Evaluation and improvements of the short circuit test at ABB Machines

Nils Englund
Abstract

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This thesis was performed to improve the test procedure for large electrical machines at ABB Machines in Västerås. New measurement equipment for the three phase short circuit test was evaluated by implementation and benchmarking against currently used system. A new position for the new measurement system was found to ensure good repetitively between tests, while at the same time reduce setup time from up to an hour to zero. The results from the new position and equipment were satisfactory and further implementation is advised. A new program for evaluation of dynamic parameters (X'd, X''d, T'd, T''d) was done in accordance to IEEE, IEC and currently implemented software, using Matlab. By controlling the phase angle of the voltage at time of the short circuit, the current can be minimized. Investigations of how that can be achieved were conducted and test of one solution was performed successfully on a machine.
Sammanfattning

ABB Machines Sweden tillverkar fyra- och sexpoliga synkronmaskiner i spannet 10-70 MVA. Dessa används bland annat som generatorer i värme kraftverk och som motorer till raffinörer och inom pappersmassa-industrin.

Kraven på hur roterande elektriska maskiner förväntas uppträda under transienta förlopp har höjts markant vilket har ökat efterfrågan på noggranna simuleringsmodeller för dessa förlopp. Modeller kan dock aldrig bli noggrannare än motsvarande mätning, därför har ABB Machines nyligen investerat i ett nytt mätsystem för kortslutnings– och startströmmar baserade på Rogowskispolar.


Resultaten i det här examensarbetet visar att den nya mätutrustningen och dess nya position fungerar tillfredsställande och i likhet med nuvarande utrustning, och bör därför kunna implementeras i alla provrum. En utredning visar också att en fasvinkelkontroll är möjlig utan att byta brytare, vilket bidrar ytterligare till en mer standardiserad testproceduren. Spridningen mellan mätningar till följd av fasvinkelkontrollen förbättrades för de subtransienta dynamiska parametrarna, medan de transienta parametrarna förblev lika eller något större. En hypotes om att minskad mättnad, till följd av lägre ström genom fasvinkelkontrollen, skulle bidra till noggrannare subtransienta reaktanser utvärderades också. Resultaten visade ingen påtaglig skillnad i reaktanserna jämfört med innan. En anledning kan vara att maskinen som testades var relativt liten. Därför bör vidare utvärdering med fler tester utföras.
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<th>Description</th>
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<tr>
<td>(E_{\text{pmax}})</td>
<td>[V]</td>
<td>No load voltage</td>
</tr>
<tr>
<td>(H)</td>
<td>[A/m]</td>
<td>Magnetic field strength</td>
</tr>
<tr>
<td>(I_k)</td>
<td>[A]</td>
<td>Short circuit current, simplified</td>
</tr>
<tr>
<td>(I_A)</td>
<td>[A]</td>
<td>Current in phase A</td>
</tr>
<tr>
<td>(I_b)</td>
<td>[A]</td>
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</tr>
<tr>
<td>(I_{ss})</td>
<td>[A]</td>
<td>Steady state current</td>
</tr>
<tr>
<td>(i_s)</td>
<td>[A]</td>
<td>Stator current</td>
</tr>
<tr>
<td>(I')</td>
<td>[A]</td>
<td>Transient current</td>
</tr>
<tr>
<td>(I'')</td>
<td>[A]</td>
<td>Subtransient current</td>
</tr>
<tr>
<td>(k_{dc})</td>
<td></td>
<td>Gradient of straight-line representation of DC comp.</td>
</tr>
<tr>
<td>(k')</td>
<td></td>
<td>Gradient of straight-line representation of transient comp.</td>
</tr>
<tr>
<td>(k'')</td>
<td></td>
<td>Gradient of straight-line representation of subtransient comp.</td>
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<tr>
<td>(l_d)</td>
<td>[p.u.]</td>
<td>Synchronous inductance, d-axis</td>
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<td>(l_q)</td>
<td>[p.u.]</td>
<td>Synchronous inductance, q-axis</td>
</tr>
<tr>
<td>(l'_d)</td>
<td>[p.u.]</td>
<td>Transient inductance, d-axis</td>
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<td>(l'_q)</td>
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<td>(l''_d)</td>
<td>[p.u.]</td>
<td>Subtransient inductance, d-axis</td>
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<td>[p.u.]</td>
<td>Subtransient inductance, q-axis</td>
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<tr>
<td>(m')</td>
<td>[p.u.]</td>
<td>X-axis zero crossing in straight-line representation of transient comp.</td>
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<td>(m'')</td>
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<td>X-axis zero-crossing in straight-line representation of subtransient comp.</td>
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<tr>
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<td>[Ω]</td>
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<td>(V_b)</td>
<td>[V]</td>
<td>Base voltage</td>
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<tr>
<td>(u_s)</td>
<td>[V]</td>
<td>Stator voltage</td>
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<tr>
<td>(V_0)</td>
<td>[V]</td>
<td>Voltage level when test is conducted</td>
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<tr>
<td>(v_{\text{coil}})</td>
<td>[V]</td>
<td>Output voltage from Rogowski coil</td>
</tr>
<tr>
<td>(Z_a)</td>
<td>[p.u.]</td>
<td>Phase impedance</td>
</tr>
<tr>
<td>(\alpha - \gamma_k)</td>
<td>[']</td>
<td>Angle of voltage in relation to steady state current</td>
</tr>
<tr>
<td>(\Delta l'(t))</td>
<td>[p.u.]</td>
<td>Increment which represent the transient current comp.</td>
</tr>
<tr>
<td>(\Delta l''(t))</td>
<td>[p.u.]</td>
<td>Increment which represent the subtransient current comp.</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>[T]</td>
<td>Flux</td>
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<td>[rad/s]</td>
<td>Base value of frequency</td>
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### Dynamic Parameters

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<th>Unit</th>
<th>Description</th>
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<tr>
<td>(T_a)</td>
<td>[s]</td>
<td>Synchronous time constant</td>
</tr>
<tr>
<td>(\tau'_d)</td>
<td>[s]</td>
<td>Transient time constant</td>
</tr>
<tr>
<td>(\tau''_d)</td>
<td>[s]</td>
<td>Subtransient time constant</td>
</tr>
<tr>
<td>(x'_d)</td>
<td>[p.u.]</td>
<td>Transient reactance</td>
</tr>
<tr>
<td>(x''_d)</td>
<td>[p.u.]</td>
<td>Subtransient reactance</td>
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1 Introduction

ABB Machines Sweden produces 4 and 6 pole synchronous machines in the power range 5-70 MVA. Common applications for their generators are steam and gas power plants, while compressor drives for air separation, pulp and paper, and oil and gas industries are common applications for the motors. Most machines are made for a specific application and thus the design between machines differ. ABB Machines in Västerås employs about 400 people and is a part of the Discrete Automation and Motion division.

Since these machines often are “one-offs”, different tests are usually carried out to ensure the theoretically calculated and simulated performance is consistent with the real machine. Customers can also have requests for specific tests. Some of the test procedures which are done before a machine is approved for delivery include vibration, heat and short circuit tests. This thesis will focus on the test procedure of the short circuit test where demands on how a machine behaves during a transient process have been raised considerably. This has led to an increased demand for accurate prediction of the transient performance. New test equipment for short circuit tests incorporating Rogowski coils has been purchased and needs to be evaluated. A Matlab program for deriving dynamic parameters is also sought after. Additionally, a device for controlling the voltage phase angle for when the short circuit occurs, and thereby minimizing the short circuit current, is desired.

1.1 Current Situation

The short circuit test is performed to evaluate the transient behavior of a machine and from this test the dynamic parameters can be derived. The short circuit current is today measured using current shunts at the neutral point. The current shunt is large and bulky and sometimes needs to be placed on top of the tested machines because of space limitations on the floor. Since there is only one shunt setup, it’s moved between the test boxes when it’s needed. It is also very expensive, which is one reason for why there’s only one of them. The setup time for the current shunt can take up to an hour. This is partly due to its weight and thereby the need for an overhead crane, and partly due to the time it takes connecting it. This is one reason why a new measurement system is sought after.

Today the short circuit is manually triggered by visually observing the frequency and then short circuit by a switch where there’s no control of phase angle of the voltage. A way of controlling when the short circuit should occur is therefore of interest.

1.2 Problem Formulation

The proposed new measurement method is to be a permanent installation where it’s out-of-the-way and as the coils are considerably cheaper than the current shunt, it enables installation in all test boxes. That would reduce the setup time to zero. The main advantage of the new measurement system is that the test procedure will be more standardized where the measurements will be done in the same way each time. Earlier work has shown that movement of the shunts could affect the consistency of the testing procedure, where surrounding equipment may interfere with the measurements. Also, contact surfaces of the connections may be affected after several connections/disconnections.

Another way of having a more standardized test procedure would be to have a short circuit breaker that shorts the machine at the exact same phase angle and frequency each time.
Additionally, the voltage phase angle has a very strong correlation to the magnitude of the short circuit current. By controlling the phase angle, minimized mechanical strain on the machine could be achieved while at the same time making the procedure more standardized.

1.3 Aim of the Thesis and Limitations

The main goal of this thesis is to improve and/or get a more standardized test procedure through two new approaches. The thesis is divided into two parts where the first will focus on evaluating the new measurement equipment versus the old. The evaluation will partly be done by writing a program in Matlab, which derives the dynamic parameters of the synchronous machine. The other part is to find or construct equipment which enable choosing where on the voltage curve the short circuit is done.

A limitation is to only focus on the three phase short circuit test. Although there are six test boxes, any improvements from the investigation will be implemented in one test box.

