Vibration Frequencies as Status Indicators for Tensegrity Structures

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Licentiate Thesis in Engineering Mechanics

June 2014
Technical Reports from
Royal Institute of Technology
Department of Mechanics
SE-100 44 Stockholm, Sweden

Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan i Stockholm framlägges till offentlig granskning för avläggande av teknologie licentiatexamen fredag den 13 juni 2014 kl 10.15 i sal E31, Lindstedtsvägen 3, Kungliga Tekniska Högskolan, Vallhallavägen 79, Stockholm.

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Abstract

Applications of vibration structural health monitoring (VHM) techniques are increasing rapidly. This is because of the advances in sensors and instrumentation during the last decades. VHM uses the vibration properties to evaluate many civil structures during the design steps, building steps and service life.

The stiffness and frequencies of tensegrity structures are primarily related to the level of pre-stress. The present work investigates the possibilities to use this relation in designing, constructing and evaluating the tensegrity structures.

The first part of the present work studies the improvement of current models for resonance frequency simulation of tensegrities by introducing the bending behaviour of all components, and by a one-way coupling between the axial force and the stiffness. From this, both local and global vibration modes are obtained. The resonance frequencies are seen as non-linearly dependent on the pre-stress level in the structure, thereby giving a basis for diagnosis of structural conditions from measured frequencies. The new aspects of tensegrity simulations are shown for simple, plane structures but the basic methods are easily used also for more complex structures.

In the second part, the environmental temperature effects on vibration properties of tensegrity structures have been investigated, considering primarily seasonal temperature differences (uniform temperature differences). Changes in dynamic characteristics due to temperature variations were compared with the changes due to decreasing pre-tension in one of the cables. In general, it is shown that the change in structural frequencies made by temperature changes could be equivalent to the change made by damage (slacking). Different combinations of materials used and boundary conditions are also investigated. These are shown to have a significant impact on the pre-stress level and the natural frequencies of the tensegrity structures when the environment temperature is changed.

Descriptors: Tensegrity, Pre-stress, Vibration, Health monitoring, Buckling, Temperature effect, Vibration health monitoring VHM

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Sammanfattning

Användandet av vibrationsbaserade hälsokontrollsmetoder (VHM) för strukturer ökar snabbt. Detta har möjliggjorts av utvecklingen inom mätmetoder och mätutrustning under de senaste decennierna. Dessa metoder använder sig av de uppmätta eller simulerade vibrationsegenskaperna under design-, uppbyggnads- och nyttjandestadierna hos många slag av byggnadsverk.

Styvheten och resonansfrekvenserna hos tensegritets-strukturer är i hög grad beroende på den aktuella förspänningsnivån. Föreliggande arbete undersöker möjligheterna att använda detta beroende i konstruktion, byggande och utvärdering av sådana strukturer.

Den första delen av föreliggande arbete studerar förbättringar av de vanligen använda modellerna för simulering av resonansfrekvenser hos tensegritetsstrukturer genom att införa de ingående komponenternas böjningsegenskaper, och genom att i en riktning koppla normalkraften till böjstyvheten. Genom detta kan såväl lokala som globala vibrationsmoder hittas. Resonansfrekvenserna ses därmed som ickelinjärt beroende av förspänningsnivån i strukturen. Detta ger därmed möjligheter att diagnosticera strukturens kondition från uppmätta frekvenser. De nya simuleringsmöjligheterna demonstreras för enkla, plana strukturer, men de utvecklade metoderna kan också lätt anpassas till mera komplexa fall.

Den andra delen av arbetet undersöker hur strukturernas vibrationsegenskaper är beroende på temperatureffekter i omgivningen. I första hand beaktas säsongsvisa (likformiga) temperaturvariationer. Förändringar i de dynamiska egenskaperna beroende på temperaturförändringar jämfördes med dem som beror på en minskande förspänning hos någon av de ingående kablarna. I allmänhet gäller att förändringarna i resonansfrekvenser kan vara av samma storleksordning som de som beror på skador (minskad förspänning). Olika kombinationer av material, och olika upplagsförhållanden undersöktes. Dessa egenskaper visades ha en betydande effekt på förspänningsnivån, och därmed också på resonansfrekvenserna, hos tensegritetsstrukturerna som utsätts för temperaturvariationer.

Nyckelord: Tensegritet, Förspänning, Vibration, Hälsokontroll, Knäckning, Temperatureffekt, Vibrationsbaserad hälsokontroll

Preface

This thesis improves the current models for resonance frequency simulation of tensegrities by introducing the bending behavior of all components. Thereby it gives the basics for vibration health monitoring of tensegrity structures. A brief introduction to the basic concepts and methods is presented in the first part. The second part contains two articles. The papers are adjusted to conform with the present thesis format for consistency, but their contents have not been changed as compared with their original counterparts.

