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Examining GARCH forecasts for Value-at-Risk predictions

Abstract

In this thesis we use the GARCH(1,1) and GJR-GARCH(1,1) models to estimate the conditional variance for five equities from the OMX Nasdaq Stockholm (OMXS) stock exchange. We predict 95% and 99% Value-at-Risk (VaR) using one-day ahead forecasts, under three different error distribution assumptions, the Normal, Student's t and the General Error Distribution. A 500 observations rolling forecast-window is used on the dataset of daily returns from 2007 to 2014. The empirical size VaR is evaluated using the Kupiec's test of unconditional coverage and Christoffersen's test of independence in order to provide the most statistically fit model. The results are ultimately filtered to correspond with the Basel (II) Accord Penalty Zones to present the preferred models. The study finds that the GARCH(1,1) is the preferred model when predicting the 99% VaR under varying distribution assumptions.

Key words: Value-at-Risk, Unconditional coverage, Christoffersen's, GJR-GARCH

1 Introduction

In econometrics, and especially in financial econometrics, the assumption of homoscedasticity of the error term is implausible since the volatility of a security tend to vary over time. Observations containing low (high) volatility are often followed by another low (high) volatility observation. In financial econometrics, this is called volatility clustering which means the data suffers from heteroscedasticity. Engle (1982) developed the ARCH model where homoscedasticity is no longer an assumption and the time varying conditional variance is treated as a variable to be modelled. It was not until a few years later when this methodology was applied in the financial sector that the model proved useful forecasting risk. Since then, a whole branch of new models emerged with the GARCH model as a starting point. (Bollerslev, 1986)

Within risk management the VaR is a widely used measurement, predicting the potential loss of a portfolio or a specified asset. It is a regulatory framework applied by the Basel committee on Banking Supervision, implemented in the committee's second accord, Basel II. The committee's mission is global financial stability and one of the three fundamental pillars is capital requirements to cover market risk. As return is associated with risk, an investor taking much risk implies potential large losses but taking too little risk implies an opportunity cost. The Basel accords therefore specify capital requirements in relation with the risk held by the investor, resulting in monitored capital coverage and limited maximum risk-taking.

VaR can be defined as the loss that a given portfolio will not exceed given a certain level of confidence within a specified timeframe. The regulatory framework does not specify how the calculations are to be done. The obvious opportunity to pinpoint the correct VaR drives the institutions to create models providing better predictions. This leaves room for research and empirical testing to conclude the superior forecasting procedure (Basel III, 2010). Having in mind the potential short term benefits of underestimating the predicted VaR, the Basel Accord Penalty Zones was developed to counteract bias in reported held risk and avoid insufficient capital coverage (Jiménez-Martín et. Al 2009).

When applying the GARCH models to financial time series the error term distribution must be assumed. When Engle (1982) first presented the ARCH model the error term (ε_t) was assumed to be normally distributed, while Bollerslev (1987) later argued for the error term to be t distributed. Also the General Error Distribution, suggested by Harvey (1981), has proven suitable for the error term in the use of GARCH models. This study uses all three different error distribution assumptions to assess the preferred conditional model for the chosen financial time series. Two different types of GARCH models are used to predict the VaR for the chosen securities. The GARCH and the GJR-GARCH. GJR-GARCH is an early development of the ARCH model. It has proven to stand the test of time while still parsimonious. It controls for eventual asymmetry in the data set with the incorporated leverage term (Teräsvirta, 2006). The data set examined consists of the closing price for five large cap equities from the OMX Nasdaq Stockholm Exchange for the time period January 1st 2007 until May 15th 2014.

Our main objective in this thesis is to answer the following question.
Which model, given the different error term assumptions, provides the most efficient VaR prediction for the examined dataset?

1.1 Earlier Research

Since the studies by Bollerslev (1986 & 1987) an extensive amount of variations of the GARCH model has been developed and tested. Today there are over 100 specifications of the GARCH model. However, earlier models, GARCH and GJR-GARCH have proven to be successful and widely used over time. Also higher order GARCH models with additional lag terms have been analysed. Engel (2002) finds GARCH (2,2) to be useful when a longer span of data is used such as hourly observations. Different distributions for the error term have been tested. Usually the normal, student-t and GED have been used. Several studies conclude that Leptokurtic distributions perform better than the Normal distribution (Angelidis et al., 2004; Engle, 2002; Orhan and Köksal, 2011). In the literature, there is no definite answer to which specific GARCH model performs the best. Extensive empirical work has been done forecasting different types of equites and markets to predict VaR. To name some of the recent studies Angelidis et al. (2004) concludes that TARCH and EGARCH perform better than the original GARCH model. Orhan and Köksal (2011) compare a comprehensive amount of GARCH models to calculate VaR. Their conclusion is that the ARCH(1) is the preferred model.

