Stability Analysis of Embankments Founded on Clay

- a comparison between LEM & 2D/3D FEM

Zhaleh Habibnezhad

Master of Science Thesis
Division of Soil- and Rock Mechanics
Department of Civil and Architectural Engineering
Royal Institute of Technology
Stockholm 2014
Foreword

The work presented in this thesis was carried out between 2012 and 2013 at Grontmij AB, Stockholm and the Royal Institute of Technology (KTH), division of Soil and Rock Mechanics.

I have to a large extent worked independently on this thesis and at the same time started working at Grontmij, but could not manage it without the support i have received from KTH and Grontmij. First and foremost i would like to thank my supervisor at school, Prof. Stefan Larsson. I also like to thank Dr. Rasmus Muller that helped me with the parameters data. I would like to express my gratitude to Dr. Kenneth Viking, my colleague at Grontmij that helped me with improving the structure of the thesis.

Finally I would like to thank my family and Daniel for their patience and encouragement under work period.

Stockholm, February 2014

Zhaleh Habibnezhad
Summary

Rapid constructed embankments founded on soft deposits have a negative influence on the short term stability. Many engineering constructions such as road and railway embankments are often constructed on soft clay deposits. In stability analysis calculation of safety factor (SF), as the primary design criteria can be evaluated through different numerous methods such as limit equilibrium method (LEM) and finite element method (FEM). It is of particular interest to determine/estimate appropriate stability of the specified embankment which is highly dependent on the analysis method used. Therefore, it is a challenge for geotechnical engineers to judge which analysis method can simulate better the reality.

The aim of this thesis is to increase understanding applicability of the three applied programs; Plaxis2D, Plaxis3D and Slope/W in simulating and stability analysis/estimation of embankments founded on clay deposits.

The work has involved analysis and comparison of the stability through estimate of the SF and the critical failure surfaces obtained through 2D and 3D programs. Four case configurations were studied for the stability analysis. In each case variation in plastic parameters of clay (\(\phi - c\)) or load geometry, was the scenario to make the comparison analysis. Moreover, application FEM3D offers an attractive alternative to traditional approaches to the problem (especially for LEM).

The main conclusions from this study are the following:

1. Concerning the three applied programs, FEM3D has the minimum SF sensitivity to change in plastic parameters of clay deposit.
2. For embankments founded on clay deposit, the 3D failure surfaces are easily found via the FEM3D analysis program, which is closer to reality, while failure results of 2D analysis programs can never occur in reality.
3. Using 2D analysis method instead of 3D, to investigate the stability of 3D embankment model tend to give higher SF results up to 14% for embankments founded on undrained clay deposit.
4. The failure surfaces in 3D analysis are likely to be shallower than in the corresponding 2D model.
5. Results from the 3D analysis through hand calculation and program calculation do not correspond with each other for embankment founded on soft clay deposit. The first reason is rooted in limitation of the hand formula. The formula is suitable for embankment founded on one layer deposit (soil); however an embankment founded on 3 layers of deposit (soil) was analyzed in this study. The second reason is related to applied method of calculation. 3D hand calculation formula is based on method of slices however; analysis method in program calculation is based on FEM.

Keywords: stability, clay, finite element, limit equilibrium, embankment, three dimensional.
Sammanfattning

Snabbt konstruerade banker byggda på leravlagringar har en negativ inverkan på den kortsiktiga stabiliteten. Många tekniska konstruktioner såsom väg och järnvägsbanker är ofta konstruerade på leravlagringar. Vid analyser av stabiliteten är beräkningar av säkerhets faktorer (SF), det primära design kriteriet. Utvärderingar kan utföras genom olika metoder såsom limit equilibrium method (LEM) and finite element method (FEM). Det är av särskilt intresse att bedöma och uppskatta stabiliteten för den specifika banken, detta är starkt beroende av vilken analysmetod som används. Därför, så är det en utmaning för geotekniska ingenjörer att bedöma vilken analys metod som bäst kan simulera verkligheten.

Målet med denna uppsats är att öka förståelsen för de tre tillämpade programmen: Plaxis2D, Plaxis3D and Slope/W för simulering och analys/bedömning av banker på leravlagringar.

Arbetet har involverat analyser och jämförelser av stabiliteten genom uppskattningen av SF och kritiska brott i glidytan genom användandet 2D och 3D program. Fyra konfigurerade fall har studerats. I varje fall har variationen i plastiska parametrar av lera (\(\varphi\) - c) eller last geometri, varit sceneriet för att utföra den jämförande analysen. Ytterligare, tillämpningen av FEM3D erbjuder ett attraktivt alternativ till traditionella metoder för att bemöta problemet (speciellt för LEM).

De huvudsakliga slutsatserna från denna uppsats är följande:

1. Beträffande de 3 tillämpade programmen, så har FEM3D lägst känslighet på säkerhetsfaktorn pga. förändringar i lerans plastiska parametrar.
2. För banker på leravlagringar, 3D brott i glidy tan är enkelt att identifiera genom FEM3D analys program, vilket är närmare verkligheten, medans brott i glidytan i 2D analys program aldrig kan återge verkligheten lika tydligt.
3. Tillämpningen av 2D analysmetod istället för 3D- för att undersöka stabiliteten av banker byggda på odränerad lera, över- eller underskattar säkerhetsfaktorns resultat med upp till 14 %.
4. Brott i glidytan i 3D analys är sannolikt ytligare än i korresponderande 2D model.
5. Resultat från handberäkningar och programberäkningar av 3D-analyserna överensstämmer inte med varandra, för banker byggda på mjuka leravlagringar. Första orsaken är rotad i begränsningen av beräkningsformeln för handberäkning. Formeln är anpassad för banker på ett lager lera: i denna studie är en bank på 3 lager av lera analyserad. Den andra orsaken är relaterad till tillämpad metod för beräkning. 3D handberäkningsformeln är baserad på metoder av ”slices”: medan analysmetoden i programberäkning är baserad på FEM.

Nyckelord: stabilitet, lera, finite element, limit equilibrium, banker, tredimensionell.
List of Symbols and Abbreviations

**Roman letters**

- \( A \) Area of total failure surface
- \( B \) Footing width
- \( C \) Cohesion
- \( D \) Load distance from embankment edge
- \( E \) Modulus of elasticity
- \( E_i \) Inclination of forces
- \( E \) Modulus of elasticity
- \( H \) Load height
- \( K_0 \) Coefficient of permeability
- \( L \) Embankment length
- \( M \) Center of slip surface segment
- \( N_i \) Normal force
- \( n \) Mode number
- \( q \) Distributed load
- \( Q \) Point load
- \( r \) Radius of slip surface
- \( U \) Pore water pressure
- \( w \) Total weight of slip surface
- \( X \) Embankment width
- \( Z \) Embankment length in third dimension

**Greek letters**

- \( \alpha \) Slope angle
- \( \beta \) Inclination angles
- \( C_u \) Undrained shear strength
- \( f(\alpha_i) \) Half-sine function
- \( W_i \) Block weight
- \( \tau_{fu} \) Reduced undrained shear strength
- \( \Phi \) Friction angle
- \( \gamma_{\text{un-sat}} \) Unsaturated soil weight
- \( \gamma_{\text{sat}} \) Saturated soil weight
- \( \nu \) Poisson ratio
- \( \psi \) Dilatancy angle

**Abbreviations**

- 2D Two dimensional
- 3D Three dimensional
- CLA Computational Limit Analysis
- Emb Embankment
- FEM Finite Element Method
- \( F_{2-Dim} \) 2Dimensional safety factor
- \( F_{3-Dim} \) 3Dimensional safety factor
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLE</td>
<td>General Limit Equilibrium</td>
</tr>
<tr>
<td>LE</td>
<td>Limit Equilibrium</td>
</tr>
<tr>
<td>LEM</td>
<td>Limit Equilibrium Method</td>
</tr>
<tr>
<td>ΣMSF</td>
<td>Total multiplier for safety calculation</td>
</tr>
<tr>
<td>SRM</td>
<td>Strength reduction method</td>
</tr>
<tr>
<td>SF</td>
<td>Safety Factor</td>
</tr>
<tr>
<td>SSC</td>
<td>Soft Soil Creep</td>
</tr>
</tbody>
</table>
Table of contents

Foreword.................................................................................................................. iii
Summary .................................................................................................................. v
Sammanfattning ....................................................................................................... vii
List of Symbols and Abbreviations ........................................................................ ix
1 Introduction .......................................................................................................... 1
   1.1 Background ..................................................................................................... 1
   1.2 Aim and Scope .............................................................................................. 2
   1.3 Limitations ..................................................................................................... 2
   1.4 Structure of thesis .......................................................................................... 2
2 Literature Survey ................................................................................................ 5
   2.1 Introduction .................................................................................................... 5
      2.1.1 Stability analysis vs. historical development ........................................... 5
      2.1.2 Methodology ........................................................................................... 5
   2.2 Analysis programs .......................................................................................... 9
      2.2.1 SLOPE/W (LEM) ....................................................................................... 9
      2.2.2 PLAXIS 2D (FEM) .................................................................................. 10
      2.2.3 PLAXIS 3D (FEM) .................................................................................. 10
   2.3 2D vs. 3D ....................................................................................................... 11
   2.4 Load (drives the instability) ........................................................................... 12
   2.5 Drained vs. Undrained ................................................................................... 12
   2.6 Previous studies ............................................................................................ 12
3 Problem definition ............................................................................................... 14
   3.1 Introduction .................................................................................................... 14
   3.2 Case configurations ....................................................................................... 14
      3.1.1 Case1: Short embankment- undrained clay ............................................ 15
      3.1.2 Case2: Long embankment- drained clay ............................................... 16
      3.1.3 Case3: Short embankment- drained clay ............................................... 17
      3.1.4 Case4: Long embankment- inconsistent clay layer thickness .............. 18
   3.3 Embankment and surcharge load ................................................................... 19
   3.4 Geometry ....................................................................................................... 20
   3.5 Soil properties ............................................................................................... 20
   3.6 Program modeling ........................................................................................ 21
      3.4.1 Slope/W .................................................................................................... 21
      3.4.2 Plaxis2D ................................................................................................. 21

vii
# Table of contents

3.4.3 Plaxis3D ........................................................................................................ 22

4 Results .................................................................................................................. 23

4.1 Introduction ......................................................................................................... 23

4.2 Case study results .............................................................................................. 23

4.2.1 Case1: Short embankment- undrained clay ............................................... 23

4.2.2 Case2: Long embankment- drained clay ...................................................... 23

4.2.3 Case3: Short embankment- drained clay ...................................................... 24

4.2.4 Case4: Long embankment- inconsistent clay layer thickness ....................... 24

5 Analysis, Discussion, Validation ........................................................................ 27

5.1 Introduction ......................................................................................................... 27

5.2 Case study analysis ............................................................................................ 27

5.2.1 Case1: Short embankment - Undrained clay ............................................... 27

5.2.2 Case2: Long embankment- drained clay ...................................................... 28

5.2.3 Case3: Short embankment- drained clay ...................................................... 29

5.2.4 Case4: Long embankment- inconstant clay layer thickness ....................... 29

5.3 Validation .......................................................................................................... 30

5.3.1 Hand calculation .......................................................................................... 30

5.4 Discussion ......................................................................................................... 34

6 Conclusions .......................................................................................................... 36

6.1 Conclusions ....................................................................................................... 36

6.2 Proposal for further research ........................................................................... 37

References ............................................................................................................... 39

Appendix .................................................................................................................. 42
1 Introduction

1.1 Background
Rapid constructed embankments on soft soil have a particularly negative characteristic on the stability concept. Lots of engineering constructions such as; roads and railway embankments are often constructed on soft clay deposits. Over the past decades, many embankments with the behavior of clay deposits and often with failure of them caused a big uncertainty in the field of stability analysis. In the recent years, with development of traffic system, number of these types of constructions is increasing rapidly. Therefore, it is a common challenge for the geotechnical engineers to estimate stability of the embankment and evaluate certainty of the stability calculation results.