Paper 1. NASSERADEEN ASHWEAR & ANDERS ERIKSSON, 2014
Natural frequencies describe the pre-stress in tensegrity structures. *Published on-line in Computers & Structures.* doi:10.1016/j.compstruc.2014.01.020.

Paper 2. NASSERADEEN ASHWEAR & ANDERS ERIKSSON, 2014
Influence of Temperature on the Vibration Properties of Tensegrity Structures. *Submitted manuscript.*

Division of work among authors

This project was first introduced by Anders Erikson (AE) who is the main supervisor of this work and Dr. Gunnar Tibert (GT) who has functioned as assistant supervisor. Nasseradeen Ashwear (NA) has discussed the results and progress with AE in regular weekly meetings, and occasionally with GT and Dr. Seif Dalilsafaei (SD).

Paper 1

NA built up the numerical model in MATLAB and performed the simulations with weekly discussions with AE and feedback from GT. The paper was written by NA with help from AE and input from GT and SD.

Paper 2

NA built up the numerical model in MATLAB and performed the simulations with feedback from AE. The paper was written by NA with help from AE and input from GT.

Contents

Part I

Overview

CHAPTER 1

Introduction

1.1. Background

Tensegrities are highly sophisticated load-carrying structures, where high mechanical efficiency is created from a clear distinction between components in compression and in tension. They thereby consist of dedicated elements, compressive bars and tensioned cables (Wroldsen 2007). There is a dispute about the invention and origin of the ideas behind class of structures, but it is now known that Buckminster Fuller and Kenneth Snelson are the inventors of the concept 'tensegrity' (Snelson 2012).

1.1.1. *Definitions*

In literature there are several definitions of tensegrity structures, of which a few will be included here.

The first article ever using the term tensegrity was written by Fuller (1961), but he did not give any definition in specific words. Later, he defined tensegrity structure as: *Islands of compression in a sea of tension* (Fuller 1975).

Skelton *et al.* (2001) define a K class tensegrity structure as *A stable structure composed of tension and compression elements with a maximum of K compressive members connected at node(s).*

Depending on the author's perspective, further and more complex definitions were given. Kanchanasaratool & Williamson (2002) state that *A tensegrity system is a stable connection of axially-loaded members. A Class k tensegrity structure is one in which at most k compressive members are connected to any node.*

Motro (2003) gave two definitions, the "patent based"; *Tensegrity systems are spatial reticulate systems in a state of self-stress. All their elements have a straight middle fibre and are of equivalent size. Tensioned elements have no rigidity in compression and constitute a continuous set. Compressed elements constitute a discontinuous set. Each node receives one and only one compressed element*, and the "extended" definition; *Tensegrity system is a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.*

1.1.2. *Tensegrity structures applications*

Only some of the applications that are using the concept of tensegrity are discussed in this section. There are many other applications that are not given here but can be found in literature, e.g., Tibert (2002) and Jáuregui (2004).

1.1.2a. *Space applications.* The tensegrity concept is used in space applications such as masts and antennas (Tibert 2002). Recently, the National Aeronautics and Space Administration (NASA) has initiated a project named *SuperBallBot* where the plan is to use a tensegrity system (Fig. 1.1) for planetary landing and exploration.

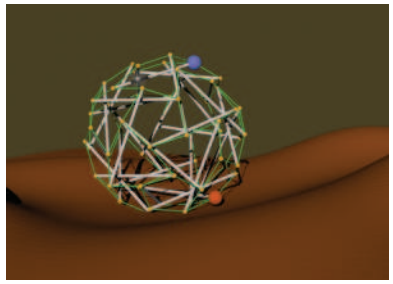

Figure 1.1: Super Ball Bot structure from http://ti.arc.nasa.gov/

1.1.2b. *Architecture and civil structures.* One of the reasons for using tensegrity structures in architecture and civil engineering is the possibility to easily reshape them under different load conditions to adapt to environmental changes (Wroldsen 2007). Thereby they are representatives of *responsive architecture* (Negroponte 1975). Such applications of tensegrity were achieved by d'Estrée (2003) where he created a tunable structure, adapting to environmental changes (Fig. 1.2). Tensegrity structures are also found within civil engineering, such as stadium roofs and tents (Fig. 1.3).