2 Theoretical framework

In financial econometrics the return is defined as,

$$r_t = 100 * ln(p_t/p_{t-1}), \quad (1)$$

where p_t is the prize at time *t* and r_t is the daily return.

2.1 GARCH models

Financial returns under the influence of heteroscedasticity is commonly represented as an autoregressive process of order one, where *u* is assumed zero and an error term is dependent on time *t* as

$$r_t = u + \varepsilon_t. \quad (2)$$

The error term can be defined consisting of two parts as

$$\varepsilon_t = \sigma_t z_t, \quad (3)$$

where z_t is an independent and identically distributed sequence with zero mean and variance one, σ_t is the conditional standard deviation in the period *t* under the assumed error distribution stated beforehand. Engle (1982) suggested the Auto Regressive Conditional Heteroscedasticity model, ARCH, as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2. \quad (4)$$

To model the variance at time *t* conditioned upon the lagged squared return. This under the assumption that α_0 and α_i is strictly positive, where *i = 1, 2, ... , q.* Bollerslev (1986) later Generalized the ARCH model adding the lagged variance according to

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \quad (5)$$

where α_0, α_i and β_j have to be strictly positive for all i and j to guarantee positive variance. The GARCH(1,1) is found if we let *q = p = 1* , for covariance stationarity we must have that $\alpha_i + \beta_j < 1$. The forecast expression for the 1-day-ahead is expressed as

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \beta_1 \sigma_t^2. \quad (6)$$

When the forecast horizon goes to infinity the conditional variance converges to the unconditional variance as (Bollerslev 1986, Taylor 1986)

$$\sigma^2 = \frac{\alpha_0}{1-(\alpha_1+\beta_1)}. \quad (7)$$

Many variations of the GARCH have been presented after its introduction but few have stood the test of time. The GJR-GARCH developed by Glosten, Jagannathan and Runkte (1993) taking the asymmetric affect of the lagged negative return into account. It is expressed as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i r_{t-i}^2 + \sum_{k=1}^{r} \gamma_k r_{t-k}^2 I_{t-k} + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \quad (8)$$

where I_{t-k} is an indicator variable taking the value one if the innovation is smaller than zero, and zero otherwise, as

$$I_t = \begin{cases} 1, if\ r_t < 0 \\ 0, otherwise \end{cases}. \quad (9)$$

The GJR-GARCH allows for asymmetric impacts of the lagged return, allowing the effect of the negative lagged returns to have a different magnitude then the positive lagged returns. This is enabled by the leverage term activated by the indicator function. The GJR-GARCH(1,1) is given by the model restricted to $q = \gamma = p = 1$, in the one day ahead forecasts as

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1 r_t^2 + \gamma_1 r_t^2 I_t + \beta_1 \sigma_t^2. \quad (10)$$

2.2 Error distributions

In order to estimate the model we need $z_t \sim N(0,1)$ as

$$f(z) = \frac{1}{\sqrt{2\pi}} exp\left\{-\frac{z^2}{2}\right\}. \quad (11)$$

Furthermore, the error term can be assumed to follow different distributions. This study will consider the Normal, Student's t and General Error distribution.

Equation 12 gives the normal probability density function as

$$f(\varepsilon) = \frac{1}{\sigma\sqrt{2\pi}} exp\left\{-\frac{1}{2}\left(\frac{\varepsilon-\mu}{\sigma}\right)^2\right\}, \quad (12)$$

where μ is the mean and σ is the standard deviation. The t distribution assumed by Bollerslev (1987) has the probability density function given by

$$f(\varepsilon) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi(v-2)\sigma_t^2}}\left(1+\frac{\varepsilon_t^2}{(v-2)\sigma_t^2}\right)^{-\left(\frac{v+1}{2}\right)} \quad v > 2, \tag{13}$$

where the lagged conditional volatility and degrees of freedom standardization $\left(\sqrt{(v-2)/v}\right)$ are introduced to the Student's t distribution. As $v \to \infty$ the t distribution converges asymptotically to the normal distribution. Harvey's (1981) General Error Distribution's (GED) is given by below, where β alters the skewness

$$f(\varepsilon) = \frac{\beta}{2\sigma\Gamma\left(\frac{1}{\beta}\right)} exp\left\{-\left(\frac{|\varepsilon-\mu|}{\sigma}\right)^{\beta}\right\}. \tag{14}$$

When $\beta = 2$ the GED distribution is symmetric and equal to the normal distribution.

