_j + \sum_{j=1}^{s} B_j \Sigma_{t-j}B_j
\]

where \( C \) is lower triangular and \( A \) and \( B \) are \( k \times k \) matrices with \( k \) being the number of assets. (Tsay, 2005)
Estimating multivariate GARCH models is often challenging since the optimization of the likelihood function is subject to increased difficulty as the parameter space increases. The so called “curse of dimensionality” quickly becomes an obstacle in this setting due to the sheer amount of parameters necessitated in the likelihood function and the recurring occurrence of local optima instead of a global optimum which give varying results in response to different starting values in optimization. (Alexander, 2008)

The diagonal BEKK model and the scalar BEKK model are two models that impose restrictions which can relieve financial time series from overparametrization. Given the full first order BEKK model, see (11), and putting the restrictions $B = AD$ where $D$ is a diagonal matrix we get the diagonal BEKK model which is a restricted VEC model. A further restricted model is the scalar model which assumes that $A = aI$ and $B = bI$. Each of the BEKK models are unique generalizations of the VEC models and guarantee positive definite matrices. (Silvennoinen and Teräsvirta, 2009)

These types of modelling approaches seems reasonable since we are modelling the same type of underlying asset but with different maturities in which the same news affects all of the forward contracts. This may suggest correlation clustering over time, especially with regard to the Samuelson effect.

3.2.4 DCC-GARCH Model

To reduce the amount of volatility equations present Bollerslev (1990) specifies the Constant Conditional Correlation (CCC) model in which the conditional correlation matrix $R_t$ is assumed to be constant over time, that is $R_t = R$. In such a setting the conditional covariance matrix $H_t$ evolves according to

$$H_t = D_t R D_t$$

(12)

where $D_t$ is a diagonal matrix containing the univariate GARCH volatilities which varies with time. A major flaw to the CCC model is its inherent assumption of constant conditional correlations as it is often not practical to assume. (Tsay, 2005)

As an extension of the CCC model by adding dynamic correlations, Engle and Sheppard (2001) were the first to consider the dynamic conditional correlation GARCH (DCC-GARCH) model where

$$H_t = D_t R_t D_t$$

(13)

with $R_t$ now being time dependent. In contrast to the degenerate case of CCC for estimating correlation, the path to calculating $H_t$ now involves applying maximum likelihood estimation not only to estimate $D_t$ but also in calculating $R_t$ given $D_t$. In estimating $R_t$ it may be further decomposed into

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2}$$

(14)
where $Q_t$ is the conditional covariance matrix satisfying

$$Q_t = (1 - \theta_1 - \theta_2) R + \theta_1 \epsilon_{t-1} \epsilon_{t-1}' + \theta_2 Q_{t-1}$$

(15)

where $\epsilon_t$ is the standardized residuals and $\theta_1$ and $\theta_2$ is non-negative parameters which satisfies $0 < \theta_1 + \theta_2 < 1$. (Tsay, 2005)

### 3.3 Portfolio optimization

When searching for an optimal portfolio there are several factors which affect the portfolio choice such as the market risk, time to maturity, volatility of the spot price and the different assets which constitutes the portfolio. Imagine a portfolio with $n$ assets and the vector of weights $w = (\omega_1, \ldots, \omega_n)'$ and expected returns $E(r) = (E(r_1), \ldots, E(r_n))'$ where each weight is applied to a certain asset according to $w'E(r)$. The optimization problem formulated as a mean-variance analysis was originally introduced by Markowitz (1952). One minimizes the variance with respect to the asset weights, that is

$$\min_{w_t} \ w_t' \Sigma_t w_t \quad \text{subject to} \quad w_t' I = 1$$

(16)

where $I$ is a $k$ by $1$ vector of ones. With forecasted covariance matrices in hand one can then readily calculate the unconstrained problem as

$$w_t = \Sigma_t^{-1} I / \Sigma_t^{-1} I^{'}. \quad (17)$$

One can further improve upon the minimum variance portfolio by taking into account the risk-free rate $R_f$ whereas the problem now becomes

$$\min_{w_t} \ w_t' \Sigma_t w_t \quad \text{subject to} \quad w_t' E(r_t) + (1 - w_t' I) R_f = E(r_{pt}). \quad (18)$$

Note that the restriction $w_t' I = 1$ is removed due to $(1 - w_t' I)$ being invested in the risk-free asset. In this case the problem is solved by

$$w_t = \Sigma_t^{-1} (E(r_t) - R_f I) / I' \Sigma_t^{-1} (E(r_t) - R_f I) \quad (19)$$

which corresponds to the tangency portfolio. One should note that these approaches are based on an assumption of multivariate normality where a known covariance matrix along with expected return is all that is required.
3.4 Backtesting Value-at-Risk

Although VaR may be stated in risk analysis through various parametric or nonparametric approaches the measure itself does not tell us if it is accurate. To achieve a measure of accuracy regarding VaR the concept of backtesting has been developed as a framework for comparing past portfolio losses with the VaR forecasts. Depending on the selected method from this framework, in which VaR measures is being compared to realized losses, the user may be able to draw conclusions to improve the model if it does not achieve a certain standard or reject the choice of model if competing options is favored. The Basel committee has adopted backtesting when implementing VaR measures for capital requirements, and rightly so as banks may be inclined to have incentives to understate their risk. Practitioners at a private firm may not be subject to regulatory constraints when providing management with stated VaR measures but should nonetheless apply backtesting to verify if the stated measure is accurate.