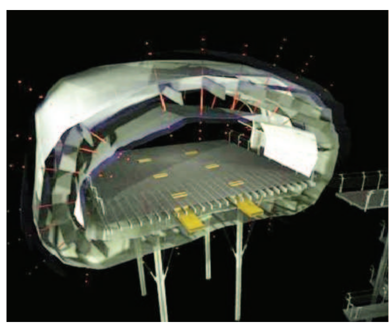

Figure 1.2: An adaptive building envelope Courtesy of d'Estrée Sterk

1.1.2c. *Biomechanics and biology.* Swanson (2013) stated that, biotensegrity provides a conceptual understanding of the human body. In industry, the tensegrity concept is used in modeling of many of the human body segments (Fig.1.4). Chen & Ingber (1999) and Ingber (2008) argued that the principle of tensegrity is also used in many viruses, enzymes and cells to arrange themselves, to minimize energy and mass.

1.1.2d. *Furniture.* Because of their light weight, applications of the tensegrity concept are increasing in the design of furniture and some of the household needs (Fig. 1.5)

Figure 1.3: Tensegrity tent (left) from www.marsonearth.org, and
La Plata Stadium from www.birdair.com (right)

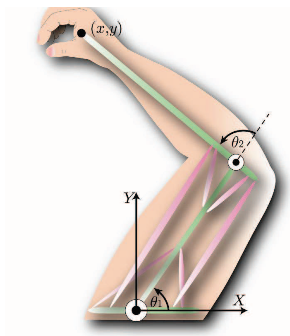

Figure 1.4: Human arm reproduced from www.fit.ac.jp

Figure 1.5: Tensegrity furniture. Reproduced from
www.intensiondesigns.com

1.2. Aim and outline of the thesis

The main objectives of this thesis is to give the basics for using the vibration frequencies in several aspects of designing, constructing, and monitoring the status during the life time of tensegrity structures. The first part of this thesis focusses on improving the existing finite element formulation so that both global and local vibration modes are included, while the second part investigates the effect of temperature changes on the natural frequencies of tensegrity structures. The temperature effects are compared to the effects of damage, represented by slacking of one of the cables.

In designing and production of tensegrity structures and with respect to applications, the pre-stressing of the structure is seen in two different contexts. The first context is a design consideration, when the frequencies are evaluated for a final geometry and topology as a function of the pre-stress level. Studying the response as function of the pre-stress level, this is denoted as *synchronous pre-stressing* of all unstressed lengths of the components. The calculated resonance frequency spectrum can thereby be seen as a signature for the correctly assembled and pre-stressed structure. Faroughi & Tur (2013) used the vibrational properties in the design process of tensegrity structures and concluded that, the designer has to consider the dynamic criteria in the design process.

The second context is a production viewpoint. In a *tuning pre-stressing*, it is assumed that a design of geometry and pre-stress is chosen, thence all unstressed lengths are known. By shortening or lengthening the designated control components, the designed configuration and pre-stress is gradually reached. The frequencies are thereby functions of the level to which the structure is yet pre-stressed on its route to the designed state. These functions thus start at a state where most of the structure is assembled from unstressed components, and the designed pre-stress state is successively obtained while length-adjusting the control components to (and perhaps, beyond) the design lengths. The spectrum of resonance frequencies can then be seen as a progress indicator of the control process.

A pre-stress pattern is established when assembling the structure. It is a fundamental aspect that the components before assembly are of other measures than the (deformed) length they have in the nominal geometry. This obviously means that the structure cannot be assembled without special methods, where the components must be forced to their nominal deformed lengths by special equipment. This can be achieved by making some of cables or some of bars to length-adjustable components, or the joints must contain special mechanisms for tensioning some of the cables. The precise mechanisms for these adjustments are out of the scope of this thesis, but the assumption is that some components can be length adjusted in some way.

In building the tensegrity structures, the main idea motivating this research stems from the concept of tuning, e.g., a guitar, where its strings are tuned to their right resonance by the introduction of an axial force. This requires a coupling between axial and bending behavior and stiffnesses. This coupling has here been achieved by the augmenting Euler-Bernoulli beam elements (Argyris & Mlejnek 1991; Paultre 2010, chap. 14, chap. 3). This formulation has been developed for the tensegrity context in the first paper.

In the present work, monitoring the status of tensegrity structure is based on the assumption that, the assembled structure will be tested by exciting a wide spectrum of frequencies, and registering the resonances. The complete resonance spectrum of the physical structure is thereby a signature for the structure in its current state including the pre-stressing. This can be used immediately after initial assembly of the structure, but also as a monitor of status during the life time of the structures. More details are given in the second paper where a comparison between healthy (no slacking) and unhealthy (cable slacking) situations is given in terms of frequency change. The effect of environmental temperature on the pre-stress level and natural frequencies has also been investigated.