3 Value at Risk

$VaR(1-\alpha)$ is defined as

$$VaR(1-\alpha) = -\sigma_t Z_\alpha. \tag{15}$$

σ_t is the conditional standard deviation at time t and Z_α is the quantile of the assumed distribution. The distribution function is either Normal, Student t or the GED. This is the upper boundary of the left tail in the assumed error distribution given the confidence level α (in this study 1 % or 5 %). The distribution determines the critical value i.e. -1,645 for 5 % in the normal distribution. Notice the negative sign since we are only interested in potential losses. This implies only positive values of VaR. $1-\alpha$ is the confidence level according to

$$\Pr\left(R_t < VaR(1-\alpha)\right) = \alpha. \tag{16}$$

α is referred to as the nominal size and this value is to be exceeded precisely $100 * \alpha$ times out of a 100 trails. For example if we want to calculate a 3 % Value-at-Risk with a 95 % confidence given a portfolio of $100.000, the portfolio would expect to lose at least $3000 (=3%*$100.000) one day out of twenty (5 %).

Each time the daily return (in absolute numbers) exceeds the VaR prediction the observation is labeled as a violation (v), using the indicator function (Equation 17). The number of violations are summed and divided with the total number of VaR predictions, which gives a proportion called the empirical size. (Orhan & Köksal, 2011)

$$J_t = \begin{cases} 1, if\ r_t < VaR\ \ (Violation) \\ 0, otherwise\ \ (Hit) \end{cases}. \tag{17}$$

To evaluate our forecasts we will use the two different tests developed by Kupiec (1995) and Christoffersen (1998), outlined in the following section.

3.1 Kupiec's test for unconditional coverage

The Kupiec test allows us to test the empirical size against the nominal size using a likelihood ratio test.

$$LR_{uc} = 2\ln\left[\left(1-\frac{v}{n}\right)^{n-v}\left(\frac{v}{n}\right)^{v}\right] - 2\ln((1-\alpha)^{n-v}\alpha^{v}), \tag{18}$$

where $LR_{uc} \sim \chi^2{}_{(1)}$ and the null hypothesis $\frac{v}{n} = \alpha$ against the alternative $\frac{v}{n} \neq \alpha$.

The more the empirical size $\left(\frac{v}{n}\right)$ deviates from the nominal size (α) the larger test statistic. A 5% significant level is used with a corresponding critical value of 3.841. A score exceeding the critical value lead to rejection of the null hypothesis, thus concluding that the model is not suitable for capturing the VaR at the nominal size. In a perfectly specified model the violations would occur with α percent probability. If the violation occurs more often in relation to the nominal level, the model underestimates the risk, implying investors take on too much risk, in comparison the their coverage. Vice versa if the violations occur less often in relation to the nominal level the model overestimates the risk. This implies that investors take on lesser risk in relation to the coverage. (Orhan & Köksal, 2011)

3.2 Christoffersen's test of independence

A drawback to the Kupiec test is its inability to take into account the sequence of violations, still assuming violations to be independent. Christofferen (1998) developed a test to consider clusters of violations when the volatility is varying and assessing how well the proposed model compensate for the sudden cluster. The purpose of this test is to examine the violations' independence. For the complete specifications regarding the needed proportions, in Equation 19 (as π s), see Christoffersen (1998).

$$LR_{ind} = -2\ln\left[(1-p)^{T-N}p^{N}\right] + 2\ln\left[(1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\right], \tag{19}$$

where $LR_{ind} \sim \chi^2_{(1)}$. The null hypothesis is that the violations are independently distributed, therefore rejecting the null implies that the violations are clustered and thus that the independence does not hold. (Christoffersen, 1998)

4 Data

The analyzed data is collected from OMXS large cap list and five equities are chosen from five different sectors (Table 1). The data comprise daily closing price spanning from January 1st 2007 until may 15th 2014 and consists of 1853 observations. During that period the market experienced periods containing volatility clustering.