1.4 Thesis Layout

Chapter 1-2 describes the background, the aim of the thesis and the current situation. Chapter 3 includes explanations of parameters and an estimation of the magnitude of the current in a three phase short circuit. In Chapter 4, the new measurement system is described together with how to extract the dynamic parameters from measurements. It also describes how the benchmark was performed. Chapter 5 is dedicated to the phase angle control of the voltage, including pre-study, approach and theory. Chapter 6 shows the results from chapter 4 and 5. In Chapter 7 the results are discussed together with conclusions and future work. Appendix A include a derivation of the short circuit current using the phase-variable model. Appendix B is a picture of the LabView program.
2 The Three Phase Short Circuit

When a three phase short circuit occur in a synchronous generator at no load, a current with descending AC and DC components is produced. The decaying AC component is characteristic for a short circuit near any rotating machine. The DC component, also called asymmetrical fault current, has the features of a discharging inductor. In Figure 1 a typical short-circuit current from a synchronous machine is presented. The figure illustrates the decay of the AC component, and also the initial DC-offset which usually disappears in the first few cycles (Reimert, 2010). Initially the AC component is very high, multiple times the rated current and then, after 2-5 seconds, the current decrease to its synchronous value of around 0.4-1.2 times rated value(Reimert, 2010), assuming that the field excitations remain constant. The temperature rise related to the high currents is usually not a problem since the current decline rapidly and the machine is usually disconnected as soon as a short circuit has occurred. However, what is a problem is the mechanical stress these currents put on the machine, since the forces are proportional to the current squared. This could lead to deformation in stator end windings, couplings, and fundament and so on.

![Figure 1 Overview of the different stages during a short-circuit. Figure from(Reimert, 2010)](image)

2.1 Parameters

Synchronous, transient, subtransient currents, reactances and time constants are terms that describe different parameters at various time-intervals of the decay of the AC component of the fault-current. The subtransient current, reactance and time are parameters derived from the first few cycles of the fault. The synchronous (or permanent in the figure) term is the final or steady-state current. The transient parameters determine the current in between the subtransient and the synchronous states. For generator applications, it’s generally desired to have a large subtransient reactance as it will reduce the short circuit peak current. The peak current in turn sets the limit for what the main circuit breaker in connection to the machine need to be rated for. At the same time, a low transient reactance is desired as it will contribute to minimized voltage drops during transients under normal operation. These two statements contradict each other as the two reactances are closely related and are derived from the same current. It’s thus up to the customer to decide what is most important to them. Similar situation arise for motor applications, where it’s desired to have a high subtransient reactance to reduce the in-rush current (which is somewhat similar to the short circuit current). While a high subtransient reactance on the other hand contributes to a lower starting torque.
2.2 Constant Flux Theorem

The large short circuit currents in a sudden short circuit relate to that the induction flow cannot immediately be brought down to the current level of a stationary short circuit; its decline is determined by the magnetic circuit time constant \( \frac{X}{R} \). Assuming the rotor winding resistance is zero, the time constant in question would become very large (infinite) and this effectively means that the flux never declines. This is due to every change in the flux induces a current in the short circuited winding which in turn induces a current which opposes the change in flux. It can also be thought of as in an inductance, where the current cannot change discontinuously. And since the flux is a function of the current, the flux cannot change discontinuously either. This leads to the flux linkage immediately after a short circuit of a winding is the same as before the fault. After the short circuit, the voltage in the stator referred to in the rotor reference frame can be described as:

\[
\mathbf{u}_s = R_s \mathbf{i}_s + \frac{d\Psi}{dt} = 0
\]

Where \( R \) is the resistance, \( I \) is the current in the stator and \( \Psi \) is the stator flux. Neglecting the resistance the equation becomes

\[
\frac{d\Psi}{dt} = 0
\]

In other words, the flux after a short circuit is equal to the flux immediately before the short circuit. As the resistance is at this moment considered to be zero, the voltage and flux changes are zero.

2.3 Short Circuit Current

It is important to consider the effects of a sudden short circuit when designing a machine due to the large currents created. Also, at ABB Machines, the three phase short circuit test is performed to derive the dynamic parameters of a machine. The magnitude of the current can be estimated using (2), i.e. the flux is constant and that the rotational speed is constant. If it’s initially also assumed that the rotor magnetic leakage flux is zero, i.e. all of the flux from the rotor will go through the stator winding and nothing between the poles, the induced voltage in the stator will also remain the same. In this case, the short circuit current in the stator will be determined only by the stator winding resistance \( R_a \) and its leakage reactance \( X_a \).

Before the short circuit, in the no-load case, the current in the stator winding is zero. Because of the leakage inductance in the stator windings, the current cannot instantly change. Thus, in addition to the stationary current a transitional current has to be superimposed (Alm, 1943). Because of mutual induction between the stator and rotor windings, when leakage in the rotor is neglected, one should not include the stator windings self-inductance \( L_a \), but its leakage inductance(Alm, 1943). A very simplified description of the short circuit then becomes,

\[
I_k = \frac{E_{\max}}{Z_a} \left[ \sin(\omega t + \alpha - \gamma_k) - \sin(\alpha - \gamma_k) e^{-\frac{t}{\tau_a}} \right]
\]

As seen in the equation above, the dc-component (second between the parenthesis) of the current is dependent on \( \alpha \) which is the angle position of the short circuit relative to steady state current represented by the first term in the equation. If \( \alpha = \gamma_k \), i.e. if the short circuit
occurs when the voltage is $E_{p\text{max}} \sin(\omega t)$ the dc-component will be zero and this means the total current becomes the same as the steady state current. However, if $\alpha - \gamma_k = \pm 90^\circ$, i.e. when the voltage is at a zero-crossing, the current reaches its maximum,

$$i = \pm \frac{E_{p\text{max}}}{Z_a} \left( \cos(\omega t) - e^{-\frac{t}{T_a}} \right)$$

(4)

In Figure 2, such a case is presented, $i_s$ is the steady state current and $I$ is the combined at $\alpha - \gamma_k = \pm 90^\circ$.

![Figure 2 Short circuit current displaying Symmetrical, asymmetrical and dc component. Figure from (Alm, 1943)](image)

For $\omega t = 180^\circ$ and Maclauren expanded with the only the first two terms included,

$$i_{\pi} \approx \pm \frac{2E_{p\text{max}}}{Z_a} \left( 1 - \frac{\pi}{2\omega T_a} \right)$$

(5)

If $T_a$ was to be very large, the instantaneous short circuit current would become double the steady state current. $T_a$, for larger generators, has a value in the region 0.025 and 0.05 seconds, at 50 Hz (Alm, 1943). This leads to the term between the parentheses in (5) becomes 0.8 and 0.9 respectively. Assuming a voltage drop, $I_aZ_a$, of normal current represent a 10% voltage drop of normal voltage, the relative impedance voltage drop would become,

$$\frac{I_aZ_a}{E_{p\text{max}}} = 0.1$$

(6)

Assuming that the impedance $Z_a$ remains the same after the short circuit and by substituting $E_p$ into (5), the current right after would become 16 and 18 times larger than the steady state current. Although this is very simplified, and the real current will be less, it shows that the current which arise during a short circuit is significant and should be considered. Since the forces are proportionally to the current squared, such a large current will put serious mechanical strain on the stator windings of the machines and could deform them unless they are properly constructed.
2.4 Phase Variable Model

A more in-depth description of the physics and a full derivation of the short circuit current is presented in Appendix A. The current is derived through the phase variable model, using the d-q transformation. The result represents a current close to a real machine and is described through (eq. (59)),

\[
I_A(t) = -v_{q0} \left( \left( \frac{1}{l_d} + \frac{1}{l_{d q}} + \frac{1}{l_{d q}'} \right) e^{-\frac{t}{\tau_d}} + \left( \frac{1}{l_{d q}''} + \frac{1}{l_{d q}''} \right) e^{-\frac{t}{\tau_{d q}}} \right) \cos(\omega_b t + \gamma_0) \\
- \frac{1}{2} \left( \frac{1}{l_{d q}''} + \frac{1}{l_{d q}''} \right) e^{-\frac{t}{\tau_a}} - \frac{1}{2} \left( \frac{1}{l_{d q}''} + \frac{1}{l_{d q}''} \right) e^{-\frac{t}{\tau_a}} \cos(2\omega_b t)
\]

The large subtransient component, together with the dc-offset and the synchronous parts of the total short circuit current become obvious when plotted, see Figure 3.

![Figure 3 Simulated short circuit current.](image-url)
3 Evaluation of Measurement Systems

This Chapter describes the evaluation of the Rogowski coils compared to the currently used system. The theory of the test equipment is presented together with a section of how the dynamic parameters are derived for the Matlab program. The Chapter also includes a description of how the experiment was conducted and the machine tested.

3.1 Rogowski Coil Theory

A Rogowski coil is a device for measure alternating current and/or transients. It consists of an air-cored coil where one of the ends is returned through the center of the coil. The result is a coil with both terminals at the same end which can be enclosed around a conductor. This makes the device easy to handle as it does not require interference with the conductor to measured, and thus good for retrofitting. The magnetic field in the conductor induces a voltage in the coil which is proportional to the rate of change of the current in the conductor. By integrating the rate of change a voltage proportional to the conductor current can be achieved, i.e. an integrator circuit is necessary for the device. An advantage of the Rogowski coil is that it isolates the measurement galvanically from the conductor. As it is air-cored, there’s no risk of saturation in the coil (as opposed to current transformers). Saturation-problems that may arise are if the induced voltage is too high causing a breakdown of the windings. Also, even though the integrator is linear within predictable limitations, it can become saturated. Other limitations in the integrator are if the output voltage becomes too large and slew-rate (Ward & Exon, 1993).

\[ \int H \cos \alpha \, dl = i \]  

(7)

Where H is the magnetic field and \( \alpha \) is the angle between the direction of the field and a small element dl, as can be seen in Figure 5.
If $A$ is the area enclosed by the rogowski coil, and the coil has $n$ turns in a section of length $dl$, the magnetic flux becomes:

$$d\phi = \mu_0 A ndl \cos a$$ (8)

By integrating along the coil, and combining the equation (7) and (8), the output voltage from a Rogowski coil enclosing an alternating current is:

$$v_{coil} = -\mu_0 A n \frac{di}{dt}$$ (9)

The output voltage is independent of how the coil is wrapped around the conductor, as long as the two ends of the coil are put together.