1.3. Form and force finding

Form-finding is defined as the process of finding an equilibrium and a stable geometry (Faroughi & Tur 2013). Tibert (2002) classified the form-finding methods into two groups, kinematic and static methods. A review of these methods can be found in (Tibert & Pellegrino 2002). Form-finding, however, can be seen from different viewpoints, where some methods require the topology and coordinates of the nodes. This is the case when using non-linear programming and force density methods while other methods use other information sets. Recently, Koohestani (2013) proposed an unconstrained optimisation approach for form-finding which requires only the connectivity data and a random set of force densities.

Form-finding in general, can be classified into two branches: shape and force finding (Xu & Luo 2010). Force-finding or pre-stress design, is the process of finding the initial member forces for a specific geometry that is known in advance. Force finding is also defined as the process of searching for feasible pre-stress (Yuan & Dong 2002) or pre-stress optimization (Masic & Skelton 2006).

In the present work, the performed simulations are based on the following assumptions concerning the design. A final geometry is defined, which is connected to a specified pre-stress state, i.e., components are lengthened or shortened from an unloaded length, introducing axial forces when reaching their design lengths. The nominal design thereby includes a specified exact pre-stress force distribution for a specified exact geometry.

CHAPTER 2

Vibration analysis

Engineering problems are normally classified into three categories: equilibrium problems, eigenvalue problems and propagation problems (Crandall 1956). Resonance and buckling are examples of the eigenvalue problems.

Modal testing and eigenvalue analysis are primarily concerned with finding the mode shapes (eigenvectors) and modes (eigenvalues). Eigenvalue analysis is used to analytically find the eigenvalues from the governing equations or from the given mass and stiffness matrices. On the other hand, modal testing deals with finding the eigenvalues and eigenvectors experimentally by exciting the structure by a white noise spectrum. White noise is defined as a forcing function that contains all possible frequencies in the same measure dealing always with spectral representations of studied quantities. In this thesis, the eigenvalues analysis has been adopted after evaluating the mass and stiffness matrices. More information is given in section 2.1.1. The idea is to compare resonance peaks from experiments with computed eigenfrequency from simulations.

2.1. Dynamics of tensegrity structures review

Dynamics of tensegrity structures has become an interesting research topic recently. The majority of research performed has been concerned with the relationship between the pre-stress level and the natural frequencies of the tensegrity structures. Sultan *et al.* (2002) and many others investigated this relationship. In general, they concluded that, the natural frequency increases with the pre-stress level. Ali *et al.* (2010) investigated a tensegrity-based footbridge and concluded that the fundamental frequency is not directly effected by the pre-stress level. The dynamic behavior has been investigated by many researchers such as Moussa *et al.* (2001) and Oppenheim & Williams (2001). Modal analysis of some tensegrity modules was considered by Murakami (2001*a*,*b*). An important finding was given by Motro *et al.* (1987) where they showed that the linearized equation of motion around an equilibrium configuration can be used instead of the complete nonlinear dynamic model. Later, the results by Motro *et al.* (1987) was extended to more complex configurations by Murakami & Nishimura (2001), and Sultan *et al.* (2002).

Different analytical methods are available in literature for the simulation of tensegrity structures. Barnes (1999) classified these methods into incremental, iterative and minimization methods. Incremental and iterative methods use the matrix formulation as in Furuya *et al.* (2006) while for the minimization method, the dynamic relaxation method is widely used, cf. Domer *et al.* (2003).

2.1.1. *Spectral analysis*

Modal analysis in this thesis was conducted after the tangent stiffness $\boldsymbol{K}_T$ and mass $\boldsymbol{M}$ matrices were evaluated at a converged equilibrium state, where $\boldsymbol{K}_T = \boldsymbol{K}_E + \boldsymbol{K}_G$, with $\boldsymbol{K}_E$ the elastic stiffness matrix and $\boldsymbol{K}_G$ the geometric stiffness matrix. The tangent stiffness matrix is including the effect of pre-stress through the geometric stiffness matrix, and is positive definite for stable tensegrity structures. The generalized eigenproblem is obtained from

$$-\omega^2 \boldsymbol{M}\boldsymbol{\phi} + \boldsymbol{K}_T\boldsymbol{\phi} = 0 \tag{2.1}$$

where ω^2 is an eigenvalue and $\boldsymbol{\phi}$ the corresponding eigenvector. The spectral decomposition thereby gives n natural angular frequencies ω_i and the related eigenvectors $\boldsymbol{\phi}_i$ of the structure at the considered equilibrium state. The extracted frequencies normally are obtained as sets of very close values, which is a known property of tensegrity structures, essentially depending on the symmetry and repetitivety of the structure. It is also known that the frequencies are non-linearly dependent on the pre-stress forces for a given final geometric design, and dependent on the discretization used in the structures model. In the present work, with the tangent stiffness and mass matrices assembled, the problem can be linearized, and the small free undamped vibrations of a structure around the evaluated equilibrium state are obtained from the generalized eigenproblem.

