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Informative Data for Model Parameter Updating

Andreas Linderholt¹, Thomas Abrahamsson¹ and Nam Hoang²

1) Department of Solid Mechanics, Chalmers University of Technology, S-412 96 Göteborg, Sweden
2) Structural Engineering Program, School of Civil Engineering, Asian Institute of Technology,
    P.O. Box 4 Klong Luang, Pathumthani 12120, Thailand

ABSTRACT

Before an error localization is to be carried out a preparatory error localization, using only data from an FE-analysis, is justified. The purpose of such preparatory work is both to decide what parameters to use to quantify model errors and to design optimal tests for the error localization. A reasonable requirement on the parameterization is that the test data are informative with respect to the parameters. That implies that a change of a certain parameter should give a detectable change in the model’s dynamic behaviour.

The aim of this study is to examine data informativity with respect to physical parameters, used in error localization and model updating. The data informativity is here quantified by the use of the Fisher information matrix. It is shown that the informativity depends on both excitation and measurements. It is reasonable to believe that parameters of which test data have low informativity, are of no use for error localization and should not be used for model updating purposes. Such parameters should be excluded from the parameters set or the test aimed for its determination should be re-designed.

1 INTRODUCTION

In recent years, finite element (FE) model updating has been an area of active research. This relates to the requirements of valid models for predictions and parametric studies. Most FE-models of practical use today involve such a multitude of physical parameters, possibly in error, such that updating them all is impractical. Thus one has to select, from a large set of parameters, those that are considered to be most prone to errors. Such selection procedure, in which candidates for error localization are identified, should be based on theoretical and engineering insight. To assess the likelihood of obtaining the distribution and quantification of model errors from given test data, the parameter identifiability and experimental data informativity should be considered. In a pre-test planning phase, the vibration test should be designed as to make likely that the test data informativity will be at hand for the candidate set of parameters when test data later will become available. A similar requirement for parameter identifiability has been treated earlier[¹, ²].

Should the test data be non-informative with respect to a certain parameter, a model updating could result in an indifferent solution of that parameter. One assumption that seems reasonable, is that a model parameter perturbation should result in a noticeable change in such measurable quantities that are informative with respect to that parameter. Hence, the data informativity could be assessed in a preparatory study. Two possibilities exist if the test data suffers from lack of informativity of a parameter. The first one is to re-parameterize, e.g. by omitting parameters or by clustering
groups of initial parameters. The second possibility is to re-design the test, e.g. move or add sensors, move or add actuators or change the excitation(s).

2 INFORMATIVE DATA

The Fisher Information matrix (FIM)\textsuperscript{3,4} plays an important role in the theory and practice of statistical estimation. It is the theory of the Maximum Likelihood estimation\textsuperscript{4} that is underlying this concept of estimating model parameters. The FIM quantifies the amount of information that is present in a noisy measurement with regards to model parameters. The inverse of the FIM, known as the Cramer-Rao bound, establishes a lower bound on the error covariance matrix for any unbiased estimator of the parameters.

Consider a model for which the measurement at time $t$ are taken

$$y_t = x_t(\theta) + v_t, \quad t = 1, 2, ..., N$$

(1)

where $\theta$ is a $p$-dimensional vector of deterministic but unknown parameters. The noise contribution is denoted by $v_t$. The noiseless measurement $x_t$ is a vector function of $\theta$ and the system’s input. The vector $y_t$ consists of $M$ independent system measurements.