### 3.2 Dynamic Parameter Extraction Theory

In order to extract the dynamic parameters from a short circuit test, standard procedures exist. For this project, test procedures from IEEE std. 115 and IEC 60034-4 was implemented. The currents in d-axis from a short circuit were derived in (58) as,

$$I_d(t) \approx -v_q \left( \frac{1}{l_d} + \left( \frac{1}{l'_d} \right) e^{-\frac{t}{\tau_d}} + \left( \frac{1}{l''_d} \right) e^{-\frac{t}{\tau''_d}} - \frac{1}{l'_d} e^{-\frac{t}{\tau_a} \cos \omega_b t} \right)$$

Or, the total short circuit alternating current is the sum of the synchronous, transient, and subtransient current,

$$I_{total} = I_{ss} + I' + I''$$ (10)

The currents, separated from each other are shown in Figure 6. Ohm's law can then be used for the actual values of transient and subtransient reactances.

$$X_d = \frac{E}{I_s}$$

$$X'_d = \frac{E}{I'}$$

$$X''_d = \frac{E}{I''}$$ (11)
Where E is the average open-circuit voltage at normal frequency the instant prior to the short circuit. The problem is that the currents in the equations above needs to be estimated by processing the measured data from a short circuit test.

To derive the currents, the steady-state (or permanent) current is first determined, and it’s the final current reached after a short circuit. It can be obtained from the measured data if the sample time is long enough, generally though, it is taken from the OCC/SCC curve. The envelope is the curve connecting all the peaks minus the curve connecting all the lows (as seen in Figure 6). That current is then deducted from the envelope. The transient and subtransient currents are then derived from an extrapolation of the envelope of the short circuit oscillogram (minus the synchronous and the dc-offset) back to the instant of the short circuit (t = 0). Now adding the steady-state current again, the direct-axis transient and subtransient reactances can be estimated. Oscillograms for the three phases are needed as well as the pre-fault armature voltage and the final steady-state field current.

3.2.1 Subtransient, Transient and DC Component

By subtracting the symmetrical AC component and the DC component from the envelope of the fault current, the transient current can be estimated. When plotting the result on a semi-logarithmic scale the line should be almost a straight line, apart from the first 20 samples which are exponential (Curve B in Figure 7). By extrapolate the expected straight line, estimated from cycle ~20 to 150-200, back to time zero, the initial transient current can be found (Curve C in Figure 7). To find the subtransient current, curve C is subtracted from curve B, that is, the logarithmic value of the envelope minus the straight line estimation. The result should be a rapidly descending curve. Extending a straight line, based on the first few cycles (3-5 cycles) of the curve, back to time zero will give the initial value of the
subtransient current (Curve D in Figure 7). The values for the transient and subtransient currents from the three phases are then averaged to obtain the value of I’ and I’’ respectively. It is assumed that for cycle ~0-4 and ~20-150 the subtransient and transient effects respectively dominate over the armature dc offset, which is why it is used for the estimations (115, 2010). For the dc component the cycles in-between the transient and subtransient is used. Time constant \( \tau' \) is defined as the time it takes for the extrapolated current (curve C) to reach a value of 1/e of its initial value or one divided by the gradient of the straight line times the initial value. \( \tau'' \) is defined in a similar fashion, but for the subtransient current (curve D).

![Figure 7 Transient and subtransient current curves. Figure from (115, 2010)](image)

### 3.2.1.1 Transient Components

The straight line representation equation for obtaining the transient current at time zero is presented below:

\[
\ln \Delta i'(t) = \ln i(t) = k't + m'
\]  
\[(12)\]

For \( t \) from cycle 20 to 150-200. The \( m' \) in the straight line equation is where the curve crosses the y-axis i.e. the initial transient current. According to (115, 2010) the following equations can be used to obtain the transient parameters:

\[
\tau' = \frac{1}{k'} \quad [s]
\]

\[
\Delta i'(0) = e^{m't}
\]

\[
X_d'(t) = \frac{V_0/V_b}{|\Delta i'(0) + I_{ss}/I_b|} \quad [p.u]
\]

Where \( V_b \) is the base voltage, \( I_b \) base current, \( I_{ss} \) the steady state current reached after the short-circuit.
3.2.1.2 Subtransient Components

The straight line representation of the subtransient component is derived from the difference between the straight line representation of the transient component and the current, by the following equation:

\[ \Delta l''(t) = l(t) - \Delta l'(t) \]  \hspace{1cm} (14)

Similar to the transient component the straight line representation of the subtransient component is presented in (15), it is assumed that the subtransient effects are much larger than the dc offset in the first few cycles of the short circuit time course:

\[ \ln \Delta l''(t) = k''t + m'' \] \hspace{1cm} (15)

For \( t \) during the first 3-5 cycles.

Subtransient parameters are then:

\[ \tau''_d = -\frac{1}{k''} \text{ [s]} \] \hspace{1cm} (16)

\[ \Delta l''(0) = e^{m''} \]

\[ X_d''(t) = \frac{V_0/V_b}{[\Delta l'(0) + \Delta l''(0) + I_{ss}/I_b]} \text{ [p.u]} \]

Example plots of the two curves are shown in Figure 8 with their respective straight line interpolations. The subtransient straight line extrapolation is straight for the first few measurement points, and its shape is due to the logarithmic scaling.

![Figure 8 Transient and subtransient current, along with extrapolated lines](image-url)
3.2.1.3 DC Component

Since the subtransient and transient components dominate cycle 0-4 and 20-150 respectively it can be assumed that the armature effects, which dictate how the dc component behave, is dominating the cycles between 4-20. A similar procedure as previously described can be used for determining the synchronous time constant.

\[ \ln \left( \left| \text{abs} \left( i_{dc}(t) \right) \right| \right) = k_{dc} t + m_{dc} \]  \hspace{1cm} (17)

For t between ~4-20 cycles.

\[ T_a = -\frac{1}{k_{dc}} \text{[s]} \] \hspace{1cm} (18)

\[ i_{dc}(0) = e^{m_{dc}} \]

3.2.2 Dynamic Parameter Determination at ABB Machines

At ABB Machines three short circuit tests, at three different voltage levels, are conducted when the dynamic parameters are derived, all according to IEEE Std. 115 and IEC 60034-4. The three voltage levels are 30 %, 50 % and 70 % of rated voltage of the machine. The machine is assumed to be saturated at 100 % of rated voltage, but to avoid the very large currents and the mechanical strain it put on the machine, no test is done at rated voltage. Instead, to estimate the saturated parameters, a straight line extrapolation of the three test up to 100 % of rated value is done. In the test procedure at ABB Machine, the unsaturated parameter value is taken from 30 % test. In the figure below the three tests are shown together with the extrapolated line,

![Figure 9 Unsaturated and saturated values derived through linear extrapolation.](image)

The straight line representations for the transient and subtransient current are done using two points on the envelope curve, this is according to IEC standard 60034-4 as compared to several points are used in the IEEE standard.
3.3 Experimental Setup

During a survey of the test-room and after discussions with the test-engineers, it was discovered that placing the coils in the locked cabinet by the short circuit breaker was the best position for installation. It would ensure that the measurements would be done in the same way each time, since the position is not in the way of anything else. The position might also provide some shielding and the problems with different result from different position of the measurement system in the test box would be eliminated. Additionally, the setup time would be reduced to zero. Therefore, an evaluation of both the coils and the new position was necessary. The new position is schematically shown in Figure 10. The approach was to have both measuring systems in parallel to be able to compare the two. According to the data sheet, these Rogowski coils are isolated up to 10 kV while the machines to be tested are sometimes tested above that. Adding additional isolation around the coils could solve this problem. The Matlab program will be constructed according to IEEE std 115 (115, 2010), IEC (International Electrotechnical Commission, 2008) and the currently used evaluation software.

![Figure 10 Schematic figure of the machine and short circuit switch together with the measurement systems.](image)

3.3.1 Rogowski for Short Circuit Tests

The Rogowski coil is galvanically isolated from the conductor to be measured and has a very linear output due the lack of iron core. This makes it very suitable for transient measurements where high currents occur.

Almost all transient measurements include an asymmetrical component, also called DC offset. This causes problems for current measurements with current transformers since it can cause short-term saturations of its iron-core. This requires the transformer-core to be larger than it would otherwise have to be. The Rogowski does not suffer from saturation problems, and can therefore (given a proper integrator-design is implemented and wide bandwidth) reproduce asymmetrical transients accurately (Ward & Exon, 1993). Compared to current shunts, the Rogowski coil greatest advantages are that they’re cheaper and require less space, making it simple to retro-fit.

For this project, three CWT150 PEM Rogowski coils are to be used(Power Electronic Measurements Ltd, 2014). These coils are constructed for transient measurements only, not continuous high voltage measurements. The lower limit of the bandwidth for this system is 0.1 Hz which is well in the range for capturing the DC-component of a short circuit test. Small variations in the coil cross-section area and the winding density can cause the
transducer output to vary to some extent. The variation of the accuracy within the coil loop is shown in Figure 11. The conductor should not be placed in the grayed out area near the junction in the figure due to some discontinuity. The cost for three of these was around £ 2350 in 2011.