Company	Sector
ABB	Industrial gods & services
Boliden	Mining, basic resources
MTG	International multimedia
SEB	Banking industry
StoraEnso	Paper & pulp, basic resources

Table 1 Description of Equities' sectors. The equities are chosen with a controlled sampling with the purpose to enable different businesses to experience different cycles and exogenous effects influencing the companies' evaluation.

Table 2 shows summary statistics for the five different equities. The high values of the Kurtosis and Jarque-Bera statistics suggest that the returns for all equities are largely deviating from a normal distribution. The returns of the equities seem to follow a leptokurtic distribution. The experienced skewness can have an effect on the model fit when comparing symmetry and leverage in the estimation model.

Company	Mean	St. D	Skewness	Kurtosis	Min	Max	Jarque-Bera
ABB	0,01	0,87	-0,073	15,458	-9,572	6,158	12121,9
BOLIDEN	-0,015	1,395	0,322	8,937	-8,313	9,342	2750,265
MTG	-0,008	1,327	-0,902	14,867	-12,223	7,058	10812,98
SEB	-0,0213	1,403	-2,333	48,112	-23,006	10,083	158635,2
StoraEnso	-0,011	1,0855	0,149	6,271	-5,595	6,488	830,9085

Table 2 Descriptive statistics for the five equities. It is clear that the data is leptokurtostic and somewhat skewed, confirmed in the Jarque-Bera test on normality (all rejected). Mean is about zero and the standard deviation is varying depending on both the sector and the individual equity.

Looking at the plots in Figure 1 the returns data clearly shows volatility clustering and the bell-shaped returns succeeding a larger shock. All series indicates a high volatility period in 2009 and 2010, as well as in 2012.