Assume that the noise contribution $v_t$ is an independent Gaussian sequence with zero mean and a diagonal covariance matrix $R$ equal to $\sigma^2 I$. As a result, the measurement $y_t$ is a sequence of independent Gaussian distribution random variables having the mean value vector $x_t(\theta)$ and a covariance matrix $R$ which stems from the noise. The joint probability density function for a random sample $y = (y_1, y_2, ..., y_N)$ is then\textsuperscript{3}

$$f_\theta(y) = \prod_{t=1}^{N} f_\theta(y_t) = (2\pi)^{-MN/2} |R|^{-N/2} \times$$

$$\exp(-\frac{1}{2} \sum_{t=1}^{N} [y_t - x_t(\theta)]^T R^{-1} [y_t - x_t(\theta)])$$

(2)

The Fisher information matrix associated to this distribution is defined to be \textsuperscript{4}

$$J(\theta) = E \left[ \frac{\partial}{\partial \theta} \ln f_\theta(y) \right] \left[ \frac{\partial}{\partial \theta} \ln f_\theta(y) \right]^T$$

(3)

in which $E$ denotes the expectation.

When measured data do not contain any information about certain parameters, it indicates that those parameters can be varied without changing the measured transfer function of the system. By Fisher formula (3), the entries of the FIM are expressed in terms of the partial derivatives of the density function of the system’s output. This shows that local non-informativity will lead to singularity of the FIM.

For the noise model here considered (i.e. $R = \sigma^2 I$), the Fisher information matrix becomes

$$J(\theta) = \frac{1}{\sigma^2} \sum_{t=1}^{N} \frac{\partial x_t^T}{\partial \theta} \frac{\partial x_t}{\partial \theta}$$

(4)

From a noisy measurement $y$, an estimate of the unknown parameters $\hat{\theta}$ can be made. Such estimate is here denoted $\hat{\theta}$. Suppose that this estimate is unbiased, i.e. its mean value converges to the exact value. Then its error covariance matrix is lower bounded by the inverse of the FIM

$$E[\hat{\theta} - \theta] [\hat{\theta} - \theta]^T \geq J^{-1}(\theta)$$

(5)

The right-hand side of eq. (5) is the Cramer-Rao (CR) lower bound. The existence of such bounds implies that irrespective of the method used to quantify the parameters from the data, there is a lower bound on the precision that cannot be overcome. It is noted that equation (4) contains the model parameters. Thus, the calculated FIM or the CR bound associated with the estimated parameters will only be reliable if the values of these parameters are not deviating too much from their true values. However, it is assumed that the parameter values of an FE-model, modelled by a skilled engi-
neer with insight into the structure, are close to correct.

The analytical calculation of the Fisher matrix is often difficult or impossible in any nonlinear problem. In this study, it suffices to calculate the Fisher matrix numerically. The calculation is illustrated by two examples. The first concerns a simple 2-dof system. The second treats a multi-degree-of-freedom plate structure.

3 A 2-DOF EXAMPLE

A two-degree-of-freedom (2-dof) structure, see Figure 1, is chosen to show the concept of data informativity with respect to physical parameters. The masses $m_1$ and $m_2$ together with the stiffnesses $k_1$ and $k_2$ constitute the set of physical parameters of the system. In this example, the exact parameters of the structure are given as $m_1=m_2=1$ kg, $k_1=100$ N/m and $k_2=1$ N/m. These numerical values ensure a significant difference in response of the two masses.

Now suppose that the masses $m_1$ and $m_2$ are unknown parameters. When the rightmost mass of the structure is excited, its response can be measured. From the noisy measurement, these unknown masses will be identified. The estimator is here assumed to yield the exact values. However, in the context of this paper, the question whether the measured data are informative to parameters to be estimated is just concerned. It may be answered by evaluating the FIM.

The response measurement of the structure is assumed to follow the model given by equation (1), that is contaminated by a zero mean, unit variance ($\sigma = 1$) Gaussian white noise sequence. The Fisher information matrix of the measurement for two unknown masses is thus obtained from the equation (4). Two cases of excitation of the same duration of 10 s are considered: (a) a transient sinusoidal function of frequency $\pi$ rad/s with 0.002 s time step and (b) a sample of Gaussian random white noise with zero mean, a variance of 0.5 N$^2$, time step of 0.1 s and low pass filtered (Butterworth filter) with cutoff frequency at $2\pi$ rad/s. Both analyses start with homogeneous initial conditions.