There are different factors that can influence stability of specified embankment (problem) in terms of reliability and probability. Among them, shear resistance of clay is of the most important factors which in the assessment of short-term stability is considered as undrained shear strength, \( c_u \). This factor can affect locally the material behavior and globally geo-structural response. Moreover, variability of external loads and geometry of the model are of the other effective parameters in stability concept. In addition to mentioned uncertainties that are called parameter uncertainty, the total uncertainty within stability concept is how well the real field embankment can be modeled and analyzed in the analysis program. This type of uncertainty is called model uncertainty.

For stability analysis, calculation of safety factor as the primary design criteria can be evaluated through different numerous methods. Numerical methods have been conducted since 1970, mainly through Limit Equilibrium Method (LEM). LEM has been widely used by engineers and is considered as a traditional, well established method. Although it does not consider the stress-strain relation of soil, but can provide an estimation of SF without the knowledge of soil's initial plastic parameters. The method is statically indeterminate and assumptions on the distributions of internal forces are required for the solution of the SF. However, as mentioned by Cheng et al. (2006), LEM has been used for simple problems but its application in complicate problems; i.e. complex geometries is limited. On the other hand, newer numerical methods, such as FEM2D, is used in analysis as a viable alternative to LEM. Finite Element Method (FEM) uses stress-strain behavior of the soil and removes the assumptions applied in LEM to change static-indeterminate problem to a statically determinate one. It is well established to predict material behavior of the ground, water and structure far better than LEM (Heibaum et al. 2009). As highlighted by Duncan (1996), it is a general purposed method for calculating stability without pre-assumption of the potential failure surface.

At the other hand according to Hicks and Spencer (2010), no slope is truly 2D: “The presence of heterogeneity means that most slope failures are 3D which have a significant influence on the predicted reliability and computed response”. It is also pointed out by Gens et al. (1988) that estimation of shear strength, derived from the 2D analysis will be unsafe, in order to account for stability analysis of a three dimensional slope. In fact, one of the most common problems of 2D analysis methods is ignoring 3rd dimension of the model and applying 2D back analysis shear strength for a 3D model. How much would results of stability calculation differ within 2D and 3D analysis? Which method of analysis can better simulate and analyze the reality?
It seems that for complex problems such as models with complex geometry, FEM3D can be a good solution. However, experience of stability analysis through FEM3D are limited, therefor reliability and efficiency of 3D analysis is still a point of consideration in Geotechnical calculations. Even though, it seems that 3D analysis has a better point of chance to represent the real soil behavior, but there are some complicated situations that handling of this analysis type becomes a point of doubt!

1.2 Aim and Scope
The aim of this thesis is to increase understanding applicability of the 3 applied programs; Plaxis2D, Plaxis3D and Slope/W in simulating and stability analysis/estimation of embankments founded on clay deposits. The work has involved analysis and comparison of the stability through estimate of the safety factors (SF), and the critical failure surfaces obtained through the 2D and 3D numerical calculations. In case configuration/parametric study plastic parameters of clay varied over the range and sets of stability charts were provided for the long and short embankments via the 2D and 3D programs. The models analyzed involve a comparison between the application of the Limit Equilibrium Model and the Finite Element 2D and 3D Model.

Scope of thesis involves 2D and 3D stability charts and failure shapes of the following four cases:
- Short embankment founded on drained clay deposit.
- Long embankment founded on drained clay deposit.
- Long embankment founded on undrained clay deposit.
- Long embankment with inconstant undrained clay layer thickness.


Case 3 is inspired by part of the study done by Cheng et al. (2006): “Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods”.

Case 4 is inspired the common question faced by geotechnical engineers; “What type of analysis is best for embankment with complex geometry of deposits, 2D or 3D?”

1.3 Limitations
This thesis is limited to study the stage constructed embankments founded on specific type of soft soil but in order to simplify the modeling process, time domain is not considered in staged construction of the embankments.

Moreover, all studied cases are inspired from past researches that have been done in different part of the world. Since this research is studied in Sweden, model geometry and the input soil parameters are taken from Swedish field measurements.

1.4 Structure of thesis
The thesis work is divided into the six following parts:
Chapt. 2. Survey of Literature: survey of found relevant literature related to stability calculation, review of past researches and historical development of the stability analysis, basic description of LEM and FEM and also mechanism of safety calculation in Plaxis 2D, Plaxis 3D and Slope/W.

Chapt. 3. Problem definition: description of embankments geometry, Modell simulation in Plaxis 2D, Plaxis 3D and Slope/W through four different case studies namely: short embankments founded on drained clay deposit-long embankment founded on drained clay deposit-short embankment founded on undrained clay deposit-long embankment founded on complex geometry of clay deposit.

Chapt. 4. Results: presentation of numerical modeling results via stability charts (via SF) and failure surface shape,

Chapt. 5. Analysis, Verification, Discussion: Analysis of the obtained results, discussion of the differences between analysis results of three programs, evaluation of programs sensitivity against soil parameters, models geometry and external load. For verification purpose, in order to evaluate results of applied analysis programs, a hand calculation is handled for all the analyzed models of studied Case2.

Chapt. 6. Conclusions: presentation of conclusions achieved through this study, some suggestions is also advised here for future researches.
Introduction
2 Literature Survey

2.1 Introduction

The literature survey aims at providing a summary of stability analysis vs. historical development and methodology.

2.1.1 Stability analysis vs. historical development

Since long ago, estimating stability of slopes remained a classical and important problem for geotechnical engineers that have drawn attention of many researchers (Merifield and Lyamin 2009). Stability calculation is performed to assess the safe design of human-made or natural slopes like embankments and respectively the equilibrium conditions. The term stability analysis can be explained as the resistance of inclined surface to failure by sliding or collapsing. The main interest of slope stability analysis are determination of SF against slope failure, designing of optimal slopes with regard to SF, estimation of models stability and investigation of potential failure mechanisms.

Before 1970, stability analysis was accomplished through hand calculation. Today there are lots of possibilities for engineers to use analysis software, Choices like traditional limit equilibrium techniques through computational limit analysis to newer numerical solutions such as finite element methods.

Traditional LEM is still widely used in practice while at the same time more and more attention has been directed to FEM for stability analysis. FEM is well established to predicting the material behavior of the ground and the interaction of ground, water and structure.

2.1.2 Methodology

Stability analysis as the primary design criteria for stability calculation can be evaluated through different methods such as: Limit Equilibrium Method (LEM) and Finite Element Method (FEM). The following parts encompass an overview of the assumption and mentioned methods work and how the SF is calculated under these methods.

LEM and safety

Classical method of slices based on LEM has proved to be fairly efficient in geotechnical analysis and is still being widely used in practice. However, the main problem with this method is disregarding stress-strain behavior of the soil in calculation. The basic theory of the method is to divide soil mass into slices and define shear and normal inter-slice forces for each slice to satisfy all the static equilibrium conditions. The most advantages of LEM can be briefly named as follow:
1. LEM can provide an estimate of SF without the knowledge of initial condition (Cheng et al. 2006).

2. The importance of interstice force function depends on a large extent to the amount of contortion that potential sliding mass must undergo to move (Krahn 2003).

3. No restrictions are imposed on the shape of slip surface (Zhu 2001).

Duncan et al. (1996) has done a review over equilibrium methods of slope stability analysis which include: Bishop Method, Force equilibrium method, ordinary method of slices, Janbu method, Morgenstern-Price method and Spencer method. Spencer’s and Morgenstern and Price’s methods are more practically used since they satisfy all statistical conditions. In this study, Morgenstern-Price method was used as the applied calculation method in LEM for computation of SF. Morgenstern-Price method or method of slices is one of the most popular analysis methods in geotechnical stability calculations. Morgenstern and Price (1967) developed the Spencer’s method and their creativity was to allow different user-specified inter slice force function. This method satisfies all conditions of equilibrium and is applicable to any shape of slip surface. It assumes inclination of side forces follow a prescribed pattern, called \( f(x) \). Inclination of slice forces can be the same or vary from slice to slice and are calculated in the process of solution so that all conditions of equilibrium are satisfied. Morgenstern and price is an accurate method based on 3N equations and unknowns. Forces acting on individual blocks are displayed in Fig.1.1:

![Static scheme - Morgenstern-Price method](image)

**Fig.1.1.** Acting forces on individual blocks, Morgenstern-Price method (GEO-SLOPE Int. Ltd. 2010).

1. Each block is assumed to contribute due to the same forces as in Spencer method. The following assumptions are introduced to calculate the limit equilibrium of forces and moment on individual blocks:
   2. Dividing planes between blocks are always vertical.
   3. The line of action of weight of block, \( W_i \), passes through the center of the segment of slip surface represented by point \( M \).
   4. The normal force \( N_i \) is acting in the center of the segment of slip surface, at point \( M \).
   5. Inclination of interslice normal forces, \( E_i \), acting between blocks (\( \delta_i \)) is different on each block, at slip surface end points is \( \delta = 0 \).

Choice of inclination angles, \( \delta_i \), of forces, \( E_i \), acting between the blocks is realized with the help of Half-sine function. One of the functions in the following figure is automatically chosen. This choice of the shape of function has a minor influence on final results, but suitable choice can improve the convergence of method. Functional value of Half-sine function, \( f(x_i) \), at boundary point, \( x_i \), multiplied by parameter lambda, \( \lambda \), which is considered as percentage of the function used, results in the value of \( \delta_i \).
\[ \delta_{i+1} = \lambda \cdot f(x_i) \]  \hspace{1cm} \text{(Eq. 1)}

The factor of safety can be calculated through the following formula:

\[ \text{FS} = \tan \varphi_i \cdot \tan (\delta_{i+1} - \alpha_i) \]  \hspace{1cm} \text{(Eq. 2)}

The initial value of angles \( \delta_i \) is set according to the Half-sine function and must be found in the interval \((-\pi/2; \pi/2)\). Another check preventing numerical instability is verification of parameter; \( m\alpha \), following condition must be satisfied:

\[ m\alpha = \cos \alpha_i + \frac{\sin \alpha_i \cdot \cos \varphi_i}{\text{FS}} \]  \hspace{1cm} \text{(Eq. 3)}

Therefore, before iteration run it is required to find the highest of critical values SF min satisfying above mentioned conditions. Values below this critical value are in area of unstable solution, therefore iteration begins by setting SF to a value "just" above SF min and all result values of SF from iteration runs are higher than SF min. Fig.1.3. shows more clear Morgenstern-Price analysis with a half-sine function.

Fig.1.2. Half-sine function (GEO-SLOPE int. Ltd. 2010).