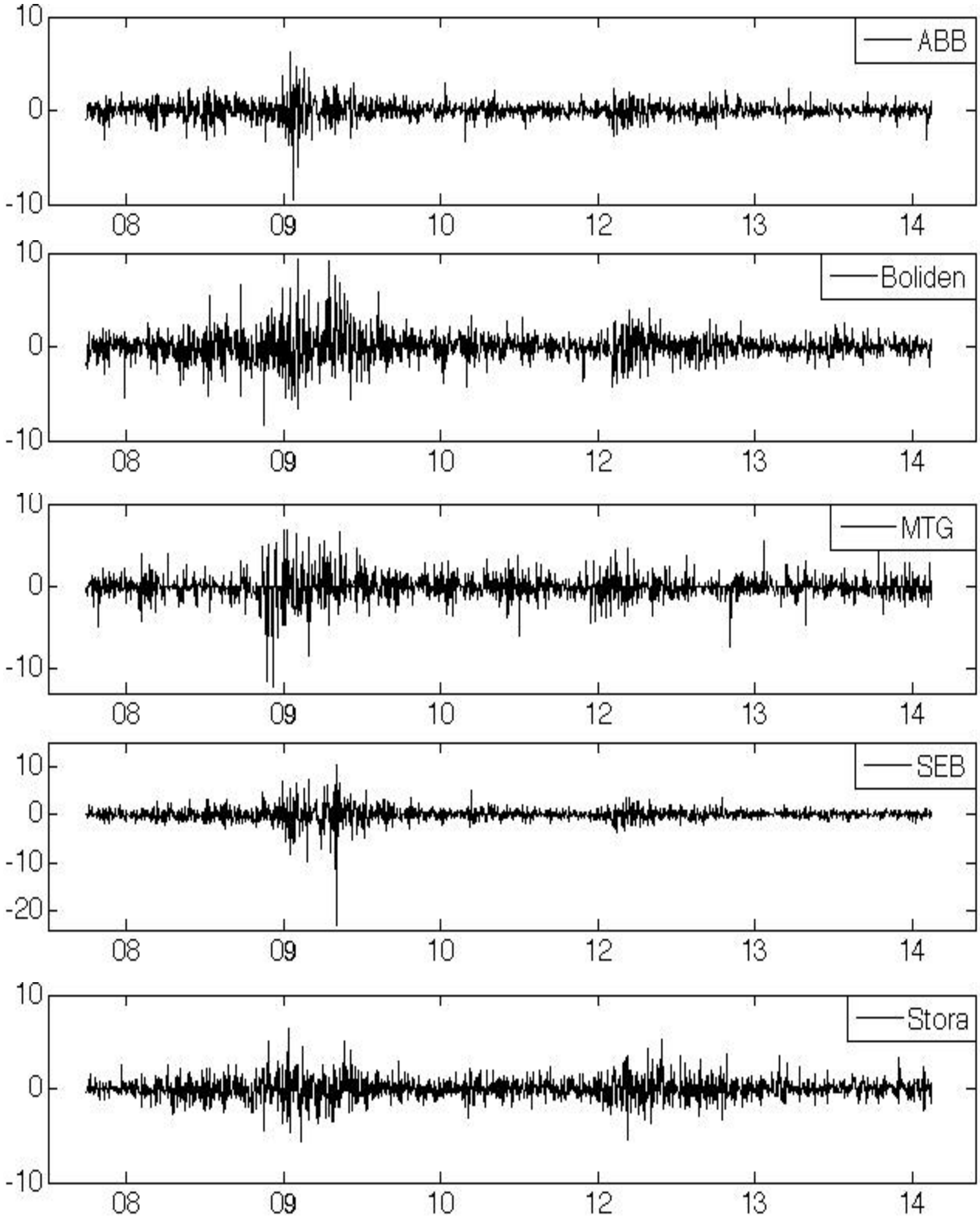

Figure 1 Illustrating returns exhibiting heteroscedasticity and the bell shaped clusters of volatility varying in different periods. The returns also indicate zero mean and stationary. The five described data series all experience more volatile periods around 2008 – 2009 and less volatile periods around 2012, depending on sector and individual events.

5 Methodology

A 1 day-ahead out of sample forecast is used to estimate the GARCH models and calculate the VaR. The 1-day ahead forecast is obtained by setting a window of 500 observations to use as a rolling forecast. These are the previous 500 observations in the dataset for each stock consisting of 1853 observations. At time *t* a forecast is made for time *t+1* using the first 500 observations. VaR is then calculated and the observation is classified as a violation or not. At time *t+1* the same procedure is repeated but with the *t-499th* observation dropped and observation *t+1* added. Our window moves one step for each observation until the end of the dataset. This process has been set up using Matlab version 2013b "Toolbox MFE toolbox", as for the empirical VaR estimations we decided to use a 95% and 99% confidence level. As the rolling forecast window reaches the end of the dataset, the number of violations is summarized and the result evaluated by using Kupiec's test for unconditional coverage and Cristoffersen's test for conditional coverage.

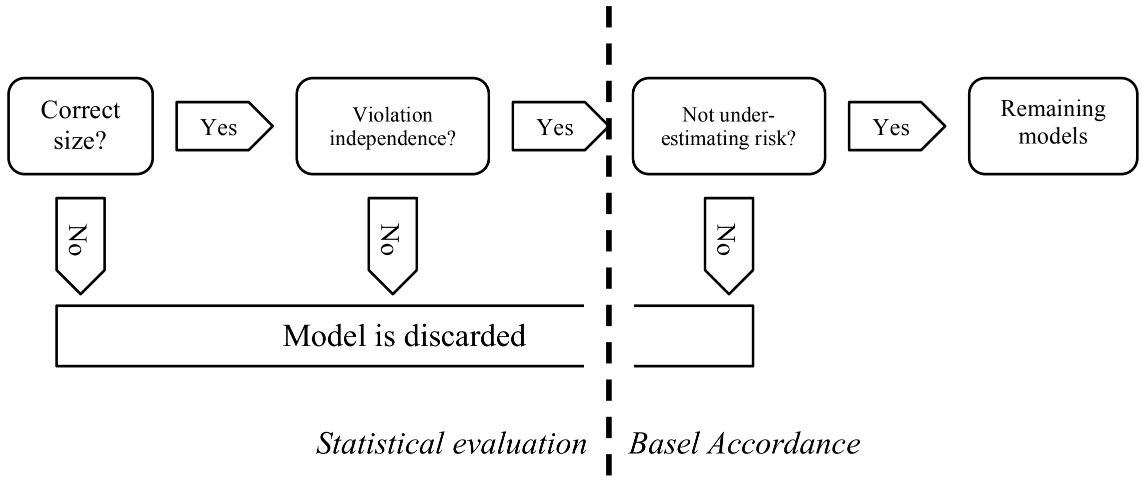

Figure 2 Flowchart of the test procedure for the estimated models to determination of the preferred models

6 Results

In this section the results and analysis is presented. For each tested series a rolling forecast has been made using the procedure described in section 5. All estimated models are tested using Kupiec's test for unconditional coverage, testing the empirical size in relation to the nominal size. Models with an empirical size significantly different from the nominal size are dropped. The remaining models are tested using Christoffersen's test of independence. Estimated models rejecting independence are discarded, leaving only the models containing independent. The models unable to reject the size or independence are the statistically efficient models predicting the VaR. Presented in Table 3 for each tested series over the given level of confidence.

