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European mathematics curricula and classroom practices
Paul Andrews

Core readings

<table>
<thead>
<tr>
<th>The Core readings addressed in this chapter are:</th>
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Introduction

Increasingly research is showing that mathematics and its teaching vary from one system to another, not least because education reflects culturally normative values about what is good, right, and desirable in a society (Schwartz 1999, p. 25). If the purpose of education is to induct learners into those beliefs and practices that history has shown to be effective for the maintenance of a society (Hofstede 1986; Triandis and Suh 2002), then that society will present a particular, culturally determined, perspective on mathematics.

The first three papers discussed in this chapter examine the cultural construction of school mathematics in England, France, Germany and Russia, countries that have influenced greatly curricular traditions around the world (Cummings 1999). This discussion is prefaced by a brief socio-historical commentary on those four curricular traditions, which acknowledges that all curricula are founded on a system’s conception of an ideal person (Cummings 1999), represent a form of social regulation (Popkewitz 1997) and are substantially more than the documentary presentation of the knowledge and skills students are expected to learn. The final paper, in considering the curriculum intersection of curriculum and classroom practice in two other European systems of interest to the mathematics education researcher, Flanders and Hungary, presents a culturally-based theorisation of teacher knowledge (Rowland, this volume) that helps explain further the issues raised in the first three papers.

A socio-historical summary of education in England, France, Germany and Russia

English education, in its emphasis on Protestant values and the importance of sport in the development of a healthy mind and body (Cummings 1999), so highlights personal morality and experiential knowledge over rationality (Pepin 1999) that “the chapel and the playing field were in many ways more important… than the classroom” (Lauwerys 1959, p. 288). In drawing on the English public school’s liberal arts tradition, and its rejection of science and engineering, it offers an education appropriate for gentlemen (Holmes and Mclean 1989) and, although modern emphases on employment (Ernest, this volume) have compromised this
tradition in mass education, “its usefulness in the business of earning a living is rejected quite decisively” (Lauwerys 1959, p. 289). Thus, the English tradition accords with Kamens et al.’s (1996) description of an arts and humanities curriculum predicated on the maintenance of both an intellectual elite and an established high culture.

French education draws on Enlightenment principles that education should not only be separated from superstition but include all human knowledge in ways that emphasise rationality (Holmes and McLean 1989; Cummings, 1999) and that not only are truth and goodness closely related but also that if it is not rational then it is not education (Lauwerys 1959). Post-revolutionary principles of égalité and laïcité underpin a curriculum focused on the removal of social inequalities and the expectation that moral issues will be addressed at home (Pepin 1999). In sum, the French tradition resonates closely with classical curricula whereby an “intellectually demanding and character-enhancing” experience produces “well-rounded generalists rather than highly trained specialists” to form a political and social elite whose responsibility is the maintenance of the natural social order (Kamens et al. 1996, p. 119).

Falling somewhere between the English and the French traditions, German education, rooted in Lutheran notions of justification by faith (Lauwerys 1959), emphasises both an encyclopaedic perspective on knowledge and personal piety (Cummings 1999; Prange 2004). Drawing on the humanistic tradition, or bildung, it emphasises a “spiritual awareness of the inwardness of the world of men and of things” (Lauwerys 1959: 290). Based on an underlying belief that every occupation has dignity, the tripartite structure of German schools, with its equal privileging of academic and practical knowledge, poses no barrier to higher education (Pepin 1999). In sum, the German tradition can be seen as resonant with the comprehensive curriculum that aims not only to allow all children, irrespective of background, similar opportunities to learn and achieve but also “to produce competent citizens and productive workers rather than technical specialists” who are endowed with “certain rights and duties” (Kamens et al. 1996, p. 120).

Finally, Russian education draws on soviet principles of egalitarianism and locates vocationalism and an encyclopaedic model of knowledge within a socialist moral philosophy (Cummings 1999). Described as polytechnicalist (Holmes and McLean 1989), it reflects a “desire to use education as an instrument for changing the conventional attitude to work”, with not only manual labour to be “as highly considered as intellectual or clerical work” but also every subject “considered from the point of view of the help which it is capable of giving to the … builder of a socialist society” (Lauwerys 1959, p. 291). Such a tradition, alongside emphases on industrial and military growth, fit well with notions of mathematics and science curricula, which developed to facilitate a rapid growth from an agrarian to an industrial society (Kamens et al. 1996).