The dependence of vibration properties on temperature is one of the topics considered in the present work. This can be seen as one important aspect of the relation between the vibration properties simulated for a structural model and the ones measured for a physical structure.

The natural frequencies ω_i from simulations are corresponding to the peaks from experiments in the frequency spectrum. A comparison between simulation and experimental procedures is given in Fig. 2.1. The figure compares practical and theoretical methods for finding natural frequencies. In practice, normally the natural frequencies of the structures are extracted from the spectrum. Structure parameters such as mass, stiffness and damping can also be computed from the experiment results. In numerical simulations, mass, stiffness and damping matrices are computed first. Then, from the solution of the eigenvalue problem the natural frequencies can be calculated.

Measurements are not further considered here, but are a necessary topic for the future work; more details are given in section 5.2.

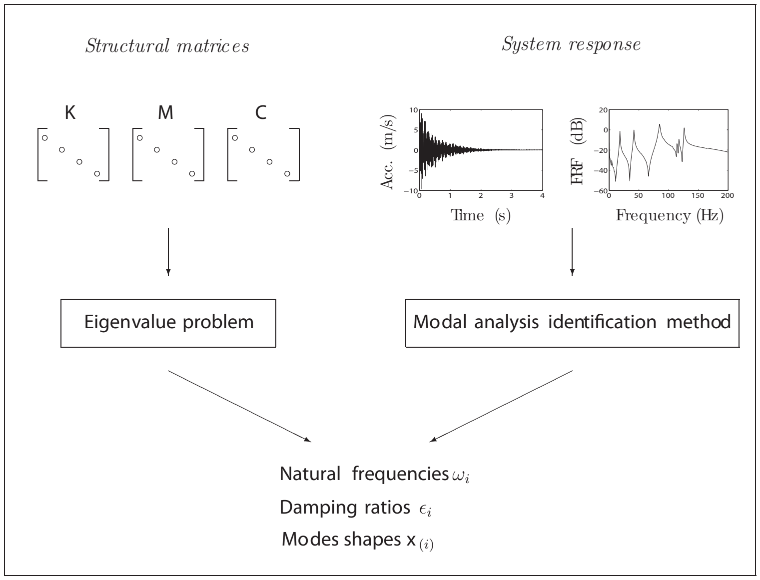

Figure 2.1: Theoretical and experimental modal analysis reproduced from www.ltas-vis.ulg.ac.be

CHAPTER 3

Vibration health monitoring

3.1. Structural health monitoring

The ability to monitor a structure and detect the damage at the earliest possible stage is very important throughout the civil, mechanical, and aerospace engineering communities. There are many methods used in structural health monitoring ('SHM'). These methods vary according to the structure type and the purpose of monitoring. The purpose of SHM is often to give at every moment during the service life of the structure diagnosis of its state of the different parts and of the full assembly of these parts constituting the structure as a whole (Balageas 2006). The SHM is a non-destructive evaluation ('NDE') method which allows continuous and remote monitoring (Inam *et al.* 2014). The SHM technics are increasingly used in industry as a quality assurance tool (Ciang *et al.* 2008).

The results of this monitoring process will be compared with the properties of a known healthy structure simulated during the design stage or measured in a healthy stage of service life. In many cases, the purpose is to make the state of the structure remain in the domain specified in the design despite the fact that this can be altered by the normal aging due to usage, the action of the environment and accidental events. The time-dimension of monitoring makes it possible to consider the full history database of the structure. Many aerospace, civil and mechanical systems continue to be used despite ageing and the associated potential for damage accumulation. Therefore, the ability to monitor the structural health is becoming increasingly important (Farrar *et al.* 2001).

3.2. Vibration health monitoring

Vibration-based health monitoring ('VHM') is a global non-destructive testing methods based on the fact that any changes introduced in a structure (including damage) will change its physical properties, which in turn will change the structural vibration response (Guechaichia & Trendafilova 2011). The VHM techniques have been developed and employed to assess the global condition (health) and detect damage in bridges, buildings, aircraft, and rotating machinery (Mooney *et al.* 2005).

3.2.1. *Damage definition*

Damage in a structure is often combined with a change in its state such as vibrational properties during the service life. In the most general terms, damage can be defined as changes introduced into a system that adversely affects its current or future performance (Farrar & Worden 2007) and (Doebling *et al.* 1998). There are several methods used to detect the damage. These methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnetic field methods, radiographs, and thermal field methods. Ciang *et al.* (2008) give a review of these methods.