Fig.1.3. Morgenstern-Price safety factors with half sin function (GEO-SLOPE int.Ltd. 2010).
As it is shown in Fig. 1.1 and Fig. 1.4, the inter slice force functions that are specified and applied are separately illustrated. The other curve shows the applied function which has almost the same shape as specified one but is scaled through, $\lambda$ factor. The good point of this method of analysis is force polygon closure; since it shows clearly the shear and normal inter slice forces. Totally it can be stated that SF through LEM is the ratio between the Resisting moments/forces ($R$), to the Motivating forces/moments ($M$).

$$SF = \sum R / \sum M$$  \hspace{1cm} (Eq. 4)

**FEM and safety**

Whereas computational techniques are developing quickly, newer numerical methods such as FEM are becoming more popular in the world of slope stability analysis. As highlighted by Duncan (1996), FEM is a general purpose method which can be used to calculate stresses, movements, pore pressure and other characteristic of earth mass during construction (Lane and Griffiths 2000; Zheng et al. 2005) without previously assuming the potential sliding surface. The most remarkable advantage of this method is using stress-strain behavior of the soil and removing the assumptions applied in LEM to change static-indeterminate problem to a statically determinate one. FEM seems to deal well with problems of slope analysis and is therefore used more and more by engineers. The solution and the results of analysis by FEM in 2D are reliable and valid. However, reliability and validity of 3-dimentional analysis through finite element method is still a point of consideration. 3D modeling in FEM is more complicated with respect to both solution method and realistic boundary conditions in the third dimension (Hicks & Spencer 2010). Moreover, Hammah et al. (2004) and Wei et al. (2010) concluded in their researches that FE can capture well with relatively deep slip surfaces. The most advantages of FEM can be briefly summarized as follow:

2. There is no pre assumption about failure shape and location of the failure surface.
3. Advanced and complex behavior of the soil or rock can be modeled in material properties.
4. Since there is no concept of slices in the finite element approach there is no assumption about slice side forces. (Griffiths and Lane 1999).
FEM uses strength reduction method (SRM), to calculate/simulate failure limit state of slope and safety factor. The SRM is based on progressive reduction of strength parameters of soil; $\varphi$ and $c$, until the failure of slope occurs. The dilatancy angle is enabled during this process and the total pore pressure contains two separate pressures:

1. Steady state pore pressure: generated according to the water situation in the existing soil layers.
2. Excess pore pressure: generated in the undrained soils.

The reduction in strength parameters has limitation. The friction angle cannot be smaller than the dilatancy angle, $\psi$, and in the case that analysis need to reduce the friction angle again the dilatancy angle starts to decrease to the same amount.

The factor; Total Multiplier ($\sum MSF$) represents soil strength parameters in each process or phase of construction.

$$\sum MSF = \frac{\tan \alpha_{input}}{\tan \alpha_{Reduced}} = \frac{c_{input}}{c_{Reduced}} \quad (\text{Eq. 5})$$

$MSF$ is set to 1 at the start of calculation after that the analysis for safety calculation starts through load advancement number of steps procedure. The incremental multiplier with the default number of 0.1 at the beginning is applied in order to determine amount of the strength reduction and the reduction in strength parameters continues until all the additional steps are calculated and analyzed. Finally at the failure stage of the slope, the total safety factor is given as follows:

$$SF = \frac{\text{Available strength}}{\text{Strength at failure}} = \text{value of} \sum MSF \text{ failure} \quad (\text{Eq. 6})$$

### 2.2 Analysis programs

#### 2.2.1 SLOPE/W (LEM)

SLOPE/W is a Limit Equilibrium software product, used for stability analysis of earth slopes through Limit equilibrium method. It can effectively analyze problems for a variety of slip surface shapes, pore-water pressure conditions, soil properties, analysis methods and loading conditions. Using limit equilibrium, SLOPE/W can model heterogeneous soil types, complex stratigraphic and slip surface geometry, and variable pore-water pressure conditions using selection of soil models. Slope stability analyses can be performed using deterministic or probabilistic input parameters. Stresses computed by a finite element stress analysis may be used in addition to the limit equilibrium computations. It is one of the most complete slope stability analysis programs available. Beginning an analysis in this program is through definition of the geometry by drawing regions and lines that identify soil layers. Then analysis method, soil properties and pore-water pressures can be chosen and applied. After application of reinforcement loads, trial slip surfaces are created. In the next step stability analysis can be run and the results are presented through display of the minimum slip surface and factor of safety.
2.2.2 PLAXIS 2D (FEM)

Plaxis 2D is a finite element software, specifically used for stability and deformation analysis in geotechnical applications. The program uses a convenient graphical user interface that enables users to quickly generate a geometry model and finite element mesh based on a representative vertical cross section of the situation hand. The problem can be modeled either by a plane strain or an axisymmetric model. The program has advantageous feature that enable user to choose different soil model which is dependent on mechanical deformation behavior of soil for the simulation. The models include Mohr-Coulomb, joint rock, hardening soil, soft soil and modified cam-clay model. Standard boundary conditions are automatically generated by the program. Finite element mesh is easily generated from the input 2D geometry model. Automatic mesh generator with the bandwidth optimizer for the finite-element discretization allows generating finite element mesh (of thousands of element) with option for mesh refinement.

The calculation program is the part of the whole simulation where the analysis of the generated model is performed. The procedure is through definition/calculation of the staged construction step (steps that the model is build up). The program offers three types of calculation for the user in each construction phase: plastic, consolidation and safety. Before final calculation (whole problem), the user can choose specific points that load-displacement curves, stress path and stress strain curves can be generate for those points in output part. The program produces outputs of: deformed mesh of the model, different types of deformation and strain, effective and total stress. Complex finite element models can be generated easily through the program due to relatively simple graphical input procedure and the enhanced output facilities make available a detailed presentation of computational results. (Reference; text above are taken from Plaxis manual)

2.2.3 PLAXIS 3D (FEM)

Plaxis 3D is a finite element package intended for 3-dimensional analysis of deformation and stability in geotechnical engineering. It is equipped with features to deal with various aspects of complex geotechnical structures and construction processes using robust and theoretically sound computational procedures. Complex geometry of soil and structures can be defined in two different modes. These modes are specifically defined for Soil or Structural modeling. Independent solid models can automatically be intersected and meshed. The staged constructions mode enables a realistic simulation of construction and excavation processes by activating and deactivating soil volume clusters and structural objects, application of loads, changing of water tables, etc. Since the program is quit new program in geotechnical engineering field a detailed tutorial of the programs is explained in this part. The program offer flexible and interoperable geometry, realistic simulation of construction stages, a robust and reliable calculation kernel, and comprehensive and detailed post-processing.

New features with the Plaxis 3D include the K0 procedure, consolidation analysis, the soft soil creep (SSC) model, the possibility to prescribe volumetric strains in soil clusters an improvements in the pore pressure generation procedure. Further features include the output of results in stress points and the generation of curves. The geometry is modeled using a top view approach. The input of soil data, structures, construction stages, loads and boundary conditions is based on convenient CAD drawing procedures, which allows for a detailed and accurate modeling of the major geometry. From this geometry a 3D finite element mesh is generated.

Soil layers are defined by means of boreholes. Multiple boreholes can be placed in the geometry to define a non-horizontal soil stratigraphy or inclined ground surface. Plaxis automatically interpolates layer and ground surface positions in between the boreholes. Structures are defined in horizontal work planes. The program allows for an automatic generation of unstructured 2D finite element meshes based on the top view. The 2D mesh generator is a special version of the
triangle generator. There are options for global and local mesh refinement. From this 2D mesh, a 3D mesh is automatically generated; taking into account the soil stratigraphy and structure levels as defined in the bore holes and work planes. Quadratic 15-node wedge elements are available to model the deformations and stresses in the soil. The program allows for various types of loads (point loads, line loads, and distributed loads) that could be applied in the model. Different loads and load levels can be activated independently in each construction stage.

Behavior of the structure may be defined as linear elastic material orthotropic or as non-linear elastic force-deformation curves. This applies to embankment in this case study. Pore pressure distributions may be generated on the basis of the input of water levels or pore pressure distributions in the bore holes.

Plaxis3D distinguishes between drained and undrained soils to model permeable sands as well as almost impermeable clays. Excess pore pressures are computed when undrained soil layers are subjected to loading. The program is automatic step-size selection mode. This avoids the need for user to select suitable load increments for non-linear calculations by themselves and it guarantees an efficient and robust calculation process.

The program enables a realistic simulation of staged construction by activating and deactivating clusters of elements, application of loads, changing of water pressure distributions, etc. This procedure allows for a realistic assessment of stresses and displacements as caused, for example, by the construction and loading of an embankment founded on deposit soils. Moreover the program has the ability for consolidation analysis; the decay of excess pore pressures with time can be computed using a consolidation analysis. A consolidation analysis requires the input of permeability coefficients in the various soil layers. Automatic time stepping procedures make the analysis robust and easy-to-use. Presentation of results is through 3D graphical features for displaying computational results. Exact values of displacements, stresses, strains and structural forces can be obtained from the output tables. (Analysis of deformations in soft clay due to unloading, Ismail & Teshome, 2011)

2.3 2D vs. 3D

Beside different methods of stability analysis, dimension of analysis is another important concept that can significantly affect the result of calculation. Regarding complexity of model geometry and diversity of material properties and behavior, significant or negligible differences can occur between 2D and 3D analysis results for safety factor. Because failure of almost all natural embankments and slopes, occur 3-dimensional, 2-dimensional assumption of failure surface (for these cases) would be a conservative and considerable simplification of the reality. Therefore, 3D modeling and analysis type find its added value in engineering calculations. In 2D modeling, the primary assumption to simulate a 3D embankment is to apply an infinite width of the model in Z direction and naturally any 3-dimensional behavior of failure surface can be easily neglected. However, it is seems that by neglecting real geometry of embankment in modeling program, it would not result in real geometry of failure surface in analysis results. Since properties of soil are not homogeneous in the 3rd direction (despite what is assumed in 2D programs), it is more realistic to create and analyze 3-dimensional models in order to reflect the real behavior of slope. However, Duncan (1992) stats that in general, 2-dimensional analysis due to yielding a conservative estimation of the safety factor is more appropriate for slope design. It is conservative because the end effects are not included in 2D estimation of safety factor (51th Annual Geotechnical engineering conference, Timothy D.strak).
Since 3D type of modeling and analysis is a new calculation step for geotechnical engineers and its validity is not in practice well examined and also as mentioned before 2D results seems almost reliable and valid for most of the problems, therefore 3D analysis is suggested by engineers as the back calculation for 2D one.

Auvinet et al. (2000) mentioned in their research to the situations that 3D analysis can be required to be performed:
1. Short slopes for which boundary conditions cannot be ignored such as earth dams built in a narrow valley.
2. When soil is submitted to concentrate loading.
3. When soil properties vary significantly along the longitudinal direction of the slope.
4. When potential failure surface is irregular.

2.4 Load (drives the instability)

Loads on slopes can be imposed by either construction activities or operational conditions. There are two types of external loads: short term loads such as traffic load, excavators, piling equipment and long term load such as construction of a building. Every type of load depending on its long lasting will affect drainage condition of the soil.

2.5 Drained vs. Undrained

The concepts of drained and undrained conditions are of fundamental importance in the mechanical behavior of soil. The difference between drained and undrained condition is time domain. Undrained signifies a condition where changes in loads occur more rapidly than water can flow in or out of the soil. The pore pressure increase or decrease in response to the changes in loads. (Soil strength and slope stability, Duncan & Wright 2005)

Drained signifies a condition where changes in load are slow enough, or remain in place long enough, so that the water is able to flow in or out of the soil, permitting the soil to reach a state of equilibrium with regard to water flow. The pore pressures in the drained condition are controlled by the hydraulic boundary conditions, and are unaffected by the changes in load. (Soil strength and slope stability, Duncan & Wright 2005)

Modeling undrained behavior of soil is a difficult issue in Plaxis; different options exist to model the soil:
1. Undrained A: Uses an effective stress approach, but the strength is modeled with effective parameters.
2. Undrained B: Uses an effective stress approach, but the strength is modeled as undrained shear strength.
3. Undrained C: Uses a total stress approach in which all parameters are defined undrained.