Table 1 shows Fisher information numbers, which are diagonal elements of the FIM corresponding to parameters of interest, for different kind of measured data: acceleration ($J_a$), velocity ($J_v$) and displacement ($J_d$) responses. From this table, it is obviously deduced that the measurement is very informative to the mass $m_2$ relative to the other mass $m_1$. This seems reasonable since the measurement is carried out at the rightmost mass and the excitation frequency in both cases are far from the natural frequency of the left spring-mass subsystem. Hence the Fisher information matrix shows to be a very effective index for the informativity of data to the parameters in question. In addition, in this example, the Fisher information numbers are most sensitive to acceleration data.

Another way to interpret the informativity of observed data for physical parameters used in model updating is described in the following. Assume that the model is to be updated by use of the response to the given harmonic loading with accelerations measured at $m_2$. Here, experimental data are mimicked by the output of the nominal model, $Y_A^{\text{nom}}$ combined with Gaussian white noise according to

Table 1. Fisher information numbers for two mass parameters of the 2-dof structure. a) Harmonic excitation, b) Random excitation

<table>
<thead>
<tr>
<th></th>
<th>$J_a$</th>
<th>$J_v$</th>
<th>$J_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $m_1$</td>
<td>150x10^{-5}</td>
<td>3.98x10^{-5}</td>
<td>1.50x10^{-5}</td>
</tr>
<tr>
<td></td>
<td>5.33x10^{-3}</td>
<td>2.33x10^{-3}</td>
<td>1.46x10^{-3}</td>
</tr>
<tr>
<td>b) $m_1$</td>
<td>279x10^{-3}</td>
<td>3.90x10^{-3}</td>
<td>1.20x10^{-3}</td>
</tr>
<tr>
<td></td>
<td>1.16x10^{5}</td>
<td>1.15x10^{5}</td>
<td>1.16x10^{5}</td>
</tr>
</tbody>
</table>

Figure 1. Two-dof structure
\[ Y_X = Y_A^{\text{nom}} + \nu \]

The noise \( \nu \) has a zero mean, a unit variance and a magnitude such that the noise to signal ratio becomes less than or equal to five percent.

Analytical data \( Y_A \) are noise-free accelerations at \( m_2 \) calculated from a model that differs from the nominal in the way that one parameter at the time is treated as a variable. The value of the variable that minimizes \( (Y_A - Y_X) \) is calculated by using a constrained minimization procedure. The variable is allowed to vary between zero and 200 percent of its nominal value. The minimization is repeated 500 times, each time with a different random noise realization \( v \). Firstly, the mass \( m_1 \) is considered the only parameter. The distribution, grouped in 20 bins, is shown in Figure 2a. The mean value and the variance of the estimates become equal to 0.92 kg and 0.59 kg\(^2\) respectively. Secondly, the mass \( m_2 \) is allowed to vary. The mean value of the 500 estimates becomes equal to 1.00 kg while the variance equals \( 236 \times 10^{-6} \) kg\(^2\). The distribution is shown in Figure 2b. This clearly shows that estimates from data that suffer from lack of informativity of the parameters in question, are of no use. Hence, such parameters should not be included in an error localization or a model updating.

4 MULTIPLE DOF EXAMPLE

We now turn our interest into a multiple degree-of-freedom (mdof) problem. The structure, shown in Figure 3, consists of two steel plates connected to each other by a brass string. The plates’ dimensions are 220 x 420 x 1 mm and 200 x 400 x 1 mm, respectively. The brass connection is 60 x 20 x 0.2 mm. The structure is modelled with free boundaries and by use of plate elements. The Matlab based FE program Calfem\(^5\) was used. The dynamics of the two plates are weakly coupled. The first eight flexible body eigenfrequencies are listed in Table 2.