![Figure 11 Variation in accuracy depending on position of conductor. Figure from (Power Electronic Measurements Ltd, 2014).](image)

### 3.3.2 Evaluation Program

To analyze and extract the dynamic parameters from the oscillogram of the short circuit the procedures described in the sections above can be implemented in a computer program. A basic program layout can be described as follows:

1. Read measurement file
2. Find the peaks and lows in the current wave in the graph
3. Synchronize peaks and lows to create envelope
4. Calculation/input of steady state envelope
5. Calculation of transient parameters by straight line representation and linear regression
6. Calculation of subtransient parameters by straight line representation and linear regression

The process is then repeated for each phase. The peaks and lows values are identified and so is their respective time-stamp. Since the peaks and lows do not occur at the same time, it is not possible to subtract the two directly to get the envelope lines. It is therefore necessary to synchronize them to the same time-basis. This can be done by cubic spline interpolation, spline, in MatLab. A spline interpolation envelope is shown in Figure 12 where the lower envelope is fitted to the time stamp of the upper envelope.
3.3.3 Machine
Tests were performed on a AMS 900 machine. It’s a four pole synchronous machine with an axis height of 900 mm. The machine is rated at 13.8 kV, 60 Hz, while the tests were performed at 11 kV, 50 Hz. The tests were done at 30, 50 and 70 percent of 11 kV, and five tests were made at each voltage level. Since no measured data for the steady state current was available, it was simulated using ABB simulation software.

3.3.4 Measurement Error Estimation
Measurements for all tests, and both measurement systems, were taken on a HIOKI 8855 oscilloscope using HIOKI 8950 analog input units at a frequency of 10 kHz. The sample frequency is sufficient since the oscillations during a short circuit are relatively slow. The accuracy of the input units are 0.1 % and the resolution is 12 bits (HIOKI E.E CORP., 2014). The accuracy of the Rogowski coil, cable and the integrator is typically ± 0.2 % when the conductor is placed within area A in Figure 11 (Power Electronic Measurements Ltd, 2014). The shunts are from Thermovolt and the accuracy is 0.1 %. The measurement equipment at ABB Machines is regularly calibrated by professionals.

The maximum measurement error is thus small, 0.2 % and 0.3 % respectively for the two measurement systems. The likely maximum systematic measurement error, using the root-sum-square method is $\pm \sqrt{(0.1)^2 + (0.1)^2} = 0.14 \%$ for the shunts and ±0.22 % for the Rogowski coils. The largest error is likely to be from how the extraction of the dynamic parameters from the oscillograms is done. Since the current will be different each time, due to no phase voltage control, the extrapolations and interpolations are most probably a larger source of error. After discussions with my supervisor, the Matlab program is allowed to differ slightly from the currently used program. This is due to the possible interpretation differences and calculation differences in Matlab vs. Excel.
3.4 Implementation

The evaluation of the position of the coils was done by having the two measurement systems in parallel at the same place, while the new position was evaluated by having the coils in the new placement while keeping the current shunts in its normal position. The data was then analyzed using Matlab and the currently implemented evaluation software (Excel). After discussions with knowledgeable people at ABB Machines, it was decided that the air gap between the cable to be measured and the Rogowski coil is isolate enough so that no additional isolation was necessary. Additionally, as a result of the placement of the coils at the neutral point they will never experience a high voltage. The first test-implementation was thus a straight forward operation. The transient and subtransient current for the Matlab script was derived using the straight line representation from the same sample points as the Excel program. The picture below show the shunt and the Rogowski coils during the evaluation of the same placement. At the time only two coils were installed.

Figure 13 Picture from implementation when the Rogowski coils and shunts were at the same place.
4 Phase Angle Control

All actions to get more accurate measurements are of course desirable, and this phase controlling device is a step in that direction. As can be seen in Table 2 from the result section, the saturated values display quite large deviations from the calculated values. According to (3) from the theory section, it’s implied that by short circuit the machine at 90 degrees from the zero-crossing of the voltage, the dc-component will be zero and the total short circuit current will be at a minimum for that phase. Maximum current would be if a short circuit occurs at the zero-crossing. The difference can be seen in Figure 14, where two simulations were done. One at the zero-crossing and one at 90 degrees from the zero-crossing of the voltage. Although the average theoretical reactances should be the same, minimizing the short circuit current could cause less saturation in the stator that lead to subtransient reactance values closer to calculated values. The calculations of the dynamic parameters might also be more accurate since the current wave forms will have the same shape for each test.

![Simulated current for a short circuit at 0 and 90 degrees respectively.](image)

In order to control where on the voltage curve the short circuit occur, a few requirements of the equipment need to be met,

- The time from when the signal is sent to the breaker to the when it’s closed needs to be the same each time the test is performed. The breakers in the test room are quite old and controlled by springs. These are not designed to be timed and controlled in the way sought after here.
- The control system needs to be fast in order to have the accuracy desired. To close the breakers in a range of 3 degrees on the voltage curve, the time from when the data is collected, processed and an output signal is sent to the breaker needs to be
  
  \[
  \frac{20 \text{ ms period time}}{360 \text{ degrees in a period}} \times 3 \text{ degrees range} \approx 0.16 \text{ ms}
  \]
- Meet safety standards of the test room.
Not be too expensive.

4.1 Test Procedure and Experimental Setup

During the no load three phase short circuit test the speed of the machine is around 10 % over its rated speed, and run as a motor. The supply voltage is then disconnected and the machine is free-wheeling and acts as a generator at no load, as its speed slowly decreases. When the speed has reached the machines rated value (usually 1500 rpm @ 50 Hz) again on its way down, the machine is short circuited. Because of the high weight of these machines, the inertia is also large. This means that the frequency transition from 55 Hz down to 50 Hz can take up to 30 seconds, which enables a fairly accurate manual switching close to 50 Hz.

The approach to this part of the thesis is straight forward. The control of phase angle is highly dependent on the short circuit switch repeatability characteristics. For example, if the switching times differ only 2 ms from one test to another, which would represent 10 % of a period or 36 degrees. That level of accuracy is not acceptable. Thus, the first step is to evaluate the repeatability of the switch. For continuation of the project, acceptable time range was set to 0.4 ms, or around 7 degrees. The next step is to survey and evaluate controlled switching devices, or build one. The desired function will be considered first, then costs and possible customizations. The third step is to implement and test the chosen equipment with a real machine and evaluate the results. Once the functionality of the procedure is validated, a next step would be to investigate if there’s a better/cheaper way to achieve the same, or better, results.

4.1.1 HKK 24

The HKK 24 is a circuit breaker built by ASEA and is used for short circuiting the machines at ABB Machines in Västerås. When the breaker receives a 110 VDC signal, a relay triggers a charged spring which forces the lower and upper part of the connecting outlets (see Figure 15) together and close the breaker. The coil is enclosed in an oil-filled cylinder.

Figure 15 ASEA HKK circuit breaker. Figure from (ABB India Ltd, 2014)

4.1.2 Control Equipment

To control the phase angle of the short circuit, the voltage of one of the phases is necessary to be able to indentify its zero-crossings. From several zero-crossings, the period and the time-delay corresponding to a certain phase angle can be calculated. At a predetermined time, the device should send a signal to the breaker to close. The procedure is illustrated in the figure below.
4.1.2.1 SwitchSync
The SwitchSync is a device manufactured by ABB, and is usually used to avoid transients when connecting shunt reactors or capacitor banks to a transmission line (ABB AB High Voltage Products, 2014). It controls the switch similar to that of Figure 16, but it’s not design for controlled short circuits, and cannot adjust for frequency changes. That means the short circuit has to be manually triggered close to 50 Hz. The Switchsync is however very expensive.

4.1.2.2 cRIO and LabView
A Compact Rio from National Instruments is a programmable device which include a real-time module, an FPGA and I/O slots for different modules. Each FPGA-chip (field-programmable gate array) is made up of a reprogrammable integrated circuit. It is good for fast high-speed signal processing and minimized latency(NI, 2014). The device can be programmed using NI LabView.
5 Results

Due to this thesis will be published, some values in this section are not presented in numbers. Instead relations, e.g. results from two positions, are presented. In the tables, the spread represent the deviation derived through a linear regression of all possible combinations of parameters. The two lines shown in the graphs of Figure 24 represent the highest and lowest range from all these combinations. The average value is the average between those two lines and the spread is the distance from the average to the lines. A value below 1 indicates that the Rogowski has a narrower spread than the shunt, which is good.

5.1 Evaluation of Measurement Systems

5.1.1 Offset and current measurement

The data from both measurement systems does have an offset present. It is probable that the oscilloscope was not configured properly since there should not be an offset on the shunts, as they are completely passive. The offsets are taken care of in the evaluation program. The signal from the Rogowski coil for phase T experiences some small noise.

The very first short circuit measurement tends to give rise to a very large offset for the Rogowski coil, which disappears for the second test, as can be seen in Figure 17. It seems like an initial reference problem in the integrator and can be eliminated by having the integrators turned on for a few minutes before the measurements start.

![Figure 17 The integrator can cause a large offset and tilt error in the beginning.](image)

If the integrators are not allowed to “settle”, the result will not only be an offset but the curve from the Rogowski coil will also be tilted and differ significantly from the shunt measurement towards the end. Since the dynamic parameters are based primarily on the beginning of the measurement, where the difference is less, the impact on the reactances are limited. But to be sure to get an accurate measurement, it is crucial to have the integrators turned on a few minutes prior to the tests.
Figure 18. Result of the error caused by the integrator.

The short circuit current from the two measurement systems, apart from the first measurement shown above, displays similar results. For the test when the Rogowski coils were placed in the cabinet while the shunts where placed on the floor, the differences between the peaks (and the lows) are largest for phase S, followed by phase R. The first peak of the Rogowski measurement is around 3 % larger than the shunt in the 70 % test for phase S and is zoomed in in Figure 19. Phase T, which has the best resemblance between the two systems, differ in the range of 0.9 % between the first peaks. With the Rogowski coils at the same place as the shunts, similar results was obtained, but with even greater differences in the current measurements. The Rogowski coils are consistently giving a higher value than the current shunts.

Figure 19 Current measurement Rogowski vs Shunt, when the Rogowski coils were place in the cabinet.