3.4.1 Unconditional coverage

The standard routine for backtesting is to compare past losses with their corresponding forecasted VaR. One soon realizes this is a statistical issue rather than a deterministic one as the VaR measure does not pinpoint the subsequent loss but rather states a figure to which the loss may be exceeded with a predetermined confidence level. The count of exceptions \( x \) expressed as a rate to the sample period \( N \) may be termed failure rate (or success rate if one prefers, as will be apparent further below). A straightforward way to employ backtesting is to simply use the failure rate within the binomial probability distribution

\[
f(x) = \binom{N}{x} p^x (1 - p)^{N-x}
\]

where \( p \) is the probability for which the VaR measure is stated and it is commonly known that \( E(x) = Np \) and \( V(x) = p(1 - p)N \). When \( N \) is deemed large enough the binomial distribution may be approximated using the central limit theorem with the normal distribution according to (Jorion, 2007)

\[
Z = \frac{x - Np}{\sqrt{p(1 - p)N}} \sim N(0, 1).
\]

Kupiec (1995) gives the corresponding chi-square log-likelihood ratio under the same asymptotic assumptions as above under the null that \( p \) is true

\[
LR_{uc} = -2\ln[(1 - p)^{N-x} p^x] + 2\ln\left\{\left[1 - \frac{x}{N}\right]^{N-x} \left(\frac{x}{N}\right)^x\right\} \sim \chi^2(1)
\]

which enables us to evaluate the null. There are some key concerns though, primarily the sample size and the probability chosen for the VaR measure under consideration.
Sample size is not only crucial for the asymptotic behaviour of the log-likelihood ratio but is also, as in many statistical circumstances, an important factor contributing to a reliable result as an increased sample narrows the confidence band upon which inference is made. A lower value of $p$ has the adverse effect of relying on rare events that an insufficient sample may not be able to record, lack of exceptions may even prevent a numerical solution to prevail. This may be remedied by extending the sample or coming to terms with reducing $p$ to a more manageable level, like the more conservative 5\% measure. (Jorion, 2007)

3.4.2 Conditional coverage

In the setting described above concerns are raised as to how the sample and probabilities may affect the rejection region of the tests, faulty rejection or acceptance of the null that are measured through type I and type II errors. Besides these concerns there are also drawbacks to the unconditional approach as it does not differentiate between samples where losses are incurred during clustering of exceptions, which ties closely to the volatility clustering often found significant in GARCH specifications. Can this effect be proven with reasonable confidence it may give an indication to choose models containing such clustering if not already utilized. (Jorion, 2007)

As an addition to the unconditional coverage test to deal with this problem was introduced by Christoffersen (1998), the idea is to add a independence test for the exceptions. The test in conditional in the sense that it accounts for time variation in the sample and conditions on the prevailing conditions.

To derive the independence test first assign indicators for failure and success of the exception, here we denote failure as 1 and success as 0. Then denote $T_{ij}$ as the count from the backtest sample which satisfies that it has indicator $i$ the previous day and indicator $j$ at the current day. Also denote $\pi_i$ as the probability of observing an exception at the current day conditional on the indicator $i$ occurring the previous day.

<table>
<thead>
<tr>
<th>Conditional Day Before</th>
<th>No exception</th>
<th>Exception</th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current Day</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Exception</td>
<td>$T_{00} = T_0(1 - \pi_0)$</td>
<td>$T_{10} = T_1(1 - \pi_1)$</td>
<td>$T(1 - \pi)$</td>
</tr>
<tr>
<td>Exception</td>
<td>$T_{01} = T_0\pi_0$</td>
<td>$T_{11} = T_1\pi_1$</td>
<td>$T\pi$</td>
</tr>
<tr>
<td>Total</td>
<td>$T_0$</td>
<td>$T_1$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

Table 1: Exception outcome conditional on the previous day exceptions outcome.

Source: Jorion (2007)
Table 1 gives the full set of outcomes. Should the first and second column be found to be identical the conclusion is that the exceptions are independent and the test conforms to the unconditional coverage log-likelihood. (Jorion, 2007)

The test statistic for the hypothesis that exceptions are independent across days is

\[
LR_{ind} = -2 \ln \left[ (1 - p)^{(T_{00} + T_{10})} p^{(T_{01} + T_{11})} \right] + 2 \ln \left[ (1 - \pi_0)^{T_{00}} \pi_0^{T_{01}} (1 - \pi_1)^{T_{10}} \pi_1^{T_{11}} \right]
\]

which is approximately chi-square distributed with 1 degree of freedom. Through simple addition we then have the relevant test statistic for the conditional coverage

\[
LR_{cc} = LR_{uc} + LR_{ind} \sim \chi^2(2)
\]

which is chi-square distributed with 2 degrees of freedom since a linear combination of chi-square distributed variables simply adds the individual degrees of freedom. (Jorion, 2007)

### 3.4.3 Comparative Predictive Ability (CPA)

The conditional and unconditional coverage criteria are helpful in testing if a model is adequate to assess the market risk, but they fall short when the question arises as to which of several competing models should be decided upon. Clearly a rejected model is less favored to an unrejected model, but the testing procedures mentioned are not suitable for ranking the remaining options. The Conditional Predictive Ability (CPA) test by Giacomini and White (2006) provides a testing procedure to address this issue. It should be clarified that the conditional approach in this circumstance is not meant as a substitute for the unconditional approach, as the former tells us which model should be suitable for a specific date using information currently available while the latter tells us which model is more generally suitable based on past experiences. Giacomini and White (2006) points out that their test also can involve the unconditional test of Diebold and Mariano (2002) and West (1996) (DMW) nested within the framework of the CPA and therefore the CPA becomes an extension of these findings. Despite this Giacomini and White (2006) choose to refer to the test as conditional even though it incorporates both procedures.