Inevitably such descriptions are prone to revision. For example, Russian education, while retaining elements of its soviet legacy, is undergoing change as part of its shift towards a market economy. These changes have included a decentralisation of responsibilities, reforms to curriculum and assessment (Mitter 2003) and the introduction of parental choice (Laczik 2006). In similar vein, post-war reforms in France have addressed, in particular, concerns with respect to a “scientific and technical lag” (Resnik 2007, p.157). However, despite such ongoing changes sufficient remains of the above traditions to facilitate not only our understanding of research reports located in those countries but also our abilities to evaluate the veracity of claims made. The remainder of this chapter, therefore, is given to examining how researchers have characterised different aspects of mathematics in these countries.
Mathematics teaching in England and Germany

Kaiser’s paper is atypical and, despite the rather strained English, exemplary. Unlike most comparative studies of mathematics teaching she pays considerable attention to framing her ethnography within a historical analysis of the two countries’ curricular traditions. To do this she draws, in the first instance, on McLean (1990) and presents summaries, not dissimilar to those above, of the English, French and German traditions. It is interesting, in this respect, that she tends to accept without comment McLean’s analyses of the English and French traditions although she makes explicit reference to elements of his account of the German with which she disagrees. Had she been French, would she have critiqued McLean’s analysis of the French system and accepted his summary of the German? Following this general introduction she offers detailed accounts of the English, continuing to draw on McLean, and the German, drawing extensively on Blankertz 1982). Both summaries are extensive and helpful in alerting the reader to the origins of the two systems’ philosophical and pedagogical traditions. In so doing she highlights English emphases on morality, individualism, specialism and the privileging of inductive rather than deductive knowledge, and German emphases on virtue, the importance of deep and connected knowledge and the equal privileging of all forms of legitimate human activity. However, it was a little disappointing that each review drew, essentially, on a single piece of writing, not least because the Lauwerys (1959) paper addresses both particularly well.

In describing her project Kaiser offers an extensive discussion of the theoretical underpinnings of ethnography in educational research. In so doing not only does she make an interesting and valuable contribution to the methodology of comparative education research but also highlights three key considerations; the long term presence of the researcher in the field, flexibility of data collection and analysis, and detailed field notes. This is followed, drawing on more than 300 observed lessons, by her analysis of the two systems’ didactic traditions as reflected in lessons taught primarily to students in grades 8 to 10. Acknowledging the impossibility of ethnography yielding generalisable outcomes she locates her analysis in the sociological tradition of the ideal type developed in the early years of the twentieth century by Max Weber. This seeks to stress recurrent patterns of behaviour in the construction of a model that reflects some sense of the typicality of the phenomenon under scrutiny. However, the word ‘ideal’ does not connote any moral evaluation but frames the ordering of the social world.

Through her ideal types she shows well how the different socio-cultural underpinnings of the two systems play out in classrooms. For example, in German mathematics teaching “theoretical mathematical considerations are of great importance” while English mathematics teaching privileges a “pragmatic understanding of theory” (p. 249). In Germany mathematics is taught in an order defined by the structure of the subject with large thematic fields taught independently of each other, while in England a spiral curriculum enables topics, typically taught over a small number of lessons, to be introduced at an elementary level, picked up again later and taught independently of any obvious sense of structure. In German lessons new topics or methods are given high priority and introduced by means of class discussions and, often illustrated by real-world examples, while in England they have a low importance and are often “given by the teacher just as information or in the style of a recipe” (p. 250) or inductively through investigation. The role of proof in German mathematics, while important, varies according to school type. In England formal proof is rare with theorems warranted by experiment, example or teacher assertion. German lessons are progressed by high expectations of students’ correct and confident execution of algorithms, with clear expectations that each is
undertaken in well-defined and predetermined ways. In English lessons “rules and standard algorithms (are) of minor importance” (p. 252). Priority is given to students’ own solutions rather than systematically introduced approaches. German teachers place great importance on precise mathematical language in all official discourse, while English teachers view such matters as of minor importance. German teachers exploit real-world problems rarely, and when they do they tend to be artificial examples used in the introduction of new concepts or methods. English place a high importance on such problems, particularly in respect of applying mathematics to extra-mathematical problems. Such descriptions, even if not typical of all lessons, resonate with the literature’s summaries of the two traditions and confirm how culture influences in largely hidden ways classroom participants’ enacting of their roles.

