Experimental Nuclear Structure Studies in the Vicinity of the $N = Z$ Nucleus $^{100}\text{Sn}$ and in the Extremely Neutron Deficient $^{162}\text{Ta}$ Nucleus

FARNAZ GHAZI MORADI

Doctoral Thesis in Physics
Stockholm, Sweden, 2014
Abstract

This work covers spectroscopic studies of nuclei from different regions of the Segré chart whose properties illustrate the delicate balance between the forces in the atomic nucleus. Studies of nuclei far from stability offer new insights into the complex nucleon many-body problem. In nuclei with equal neutron and proton numbers ($N = Z$), the unique nature of the atomic nucleus as an object composed of two distinct types of fermions can be expressed as enhanced correlations arising between neutrons and protons occupying orbitals with the same quantum numbers. The bound $N = Z$ nuclei with mass number $A > 90$ can only be produced in the laboratory at very low cross sections. The related problems of identifying and distinguishing such reaction products and their associated gamma rays have prevented a firm interpretation of their structure even for the lowest excited states until recently. In the present work the experimental difficulties of observation of excited states in the $N = Z = 46$ nucleus $^{92}$Pd have been overcome through the use of a highly efficient, state-of-the-art detector system; the EXOGAM-Neutron Wall-DIAMANT setup, and a prolonged experimental running period. The level spacings in the ground state band of $^{92}$Pd give the first experimental evidence for a new spin-aligned neutron-proton ($np$) paired phase, an unexpected effect of enhanced $np$ correlations for $N = Z$ nuclei in the immediate vicinity of the doubly magic nucleus $^{100}$Sn.

Excited states in $^{94}$Ru and $^{95}$Rh nuclei close to the double magic shell $Z = N = 50$ have been studied in order to untangle the ambiguity of the spin and the parity of the lowest-lying states. The observed yrast structures are compared to results of large-scale shell model (LSSM) calculations and the strengths of hindered E1 transitions are used as a sensitive test of the LSSM parameters. The effect of single-particle-hole excitations is discussed in terms of the strength of hindered E1 transitions.

Excited states of the odd-odd nucleus $^{162}$Ta have been observed using the JUROGAM/RIPT experimental set-up. This nucleus is located in a transitional region in the nuclide chart which is between near-spherical nuclei and well-deformed nuclei, offering the possibility to study the emergence of collective phenomena and nuclear deformation (in particular the degree of triaxiality). The results, which are interpreted in the framework of the cranked shell model with total Routhian surface calculations, suggest an almost axially symmetric nuclear shape. The energy staggering between the signature partners of the yrast rotational bands has been deduced for eight odd-odd isotopes in the neighborhood of $^{162}$Ta nucleus and the special observed feature of signature inversion for these nuclei is discussed.
List of Publications

The author has been part of experimental collaborations resulting in the papers listed below. This thesis is based on the first four papers in the list.

1. Evidence for a spin-aligned neutron-proton paired phase from the level structure of $^{92}$Pd
   Nature Journal 469, 68 (2011)

2. High-spin study of $^{162}$Ta

3. Character of particle-hole excitations in $^{94}$Ru deduced from $\gamma$-ray angular correlation and linear polarization measurements
4. Spectroscopy of the neutron deficient \( N = 50 \) nucleus \(^{95}\text{Rh}\)


5. Lifetime measurement of the first excited \( 2^+ \) state in \(^{108}\text{Te}\)


6. Transition probabilities near \(^{100}\text{Sn}\) and the stability of the \( N = Z = 50 \) shell closure

T. Bäck, C. Qi, B. Cederwall, R. Liotta, **F. Ghazi Moradi**, A. Johnson, R. Wyss, R. Wadsworth.

7. \( \gamma \)-ray linear polarization measurements and \((g_{9/2})^3\) neutron alignment in \(^{91}\text{Ru}\)

# Contents

## 2 Theoretical Background

2.1 The nuclear shell model ........................................... 5  
2.2 Spherical shell model calculations ............................... 6  
2.3 Identical and non-identical nucleon systems ................. 8  
2.4 Nuclear deformation parametrization in collective models .... 10  
2.5 Deformed shell model ........................................... 12  
2.6 The rotational nuclear motion ................................... 13  
  2.6.1 The cranked shell model .................................... 15  
  2.6.2 Nucleon-nucleon pair correlations ......................... 17  
  2.6.3 TRS calculations ............................................ 19  
  2.6.4 $B(M1) / B(E2)$ calculations .............................. 20  
2.7 Directional correlation and polarization of successive $\gamma$-rays .. 22  
  2.7.1 Gamma-gamma directional correlation ..................... 22  
  2.7.2 Direction-polarization correlation ......................... 25  

## 3 Experimental Techniques

3.1 Heavy-ion fusion evaporation ................................. 27  
  3.1.1 Beam selection and target thickness ....................... 28  
3.2 Experimental set-up to study nuclear structure in the vicinity of $N = Z = 50$ ............................................ 29  
  3.2.1 EXOGAM ........................................... 29  
  3.2.2 Neutron Wall ........................................ 30  
  3.2.3 DIAMANT ........................................ 32  
  3.2.4 Trigger condition ................................... 32  
3.3 In-beam spectroscopy of extremely neutron deficient nuclei in the $A \sim 160$ mass region ...................................... 34  
  3.3.1 JUROGAM ........................................ 34  
  3.3.2 The gas-filled recoil separator RITU ..................... 35  
  3.3.3 The focal-plane spectrometer GREAT ..................... 35
CONTENTS

3.3.4 Data Acquisition ............................................. 36

4 Data Analysis .................................................. 37
  4.1 $^{92}$Pd, $^{94}$Ru and $^{95}$Rh: Data acquisition and sorting .............. 37
    4.1.1 Channel identification and gating ................................ 38
    4.1.2 Discrimination of neutrons and $\gamma$-rays ....................... 41
    4.1.3 Neutron multiplicity correction ................................. 42
    4.1.4 Deducing the level scheme of $^{92}$Pd ............................ 45
    4.1.5 Compton polarimetry using EXOGAM .............................. 46
    4.1.6 Spin-parity assignments of excited nuclear states in $^{94}$Ru and
         $^{95}$Rh .......................................................... 48
  4.2 Gamma-ray spectroscopy of $^{162}$Ta ................................ 51
    4.2.1 Recoil identification and gating ................................ 51
    4.2.2 Constructing the $^{162}$Ta level scheme and spin assignment ... 52
    4.2.3 Experimental Routhians and $B(M1)/B(E2)$ ratios .............. 53

5 Discussion ....................................................... 57
  5.1 Neutron-proton interaction in $g_{9/2}$ orbitals ....................... 57
  5.2 Particle-hole excitations and strengths of E1 transition ............ 59
  5.3 Signature inversion .......................................... 60

6 Summary of Papers .............................................. 63
  6.1 Paper I ....................................................... 64
  6.2 Paper II ..................................................... 64
  6.3 Paper III ..................................................... 64
  6.4 Paper IV ..................................................... 65
  6.5 Author’s Contributions ........................................ 65

Bibliography ....................................................... 71
Chapter 1

Introduction

The picture of the atomic nucleus as a hadronic many-body system was unknown until Ernest Rutherford proposed its existence in 1911 [1]. A few years later he performed the first artificial nuclear reaction experiment which led to the discovery of fast proton emission. These discoveries, together with the later investigations of James Chadwick proving the existence of neutrons in 1932 [2], were fundamental steps towards understanding of the properties of the atomic nucleus as a dense core consisting of smaller building blocks, protons and neutrons. These significant milestones expanded the horizons of physics to investigate the level structure of the nucleus as a system of elementary particles. The exploration of the structure of different nuclei as nucleonic systems, including a vast number of experiments, has played a prominent role in the developments of nuclear physics and its application in many other scientific fields such as medical physics, material science, archeology, and nuclear energy production. Although in our present understanding the nucleus constitutes systems of quarks embedded in nucleons it can, for many purposes, be regarded as a complex many-body system of protons and neutrons. In an atomic system electrons move in the central potential produced by the electromagnetic field from the nucleus while in a nuclear system nucleons are held together by short-range attractive nuclear forces and appear to move independently in a potential provided by the mean field of all nucleons together. This is also the basic assumption of the nuclear shell model which has been very successful in describing some properties of nuclei. In recent years much progress has been made towards understanding the evolution of nuclear structure with the focus on exotic nuclei far from the valley of stability and close to the proton and neutron drip lines. The neutron deficient nuclei in the vicinity of $^{100}$Sn with equal numbers of protons and neutrons ($N = Z$) exhibit special features due to the fact that protons and neutrons occupy orbitals with the same quantum numbers. This induces the large spatial overlap which may result in enhanced neutron-proton ($np$) interaction. The study of these nuclei has been the subject of many experimental and theoretical investigations and numerous attempts have been made to test the validity of the shell model.
near the $N = Z$ line where the impact of isospin symmetry is maximal and the effects of $np$ correlations on nuclear level structure can be observed more explicitly as the mass number increases towards the doubly magic $N = Z$ nucleus $^{100}\text{Sn}_{50}$, the heaviest self-conjugate nucleus predicted to be bound. Such correlations are generally manifested in two possible pairing schemes, namely the isovector pair and the isoscalar pair of nucleons. Their contribution plays an important role in the theoretical interpretation of the $^{96}\text{Pd}_{46}$ nucleus. Another topic of interest in nuclear structure studies of neutron deficient nuclei along the $N = Z$ line is the underlying structure of nuclei in the vicinity of $^{100}\text{Sn}$ and whether the stability of a shell closure which is found close to the valley of $\beta$ stability is preserved when approaching the proton dripline. The low lying yrast states of $Z, N \leq 50$ nuclei are rather well produced in a model space consisting of a rigid core on top of which particle-hole excitations can be described by adding a few particles (or holes). The presence of the core excited states in several $Z < 50$ neighbors of $^{100}\text{Sn}$ has been extensively studied in recent years [3, 4, 5, 6, 7]. The spin assignment of many studied nuclei in this mass region is tentative and for instance in $N = 49$ and $N = 50$ neutron deficient isotones $\text{Tc}$, $\text{Ru}$ and $\text{Rh}$ the assignment of level spins are only based on angular distribution measurement. In contrast to nuclei which are in the vicinity of the closed shells $N = Z = 50$ the neutron deficient nucleus $^{162}\text{Ta}_{89}$ is located in a mass region below the proton shell $Z = 82$, and between the neutron mid-shell and neutron closed shell at $N = 82$ (see Fig. 1.1). The light neutron deficient tantalum isotopes in this region of the nuclide chart lie in a transitional zone between near-spherical nuclei and well-deformed nuclei. They are predicted to show near-prolate deformation at $\beta_2 \approx 0.2$. This is evident for the tantalum neutron deficient nuclides [8, 9, 10, 11] down to the $N = 88$ nucleus $^{161}\text{Ta}$ [12] as the neutron number approaches the $N = 82$ shell closure. In the odd-odd nucleus $^{162}\text{Ta}_{89}$ the residual interaction between the last valence proton and neutron may influence the rotational band structure directly depending on the quasiparticle configuration. The residual interaction also influences the structure of Ta isotopes by polarizing the nuclear shape.

This doctoral thesis is divided into six chapters: following this introduction, chapter 2 gives a brief overview of the theoretical methods utilized to explain the experimental results. Chapter 3 covers a more detailed description of the experimental set-ups that were only briefly described in the papers. In chapter 4 the methods of data analysis are explained. After a brief discussion of the results in chapter 5, a summary of papers I, II, III and IV is given in chapter 6.
Figure 1.1: The nuclide chart. The black and grey areas indicate stable and unstable isotopes, respectively, and magic numbers are marked by red lines. The arrows pointing at the white squares indicate the nuclei studied in this work (courtesy of T. Bäck).
Chapter 2

Theoretical Background

This chapter outlines the theory used to interpret the experimental results in the present work. The first three sections give a general introduction to the nuclear shell model and a few basic notions about the shell model calculations and neutron-proton pairing correlations that were used to interpret the experimental data of $^{92}$Pd (paper I). The parametrization and structure of stable deformed nuclear shapes and energy levels are explained in section 2.4 and 2.5 in terms of the deformed shell model. In section 2.6 some properties of the nuclear motion are described and theoretical approaches which are used for the interpretation of results in paper II are briefly explained. Finally, section 2.7 presents some remarks on the theory of directional correlations and polarization used for spin-parity assignments in papers III and IV. The aim of this chapter is to give a brief description of the models that are used in this work rather than to present a detailed review of the existing theoretical approaches.

2.1 The nuclear shell model

Following the pioneering work of Gamow in proposing the liquid drop model of the nucleus in 1928, Bohr and Wheeler developed a theoretical approach of the atomic nucleus based on this model [13]. This liquid drop description of the atomic nucleus was used by Meitner and Frisch [14] to give a clear physical explanation of the experimentally observed fission phenomenon [15, 16]. Using this analogy one could interpret important features of the nucleus such as nuclear binding energies. It also made it possible to explain macroscopic properties such as collective processes taking place in nuclei. Yet, it could neither explain the variation of ionization energies nor the sudden change of nucleon separation energies that had been observed for sequences of isotopes and isotones. The occurrence of certain magic numbers in nuclei (as 2, 8, 20, 28, 50, 82 and 126), which has been one of the incentives to develop the nuclear shell model, could be understood as the result of the shell structures that arise from the fermionic character of the nucleons. It is equivalent to the sit-
uation in atomic shells that, when certain shells are completely filled, result in the appearance of noble gases. The nuclear shell model has been successful in explaining the variation of neutron and proton separation energies and in predicting the observed properties of nuclei near the shell gaps such as spins, parities and nuclear electromagnetic moments. Different versions of this model have been extensively used to explain the properties of nuclei in different regions of the nuclide chart. The essential assumption of this model is that neutrons and protons move independently in an average potential, interacting with each other through a residual interaction of a two-body character. The first step in the application of the model is to determine the mean field in which the nucleons move or, in other words, to determine a representation to be used in solving the nuclear many-body problem. The best way of doing this is by choosing a realistic potential. For the radial term a good approximation is the Woods-Saxon potential [17] which has an intermediate form between the harmonic oscillator and the infinite well potential (which both reproduce the shell gaps at 2, 8 and 20). A reformulation of the nuclear potential was introduced in 1949 by Mayer, Haxel, Suess and Jensen [18, 19] by including a spin-orbit interaction term of the form \( f(r) \vec{l} \cdot \vec{s} \). This splits the high-\( j \) shells and squeezes the \( \vec{l} + \vec{s} \) state down from a major shell \( N \) into the shell \( N - 1 \), leading to the reproduction of all remaining shell gaps (28, 50, 82, 126). That is, one assumes that the nucleus is a Fermi gas (as in atomic physics) in which nucleons occupy the shells in increasing order up to the Fermi level. If this level is a magic number, then one has reached a large gap and the next level is high in the spectrum. A realistic potential reproducing all magic numbers consists of a general radial term, the spin-orbit term and a Coulomb potential term which enters only for protons. With this approximation the Hamiltonian can be expressed as:

\[
H = -\frac{\hbar^2}{2m} \nabla^2 + V_{WS}(r, \theta, \phi) + V_{LS} + V_C
\]

(2.1)

where \( V_{WS} \) is the Woods-Saxon potential, \( V_{LS} \) is the spin-orbit interaction and \( V_C \) the Coulomb potential. The low-lying nuclear excitations in spherical nuclei can in most cases be described within the spherical shell model considering nuclei with neutrons and protons filling nuclear shells. A generalized spherical Wood-Saxon potential often can reproduce the single particle energies better in heavier nuclei.