3.2.2. *VHM of tensegrity structures*

All structures and components are susceptible to damage. Each structure also has its typical dynamic behavior, which may be addressed as its 'vibrational signature' which is defined as the measurements of normal operating vibrations.

For tensegrity structures, damage is normally expected to occur in the form of buckling (or at least stiffness reduction close to this instability) of one or more bars and/or complete or partial slacking of one or more cables, i.e., a force reduction in and redistribution in the components. Shekastehband *et al.* (2012) numerically and experimentally investigated if a bar collapse or cable rupture will lead to collapse of the whole system. Vibrations frequencies as an indicator of tensegrity damage has not been explicitly reported in literature. Korkmaz *et al.* (2011) used the displacements as an indicator of the damage in an active tensegrity. Shekastehband *et al.* (2012) used the axial forces and displacements as indicators of damage in their investigation.

3.2.3. *VHM as a maintenance tool for tensegrity structures*

Tensegrity structure VHM can be used to check the condition by using the measured global resonance frequencies of the structure and comparing it with a data index stored during previous health condition measurements. The objective is to investigate if there is any difference compared with the present measured dynamic properties, such as natural frequencies, mode shapes and damping. In industry this is known as preventative maintenance, which will reduce the operational cost of the structure and hence has an economic impact (Fritzen 2005). Based on an accurate comparison, a conclusion can be produced describing the current condition of the tensegrity. Then, these records can be used when monitoring the tensegrity, and the result can be interpreted and translated. After that the decision and the right action can be taken.

3.2.4. *VHM as a design tool for tensegrity structures*

Vibration health monitoring in the design process of tensegrity structures can be seen from two points of view: form finding or the evaluation of the pre-stress where the unstrained lengths of all components are introduced based on a predefined geometry. Recently, the dynamic properties for tensegrity structures have been considered as a design tool by many researchers, such as Koohestani (2013) for the form finding or by Dubé & Angellier (2013) for identifying the pre-stress level. Faroughi & Tur (2013) examined the influences of vibrational properties on the design of tensegrity structures and developed an algorithm to design the tensegrity structures using their dynamic behavior.

Oppenheim & Williams (2001) tested the effect of pre-stress on the natural damping of the cables of a simple elastic tensegrity structure and concluded that, the natural geometric flexibility of tensegrity structures at equilibrium leads to a much slower rate of decay of amplitude of vibration than conceivably expected.

CHAPTER 4

Summary of papers

Paper 1

Natural frequencies describe the pre-stress in tensegrity structures

The aim was to study the resonance frequencies in pre-stressed non-linear tensegrity structures. The existing formulation was improved by using finite beam elements with axial-transversal coupling. The axial-bending coupling emphasizes the non-linear dependence of the natural frequencies on the pre-stress state, thereby giving a basis for diagnosis of structural conditions from simulated natural frequencies. Two-dimensional models were used, but the formulation is also relatively easily implemented for more complex structures.

With respect to applications, the pre-stress was seen as either synchronous, considering a variable final pre-stress design or as tuning, when increasing pre-stress is followed in a planned construction sequence.

A quantitative agreement between the results from simulation and the analytical solution of the simply supported pre-stressed beam was observed. With the bending moment released at the physical joints, it is found that vanishing of the lowest natural frequency of the system is related to the critical buckling load of one or several compressed components. The new aspect in this study is that both local and global modes are obtained when using this formulation. Components with the relaxed pre-stress will most likely reveal themselves by their local transversal vibration behavior. Hence, this work will give possibilities to improve the VHM evaluation techniques and interpretation.

The methods in this study can be used to plan the tuning of the considered tensegrity structure towards the design level of pre-stress, and as health monitoring tools.

Paper 2

Influence of temperature variation on tensegrity structure vibration properties

The aim was to explore how the temperature changes affect the natural frequencies of tensegrity structures. Natural frequencies and internal forces are not only sensitive to damage, but also to other factors such as external load applications and environmental conditions. The VHM investigators should have the ability to recognize whether the change in the measured vibration frequencies is related to damage occurring, or because of the environmental temperature change.

Two parameters were considered in this study: the changes to the expansion coefficient and the elastic modulus with temperature changes. A comparison between the effects of damage and of environment temperature changes on the natural frequencies and internal forces of tensegrity structures was given. In this study, the same formulation as in the first study was used. Different support conditions and sizes of tensegrities were considered. For the simple case, a string in a tube, it was possible to express the internal forces as a function of temperature changes. Hence, the results from the simulations were first validated using analytical formula. It was concluded that the internal forces and natural frequencies of tensegrity structures are sensitive to change in the environment temperature, which must be recognized in VHM.