Textbooks and the teaching of angle in England, France and Germany

Haggarty and Pepin focus on the nature and use of text books (Rezat and Straesser, this volume) in the teaching of lower secondary mathematics in England, France and Germany. In introducing the topic they highlight not only the importance of text books in the construction of children’s mathematical competence but also the role of culture in determining both a textbook’s content and the manner of its use. Their literature review alerts readers to different analytical frameworks, as in Van Dormolen’s (1986) didactical analysis, Schmidt et al.’s (1997) topic complexity analysis and Dowling’s (1996) sociological analysis, reminding us that textbooks are neither value-free in their authorship nor independent of the teacher in the manner of their use. This latter point leads to an extensive summary, framed against five sub-headings, of how textbooks are used in classrooms. The first of these, the authority of the textbook, is particularly interesting in its offering three distinctive perspectives on authority; authority as reflected in a book’s presentation of societally valorised knowledge, authority as reflected in teachers’ acquisition of author status by dint of how they use the texts, and authority as reflected in teachers’ mediation of a text’s content.

Research methods are discussed briefly and the reader reminded that qualitative research does not seek to generalise but facilitate understanding. However, they indicate that their analysis “uses a schedule which draws on the range of ideas in the literature” but say nothing with respect to its precise content or the “comprehensive set of questions” (p. 574) that underpin it. Also, nothing is offered to indicate the means by which data were analysed. Despite such criticisms, their analyses confirm the extent not only to which culture influences textbook content and use but also the robustness of the curricular traditions described above.

Examining textbooks, focused on students in grades 6-8, through the framing lens of angle they found that French texts were comprehensive and cognitively challenging. They incorporate extensive explanatory text, sufficient for students to “do the questions … without additional support” (p. 576), with technical vocabulary used throughout. Exercises included opportunities for students to speculate and links to other topic areas. The German texts were differentiated according to school type although, fundamentally, few differences were found. They attempted to establish links “between everyday situations and what pupils are to learn” (p. 578). Detailed explanatory text is included and technical vocabulary is used consistently. “Pupils are not required to speculate and most questions require low-level applications of the skill”. English texts were “less densely packed and contain fewer examples than textbooks in either France or Germany” (p. 579). They incorporate no explanatory text, thereby ensuring some teacher mediation, and little emphasis on technical vocabulary. Exercises included opportunities for students to estimate before measuring, although questions “are of a low level” with “no obvious scope for extending them” (p. 582). In the two books analysed only one question was put in context.
French teachers used textbooks primarily for exercises, preferring to devise their own topic presentations, although students were encouraged to learn how to use the book as resource for their learning. The mixed ability nature of French classes meant that all students “were given the same questions to do” and “exposed to the same cognitive and language demands as each other” (583), although teachers spent time with individuals if they thought they needed additional support. While all German teachers used textbooks to plan their teaching classroom use varied according to school type. Gymnasium teachers used them less than Hauptschule teachers although typically any use was as a source of exercises. Teachers in the former tended to make assumptions about the maturity of their students and adopted more discursive approaches to teaching than their Hauptschule colleagues, who tended to assume that their students would not be “able to cope with too many deviations from the straightforward mathematical algorithms” (p. 584) or word problems. English teachers used textbooks regularly as both a source of exercises and ideas for presentation. All “considered that it would be impossible to use the same textbook with all pupils in a year group” (p. 584) and spoke of the need for different ability students to be offered different types of problem, not least because of lower ability students’ linguistic deficiencies and their need for “plenty of straightforward questions practising particular skills or techniques”. In reality, few English pupils experienced an “opportunity to develop their reading and comprehension skills in mathematics”.

**Teacher-pupil interactions in the mathematics classrooms of Russia and England**

The paper by Wilson, Andrew and Below focuses on teacher-pupil interactions in primary mathematics classrooms in St Petersburg, Russia and the North East of England. Their descriptions of their locations were deliberate as, with all case study research, a small number of schools researched in one location should not be construed as representative of the schools generally. Their paper is framed against curriculum reform in both countries. In England reform has encouraged teachers to introduce regular whole class teaching into a tradition largely dominated by individual work, while in Russia, somewhat ironically, reform has focused on making more individual a tradition based on whole class work. In framing their study, no allusion was made to the socio-historical underpinnings of the two countries’ didactic traditions.