### 2.2 Spherical shell model calculations

For a many-body system consist of \( A \) nucleons the solution of the Schrödinger equation:

\[
H \Psi(r_1, r_2, \ldots, r_A) = E \Psi(r_1, r_2, \ldots, r_A)
\]

(2.2)

corresponds to the energy of the nuclear state, \( E \), with the many-body wavefunction \( \Psi(r_1, r_2, \ldots, r_A) \). The Hamiltonian is given as the sum of the kinetic energy of each
nucleon, $T_i$, and the interaction between any two nucleons, $V_{ij}$, as:

$$H = \sum_{i=1}^{A} T_i + \sum_{i \neq j} V_{ij}. \quad (2.3)$$

The calculation starts by introducing a complete set of orthonormal basis states, $\Phi_k(r_1, r_2, \ldots, r_A)$, and rewriting the eigenfunctions, $\Psi(r_1, r_2, \ldots, r_A)$, as a linear combination of a number of these basis states. The eigenvalue problem is then solved in matrix representation by diagonalizing the Hamiltonian, $H$. For the calculation of many-body wavefunctions the basis states, $\Phi_k$, can be obtained from the products of single-particle wavefunctions, $\phi_k(r_i)$. To account for the antisymmetric property of the wavefunction the basis state is written in the form of a Slater determinant. As already indicated, one of the main assumptions of the shell model is that a nucleus with neutron and proton numbers corresponding to magic numbers are inert cores. This nucleus is the “vacuum” of excitation. Thus the $n$ nucleons outside the core determines the spectrum. In addition, for the lowest many-body excitations one assumes that these $n$ nucleons move in the shells located just around the Fermi level. These are called valence shells, again in analogy with atomic physics. Therefore the task is just to diagonalize the many-body Hamiltonian matrix in the representation of the Slater determinants mentioned above within the space determined by the valence shells. With increasing mass the number of different shells that are partly filled with nucleons increases, leading to a larger shell space. For such “large scale” shell model calculations the number of Slater determinants rapidly increases and the diagonalization of the full matrix is not possible for very large shell model dimensions due to the present limitations of the computer sizes and speeds. At present, one can deal numerically with dimensions of up to $10^{10}$. In order to restrict the active shell space calculation to a manageable size a set of single-particle states is selected to truncate the Hilbert space. This is done by taking the eigenfunction of a single-particle Hamiltonian as:

$$h(r_i)\phi_k(r_i) = \epsilon_k \phi_k(r_i) \quad (2.4)$$

where $\epsilon_k$ is the observed energy level of single particle states in the region of interest. The diagonalization of the single-particle Hamiltonian provides a representation, i.e. a complete set of single-particle states which form the basis to describe the calculated many-body states. The Hamiltonian can be expressed in terms of such a single-particle Hamiltonian as:

$$H = \sum_{i=1}^{A} h(r_i) + \sum_{i \neq j=1}^{A} V(r_i, r_j) \quad (2.5)$$

where $V(r_i, r_j)$ is the residual two-body interaction corresponding to the nucleon-nucleon interaction, $V_{ij}$, minus the contributions that are already included in single-particle Hamiltonian $h(r_i)$. In spherical shell model calculations the derivation
of the effective nucleon-nucleon interaction $V_{NN}$ generally includes several terms, such as central term, spin-orbit term, spin-spin and tensor terms, etc. These terms may be important in describing some features of the nuclear levels. At present, there is an intense theoretical activity in order to get a detailed expression of the tensor term. There are also efforts to introduce 3-body forces into the shell model Hamiltonians \[20\]. One method to define the realistic effective interactions is to determine empirically the two-body matrix elements from a fit to experimental energy levels. By using these matrix elements one can find the solution to the Schrödinger equation in the valence shell space. That is, one obtains the theoretical level scheme of the nucleus corresponding to the different angular momenta and isospins of interest. For nuclei in the vicinity of $^{100}$Sn the realistic Two-Body Matrix Elements (TBME) are mostly derived from the charge-dependent nucleon-nucleon potential which is known as “CD-Bonn” potential \[21\].

2.3 Identical and non-identical nucleon systems

The exchange symmetry between neutrons and protons is based on the fact that the attractive nuclear force with a good approximation can be considered to be invariant with respect to nuclear electric charge. Although slight deviations from charge symmetry and charge independence of the attractive nucleon-nucleon interaction have been recently observed \[22\], still, for many applications, this is a good approximation to describe many aspects such as pairing in atomic nuclei. In a two-particle system the coupling of a nucleon in orbit $j_1$ with a nucleon in orbit $j_2$ results in an angular momentum $J$ and in the absence of a residual interaction, $V_{12}$, all $J$ states are degenerate. The energy shifts induced by the residual interaction are given by:

$$\Delta E(j_1,j_2;J) = \langle j_1,j_2;JM|V_{12}|j_1,j_2;JM \rangle$$  \hspace{1cm} (2.6)

and the two-particle states $\Psi(j_1,j_2;JM)$ will split accordingly. For non-identical nucleons $(pn)$ there is no Pauli principle restriction and we have:

$$\Psi(j_1,j_2;JM) = \sum_{m_1,m_2} \langle jm_1,jm_2|JM \rangle \phi_{j_1,m_1}(1)\phi_{j_2,m_2}(2).$$  \hspace{1cm} (2.7)

For identical nucleons $(pp-\text{nn})$, the two-particle configurations can be constructed for two cases of $j_1 = j_2$ and $j_1 \neq j_2$.

If $j_1 = j_2$, the antisymmetrized two-particle wave function can be written in terms of the Clebsch-Gordon coefficients as:

$$\Psi(j^2;JM) = N \sum_{m_1,m_2} \langle jm_1,jm_2|JM \rangle \left[ \phi_{jm_1}(1)\phi_{jm_2}(2) - \phi_{jm_2}(1)\phi_{jm_1}(2) \right]$$

$$= N \left[ 1 - (-1)^{2J-J} \right] \sum_{m_1,m_2} \langle jm_1,jm_2|JM \rangle \phi_{jm_1}(1)\phi_{jm_2}(2)$$  \hspace{1cm} (2.8)

where $N$ is the normalization factor. Hence only even $J$ values are allowed for identical nucleons in equivalent orbitals ($J = 0, 2, 4, \ldots, (2j-1)$) and $N = 1/2$. 
2.3. IDENTICAL AND NON-IDENTICAL NUCLEON SYSTEMS

If \( j_1 \neq j_2 \), we have:

\[
\Psi(j_1, j_2; JM) = N \sum_{m_1, m_2} \langle j_1 m_1, j_2 m_2 | JM \rangle [\phi_{j_1 m_1}(1) \phi_{j_2 m_2}(2) - \phi_{j_1 m_1}(2) \phi_{j_2 m_2}(1)]
\]

with \( N = 1/\sqrt{2} \). The detailed description of the two-particle configurations can be found in Ref. [23].

As an alternative way, the neutron-proton exchange invariance can also be comprehensively explained within the concept of isospin. In this formalism, for a system of \( A = N + Z \) nucleons, neutrons and protons are manifested as two different isospin states of the nucleon and are distinguished by an isospin quantum number indicating whether the nucleon is a proton or a neutron. The isospin operator is defined as \( \vec{t} = \vec{\tau}_2 \) with Pauli isospin matrices \( \vec{\tau}_x, \vec{\tau}_y, \vec{\tau}_z \).

By definition the value of \( \tau_z \) distinguishes between protons and neutrons. The total isospin vector, \( \vec{T} \), is given as the vector sum of the isospins of individual nucleons as:

\[
\vec{T} = \sum_i \vec{t}_i.
\]

The two-nucleon isospin wave functions can then be reconstructed using this formalism. For a system of nucleon-nucleon pairs the pairing between nucleons can also be manifested in two ways. On the one hand one has the, normal, isovector \( T = 1 \) pairs with anti-parallel spins, where nucleons move in time reversed orbitals, that is, each nucleon pair is coupled to 0 angular momentum (which may give rise to nuclear condensation, equivalent to superconductivity in solids). In this case the two-particle isospin wave function consists of a symmetric isospin part and thus an antisymmetric space-spin part and in analogy to the preceding \( pp - nn \) representation only even \( J \) values 0, 2, 4, \ldots, \((2j - 1)\) are obtained. But on the other hand one may also have isoscalar \( T = 0 \) neutron-proton pairs which can be described by a two-particle isospin wave function consisting of an antisymmetric isospin part and a symmetric spin-part. Again by a derivation analogous to the two-particle wave functions mentioned above one finds that \( T = 0 \) neutron-proton system contains only odd \( J \) values. Thus the total nuclear wavefunction can be constructed and the angular momentum couplings result in all \( J \) from 0 to 2\( j \). For nuclei close to stability mainly like-particle pairing is considered as the Fermi levels for neutrons and protons are very different. However, this pairing pattern may remarkably change for nuclei with neutrons and protons in equivalent orbitals. Just as in condensed matter physics pairwise correlations between the fermions in the nucleus may also give rise to properties similar to superfluidity and superconductivity.
2.4 Nuclear deformation parametrization in collective models

The shape of a deformed nucleus can be parametrized by representing the nuclear surface via expansion of the spherical harmonics, \( Y_{\lambda \mu} \), as:

\[
R(\theta, \phi) = R_0 \left( 1 + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi) \right) \quad (2.11)
\]

where \( R(\theta, \phi) \) describes the radius length as a vector from the center of the nucleus to a point at the surface with polar coordinates \((\theta, \phi)\) and \( R_0 \) corresponds to the radius of a spherical nucleus with the same volume. For a shape parametrization inferred from internal nuclear forces only terms with \( \lambda \geq 2 \) are considered. The term \( \lambda = 1 \) corresponds to displacement of the center of mass and is eliminated assuming a fixed center of mass. By choosing the next lowest order, \( \lambda = 2 \), the parametrization of quadrupole deformation is written as:

\[
R(\theta, \phi) = R_0 \left( 1 + \sum_{\mu=-2}^{2} \alpha_{2\mu} Y_{2\mu}(\theta, \phi) \right). \quad (2.12)
\]

Depending on the shape of the nucleus with respect to the rotational motion one can reduce the nuclear deformation parameters in Eq. 2.11. For example for the case of an axially symmetric nucleus where the collective rotation is perpendicular to the intrinsic symmetry axis, \( \mu = 0 \) and \( \alpha_{30} \) are denoted as \( \beta_{\lambda} \). It is customary to express quadrupole deformations in a body-fixed reference frame with axes 1, 2, 3. If we choose the axes of the coordinate system to coincide with the axes of the body-fixed system then, \( \alpha_{21} = \alpha_{2-1} = 0 \) and \( \alpha_{22} = \alpha_{2-2} \). Therefore the non-vanishing coefficients \( \alpha_{20} \) and \( \alpha_{22} \) are sufficient to describe the shape of the nucleus. These parameters are often expressed in terms of the Hill-Wheeler [24] parameters \( \beta_2 \) as the measure of nuclear deformation and \( \gamma \) as the degree of triaxiality:

\[
\alpha_{20} = \beta_2 \cos \gamma, \quad \alpha_{22} = \frac{1}{\sqrt{2}} \beta_2 \sin \gamma. \quad (2.13)
\]

Equation 2.12 is then simplified to:

\[
R = R_0 \left[ 1 + \beta_2 \cos \gamma Y_{20}(\theta, \phi) + \frac{\sqrt{2}}{2} \beta_2 \sin \gamma (Y_{22}(\theta, \phi) + Y_{-2}(\theta, \phi)) \right]. \quad (2.14)
\]

In the Lund convention of describing the nuclear deformation the rotation takes place around the small axis in the \( 0^\circ \leq \gamma \leq +60^\circ \) sector and the large axis in the \( -120^\circ \leq \gamma \leq -60^\circ \) sector. Assuming the direction of the principal axes 1, 2, 3 as depicted in Fig. 2.1 and by inserting the spherical harmonics functions in the above
2.4. NUCLEAR DEFORMATION PARAMETRIZATION IN COLLECTIVE MODELS

equation the length of the three semi-axes 1,2,3 for collective and non-collective deformed shapes are given by:

$$R_i = R_0 \left[ 1 + \sqrt{\frac{5}{4\pi}} \beta_2 \cos(\gamma - \frac{2\pi i}{3}) \right], \quad i = 1, 2, 3.$$  \hspace{1cm} (2.15)

**Non-collective oblate** $\gamma = +60^\circ$

**Collective prolate** $\gamma = +0^\circ$

**Collective oblate** $\gamma = -30^\circ$

**Non-collective prolate** $\gamma = -120^\circ$

Figure 2.1: Schematic illustration of the Lund convention to depict the $\beta_2$ and $\gamma$ deformation parameters. The intrinsic symmetry axis is denoted as “S”.

**prolate**: \[ R_{1C} = R_{2C} = R_{1NC} = R_{3NC} = R_0(1 - \sqrt{\frac{5}{16\pi}} \beta_2) \]  \hspace{1cm} (2.16)
\[ R_{3C} = R_{2NC} = R_0(1 + 2\sqrt{\frac{5}{16\pi}} \beta_2) \]  \hspace{1cm} (2.17)

**oblate**: \[ R_{1C} = R_{3C} = R_{2NC} = R_{3NC} = R_0(1 + \sqrt{\frac{5}{16\pi}} \beta_2) \]  \hspace{1cm} (2.18)
\[ R_{2C} = R_{1NC} = R_0(1 - 2\sqrt{\frac{5}{16\pi}} \beta_2) \]  \hspace{1cm} (2.19)
CHAPTER 2. THEORETICAL BACKGROUND

Here each set of semi-axes parameters for collective and non-collective shapes are denoted as C and NC, respectively.

2.5 Deformed shell model

Although the spherical nuclear shell model has been successful in describing nuclei near magic numbers it fails to describe the structure of nuclei which have many valence nucleons outside closed shells. There are important consequences of the residual interactions between such nucleons, most notably collective motion and nuclear deformation. One of the important effects of the nuclear deformation on shell model states is that the $2j + 1$ degeneracy of shell model states no longer holds. Nilsson and Mottelson developed a unified model to account for the effects of deformation on the shell-model states [25, 26]. An anisotropic oscillator potential was first introduced by Nilsson to be used as a modified shell-model potential which well reproduced the single-particle states in deformed nuclei. For nuclei far from closed shells, e.g. the rare earth nuclei or the actinides, strongly enhanced $B(E2)$ values indicate the existence of stable quadrupole deformations and the low-lying states closely follow those of quantum rotors. This is also true for the neutron deficient isotopes which lie in the mass region below the closed $Z = 82$ shell and in the neutron shell above $N = 82$ where slightly deformed rotational structure has been observed. Hence in these regions a deformed potential is a good assumption justified by the fact that nuclear deformation appears to be present although it may easily be susceptible to changes induced by various excitations. In its simplest form we restrict ourselves to the case of cylindrical symmetry in a body-fixed coordinate system with the $z$-axis as the symmetry axis (also noted as 1,2,3 with 3-axis as $z$-axis). The Nilsson modified potential can be written as:

$$V = \frac{M}{2} \left[ \omega_x^2 (x^2 + y^2) + \omega_z^2 z^2 \right] + C \vec{l} \cdot \vec{s} + D \vec{l}^2$$  \hspace{1cm} (2.20)

where the first term is the anisotropic harmonic oscillator with oscillator frequencies:

$$\omega_x^2 = \omega_y^2 = \omega_0^2 (\delta) \left(1 + \frac{2}{3} \delta\right), \quad \omega_z^2 = \omega_0^2 (\delta) \left(1 - \frac{4}{3} \delta\right).$$  \hspace{1cm} (2.21)

The parameter $\delta$ which is named the parameter of deformation by Nilsson can be considered as the measure of elongation of the potential along the nuclear $z$-axis and is related to the previously mentioned $\beta_2$ parameter to first-order as $\delta = \frac{3}{4} \sqrt{\frac{5}{2}} \beta_2$.