5.1.2 Data Comparison Using Excel Program

In the tables below, the Excel program currently implemented was used to compare the Rogowski coils to the shunt measurements. Parameters from each test was compiled and compared to each other. In this case, the values represent a simple average of the unsaturated and the saturated values from the five tests performed.
As a result of the consistently higher current measured from the coils compared to the shunts, lower values for the reactances were obtained, i.e. a value below 1 for the reactance values. This was the case for both placements of the coils. The reactance values from the two positions match each other well. The new position show a little higher spread for the unsaturated transient reactance and the saturated subtransient reactance, while the unsaturated subtransient and the saturated transient reactance spreads were better. The majority of the spreads indicate that the Rogowski coils are more even between tests than the shunts. These results indicate that the new position does not display any major deviations and is thus feasible. The project could therefore go on with the Matlab script for further analysis.

### Table 1 Dynamic parameter comparison Excel.

<table>
<thead>
<tr>
<th>Unsaturated Values</th>
<th>Saturated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Values</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Same Placement</td>
<td></td>
</tr>
<tr>
<td>Xd'</td>
<td>0,977</td>
</tr>
<tr>
<td>Xd''</td>
<td>0,975</td>
</tr>
<tr>
<td>Td'</td>
<td>1,004</td>
</tr>
<tr>
<td>Td''</td>
<td>0,995</td>
</tr>
<tr>
<td>Rogowski in Cabinet</td>
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<td>Xd'</td>
<td>0,979</td>
</tr>
<tr>
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<tr>
<td>Td'</td>
<td>1,006</td>
</tr>
<tr>
<td>Td''</td>
<td>0,995</td>
</tr>
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</table>

A Matlab script was constructed in accordance to the Excel program. This was a part of the thesis, but it also made it easier to compare the two measurement systems in greater detail. Figure 20 show the reactances for each phase for the Rogowski coil divided by the value for the reactance for each phase obtained from to the shunt measurements. As a result of the difference in amplitudes in Figure 19, there’s also a difference in the reactances between the phases. The calculated reactances for phase T have the smallest deviations. Figure 20 is when the coils were placed in the cabinet. In the figures, the first test-run are the first 30, 50, 70 [%] the second test-run are the second set and so on.
Similar results were obtained when the Rogowski coils were placed at the same place as the shunts. It was thus necessary to investigate if the shunts were giving larger reactances or if the Rogowski’s were giving lower. The average reactances for the three phases were therefore plotted (reactances from 30, 50 and 70 % were averaged, as seen in Figure 21). Ideally, the reactances should be the same for all phases. From the plot below it becomes clear that phase S of shunt measurements give larger reactance values compared to the other phases. This indicates that the shunt for phase S, i.e. the phase in the middle, is influenced by the two surrounding shunts. The Rogowski coils display much better results in this aspect.
Figure 21 Reactance phase difference between the two measurement systems.

Similar to Figure 20, the average relative reactances were plotted. Figure 22 is when the coils were placed in the cabinet and Figure 23 when they were placed at the same location. These plots indicate that the placement of the Rogowski coils in the cabinet is a slightly better match to the shunts than when they were placed at the same place. This could be a result of other equipment interfering with the coils when placed outside the cabinet.

Figure 22 Rogowski vs shunt when the Rogowski measurement system is in the cabinet. The five tests are displayed after each other, i.e., test one are the first 30, 50, 70. Test two is the second set and so on.
5.1.3.1 Unsaturated and Saturated values
For this comparison, all reactances from all five tests were plotted in all possible combinations, along with their respective linear regressions for the reactances. The largest and lowest value of the regressed line at voltage level 0 and 1 p.u. was then used to create the maximum interval the reactances spanned during the tests. This should therefore not be directly compared to what’s presented in Table 1, where the deviation and the average values are taken directly from the end result of the evaluation.
Figure 24 shows the reactances for when the rogowski coils were placed in the cabinet, the same procedure was done for when the two measurement systems were at the same place. The table below compares the dynamic parameters from both measurement systems along with deviations from the calculated value, derived from the Matlab script. A column showing which measurement system display values closest to calculated values is also included. As been mentioned before, the spread is the relation between the deviations from the two measurement systems, a value below 1 indicates a more accurate spread for the Rogowski measurement.

<table>
<thead>
<tr>
<th>Unsaturated Values</th>
<th>Saturated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Values</td>
</tr>
<tr>
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<td>Rogowski/Shunt</td>
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<tr>
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<td>X</td>
</tr>
<tr>
<td>Xd'</td>
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</table>

Table 2 Dynamic parameter comparison using the Matlab script.

As seen above, the Rogowski measurements are closer to the calculated for all unsaturated values for the new position of the coils in the cabinet, while all except one measurement indicate the same for the saturated values. The spread become better in the new position compared to the old one, which most likely is a result of the enclosure of the measurement system. In the old position, with this comparison method, the spread was narrower for the shunt. In the new position the shunt and the Rogowski were more equal, where the Rogowski had lower spread in four cases. Where the shunt had lower, it was closer to 1 than for the old position. A notation on the time values is that they were derived assuming a temperature of 95 °C, by adding a specific constant. There were no temperature measurements available during these tests. Considering the machine was short circuited more than 30 times, the temperature is likely to have been higher towards the end, which could affect the results for the time parameters.

5.1.4 Measurement Error Estimation

The possible error from the measurement equipment was not considered when evaluating the measurement systems. Considering the spread differs from different parameters extracted from the same set of data indicate that the process of extracting parameters is a larger source of error.
5.2 Phase Angle Control
The switch in question is a HKK 223, controlled by springs and relays, produced in the seventies and not constructed for timed switching. Considering these conditions, several tests of the switching time had to be performed to evaluate if a retrofitted phase controller device was possible or if a whole new circuit breaker was necessary.

5.2.1 Timing Test of Breaker
The breaker was tested by measuring the time from when the trigger signal was sent to when the breaker was completely closed. As there are three breakers within the breaker, one for each phase, all three were tested to get the time for each one. In Figure 25 the switching time for one phase is presented.

![Figure 25 Short circuit breaker turn on time.](image)

The figure indicates a rather large range of almost 0.5 ms, representing 9 degrees on the voltage curve, between the different tests. However, further analysis of the results show that the first 10 test range 0.3 ms or 6 degrees and that the last two measurements was the last of around 25 tests under a short period of time which could affect the switch’s temperature and thereby its switching time. The results can be seen in Table 3. The consistency was thus within earlier set limits.

<table>
<thead>
<tr>
<th>Phase/Test #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
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<tr>
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<td>73.6</td>
<td>73.5</td>
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<td>73.5</td>
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<td>73.6</td>
<td>73.4</td>
<td>73.2</td>
<td>73.2</td>
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<td>Phase b</td>
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<td>73.5</td>
<td>73.5</td>
<td>73.4</td>
<td>73.5</td>
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<td>73.4</td>
<td>73.5</td>
<td>73.4</td>
<td>73.2</td>
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<tr>
<td>Phase c</td>
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<td>73.6</td>
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<td>73.5</td>
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<td>73.6</td>
<td>73.4</td>
<td>73.3</td>
<td>73.3</td>
</tr>
</tbody>
</table>

Table 3 Measurement results of the short circuit breaker, in ms.
5.2.2 Controlled Switching
The large inertia of the machines enables a fairly accurate manual short circuit close to 50 Hz. According to the test engineers, an error in the range ± 10 rpm on a 1500 rpm machine, or 1.3 %, is normal. Old test-protocols include 3-5 periods of the voltage prior to the short circuit, and a review of the frequency at the short circuit could therefore be done. Several protocols were evaluated and the result indicates that the test engineers timing is even more accurate with an error of just around ± 3 rpm, or 0.4 %. It was thus concluded that timing is not a large source of error and an automatic frequency sensor implementation is not a high priority issue for the control device.

5.2.2.1 Switchsync - Initial Testing
As the Switchsync is not designed for this application, the device was first tested using a accurate function generator for a 50 Hz reference voltage and instead of short circuit a machine, a battery was used to be able to record when the short circuit occurred. This was also necessary due to the actual machines frequency vary, which makes it hard to determine if a deviation was caused by a frequency variation or if the problem lies in the switching device (Switchsync). The results of ten tests are plotted in the same graph in Figure 26, where the blue curve is the battery voltage before the short circuit and after, i.e. when the breaker is closed. The oscilloscope is set to trigger on the switch signal, which is why there’s a time reading <0.

![Figure 26 Switchsync initial testing without machine.](image)

By zooming in on the zero-crossing the largest deviation can be found to be around 0.15 ms between the tests, as can be seen in Figure 27. This represents around 3 degrees accuracy for the phase angle, which is within acceptable measures.
5.2.2.2 Switchsync - Machine Testing

Ten short circuits were done at each voltage level (30, 50, and 70). The resulting reactances and time constants were plotted in all possible combinations. In this case, the earlier measurements from when the Rogowski coils were placed in the cabinet was compared to the values obtained using the Switchsync. The same relations as described earlier where used. A spread lower than 1 in this case indicate a narrower spread for the phase voltage controlled measurement. The result shows no clear benefit of the phase control for this machine. The values were closer to calculated in four cases for the phase control compared to three when no phase control was used. In one case the result was equal. The spread was better in the subtransient cases with the phase control and worse when it came to the unsaturated cases.

<table>
<thead>
<tr>
<th>90° vs Random phase angle</th>
<th>Unsaturated Values</th>
<th>Saturated Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>Closer to Calculated</td>
<td>Spread</td>
</tr>
<tr>
<td>90°/Random</td>
<td>X</td>
<td>1,667</td>
</tr>
<tr>
<td>Xd'</td>
<td>0.994</td>
<td>1.022</td>
</tr>
<tr>
<td>Xd''</td>
<td>0.997</td>
<td>0.829</td>
</tr>
<tr>
<td>Td'</td>
<td>X</td>
<td>2,000</td>
</tr>
<tr>
<td>Td''</td>
<td>1.057</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Dynamic parameter comparison with phase control.