Company	1%	5%
ABB	GARCH -N -t -GED	GARCH -t, GJR -N -t
Boliden	GARCH -t -GED	GJR -N -t -GED
MTG	GARCH -N-GED	GARCH -GED GJR -N -GED
SEB	GARCH-t -GED	all except GARCH-t
Stora Enso	GARCH -N -t -GED	all except GJR -N

Table 3: The most précis model or models are presented with the assumed error distribution under the specified confidence level. The Kupiec likelihood ratio test is used to discard statistically differing empirical size models. The Christoffersen test of independence is non-significant for all but four of the presented models. Those four models are discarded.

The statistically efficient models are separated with respect to risk aversion in line with the Basel Accord Penalty Zones. The underestimating models are dropped (where the empirical size is larger than the nominal size), as the potential lesser capital coverage is conflicting the Basel committee's mission of financial stability. The models overestimating the risk are promoted and presented in Table 4.

Company	1%	5%
ABB	GARCH -t -GED	GARCH -t, GJR -N -t
Boliden	GARCH -t -GED*	GJR -N -t -GED
MTG	GARCH -N -GED	GARCH -GED, GJR -N
SEB	GARCH -t	GARCH -N -t -GED
Stora Enso	GARCH -t -GED	all execpt GJR -N

Table 4: Selection of the preferred models based on a risk avert approach promoted in the literature of the Basel accords. The risk underestimating models are here discarded.
**** In the case of 1%–Boliden the models that withstood the evaluation test were both underestimating the risk.***

Table 4 presents the main results. The multiple confirmed models arise from the fact that the Kupiec test was not rejected in several cases. The Christoffersen test was only able to reject a small number of statistics among the models selected from the Kupiec test[1]. There are known bad small sample properties (regarding the small number of violations) in the independence test that a larger sample could have overcome.

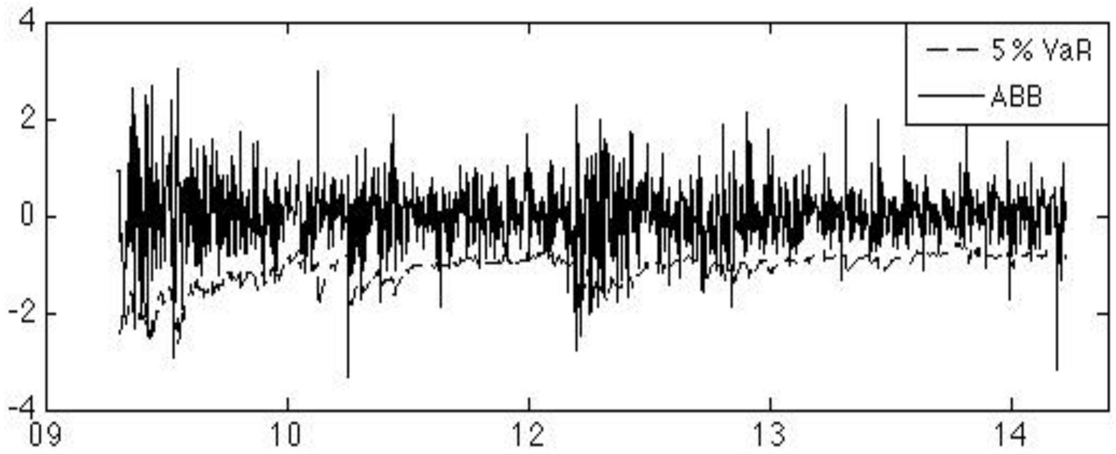

Figure 3 Illustrating example of the graphical result predicting 5 % VaR using GARCH-t for ABB. The broken line is the predicted VaR. The plot starts at time t+499 because of the rolling window estimating the GARCH. The empirical size is calculated to 3,84 % overestimating the nominal 5 % and rejecting the null hypothesis.