They present their methods clearly and, in so doing, highlight well how several recurrent issues in comparative mathematics education research were addressed. For example, five consecutive days of mathematics lessons were observed in a range of year groups in two primary schools in each country, which “enabled observers to record the story of the mathematical experience of each class during the week and counteracted the possible effect of observation of a single atypical lesson with any one class” (p. 413). Typically lessons were observed by two English and one English-speaking Russian researcher and three forms of data were collected. These derived from a semi-structured observation schedule designed to produce “a time-related description of specific features of the lesson at each stage” (p. 413), a lesson narrative, constructed in real time, to record “as fully as possible the events of the lesson in chronological order”, and an interaction map used to record the location of any pupil involved in an interaction with the teacher and whether it was private or public. With regard to the interactions, attempts were made to “record the duration of each interaction” although, as they concede, this proved difficult.

Their analyses highlight some interesting patterns in the ways that interactions played out in project lessons. On the one hand, English lessons, typically, were structured around four or
five sections. Within that structure they comprised large numbers of private interactions, which were typically located between introductory sections of public interaction and closing sections of public interaction, highlighting the increasing prevalence of the three part lesson recommended by the English authorities. Mixed periods of interaction were rare. Russian lessons, on the other hand, comprised relatively few private periods of private interaction. Solely public interactions were seen to dominate, “occasionally punctuated by … a mixture of public and private exchanges” (p. 421). Russian public interactions were found to be longer than the English. Indeed, the limited evidence available indicated that around 30 per cent of Russian public interactions lasted more than 3 seconds and a further 12 per cent that lasted more than ten seconds. In contrast, of the English around 7 per cent lasted longer than three seconds and only 3 per cent lasted longer than ten.

In both countries the typical public interaction involved two elements, the teacher initiates and the class responds. However, the manner of these class responses varied. Russian teachers expected students to “focus on repetition of rules and procedures whereas English teachers concentrated on counting and chanting number sequences”. That is, the Russian responses focused on the general concepts or algorithms underlying the question while the English focused on the particular numerical operations necessary to address it.

When compared with their earlier study (Wilson et al. 2001), the data reported in this paper confirm an increase in private interactions in the Russian classrooms and public interactions in the English, in line with reform objectives in both countries. Importantly, when a Russian student responds in a public interaction, he or she “appears to do so on behalf of the class as a whole and is expected to articulate the standard answer; the rest of the class is expected to participate by silently rehearsing the same answer while listening” (p. 431). When an English pupil responds, he or she may be reporting a method unfamiliar to other pupils. This means not only that students may not be able to “predict the response and collectively mentally rehearse the procedure” but also that “they are required to make sense of … a procedure which may not resonate with the method they have themselves chosen” (p. 431).

The intersection of mathematics curricula and teaching in Flanders and Hungary

The final paper is framed against recent categorisations of teachers’ mathematical knowledge for teaching (MKT) that is, Andrews argues, typically construed as an individual rather than a cultural construction. He outlines a tripartite curricular framework for analysing teachers’ mathematical knowledge that explicitly acknowledges “the cultural discourse in which mathematics teaching and learning occur” (p. 100). According to Andrews, teachers work within an intended curriculum, as conceived by the second international mathematics study (Garden, 1987), reflecting the knowledge and skills privileged by the system in which teachers operate. They work within a received curriculum, amenable only to inference, reflecting the hidden and culturally derived beliefs and practices teachers acquire by dint of being who they are. Finally, they work within an idealised curriculum, which is articulable, reflecting individual teachers’ personal and experientially informed beliefs. Having proposed the framework, he shows, through an analysis of a sequence of lessons taught on linear equations in each of Hungary and Flanders, how the tripartite curriculum plays out. In so doing he summarises the two systems’ curricula’s statements with respect to linear equations, highlighting both differences and similarities in the ways in which the topic is framed. In respect of differences the Flemish curriculum comprises a general statement for each grade, while the Hungarian comprises detailed statements for each grade alongside indications as to solution processes. In terms of similarities both highlight the solution of equations of the first
degree and, more generally, locate the topic in a systemic desire to encourage students to translate text into equations for solving. In addition, he frames his analyses of the two sequences against literature on linear equations and, in particular, the distinction between arithmetic equations and algebraic equations (Filloy and Rojano 1989; Kieran 1992).

The two sequences of lessons were derived from a larger project involving four case study teachers from each of England, Finland, Flanders, Hungary and Spain. Andrews reports on the ways in which videotaped data were collected and, with respect to quantitative analyses, discusses briefly the application of a coding schedule developed during a year of live observations in the year prior to data collection. This comprised seven generic and inferable mathematical learning outcomes and ten generic and inferable didactic strategies. He reports that on differences and similarities in the codes applied to the lessons of the two countries under scrutiny. In general, Hungarian teachers privileged higher order learning outcomes and exploited didactic strategies commensurate with those outcomes more frequently than their Flemish colleagues.