Two cases of excitation, both applied to the left plate according to Table 3, are examined. Discrete nodal weights at three different locations according to Table 3, are chosen to be the physical parameters noted as \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \). The transverse accelerations of all 200 inner nodes of the left plate are considered measurement quantities.

The first case is a random load which consists of 4096 samples 5\( \times 10^{-4} \) s apart of uniform distribution. The load components above 500 Hz are suppressed by low-pass filtering. The Fisher numbers for the three parameters, are given in Table 4.

![Figure 2. Distribution of parameter estimates of 2-dof structure shown in Figure 1. Note the different abscissa scales. a) parameter \( m_1 \), b) parameter \( m_2 \).](image)

![Figure 3. Mesh of mdof structure consisting of two steel plates connected with a brass string. Weight and exciter locations are shown.](image)
From the computed Fisher numbers, it is found that parameter $\theta_1$ and $\theta_3$ have strong impact on the measured data. Hence real measured data, when available, will be informative with respect to these parameters. The test data contains less information about the second parameter. This shows that the locations of accelerometers, or other measurement equipment, in relation to the location of the excitation are of great importance for the data informativity.

In the second case, a transient sinusoidal excitation (at frequency 79.32 Hz) is used to strongly excite the eighth flexible mode of the plate assembly (Table 2). This is the vibration mode which exposes distinguishable uncoupling between the two plates, as depicted in Figure 4. The Fisher numbers for this load case are displayed in Table 4.

From this harmonic load case it is found that the test data are informative with respect to the first parameter and much less informative with respect to the second parameter. The informativity is smaller for parameter number three than for parameter one. This is due to the sinusoidal load used here, which hardly excite the right plate (see Figure 4). This shows that also the type of excitation is important for the data informativity.

## 5 CONCLUSIONS

The data informativity can be quantified by the use of Fisher information matrix. It is considered useless to try to locate errors of parameters for which

Table 2. First eight flexible body eigenfrequencies of the plate assembly.

<table>
<thead>
<tr>
<th>No.</th>
<th>$f$ (Hz)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.38</td>
<td>Mostly torsion of brass string</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>Mostly bending of brass string</td>
</tr>
<tr>
<td>3</td>
<td>6.92</td>
<td>Mostly bending of brass string</td>
</tr>
<tr>
<td>4</td>
<td>30.50</td>
<td>Mostly bending of leftmost plate</td>
</tr>
<tr>
<td>5</td>
<td>33.55</td>
<td>Mostly bending of rightmost plate</td>
</tr>
<tr>
<td>6</td>
<td>36.16</td>
<td>Mostly torsion of leftmost plate</td>
</tr>
<tr>
<td>7</td>
<td>41.67</td>
<td>Mostly torsion of rightmost plate</td>
</tr>
<tr>
<td>8</td>
<td>79.32</td>
<td>Bending and torsion of leftmost plate</td>
</tr>
</tbody>
</table>

Table 3. Co-ordinates of excitation and mass parameters of the plate structure.

<table>
<thead>
<tr>
<th>$\theta_1$ and exciter</th>
<th>$x$ (mm)</th>
<th>$y$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>300</td>
<td>380</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>300</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 4. Fisher Information numbers for $\theta_1$, $\theta_2$ and $\theta_3$ of the plate structure.

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Parameter</th>
<th>$J_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td>$\theta_1$</td>
<td>1.79x10$^9$</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$</td>
<td>3.08x10$^5$</td>
</tr>
<tr>
<td></td>
<td>$\theta_3$</td>
<td>5.16x10$^8$</td>
</tr>
<tr>
<td>Harmonic</td>
<td>$\theta_1$</td>
<td>2.78x10$^7$</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>$\theta_3$</td>
<td>5.93x10$^4$</td>
</tr>
</tbody>
</table>

Figure 4. The eighth flexible mode of the plate assembly
the test data are not informative enough. Thus, if it is found that the data informativity is low, one is left with two possibilities. One possibility is to re-design the test to get complementary information. The other is to re-parameterize the model.

6 REFERENCES