5.2.2.3 cRIO Testing

A NI cRIO-9076 combined with one NI-9201 analog voltage input module and one NI-9401 digital output module was used for testing. The initial tests were conducted without a machine where a function generator was used instead. A simple LabView FPGA program was constructed for measuring the input signal and send out a digital out at a predetermined time delay from a negative-to-positive zero-crossing (picture of the program in Appendix B). This signal could then be used to trigger the short circuit breaker. The result was satisfactory and worked as planned. As this solution is easy to customize through LabView-programming, a
A tailored-made solution with frequency measurement can be achieved, as compared to the Switchsync. Due to time limitations, no testing was performed on a real machine.

Figure 28 Readings of the digital output with no delay time vs. a 50 Hz input signal.
6 Discussion

6.1.1 Evaluation of Measurement Systems

As the result of the higher current measured in the Rogowski coils, as shown in figure 1, the reactances derived from it will naturally be lower, which can be seen in table 1. The deviations between the five tests for the Rogowski coils are showing similar results to the shunts. As for the reactance values, the Rogowski coils are measuring closer to the calculated values for all except the saturated subtransient using the new position. The absolute value of the time parameters differs from calculation because of wrong temperature correction. I was not aware of the temperature impact on the determination of the parameter at the time of testing, and thus are the values themselves way off. However, the relation between other positions or other measurement systems is still of interest as the temperature impact is linear. Another error in the subtransient time constants is, at least partly, likely to be caused by calculation difficulties rather than that it actually differ that much. This might be because the time constants are derived from the slope of the extrapolated subtransient current curve’s two sample points. Overall, the subtransient parameters are more difficult to reproduce as accurately as the transient parameters.

The placement of the Rogowski coils in the cabinet where the short circuit breakers are seem to provide some shielding which reduce the disturbances from the surroundings and could cause the difference seen between figure 11 and 12. The problem with the initial large tilted offset reading which can be solved by letting the integrator be turned on a few minutes prior to a test, was unexpected. They took even longer time to “settle” when the rechargeable batteries were changed to regular batteries. The Matlab script calculation results are similar to the old evaluation software although some differences in the subtransient parameters exists. Compared to the Excel based evaluation program, the Matlab approach is in my opinion more intuitive and it’s easier to extract specific plots and data from it.

6.1.2 Phase Angle Control

The project was successful in achieving the desired function of short circuiting the machines at a certain phase angle, and the short circuit test can be controlled to be done in the same way each time. The accuracy between different tests for the breaker, as shown in Figure 27, was not anticipated considering the breaker was far from ideal for this purpose. And because of this, prior to any possible implementation, the switches in the other test boxes need to be evaluated to see if they show similar characteristics.

The saturated subtransient reactance, time parameter and spread values was better compared to the calculated values when short circuiting at 90 degrees, although not as much as initially hoped. It’s interesting though, as the subtransient parameters are derived from the highest currents, where there’s a risk of saturation. The lower spread indicates that the new phase control makes the determination of dynamic parameters more accurate, which was one reason for why this was investigated. The transient parameters and spread show the opposite, but the differences are quite small in this case too. The tests were performed on one of the smaller sized machines from ABB Machines (AMS 900). It’s possible the impact of saturation is more visible in a larger machine with higher currents, and minimizing that current could have a more apparent effect.

The LabView implementation provides more customization possibilities. It makes it possible to have an automated short circuit control, where the short circuit switch is triggered when the
machine has reached desired frequency. The equipment from NI for this test was unnecessarily expensive because cheaper equipment exists which would produce the same results. The test room has several NI USB-6215 I/O devices, which might work if they were connected to an external clock for synchronization.

7 Conclusions
A cheaper, more efficient test procedure has been achieved using the Rogowski coils. From the results presented in this report, I would recommend the Rogowski coils to be permanently implemented in the cabinets. The tests suggest that the new placement is comparable to the placement of the currently used measure system in terms of consistency between tests, and accuracy. One major benefit is that the test procedure will also be more efficient due to the setup time is eliminated, which earlier could take up to an hour. A permanent installation also ensures that the measurements are performed in the same way each time. The difference in the current between the phases using the shunts can largely be avoided if the Rogowski coils are used instead. For the phase control, the subtransient parameters and spread show opposite result compare to the transient ditto, which makes it hard to draw any conclusions other than more tests should be performed for further investigation. However, the lower spread for the saturated values indicates a more accurate parameter calculation due to similar current waves from each test. The Switchsync is very expensive and I would recommend more tests to investigate why the transient spread did not improve as the subtransient. A cheaper method with the same features could be implemented.

8 Future Work
As this entire project was done in just one test box, additional Rogowski coils would need to be purchased for installation in the other boxes. Similar, but with smaller radius and more robust coils could then be bought, as the robustness of the coils has been questioned by the test room personnel. The phase controlling could be done cheaper with the same result. In this thesis the main focus of that part was to see if it was possible and what the result was. A continuation could thus be to find a cheaper/better way to do it. Further tests with higher-current machines could show a more apparent effect of decreasing the dc-component.
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Appendix – Derivation of Short Circuit Current using the Phase-variable Model

A1.1 Phase-variable Model

The phase-variable model is a circuit model to describe the synchronous generator as three stator circuits coupled together through motion with two, or multiples of two, damper windings, and a field winding (along the d-axis in Figure 29 i.e. where the permeance is at its maximum).

![Circuit model using the Phase-variable model](image)

**Figure 29** Circuit model using the Phase-variable model. Figure from (Boldea, 2006)

Of course, the stator and rotor windings are coupled magnetically to each other. Phase-voltage equations for the different circuit-equivalents in Figure 29 are presented below (rotor equations in rotor coordinates, stator equations in stator coordinates):

\[
\begin{align*}
    i_A R_s + v_a &= -\frac{d\Psi_A}{dt} \\
    i_B R_s + v_b &= -\frac{d\Psi_B}{dt} \\
    i_C R_s + v_c &= -\frac{d\Psi_C}{dt} \\
    i_D R_D &= -\frac{d\Psi_D}{dt} \\
    i_Q R_Q &= -\frac{d\Psi_Q}{dt} \\
    i_f R_f + v_f &= -\frac{d\Psi_f}{dt}
\end{align*}
\] (19)

Including the self-inductance and mutual inductances the total flux in phase A can be written as:

\[
\Psi_A = L_{AA}i_A + L_{AB}i_B + L_{AC}i_C + L_{AF}i_f + L_{AD}i_D + L_{AQ}i_Q
\] (20)

The same equation can describe the flux for the other phases, only that they’re 120 electrical degrees apart. A large number of circuits are involved in determining the mutual and self-
inductances which, in addition to the inductances change with the rotor position, make the synchronous machine equations complicated. The variation in the inductance is caused by the varying permeance due to the non-uniform air-gap of the salient pole machine. The varying permeance in relation to rotor position is presented in Figure 30 below. Assuming the winding distribution to be sinusoidal, the current in phase A creates a mmf in the air gap which can be divided in to components corresponding to the direct and quadrature axes \((\cos(\Theta)\) and \(-\sin(\Theta))\)\textsuperscript{(Concordia, 1951)}, more on the d-q transform is describe further down. These mmf’s in turn creates a corresponding flux with space fundamental components in magnitudes of \((\phi_d = P_d\cos(\theta_d))\) and \((\phi_q = P_q\cos(\theta_q))\), where \(P\) represent the effective permeance in the two axes (d and q). Space harmonics are also created, but they’re not link to the stator and thus not considered at this point. The linkage in a phase caused by the flux can therefore be described on the form,

\[
\phi_d \cos(\theta_{er}) - \phi_q \cos(\theta_{er}) = P_d\cos^2(\theta_{er}) + P_q\sin^2(\theta_{er}) = \frac{P_d + P_q}{2} + \frac{P_d - P_q}{2}\cos(2\theta_{er}) \rightarrow P(\theta_{er}) = P_0 + P_2\cos(2\theta_{er})
\]  

(21)

As is described by (21), the permeance has a double frequency component. In a salient pole machine, the angle teta is the angle of direct axis, d, of the rotor ahead of the axis of phase a.

Figure 30: Air gap permeance in relation to rotor position. Figure from (Boldea, 2006)

The self-inductance of phase A, due to air-gap flux is

\[
L_{AAg} = L_0 + L_2\cos(2\theta_{er})
\]  

(22)

For the total phase A self-inductance, the phase leakage inductance \(L_{sl}\) is added:

\[
L_{AA} = L_{st} + L_0 + L_2\cos(2\theta_{er})
\]  

(23)

For phase b and c, 120 degrees separated the self-inductances are:

\[
L_{BB} = L_{st} + L_0 + L_2\cos\left(2\theta_{er} + \frac{2\pi}{3}\right)
\]  

(24)

\[
L_{CC} = L_{st} + L_0 + L_2\cos\left(2\theta_{er} - \frac{2\pi}{3}\right)
\]  

(25)
Self-inductances are considered to be related to the air-gap permeans only. If the windings are sinusoidal distributed in the stator the mutual inductance between stator windings, e.g. \(L_{AB}(\Theta)\), varies with \(\Theta\) as in equation \((25)\). The mutual inductance \(L_{AB}\) also experience the double frequency harmonic as in the permeance, due to the shape of the rotor.

\[
L_{BA} = L_{AB} = L_{AB0} + L_{AB2} \cos \left(2\theta_{er} - \frac{2\pi}{3}\right)
\]  

(26)

As the three phases are 120 degrees shifted, \(L_{AB0}\) must also be 120 degrees shifted;

\[
L_{AB0} \approx L_0 \cos \left(-\frac{2\pi}{3}\right) = -\frac{L_0}{2}
\]  

(27)

The resemblances between the variable part of the mutual and self-inductances ((25) and (26)) leads to, \(L_{AB2} = L2\), additionally,

\[
L_{CA} = L_{AC} = -\frac{L_0}{2} + L_2 \cos(2\theta_{er} + \frac{2\pi}{3})
\]

\[
L_{BC} = L_{CB} = -\frac{L_0}{2} + L_2 \cos(2\theta_{er})
\]  

(28)

Where

\[
L_0 = \frac{(L_{dm} + L_{qm})}{2}
\]

\[
L_2 = \frac{(L_{dm} + L_{qm})}{2}
\]  

(29)

Where \(L_{dm}\) and \(L_{qm}\) are the magnetizing inductances.