Two observations were made, concerning the changes in the resonance spectrum with temperature:

(i) When the change in internal forces caused by different expansion coefficients for bars and cables dominates the change due to E modulus, the natural frequencies decrease/increase with increasing/decreasing internal forces, as long as the structure has bar dominated vibration. The opposite is true for structures that have cable dominated vibration.

(ii) When the change in the internal forces due to a decreasing elastic modulus dominates the change due to the difference between the thermal expansion coefficients of bars and cables, the lowest natural frequencies decrease with increasing temperature.

CHAPTER 5

Conclusions and outlook

5.1. Conclusions

The beam element formulation used in this thesis emphasizes the non-linear dependence of the resonance frequencies on the pre-stress level in the tensegrity structures. Thereby, it gives a basis for diagnoses of structural conditions from measured frequencies.

Effect of the axial force on the bending stiffness has been established by using Euler-Bernoulli beam elements which include the effect of the axial force on the transversal stiffness. It is well known that a string is tuned to its right resonance by introduction of an axial force (such as when tuning the guitar). Similarly, the resonance is affected by a compressive force, and the resonance frequency lowered with increasing force magnitude, until the buckling load, where transversal stiffness disappears and infinite non-periodic displacements are obtained.

It is shown that frequencies going down will correspond to approaching an instability of the structure. This instability, however, is also manifested by a tangent stiffness matrix of the structure approaching singularity.

The pre-stress in this thesis was seen in two different contexts. The first context is the design consideration when the frequencies are evaluated for a final geometry and topology as a function of pre-stress level. The calculated resonance frequency spectrum can thereby be seen as a signature for the correctly assembled and pre-stressed structure. The second context is a production viewpoint, where all unstressed lengths are known. By shortening or lengthening the designated control components, the designed configuration and pre-stress is gradually reached.

The new aspects of this research are the consideration of the axial force also in the compressed components and the introduction of bending stiffness for all components. With this formulation, it is shown that the lowest natural frequencies either increase or decrease with increasing of pre-stress level. It is found that this is depending on whether the tensegrity structure can be said to have a cable or a bar dominated behavior.

In a health monitoring context, where essentially the resonance spectrum is triggered by external excitation, the most visible modes will, however, be the transversal, more or less localized, cable vibration modes, corresponding to bending of the components. This consideration must see the components as having a bending behavior and the member stiffness affected by the axial force. A beam model with a one-way coupling between axial force and both axial and bending stiffness is introduced in the present work, in order to include vibration modes primarily consisting of member bending and to reflect an approaching member buckling type of instability.

The above formulation was used to investigate the effect of temperature on the vibration properties of tensegrity structures. It was shown that the internal forces and the natural frequency of tensegrity structures are sensitive to changes in the environment temperature, in particular when the structure is externally statically indeterminate, is of irregular geometry or under other circumstances when a thermal expansion effect causes a change in structural geometry. A strategy for how to predict the change in resonance frequencies with temperature has been developed.

The change in the elastic modulus with temperature was found to be an important factor affecting the VHM results. It was concluded that VHM must consider the temperature effects.

5.2. Outlook and future work

The finite element formulation developed in this thesis improves the existing work. However, the proposed formulation can be further improved by considering a two way coupling between the axial and bending behaviors rather than one way coupling. This will create an even higher degree of non-linearity in the vibration response. In addition, developing a 3D version of the dynamic formulation for a prestressed tensegrity is one of the aims.

The following studies could be interesting and achievable, based on the current work:

- Developing an VHM algorithm to explore the effects of choosing different (in case there are more than one) designed self-stress vectors on the natural frequencies and mode shapes of the tensegrity structures. The existence of multiple internal equilibrium states obviously will affect the dynamic response. This must then be seen as a result of different criteria for the choice of pre-stress in the null space of equilibrium states.

- Damage detection and localization in tensegrity structures, with consideration of the application of static and dynamic external loads. The idea is to increase the sensitivity of the lowest natural frequencies to damage by applying static or dynamic external loads in connection with response measurements. The primary type of damage is defined in section 3.2.1.

- The future work will be focused on the construction and service life stages. Investigating of how the vibration monitoring can be used to control the quality of a construction of tensegrity structures could be an interesting topic. In practice, there are many difficulties in this, i.e., reaching the designed pre-stress level and geometry. This will include comparisons between simulations and experimental results.

Basically, the future work will focus on vibration measurement and testing where a knowledge of the instrumentations used is required. These instrumentations can be very different for different applications, but will, typically, include, an actuator called exciter, sensors, a data acquisition system and an analyser to acquire and process the signals from sensors. A wide variety of vibration and shock exciters for modal analysis have been developed. They are in general either shakers, Fig. 5.1, or impulse exciters, Fig. 5.2. However, the size of the shakers and impulse exciters vary according to the size, mass and stiffness of the object under investigation.