Andrews then presents qualitative analyses of the two teachers’ lesson sequences. He highlights how they both exploited realistic word problems in their teaching; deferred introducing analytical solutions, which were based on the balance scale, until they presented their students with algebraic equations; frequently invited their students to solve algebraic equations involving brackets, negatives and fraction coefficients. However, their exploitation of word problems varied considerably, with only a single example in Pauline’s sequence and many in Eva’s. The extent to which they sustained the balance varied, with Pauline mentioning it in passing while Eva sustained it through several examples, including drawing it on the board. Pauline spent gave much time in three of her five lessons to exercises in which students worked individually while Eva never asked her students to work on more than one question before sharing publicly the solutions obtained. Finally, he discusses each teacher’s teaching against the tripartite curriculum. For example, he writes that Eva’s teaching showed a clear adherence to Hungarian intended curricular expectations, both in terms of the content expected of grade 7 students and her exploitation of non-routine problems. Secondly, the explicit manner in which she exploited and sustained the balance, her “invocation of brackets, negatives and fractions” accorded with “earlier findings that Hungarian teachers operate with the general rather than the particular”, and her close adherence “to a previously observed cycle of problem posing, solving and sharing” (p. 114) all indicated a close adherence to a well-established received curriculum. However, her use of word problems, which earlier observers of Hungarian teaching had noted as a rarity, alluded to a particular idealised curriculum.

**Discussion**

I began this chapter by examining the distinctive socio-historically derived characteristics of the English, French, German and Russian educational traditions. It is my conjecture, supported by the evidence of the first three papers examined, that these traditions permeate all aspects of mathematics teaching. Whether one is examining the text books available for teachers to use, the classroom interactions of teachers and students, or the forms of mathematics privileged in different classrooms, the evidence suggests that all are prone to largely hidden cultural influences that make the mathematics classrooms of one country unique. For example, the first three papers highlight an intellectually impoverished English mathematics that permeates both text books (Haggarty and Pepin 2002) and classroom expectations (Kaiser 2002) and which is exacerbated by the idiosyncratic ways in which classroom interactions, located in a tradition of individualism, play out (Wilson et al. 2006). Such traditions contrast greatly with the evidence from the other three key systems – whether
the mathematical formalism of German classrooms (Kaiser 2002), the intellectual rigour of
French textbooks (Haggarty and Pepin) or the well-rehearsed collective interactions of
Russian classroom (Wilson et al. 2006). In short, the differing mathematics education
traditions highlighted in the first three papers are clearly manifestations of cultural
expectations with respect to the ideal person. In this respect, whether one’s particular cultural
lenses lead one to feel more comfortable with one system’s perspectives than another’s is not
the issue, all three systems clearly operate a distinctive control over what children are
expected to learn and how they learn it.

The fourth paper, while not located in analyses of the four major traditions, attempts to
explain how much of what teachers do in their classrooms is so culturally embedded as to be
hidden and beneath teachers’ immediate articulation. Indeed, Andrews would argue that the
interactions of the idealised, received and intended curricula confirm the extent to which
teachers are products of the cultures in which they are raised and work. Such interactions
structure not only the ways in which textbooks are produced but also the ways in which they
are used; they underpin the ways in which individual teachers, as members of a particular
collective, structure the classroom interactions characteristic of a system.

All four papers confirm that mathematics and its teaching are elements of a collective
psychological conditioning (Triandis and Suh 2002) or collective mental programming of a
particular group of people (Hofstede 1986). Of course, there are those who challenge the
extent to which mathematics teachers’ work is culturally determined on the basis that
teachers’ core teaching practices show little cross-national variation (LeTendre et al. 2001).
However, Andrews (2009) has argued that similarity of broad categories of didactical strategy
like, for example, explaining, are inevitable and that characteristic differences lie in other
practices such as the use of high level questioning or the public sharing of student ideas.

Further Reading

Andrews, P., 2007. Mathematics teacher typologies or nationally located patterns of
Aubrey, C., Tancig, S., Magajna, L. and Kavkler, M., 2000. The development of numeracy in

Additional References

potential of low inference generic descriptors. *Comparative Education Review, 53* (4), 559-
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