The second term in Eq. 2.20 is the well known spin-orbit term needed to reproduce the magic numbers. The last term is added to produce a more realistic potential that accounts for the fact that for high $l$-values the nucleons experience a deeper potential as compared to the harmonic oscillator and consequently the energy levels of higher $l$-values are shifted down in energy. The Nilsson single-particle Hamiltonian is then written as:

$$H = \frac{\hbar^2}{2M} \nabla^2 + \frac{M}{2} \left[ \omega_x^2 (x^2 + y^2) + \omega_z^2 z^2 \right] - 2\hbar \omega_0 \mu \vec{l} \cdot \vec{s} - \hbar^2 \omega_0 \mu \vec{l}^2.$$  \hspace{1cm} (2.22)
The parameters $\kappa$ and $\mu$ can be viewed as the strength of spin-orbit force and the surface diffuseness depth and vary from shell to shell. For the case of large deformations, the eigenenergies and the asymptotic wavefunctions can be calculated by diagonalizing the anisotropic oscillator potential while the $\vec{l}^2$ and $\vec{l} \cdot \vec{s}$ terms are treated as perturbations. The energy states are then labeled by quantum numbers: $N$ the total number of oscillator quanta, $n_z$ the number of oscillator quanta along the intrinsic symmetry axis, $\Sigma$ the projection of intrinsic spin and $\Lambda$ the projection of orbital angular momentum along the intrinsic symmetry axis with $\Omega = \Sigma + \Lambda$ (see Fig. 2.2). The examples of Nilsson diagrams which can be used to classify the observed single particle energies in paper II can be found in Ref. [27] where the single particle energies are plotted as a function of a slightly changed deformation parameter, $\varepsilon_2$ which is related to $\delta$ as:

$$\varepsilon_2 = (\delta + \frac{1}{6} \delta^2 + \frac{5}{18} \delta^3 + \cdots).$$

(2.23)

The calculated single-particle energies using different potentials such as deformed Woods-Saxon potential give very similar results.

**2.6 The rotational nuclear motion**

Many empirical observations such as low-lying rotational states and very large electric quadrupole moments of even $Z$-even $N$ nuclei that lie in the mass region $150 < A < 190$ have confirmed the existence of permanent nuclear deformation.

![Figure 2.2: Graphical representation of vector couplings of angular momenta and asymptotic Nilsson quantum numbers for a non-axial symmetric nucleus. For an axially symmetric shape $K = \Omega$.](image)
in nuclei far from closed shells. In the collective model the nucleonic motion is quantized by assuming that the even-even core is an incompressible nuclear matter in the form of a deformed liquid drop. A distinct feature is that the coherent motion of all nucleons contributes in the total angular momentum of the system. Another important property of collective rotational motion is that the angular momentum of the odd nucleon is no longer a conserved quantum number and for the conservation of the total angular momentum of the system the core must have an angular momentum coupled to the single particle angular momentum. This is explained within the particle-plus-rotor model which was proposed by Bohr and Mottelson by means of the coupling of a few valence nucleons outside a rotating rigid core [28]. For an axially symmetric rotor the total Hamiltonian consists of intrinsic and collective parts. The energy levels of rotational states in odd-$A$ nuclei are obtained by [29]:

$$E = e_K + \frac{\hbar^2}{2J} [I(I+1) - K^2]$$  

(2.24)

where $e_K$ is the single-particle energy, $\hbar\sqrt{I(I+1)}$ is the total angular momentum of the nucleus, $\hbar K$ is the angular momentum component along the symmetry axis and $J$ is the component of the moment of inertia perpendicular to the symmetry axis. The intrinsic property of the system as being invariant to rotation by an angle $180^\circ$ about an axis perpendicular to the symmetry axis gives rise to two-fold degenerate $\Omega$ states that are filled pairwise. The ground state band in even-even nuclei has positive parity and $K = 0$. The energies then reduce to:

$$E = \frac{\hbar^2}{2J} [I(I+1)].$$  

(2.25)

Such low-lying rotational states are characterized by spin sequence $I = 0, 2, 4, 6, \ldots$ In odd-$A$ nuclei parity and angular momentum in the band head is $K^\pi = \Omega^\pi$ corresponding to the odd nucleon. One can also discuss different degrees of coupling of the odd nucleon to the collective axially symmetric rotor [30]. In the strong coupling limit (deformation alignment) the orientation of the rotating deformed core is a leading factor to determine the motion of the valence nucleons. This is the case when the deformation is large and the Coriolis force is weak and consequently the large quadrupole deformation causes the odd nucleon to couple to the deformed core ($K = \Omega$). The spin values of the rotational states are then given as $I = K, K + 1, K + 2, \ldots$ ($K = \Omega$). In the decoupling limit (rotation alignment) the Coriolis force largely dominates the motion of the valence nucleon and the angular momentum of the band head is not necessarily the same as the $K$ value. This is the case for nuclei with high-$j$ orbitals and low-$\Omega$ values where the Coriolis force favors the alignment of the angular momentum $j$ of the odd particle with the rotating core. The spin sequence of the band members is then given by $I = j, j + 2, j + 4, \ldots$ and the energies of the rotational states can be calculated by considering the projection of $j$ on the rotational axis (denoted as $j_z$). The complete alignment of $j$ along the rotation axis, i.e. $j = j_z$ generates the lowest-lying rotational band which is
often termed as a favored band. The spin values of a rotational band with less alignment is \( I = j - 1, j + 1, j + 3, \ldots \) (unfavored band). This approach has been successful in describing the rotational bands in well-deformed odd-\( A \) nuclei as well as the backbending phenomena as a consequence of breaking time-reversed nucleon-nucleon pairs due to the Coriolis interactions with the rotating core. In a heavy-ion fusion evaporation experiment, where a large amount of angular momentum (up to 80 \( \hbar \)) is transferred to the nucleus, a stable nuclear deformation could be characterized by coherent movement of many nucleons. Such collective rotational excitations are experimentally observed over a wide range of nuclei. The rules governing the angular momentum couplings of protons and neutrons in odd-odd nuclei were studied by Nordheim in 1950 who proposed two coupling rules known as strong and weak rules [31]. Later Brennan and Bernstein performed an empirical analysis over a large range of odd-odd nuclei and replaced the Nordheim rules with new revised rules [32]. In brief, for configurations in which both the odd proton and neutron are particles (or holes) the spins of the lowest states, \( J_{gs} \), can be obtained as:

**strong rule :** \( J_{gs} = |j_p - j_n| \) for \( j_p = l_p \pm \frac{1}{2} \) and \( j_n = l_n \pm \frac{1}{2} \)

**weak rule :** \( J_{gs} = |j_p \pm j_n| \) for \( j_p = l_p \pm \frac{1}{2} \) and \( j_n = l_n \pm \frac{1}{2} \)

However, in the case of an odd-odd nucleus with many nucleons outside the core the complexity of the coupling of the valence nucleons to the core usually prohibits a clear description of angular momentum coupling scheme.

### 2.6.1 The cranked shell model

One of the successful microscopical approaches to understand the rotation of the nucleus is the cranking model which was first derived by Inglis [33, 34]. This model describes the collective angular momentum as a sum of single-particle angular momenta. The basic idea of this model is to consider independent particle motions in a potential rotating with frequency \( \omega \). The calculation is performed by rotating (cranking) a body-fixed coordinate system with respect to the nuclear potential. For collective rotations we can simplify the calculations by choosing the symmetry axis to coincide with the 3-axis and the laboratory axis \( x \) coincide with the 1-axis of the body-fixed coordinate system. The single-particle cranking Hamiltonian of rotating nucleons is given as:

\[
h_{s.p.}^\omega = h_{s.p.} - \omega j_1
\]

where \( h_{s.p.} \) is the time-independent single-particle Hamiltonian in the body-fixed system and \( j_1 \) represents the projection of single-particle angular momentum on the axis of rotation. The second term is classically equivalent to the centrifugal and Coriolis forces and is obtained from the transformation of the time dependent
Schrödinger equation from the laboratory system to the intrinsic rotating system. The lowest eigenstates of $h_{s.p.}$ correspond to the yrast states. The eigenvalues, which are often called single-particle Routhians, are obtained as:

$$e_\omega^i = \langle i | h_{s.p.} | i \rangle - \omega \langle i | j_1 | i \rangle. \quad (2.27)$$

The expectation value of the operator $j_1$, which is now equal to the aligned angular momentum along the rotation axis, is obtained from the derivative of $e_\omega^i$ with respect to $\omega$:

$$i_x = i_1 = -\langle i | j_1 | i \rangle. \quad (2.28)$$

The single particle Hamiltonian has two symmetry properties: invariance under the space inversion and invariance under $180^\circ$ rotation around the cranking axis (1-axis). The rotation operator is given by:

$$\mathcal{R}_1 = e^{-i\pi j_1}. \quad (2.29)$$

The single-particle state, $|\alpha i\rangle$, can then be identified according to this conserved property as:

$$\mathcal{R}_1 |\alpha i\rangle = r |\alpha i\rangle \quad (2.30)$$

with eigenvalues $r = e^{-i\pi \alpha}$. The new quantum number $\alpha$ which is known as signature is preserved due to this symmetry (see Ref. [35]). The signature quantum number is related to the total angular momentum with:

$$I = \alpha \mod 2. \quad (2.31)$$

For an even-$A$ system:

$$I = \begin{cases} 0, 2, 4, \ldots \quad (\alpha = 0, r = +1) \\ 1, 3, 5, \ldots \quad (\alpha = 1, r = -1) \end{cases} \quad (2.32)$$

while for an odd-$A$ system:

$$I = \begin{cases} 1/2, 5/2, 9/2, \ldots \quad (\alpha = +1/2, r = -i) \\ 3/2, 7/2, 11/2, \ldots \quad (\alpha = -1/2, r = +i) \end{cases} \quad (2.33)$$

For an axially symmetric nucleus the cranking Hamiltonian can be calculated by summation over all independent particles of the system. The transformation of the wave function and Hamiltonian into the body-fixed coordinate system is performed by means of a rotation operator, $\mathcal{R} = e^{-i\omega t J_1/\hbar}$, with $J_1$ as the sum of angular momentum projections of all particles on the 1-axis. An expression for the total cranking Hamiltonian is obtained by inserting the transformed wave functions and Hamiltonians into the time dependent Schrödinger equation as:

$$H_\omega = H - \omega J_1. \quad (2.34)$$
2.6. THE ROTATIONAL NUCLEAR MOTION

The cranking Hamiltonian can be chosen as deformed Wood-Saxon or Nilsson Hamiltonian. The total Routhian which is summed over the $N$ single-particle Routhians is:

$$E_\omega = \sum_{i=1}^{N} \langle i | h_{s.p.}^\omega | i \rangle = \sum_{i=1}^{N} e_\omega^i.$$  \hfill (2.35)

The total angular momentum projection onto the rotation axis can be used to calculate the total energy in the laboratory system as:

$$I_x = \sum_{i=1}^{N} \langle i | j_1 | i \rangle,$$

$$E_{lab} = E_\omega + \omega I_x.$$  \hfill (2.36)

It is shown by Bohr and Mottelson that the calculated moment of inertia derived from the cranking formalism is approximately equal to that of a rigid body [36]. The single-particle Routhians of the $^{162}$Ta nucleus discussed in paper II have been calculated using the cranked Wood-Saxon Hamiltonian and some features like the negative slopes of the quasiparticle levels are extracted and used in the theoretical formulation of $B(M1)/B(E2)$ ratios (see below).

2.6.2 Nucleon-nucleon pair correlations

The results of diagonalizing the cranked shell model Hamiltonian are often incomplete without the inclusion of the pairing interaction. Pairing results from the short-range part of the nucleon-nucleon residual interaction. For nucleon pairs that are sufficiently close to the Fermi surface this attractive interaction manifests itself in scattering of pairs of nucleons from occupied time-reversed orbitals into excited states with pairs in time-reversed orbitals. The fact that all even-even nuclei couple to $0^+$ ground states is a clear evidence of the occurrence of the pairing interaction. The existence of pairing correlations is also strongly supported by other empirical observations such as the energy gap in the spectra of even-even nuclei and odd-even nuclear mass differences. The observation of the reduced moments of inertia of rotating deformed nuclei also suggests the presence of a nuclear pairing effect. The pairing force is taken into account by adding a two-body interaction of the form:

$$H_{pair} = -G \sum_{\mu,\nu>0} a_\mu^\dagger a_\mu a_\nu^\dagger a_\nu$$  \hfill (2.37)

to the single-particle Hamiltonian. Here $a_\mu^\dagger$ and $a_\nu$ are the fermion creation and annihilation operators acting on state $\mu$ and $\nu$, respectively, and $G$ is the interaction strength. The indices $\tilde{\mu}$ and $\tilde{\nu}$ refer to the time-reversed states of $\mu$ and $\nu$. The total Hamiltonian then reads:

$$H = \sum_\nu \epsilon_\nu a_\nu^\dagger a_\nu - G \sum_{\mu,\nu>0} a_\mu^\dagger a_\mu a_\nu a_\nu$$  \hfill (2.38)
CHAPTER 2. THEORETICAL BACKGROUND

with \( \varepsilon_\nu \) as single-particle energies. For a system of many pairs moving in different orbitals outside a closed core the pairwise correlations between the fermions in the nucleus may give rise to properties similar to superfluidity and superconductivity in condensed matter physics. Such nuclear “pairing” may also be described within the Bardeen-Cooper-Schrieffer (BCS) formulation \[37\]. In analogy to the formalism used to find the ground states of a superconductor one can obtain the ground state of such a system of nucleons by introducing a trial wavefunction as:

\[
|\Psi_{BCS}\rangle = \prod_{\nu > 0} (u_\nu + v_\nu a_\nu^+ a_\nu^+) |0\rangle.
\]

The \( |0\rangle \) here is the BCS vacuum state. The parameters \( u_\nu^2 \) and \( v_\nu^2 \) are the occupation probabilities which are determined by minimizing the expectation value of the Hamiltonian. The normalization condition requires that \( u_\mu^2 + v_\mu^2 = 1 \). The BCS formalism within this description does not have definite a number of particles and the minimization procedure should be performed with a constraint that fixes the average number of particles\(^1\) to \( N \). This is achieved by choosing a Lagrangian multiplier \( \lambda \) and rewriting the Hamiltonian as:

\[
\mathcal{H} = H - \lambda \hat{N}.
\]

The “quasiparticle” (qp) states are introduced by a unitary transformation that provides new sets of operators from the particle creation-annihilation operators. The nucleonic motion in such a system can then be described by the Hamiltonian:

\[
\mathcal{H} = \sum_{\nu > 0} (\varepsilon_\nu - \lambda)(a_\nu^+ a_\nu + a_\nu^+ a_\nu^+) - G \sum_{\mu, \nu > 0} a_\mu^+ a_\nu^+ a_\nu a_\mu.
\]

The minimization problem should be solved for a set of equations simultaneously to obtain the energy, average particle number and occupation probability. The detailed description of the procedure can be found in e.g. Refs \[38, 39\]. The quasiparticle energy in the BCS approximation is given as:

\[
E_\nu = \left[ (\varepsilon_\nu^\prime - \lambda)^2 + \Delta^2 \right]^{1/2}
\]

with the pairing energy gap parameter \( \Delta \) defined as:

\[
\Delta = G \sum_{\nu > 0} u_\nu v_\nu.
\]

\(^1\)The total number operator in the second quantization formalism is defined as \( \hat{N} = \sum_i a_i^+ a_i \).
The strength of the pairing interaction plays an important role in the structure of the nuclear systems with finite number of nucleons especially in weakly bound nuclei approaching the proton dripline. In the quasiparticle representation for an even-even nucleus the lowest excited states result at a minimal value of $E_x \approx 2\Delta$ with a typical value of $\Delta \approx 1$ MeV. In an odd nucleus the lowest-lying excited states correspond to 1qp excitations at even-even doubly-closed shell nucleus. It is also found that the moment of inertia calculated within the cranked shell model formalism is very close to the rigid-body moment of inertia. Experimentally deduced values show, however, that the moments of inertia are about 30%-50% less than that of the rigid moment of inertia. The inclusion of a residual two-body interaction in the single-particle cranked Hamiltonian greatly improves the calculation and reproduces the experimentally deduced moment of inertia with good precision. In a rotating deformed nucleus with a few valence nucleons there is an interplay between the pairing force which keeps the nucleons in occupied paired states and the Coriolis force which tends to break a pair of nucleons and align the single particle angular momenta with the rotation axis. This type of calculation is used to interpret the experimental results of paper II.