Filtering the underestimating models there are still several cases of multiple preferred choices to predict the equities. For the 1% nominal size, the GARCH model outperforms the GJR in large but under varying distributions, the GARCH-t and GARCH-GED are the preferred models for four out of five equities. Regarding the results covering the 5% nominal size there is no clear trend where one of the models or distributions are preferred, or the combination of the two, over the others.

7 Conclusions

The purpose of this paper has been to evaluate different GARCH models predicting VaR under different distributional assumptions for five picked equities from the OMX Nasdaq Stockholm Exchange. The data set ranges from January 1st 2007 to May 15th 2014 and contains both high and low volatile periods. The study compared GARCH and GJR-GARCH, under the Normal, Student's t and General Error Distribution assumptions predicting Value-at-Risk (1% and 5% nominal size) using a rolling window of 500 observations. For the 1% the GARCH was preferred over the GJR-GARCH under the Student's t and GED distributional assumption. Regarding the 5% nominal size the result is inconclusive since no trend is found in the results. For the

[1] For test statistics and p-values of the Kupiec and Christoffersen tests, consult the Appendix.

five tested equities there are multiple preferred models under varying distributional assumptions. Interesting to note is the lack of difference between the GARCH and GJR-GARCH, thus proposing that controlling for leverage is not an advantage. In the 1% case the results indicate that the leverage accounted for using the GJR-GARCH does not yield an advantage. The take from this study is that there is no overall preferred model and distributional assumptions. Being able to simulate the most efficient fit from a range of models under different assumptions is the preferred strategy to a successful risk management team.

7.1 Recommendations for further studies

This study only covers a handful of cases and assumptions. Expanding the range of statistical forecasting model and model assumptions, prolonging the forecasting range and controlling for the expected shortfall (extreme value theory) on a wider range of financial time series is our recommendations for the continued research.

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9 Appendix

Estimation results presented over company and estimated model. These are the results we focused on when comparing and confirming the preferred model for each tested equity.

	GARCH					
ABB	Normal		Student's t		GED	
Nominal size	1%	5%	**1%**	**5%**	**1%**	5%
Empirical size	1,18%	3,84%	**0,74%**	**4,51%**	**0,74%**	3,84%
Kupiec	0,43	4,13	**1,02**	**0,71**	**1,02**	4,13
	(0,51)	(0,04)	**(0,31)**	**(0,40)**	**(0,31)**	(0,04)
Christoffersen	0,00	0,47	**0,00**	**0,02**	**0,00**	0,47
	(1,00)	(0,49)	**(1,00)**	**(0,88)**	**(1,00)**	(0,49)

Table 5 GARCH for ABB. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

	GJR-GARCH					
ABB	Normal		Student's t		GED	
Nominal size	1%	**5%**	1%	**5%**	1%	5%
Empirical size	1,30%	**4,07%**	0,96%	**4,58%**	0,96%	3,91%
Kupiec	53,53	**2,65**	68,71	**0,51**	68,71	4,13
	(0,00)	**(0,103)**	(0,00)	**(0,475)**	(0,00)	(0,042)
Christoffersen	0,00	**1,24**	0,00	**0,46**	0,00	1,71
	(1,00)	**(0,26)**	(1,00)	**(0,50)**	(1,00)	(0,19)

Table 6 GJR-GARCH for ABB. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

	GARCH					
Boliden	Normal		Student's t		GED	
Nominal size	1%	5%	**1%**	5%	**1%**	5%
Empirical size	1,62%	4,06%	**1,33%**	4,58%	**1,33%**	4,06%
Kupiec	4,49	2,67	**1,34**	0,52	**1,34**	2,67
	(0,03)	(0,10)	**(0,25)**	(0,47)	**(0,27)**	(0,10)
Christoffersen	3,86	4,88	**1,40**	4,92	**1,40**	4,88
	(0,05)	(0,03)	**(0,24)**	(0,03)	**(0,24)**	(0,03)

Table 7 GARCH for Boliden. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GJR-GARCH						
Boliden	Normal		Student's t		GED	
Nominal size	1%	**5%**	1%	**5%**	1%	**5%**
Empirical size	1,62%	**4,28%**	1,33%	**4,95%**	1,40%	**4,28%**
Kupiec	43,55	**1,53**	53,61	**0,01**	50,94	**1,53**
	(0,00)	**(0,22)**	(0,00)	**(0,93)**	(0,00)	**(0,22)**
Christoffersen	0,81	**0,86**	1,40	**0,15**	1,24	**0,86**
	(0,37)	**(0,35)**	(0,24)	**(0,70)**	(0,27)	**(0,35)**