However, in Slope/W program, the undrained strength option is a convenient way of setting $\phi$ to zero in the Mohr-Coulomb model. With this option, the shear strength of the material is only described by the $C$ value and the pore-water pressure has no effect to the shear strength of the material.

2.6 Previous studies

This section provides a summary of the past researches and studies related to two or three dimensional slope stability analysis.
A detailed summary of study over stability of landfill cap, which is an important issue in landfill design, have been conducted by Belczyk and Smith (2012). A generally applicable limit equilibrium (LE) analysis that can account for local slope failure was presented and calibrated against LE analysis according to Keener and Soong (2005) and with computational limit analysis (CLA) and FE analysis. The purposed scenario encompasses a cover of uniform thickness, a cover of tapered thickness, a buttressed cover, the effect of seepage forces and construction equipment. The results showed that the LE method presents a reasonable estimate of landfill cap stability for most of cases but not a conservative result in comparison with FEM and CLA. They concluded that in complex geometries, numerical FE method provides a better approximation of SF and failure mechanism.

Yang et al. (2012) have performed a study over stress distribution and development within Geo synthetic-reinforced soil slopes. Numerical simulations were done to search distribution of backfill stress and reinforcement tensile loads in geo synthetic-reinforced soil slope. The main achieved results stated in their paper were mentioned as: 1.) "backfill stress increases with load and propagates along the potential failure surface.” 2.) “Mobilization of stress was non-uniform along the failure surface.” 3.) "Numerical results show that the initiation of soil softening and the failure of the slope occurred earlier in the slope model with low backfill density.” 4.) ” The limit equilibrium analysis over estimates the SF at each loading increment, compared with those obtained by finite element analysis. The use of actual mobilized reinforcement loads in finite element analysis provides a more realistic calculation of the FS used to represent mobilization of soil strength.”

Salokangas and Vepsalainen (2009) performed a comparative study to investigate stability of an old railway embankment built on very soft ground. Stability of this embankment which is located in southern Finland was investigated by increasing axel load and calculation and comparison of safety factors, through LEM and FEM. All calculations were done under undrained shear strength parameters of the soft layer and Plaxis and Slope/W were chosen as the analysis program. The challenging point of this project was location of the embankment over very soft clay ground which made the analysis more complicated and sensitive. Result of the study provided noticeable outcome in the embankment stability concept. It specified that independent of method or type of analysis, soil strength as a unique factor has the most important role in stability of embankment and slopes containing undrained shear strength of the soil are less stable than slopes containing effective strength parameters. They also faced a confusing matter related to study of LE calculations based on effective parameters. In LE type of modeling, the pore pressure was kept constant and determined at the measuring instant, during the train loading however; in reality, this pressure would increase during this time up to failure occurs. As a solution and based on the results of this study Plaxis, due to ability of defining the excess pore pressure of water in fully saturated conditions, was recommended as a suitable program.

Another different but interesting applied project in slope stability analysis through FEM names: “slope stability analysis of volcano sediments undercut by cellars with FEM analysis”. In this research the engineering geological evaluation of the steep slopes of the volcanic avers hill; an urban area in north Hungary, was modeled and analyzed through FEM. This volcanic area is densely built-in with small houses. What increases the geological risk of this area is extensive distribution of cellars and heterogeneity of geological formations. The laboratory analysis and field tests have been done in order to achieve the primary input parameters needed for the computer modeling. Stability of slope was analyzed on the basis of FE method by Plaxis 8.2. The reason is rooted in ability of the program in considering the plasticity and heterogeneity in the material, though it could examine the failure probability of the considered slope. Results of the analysis showed that:” geological layers weakened by cavities are affected significantly by the
The safety factor of the geological layers in the dry stage concerning the calliers is 1.3, while slopes without any cavities have the safety factor of 2.1” (Vamos and Kozak 2011). The main approach of the study was made clear by succession of identifying the landslide prone zone of a slope containing tuffitic and sandstone layers, weakened by cavities and cellars.

In addition, the undrained bearing capacity of a strip footing resting near the edge of a slope has been studied by Shiau et.al. (2011). Rigorous bounds on the bearing capacity of footings on slopes with a large variation of parameters have been studied by FE limit analysis formulations of the lower and upper bound theorems. The results have been presented through the factors of unit weight, γ and footing width, B. The results indicate that: “There exists a critical value of strength ratio that separates two types of failure: bearing capacity failure and slope failure. This critical value of \( \frac{c_u}{\gamma B} \) is an important parameter in the design of foundations located near slopes.”

In the last decades, most slope stability analyses were performed using (2D) method. The simplicity assumed in this method is expecting width of slip surface infinite and neglecting (3-D) effects of slide mass. Obviously, slopes are not infinitely wide and 3-D effects influence the stability of most, if not all, slopes. Influence of heterogeneity of undrained shear strength on the reliability and failure of a long 3D slope in clay was the research done by Hicks et al. (2010). A random field theory was used to define the concept of heterogeneity and FEM was applied to analyze the slope reaction (behavior). The results showed 3 different failure modes depending on the ratio of horizontal fluctuation scale to the dimension of the slope, length of slope and height of it:

1) Small ratio: The failure happens along the entire length of the slope and there is no significant difference in results of 2D in comparison with 3D.
2) Intermediate ratio: Discrete failures happen and stability is dependent on the length of the slope
3) Large ratio: The variability takes on a layered appearance and the result is equivalent to a 2D stochastic analysis (Cheng et al. 2006)

As a final point, they have argued that independent of dimension of slopes geometry, load condition and mean strength profile, most slope failures are 3D due to presence of heterogeneity.

3D analysis of stone columns to support a roadway embankment on soft soil is another interesting study being examined by Koch (2011). Reinforcement effects of stone columns in a 3D setting were investigated through a 3-dimensional FE modeling program names Novella. Undrained shear strength was varied under a specific range while, slope angle, material properties, thickness of soft band and embankment’s height were kept constant for all cases. The results were logical and acceptable for the relation between SF and surface of failure. They concluded that:” Up to certain shear strength of the subsoil an undercutting slip plane is the dominant failure mechanism, beyond a threshold shear strength the failure occurs in the slope.”

Cheng et.al (2006) investigated a comparative slope stability study, where FE analysis was compared with strength reduction method (SRM) for a long slope. The comparisons were been done between the safety factor and critical failure surface results of the slope through these two methods. The results from FEM were in good agreement with SRM, except when \( \phi \) was zero. Additionally, some parameters of soil such as soil module and domain size were changed also, but SRM results of SF did not show any sensitivity. Finally it as concluded that SRM is sensitive to none linear solution algorithm for the case of fifth soft band with frictional material.

Consequentially, it can be seen that great progress had been made within past years in development of geostatistical methods and design methods. The common task through past researches was investigating positive and negative points of (stability) analysis methods for different slope conditions especially for short term stability of embankments founded on soft clay. In fact, the
main uncertainty behind all these examinations was reliability of stability calculation results with accordance to the methods used. It also shows that recently, with development of computational programs more studies are focusing on challenging new 3D methods of analysis in calculations. (Hence, this research has laid the ground work on extension of 2dimentional methods of slope stability analysis to the 3dimentional case). A comparative study through Plaxis2D, Plaxis3D and Slope/W program is performed to investigate stability of slopes with soft layer.
3 Problem definition

3.1 Introduction
There are many ongoing infrastructural projects on typical Scandinavian geology; i.e. clay deposit based on frictional soil on top of bedrock. Such structures include road or railway embankments. It is also often the cases that these embankments are rapidly constructed, which particularly have a negative influence on the short term stability. Therefore, it is of particular interest to determine/estimate appropriate stability of the specified embankment which is highly dependent on the analysis method used. Hence, it is a common task faced by geotechnical engineers to judge which analysis method can simulate better the reality. These problems are central in geotechnical engineering and involve together a main part of relevant questions and complement each other in covering the subject.

The ambition of this thesis was to illustrate and examine the capability of the three numerical programs: Slope/W, Plaxis2D and Plaxis3D to simulate and stability analysis of embankments on soft soil deposit. It deals with models of different accuracy and precision. Moreover, application of Plaxis3D through using FEM3D offers an attractive alternative to traditional approaches to the problem (especially for Slope/W). Plaxis3D uses recent advances in FEM to perform analysis of the problem. Since the thesis work is performed in Sweden the input soil parameters are in the range of Sweden’s field measurements.

Three following factors were studied more in depth within the comparison analysis:

1) Method of analysis: FEM, LEM
2) Geometry of modeling and analysis: 2D, 3D
3) Drainage condition: drained, undrained

3.2 Case configurations
In this part the total four analyzed cases are presented for embankments founded on soft clay deposit. Two types of long and short embankments were created. In cases 1, 2 and 3 embankments of uniform cross section and in case 4 embankment of inconsistent cross section were created. Computations were performed via modeling of each model case through 3 specified programs. Dependent on the studied cases, range of plastic parameters for clay, load geometry/quantity and embankment length were applied through different model cases and obtained SF via LEM, FEM2D and FEM3D were compared through sets of stability charts provided. The four cases analyzed are presented namely as follow:

Case1: Short Emb. ; ud clay; q = 20 kPa: Fig. 3-1.
Case2: Long Emb. ; d. clay; q = 20 kPa: Fig. 3-2.
Case3: Short Emb. ; d. clay; q = 20 kPa: Fig. 3-3.
Case4: Long Emb. ; ud clay; inconsistent clay thickness; q = 50 kPa: Fig.3-4.

Emb. is an abbreviation of embankment, ud. as undrained and d. as drained condition of the soil. The adopted parameters for the analysis of four specified cases of this study are summarized in
Problem definition

Tab. 3-1. and further described in detail in following sections. Tab. 3-1. presents a summary of the studied cases in relation to the varied parameters for the comparison of how parameter affects safety analysis. The varied parameters are presented as $f(xI)$ as a function of SF. Tab. 3-2 presents the summary of the case configurations in relation to the analysed methods.