2.6.3 TRS calculations

The basic idea of the Total Routhian Surface (TRS) calculations is to calculate the energy in the rotating coordinate system as a function of the deformation and rotational frequency. It merges the macroscopic liquid drop model which accounts for the bulk properties of the nucleus and the mean field approach which is the basis of the shell model to describe microscopic properties of nuclei in the vicinity of closed shells. The total Routhian $E^\omega(Z,N,\hat{\beta})$ of a nucleus with $Z$ protons and $N$ neutrons and rotational frequency $\omega$ is calculated in $\hat{\beta} = (\beta_2, \beta_4, \gamma)$ deformation space as:

$$E^\omega(Z,N,\hat{\beta}) = E^\omega_{\text{macr}}(Z,N,\hat{\beta}) + \delta E^\omega_{\text{shell}}(Z,N,\hat{\beta}) + \delta E^\omega_{\text{pair}}(Z,N,\hat{\beta}). \quad (2.45)$$

Here the first term is the sum of the macroscopic liquid drop energy, the second term is added to account for the shell correction energy and the last term is added to include the pairing correction energy. Eq. 2.45 can be rewritten as:

$$E^\omega(Z,N,\hat{\beta}) = E^{\omega=0}(Z,N,\hat{\beta}) + \left[ \langle \Psi^0 | \hat{H}^\omega(Z,N,\hat{\beta}) | \Psi^\omega \rangle - \langle \hat{H}^{\omega=0}(Z,N,\hat{\beta}) \rangle_{\text{BCS}} \right] \quad (2.46)$$

where the first term, $E^{\omega=0}(Z,N,\hat{\beta})$, consists of the liquid drop energy, the shell correction energy (calculated from the Strutinsky shell correction method [40]) and the BCS pairing energy (calculated using the self-consistent BCS equations at $(\omega = 0)$). The second term accounts for the energy change induced by the rotation. In order to determine the equilibrium deformations the total Routhian is minimized with respect to the shape parameters and is then transformed into
CHAPTER 2. THEORETICAL BACKGROUND

Cartesian coordinates, \( X = \beta_2 \cos(\gamma + 30^\circ) \) and \( Y = \beta_2 \sin(\gamma + 30^\circ) \). The minimum of the Routhian at a fixed frequency \( \omega \) corresponds to the solution for an yrast state and the results are often depicted in contour maps of the energy in the \( \beta_2 - \gamma \) plane known as TRS plots. This approach has been successful in describing the shape-driving properties of deformed states. For the light neutron deficient nuclei in the transitional mass region \( A \approx 160 - 180 \) the occupied high-\( j \) orbitals can have large polarizing effects with the degree of polarization depending on the softness\(^2\) of the core. The deformation parameters obtained by TRS calculations for different rotational frequencies of the \( ^{162} \text{Ta} \) rotational band are shown in Fig. 2.3.

![Deformation parameters calculated for the \( ^{162} \text{Ta} \) rotational band corresponding to the configuration proton(\( \pi, \alpha \)) = (\( -\), \(-1/2 \)) \( \otimes \) neutron(\( \pi, \alpha \)) = (+, +1/2) at four rotational frequencies.](image)

2.6.4 \( B(M1)/B(E2) \) calculations

The calculation of \( \gamma \)-ray transition probabilities within a cranking model is not straightforward due to the complication of describing the angular momentum properties in this framework. In a direct method proposed by Dönau \cite{41} the axially symmetric rotor-plus-particle system is considered as an appropriate regime for treating the angular momentum. The cranking approximation is formulated to calculate the transition amplitudes of the electromagnetic radiation in a rotating nucleus. The method is applied to a single \( j \)-shell quasiparticle in a rotating axially deformed potential to specifically determine the \( M1 \) transition strength which is extracted from the \( M1 \) reduced transition matrix elements. In a semiclassical approach Dönau and Frauendorf derived a relation between the magnetic moment

\(^2\)The term softness refers to the polarizability of the nuclear shape with respect to the shape deformation parameters. For example for the Routhian minimum in the TRS plot a range of deformed shapes are taken into consideration at a rather constant energy.
vector and the quasiparticle angular momentum \([42]\). The coupling scheme of two quasiparticles plus a rotor (reference) for an axially symmetric system is illustrated in Fig. 2.4. The intrinsic system is rotating with an angular frequency \(\omega\) about the vector \(\vec{I}\) that is fixed in the lab system. The quasiparticle 1 is deformation aligned with the angular momentum component, \(i_1\), along the x axis and quasiparticle 2 is rotation aligned and has only a component, \(i_2\), along the x axis. The total angular momentum, \(\vec{I}\), is given as the sum of the quasiparticle angular momenta, \(\vec{j}_1\) and \(\vec{j}_2\), and the collective angular momentum \(\vec{R}\). The magnetic moment \(\mu\) can be considered to precess about the total angular momentum vector \(\vec{I}\) and only its component perpendicular to \(\vec{I}, \mu_\perp\) is a constant of motion. The \(M1\) transition strength is generated by \(\mu_\perp\) and hence only depends on the perpendicular component of \(\vec{j}_1\) and \(\vec{j}_2\). By representing the components in terms of trigonometric functions and combining with the quadrupole tensor components the ratio of the reduced transition probability, \(B(M1)/B(E2)\), is given by:

\[
\frac{B(M1, I \rightarrow I - 1)}{B(E2, I \rightarrow I - 2)} = \frac{12}{5Q_0^2 \cos^2(\gamma + 30^\circ) \cos^2(\gamma + 30^\circ)} \left(1 - \frac{K^2}{(I - 1/2)^2}\right)^2 \frac{K^2}{I^2} \times \frac{1}{\sqrt{I^2 - K^2 - i_1} - (g_2 - g_R)i_2}^2. \quad (2.47)
\]
Here the gyroscopic factors, $g_1$ and $g_2$, are estimated with the Schmidt relation \[43\] and the quadrupole moment of the charge distribution is given by:

$$Q_0 = \frac{3}{\sqrt{5\pi}} R^2 Z \beta_2 (1 + 0.16 \beta_2)$$

where $R$ is the nuclear radius, $Z$ the proton number and $\beta_2$ the deformation parameter. In the case where the deformation alignment is not ideal one should consider the contribution of the signature splitting term, $\Delta e'$, to the perpendicular magnetic vector, $\mu_\perp$.

### 2.7 Directional correlation and polarization of successive $\gamma$-rays

It is well known that in compound nucleus reactions a large degree of alignment is present after particle evaporation and that the angular momenta of the reaction products, being in excited states, are strongly oriented. For such an ensemble of aligned nuclei we deal with quantum mechanical systems which consist of many decaying nuclei and therefore we can typically measure only statistically how the initial and the final nuclear states are populated. The directional angular correlation measurement of $\gamma$-rays provides information about the multipole order of the radiation but does not resolve the ambiguity regarding the relative parity of nuclear states between which the transition occurs. The polarization state of electromagnetic radiation is associated with the electric or magnetic character of it. The knowledge of the polarization of $\gamma$-rays can remove the ambiguity about the parity of initial and final nuclear states. Moreover, in the case of mixed multipole radiations the results of a polarization measurement can reduce the ambiguities regarding the mixing ratio. There are various polarization sensitive processes through which linearly or circularly polarized $\gamma$-rays emitted from the nuclei are studied and numerous experiments have been performed in order to measure the polarization of $\gamma$-rays. The linear polarization of $\gamma$-rays has been studied by means of the Compton effect, Coulomb excitations, nuclear reactions and the photoelectric effect. The circular polarization of $\gamma$-rays is also studied by the Compton effect, $\beta - \gamma$ circular polarization correlation and circular polarization of neutron capture $\gamma$-rays. The theoretical aspects of angular correlation and polarization of $\gamma$-rays are studied in several pioneering works with various formulations \[44, 45, 46\]. The method has also been applied to $\alpha - \gamma$ and $\beta - \gamma$ correlation experiments.

#### 2.7.1 Gamma-gamma directional correlation

In the theory of directional correlations of successive radiations from excited nuclear states which was introduced by Hamilton \[47\] it was shown that there is a correlation between the propagation directions of quanta which are emitted successively, depending on their multipolarity. Experimentally one often observes the directions
2.7. DIRECTIONAL CORRELATION AND POLARIZATION OF SUCCESSIVE $\gamma$-RAYS

of propagation of radiations. Bradly and Deutsch succeeded in observing angular correlation of successive $\gamma$-rays for the first time [48]. The angular correlation between two successive radiations is expressed as:

$$W(\Theta) = \sum_{\nu} A_{\nu} P_{\nu}(\cos \Theta)$$

(2.49)

where the summation over $\nu$ is restricted to even values including zero and the coefficients $P_{\nu}(\cos \Theta)$ are the Legendre polynomials with $\Theta$ being the angle between the direction of emission of the two radiations. The calculation of the expansion coefficients $A_{\nu}$ is based on the fact that they can be broken up into two factors where each factor only depends on one transition of the cascade [49]. For a cascade of two successive $\gamma$-rays we assume that the intermediate state resulting from the first transition is characterized by a unique spin and parity and is not changed before the emission of the second radiation. The correlation function, $W(\Theta)$, is then the probability that the second $\gamma$-ray is emitted with a direction of propagation making an angle $\Theta$ with that of the first $\gamma$-ray. The direction of propagation of the first $\gamma$-ray is taken as the axis of quantization. With these assumptions the angular correlation function for a cascade $I_{i} \rightarrow I_{f}$ by successive emission of $\gamma_{1}$ and $\gamma_{2}$ with the first radiation $I_{i} \rightarrow I_{1}$ of multipolarity $L_1$ mixed with $L_1' = L_1 + 1$ and the second radiation $I_{1} \rightarrow I_{f}$ of multipolarity $L_2$ mixed with $L_2' = L_2 + 1$ is given by:

$$W(\Theta) = \sum_{\nu} A_{\nu}(L_1 L_1' I_I I) A_{\nu}(L_2 L_2' I_f I) P_{\nu}(\cos \Theta).$$

(2.50)

The expansion coefficients are given by:

$$A_{\nu}(L_1 L_1' I_I I) = \left( \frac{1}{1 + \delta_1^2} \right) \left( F_{\nu}(L_1 L_1 I_I I) + 2 \cdot \delta_1 F_{\nu}(L_1 L_1' I_I I) + \delta_1^2 F_{\nu}(L_1 L_1' I_I I) \right),$$

(2.51)

$$A_{\nu}(L_2 L_2' I_f I) = \left( \frac{1}{1 + \delta_2^2} \right) \left( F_{\nu}(L_2 L_2 I_f I) + 2 \cdot \delta_2 F_{\nu}(L_2 L_2' I_f I) + \delta_2^2 F_{\nu}(L_2 L_2' I_f I) \right).$$

(2.52)

The $F$-coefficients are symmetric in multipole orders ($F_{\nu}(LL' I' I) = F_{\nu}(L' L I'I)$) and for $\nu = 0$, $F_{0}(LL' I'I) = \delta_{L'L}$. The numerical tables of ordinary $F$-coefficients are given in Ref. [50]. Finally, the range of allowed angular momenta for $L$ and $L'$ are:

$$|I_i - I| \leq L_1, \quad L_1' \leq (I_i + I), \quad L_1, L_1' \neq 0,$$

$$|I - I_f| \leq L_2, \quad L_2' \leq (I + I_f), \quad L_2, L_2' \neq 0.$$
CHAPTER 2. THEORETICAL BACKGROUND

The directional correlation of $\gamma$-rays that are emitted from oriented nuclei can be calculated in the same manner as the directional correlation of a cascade of successive $\gamma$-rays. In the former case the degree of orientation provides vital information about the population of magnetic substates of the initial oriented nuclear state while in the latter case the observation of the direction of emission of the first $\gamma$-ray gives information about the population of the substates of the intermediate nuclear state. If we choose the beam direction as the axis of quantization the impinging particle brings orbital angular momentum to the system but only in magnetic substates with $m = 0$ and therefore the residual nuclei have their spins aligned in a plane perpendicular to the beam axis. The population can be approximately represented by a Gaussian distribution\(^3\) about this aligned direction. To extract information about the alignment one can also measure the angular distribution of de-exciting $\gamma$-rays with respect to the beam direction [51]. If the parity change is fixed the lowest possible multipolarities of $\gamma$-rays emitted in a cascade obey the following selection rules:

$$
\begin{align*}
M1, E2, M3, E4 & \quad \text{for } \pi_1 = \pi_2, \\
E1, M2, E3, M4 & \quad \text{for } \pi_1 \neq \pi_2.
\end{align*}
$$

(2.53)

As multipole order $L$ increases the transition probabilities rapidly decrease and it is therefore often appropriate to assume that except for the two lowest multipoles ($L = 1, 2$) the higher orders are negligible. Transitions may also show an admixture of multipoles and the multipole mixing ratio should also be considered in intensity analyses of certain $\gamma$-ray transitions. In the theory of directional correlation of $\gamma$-rays emitted from oriented nuclear states, known as DCO, the two successive $\gamma$-rays are emitted at angles $\theta_1$ and $\theta_2$ with respect to the beam direction. The angle $\Delta \phi$ is the angle between the planes spanned by the direction of propagation of each $\gamma$-ray and the beam direction. The details of the theory are discussed in Ref. [52] and the generalized $F$-coefficients are tabulated. For multidetector systems such as the EXOGAM and EUROGAM arrays different combinations of detector pairs and subsequent symmetries should be considered in building the angular correlation function. Experimentally the DCO function can be evaluated by constructing the asymmetrical $E_{\gamma \theta_1} - E_{\gamma \theta_2}$ correlation matrix where $\gamma$-ray energies detected at an angle $\theta_1$ are sorted against $\gamma$-ray energies detected at an angle $\theta_2$ where $\theta_1 \neq \theta_2$. The experimental DCO ratios are obtained from such correlation matrices and are defined as:

$$
R_{DCO} = \frac{T_{E_1}^{\theta_2} (\text{gated by } E_{\gamma \theta_1})}{T_{E_2}^{\theta_1} (\text{gated by } E_{\gamma \theta_2})}.
$$

(2.54)

\(^3\)This distribution is often centered around substate $m = 0$ with a half width of $\sigma$. For substates with short lifetimes the $\sigma/I$ ratio is relatively constant over a wide range of spin. This ratio can be determined experimentally if there are previously known $\gamma$-ray transitions with known multipolarities.
2.7. DIRECTIONAL CORRELATION AND POLARIZATION OF SUCCESSIVE γ-RAYS

For example by choosing the gating transition as a pure stretched $E2$ type one can distinguish between a pure stretched $M1$ and a pure stretched $E2$ transition from their corresponding DCO ratios.