The null hypothesis of the CPA for two models is

\[
H_0 : E[\mathcal{L}_1^\alpha(e_{t,1}) - \mathcal{L}_2^\alpha(e_{t,2}) | \mathcal{F}_t] = E[\mathcal{L}D_1^\alpha | \mathcal{F}_t] = 0
\]

which signifies the circumstance when both models have equal predictive ability, \(\mathcal{L}^\alpha\) is the loss function for each respective model, \(\mathcal{L}D_1^\alpha\) is the loss difference and \(\mathcal{F}_t\) is the information which the expectation is conditional on. In case the conditioning set \(\mathcal{F}_t\) is disregarded the hypothesis coincides with that of DMW. Another key difference between
the DMW unconditional test and the CPA is that the DMW test, due to dependance on probability limits of the population parameters, makes statements about forecasting models. The CPA on the other hand depends on the estimated parameters, which makes a statement about what they refer to as the forecasting methods which does not only involve the models but also estimation procedures and choice of estimation window. (Giacomini and White, 2006)

To perform the test the following asymmetric linear loss function is used
\[ L^\alpha(e_t) = (\alpha - I(e_t < 0))e_t \] (26)

where \( \alpha \) is the order of the function which corresponds to the \( \alpha \) quantile forecast and \( e_t = y_{p,t} - VaR_{\alpha,t} \). This loss function is applicable when the forecast of a quantile is of interest which makes it suitable for VaR applications. (Santos et al., 2013)

A Wald test is then conducted
\[ CPA^\alpha = n(n^{-1} \sum_{t=m}^{T-1} \mathcal{I}_t \mathcal{LD}_{t+1}^a) \hat{\Omega}^{-1}(n^{-1} \sum_{t=m}^{T-1} \mathcal{I}_t \mathcal{LD}_{t+1}^a) \] (27)

where \( n = T - m \) is the out-of-sample size and \( \hat{\Omega} \) is a 2 \( \times \) 2 matrix containing the consistent estimates of the variance of \( \mathcal{I}_t \mathcal{LD}_{t+1}^a \). We assume according to Giacomini and White (2006) that \( \mathcal{I}_t = (1, \mathcal{LD}_t^a) \), the null hypothesis is rejected at significance level \( \theta \) if \( CPA^\alpha > \chi^2_{2,1 - \theta} \). The authors note that the test statistic \( CPA^\alpha \) can alternatively be computed as \( n \tilde{R}_2 \) in which \( \tilde{R}_2 \) is the uncentered squared multiple correlation coefficient of the regression of the constant unity on \( (\mathcal{I}_t \mathcal{LD}_{t+1}^a) \) over the out-of-sample period. Alternatively if one can assume conditional homoscedasticity of \( \mathcal{LD}_{t+1}^a \) the statistic instead corresponds to \( n \tilde{R}_2 \) retrieved from the regression of \( \mathcal{LD}_{t+1}^a \) on \( \mathcal{I}_t \) over the out-of-sample period. (Giacomini and White, 2006)

To conduct a unconditional test of the hypothesis
\[ E[\mathcal{LD}_{t+1}^a] = 0 \] (28)

within the same framework the test statistic is
\[ t_{m,n,1} = \frac{\overline{\mathcal{LD}}_{m,n}^a}{\hat{\sigma}_n / \sqrt{n}} \] (29)

where \( \overline{\mathcal{LD}}_{m,n}^a = n^{-1} \sum_{t=m}^{T-1} \mathcal{LD}_{m,t+1}^a \) and for example
\[ \hat{\sigma}_n^2 = n^{-1} \sum_{t=m}^{T-1} \mathcal{LD}_{m,t+1}^a + 2 \left[ n^{-1} \sum_{i=1}^{p_n} w_{n,i} \times \sum_{t=m+i}^{T-1} \Delta \mathcal{LD}_{m,t+1}^a \mathcal{LD}_{m,t+1-i}^a \right] \] (30)

with \( \{p_n\} \) a sequence of integers such that \( p_n \to \infty \) as \( n \to \infty \) and \( \{w_{n,i} : n = 1, 2, ..., i = 1, ..., p_n\} \) with \( |w_{n,i}| < \infty \). The null of equal unconditional predictive ability is rejected when \( |t_{m,n,1}| > z_{\alpha/2} \). (Giacomini and White, 2006)
Provided that a model comparison as described above will result in a rejected null
a means of assessing which model is better is also outlined in Giacomini and White
(2006). This decision rule for selection of forecast is based on the approximation
\( \delta_n I_T \approx E[LD^n_t|\mathcal{F}_T] \) where \( \delta_n \) is the regression coefficients from the regression of \( LD^n_{t+1} \) on \( I_t \)
over the out-of-sample period, if \( \delta_n I_T > c \) choose model 1 and if \( \delta_n I_T < c \) choose model
2 with c being specified by the user, we use \( c = 0 \). The plot of \( \{\delta_n I_t\}_{t=m}^{T-1} \) also generally
proves useful in determining which model to decide upon. We should additionally
clarify that the CPA procedure also applies to longer forecast horizons although we
restrict our attention to 1-day ahead forecasts. (Giacomini and White, 2006)

3.4.4 Backtesting Expected Shortfall

In 2013 the Basel Committee proposed the Basel III regulations to overhaul Basel II
which is currently employed. Basel II has shortcomings especially considering the tail
risk, one of the proposed changes in Basel III were to replace the 99% VaR with a 97.5%
ES (Kou and Peng, 2014). Although this seems like a sound choice there are reasons why
this has not already taken place in previous Basel accords, one reason is the difficulty
inherent in backtesting ES. (Kerkhof and Melenberg, 2004).