**Tab. 3-1 Summary of case studies in relation to varied parameters.**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Varied parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Emb. ud. clay</td>
<td>$(f(c_u))$</td>
</tr>
<tr>
<td>$q=20$ kPa</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>Varied parameters</td>
</tr>
<tr>
<td>Long Emb. d. clay</td>
<td>$(f(B))$</td>
</tr>
<tr>
<td>$q = 20$ kPa</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>Varied parameters</td>
</tr>
<tr>
<td>Long Emb. d. clay</td>
<td>$(f(c', q'))$</td>
</tr>
<tr>
<td>$q = 20$ kPa</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>Varied parameters</td>
</tr>
<tr>
<td>Short Emb. ud. clay</td>
<td>$(f(y))$</td>
</tr>
<tr>
<td>$q=50$ kPa</td>
<td></td>
</tr>
</tbody>
</table>

**Tab. 3-2 Summary of Case configurations analysed.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Load [kPa]</th>
<th>Embankment height [m]</th>
<th>Geometry</th>
<th>Drainage condition</th>
<th>LEM</th>
<th>FEM2D</th>
<th>FEM 3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>2</td>
<td>Short</td>
<td>ud.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2</td>
<td>Long</td>
<td>d.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>2</td>
<td>Short</td>
<td>d.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>2</td>
<td>Long</td>
<td>ud.</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

3.1.1 Case1: Short embankment- undrained clay

The stability analysis of a short embankment over two-layer deposit soil including clay was carried out, see Fig. 3.1. A distributed load of $q \sim 20$ kN/m$^2$ was imposed with width B $\sim 5$ m and height H $\sim 1$ m at distance D $\sim 1$ m from the edge of the slope. The clay layer is assumed to be undrained. Sand and embankment are assumed as drained (which is the same for all case studies). Ranges of undrained shear strength were considered (for clay) and corresponding stability chart were produced. In parametric study stability analysis carried out for 6 cases, value of $c_u$ (for clay) was applied over the range of 5 - 30 kPa. Meaning that in each case the value of $c_u$ for undrained clay was increased 5 kPa starting from $c_u \sim 5$ kPa. Then 3 mentioned programs were applied to analyses the stability. This study has extended a part of previous research by Cheng et al. (2006) about 2-dimentional slope stability analysis by LEM and SRM.
3.1.2 Case2: Long embankment- drained clay

The stability analysis of a long embankment over two-layer deposit soil including clay was carried out that’s illustrated in Fig. 3.2. All soil materials inclusive clay layer are in drained condition and $C'$ is constant for clay in all cases. A distributed surcharge load of $q \sim 20 \text{kN/m}^2$ is imposed with width $B \sim 4$ and height $H \sim 1 \text{m}$ at distance $D \sim 1 \text{m}$ from the embankment crest. Load length is 30 m in $Z$ direction. In parametric study stability analysis carried out for 6 parameters, in each case load was located in distance $L$ from the embankment edge (in z direction). The distance varied over the range of $L/Z \sim 0.1$ till 0.6, which $Z$ is length of the embankment in third dimension (see Fig 3.3) and corresponding stability chart were produced for the specified range of $(L/Z)$. The aim of analysis was to determine the relation between $(L/Z)$ and SF. By changing $L/Z$ in a range of: 0.5, 1 and 1.5.

It should be noted that Plaxis 2D and Slope/W due to 2D (Plain strain) simulation of the real 3D model can create and analyze just one case(x-y view) however, a number of 6 models were created in FEM3D.

This part of study was inspired by Merifield and Lyamin’s, research about:” Limit analysis solution for three dimensional undrained slopes”. For a range of depth factor $(d/H)$ stability of a long 3D slope was analyzed.
3.1.3 Case 3: Short embankment- drained clay

The stability analysis of a short embankment founded on two-layer deposit soil including drained clay was carried out as illustrated in Fig. 3.2. A distributed load of $q \approx 20$ kN/m$^2$ was imposed with width $B \approx 3$ m and height $H \approx 1$ m at distance $D \approx 1$ m from the edge of the slope. All soil materials are in drained condition (Model geometry and load situation were the same as Case 1).

In the parametric study, 6 different models were considered and different shear strength properties were used for clay. For each value of $c'$ for clay value of $\varphi'$ was varied from 25, 28, 30 and 33-35 which is the common range for $\varphi'$ in Sweden. The cohesion $c'$ of clay varied from 0.5, to 3 kPa. Finally FEM2D, FEM3D and LEM analysis were carried out for 30 modeled cases of this study. Corresponding stability chart were produced. This study has extended a part of previous research by Cheng et al. (2006) about two dimensional slope stability analysis by LEM and SRM. In that study, the factor of safety and location of critical failure surfaces obtained by LEM and SRM were compared for various slopes through variation in value of soil parameters.
3.1.4 Case 4: Long embankment- inconsistent clay layer thickness

A study on the effect of the thickness of horizontal soil deposit profile on stability and failure mechanism was performed by varying the thickness of clay from 2 to 4 m. Stability analysis of a long embankment over two-layer deposit soil with inconsistent thicknesses of the soil layers along Z direction was studied, see Fig. 4.4. The soft band’s thickness; Clay, was 2 m at Z=0 m and increased to 4 m at the other side of the model; Z=100 m. Soil properties were presented in Tab. 3-3. A distributed load of $q=50 \text{kN/m}^2$ affected over the embankment with width $B\approx4 \text{ m}$ and height $H\approx1 \text{ m}$ at distance $D\approx1 \text{ m}$ from the crest of the embankment. Load length is 50 m located from $Z\approx20 \text{ m}$ till 70 m. Clay is in undrained condition; sand and embankment are in drained condition.

The problem of modeling and stability analysis of a 3D slope with inconstant soil layers thickness is considered in this part. Since soil layers geometry change along Z dimension and with acknowledge to the fact that in 2D analysis due to plain strain simulation (x-y view) and infinite assumption of models third dimension with consideration of the constant section profile, analysis of this problem goes through problem. In order to solve the problem in 2D analysis, vertical cut in every 10 m of the model length was created and SF was calculated for each cut. Applying several cross sections for 2D analysis provided a reasonable assessment of 3D effect. At the other hand in Plaxi3D program, FEM3D dealt up well with the problem with complex geometry and the whole structure was modeled and analyzed in one case.

A range of $(L/Z)$ was considered and corresponding stability chart were produced. Z is length of the embankment in third dimension and L is distance of the cut in Z direction. The aim of
3.3 Embankment and surcharge load

Studied embankment resembles a situation like new road or railway embankment. It is made of crushed rock, raised to a height of 2 m above the subsoil surface. Angle of $\beta \sim 29.7^\circ$ is considered for the embankment toe. The embankment is assumed to rest on a firm base and is characterized by 4 m clay and 2 m sand under it. It should be noted that, even though embankment’s load does impose stress increase in the deposit soils and also construction type is considered as staged construction (which lead to drainage and consolidation with time), time domain was not considered in analysis to simplify calculation process.

The loads were placed along the axis of symmetry; 1 m behind crest of the embankment. In case 1 to 3 distributed loads of 20 kN/m$^2$ with different geometries and influence surfaces were imposed to the embankment as depicted in Fig. 3.5. In case 4, distributed loads of 50 kN/m$^2$ was assigned to the embankment, in order to affect more on the stability and safety factor. In Plaxis 2D and Slope/W due to plain strain simulation of the model surface load were applied to simulate the reality; however, in Plaxis 3D surface load were created to simulate the loads. Tab. 3-3 contains a summary of geometry and quantity for the applied loads.
Summary of loads applied to different cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Load</th>
<th>Z [m]</th>
<th>H [m]</th>
<th>B [m]</th>
<th>D [m]</th>
<th>Load length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>20</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Case 2</td>
<td>20</td>
<td>100</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>Case 3</td>
<td>20</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Case 4</td>
<td>50</td>
<td>100</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>50</td>
</tr>
</tbody>
</table>

Fig. 3.5 Embankment and load geometry.

### 3.4 Geometry
Geometry is according to Cartesian X, Y, Z coordinate system for 3D program and x, y coordinate system for 2D programs as depicted in Fig 3.5. Water level is 1 m below the clay top. All soil types are modeled through Mohr-Coulomb yield criterion in Plaxis. In Geo slop all soils are modeled as Mohr-Columb except in undrained condition of clay which was modeled as undrained (\(\phi=0\)). Embankment width; X, is 37 m with 2 m height. Length of embankment; Z, is 100 m and 5 m respectfully for long and short embankments. In order to decrease the total number of elements symmetry is exploited for modeling cases.

### 3.5 Soil properties
The soils are modeled as linear elastic perfectly plastic utilizing Mohr-Coulomb yield criterion. Table 3-3. contains list of mechanical properties for soil units. As it is presented, embankment and sand properties are kept constant for all studied cases. The soil parameters are obtained from field and laboratory tests and different empirical approaches in Sweden.
Problem definition

Tab 3-3. Mechanical properties of soil units.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma_{s\text{at}}$ [kN/m$^3$]</th>
<th>$\gamma_{u\text{un-sat}}$ [kN/m$^3$]</th>
<th>$\phi$ [$^\circ$]</th>
<th>$E$ [kPa]</th>
<th>$\nu$ [-]</th>
<th>$C$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embankment</td>
<td>19</td>
<td>18</td>
<td>35</td>
<td>35000</td>
<td>0.33</td>
<td>1</td>
</tr>
<tr>
<td>Sand</td>
<td>20</td>
<td>17</td>
<td>34</td>
<td>30000</td>
<td>0.34</td>
<td>2</td>
</tr>
<tr>
<td>Clay-case1</td>
<td>17.5</td>
<td>17</td>
<td>0</td>
<td>2500</td>
<td>0.35</td>
<td>5-30</td>
</tr>
<tr>
<td>Clay-case2</td>
<td>17.5</td>
<td>17</td>
<td>28</td>
<td>2500</td>
<td>0.35</td>
<td>3</td>
</tr>
<tr>
<td>Clay-case3</td>
<td>17.5</td>
<td>17</td>
<td>25-35</td>
<td>2500</td>
<td>0.35</td>
<td>0.5-3</td>
</tr>
<tr>
<td>Clay-case4</td>
<td>17.5</td>
<td>17</td>
<td>0</td>
<td>2500</td>
<td>0.35</td>
<td>25</td>
</tr>
</tbody>
</table>

3.6 Program modeling

Three different computer programs: Slope/W, Plaxis 2D v8.6 and Plaxis 3D were used to calculate stability of embankments located over a soft layer cohesive soil. Different load, material and geometry conditions were defined to make the comparison analysis. Due to assumed symmetry condition, only one half of the problem was modeled for analysis.

3.4.1 Slope/W

The analysis through Slope/W was based on Morgenstern-Price half-sine function method that deals with both moment and force equilibrium equations. The calculation was done via $\phi = 0$ and $C - \phi$ methods dependent on the drained or undrained condition of the clay layer. Effective strength parameters and effective cohesion were applied through application of $C - \phi$ method. A distributed line load was also imposed over the embankment but its location and quantity varied, depending on the case study situation. A piezometric line with RU bar was applied to model pore water pressure and the tolerance for location of the critical failure surface was assigned 0.0003, which was sufficiently accurate for the present study. Grid-radius function was selected as the slip surface method of analysis. $SF$ distribution was considered constant and no crack tension was allowed.

3.4.2 Plaxis2D

Analysis in Plaxis2D includes 3 phases: input, calculation and output phase. In this section the first two phases are presented and the last phase is discussed in the result section. Following assumptions were considered for modeling and analysis within the 2D program:

**Input phase:** Elasto-plastic Mohr-Columb model was applied to simulate soil behavior and material model. For undrained analysis in case 1 and 4, drainage condition of clay layer was selected as “undrained B”. The K0 determination was selected as automatic option. The strength interfaces were modeled using the option “rigid interface” available in FE software. Creation of embankment was done in the same step after soil layer creation and respectively external load was modeled in the same stage after creation of embankment. Standard fixities were applied for the bottom and sides of the model geometry. In other words, boundary conditions were set up in a mode that vertical and horizontal displacements were zero at the bottom of model (pinned) and free at top surface of it. Interface between the soil and embankment was assumed rigid.