2.7.2 Direction-polarization correlation

In a $\gamma$-$\gamma$ directional correlation measurement, one often observes the directions but not the polarizations of the two emitted $\gamma$-rays and the correlation function does not depend on the parities of the nuclear states. For example the experimental DCO ratio of a pure stretched $M1$ transition is identical to the DCO ratio of a pure stretched $E1$ transition if we assume the gating $\gamma$-ray is a pure stretched $E2$ type. Such ambiguities can be resolved if the polarization of the radiation, that is, if the direction of the electric vector of the $\gamma$-ray (often measured relative to a reference plane) is known. The correlation between the direction of propagation of one photon and the polarization state of another photon for the case of two successive $\gamma$-rays was calculated by Hamilton [53] and shortly after was observed by Deutch and Metzner [54]. They also studied the Direction-Polarization Correlation (DPC) by measuring the polarization of successive $\gamma$-rays of $^{46}$Sc, $^{60}$Co, $^{104}$Rh and $^{134}$Cs with scintillation counters [55]. This technique of detecting the linear polarization is based on polarization measurement in a $\gamma$-$\gamma$ cascade in which the polarization of one $\gamma$-ray is determined relative to the direction of the other $\gamma$-ray thereby fixing the relative parity. This method of $\gamma$-ray polarimetry has been extended with the recent development of segmented Ge detectors. The application of composite $\gamma$-ray detectors as Compton polarimeters has been investigated for large segmented Ge detector arrays such as EUROGAM and EXOGAM [56, 57]. The polarization correlation measurement can be described within the same formalism as a $\gamma$-$\gamma$ angular correlation measurement. We restrict the measurement to a combination of a polarization-insensitive detector to detect the direction of one $\gamma$-ray and a polarization-sensitive detector for the analysis of the polarization state of the other $\gamma$-ray with the assumption that the first $\gamma$-ray is detected in the polarization-insensitive detector. In order to obtain the direction of linear polarization in a DPC experiment each $A_{\nu}$ term of Eq.(2.50) that is characterized by $(L_2, L_2')$ is replaced with the following terms:

\[ 2L_2' - \text{pole Electric} : \quad A_{\nu}(L_2 L_2') + \cos(2\gamma)\zeta_{\nu}(L_2 L_2'), \]

\[ 2L_2' - \text{pole Magnetic} : \quad A_{\nu}(L_2 L_2') - \cos(2\gamma)\zeta_{\nu}(L_2 L_2'), \]

(2.55)

with $\gamma$ being the angle between the direction of the electric field vector $E$ and the plane of reaction. The linear polarization correlation function is thus written as:

\[ W(\Theta, \gamma) = W(\Theta) \pm \cos(2\gamma) \sum_{\nu=0,2,4} a_{\nu}(L_2 L_2') P_{\nu}(\cos \Theta). \]  

(2.56)
Table 2.1: The degree of the polarization for the case of a cascade of pure dipole and quadrupole radiations. If we denote $1 \rightarrow L_1 L_1 I_I$ and $2 \rightarrow L_2 L_2 I_I$ then $A22 = A_2(1)A_2(2)$ and $A44 = A_4(1)A_4(2)$.

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Spin sequence</th>
<th>$P(\Theta = 90^\circ)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic dipole</td>
<td>$0^+ \rightarrow 1^+ \rightarrow 0^+$</td>
<td>$+ \frac{A22}{2}$</td>
</tr>
<tr>
<td>Electric dipole</td>
<td>$0^+ \rightarrow 1^- \rightarrow 0^+$</td>
<td>$- \frac{A22}{2}$</td>
</tr>
<tr>
<td>Electric quadrupole</td>
<td>$0^+ \rightarrow 2^+ \rightarrow 0^+$</td>
<td>$+ \frac{A22+1.25A44}{2}$</td>
</tr>
<tr>
<td>Magnetic quadrupole</td>
<td>$0^+ \rightarrow 2^- \rightarrow 0^+$</td>
<td>$- \frac{A22+1.25A44}{2}$</td>
</tr>
</tbody>
</table>

where $W(\Theta)$ is the relative probability of the second $\gamma$-ray being emitted at an angle $\Theta$ with respect to the first. The coefficients $P_\nu(\cos \Theta)$ are normalized associated Legendre polynomials. For practical reasons the $\nu$ values are restricted to $\nu = 0, 2, 4$. The numerical values of $a_\nu(L_2L_2)$ coefficients can be found in Ref. [45].

Theoretically one is interested in polarization excess in which two extreme cases of $\gamma = 0^\circ$ and $\gamma = 90^\circ$ (normalized to the total unpolarized intensity) are considered. This can be written as:

$$P(\Theta, \gamma) = \frac{W(\Theta, \gamma = 0^\circ) - W(\Theta, \gamma = 90^\circ)}{W(\Theta)}. \quad (2.57)$$

Inserting the angular correlation function from Eqs.(2.56) we obtain:

$$P(\Theta) = \frac{\langle \pm \rangle L'_2 \sum_{\nu=0,2,4} a_\nu(L_2L'_2)P_\nu(\cos \Theta)}{W(\Theta)}. \quad (2.58)$$

The formula to calculate the degree of polarization is given for the most simplified cases of pure magnetic and electric dipole and quadrupole transitions in Table 2.1. As can be seen the sign of polarization for transitions of the same multipole order alters if transitions are electric or magnetic in character. Thus, if we determine the multipole order of the transitions in a cascade from angular correlation measurement then the sign of $W(\Theta, \gamma = 0^\circ) - W(\Theta, \gamma = 90^\circ)$ determines if those transitions are of electric or magnetic character.
Chapter 3

Experimental Techniques

The study of nuclides far off the stability line has been extended in recent years with the rapid development of multi-detector arrays and the application of selective tagging techniques. In parallel, accelerator facilities for radioactive ion beams are also pushing the experimental limits. Thanks to highly efficient gamma-ray spectrometers with high granularity it has been possible to measure the excitation energies of high-spin states of these nuclei, investigate nuclear properties in terms of collective and non-collective structures and probe the effect of protons and neutrons occupying nuclear orbitals in those high-spin states. This chapter is divided into two parts explaining two different in-beam spectroscopic techniques: a multi-detector system which was adapted for the spectroscopy of heavy $N \approx Z$ nuclei in the vicinity of the double closed shell nucleus $^{100}$Sn in the first part and the Recoil-Decay Tagging (RDT) technique which was used to identify the excited states of heavy neutron deficient nuclei located in the mass region $A \approx 160$ in the second part.

3.1 Heavy-ion fusion evaporation

The fusion-evaporation reaction is the principal reaction used in spectroscopic measurements of nuclear properties in the neutron deficient regions of the nuclear chart as it enables the production of reaction products at high angular momenta. In this reaction, which can be considered as a two-step process, the fusion of the projectile and the target nuclei produces a compound nucleus which lives for a short time ($\approx 10^{-18}$ s) and then may decay by emission of charged particles (protons, $\alpha$-particles, etc) and neutrons. The residual nucleus is then left in an excited state and de-excites to the ground state by emitting a cascade of $\gamma$-rays. The feasibility of the reaction depends on the kinetic energy in the center-of-mass reference frame being higher than the Coulomb barrier of the target-projectile system. A schematic view of the production of the compound nucleus $^{94}$Pd and four possible reaction channels after particle evaporation is depicted in Fig. 3.1.
3.1.1 Beam selection and target thickness

In order to populate the excited states of extremely neutron deficient nuclei a proper selection of the ion beam and the target is essential. Before the experiment the production cross section of the reaction channel of interest and the most competing neighboring reaction channels are estimated by comparison with other relevant spectroscopic measurements and by running statistical simulation codes such as PACE4 [58] and LISE++ [59]. A higher beam energy combined with a thicker target may produce a larger number of nuclei of interest but at the cost of an enhanced production of other strong reaction channels potentially affecting the quality of the \( \gamma \)-ray energy spectra of interest. When the main objective is to study low-lying states of the weak \( 2n \)-evaporation reaction channel, the energy of the beam should be estimated in a way so that it is only slightly higher than the Coulomb barrier to reduce other unwanted reaction channels as much as possible. By considering different beams and targets and comparing the estimated relative yield of the nucleus of interest to the total fusion evaporation yield the beam-target selection should be optimized. The beam energy is also optimized based on the stopping power of the target. In the \( ^{92}\text{Pd} \) experiment (paper I) the beam energy and target thickness were chosen such that the cross section for \( ^{92}\text{Pd} \) production was as high as possible and the compound nucleus was stopped in the target. In the experiment resulting in paper II the main focus of the experiment was to populate high spin excited states of \( ^{163}\text{Re} \) nucleus so the cross section for \( ^{162}\text{Ta} \) production was not used to optimize the beam energy [8]. The beam-target combination was chosen in a way so that the recoils travelled downstream from the target to the recoil separator and were transmitted to the focal plane for alpha decay tagging measurements. In the experiment resulting in paper III and IV \( ^{40}\text{Ca} \) ions were delivered by the CSS1 cyclotron of GANIL (Grand Accélérateur National d’Ions
3.2 EXPERIMENTAL SET-UP TO STUDY NUCLEAR STRUCTURE IN THE VICINITY OF $N = Z = 50$

Lourds), in France. The occurrence of some technical difficulties in deliverance of the beam resulted in only 25% of the total beam time which was allocated for identifying the excited states of $^{96}$Cd.

3.2 Experimental set-up to study nuclear structure in the vicinity of $N = Z = 50$

The region of nuclei immediately around the presumed doubly-magic nucleus $^{100}$Sn where the $N = Z$ line coincides with the proton dripline is unique and therefore is expected to show special structure features. Nuclei with $N \approx Z$ are likely to show enhanced neutron-proton ($np$) correlations as the valence neutrons and protons occupy identical orbitals. Strong $np$ correlations could considerably alter the level structures compared with predicted nuclear properties closer to the line of stability. So far the closest observed even mass neighbors of $^{100}$Sn for which the nuclear excited states have been identified are $^{102}$Sn [60] and $^{98}$Cd [61] which were observed using EUROBALL Ge cluster detectors and NORDBALL ancillary detectors. The heaviest even-even $N = Z$ nuclei with known low-lying yrast level structures, $^{84}$Mo and $^{88}$Ru, were studied using the GASP Ge array and the ISIS silicon ball [62, 63].

In the experiment resulting in paper I the principal goal was to measure the excited states of $^{92}$Pd and to seek evidence for a spin-aligned $T = 0$ np coupling scheme by comparing the observed level energies with the shell model predictions. In this experiment, which was performed at GANIL, the assignment of $\gamma$-rays was done using the EXOGAM $\gamma$-ray spectrometer array and the particle detection was achieved by ancillary detectors consisting of the DIAMANT detector system for charged particle identification and the Neutron Wall for detection of neutrons. This detector set-up was also used in another experiment with the original aim of searching for an isoscalar spin-aligned $np$ coupling scheme in $^{96}$Cd nucleus via $^{58}$Ni($^{40}$Ca,2$n$) reaction. The fusion-evaporation cross section for populating $^{96}$Cd in the $2n$ reaction channel was estimated to be about 1.0 $\mu$b and although it was not possible to obtain enough statistics for this nucleus still other prolific reaction channels were produced with sufficient statistics. The primary goal of the experiment could therefore not be achieved. Nevertheless, the data analysis led to the results reported in paper III and IV.

3.2.1 EXOGAM

The emitted $\gamma$-rays from the reaction products were detected using the EXOGAM Ge-detector array comprised of 11 clover detectors. Each clover consists of four germanium crystals and each crystal is segmented in four quadrants of equal volume. Seven clover detectors were placed at an angle of 90° and four detectors at an angle of 135° relative to the beam direction. The composite detectors were surrounded by escape suppression shields consisting of BGO (bismuth germanate) scintillators to suppress the background caused by Compton scattering out of the
CHAPTER 3. EXPERIMENTAL TECHNIQUES

Figure 3.2: A photograph and a schematic illustration of EXOGAM (left) and the Neutron Wall (right) detector array. The DIAMANT charged particle detector array is inside a vacuum chamber located around the target (Photo: F. Ghazi Moradi. Figure: courtesy of T. Bäck).

germanium crystals. In order to increase the total $\gamma$-ray detection efficiency, parts of the Compton suppression shields were removed from the clover detectors and the germanium detectors were brought closer to the reaction target. The total photo-peak efficiency of EXOGAM was 11% at 1332 keV. The high efficiency of the array, the excellent Ge detector energy resolution (about 2.2 keV at 1332 keV) and the effective background reduction of the Compton suppression shields made it possible to get clean energy spectra. The detected $\gamma$-ray energies were sorted off-line into two-dimensional histograms ($E_\gamma - E_\gamma$ coincidence matrices) after setting relevant gates on charged particles and neutrons. A cross section of the detector array is illustrated in Fig. 3.3. The geometrical configuration of the array covers a solid angle of about 3$\pi$ allowing room for the Neutron Wall detector array at the forward angles.

3.2.2 Neutron Wall

The detection of neutrons emitted following the production of the reaction channel of interest was a crucial part of the experiment. The Neutron Wall detector array, from the EUROBALL project, consists of 50 organic BC501A liquid-scintillator detectors mounted in 16 detector modules in hexagonal and pentagonal geometrical configurations. The array covers 1$\pi$ solid angle in the forward direction. The kinematics of the reaction focuses the emitted neutrons towards the forward angles thus increasing the detector efficiency for neutrons. The thickness of each detector is 15 cm and the distance from the target to the center of the front face of neutron detectors is 50 cm. The special character of this type of scintillator is that for each type of particle there is a distinct response of the detector in producing the pulse shape. The front-end electronics, which uses the zero-crossing technique [64,
3.2. **EXPERIMENTAL SET-UP TO STUDY NUCLEAR STRUCTURE IN THE VICINITY OF $N = Z = 50$**

65, 66, 67, has two inputs; the PMT anode pulse and the external time reference signal. The measured quantities (appearing as output signals) for each individual neutron detector are the zero-crossing time (ZC), the time-of-flight (TOF) and the energy spectrum of neutrons and gammas detected in each detector. Before the experiment the hardware gain matching of the anode signal was accomplished by adjusting the applied high voltage to each detector. To avoid noise the CFD threshold of each pulse shape discrimination (PSD) unit was adjusted and hardware time alignment was done to make sure that the centroid of the time peak of all detectors matched. The Neutron Wall has a time resolution of about 1 ns enabling it to discriminate between neutron and gamma events based on the difference in time-of-flight from the target to the Neutron Wall. In the $^{92}$Pd experiment the radio frequency (RF) signal from CIME cyclotron was used as the external time reference for the TOF signal. The precision of this signal (about 3.5 ns) was monitored during the experiment by measuring the time between the RF signal and a signal from a BaF$_2$ detector mounted in the EXOGAM frame. The Neutron Wall detector array is shown in Fig. 3.4. In the $^{92}$Pd experiment the $ln$ detection efficiency was found to be about 23% while in paper III and paper IV it was estimated to 14%. These differences can mainly be explained by the poor beam quality resulting from technical difficulties in the accelerator.

Figure 3.3: Left: A close-up photograph and right: a schematic drawing of the EXOGAM array (Photo: A. Khaplanov).
3.2.3 DIAMANT

The DIAMANT charged particle detector system is a $4\pi$ detector array consisting of 84 CsI(Tl) scintillators coupled to PIN-photodiodes. In CsI(Tl) the relative intensity of the fast and the slow light emission components depend on the energy loss of the particle ($dE/dx$). Therefore the overall decay time of the emitted light pulse is different for protons and $\alpha$-particles making it possible to distinguish between them. The array is arranged in a polyhedron compact geometry consisting of square and triangular shaped detectors. In order to shield the detectors from the scattered beam particles and delta electrons (produced when the beam hits the target), tantalum absorber foils of optimized thickness for each detection angle were used. The distance to the target from the detectors was about 3 cm. The measured parameters for each detector were energy, time, and a particle identification (PID) parameter. The PID signals are obtained from the pulse shapes using the ballistic deficit method [68, 69]. The typical $\alpha$-particle and proton detection efficiencies are about 50% and 70%, respectively and the typical relative $\alpha$-energy resolution at 5.5 MeV is about 2%. A schematic drawing of the detector arrangement can be seen in Fig. 3.5.