In our case we resort to a simple method of backtesting ES called the normalized
shortfall (NS), it is calculated as

\[
NS_{t,q} = \frac{r_t}{ES_{q,t}}
\]

(31)

where now \( r_t \) corresponds to the portfolio return. From the definition of ES we have
that

\[
\frac{E[R_t|R_t > VaR_{q,t}]}{ES_{q,t}} = 1
\]

(32)

and as such the average \( NS \), \( \bar{NS} \), should equal one which forms the null hypothesis.
The test tells us wether the mean of returns on days when VaR is exceeded is equal to
the average ES on those days. It is not a formal test like the coverage tests previously
mentioned as this would be a joint test of accuracy of VaR as well as the expectation
past VaR, which suggests that ES backtesting would be less reliable than VaR backtest-
ing. (Danielsson, 2011)

For more advanced methods of backtesting ES see Berkowitz and O’Brien (2002),
4 Data and descriptive statistics

The data set used in our empirical analysis was supplied by Skelleftea Kraft AB, one of the largest electricity distributors in Sweden. It contains in total 120 forward contracts maturing monthly. The study is restricted to price zone one\(^2\) with daily closing prices for each of the contracts from May 17, 2011 to October 15, 2013, the choice of price zone one over the other price zones is arbitrary as the overall characteristics are very similar between the zones such as trend and variance, but we acknowledge that future studies may need to take this into account. We decided to use four forward contracts with varying maturities \(T\) for each portfolio scenario to depict different outcomes. The reason for only using four contracts are first and foremost that of simplicity, as mentioned in Section 3 estimation of multivariate models to forecast the covariance matrix along with other parameters will quickly prove difficult with some of the proposed models. As a starting point of the analysis we have arbitrarily selected contracts with the following maturities during 2014: March, June, September and December which is called the in-sample portfolio due to it taking advantage of the full time series available when estimating models, this portfolio will be used to show how well the models fit and display some general characteristics of the data.\(^3\)

As seen in Figure 1 the electricity forward price series for the selected maturities of the in-sample portfolio tend to move together in trend but not in level, which is to be expected from the construction of the time series. As information reaches the market the different price series are prone to react in similar manners albeit having their slight differences due to different maturities. The levels of the price curves are affected by the way the forward contracts progress along the forward curve in a wave-like, sinusoidal pattern depicting its highly seasonal nature. This is obvious in Figure 1 as December clearly shows the highest price level whilst the months closer to the summer, June and September, display a distinctly lower level not intersecting the March and December curve. When transformed to log return series\(^4\) according to Section 2 we find all series stationary. We also study the ACF and PACF for various contract return series where we could not determine any recurrent pattern.

\(^2\)for more information on price zones see for example http://www.nordpoolspot.com/ How-does-it-work/Bidding-areas.
\(^3\)From here on out we will abbreviate maturity dates with YYYYMM year/month coding, for example March, 2014 will be stated as 201403 or alternatively 2014-03.
\(^4\)We will refer to our log return series as return series interchangeably from here on.
Figure 1: Plot of the electricity forward price series.
Figure 2: Difference in standard deviation for the last 50 days versus standard deviation of the preceding data for contracts maturing from Januari, 2012 to Januari, 2018.

Since we are studying the structure of the forward return series which depends on maturity $T$ we may see an increase in volatility as the contract draws closer to maturity according to the Samuelson effect. Walls (1999) has studied fourteen electricity futures contracts traded on the New York Mercantile Exchange (NYMEX) for delivery at the California-Oregon-Border (COB) network intertie for evidence of maturity effects by applying linear regression which involves volatility and volume of trade. The results of the study reveals strong evidence of maturity effects in electricity futures controlling for volume of trade, these effects also seem to be more present than for other energy futures such as crude oil, unleaded gasoline and heating oil contracts. In Figure 2 we have plotted the standard deviations of contracts maturing from 2012 to 2018 along with the standard deviations of these contracts before and during the last 50 days of data available. By the fact that the first 19 contracts in this plot have already matured they can be analyzed as to whether the Samuelson effect is present, which appears to be the case as the standard deviation is higher during the last 50 days.
Table 2: Correlation matrix of the portfolios.

<table>
<thead>
<tr>
<th></th>
<th>near maturity summer</th>
<th>near maturity winter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>201305 201306 201307 201308</td>
<td>201311 201312 201401 201402</td>
</tr>
<tr>
<td>201305</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>201306</td>
<td>0.750 1</td>
<td>201312 0.751 1</td>
</tr>
<tr>
<td>201307</td>
<td>0.538 0.615 1</td>
<td>201401 0.579 0.657 1</td>
</tr>
<tr>
<td>201308</td>
<td>0.558 0.613 0.851 1</td>
<td>201402 0.596 0.574 0.749 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>distant maturity summer</td>
<td>distant maturity winter</td>
</tr>
<tr>
<td></td>
<td>201307 201407 201506 201608</td>
<td>201401 201501 201512 201611</td>
</tr>
<tr>
<td>201307</td>
<td>1</td>
<td>201401 1</td>
</tr>
<tr>
<td>201407</td>
<td>0.551 1</td>
<td>201501 0.579 1</td>
</tr>
<tr>
<td>201506</td>
<td>0.432 0.532 1</td>
<td>201512 0.453 0.623 1</td>
</tr>
<tr>
<td>201608</td>
<td>0.369 0.507 0.427 1</td>
<td>201611 0.224 0.313 0.570 1</td>
</tr>
</tbody>
</table>

Yet another apparent pattern in Figure 2 is that contracts which mature during months closer to winter seem to be less volatile overall than those contracts which mature closer to the summer months. With this observation in mind we decide to incorporate portfolios into our analysis based on the assumption that the volatility differs in that respect. We are also interested in seeing if there are any benefits from choosing portfolios consisting of contracts near to each other in maturity or vice versa, therefore we decide to construct portfolios with these attributes as well as consisting of summer or winter months.