Once the geometry model and boundary conditions were defined, automatic mesh generation was applied with the bandwidth optimizer for the finite-element discretization refinement. The program automatically applies quadratic 3 node and 6 node triangular mesh elements to model deformations and stresses in the soil. Mesh generation type was selected as medium in order not to make the analysis time taking.
**Problem definition**

**Calculation phase:** After mesh generation, calculation phase started through building up initial stresses within the deposit soils; phase 0. For cases: 1, 2 and 3, K0-procedure was chosen as the calculation type for the initial stage. If soil layers in the profile are not horizontal, which is the situation in case 4 of the thesis; the initial case has to be generated via gravity loading option. Weight multiplier was set as 1. The ground water level has to be set in initial phase and it was set to 1 meters above the bottom surface of clay; \( y \approx 3 \). In the next phase; phase1, the embankment was added to the model and calculation type was selected as plastic or plastic drained dependent on undrained or drained condition of clay in studied cases. In order to eliminate unrealistic deformations caused by generation of initial stress, displacement was set to zero. Thereafter, phase 2 was created through imposition of distributed load to the model. Calculation type was selected plastic as in phase 1. In order to simplify the calculation, no time was allocated for the soft layer to be consolidated. In other words, \( t = 0 \) for loading time interval and consolidation. In the last phase; phase 3, calculation type was applied as safety in order to determine collapse loads and calculate the stability. The program used \( \sum MSF \) to specify increase of the strength reduction. A number of 250 additional steps were applied for all analysis to keep the calculation safety high.

**3.4.3 Plaxis3D**

Analysis in Plaxis3D includes 3 phases: input, calculation and output phase. In this section the first two phases are presented and the last phase is discussed in the result section. Following assumptions were considered for modeling and analysis within the 3D program:

**Input phase:** Three-dimensional geometry model was defined through work planes and boreholes. Elasto-plastic Mohr-Columb model was applied to simulate soil behavior and material model. For undrained analysis in case 1 and 4, drainage condition of clay layer was selected as “undrained B”. The K0 determination was selected as automatic option. The ground water level was defined at \( y \approx 5 \) (1 m below the clay top surface), through the borehole creation step. In the next phase strength interfaces were modeled using the option “rigid interface” available in FE software. In Plaxis 3D, embankment is identified as a structure which can be created after soil modeling in a separate step names; structure. However in Plaxis 2D as explained above, creation of embankment was done in the same step after soil layer creation. External load modeling was performed in same stage of embankment creation through surface load option. Standard fixities were applied for the bottom and sides of the model geometry and hydraulic condition was set up through surface drain option.

Once soil and structure steps were identified, full automatic mesh generation was applied with the bandwidth optimizer for the finite-element discretization refinement. Mesh element distribution was selected as medium in order not to make the analysis time taking. Afterward, water level and initial stress were generated in a separate step named: water level. The boundary conditions for ground water flow were set as open for \( x \) and \( y \) direction but closed for through \( z \) direction. Pore pressure calculation type was set as phreatic. In Plaxis 3D contrary to Plaxis 2D the stability of the system under specified water pressure was calculated in consistent with staged construction of model.

**Calculation phase:** After mesh and water pressure generation, calculation phase started through building up initial stresses in the calculation step. For cases: 1, 2 and 3, K0-procedure was chosen as the calculation type for the initial stage. If soil layer in the profile are not horizontal, which is the situation in case 4 of the thesis; the initial case has to be generated via gravity loading option. Weight multiplier was set as 1 in phase 1, the embankment was added to the model and calculation type was selected as plastic or plastic drained dependent on undrained or drained condition of clay in studied cases. In order to eliminate unrealistic deformations caused by generation of initial stress, displacement was set to zero. Thereafter, phase 2 was created through imposition of 3D distributed load to the model. Calculation type was selected plastic as in phase 1. Time interval was considered zero for loading input in all phases. In the last phase; phase 3, calculation type was applied as safety in order to determine collapse loads and calculate the
stability. The program used $\Sigma_{MSF}$ to specify increase of the strength reduction. Loading input for MSF was applied as 0.1. A number of 150 additional steps were applied to keep the calculation safety high.
Problem definition
4 Results

4.1 Introduction
This chapter presents results of numerical modeling through stability charts (via SF) and failure surface shapes. The 2D and 3D charts for the embankments founded on soft clay obtained from FEM2D, FEM3D and LEM are identified and explored in this part for a range of $\phi$, $c$, $B$, $Z$ and $\theta$. The failure surfaces are presented in Appendix A.

4.2 Case study results

4.2.1 Case1: Short embankment- undrained clay
Results of stability analysis for short embankment founded on undrained clay with imposition of $q=20$ kN/m$^2$ distributed load are presented in Tab. 4-1. Value of $C_u$ (for clay); was applied over the range of 5 to 30 kPa and corresponding stability chart were produced.

<table>
<thead>
<tr>
<th>$C_u$ [kPa]</th>
<th>SF [LEM]</th>
<th>SF [FEM2D]</th>
<th>SF [FEM3D]</th>
<th>SF difference with LEM [FEM2D %]</th>
<th>SF difference with FEM3D [LEM %]</th>
<th>SF difference with FEM3D [FEM2D %]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1.10</td>
<td>1.003</td>
<td>1.31</td>
<td>8.8</td>
<td>16</td>
<td>23.4</td>
</tr>
<tr>
<td>15</td>
<td>1.551</td>
<td>1.460</td>
<td>1.848</td>
<td>5.9</td>
<td>16.04</td>
<td>21</td>
</tr>
<tr>
<td>20</td>
<td>1.760</td>
<td>1.620</td>
<td>1.974</td>
<td>7.9</td>
<td>10.8</td>
<td>17.7</td>
</tr>
<tr>
<td>25</td>
<td>1.760</td>
<td>1.750</td>
<td>1.975</td>
<td>0.56</td>
<td>10.89</td>
<td>11.4</td>
</tr>
<tr>
<td>30</td>
<td>1.760</td>
<td>1.742</td>
<td>1.972</td>
<td>1.02</td>
<td>10.75</td>
<td>11.66</td>
</tr>
</tbody>
</table>

4.2.2 Case2: Long embankment- drained clay
Results of stability analysis for long embankment founded on drained clay with imposition of $q=20$ kN/m$^2$ distributed load are presented in Tab. 4-2. Variation in location of external load was the base factor to make the comparison analysis. Results of the analysis; SF, are presented as a function of $L/Z$. For the distributed load of $q=20$ kPa ratio of the loading distance; $L$, to the total length of slope; $Z$, varies from 0.1 to 0.

<table>
<thead>
<tr>
<th>$L/Z$</th>
<th>SF [LEM]</th>
<th>SF [FEM2D]</th>
<th>SF [FEM3D]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.765</td>
<td>1.75</td>
<td>1.987</td>
</tr>
<tr>
<td>0.2</td>
<td>1.765</td>
<td>1.75</td>
<td>1.989</td>
</tr>
<tr>
<td>0.3</td>
<td>1.765</td>
<td>1.75</td>
<td>1.989</td>
</tr>
<tr>
<td>0.4</td>
<td>1.765</td>
<td>1.75</td>
<td>1.99</td>
</tr>
<tr>
<td>0.5</td>
<td>1.765</td>
<td>1.75</td>
<td>1.954</td>
</tr>
<tr>
<td>0.6</td>
<td>1.765</td>
<td>1.75</td>
<td>1.46</td>
</tr>
</tbody>
</table>
4.2.3 Case3: Short embankment- drained clay

Results of stability analysis for short embankment founded on undrained clay with imposition of $q\approx 20$ kN/m$^2$ distributed load are presented in Tab. 4.3. The $c'$ varied over a range of 0.5, 1, 1.5, 2 to 3 kPa while $\phi'$ varied from 25 till 35. Results of the analysis; SF, are presented as a function of $\phi'$ and $c'$.

<table>
<thead>
<tr>
<th>$c'$ [kPa]</th>
<th>$\phi'$ [˚]</th>
<th>SF [LEM]</th>
<th>SF [FEM2D]</th>
<th>SF [FEM3D]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>25</td>
<td>1.555</td>
<td>1.492</td>
<td>1.878</td>
</tr>
<tr>
<td>0.5</td>
<td>28</td>
<td>1.623</td>
<td>1.568</td>
<td>1.983</td>
</tr>
<tr>
<td>0.5</td>
<td>30</td>
<td>1.627</td>
<td>1.623</td>
<td>1.963</td>
</tr>
<tr>
<td>0.5</td>
<td>33</td>
<td>1.75</td>
<td>1.707</td>
<td>1.968</td>
</tr>
<tr>
<td>0.5</td>
<td>35</td>
<td>1.765</td>
<td>1.722</td>
<td>1.969</td>
</tr>
<tr>
<td>1</td>
<td>25</td>
<td>1.612</td>
<td>1.507</td>
<td>1.965</td>
</tr>
<tr>
<td>1</td>
<td>28</td>
<td>1.681</td>
<td>1.634</td>
<td>1.965</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>1.73</td>
<td>1.683</td>
<td>1.967</td>
</tr>
<tr>
<td>1</td>
<td>33</td>
<td>1.765</td>
<td>1.735</td>
<td>1.974</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>1.765</td>
<td>1.727</td>
<td>1.970</td>
</tr>
<tr>
<td>1.5</td>
<td>25</td>
<td>1.669</td>
<td>1.616</td>
<td>1.937</td>
</tr>
<tr>
<td>1.5</td>
<td>28</td>
<td>1.738</td>
<td>1.626</td>
<td>1.966</td>
</tr>
<tr>
<td>1.5</td>
<td>30</td>
<td>1.765</td>
<td>1.723</td>
<td>1.971</td>
</tr>
<tr>
<td>1.5</td>
<td>33</td>
<td>1.765</td>
<td>1.725</td>
<td>1.969</td>
</tr>
<tr>
<td>1.5</td>
<td>35</td>
<td>1.765</td>
<td>1.727</td>
<td>1.971</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>1.726</td>
<td>1.663</td>
<td>1.981</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>1.765</td>
<td>1.718</td>
<td>1.965</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>1.765</td>
<td>1.725</td>
<td>1.968</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
<td>1.765</td>
<td>1.725</td>
<td>1.970</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>1.765</td>
<td>1.720</td>
<td>1.971</td>
</tr>
<tr>
<td>2.5</td>
<td>25</td>
<td>1.765</td>
<td>1.705</td>
<td>1.963</td>
</tr>
<tr>
<td>2.5</td>
<td>28</td>
<td>1.765</td>
<td>1.719</td>
<td>1.970</td>
</tr>
<tr>
<td>2.5</td>
<td>30</td>
<td>1.765</td>
<td>1.730</td>
<td>1.972</td>
</tr>
<tr>
<td>2.5</td>
<td>33</td>
<td>1.765</td>
<td>1.723</td>
<td>1.972</td>
</tr>
<tr>
<td>2.5</td>
<td>35</td>
<td>1.765</td>
<td>1.732</td>
<td>1.977</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>1.765</td>
<td>1.723</td>
<td>1.972</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>1.765</td>
<td>1.723</td>
<td>1.972</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>1.765</td>
<td>1.731</td>
<td>1.971</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>1.765</td>
<td>1.731</td>
<td>1.971</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>1.765</td>
<td>1.731</td>
<td>1.971</td>
</tr>
</tbody>
</table>

4.2.4 Case4: Long embankment- inconsistent clay layer thickness

Results of stability analysis for long embankment founded on subsoil with inconstant soil layers thickness (within $Z$ dimension) are presented in Tab. 4.4. nine vertical cut in every 10 meters of the model length were made which stability was calculated for. While soil properties were kept constant, results of the analysis; SF, are presented as a function of $L/Z$ for the distributed load of $q=50$ kPa. Ratio of the loading distance; $L$, to the total length of slope; $Z$, varies from 10 to 90.
**Tab. 4-4. Safety factors under inconstant soft layer thickness.**

<table>
<thead>
<tr>
<th>$L/Z$</th>
<th>SF [LEM]</th>
<th>SF [FEM2D]</th>
<th>SF [FEM3D]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.765</td>
<td>1.771</td>
<td>1.695</td>
</tr>
<tr>
<td>20</td>
<td>1.588</td>
<td>1.567</td>
<td>1.695</td>
</tr>
<tr>
<td>30</td>
<td>1.638</td>
<td>1.571</td>
<td>1.695</td>
</tr>
<tr>
<td>40</td>
<td>1.558</td>
<td>1.461</td>
<td>1.695</td>
</tr>
<tr>
<td>50</td>
<td>1.625</td>
<td>1.462</td>
<td>1.695</td>
</tr>
<tr>
<td>60</td>
<td>1.607</td>
<td>1.472</td>
<td>1.695</td>
</tr>
<tr>
<td>70</td>
<td>1.630</td>
<td>1.460</td>
<td>1.695</td>
</tr>
<tr>
<td>80</td>
<td>1.772</td>
<td>1.768</td>
<td>1.695</td>
</tr>
<tr>
<td>90</td>
<td>1.780</td>
<td>1.777</td>
<td>1.695</td>
</tr>
</tbody>
</table>
5 Analysis, Discussion, Validation

5.1 Introduction
Section 5.2 and 5.3 go through analysis and discussion of the results presented in chapter 4. The focus was on evaluation of programs sensitivity against soil parameters, models geometry and external load. Moreover, comparison of the achieved results with past researches has been argued also. Section 5.3 goes through verification purpose. In order to evaluate results of the analysis programs, SF3D is hand calculated driven from the formulas in “SKREDKOMMISSIONEN, Report 3:95”.