3.2.4 Trigger condition

For the $^{92}$Pd experiment a general trigger condition was set for the synchronization of the data processing. The goal was to identify each event which was detected by EXOGAM and the ancillary detector electronics during a typical event processing time of the order of a microsecond. The EXOGAM main trigger was created in
3.2. EXPERIMENTAL SET-UP TO STUDY NUCLEAR STRUCTURE IN THE VICINITY OF $N = Z = 50$

Figure 3.5: Schematic drawing of the DIAMANT array (courtesy of B. M. Nyakó).

the Master Trigger (MT) card which recognizes the events from the Ge detector multiplicity and a user-defined external logic input. This card generates a Fast Trigger (FT) signal (before the Ge pulse shapers reach a peak) as an event indication and later a validation signal which is used by the detector electronics to confirm the good event and initiate the data readout. The trigger condition was fulfilled if one or more $\gamma$-rays were registered in the Ge detectors together with at least one neutron in the Neutron Wall detector. A hardware trigger requirement on the pulse shape from the neutron detectors was set using the zero-cross-over (ZCO) time. Since the neutron signal of the Neutron Wall was required in the trigger signal a fine-tuned ZC adjustment was performed for each individual detector and the threshold was set in a way so that the majority of the gamma signals in the ZCO spectrum were avoided. This hardware trigger condition was used in the experiment resulting in paper I and was also sufficiently relaxed so that data from pure charged particle evaporation channels could be collected resulting in paper III and IV.
3.3 In-beam spectroscopy of extremely neutron deficient nuclei in the $A \sim 160$ mass region

Following the heavy-ion fusion evaporation reactions leading to the nucleus under study many other reaction channels with large cross sections are open and a large number of unwanted $\gamma$-rays are emitted near the target and are detected by the gamma detectors. The high selective power of the Recoil Decay Tagging (RDT) technique enables clean selection of a specific reaction channel and precise spectroscopic studies of nuclei produced with cross sections well below 1 $\mu$b. This method is based on separation and identification of fusion evaporation residues (recoils) and detection of their radioactive decay by means of a proper spatial and temporal correlation between them. The prompt $\gamma$-rays which are emitted at the target position are correlated with the recoil and its subsequent decay and can be associated with the reaction channel of interest. The experiment resulting in paper II, which was performed at the University of Jyväskylä Accelerator Center in Finland, employed the RDT technique to study excited states in the $^{163}$Re via the $^{106}$Cd($^{60}$Ni, $p2n$) reaction (see Fig. 3.6). Different neighboring fusion evaporation channels such as $2p1n$ leading to $^{163}$W, $3p$ leading to $^{163}$Ta and $3pn$ leading to $^{162}$Ta were also present. The high-spin excited states of the recoils de-excited to the ground state by emission of prompt $\gamma$-rays that were detected in the JUROGAM germanium spectrometer. The recoils leaving the thin target were then separated from the beam particles in the RITU gas filled separator and were transported to the focal plane detector system for gamma-correlated recoil identification and the subsequent decay detection by the GREAT spectrometer. As will be discussed in chapter 4, the decay-tagging technique was not applied for identification of excited states in $^{162}$Ta due to the unfavorably low $^{162}$Ta $\alpha$-decay branching ratio.

3.3.1 JUROGAM

Coincident $\gamma$-ray events were recorded at the target position by the JUROGAM detector array consisting of 43 Compton-suppressed high-purity germanium (HPGe) detectors with high granularity and large solid angle coverage. The detectors are of two types: single tapered detectors (type I) and clover detectors (type II) and are placed at six rings at different angles relative to the beam direction [70]. Since the first quadrupole magnet of the RITU separator is located close to the target the array has less detectors in the forward direction. The detectors operate in an energy range of between approximately 50 keV and 10 MeV. The total photopeak efficiency for JUROGAM was about 4.2% at 1332.5 keV. The energy resolutions (FWHM) of the detectors were measured to be between 2 keV and 3 keV for the 1332.5 keV peak. The peak-to-total ratio (the ratio of total photopeak area compared to the total number of detected events in the $^{60}$Co gamma spectrum) was about 25%.
3.3. IN-BEAM SPECTROSCOPY OF EXTREMELY NEUTRON DEFICIENT NUCLEI IN THE A ∼ 160 MASS REGION

3.3.2 The gas-filled recoil separator RITU

The unstable heavy rare-earth nuclei close to the proton drip-line that can be produced in heavy-ion fusion evaporation reactions are often mixed with a large background from the strongest fusion-evaporation channels as well as products emanating from other reactions such as fission, transfer reactions and Coulomb excitations. The γ-rays emitted from the nuclei of interest are therefore often buried under a high γ-ray background (mostly from fission). Because of this, a clean separation of recoils from fission fragments and beam particles is an essential factor in background suppression of the recoil-correlated γ-ray spectra. This is done by means of a recoil separator where by applying a strong magnetic field different reaction products with different magnetic rigidities are separated in-flight and fusion evaporation residues can be cleaned from the primary beam particles. In gas-filled recoil separators such as RITU a helium gas at low pressure (≈1 mbar) is injected in the volume between the target chamber and the focal plane detector. This causes atomic charge-changing collisions of the reaction products with the gas molecules which results in a change in the average charge state and in this way a higher total transmission can be achieved. The arrangement $Q_vDQ_h$ is used for RITU where D stands for the bending dipole magnet and $Q_h$ and $Q_v$ stand for horizontally and vertically focusing quadrupoles, respectively.

3.3.3 The focal-plane spectrometer GREAT

The reaction products were implanted at the focal plane of RITU where the Gamma Recoil Electron Alpha Tagging (GREAT) spectrometer is situated. This composite
detector installation enables the identification of the recoil, its subsequent decay as well as isomer spectroscopy. The major detector parts of GREAT are:

1. The Multi-Wire Proportional Counter (MWPC) which is a gas detector placed after the RITU recoil separator and before the DSSSDs that is used to measure the time of flight and the deposited energy of those fusion evaporation products that pass through it and implant into the DSSSD array.

2. An array of Si PIN diode detectors, placed in a box directly in front of the DSSSDs, which consists of 28 silicon PIN diodes with an active area of $28 \times 28 \text{ mm}^2$ and a thickness of 500 $\mu\text{m}$. These detectors are used for conversion electron measurements and for detection of those alpha particles that escape the DSSSD.

3. Two Double-Sided Silicon Strip Detectors (DSSSD) form the essential part of the GREAT spectrometer where recoils are implanted and $\alpha$-decays are detected. Each detector has a thickness of 300 $\mu\text{m}$ and an active area of $60 \times 40 \text{ mm}^2$.

4. A Double-sided planar germanium detector, which is mounted downstream from the DSSSD inside the vacuum, is used to measure X-rays and low energy $\gamma$-rays emitted from isomeric states. A high-efficiency segmented clover germanium detector, which is mounted above the GREAT vacuum chamber, is used to measure high energy $\gamma$-rays.

3.3.4 Data Acquisition

The triggerless total-data-readout (TDR) acquisition system [71] was used for data collection during the experiment. It was controlled by the MIDAS software [72]. The advantage of such a data acquisition system is that all data channels are time stamped and read out independent of any hardware trigger, reducing the dead time losses to those from pile-up in the individual detector channels. The data from individual detector channels can then be correlated temporally and spatially in the offline analysis making it possible to reconstruct physical events originating from rare reaction products even when the total event rate is quite large. The signals from the JUROGAM Ge detectors are typically registered about 0.5 $\mu\text{s}$ to 1 $\mu\text{s}$ before a DSSSD signal produced by the corresponding ion implant, depending on the velocity of the fusion products. A software trigger time setting can thus be applied to reduce storage of the JUROGAM data to those with the proper flight time from the reaction target to the RITU focal plane detectors. The physical events were reconstructed using the GRAIN [73] and RADWARE [74] software packages.
Chapter 4

Data Analysis

The data processing of an in-beam spectroscopic measurement is usually carried out online to evaluate different parameters during the experiment and offline to perform a detailed, fine-tuned analysis. This chapter covers the analysis of in-beam spectroscopy of the $^{92}\text{Pd}$, $^{94}\text{Ru}$, $^{95}\text{Rh}$ and $^{162}\text{Ta}$ nuclei. Since different techniques were used in these experiments the data analysis of each experiment is presented under a separate section.

4.1 $^{92}\text{Pd}$, $^{94}\text{Ru}$ and $^{95}\text{Rh}$: Data acquisition and sorting

The lowest excited states in $^{92}\text{Pd}$ were observed via detection of gamma rays emitted in the fusion-evaporation reaction together with detection of charged particles and neutrons in the ancillary detector system. In the experiment resulting in paper I the principal goal was to measure the excited states of $^{92}\text{Pd}$. The experiment lasted 14 days and $3.9 \times 10^9$ events were recorded as 563 files with a maximum size of 700 Mbyte each. For the experiment resulting in paper III and IV, although the original aim was the identification of the lowest excited states in $^{96}\text{Cd}$, still high statistics for the $4p$- and $3p$- exit channels allowed the spectroscopy of the $^{94}\text{Ru}$ and $^{95}\text{Rh}$ nuclei. During this experiment 90 files with a maximum size of 2 Gbyte each were recorded in total. In both experiments the output format of the files was ordered as event-by-event in data blocks with the same fixed length and contained individual events as a collection of 16 bit words. The calibration of the energy spectra and the efficiency measurements of the Ge detectors were performed using a $^{152}\text{Eu}$ source and all Ge detectors were gain-matched to obtain a good overall resolution. After the alignment of the Ge time spectra the prompt events were selected by setting a narrow time gate on the prompt peak. The proper calibration and alignment check of all TOF spectra of the Neutron Wall were also of great importance for clean selection of any populated neutron reaction channel. The calibration of the DIAMANT detector was performed online and in the offline analysis a threshold was set for each individual energy spectrum of the CsI(Tl)
detectors to avoid triggering on noise.

4.1.1 Channel identification and gating

Prompt protons and α-particles were identified by simultaneous selection criteria on the PID and energy parameters of the DIAMANT particle detector. An expression for the probability of detecting \( p \) out of \( P \) emitted protons and \( a \) out of \( A \) emitted α-particle can be written as a binomial distribution:

\[
P_{PA}^{pA} = \binom{P}{p} \binom{A}{a} \epsilon_p^p (1 - \epsilon_p)^{P-p} \epsilon_\alpha^a (1 - \epsilon_\alpha)^{A-a}
\] (4.1)

where the \( \epsilon_p \) and \( \epsilon_\alpha \) symbols are the efficiencies to detect a proton and an α particle, respectively. The total intensity of a γ-ray following the emission of \( P \) protons and \( A \) α-particles is related to the measured intensity in coincidence with \( p \) protons and \( a \) α-particles by:

\[
I_{exp} = I_{total} \cdot P_{PA}^{pA}.
\] (4.2)

The detection efficiencies were measured by comparing the γ-ray intensities belonging to a certain fusion evaporation reaction channel in spectra gated with different particle conditions. In the \(^{92}\text{Pd}\) experiment (paper 1) the α-particle and proton detection efficiencies were estimated to be 40% and 55%, respectively. Since a pure \( 2n \)-evaporation channel leading to \(^{92}\text{Pd}\) was the aim of this study a general veto condition on any detected charged particle in the DIAMANT array was applied.Gamma rays from decays of excited states in \(^{92}\text{Pd}\) were identified by comparing γ-ray spectra in coincidence with two emitted neutrons and no charged particles with γ-ray spectra in coincidence with other combinations of neutrons and charged particles. This was done by setting a two dimensional cut in the PID versus energy plot as shown in Fig. 4.1. The efficiency for detecting any charged particle then rose to 66% compared with the detection efficiency of cleanly identified individual particle types. Most reaction channels in this experiment involved emission of more than one charged particle. Thus a higher average rejection fraction was obtained in the selection of the rare \( 2n \)-evaporation events from the total number of events which were dominated by the prolific charged particle emission channels. For γ-rays that were detected in coincidence with two protons and one neutron (\( 2p1n \) leading to \(^{91}\text{Ru} \) which was the strongest reaction channel involving a neutron signal) and that passed the trigger condition this rejection fraction was 88% (see Fig. 4.2). The γ-rays from \( 2n \) channel leading to \(^{92}\text{Pd}\) are expected to be very weak, and are not expected to be visible in spectra gated by any other combination of detected particles. In the \( 1n \)-gated spectra there were more events from the \( 2n \)-evaporation channel than in the \( 2n \)-gated spectra due to the finite detection efficiency. These γ-rays were buried in the huge background from strong reaction channels that leaked into the spectra from different charged particle combinations. Since the vacuum in the beam line and the target chamber was not ideal and the fusion-evaporation cross sections for \(^{16}\text{O}\) and \(^{12}\text{C}\) were large compared with the \(^{92}\text{Pd}\) reaction channel,
Figure 4.1: Proton and α-particle distributions of the DIAMANT segment number 3. A veto condition was applied in the offline analysis in a way that γ-rays associated with any registered event above the noise threshold in DIAMANT were vetoed. This is illustrated by the wide 2D-gate (dashed).

Figure 4.2: Comparison of 2n-gated spectra before and after applying the veto condition.

the contaminating reactions involving these nuclides were also visible in the corresponding particle-gated spectra. The major contaminants in the spectra gated by
2n were from the reaction channels with one or two emitted neutrons, together with one or two protons corresponding to $^{91}$Rh [75] and $^{93}$Ru [76] and $^{46}$V [77, 78]. The last nuclide was produced from $^{38}$Ar-induced 1p1n-evaporation reactions on small amounts of carbon deposited on the targets during irradiation and its $\gamma$-rays were visible in the corresponding particle $\gamma$-ray gated spectra. Gamma rays produced in these reactions together with all $\gamma$-rays emitted in other significant reaction channels are reported in the literature. An extensive study was also done to search for the known $\gamma$-ray transitions which could be produced in the 2$n$-gated spectra originating from fusion evaporation reactions of $^{36}$Ar beam particles and also from very small percentage of impurities on the target material (such as $^{60-64}$Ni).

In the experiment resulting in papers III and IV $\alpha$-particle and proton efficiencies were estimated to 46% and 62%. For the identification of 4$p$- and 3$p$-exit channels 80 individual two-dimensional gates were set on PID and energy parameters of DIAMANT. An example of the population yield in this experiment is shown in Fig. 4.3 where the particle identification spectrum is plotted against the $PIDIndex$ number which is defined as:

$$PIDIndex = 16 \cdot P_m + 4 \cdot \alpha_m + N_m$$

(4.3)

where $P_m, \alpha_m$ and $N_m$ are proton, $\alpha$-particle and neutron multiplicities$^1$. The particle identification spectrum for each channel includes also the contribution from

$^1$Particle multiplicity is defined for each event during offline analysis where only the events that
reaction channels with larger particle multiplicities. The strongest populated reaction channels were $4p$- and $3p$ channels leading to the $^{94}\text{Ru}$ and $^{95}\text{Rh}$ residual nuclei, respectively.

### 4.1.2 Discrimination of neutrons and $\gamma$-rays

The large number of $\gamma$-ray events detected by the Neutron Wall scintillators together with the fast neutrons could be suppressed using a pulse shape discrimination technique and time-of-flight (TOF) to distinguish between detected neutrons and $\gamma$-rays. The time component of the light pulse generated by a recoil proton in a $(n,p)$ scattering process or a $(\gamma,e^-)$ process, was used to derive the TOF and the ZCO parameters for neutrons and $\gamma$-rays. Fig. 4.4 shows the TOF parameter plotted against ZCO parameter where for each event neutrons and $\gamma$-rays were discriminated with high accuracy. The probability of mis-identification of a gamma event as a neutron event in $^{92}\text{Pd}$ experiment was measured to be less than 0.3% but this number is very sensitive to the setting of the gate, as is the neutron detection efficiency.