We choose contracts based on Figure 2 and our portfolio assumptions, the contracts chosen for each portfolio are displayed in Table 2 along with the correlation structures. As we suspected contracts which are close in maturity measures higher correlation than those further apart, this may have further adverse effect on the diversification other than the fact that they are inherently positively correlated. In Table 3 we present some descriptive statistics for the portfolios. All assets clearly reject univariate normality. The skewness is mostly negative except in the portfolio distant maturity summer, the excess kurtosis is clearly higher than the 3 corresponding to normality but is similar amongst the return series. The standard deviation generally seem higher during 2013 as can also be seen in Figure 2.
Table 3: Descriptive statistics for the portfolios.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St.dev</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>p-value(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>near maturity summer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>201305</td>
<td>0.000</td>
<td>0.016</td>
<td>−0.090</td>
<td>0.072</td>
<td>−0.423</td>
<td>6.702</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201306</td>
<td>0.000</td>
<td>0.016</td>
<td>−0.070</td>
<td>0.055</td>
<td>−0.296</td>
<td>4.348</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201307</td>
<td>0.000</td>
<td>0.015</td>
<td>−0.073</td>
<td>0.056</td>
<td>−0.417</td>
<td>5.555</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201308</td>
<td>0.000</td>
<td>0.014</td>
<td>−0.077</td>
<td>0.054</td>
<td>−0.551</td>
<td>6.016</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td><strong>near maturity winter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>201311</td>
<td>0.000</td>
<td>0.016</td>
<td>−0.078</td>
<td>0.069</td>
<td>−0.401</td>
<td>6.455</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201312</td>
<td>0.000</td>
<td>0.014</td>
<td>−0.075</td>
<td>0.048</td>
<td>−0.381</td>
<td>6.343</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201401</td>
<td>0.000</td>
<td>0.011</td>
<td>−0.054</td>
<td>0.036</td>
<td>−0.250</td>
<td>4.988</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201402</td>
<td>0.000</td>
<td>0.011</td>
<td>−0.059</td>
<td>0.038</td>
<td>−0.465</td>
<td>5.585</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td><strong>distant maturity summer</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>201307</td>
<td>0.000</td>
<td>0.012</td>
<td>−0.060</td>
<td>0.049</td>
<td>−0.116</td>
<td>5.349</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201407</td>
<td>0.000</td>
<td>0.012</td>
<td>−0.051</td>
<td>0.057</td>
<td>0.116</td>
<td>5.125</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201506</td>
<td>0.000</td>
<td>0.011</td>
<td>−0.042</td>
<td>0.046</td>
<td>0.194</td>
<td>4.336</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201608</td>
<td>0.000</td>
<td>0.011</td>
<td>−0.040</td>
<td>0.051</td>
<td>0.362</td>
<td>5.564</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td><strong>distant maturity winter</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>201401</td>
<td>0.000</td>
<td>0.011</td>
<td>−0.042</td>
<td>0.046</td>
<td>0.194</td>
<td>4.336</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201501</td>
<td>0.000</td>
<td>0.009</td>
<td>−0.039</td>
<td>0.028</td>
<td>−0.327</td>
<td>4.661</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201512</td>
<td>0.000</td>
<td>0.008</td>
<td>−0.033</td>
<td>0.031</td>
<td>−0.166</td>
<td>4.472</td>
<td>&lt; 0.001</td>
</tr>
<tr>
<td>201601</td>
<td>0.000</td>
<td>0.009</td>
<td>−0.038</td>
<td>0.031</td>
<td>−0.424</td>
<td>4.826</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

\(^1\) The p-value reported is calculated from the Jarque-Bera test of composite normality based upon skewness and kurtosis.
Our initial approach will consist of the simplified case of equally weighted portfolios compared to portfolios achieved through portfolio allocation to compare differences. The main focus will then be to model more accurately the multivariate covariance matrix by the different methods described in section 2. We will try to evaluate how different models react and how well they can predict worst case scenarios by backtesting precision of the VaR measures. In each portfolio we use the amount of observations which satisfies the range of data available for the contract having the least amount of observations available, this will provide a slight difference in the amount of observations between each portfolio scenario but this is deemed fairly negligible. We will then use the last 250 observations for out-of-sample backtesting purposes, the choice of using 250 observation for the backtesting procedure is in accordance with the BASEL regulation in the banking sector where the proposed number of historical observations is at least 250 days when interpreting backtesting methodology. Of course, as in most statistical circumstances, a larger dataset would be preferred to increase the power of the tests. The backtesting procedure will consist of, for each portfolio in Table 2, 250 1-day ahead forecasted means and covariance matrices which will result in 250 corresponding VaR and ES for analysis and backtesting purposes.

Although the covariance matrix forecasts are of main concern we also decide to model the mean of the electricity forward return series as opposed to zero mean processes. To this end we employ the Bayesian Information Criterion (BIC) of Schwarz et al. (1978) to decide which ARMA($p,q$) model is most suitable. We restrict our attention to $p,q \leq 2$ to alleviate computational burdens, the BIC favors simplicity as was encountered in early runs with higher degrees of $p$ and $q$ which suggested the restriction on these parameters. The different models applied when predicting the covariance matrices will be; the DCC model, the full BEKK model along with its restricted parameter counterparts the diagonal BEKK model and the scalar BEKK model and finally the more degenerate case (in terms of model optimization) of the EWMA.
5 Results

This chapter provides the reader with the empirical results of the study based on the theory and methodology set out in the previous sections. We begin by evaluating our selected models by presenting parameter estimates, providing graphs of the time varying covariances and correlations as well as statistics and graphical description that will ascertain the fit of our selected models. Following this section we use the forecasted covariance matrices to exhibit our estimated VaR and ES for the equally weighted portfolios and the portfolios subject to portfolio allocation. With the VaR and ES readily available we move on to our backtesting procedure where the different portfolio risk measures will be compared to give an indication as to which model is preferable. The entirety of computations were performed in the MATLAB® (MATLAB, 2013) software environment with the following toolboxes required depending on application: Financial Toolbox®, Statistics Toolbox®; or the Oxford MFE Toolbox.5,6

5.1 Model evaluation

When running the BIC model choice iterations for the mean of the series we came across one instance where the moving average polynomial was non-invertible, in that case we chose to restrict our attention to a ARMA\((p, q)\) process with \(p, q \leq 1\). In Table 4 we present the model fit for our in-sample portfolio, we have chosen to disregard the full BEKK model in the rest of the results due to having dissatisfactory results in the optimization. The parameter estimated for the two remaining BEKK models and the DCC are all significant at the 1% significance level.