5.2 Case study analysis

5.2.1 Case1: Short embankment - Undrained clay
Comparing results of SF for the three applied programs on short embankment founded on undrained clay with imposition of 20 kPa load, shows that by increasing cohesion from 5 kPa to 20 kPa, value of SF increases respectively also. The smaller the cohesion is the lower the SF is (will be).

When $c_u$ varies from 5 to 20 kPa the SF increases respectively in all the applied methods. But after this content, the results seem to be little insensitive to further increase of the cohesions. It is obvious since the slip surface is located in the embankment. Moreover, SF results from LEM and FEM2D are quite more in convergence under the same combination of soil parameters. Major differences are recognized in the safety factor of FEM2D and LEM in contrast with FEM3D, especially at lower value of $c_u$. It is obvious from Tab. 4-1 that Plaxis 3D, in all cases, gives a larger SF than Slope/W and Slope/W results of SF are larger than Plaxis 2D results. Based on the results of Tab. 4-1, some points can be extracted as follow:

- Quantity of SF results for the three methods apart some exceptions conform the following order:
  - SF (FEM3D) > SF (LEM) > SF (FEM2D)

(1) While friction angle is zero, the average difference of SF between the three applied methods are:
  - SF(LEM) / SF(FEM2D) $\equiv$ 4.03%
  - SF(FEM3D) / SF(FEM2D) $\equiv$ 10.47%
  - SF(FEM3D) / SF(LEM) $\equiv$ 14.07%
(2) In smaller value of \( c_u \) (under 20 kPa), increase in rate of \( c_u \) causes noticeable changes in SF for all three methods of calculations, but as the cohesion exceeds over 20 kPa, fluctuation of SF are almost negligible. In other words, increasing of the \( c_u \) (At the value over 20 kPa), does not enhance the SF since the slip surface is shallow, see Appendix, case2.

(3) In smaller content of \( c_u \) for clay the differences in values of SF between LEM &FEM2D & FEM3D are quite large. However, for higher rates of \( c_u \), values of SF does not exceed by increasing \( c_u \).

The failure surfaces generated by LEM and FEM2D are quite similar in all the cases and the 3D representation of failure by FEM3D seems to be in good harmony with the two other applied methods. In general, shape of failure surface can be categorized into three types: face failure, toe failure and base failure. In this research in smaller cohesion of the soft band, failure surfaces are base failure within all methods. And by increasing the \( c_u \) the failures shape will change to face failure and at the high values of cohesion the failures occur as toe failure. Meaning that, the thickness and the depth of the critical failure surface reduce by increasing the cohesion in soft layer.

Totally, it is a point of interest that even though, these three different programs have quite different result of SF for every unique sample analyzed, but the critical failure surfaces which are represented through these programs are almost in a good harmony with each other.

One point of attention in the failure figures is location of the maximum stress point, see appendix, case2. Especially these areas are better shown in FEM2D and FEM3D. In FEM2D program the red area which shows the max stress point is located in right up part of failure but for the same case in 3D programs the red area is in the surface of the slope.

### 5.2.2 Case2: Long embankment- drained clay

The results are noticeable, as it can be seen in Table 4-2., results of LEM and FEM 2D are quite similar under the same load distribution. FEM2D and LEM show almost same value of SF=1.7. However, FEM3D shows quite safer perspective of stability for a long embankment founded on drained clay. Looking deeper to the results of 3D analysis shows; when the load is situated in the middle of the model, SF has its highest quantity and safest value and as it gets near to the edges of the model, value of SF decreases respectively.

Even though, 3Dimensional analysis can probably show more logical reflection of the reality for representing both failure and SF, but it seems that stability analysis through LEM3D does not show sensitivity to location of external loads (for the case of long embankment founded on drained clay). The value of SF in 3D doesn’t have any sensible variation through change in the load position since the slip surface is shallow, see appendix, case2.

At the other hand, comparing figures of failure surfaces analyzed through FEM3D shows that all the failures occur three dimensional. This means that the 3D program can be better analysis solution for this case.

Putting results of SF and failure surfaces together, makes a challenge for evaluating the best analysis method for the long embankment founded on soft drained clay. While the failure mode occur three dimensional through FEM3D (see Appendix A), the SF results of 3D program are
quid insensitive to the ratio of $B/L$. It seems that solution of the problem can be suggested as parallel analysis through both 3D and 2D programs.

5.2.3 Case3: Short embankment- drained clay

Based on the outcomes of Table 4-3, in almost all the cases, results of SF by FEM2D, FEM3D and LEM are very similar for embankment founded on cohesive frictional clay under different soil combination parameters. The SF results by FEM3D have almost an average difference of 14.4% with FEM2D and respectively 12.3% with LEM. It seems that LEM and then FEM3D have the minimum sensitivity to the soil’s plastic parameter alteration. Though, in all three methods by increasing $\phi'$, SF increases respectively. Besides that, Plaxis 3D gives the largest factor of safety in all cases that can be considered as a kind of regularity in this study.

Comparing results of FEM2D and FEM3D shows that, increasing the $c'$ at low $\phi'$ of the soil (clay), results in decreasing the difference between SF results of these two methods. Initially the SF increases with increase of the cohesion, but the incremental change in SF reduces as the functional of the increasing value of friction angle. At lower cohesion of clay, enhancement of the friction angle affects significantly the SF results, in all 3 applied methods. Referring to the Table 4-3, comparison of $SF_{\text{LEM}}/SF_{\text{FEM2D}}$ for each case shows that at lower value of $c'$ when $\phi'$ is small, this ratio has the highest content and with increase in the plastic parameters this ratio tends to reduces. However at higher $c'$, increase of the $\phi'$ does not enhance value of SF. These SF results between the 2D and 3D analysis, seems quite reasonable since, 3D has a more logical view of the problem by considering the longitudinal direction of the model (as Cavounidis (1987) mentioned in his research also). Moreover through increment of the $c'$ value, the SF results from 2D analysis methods (LEM, FEM2D), get gradually closer to those of 3D analysis, but there is still noticeable difference between the compared values. Contrary to FEM2D and LEM, the FEM3D does not show similar trend of SF change as is presented in Table 4-3.

At the year 2006 a similar study research has been done through LEM and SRM method names; “two dimensional slope stability analyses by LEM and SRM” . The result of that study, dependent on the method of analysis, were different and similar. Most of the SF obtained by SRM were slightly larger than those of LEM. However, in this study SF results of LEM are slightly larger than FEM 2D. It is also remarkable that, at small $c'$ of clay, the difference between SF results of LEM and SRM were maximum for higher $\phi'$ meanwhile, in this study the difference between SF through LEM and FEM2D, at smaller strength of the soil will be smallest at higher $\phi'$.

5.2.4 Case4: Long embankment- inconstant clay layer thickness

Comparing results of SF for the 3 applied programs on long embankment founded on undrained clay (with variation in thickness of clay from 2 m to 4 m), demonstrates that the difference between 2D and 3D analysis can range from 1% to 49.5% with the average of 24.4% difference between LEM and FEM3D and 7.3% average difference between FEM2D and FEM3D.

Totally the SF through FEM2D doesn’t change significantly; according to Table 4-4., a range of 1.46 to 1.77. But LEM analysis shows quite significant difference between SF results. The difference between the Min. and the Max. value of SF is something as double. More curious look at the chart shows that at lower thickness of clay, the SF from both 2D analysis of LEM and FEM are in good inclination and by increasing soft band thickness from 2 m till 4 m with
existence of distributed load; 50 kPa, FEM2D shows more sensitivity. However, without existence of load and while the soft band thickness reaches its maximum amount, results of SF in LEM show more sensitivity to the soft layer thickness variation. As the final step, in comparison between 2D and 3D results the 3D analysis resulted to a logical result of 1.69 and as the related failure plot shows, the failure is 3D. The differences between upper and lower bound solution are within 10%. These results are almost contrary to what M.Cala, Flisiak and A.Tajdus (2009) study result showed through Flac2D and 3D. The graph of their research showed that, further increasing of the thickness of soft soil produces decrease of difference between SF values between Flac2D and Flac3D.

The average of SF through FEM2D is 1.58 and in LEM is 2, and in FEM3D the average is 1.69. Is it logical to consider the average SF results of 2D program as a final stability factor? And compare it with single result of 3D? Or each soil cut has its own failure condition and it is a better solution to have a cross section model in every 10 meters; meaning 10 value for the SF?

### 5.3 Validation

#### 5.3.1 Hand calculation

For validation purpose and in order to evaluate results of applied analysis programs, a hand calculation is handled for all the analyzed models of case2. A three stage formula is driven from “SKREDKOMMISSIONEN, Rapport 3:95”, to hand calculate SF3D. Calculations are performed through the following instruction and are finally compared with SF results of FEM3D. The primary input data are summarized in Tab. 4-5.

<table>
<thead>
<tr>
<th>Stability Method</th>
<th>Morgenstern-Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor of Safety</td>
<td>1.066</td>
</tr>
<tr>
<td>Total Volume</td>
<td>26.839 m³</td>
</tr>
<tr>
<td>Total Weight</td>
<td>473.95 kN</td>
</tr>
<tr>
<td>Total Resisting Moment</td>
<td>2353.8 kN-m</td>
</tr>
<tr>
<td>Total Activating Moment</td>
<td>2207.7 kN-m</td>
</tr>
<tr>
<td>Total Resisting Force</td>
<td>290.64 kN</td>
</tr>
<tr>
<td>Total Activating Force</td>
<td>4.23 kN</td>
</tr>
</tbody>
</table>
1. Calculation of SF without consideration of end effect; $F_{2-Dim}$ ($F_{2-Dim}$ is considered as SF2D according to skred mission hand book).

$$F_{2-Dim} = \frac{M(t \cdot f_u \cdot r \cdot \varphi)}{M(W \cdot a + Q \cdot \varphi \cdot r)}$$

![Diagram of SF without considering face effects](image)

*Fig. 4.1 Calculation of safety factor without considering face effects (SKREDKOMMISSIONEN, Report 3:95 figure and formula).*

2. Safety factor calculation with considering flat end surfaces; $F_p$.

$$F_p = \frac{M(t \cdot f_u \cdot r \cdot L) + 2 \cdot M(t \cdot f_u \cdot A \cdot c)}{M(W \cdot a + Q \cdot b \cdot L)}$$

![Diagram of SF with flat end face](image)

*Fig. 4.2 Safety factor calculation with flat end face. (SKREDKOMMISSIONEN, Report 3:95 figure and formula).*

3. Calculation of 3Dimentional safety factor; $F_{3-Dim}$. 

31
Analysis, Discussion, Validation

Fig. 4.3  Three dimensional safety factor calculation (SKREDKOMMISSIONEN, Report 3:95).