![Figure 4.4: Neutron-gamma discrimination by means of setting a two dimensional gate on TOF versus ZCO.](image.png)

have information about any detected particles in the DIAMANT detector and the Neutron Wall detector with at least one detected $\gamma$-ray in EXOGAM are registered. The number of detected protons and $\alpha$-particles that were identified by PID and energy parameters of DIAMANT are denoted as $P_m$ and $\alpha_m$, respectively and the number of neutrons that were identified in the Neutron Wall by means of TOF-ZCO discrimination is denoted as $N_m$. 
4.1.3 Neutron multiplicity correction

As mentioned above, in the $^{92}$Pd experiment (paper I), due to the low production cross section for the $2n$ reaction channel, the excited states of the $^{92}$Pd nucleus were very weakly populated (with a relative yield of less than $10^{-5}$ of the total fusion cross section) compared with the other prolific evaporation channels. There is also a certain probability that whenever a neutron is detected in one of the Neutron Wall detectors it could scatter out into the neighboring detectors and again be detected within the same time window that is set for the Neutron Wall electronics (see Fig. 4.5). This gives rise to background emanating from the one neutron re-

![Figure 4.5: Schematic illustration of neutron multi-scattering in the Neutron Wall detectors shows single-scattering (green) and multi-scattering (yellow) of a neutron event.](image)

action channels in $\gamma$-ray spectra gated by two neutrons. For $^{91}$Ru ($2p1n$ channel) which was one of the reaction channels with highest cross section the probability of detecting a true one neutron event in the $2n$ channel was 12%. Therefore it was of utmost importance to improve the discrimination of the $1n$ scattered events from the events with two emitted neutrons. Considering that the scattering mainly occurs between adjacent detectors the time difference between true $2n$ and $1n$-scattered events are very small and by rejecting the $2n$ events in those neighboring detectors we can partly suppress the prompt $\gamma$-rays related to false $2n$ events and
obtain a cleaner 2n-gated γ-ray spectrum. Moreover, since the emitted neutrons have a finite velocity, the difference in the detected time of interaction resulting from two separate neutrons is on average smaller than for a 1n-scattered neutron. The reduction of the background from neutron scattering in 2n-gated spectra could therefore be achieved by applying a criterion on the difference in the TOF parameter relative to the distance between the neutron detectors firing. In this way, depending on the distance between the centers of those fired detectors and the TOF difference values a two dimensional cut was used to further reduce the influence of the scattered neutrons. In Fig. 4.6 the neutron detector distances are plotted versus ΔTOF and by looking at the projected ΔTOF spectra for different distances we can set a proper time gate for each detected 2n-like event. With this neutron multiplicity correction the efficiency for correctly identifying both neutrons from a two-neutron event was estimated to 3% and the rejection efficiency of 1n-scattered neutrons from the $^{58}\text{Ni}(^{36}\text{Ar},2p1n)^{91}\text{Ru}$ reaction was raised to 87% while 73% of the real 2n events were preserved. In Fig. 4.7 the charge particle vetoed 2n-gated γ-ray spectrum is shown before and after the multiplicity correction. The rejection efficiency of the $^{12}\text{C}(^{36}\text{Ar},1p1n)^{46}\text{V}$ reaction channel involving $^{12}\text{C}$ target contaminant was estimated to be lower ($\approx 75\%$) because the neutron scatter rejection factor decreases as a function of the velocity of the compound nucleus.
Figure 4.7: 2n-gated spectra following the $^{58}\text{Ni}(^{40}\text{Ar},2n)$ reaction representing prompt γ-rays in coincidence with two neutrons. Charged particle veto criterion was applied to all spectra. (a): Before neutron multiplicity correction (b): After nearest neighbor rejection correction (c) After nearest neighbor rejection and $\Delta TOF$ corrections. Symbols indicate γ-rays from known 1n and 2n reaction channels. Gamma rays from 2n reaction channels were more pronounced after suppression. (d): To magnify the difference after suppression a very small fraction of the 1n-gated spectrum was subtracted from the suppressed 2n-gated spectrum. The star symbol marks a transition for which the assignment is uncertain. The transitions assigned as the low-lying states of $^{92}\text{Pd}$ are marked as filled blue squares. The open blue squares mark transitions 857 and 954 keV that are tentatively assigned to $^{92}\text{Pd}$. 
4.1. Deducing the level scheme of $^{92}$Pd

In order to further suppress the background originating from the Compton scattered photons, the energies extracted from individual coincident pulses from the four crystals belonging to one EXOGAM clover were summed to create a single event. This procedure is commonly called add-back. After applying the charged particle veto condition, the neutron multiplicity correction, and the Compton background suppression the resulting $\gamma$-ray data events were sorted into $E_\gamma - E_\gamma$ coincidence matrices. Figure 4.8 shows the gated $\gamma$-ray spectra from the charged particle-vetoed, 2n-gated $E_\gamma - E_\gamma$ coincidence matrix. Three $\gamma$-ray transitions with energies 874 keV, 912 keV, and 750 keV which are mutually coincident have been assigned to $^{92}$Pd. Two additional $\gamma$-rays with energies 954 keV and 857 keV were also tentatively assigned to this nucleus. A comparison of 2n-gated $\gamma$-ray projected spectra of the $E_\gamma - E_\gamma$ coincidence matrices with and without applying the charged particle veto condition confirmed that these three $\gamma$-rays are not associated with emission of charged particles from the compound nucleus. A plot of the intensity ratios of these $\gamma$-rays in coincidence with two neutrons and one neutron (see figure 3d of paper I) also shows that the $\gamma$-rays assigned to $^{92}$Pd belong to the 2n-evaporation reaction channel. The $\gamma$-ray transitions assigned to $^{92}$Pd (874 keV, 912 keV and 750 keV) were ordered based on their relative intensities (see Fig. 4.8) to constitute a cascade of mutually coincident transitions in the ground state band. Because of the uncertainties in the relative intensities of these three transitions there was also
an uncertainty in their ordering. The relative intensities were normalized to the intensity of the 874 keV transition and were 100(8), 77(5) and 50(6) for 873.6(2), 912.4(2) and 749.8(3), respectively (the statistical errors are given in parenthesis).

4.1.5 Compton polarimetry using EXOGAM

The Compton effect has an ideal response to the polarization of an incident γ-ray. The differential cross section \( \frac{d\sigma}{d\Omega} \) for the scattering of a photon off free electrons is given by the Klein-Nishina formula:

\[
\frac{d\sigma}{d\Omega} = \frac{1}{2} r_0^2 \frac{E_{\gamma}^2}{E_{\gamma}'} \left( \frac{E_{\gamma}'}{E_{\gamma}} + \frac{E_{\gamma}}{E_{\gamma}'} - 2 \sin^2 \theta \cos^2 \phi \right)
\]  

(4.4)

where \( r_0 = e^2/m_0 c^2 \) is the classical electron radius, \( E_{\gamma} \) and \( E_{\gamma}' \) are the energy of the incoming and the scattered photons respectively, \( \theta \) is the scattering angle and \( \phi \) is the angle between the direction of the polarization vector of the incident photon and the plane of scattering. In order to measure the linear polarization of the incoming photon one can for example determine the difference of the Compton scattering cross sections in the two extreme cases of \( \phi = 0^\circ \) and \( \phi = 90^\circ \). Usually (and also here) the intensities corresponding to Compton scattering at \( \theta = 90^\circ \) which are parallel (\( N_\parallel \)) or perpendicular (\( N_\perp \)) to the reaction plane are measured (see Fig. 4.9). The yields in the full energy peaks in the corresponding energy spectra are given by \([79]\):

\[
N_\parallel = W(\Theta, \gamma = 0^\circ) \cdot \frac{d\sigma}{d\Omega}(\theta, \phi = 0^\circ) + W(\Theta, \gamma = 90^\circ) \cdot \frac{d\sigma}{d\Omega}(\theta, \phi = 90^\circ),
\]

\[
N_\perp = W(\Theta, \gamma = 0^\circ) \cdot \frac{d\sigma}{d\Omega}(\theta, \phi = 90^\circ) + W(\Theta, \gamma = 90^\circ) \cdot \frac{d\sigma}{d\Omega}(\theta, \phi = 0^\circ)
\]  

(4.5)

where \( W(\Theta, \gamma) \) corresponds to the probability that the electric field vector has a well-defined direction with \( \gamma \) being the angle between the reaction plane and the electric vector \( \vec{E} \). An experimental value of the degree of polarization (see also Eq.(2.57)) from a measurement of parallel and perpendicular Compton scatterings can be written as:

\[
P = \frac{1}{Q} \cdot \frac{N_\perp - N_\parallel}{N_\perp + N_\parallel}.
\]  

(4.6)

Here, \( Q \) is the analyzing power which is a measure of polarization sensitivity of a polarimeter and is defined as:

\[
Q = \frac{d\sigma_{90^\circ} - d\sigma_{0^\circ}}{d\sigma_{90^\circ} + d\sigma_{0^\circ}}.
\]  

(4.7)

with \( Q = 0 \) and \( Q = 1 \) indicating completely polarization insensitive and completely polarization sensitive, respectively. From Eq. (4.4) it follows:

\[
Q = \frac{\sin^2 \theta}{\frac{E_{\gamma}'}{E_{\gamma}} + \frac{E_{\gamma}}{E_{\gamma}'} - \sin^2 \theta}.
\]  

(4.8)
4.1. $^{92}$PD, $^{94}$RU AND $^{95}$RH: DATA ACQUISITION AND SORTING

For photons with energies less than 1 MeV the maximum value of the analyzing power is achieved at $80^\circ \leq \theta \leq 90^\circ$. For photon energies above 3 MeV the applicability of a Compton polarimeter is restricted due to the fact that the Compton scattering cross section and polarization sensitivity are both very low [80]. For an ideal polarimeter the polarization sensitivity at $\theta = 90^\circ$ is given by:

$$Q_0 = \frac{1 + \alpha}{1 + \alpha + \alpha^2}$$  \hspace{1cm} (4.9)

with $\alpha = E_\gamma/m_0c^2$ as the incident $\gamma$-ray energy in terms of the electron rest mass energy $m_0c^2$. The experimental polarization sensitivity of segmented Ge detectors has been studied in Refs. [81, 57]. The effective polarization sensitivity of a polarimeter with finite size of detectors is a fraction of $Q_0$ and is defined as:

$$Q(E_\gamma) = Q_0 \cdot (p_0 + p_1 E_\gamma).$$  \hspace{1cm} (4.10)

The fraction $\frac{N_\perp - N_\parallel}{N_\perp + N_\parallel}$ in Eq. (4.6) is often denoted as asymmetry $A$. In polarimeters where the arrangement of the Ge detectors is in such a way that the response of the detector pairs in parallel and perpendicular geometries are different the corresponding intensities should be normalized. For the EXOGAM array this normalization factor has to be considered and the asymmetry formula reads:

$$A = \frac{a(E_\gamma) N_\perp - N_\parallel}{a(E_\gamma) N_\perp + N_\parallel}.$$

The scaling factor, $a(E_\gamma)$, is obtained from the ratio of the sum of energy spectra of unpolarized $\gamma$-rays corresponding to the parallel and perpendicular Compton scatterings relative to the reaction plane and summed over all eleven clovers. An ideal Compton polarimeter is expected to give $a(E_\gamma)$ equal to one. The scaling factors in paper III and IV were obtained from such a measurement for $\gamma$-rays from
Figure 4.10: Scaling factor for the EXOGAM array as a function of $\gamma$-ray energy obtained from measurement with a standard $^{152}$Eu radioactive source. The data were fitted with $a(E_\gamma) = a_0 E_\gamma + a_1$.

a standard $^{152}$Eu source (see Fig. 4.10). In analogy to Eq. (2.57) in the theoretical chapter one finds that the asymmetry parameter, which is proportional to the degree of linear polarization $P$, is negative for pure stretched magnetic dipole radiation and positive for a pure stretched electric dipole (see for example figure 4 of paper IV). For a pure non-stretched dipole radiation the sign of $A$ is the reverse, negative for electric and positive for magnetic. For a stretched pure electric quadrupole radiation the asymmetry value is positive and for a pure magnetic quadrupole it is negative.

4.1.6 Spin-parity assignments of excited nuclear states in $^{94}$Ru and $^{95}$Rh

Spin assignments of levels in $^{94}$Ru and $^{95}$Rh have previously been performed by measuring $\gamma$-ray anisotropies [82]. In order to test these tentative assignments the asymmetry values of transitions measured here were first plotted against the reported anisotropy values (see Fig. 4.11). The results indicate that most of the spins and parities of the levels in $^{94}$Ru proposed in Ref. [82] are assigned correctly, however, with a few notable exceptions. In particular one notes the asymmetry shift of the 532 and 1868 keV transitions towards positive and negative values, respec-
In order to investigate this we further performed DCO measurements by constructing the $\gamma-\gamma$ ($3p$ and $4p$) particle gated coincidence matrices. The matrices contain events with $\gamma$-rays detected at an angle 90° on one axis versus events with $\gamma$-rays detected at angle 135° on the other axis. The final spin-parity assignments in paper III and IV are based on Asymmetry-$R_{DCO}$ measurements. These measurements revealed for example that the 532 keV transition in $^{94}$Ru feeding the 7970 level energy has E2 character and the 1868 keV transition depopulating the 6358 keV level energy is M1 rather than the previously assigned E1 character in Ref [82]. Fig. 4.12 shows the extracted experimental DCO ratios for yrast transitions in $^{94}$Ru and the plot of the anisotropies of Ref. [82] versus those $R_{DCO}$ values. The irregular shift of anisotropy and $R_{DCO}$ values of the 1268, 674, 597, 532 and 2402 keV transitions in Fig. 4.12 (b) might be an indication of a significant amount of admixture of higher multipoles. Finally typical values of $\Delta I$ and asymmetry are shown in Table. 4.1.
Figure 4.12: (a): Experimental DCO ratios for γ-rays belonging to the yrast band of the $4p$ fusion evaporation reaction channel. The dashed lines denote the expected values for pure stretched quadrupole and dipole. (b): γ-ray anisotropies plotted against the experimental DCO ratios for $\Delta I = 0, 1, 2$ transitions belonging to $^{94}$Ru.
Table 4.1: The linear polarization and $\Delta I$ values of different multipole orders where the letters sE and sM denote stretched electric/magnetic multipoles and nsM and nsE denote non-stretched magnetic/electric multipoles, respectively

<table>
<thead>
<tr>
<th>$\Delta I$</th>
<th>Asymmetry</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>sE</td>
<td>sM</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>sE</td>
<td>sM</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>nsM</td>
<td>nsE</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Gamma-ray spectroscopy of $^{162}\text{Ta}$

Data on excited states in doubly odd nuclei far from stability can be difficult to interpret due to their complex structure. The neutron deficient rare earth nucleus $^{162}\text{Ta}$ offers the possibility to study the complex structure of an odd-odd nucleus with a large number of valence nucleons by spectroscopy of its high-spin states. The excited states in $^{162}\text{Ta}$ were produced via the $^{106}\text{Cd}(^{60}\text{Ni},3\text{pn})$ reaction. Since the $\alpha$-decay branching ratio of the $^{162}\text{Ta}$ nucleus is very small (about 0.07%) it was not worthwhile to use the standard $\alpha$-decay tagging method. Therefore the assignments of associated $\gamma$-rays to this nucleus had to be made without decay tagging and only via the recoil-correlated $\gamma$-ray coincidences with characteristic tantalum X-rays. In addition, $\gamma$-rays observed in the present measurement had previously been assigned to mass $A = 162$ using the Daresbury recoil separator. In the following sections the different steps of this spectroscopic measurement are discussed. The energy calibration of the JUROGAM detectors was performed with $^{152}\text{Eu}$, $^{133}\text{Ba}$ and $^{60}\text{Co}$ sources using a second order polynomial fit. The total efficiency was measured with $^{133}\text{Ba}$ and $^{152}\text{Eu}$ sources by using the given peak intensities and by fitting efficiency curves to the data. The DSSSD was calibrated with a triple $\alpha$ source containing $^{239}\text{Pu}$, $^{241}\text{Am}$ and $^{244}\text{Cm}$ and the gain matching was performed for all detectors using the calibration spectra.

4.2.1 Recoil identification and gating

The recoil signals which were detected in GREAT were used as triggers for events in JUROGAM. The recoils emerged from the target approximately 0.5 $\mu$s to 1 $\mu$s before they were implanted in the DSSSD. A proper time gate was set off-line to select prompt $\gamma$-rays from the fusion evaporation residues by the alignment of the time differences between JUROGAM and the DSSSD events for each Ge detector. Further identification of the recoils was accomplished in the offline analysis by building a two dimensional histogram and setting a gate on the matrix of energy loss in the MWPC detector relative to the time of flight of the recoil as is shown in Fig. 4.13. Although the $\alpha$-decay branch of $^{162}\text{Ta}$ was very weak the relative
population of the $3p1n$ fusion evaporation channel was high, and it allowed for the identification of its associated prompt $\gamma$-rays by means of constructions of coincidences from the recoil-correlated $\gamma$-ray spectra. Figure 2 of paper II shows two samples of such recoil-correlated coincident $\gamma$-ray spectra.