### A1.1.2 Mutual inductances between stator and rotor circuits

The mutual inductance between the stator phases and the rotor magnetizing circuit vary with \(\cos(\Theta)\) and \(\sin(\Theta)\) as follows,

\[
L_{Af} = M_f \cos(\theta_{er})
\]

\[
L_{Bf} = M_f \cos(\theta_{er} - \frac{2\pi}{3})
\]

\[
L_{Cf} = M_f \cos(\theta_{er} + \frac{2\pi}{3})
\]

\[
L_{AD} = M_D \cos(\theta_{er})
\]

\[
L_{BD} = M_D \cos(\theta_{er} - \frac{2\pi}{3})
\]

\[
L_{CD} = M_D \cos(\theta_{er} + \frac{2\pi}{3})
\]

\[
L_{AQ} = -M_Q \sin(\theta_{er})
\]

\[
L_{BQ} = -M_Q \sin(\theta_{er} - \frac{2\pi}{3})
\]

\[
L_{CQ} = -M_Q \sin(\theta_{er} + \frac{2\pi}{3})
\]  

(30)
Where $M$ is the mutual inductance between the field and armature windings.

Now that all variables from (20) are defined, a $6 \times 6$ matrix phase-variable matrix, $[LABCDFQ(\Theta_{er})]$, can be defined. The matrix describe all the coupled inductances.

For a salient-pole rotor, an additional mutual coupling leakage inductance appears between the field winding $f$ and the $d$-axis cage winding $D$, as well as the self-inductances. The zeros in the matrix represent that there is zero coupling between orthogonal windings, given that there’s no magnetic saturation.

We now have all the inductances for the total flux in each phase, i.e.:

$$\Psi_{ABCDQ} = [\Psi_A, \Psi_B, \Psi_C, \Psi_f^r, \Psi_D^r, \Psi_Q^r]$$

(31)

Where the $r$ represent parameters referring to the rotor. Given that core losses, magnetic saturation, space harmonics and skin effects in the rotor and damper cage are neglected, the synchronous generator can be described by the following matrix equation at constant speed:

$$[I_{ABCDQ}] [R_{ABCDQ}] + [V_{ABCDQ}] = \frac{-d\Psi_{ABCDQ}}{dt}$$

(32)

$$= - [L_{ABCDQ}(\Theta_{er})] \frac{d}{dt} [I_{ABCDQ}]$$

$$- \frac{\partial [L_{ABCDQ}]}{\partial \Theta_{er}} \frac{d\Theta_{er}}{dt} [I_{ABCDQ}]$$

Where $[R_{ABCDQ}]$ is a diagonal matrix where with the last three elements are referred to the rotor and

$$[V_{ABCDQ}] = [V_A, V_B, V_C, -V_f, 0, 0]$$

(33)

This presents a very complex equation system of high order with time-varying coefficients (the inductances). To solve these kinds of systems for different transient or steady-state behavior, numerical methods such as the predictor-corrector or Runge-Kutta-Gill can be used (Boldea, 2006). This however is a very time-consuming and unintuitive operation, and
that are two reasons why the d-q model was developed. The d-q model presents a faster way to solve transient operations in a synchronous generator.

### A1.1.3 The d-q Model

The d-q model’s main objective is to remove the rotor position dependence of the inductances, which will greatly simplify the machine calculations. In order to accomplish this, the stator equations should be expressed in rotor coordinates and align to rotor axes d and q. This is because there’s no magnetic coupling between these two axes, given that no magnetic saturation exists. The rotor equations for windings f, D and Q are in rotor coordinates already, from (19). Thus, only the stator parameters (voltage, flux and current) need to be transformed. The transformation from ABC coordinates to d-q0 is known as the Park transformation and is valid for currents, flux linkage and voltage.

$$[P(\theta_{er})] = \frac{2}{3} \begin{bmatrix} \cos(-\theta_{er}) & \cos(-\theta_{er} + \frac{2\pi}{3}) & \cos(-\theta_{er} - \frac{2\pi}{3}) \\ \sin(-\theta_{er} + \frac{2\pi}{3}) & \sin(-\theta_{er} + \frac{2\pi}{3}) & \sin(-\theta_{er} - \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

So that,

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = [P(\theta_{er})] \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}$$

The inverse Park transformation is,

$$[P(\theta_{er})]^{-1} = \frac{3}{2} [P(\theta_{er})]^T$$

By transforming the flux linkage parameters, and after some trigonometric reductions of terms, the following equations for the stator flux in d-q0 coordinates are obtained,

$$\Psi_d = \left(L_{sl} + L_0 - L_{AB0} + \frac{2}{3} L_2\right) I_d + M_f I_f^r + M_D I_D^r$$

$$\Psi_q = \left(L_{sl} + L_0 - L_{AB0} - \frac{2}{3} L_2\right) I_q + M_Q I_Q^r$$

$$\Psi_0 = \left(L_{sl} + L_0 - 2L_{AB0}\right)$$

Defining new inductances, and $L_{AB0} \approx -\frac{L_0}{2}$.

$$L_d = L_{sl} + L_0 - L_{AB0} + \frac{2}{3} L_2$$

$$L_q = L_{sl} + L_0 - L_{AB0} - \frac{2}{3} L_2$$

$$L_0 = L_{sl} + L_0 - 2L_{AB0}$$

Which reduces the stator flux equations to,

$$\Psi_d = L_d I_d + M_f I_f^r + M_D I_D^r$$

41
\[ \Psi_q = L_q I_q + M_{D} I_D \]
\[ \Psi_0 = L_{s} I_0 \]

Following the same procedure for the rotor gives,

\[ \Psi_f^r = (L_{f1} + L_{fm}) I_f^r + \frac{3}{2} M_f L_d + M_{fd} I_D \]
\[ \Psi_0^r = (L_{D1} + L_{Dm}) I_D^r + \frac{3}{2} M_D L_d + M_{fd} I_f^r \]
\[ \Psi_0^r = (L_{Q1} + L_{Qm}) I_Q^r + \frac{3}{2} M_Q L_q \]

Where inductance index l and m stands for leakage and mutual respectively. From (39), one can notice that the zero component of the stator flux only depend on the leakage inductance. It does therefore not take part in the energy conversion through the fundamental components of fields and mmfs in the synchronous generator. It can thus be seen as separate, and the d-q transformation can be represented as an imagined synchronous generator where the stator axes are orthogonal and magnetically fixed to the rotor d-q axes, and is illustrated in Figure 32. The windings (i.e. the coils) in the stator are however standing still in this model and do not rotate with the rotor.

![Figure 32 Synchronous machine, d-q model. Figure from (Boldea, 2006)](image)

Assuming \( \frac{d\Psi_d}{d\theta_{eq}} = -\Psi_d \) and \( \frac{d\Psi_q}{d\theta_{eq}} = \Psi_q \), which implies that the windings d-q are sinusoidal distributed and a constant air gap with radial flux barriers along axis d. This is a valid approximation for distributed stator windings if only the fundamental air gap flux density is of interest. The voltage equations for the machine can then be derived using KVL from Figure 32. For the stator,

\[ I_d R_s + V_d = -\frac{d\Psi_d}{dt} + \Psi_q \omega_r \]
\[ I_q R_s + V_q = -\frac{d\Psi_q}{dt} + \Psi_d \omega_r \]

And the rotor,

\[ I_f R_f + V_f = -\frac{d\Psi_f}{dt} \]
\[ I_d R_d = -\frac{d \psi_d}{dt} \]
\[ I_q R_q = -\frac{d \psi_q}{dt} \]

The equation for the zero component is shown below,

\[ I_0 R_s + V_0 = -I_{st} \frac{d I_0}{dt} = -\frac{d \psi_0}{dt} \]  (43)

Where,

\[ I_0 = \frac{I_A + I_B + I_C}{3} \]  (44)

A1.1.3.1 Per Unit d-q model

Using Lapace the time derivative in per units becomes,

\[ \frac{d}{dt} \rightarrow \frac{1}{\omega_b} \frac{d}{dt}; s \rightarrow \frac{s}{\omega_b} \]  (45)

Where \( \omega_b \) is the base quantity. Combining equation (41), (42) and (45) the P.U d-q-model equations become

\[
\begin{align*}
\frac{1}{\omega_b} \frac{d \psi_d}{dt} &= \psi_q \omega_r - i_d r_s - v_d; \psi_d = l_{st} i_d + l_{dm} (i_d + i_D + i_F) \\
\frac{1}{\omega_b} \frac{d \psi_q}{dt} &= -\psi_d \omega_r - i_q r_s - v_d; \psi_q = l_{st} i_d + l_{qm} (i_q + i_Q) \\
\frac{1}{\omega_b} \frac{d \psi_f}{dt} &= -i_f r_f + v_f; \psi_f = l_{fi} i_f + l_{dm} (i_d + i_D + i_F) \\
\frac{1}{\omega_b} \frac{d \psi_D}{dt} &= -i_D r_D; \psi_D = l_{Df} i_D + l_{dm} (i_d + i_D + i_F) \\
\frac{1}{\omega_b} \frac{d \psi_Q}{dt} &= -i_Q r_Q; \psi_Q = l_{Qf} i_Q + l_{qm} (i_q + i_Q)
\end{align*}
\]  (46)

\( I_f, I_D, \) and \( I_Q, \) which is the field current and damper currents, remain the same for the d-q model and the actual machine. All values in equation 5 are in stator coordinates, and thus related to the stator base values. The P.U. Park transformation is essential in transforming the actual machine to the d-q-model voltages.