In a similar manner to Tsay (2005) we compare the time-varying variances and correlations to that of moving windows each based on 69 days, which is approximately the number of days in a trading quarter. The covariance structure and correlation structure displayed in Appendix 6 clearly shows that the covariances, like the correlations, are time dependent. The correlations for the BEKK scalar and diagonal models differs from that of the moving window in level somewhat but seem to withhold the same volatility. The DCC model instead has a substantially smoothed correlation structure over time, much in accordance to the findings of Tsay (2005), yet even more so being borderline constant. This hints that the DCC model may yield notably different results from that of the other models. The time-varying variances shows that all the models have very similar patterns and, as also in accordance to Tsay, all show that the rolling estimates are slower to react to shocks than the proposed models, which is to be expected.

5Financial Toolbox® and Statistics Toolbox® are MathWorks products (see http://www.mathworks.se/products/matlab/)

6see http://www.kevinheppard.com/MFE_Toolbox
Table 4: Estimated parameters for the scalar and diagonal BEKK and DCC models.

<table>
<thead>
<tr>
<th>Element</th>
<th>Scalar BEKK</th>
<th>Diagonal BEKK</th>
<th>DCC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>(1,1)</td>
<td>0.0017</td>
<td>0.224</td>
<td>0.963</td>
</tr>
<tr>
<td>(2,2)</td>
<td>0.0011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,3)</td>
<td>0.0009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,4)</td>
<td>0.0008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,2)</td>
<td>0.0017</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3)</td>
<td>0.0006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td>0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,3)</td>
<td>0.0016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,4)</td>
<td>0.0004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td>0.0012</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: All parameters are found significant at the 1% significance level.

5.2 Value-at-Risk and Expected Shortfall

With the models in hand we can produce estimates of VaR and ES. We begin by estimating the in-sample measures to display the general attributes of the dataset. In Table 5 we can see that the different models used estimates VaR almost equally at the different quantiles, although the EWMA measures of VaR and ES seems to be more conservative than the other models. The portfolio VaR shows us a approximate loss of 1.4% every ten days, a approximate loss of 1.7% every 20th day and so on for each model. The differences between the models appear to be small but the importance of this discrepancy increases when the portfolio value is larger.

We perform out-of-sample VaR measurements for the four portfolios according to section 4 and plot these in boxplots in Figure 3.\textsuperscript{7} As we suspected the VaR is higher during the summer months for the return series than during the winter months. Depending on wether the contracts are adjacent in maturities or not the VaR measures vary heavily, those portfolios who has contracts which are closer to each other seem to display outliers further away from the mean than those with contracts further apart.

\textsuperscript{7}Note that we will use the notations scalar BEKK and scalar as well as diagonal BEKK and diagonal interchangeably throughout our results from here on to save space for figures and tables in some instances.
Table 5: Estimated VaR & ES for the in-sample portfolio.

<table>
<thead>
<tr>
<th></th>
<th>VaR 90%</th>
<th>VaR 95%</th>
<th>VaR 99%</th>
<th>ES 90%</th>
<th>ES 95%</th>
<th>ES 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scalar BEKK</strong></td>
<td>1.362%</td>
<td>1.699%</td>
<td>2.330%</td>
<td>1.801%</td>
<td>2.086%</td>
<td>2.644%</td>
</tr>
<tr>
<td><strong>Diagonal BEKK</strong></td>
<td>1.372%</td>
<td>1.712%</td>
<td>2.348%</td>
<td>1.815%</td>
<td>2.102%</td>
<td>2.665%</td>
</tr>
<tr>
<td><strong>DCC</strong></td>
<td>1.351%</td>
<td>1.685%</td>
<td>2.310%</td>
<td>1.786%</td>
<td>2.068%</td>
<td>2.620%</td>
</tr>
<tr>
<td><strong>EWMA</strong></td>
<td>1.347%</td>
<td>1.679%</td>
<td>2.302%</td>
<td>1.780%</td>
<td>2.061%</td>
<td>2.611%</td>
</tr>
</tbody>
</table>

According to the boxplots a risk averse investor should be inclined to invest in a portfolio consisting of forwards maturing in the winter months with a distant maturity date. Regarding as to which model is preferable the boxplots provide less than adequate information, even though the models seem to differ to some extent. Most notably the EWMA fares well considering the range of VaR while the median also suggest it being somewhat more conservative in its VaR measures than the other models.

We applied portfolio theory to our portfolios as explained in chapters 3 and 4 and thus found the optimal portfolios according to these allocation restrictions. Due to the highly correlated nature of the assets we would time and again be faced with the inherent fact that when all assets would incur a predicted loss the weights would all be assigned to the risk-free asset in the portfolio allocation formulas. In order to achieve an interpretable result we instead decide to turn our attention to the MVP portfolio which disregards any prevailing risk-free rates and fully distributes the weights amongst the electricity forward contracts. In Figure 4 the difference between the estimated VaR for the MVP portfolio and the corresponding measure for the equally weighted portfolio is plotted for each of the models calculated from an out-of-sample period of 250 days as described in chapter 4 at the 95% VaR. The plots show that gains are made from optimizing the portfolio through the MVP approach by reducing the VaR in the portfolio, this is mostly in the range of 0.1 − 0.5%, which yields a approximate 20 – 30% reduced risk in the portfolios. The DCC and EWMA models exhibit some rare instances of differences in VaR of up to 1.7% between the optimized portfolio and the equally weighted portfolio further emphasizing the importance of portfolio allocation.
Figure 3: Boxplots of VaR for the different portfolios over the out-of-sample period.
Figure 4: Difference plots of the MVP portfolio VaR versus equally weighted portfolio VaR for the different models at 95% VaR.
5.3 Backtesting