In previous calculations, if soil had variation in cohesion regarding its depth, a weighted average of the cohesion should be used for the end surfaces. It should be taken into account that consideration of 3-dimensional effects within this method, is only taken in the case where the material of the end surface is consist of cohesive soil.

End surfaces stabilizing contributions, are dependent entirely on the undrained shear strength and where layers and layers of friction soil are located, count the stabilizing contributions moments only for the portion of the end surface consisting of cohesive soil in principle applies to the corresponding 3Dimensional effects also for frictional soils, but no similar simple calculation does exist. Calculating end surfaces in friction soil requires that the horizontal forces across the slope can be estimated and this in turn, requires entirely different calculation method.

\[
F_{2-Dim} = \frac{M(\tau_{fu}.l.r)}{M(w.a+Q.b)} = \frac{\text{Resistance force}}{\text{Existing force}} = \frac{2353.8}{2207.7} = 1.066 \quad (\text{Eq.9})
\]

Note: The value of resistance and exiting moments are chosen from the Geo slope based on LEM2D method results.

Since the formula is just valid for cohesive soil so the embankment cannot be considered as the part of failure soil in calculation. Therefore just the clay circular failure is considered. The radius of the circle of failure; C, is considered as 5, 5 m and =30m.

\[
F_p = \frac{M(\tau_{fu}.l.r)+2M(\tau_{fu}.A.c)}{M(w.a+Q.b)L} = \frac{(2353.8L+2M)}{(2207.7L)} = 1.12 \quad (\text{Eq.10})
\]

\[
M = (Cu.A.c) = (25)(12.8)(5.5) = 1760 \text{ kN.m} \quad (\text{Eq.11})
\]

\[
F_{3-Dim} = \left[ F_{2-Dim} + 0.75\left( \frac{F_p}{F_{2-Dim}} - 1 \right) \right] = 1.1 \quad (\text{Eq.13})
\]

32
Fig. 4.4., illustrates the results of 3D hands calculated SF according to formula of SKREDKOMMISSIONEN, Report 3:95, see Fig. 4.3., and results of FEM 3D as a function of the load position; $B/L$ (size ratio of distributed load), are compared:

![Graph showing SF3D (Hand calculated) and SF3D (FEM3D) as a function of $B/L$.]

**Fig. 4.4. Comparison between SF3D hand calculation and FEM3D calculation.**

There is a remarkable difference between hand calculation and FEM results of SF, the reason can be returned back to the method of calculation, which in hand 2D calculation, the results are driven from method of slices and also beside that the formulas above which the analysis is based on, is for slopes for one layer of cohesive soil but in this study the slope has three types of soil and the failure happens in 2 of the layers, therefore the embankment layer is omitted from the calculations since it is not a cohesive soil.

An almost relative research has been done by Cheng and Li (2009) by definition of the load length enhancing during different stages over the embankment and calculation of SF through SRM and LEM. Results of that study indicates that SF results from both methods are in good harmony with each other and SF(SRM) is always larger than SF (LEM), meanwhile LEM and FEM results of this study are in good harmony but SF (LEM) is larger than SF(FEM 2D).
5.4 Discussion

FEM3D is obviously more demanding than old traditional LEM method and for practical and economic reasons it is not always possible to analyze all the sections of poor stability through 2D analysis programs. Hence considering the whole model as a 3D structure finds its added value in the engineering calculations.

Totally the FEM2D based factor of safety is more sensitive than the LEM based one with regard to the different parameter values applied. This is understandable because in FEM the computed stress is a function of computed base normal stress which is consequential of initial stress plus incremental stress from the imposed loads. However in LEM through the procedure of SRM, the SF is determined by reduction of the soil shear strength within a constant factor, while regardless of variation in shear strength, the computed stress and stress through this method remain the same.

It is anticipated that the initial stress state plays an important role in stability evaluation. However, dependent on the mechanism of specified method, in situ stress would be calculated differently which respectively will cause difference in the value of the SF calculated.

All the studied cases of this thesis indicate that the SF obtained from the 3D analysis is always larger than to that obtained from a 2D analysis. It can be discussed that using 2D method as analysis solution is a conservative or maybe under estimation of reality, in design but is not conservative and safe when determining strength parameters from a back analysis from a failed embankment.

According to results of this study; SF3D is always greater than SF2D for the case of embankments founded on soft clay. This may lead to a conservative approach for 2D analysis. At the other hand effect of 3D analysis is mostly considered as safety reserve or back calculation in engineering design. Reason that 2D methods are found to be conservative can be assuming plain strain condition of the model which can lead to overestimation of the soils strength or due to lack of input knowledge of the soil or ignoring third dimension of the model.
6 Conclusions

6.1 Conclusions
The analysis and design of embankments founded on soft clay deposit requires an in-depth understanding of the failure mechanism in order to choose the right analysis method. In this study, stability analysis of embankments founded on clay is approached using FEM2D, FEM3D and LEM in order to increase understanding applicability of the three applied programs; Plaxis2D, Plaxis3D and Slope/W.

The work has involved analysis and comparison of the stability through estimation of the SF and the critical failure surfaces obtained through the 2D and 3D applied programs for four case configurations. In each case variation in plastic parameters of clay ($\varphi$ - $C$) was the base work to make the comparison analysis. Additionally, in this numerical simulation, influences of load position have been investigated also. Sensitivity evaluations of the embankments founded on clay deposit have been carried out to verify the feasibility of stability indices. From the study and analysis carried out and taking into account the limitations, following conclusions are drawn:

1. The failure surfaces in 3D analysis are likely to be shallower than in the corresponding 2D model.
2. Span of variation for SF all the samples analyzed by Plaxis3D is very small in comparison with other analysis programs applied.
3. For embankments founded on clay deposit, the 3D failure surfaces are easily found via the FEM3D analysis program, which is closer to reality, while failure results of 2D analysis programs can never occur in reality.
4. Concerning the three applied programs, FEM3D has the minimum sensitivity to change in plastic parameters of clay deposit.
5. Results from the 3D analysis through hand calculation and program calculation do not correspond with each other for embankment founded on soft clay deposit. The first reason is rooted in limitation of the hand formula. The formula is suitable for embankment founded on one layer deposit (soil); however an embankment founded on 3 layers of deposit (soil) was analyzed in this study. The second reason is related to applied method of calculation. 3D hand calculation formula is based on method of slices however; analysis method in program calculation is based on FEM.
6. Using 2D analysis instead of 3D, to investigate the stability of 3D embankment model tend to give higher SF results up to 14% for embankments founded on undrained clay deposit.
7. Sensitivity of safety factor became very low for embankments founded on clay deposit with high value of $C'$ (i.e. $C'>20$ kPa) in all the methods of FEM2D, FEM3D and LEM.
8. Results of this study have shown that the average ratio $(SF_{3D} - SF_{2D})/SF_{3D}$ is 12% for LEM and FEM3D and average of 14 % between FEM3D and FEM2D, for a range of short embankments founded on cohesive frictional clay.
9. It is stated in the 51st Annual Geotechnical Engineering conference (2003) that 3D analysis is beneficial in designing slopes with a complicated topography, shear strength and pore-water pressure condition. Conversely, as it is shown in case 4 of this thesis, 3D method for stability analysis of an embankment over deposit soils with inconsistent thickness of soil layer, can be a point of doubt. While 3D method calculates one SF for
the whole model, 2D methods represent 9 values of SF for each cut section of the same model. Therefore, it demands a large proportion of experience to analyze and judge the results and the conservative choice is parallel analysis with both 2D and 3D methods.

6.2 Proposal for further research

This thesis was planned as a theoretical case study. Suggestion for the future research is to validate the studied models of this thesis work with the parameters of a full scale project/field test.
References


Appendix

Appendix A- Failure surfaces

Appendix A presents the failure surfaces analysed/presented through LEM, FEM2D and FEM3D for all the studied cases of this thesis except case3. Since 90 models were analyzed within the stability analysis of case 3 it was preferred to just concentrate on the results of SF for this case.

Case1.

*Fig. A.1 Failure Surface through Plaxis2D program for $C_u = 30, 25, 20, 15, 10$ kPa*
Fig. A.1 Failure Surface through Plaxis2D program for $c_u = 30, 25, 20, 15, 10, 5$ kPa
Fig. A.2 Failure Surface through Slope/W program for $C = 30, 25, 20, 15, 10, 5$ kPa

$c_u = 5$

$c_u = 10$

$c_u = 15$

$c_u = 20$
Appendix

Fig. A.2 Failure Surface through Slope/W program for $C = 30, 25, 20, 15, 10, 5$ kPa

$C_u = 25$

$C_u = 30$

Fig. A.3 Failure Surface through Plaxis3D program for $C = 30, 25, 20, 15, 10$ kPa

$C_u = 5$
Fig. A.3  Failure Surface through Plaxis3D program for $C = 30, 25, 20, 15, 10$ kPa

Incremental displacements [$\Delta u$]
Maximum value = 0.00434 m (Element 170 at Node 942)

$c_u = 10$

Incremental displacements [$\Delta u$]
Maximum value = 0.01657 m (Element 38 at Node 357)

$c_u = 20$
Fig. A.3  Failure Surface through Plaxis3D program for $C = 30, 25, 20, 15, 10$ kPa

$C_u = 25$

$C_u = 30$
Case2.

Fig. A.4  Failure Surface through Plaxis3D program for long embankment founded on drained clay with external load of 20 kPa.
Fig. A.4 Failure Surface through Plaxis3D program for long embankment founded on drained clay with external load of 20 kPa.
Fig. A.4 Failure Surface through Plaxis3D program for long embankment founded on drained clay with external load of 20 kPa.
Fig. A.4 Failure Surface through Plaxis3D program for long embankment founded on drained clay with external load of 20 kPa.
Appendix

Fig. A.5 Failure Surface through Plaxis2D program for long embankment founded on drained clay with external load of 20 kPa.

Case 4

Fig. A.6 Surface of failure for slope cuts, via Plaxis 2D for embankment founded on inconstant clay layer thickness.
Fig. A.6 Surface of failure for slope cuts, via Plaxis 2D for embankment founded on inconstant clay layer thickness.
Fig. A.7 Surface of failure for slope cuts via Slope/W program for embankment founded on inconstant clay layer thickness.

Appendix

Fig. A.7 Surface of failure for slope cuts via Slope/W program for embankment founded on inconstant clay layer thickness.
Fig. A.8 3D Surface of failure via Plaxis 3D for slope with inconstant clay layer thickness.
Incremental displacements [$\Delta u$]

Maximum value = 0.05360 m (Element 348 at Node 5243)