Sensors are needed to measure the required signals. The most common sensors are strain gauges and accelerometers. The sensor natural frequency range should be

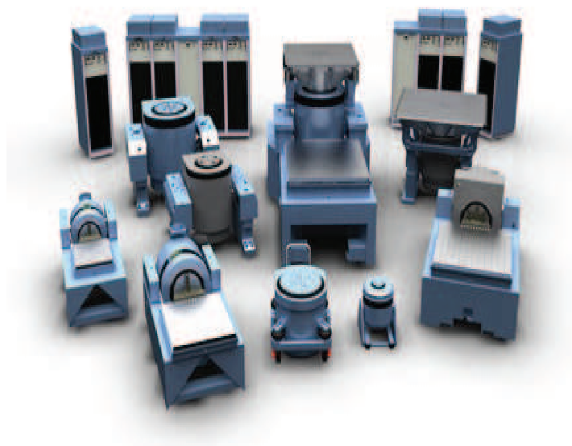

Figure 5.1: Different sizes of shakers from www.pcb.com

Figure 5.2: Different sizes of impact hammer from www.pcb.com

greater than the highest frequency to be measured. However, for small sizes of the objects, these sensors will affect the measured dynamics properties. Other sensors can be used for small objects, such as laser vibrometers which are instruments for rapid non-contact measurement and imaging of vibration. Laser vibrometer instruments and the corresponding methods seem suited for light weight structures like tensegrities. The principial idea of the laser vibrometer is based on the Doppler effect, Fig. 5.3, which occurs when light is back-scattered from a vibrating surface.

Signals generated by exciters are transmitted by sensors to analyzers which will display the plot of the frequency function, and compute the natural frequencies and mode shapes.

In order to compare the results from simulations and from measurements, one should consider that there are factors that might be existing during the test and affecting the measured results. Environmental conditions such as temperature, humidity, light and wind have significant impacts on vibration measurements. This

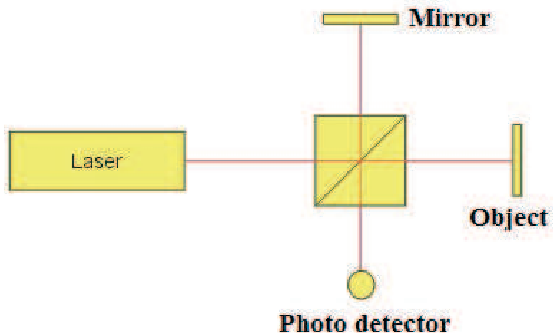

Figure 5.3: Laser Vibrometer Setup reproduced from www.applv.blogspot.se

is either through the effect of these parameters on the structure under consideration or on the instrumentations used to measure the response. For instance, laser vibrometers are sensitive to light and indoor/outdoor measurements may give different results.

Another factor is the size of the structure, which must be considered when choosing the measurement instrumentations. The tests will demand a suitable exciter for specific size of the structure. Excitation force magnitude is also related to the size of the structure under consideration.

Spectrum width must be taken into account when choosing the measuring instrumentations, in particular the vibrometers. Tensegrity structures have a wide range of their resonance spectrum with sets of frequency values very close to each other. Hence, the instrument will be used must be capable to measure a wide range of this spectrum, and to deal with clusters of resonance frequencies.

Acknowledgements

I would like to express my deep gratitude to Prof Anders Eriksson for his patient guidance, enthusiastic encouragement and for giving me the chance to do postgraduate studies. Apart from scientific topics I learned a lot from him. I learned how to deal with students and what is the meaning to be a supervisor.

I thank Dr. Gunnar Tibert for the valuable discussion and for having time for me whenever I come to his office. My occasional meetings with him were very useful and a cause of my success.

Furthermore I would like to thank Dr. Lanie Gutierrez-Farewik and Dr. Arne Nordmark for their notes and comments during the group seminars. In addition I would like to thank all my group mates for the nice environment and being always positive. With the group, it is always easy and quick to make a plan for going outside for any activity. I would also like to offer my special thanks to Erik Dijkstra for the early morning discussions and talks. Discussions with Dr. Seif Dalilsafaei, Dr. Ganesh Tamadapu, Pau Mallol and Amit Patil were also very constructive for this work.

Finally I would like to express my very great appreciation to Boshra Ben Salem, Nada Ashwear and Mohammed Ashwear for supporting me all the time and for being so patient when I stayed late at the office.

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Part II

Papers