After observing that the VaR measures differ depending on model choice and portfolio allocation we now turn to the task of backtesting. As mentioned in Section 3 and the previous section we have divided our sample into out-of-sample electricity forward return series as opposed to in-sample series used for the parameters estimates and descriptive statistics produced earlier. We have performed the backtesting procedures for the equally weighted portfolios as well as those portfolios obtained by portfolio allocation. As the backtesting results from the equally weighted portfolio closely resembles the results from the MVP portfolio allocation we choose to present the results for the latter. Table 6 presents the likelihood ratios for the unconditional coverage and conditional coverage tests along with their corresponding $p$-values for the 90%, 95% and 99% VaR of the MVP portfolios.

Our first observation is that most models, with the exception of the EWMA model, do not reject the null at 90% VaR for varying significance levels which suggests a decent fit of these models at this quantile. When examining the 95% quantile it is immediately clear that most models fail to prove an adequate fit with the exception of the EWMA model which fares well throughout except for one instance. The only strong contender to the EWMA model seem to be the DCC but this result varies depending on portfolios. At the 99% quantile all models reject the null that the $p$ is true, but this is much to be expected as the sample size is deemed fairly low, in general predictions at smaller tail probabilities is difficult. As these measures are best suited to reveal how well models perform it is now time to move on to the CPA analysis for assessing which model could be deemed most advantageous in forecasting VaR unconditionally and conditionally.
Table 6: Unconditional and Conditional Likelihood ratio tests.

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<td>7.627***</td>
<td>29.241***</td>
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<td>25.633***</td>
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<td>8.548**</td>
<td>27.887***</td>
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<td>0.415</td>
<td>19.582***</td>
<td>7.361**</td>
<td>4.502</td>
<td>21.836***</td>
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<tr>
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<td>25.633***</td>
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<td>13.046***</td>
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Note: Significance codes: *p<.10 **p<.05 ***p<.01
We now analyze the results of the CPA testing procedure and the average NS to check as to whether we can find any evidence for a preferable model amongst the alternatives. We begin by examining Table 7 where a significant $\chi^2$ value will signify that a difference between models is found in each pairwise comparison. Each element in the table which has a significant $\chi^2$ value will then be assigned an arrow pointing towards the model preferred according to the decision rule in Section 3. Most model comparisons result in a failure to reject the null of equal predictive ability, with the only exception being the EWMA outperforming the diagonal model. Moving on to the unconditional tests in Table 8 the same reasoning for interpretation applies. Here we can see a few more distinctions between the models mainly in the near maturity winter portfolio. The EWMA and the DCC models seem to perform the best in these instances while it is unclear as to wether the Scalar model outperforms the Diagonal model or vice versa. But the overall results seem to give little evidence for which model should be preferred.

The average NS is displayed in Table 9 along with the number of exceptions. What is apparent immediately is that the models are unable to capture the average ES accurately in the summer month portfolios as these are more volatile than the winter month counterparts. This is especially true for the near maturity summer portfolio which also is more volatile due to having adjacent maturities. When looking at the other portfolios we can yet again not determine any favorable model amongst the alternatives, although the results suggests that the scalar model appear to be redundant.
<table>
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<td>0.003</td>
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<td>0.139</td>
<td>1.774</td>
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<td>0.036</td>
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<td>0.514</td>
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Note: Arrows indicate which model is preferred according to the decision rule by Giacomini and White (2006), see Section 3.
Table 8: Unconditional CPA.

<table>
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<td>Scalar</td>
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<td>0.974</td>
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Note: Arrows indicate which model is preferred according to the decision rule by Giacomini and White (2006), see Section 3.
Table 9: Normalized Expected Shortfall.

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<td></td>
<td></td>
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<td><em>Scalar BEKK</em></td>
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<td>0.296(9)</td>
<td>0.532(2)</td>
<td>1.344(23)</td>
<td>1.251(15)</td>
<td>1.341(6)</td>
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<td><em>Diagonal BEKK</em></td>
<td>0.002(22)</td>
<td>0.131(12)</td>
<td>-0.111(3)</td>
<td>1.077(26)</td>
<td>1.086(18)</td>
<td>1.075(6)</td>
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<td><em>DCC</em></td>
<td>-0.181(35)</td>
<td>-0.241(21)</td>
<td>-0.128(10)</td>
<td>0.911(23)</td>
<td>0.933(15)</td>
<td>0.742(4)</td>
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<td>0.589(51)</td>
<td>0.622(33)</td>
<td>0.553(15)</td>
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<td>0.936(17)</td>
<td>0.814(9)</td>
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<tr>
<td><strong>near maturity winter</strong></td>
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<tr>
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<td>0.159(24)</td>
<td>0.125(17)</td>
<td>-0.164(5)</td>
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<td>1.079(16)</td>
<td>0.907(8)</td>
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<td>0.616(27)</td>
<td>0.839(18)</td>
<td>1.160(6)</td>
<td>1.050(25)</td>
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<td>0.975(6)</td>
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<td>1.229(20)</td>
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<td>1.074(26)</td>
<td>1.035(17)</td>
<td>0.964(6)</td>
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<td>1.087(33)</td>
<td>1.042(24)</td>
<td>0.962(13)</td>
<td>1.165(33)</td>
<td>1.098(21)</td>
<td>1.044(10)</td>
</tr>
</tbody>
</table>

Note: The values within parenthesis displays the numbers of exceptions for each corresponding value of average NS.
6 Discussion

This study was conducted with the purpose of examining general characteristics of electricity forward price series transformed to return series, supplied to us by Skellefteå Kraft AB, while also attempting to apply multivariate volatility models which would be subject to portfolio allocation and backtesting. While electricity price series as well as electricity return series in general exhibit strong seasonal patterns in the mean, the corresponding forward return series show little such evidence. Instead we depict the volatility as measured by standard deviation of each maturity contract and find evidence that the volatility, quite similarly to the mean series for electricity prices and returns, is seasonally dependant. Most notably that summer months are generally more volatile then winter months accounting for trend. Also the Samuelson effect appeared present for the contracts already matured but due to limitations of the data on these contracts we only make the observation and do not apply it to our study, a possible gain from this knowledge is that a portfolio manager might be able to reduce his risk by shorting the contract before the increased risk period commences. The mean of the return series did not show any recurrent pattern and were modelled by a model selection criteria in the backtesting procedure.