Table 8 GJR-GARCH for Boliden. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GARCH						
MTG	Normal		Student's t		GED	
Nominal size	**1%**	5%	1%	5%	**1%**	**5%**
Empirical size	**1,55%**	3,55%	2,58%	15,44%	**1,18%**	**4,51%**
Kupiec	**3,55**	6,69	23,90	204,71	**0,43**	**0,72**
	(0,06)	(0,01)	(0,00)	(0,00)	**(0,51)**	**(0,40)**
Christoffersen	**0,94**	0,33	1,05	0,44	**1,79**	**0,22**
	(0,33)	(0,56)	(0,30)	(0,51)	**(0,18)**	**(0,64)**

Table 9 GARCH for MTG. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GJR-GARCH						
MTG	Normal		Student's t		GED	
Nominal size	1%	**5%**	1%	5%	1%	5%
Empirical size	1,70%	**3,99%**	2,36%	13,29%	1,55%	5,47%
Kupiec	39,09	**3,13**	24,42	137,54	45,84	0,61
	(0,00)	**(0,08)**	(0,00)	(0,00)	(0,00)	(0,44)
Christoffersen	0,59	**0,78**	0,08	0,84	0,94	1,34
	(0,44	**(0,38)**	(0,78)	(0,36)	(0,33)	(0,25)

Table 10 GJR-GARCH for MTG. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GARCH						
SEB	Normal		Student's t		GED	
Nominal size	1%	**5%**	**1%**	5%	1%	**5%**
Empirical size	1,85%	**4,95%**	**0,96%**	4,95%	1,11%	**4,87%**
Kupiec	7,84	**0,01**	**0,02**	0,01	0,15	**0,05**
	(0,01)	**(0,93)**	**(0,88)**	(0,93)	(0,70)	**(0,83)**
Christoffersen	0,49	**1,98**	**2,53**	7,67	2,02	**2,17**
	(0,48)	**(0,16)**	**(0,11)**	(0,01)	(0,16)	**(0,14)**

Table 11 GARCH for SEB. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GJR-GARCH						
SEB	Normal		Student's t		GED	
Nominal size	1%	5%	1%	5%	1%	5%
Empirical size	1,85%	5,47%	0,81%	5,61%	0,81%	5,03%
Kupiec	36,99	0,60	75,89	1,03	75,80	0,00
p-value	(0,00)	(0,44)	(0,00)	(0,31)	(0,00)	(0,97)
Christoffersen	0,00	0,93	0,00	0,71	0,00	0,72
p-value	(1,00)	(0,33)	(1,00)	(0,40)	(1,00)	(0,40)

Table 12 GJR-GARCH for SEB. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GARCH						
StoraEnso	Normal		Student's t		GED	
Nominal size	**1%**	**5%**	**1%**	**5%**	**1%**	**5%**
Empirical size	**1,33%**	**4,43%**	**0,74%**	**4,73%**	**0,67%**	**4,21%**
Kupiec	**1,34**	**0,96**	**1,03**	**0,22**	**1,74**	**1,88**
	(0,25)	**(0,33)**	**(0,31)**	**(0,64)**	**(0,19)**	**(0,17)**
Christoffersen	**0,00**	**0,20**	**0,00**	**0,43**	**0,00**	**1,12**
	(1,00)	**(0,66)**	**(1,00)**	**(0,51)**	**(1,00)**	**(0,29)**

Table 13 GARCH for StoraEnso. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.

GJR-GARCH						
StoraEnso	Normal		Student's t		GED	
Nominal size	**1%**	5%	**1%**	**5%**	**1%**	**5%**
Empirical size	**1,40%**	3,84%	**0,81%**	**4,36%**	**0,74%**	**3,92%**
Kupiec	**50,94**	4,15	**75,89**	**1,21**	**79,61**	**3,60**
	(0,00)	(0,04)	**(0,00)**	**(0,27)**	**(0,00)**	**(0,06)**
Christoffersen	**0,00**	0,00	**0,00**	**0,00**	**0,00**	**0,00**
	(1,00)	(1,00)	**(1,00)**	**(1,00)**	**(1,00)**	**(1,00)**

Table 14 GJR-GARCH for StoraEnso. Kupiec & Christoffersen statistics and p-values included. The preferred models according to table four are highlighted in Bold.