A1.1.3.2 General Equivalent circuits

By replacing \( d/dt \) in (46) with the Laplace operator \( s/\omega_b \) and after separating the main flux linkage components \( \psi_{qm} \) and \( \psi_{dm}, \) the general circuit equations can be derived through,

\[
\begin{align*}
\psi_d &= l_{st} i_d + \psi_{dm}; \psi_q = l_{st} i_q + \psi_{qm} \\
\psi_{dm} &= l_{dm} (i_d + I_D + I_f); \psi_{dm} = l_{qm} (I_q + I_Q) \\
\psi_f &= l_{si} i_f + \psi_{dm}; \psi_D = l_{st} I_D + \psi_{dm}
\end{align*}
\]  (47)
The voltage equations are then,

\[
\left( r_f + \frac{S}{\omega_b} l_{fl} \right) I_f - V_f = - \frac{S}{\omega_b} \Psi_{dm} \\
\left( r_D + \frac{S}{\omega_b} l_{Dl} \right) I_D = - \frac{S}{\omega_b} \Psi_{dm} \\
\left( r_Q + \frac{S}{\omega_b} l_{Ql} \right) I_Q = - \frac{S}{\omega_b} \Psi_{qm} \\
\left( r_s + \frac{S}{\omega_b} l_{sl} \right) I_d + V_d - \omega_r \Psi_q = - \frac{S}{\omega_b} \Psi_{dm} \\
\left( r_s + \frac{S}{\omega_b} l_{sl} \right) I_q + V_q - \omega_r \Psi_d = - \frac{S}{\omega_b} \Psi_{qm}
\]

From (48), it follows that three of them are in parallel along the d-axis and two along the q-axis. As no magnetic saturation or frequency effects are considered, the coupling of the circuits is only done through the main flux \( \Psi_{qm} \) and \( \Psi_{dm} \). Based on these equations, the general equivalent circuits of a synchronous generator are shown in Figure 33 below.

![General equivalent circuits](image-url)
Here follows a few remarks related to Figure 33,

- In the d-q model the magnetizing current components $I_{dm}$ and $I_{qm}$ are defined as the currents along their axes,

\[
I_{dm} = I_d + I_D + I_f \\
I_{qm} = I_q + I_Q
\]

- Because of no magnetic coupling between the d and q axes, and also magnetic saturation is considered separately or ignored, the different inductances are functions of their respective currents,

\[
l_{s1}(I_s); l_{dm}(I_{dm}); l_{qm}(I_{qm}); l_{s1}(I_s); I_s = \sqrt{I^2_d + I^2_q}
\]

- If a skin effect should be present in the rotor pole solid iron or in the damper cage, additional parallel rotor circuits are added. Generally, two along the q-axis and one along the d-axis is sufficient for a solid rotor pole synchronous generator (Boldea, 2006).

- In Figure 33 the zero component of (48) is not present, this is because it doesn’t interfere with the main flux fundamental. In real life, some eddy currents from the zero component may be produced rotor cage through its third space-harmonic mmf.

### A1.1.4 Operational Parameters

A convenient way of making the machine equation more compact when calculating short circuit currents, is to eliminate the rotor currents. This could be done by first substituting the rotor flux-linkage relations ( (40)) into the voltage equations of the rotor-circuit ( (48)). Then solving the equations for the rotor currents in terms of armature currents $i_d$, $i_q$ and the field voltage $v_{fd}$, and then substituting the result into armature flux-linkage relations ( (37)). This can be a fairly complicated procedure. But if the derivative operator $\frac{d}{dt}$ is treated algebraically, which is legitimate in this case since the flux-linkage relations and all the voltage relations in the rotor-circuit are linear(Concordia, 1951), the general expression of the operational parameters for $\Psi_d$ and $\Psi_q$ in a synchronous generator, expressed in Laplace-form, becomes:

\[
\Psi_d(s) = l_d(s) \cdot i_d(s) + g(s)v_{ex}(s) [P,U] \\
\Psi_q(s) = l_q(s) \cdot i_q(s)
\]

Where,

\[
l_d(s) = \frac{(1 + sT'_d)(1 + sT''_d)}{(1 + sT'_d0)(1 + sT''_d0)} [P,U] \\
l_q(s) = \frac{(1 + sT'_q)}{(1 + sT''_q)} \cdot i_q [P,U] \\
g(s) = \frac{1}{(1 + sT'_d0)(1 + sT''_d0)} \cdot \frac{I_{dm}}{n_f} [P,U]
\]
A1.1.5 Transients

For the transient analysis, the speed is assumed constant and therefore no motion equation is needed. (46) in Laplace form together with (49) yields the following expression,

\[-v_d(s) = r_sl_d + \frac{s}{\omega_b} l_d(s)l_d - \omega_rl_q(s)l_q + g(s) \frac{s}{\omega_b} v_f(s)\]
\[-v_q(s) = r_sl_q + \frac{s}{\omega_b} l_q(s)l_q - \omega_r \left(l_d(s)l_d + g(s)v_f(s)\right)\]

For rated speed is 1. If the time variation of \(v_f(t)\), \(v_d(t)\) and \(v_q(t)\) can be transformed to Laplace forms, (51) can be solved to obtain \(i_q(s)\) and \(i_d(s)\):

\[
\begin{vmatrix}
-v_d(s) & -g(s) \frac{s}{\omega_b} v_f(s) \\
-v_q(s) & -\omega_r g(s)v_f(s)
\end{vmatrix}
= \begin{vmatrix}
  r_s + \frac{sl_d(s)}{\omega_b} & -\omega_r l_q(s) \\
-\omega_r l_d(s) & r_s + \frac{s}{\omega_b} l_q(s)
\end{vmatrix}
\begin{bmatrix}
  l_d(s) \\
  l_q(s)
\end{bmatrix}
\]

A1.1.5.1 The 3 Phase Short Circuit

For the no-load operation, the initial values of \(I_{d0}\) and \(I_{q0}\) are zero, and the field winding terminal voltage is assumed to be constant. That is,

\[v_{f0} = I_{f0} \cdot r_f\]

With \(s = 0\), \(I_{q0} = 0\) and \(I_d = 0\) the initial voltage \(v_d0\) along the d-axis become zero under no load. Along q-axis, the no load voltage occur. That along with \(v_f(s) = 0\), (52) can be solved for \(I_d(s)\) and \(I_q(s)\),

\[I_d(s) = \frac{-v_{q0}\omega_b^3 \omega_r}{s l_d(s) \left(\omega_b^2 \omega_r^2 + s^2 + s r_s \omega_b \left(\frac{1}{l_d(s)} + \frac{1}{l_q(s)}\right) + \frac{\omega_b^2 r_s^2}{l_q(s) l_d(s)}\right)}\]

To make this expression easier to handle, the \(r_s^2\) terms are neglected. Also,

\[\frac{r_s \omega_b}{2} \left(\frac{1}{l_d(s)} + \frac{1}{l_q(s)}\right) \approx \frac{1}{r_a} \approx \text{constant} \approx \frac{r_s \omega_b}{2} \left(\frac{1}{l_d} + \frac{1}{l_q}\right)\]

Using the above notions (55) result in,

\[I_d(s) = \frac{-v_{q0} \omega_b^3 \omega_r}{s \left(\omega_b^2 \omega_r^2 + s^2 + \frac{2}{r_a} s\right)} \cdot \frac{1}{l_d(s)}\]
\[I_q(s) = \frac{-v_{q0} \omega_b^2 \omega_r}{s \left(\omega_b^2 \omega_r^2 + s^2 + \frac{2}{r_a} s\right)} \cdot \frac{1}{l_q(s)}\]
1/\text{i}_q(s) and 1/\text{i}_d(s) can be rewritten using (49) to,

\[
\begin{align*}
\frac{1}{\text{i}_d(s)} &= \frac{1}{\text{i}_d} + \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) \frac{s}{s + 1/T''_d} + \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) \frac{s}{s + 1/T''_d} \\
\frac{1}{\text{i}_q(s)} &= \frac{1}{\text{i}_q} + \left(\frac{1}{\text{i}''_q} + \frac{1}{\text{i}'_q}\right) \frac{s}{s + 1/T''_q}
\end{align*}
\]

(57)

The inverse Laplace transform of \text{i}_d(s) and \text{i}_q(s) can be found assuming \text{T}'_d \text{ and } \text{T}''_q \text{ are larger than } 1/\omega_b \text{ and that rotational speed is constant, } \omega_b = 1 \text{. After some approximations and derivations the equations become,}

\[
\begin{align*}
\text{i}_d(t) &\approx -v_{q0} \left(\frac{1}{\text{i}_d} + \frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) e^{-\frac{t}{T_d}} + \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) e^{-\frac{t}{T''_d}} - \frac{1}{\text{i}'_d} e^{-\frac{t}{T_a}} \cos \omega_b t \\
\text{i}_q(t) &\approx v_{q0} e^{-\frac{t}{T_a}} \sin \omega_b t \\
\text{i}_d(t) &= \text{i}_{d0} + \text{i}_d(t) \\
\text{i}_q(t) &= \text{i}_{q0} + \text{i}_q(t)
\end{align*}
\]

(58)

The current in a phase during a three phase short circuit is then obtained by adding the two axes again,

\[
\begin{align*}
\text{i}_q(t) &= \text{i}_d(t) \cos(\omega_b t + \gamma_0) - \text{i}_q(t) \sin(\omega_b t + \gamma_0) \\
&= -v_{q0} \left(\frac{1}{\text{i}_d} + \frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) e^{-\frac{t}{T_d}} \\
&\quad + \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) e^{-\frac{t}{T''_d}} \cos(\omega_b t + \gamma_0) - \frac{1}{2} \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_d}\right) e^{-\frac{t}{T_a}} \\
&\quad - \frac{1}{2} \left(\frac{1}{\text{i}''_d} + \frac{1}{\text{i}'_q}\right) e^{-\frac{t}{T_a}} \cos(2\omega_b t)
\end{align*}
\]

(59)

An example with made up values for the inductances is plotted below,

Figure 34 Simulated short circuit current.
B1  Appendix - LabView Program