These findings led us to construct portfolios which we applied multivariate volatility models to. The multivariate models consisted of the Riskmetrics approach which is well known and often applied in practice as well as other classes of volatility models as the BEKK model which extends the GARCH specifications and the DCC model concerned with modelling dynamic correlation. All the models fit the data well, the conditional covariances appear to react faster to innovations than the corresponding moving window estimations and the correlations seem to be time-varying except in the case of the DCC model. Our purpose was initially to apply portfolio theory with a included risk-free rate to our portfolios but due to the highly positive correlation amongst the assets it proved difficult to produce any meaning result in our context, instead we focused on measuring the gains from MVP portfolios versus equally weighted portfolios to emphasize the importance of portfolio allocation regarding electricity forward contracts. We saw significant gains from the MVP portfolio regarding risk reduction, but of course acknowledge that returns could be increased with increased risk.

After the in-sample analysis was conducted we proceeded to a out-of-sample analysis. We first tried to illustrate our hypothesised seasonality patterns from Figure 2, and as expected the seasonality was strong. One interesting result is that portfolios containing assets which are close to each other in maturity seem to have both higher VaR and dispersion of this measure compared to the portfolios containing contracts which are distant in maturity. Despite the portfolio containing forward contracts maturing during
the summer months increased the volatility, we saw that the correlation structure seem to be a more important factor in increasing the risk.

In our backtesting procedure we were initially hoping to see a clear indication of which model to use when forecasting VaR and ES on the electricity forward market, unfortunately the results were somewhat inconclusive as all models fared equally well for the most part. This was the case for the LR and CPA backtesting, unconditional or not, as well as for the average NS. We believe that one reason for this result may be the short data set period, even though the industry standard is often set at 250 days this still has an adverse effect on the ability to provide reasonable amounts of observations on the higher quantiles, since all models can provide enough predictions which exceeds the 90% VaR but fail at the 95% and 99% VaR this is a likely cause. Although the result may be deemed fairly inconclusive we nonetheless would like to make model selection suggestion based upon the study, when looking at the backtesting results as a whole we see indication that the EWMA or DCC are reliable models as opposed to the BEKK models when predicting VaR and ES for electricity forward returns, this certainly becomes more true when lowering the significance level. And would we suggest only one model we would definitely go with the EWMA as it also, by the merit of its simplicity, allows for a larger portfolio than the other more complicated models suggested in this study.

As a improvement for future studies other models, such as the popular Copula and those models who incorporate asymmetric volatility, should be considered as these are out of scope for this study. The backtesting procedures, which also applies to the aforementioned models, is very versatile and practical for end users and practitioners. When performing risk analysis the backtesting of the estimated VaR and ES is, or at least should be, a crucial part of the risk management, and we believe that electricity companies and other firms would benefit greatly from applying these tests. An optimization scheme, preferably with some routine, with model choice through backtests would probably improve estimation and give some confidence in the stated VaR and ES.
A Time-varying Correlations and Variances

Figure 5: Time-varying correlation of the in-sample electricity forwards contracts for the scalar BEKK model alongside a moving window (dotted line) of the correlations based on 69 days: (a) 201403 vs 201406, (b) 201406 vs 201409, (c) 201409 vs 201412, (d) 201403 vs 201409, (e) 201406 vs 201412, and (f) 201403 vs 201412.
Figure 6: Time-varying correlation of the electricity forwards contracts for the Diagonal BEKK model alongside a moving window (dotted line) of the correlations based on 69 days: (a) 201403 vs 201406, (b) 201406 vs 201409, (c) 201409 vs 201412, (d) 201403 vs 201409, (e) 201406 vs 201412, and (f) 201403 vs 201412.
Figure 7: Time-varying correlation of the electricity forwards contracts for the DCC model alongside a moving window (dotted line) of the correlation based on 69 days: (a) 201403 vs 201406, (b) 201406 vs 201409, (c) 201409 vs 201412, (d) 201403 vs 201409, (e) 201406 vs 201412, and (f) 201403 vs 201412.
Figure 8: Time-varying variance of the electricity forwards contracts for the Scalar BEKK model alongside a moving window (dotted line) of the variances based on 69 days: (a) 201403, (b) 201406, (c) 201409, and (d) 201412.
Figure 9: Time-varying variance of the electricity forwards contracts for the Diagonal BEKK model alongside a moving window (dotted line) of the variances based on 69 days: (a) 201403, (b) 201406, (c) 201409, and (d) 201412.
Figure 10: Time-varying variance of the electricity forwards contracts for the DCC model alongside a moving window (dotted line) of the variances based on 69 days: (a) 201403, (b) 201406, (c) 201409, and (d) 201412.
References


Caporin, M. and McAleer, M. Do we really need both bekk and dcc? a tale of two covariance models. A Tale of Two Covariance Models (February 5, 2009), 2009.


MATLAB. *version 8.1.0 (R2013a)*. The MathWorks Inc., Natick, Massachusetts, 2013.