### 4.2.2 Constructing the $^{162}$Ta level scheme and spin assignment

In the off-line analysis, the level scheme was constructed by using the LEVIT8R code [83]. This code is a graphical software for the analysis of $\gamma - \gamma - \gamma$ data that uses detector efficiency and energy calibration coefficients, peak shape parameters and peak widths and a background subtraction algorithm to unpack three-fold and higher-fold coincidence data into a three-fold coincidence cube. By demanding coincidences with a double gate which is set on two axes of the cube the number of counts on the third axis can be projected into one-dimensional spectra. The assignment of $\gamma$-rays to the $^{162}$Ta nucleus was based on observation of such triple coincident photons. The level scheme which is shown in figure 1 of paper II revealed a strongly coupled rotational band structure with two nearly degenerate signature partner bands. In JUROGAM the Ge detectors were distributed in six rings at $158^\circ$, $134^\circ$, $108^\circ$, $94^\circ$, $86^\circ$ and $72^\circ$ relative to the beam direction. If there are enough statistics for each observed $\gamma$-ray transition the multipolarity and the multipole mixing ratio can be extracted by the method of Directional Correlation of Oriented states (DCO) [52]. For the multipolarity assignment of
4.2. GAMMA-RAY SPECTROSCOPY OF $^{162}$Ta

The experimental DCO ratio was obtained by building a matrix of coincidences between the detector rings at $94^\circ$ and $158^\circ$ as:

$$R_{DCO} = \frac{I_{94^\circ \gamma_2}}{I_{158^\circ \gamma_2}} \text{ (gated by } \gamma_1 \text{ at } 158^\circ)$$

(4.12)

With the assumption of no multipole mixing, if the gate is set on a pure stretched quadrupole transition (275 keV) the values for the DCO ratios are about 0.8 for a pure stretched dipole and about 1 for a pure stretched quadrupole.

4.2.3 Experimental Routhians and $B(M1)/B(E2)$ ratios

In order to compare the experimental results with the cranking shell model calculations one has to transfer observables such as experimental excitation energies and angular momenta to the rotating frame. The total angular momentum on the axis of rotation (cranking axis) is:

$$I_x = \sqrt{I(I+1) - K^2} \approx \sqrt{(I+1/2)^2 - K^2}$$

(4.13)

where $I$ is the measured angular momentum. For a $\gamma$-ray transition decaying from the level energy $E_i$ to the level energy $E_f$ the experimental rotational frequency, which is often expressed as $\hbar \omega$ in units of MeV, can be deduced from the total corresponding spins along the rotational axis as:

$$\hbar \omega = \frac{dE(I)}{dI} = \frac{E_i - E_f}{I_x i - I_x f}.$$ 

(4.14)

For any two adjacent states with $\Delta I = 2$ the rotational frequency, $\omega$, is ascribed to the mean value of the two angular momenta which are involved in transition $I + 1 \rightarrow I - 1$. Hence we write:

$$\omega(I) \approx \frac{E(I + 1) - E(I - 1)}{I_x (I + 1) - I_x (I - 1)}.$$ 

(4.15)

The experimental Routhian is defined as:

$$E_{exp}(I) = \frac{1}{2} [E(I + 1) + E(I - 1)] - \omega(I)I_x$$

(4.16)

where the energy at the intermediate value $I$ is approximated as the average values of $E(I + 1)$ and $E(I - 1)$. To extract the single-particle contributions one can subtract the collective energy and angular momentum of an assumed rotor from the corresponding excitation energy values. The moment of inertia of the reference rotor, $J_{ref}$, can be obtained using the so called Harris formula [84] as:

$$J_{ref} = J_0 + J_1 \omega^2.$$ 

(4.17)
CHAPTER 4. DATA ANALYSIS

The \( x \)-component of the angular momentum of the reference system is then written as:

\[
I_x^{\text{ref}}(\omega) = (J_0 \omega + J_1 \omega^3)
\]

where \( J_0 \) and \( J_1 \) are constant. This reference angular momentum is used to deduce the reference Routhian:

\[
E_{\text{ref}}(\omega) = - \int I_x^{\text{ref}}(\omega) d\omega.
\]

The expression that was used to obtain the reference energy as a function of rotational frequency in \textit{paper II} can be found in Ref [85]. Finally, the total experimental Routhians, \( e'(\omega) \), and the aligned angular momentum, \( i_x \), in the rotating frame are given as:

\[
e'(\omega) = E_{\text{exp}}(\omega) - E_{\text{ref}}(\omega),
\]

\[
i_x(\omega) = I_x(\omega) - I_{\text{ref}}(\omega).
\]

For the nucleus \(^{162}\text{Ta}\) different quasiparticle configurations for the yrast rotational band were tested by comparing the experimental \( B(M1)/B(E2) \) values to the theoretical predictions based on the Dönau-Frauendorf approach. The calculation of the \( B(M1)/B(E2) \) ratios is done for two ranges of rotational frequency. Before the crossing frequency \( (\hbar \omega \approx 0.3 \text{ MeV}) \) a two-quasiparticle system is considered in equation 2.47 where the rotational band is built on the \( \pi_h\frac{13}{2} \otimes \nu\frac{13}{2} \) configuration consisting of a rotation-aligned neutron and a deformation-aligned proton. After the crossing frequency and the alignment of the second and the third neutrons equation 2.47 has been rewritten for a four-quasiparticle system consisting of three rotation-aligned neutron and one deformation-aligned proton. The results, which are shown in figure 5 of \textit{paper II}, indicate that before the band crossing frequency the two-quasiparticle configuration \( \pi_h\frac{11}{2} \otimes \nu\frac{13}{2} \) shows a good agreement with the experimental values. Different values of alignment and triaxial parameter were tested with the \( \pi d_{9/2} \nu\frac{13}{2} \) and \( \pi g_{7/2} \nu\frac{13}{2} \) configurations but none of them could reproduce the measured \( B(M1)/B(E2) \) ratios. The cranked shell model calculation predicts that after the crossing the \( \pi h\frac{11}{2} \otimes \nu\frac{13}{2} \otimes \nu(f_{7/2}, h_{9/2}) \) configuration competes with the \( \pi h\frac{11}{2} \otimes \nu\frac{13}{2} \otimes \nu(\frac{13}{2})^2 \) configuration and the crossing frequency appears to be almost the same for both configurations. However, the predicted values of \( B(M1)/B(E2) \) for these two configuration shows that the \( \pi h\frac{11}{2} \otimes \nu\frac{13}{2} \otimes \nu(\frac{13}{2})^2 \) configuration agrees better with the experimental data although still not precisely enough for a firm assignment. The experimental energy and intensity values of \( M1 \) and \( E2 \) transitions were used to express the reduced transition probability ratio [86] as:

\[
\frac{B(M1; I \rightarrow I - 1)}{B(E2; I \rightarrow I - 2)} = 0.697 \left[ \frac{E_\gamma(I \rightarrow I - 2)}{E_\gamma(I \rightarrow I - 1)} \right]^{5/3} \frac{T_{\gamma}(E2)}{T_{\gamma}(M1)} \left[ \frac{\nu_N^2}{2\beta^2} \right]
\]

where the ratio \( \frac{T_{\gamma}(E2)}{T_{\gamma}(M1)} \) is obtained from the experimental \( \gamma \)-ray branching ratio taken from the \( \gamma \)-ray transition intensities. This method was used in \textit{paper II} to
compare the theoretical branching ratios of different quasiparticle configurations with the deduced experimental data.
Chapter 5

Discussion

Today’s research in nuclear physics is pushing at new frontiers aiming for an increasing understanding of fundamental properties of nuclear structure. The advanced accelerator facilities and new detector systems nowadays allow to produce and identify more unstable nuclei at the limit of nuclear existence which not only enables unraveling many fundamental questions about matter and energy but also opens up new opportunities in the future to develop new technologies in materials science, chemistry etc. In nuclear structure physics one of the open questions is whether the stability of the shell closures which are found close to the valley of $\beta$ stability are preserved when approaching the proton and neutron dripline. Of special interest in this regard is the underlying structure of nuclei in the vicinity of $^{100}$Sn which is predicted to be the heaviest bound self conjugate doubly magic nucleus. In this chapter some of the results of the attached papers are discussed and compared with suggested theoretical methods in more detail.

5.1 Neutron-proton interaction in $g_{9/2}$ orbitals

In spite of the fact that the experimental binding energies and excited states in the doubly magic $^{100}$Sn nucleus and its closest neighbors are not available yet the shell model structure in the $^{100}$Sn hole-hole space can be well established from various experimental and theoretical investigation of the $N \approx Z$ nuclei in the region just below $^{100}$Sn [87]. The observation of the high-spin isomeric excited states in many $N \leq 50$ nuclei such as $^{96}$Cd [61, 88], $^{94,95}$Ag [89, 90] and $^{94,95}$Pd [61, 91, 92] isotopes in recent years has provided a good testing ground for the study of single-particle energies, residual interactions and core-excited high-spin isomeric states. The low lying yrast states of $Z < 50$, $N \leq 50$ nuclei immediately below $^{100}$Sn can be well described within the shell model calculations in a model space consisting of a rigid core plus particle-hole excitations of a few particles or holes in $\pi\nu(f_{5/2}, p, g_{9/2})$ valence space. It is also shown that the properties of many isomeric excited states are governed by the strong hole-hole proton-neutron interaction emanating from the
large overlap of the $\pi \nu g_{9/2}$ proton and neutron wavefunctions. As an evaluation of the strength of the $np$ interaction in $\pi \nu g_{9/2}$ orbitals one can consider the $T=0$ and $T=1$ components of the two-body matrix elements (TBME) [93, 94]. In a new approach Blomqvist proposed that for $N=Z$ nuclei approaching $^{100}$Sn the shell model wavefunctions can be represented in terms of isoscalar $np$ pairs with the maximum aligned angular momentum $J=9$ that is allowed by the $0g_{9/2}$ shell. The role of the $\pi \nu g_{9/2}$ interaction in the ground state wavefunction of $^{96}$Cd, $^{94}$Ag and $^{92}$Pd is studied in Refs. [95, 96]. It is shown that the isovector pairing coupling scheme accounts for about half of the ground state wave function. The ground-state wave functions of $^{92}$Pd, $^{94}$Pd and $^{96}$Pd were studied using spherical shell model calculations. This was carried out in the $f_{5/2} p_{3/2} p_{1/2} g_{9/2}$ model space and a least-squares fit to the available binding energies was applied to obtain the two-body matrix elements of the residual interaction [97]. The $T=0$ and $T=1$ components of the interaction matrix element in the description of the low-lying states of $^{92,96}$Pd shown in Fig. 5.1 (see also figure 2 of paper I) are manifested in the evolution from the seniority structure emanating from the $T=1$ component of the $np$ interaction in $^{96}$Pd to isoscalar $np$ pairs in the spin-aligned $J^\pi = 9^+$ coupling in $^{92}$Pd emanating from the $T=0$ component of the $np$ interaction. It is especially

$$
\begin{align*}
8^+ & \quad 3127 \\
6^+ & \quad 2535, 6^+ 2466, 8^+ 2633, 4^+ 2212 \\
4^+ & \quad 1786, 4^+ 1708, 2^+ 1417 \\
2^+ & \quad 874, 2^+ 878 \\
0^+ & \quad 92Pd_{\text{exp}}, 0^+ 92Pd_{\text{SM}}, 0^+ 92Pd_{\text{no np}}, 0^+ 96Pd_{\text{exp}}, 0^+ 96Pd_{\text{SM}}, 0^+ 96Pd_{\text{no np}}
\end{align*}
$$

Figure 5.1: Comparison of experimental level energies with shell model predictions. The calculated level energies of $^{92}$Pd are given for the full shell model including neutron-proton pairing (SM) and for a shell model without neutron-proton interaction (no $np$). Energies are given in keV.

notable that the regularly-spaced level sequence observed in the low-lying structure
5.2 PARTICLE-HOLE EXCITATIONS AND STRENGTHS OF E1 TRANSITION

of $^{92}$Pd is absent in $^{96}$Pd. A recent identification of the long predicted $I = 16^+$ spin-gap isomer in $^{96}$Cd [88], together with the well known $I = 16^+$ state observed in $^{94}$Pd [98], indicates the contribution of the $J^* = 9^+$ TBME element in the high-spin part. For $^{92}$Pd, neutrons and protons mainly occupy the $g_{9/2}$ subshell with four proton holes and four neutron holes relative to the $N = Z = 50$ closed shell. The results of the calculation show that the low-lying yrast states of $^{92}$Pd are dominated by the $g_{9/2}$ single particle shell. Performing the calculation with the inclusion of $f_{5/2}$ and $p_{3/2}$ shells has little effect on the calculated level energies. The same calculations performed for $^{96}$Pd and $^{94}$Pd also revealed that the $g_{9/2}$ shell is the dominant shell that contributes in building the ground state wave function of these nuclei. Within this description the ground state of $^{92}$Pd nucleus can be represented by a wave function which is dominated by four proton-neutron hole pairs each coupled to $9^+$ angular momentum which together are coupled to $0^+$ rather than by a wave function which is emanated from proton-proton and neutron-neutron hole pairs coupled to $0^+$. This is schematically depicted in Fig. 5.2.

Figure 5.2: Schematic illustration of seniority coupling scheme (right) with the spin-aligned np paired scheme (left) for $^{92}$Pd.

5.2 Particle-hole excitations and strengths of E1 transition

The early measurements on high spin states of $N = 49$ and $N = 50$ neutron deficient isotones revealed that the observed high spin states can not be explained in shell model configurations restricted to the $(f_{5/2}, p_{1/2}, p_{3/2}, g_{9/2})$ space but also require neutron excitations of the $N = 50$ core in an extended shell model space [82, 99]. The role of $N = Z = 50$ core excitations has also been studied in several $^{100}$Sn neighboring nuclei and the size of the $N = 50$ neutron shell gap is inferred from the excitation energy of core-excited states [100, 101]. As an alternative approach one can measure the reduced transition probabilities in the vicinity of a shell closure as a measure of the role of $N = Z = 50$ core-excitations. For $^{94}$Ru and $^{95}$Rh experimental $B(M1)$ and $B(E2)$ values are extracted and compared with the shell model values in Ref. [102]. In paper III and IV a study of E1 transition strengths is used as a useful test ground for the accuracy of the shell model wave functions.
The reduced E1 transition probabilities between positive and negative parity bands in $^{94}$Ru and $^{95}$Rh were deduced from the measured branching ratios and lifetimes and were expressed in terms of single particle Weisskopf units. Fig. 5.3 shows the measured $B(E1)$ values as a function of increasing spin for E1 transitions in $^{94}$Ru. Since E1 transitions are strictly forbidden in the $p_{1/2}/p_{3/2}f_{5/2}g_{9/2}$ model space the

observed of the enhanced E1 transition strength in the decay from the $^{13}_1^+$ to the $^{12}_1^-$ state indicates a contribution from the lowest possible core-excited states in the wavefunction. The strong hindrance of transition strengths can also be observed in the decay of $^{13}_1^+$ to $^{12}_1^-$ [102]. The wavefunction of the $^{13}_1^+$ and $^{12}_1^-$ states can be described as:

$$\Psi(13_1^+) = a \cdot \Psi\{\pi(1p_{1/2}^{-1} \otimes 0g_{9/2}^{-5}) \otimes \nu(1d_{5/2}^1 \otimes 0g_{9/2}^{-1})\} + \ldots,$$

and the $^{13}_1^+ \rightarrow 12_1^-$ transition requires neutron excitations from the $d_{5/2}$ shell to $p_{3/2}$.

### 5.3 Signature inversion

Signature splitting is the energy difference between the signature partners of a rotational band. It is an important quantity which can give information about the effects of the quasiparticle configuration on the overall nuclear shape. Signature splitting normally occurs if there is an admixture of the $\Omega = 1/2$ component of a high-$j$ shell in the nuclear wave function. Such admixture can occur, for example
5.3. SIGNATURE INVERSION

in a $\Omega > 1/2$ band due to a triaxial shape of the nucleus. In order to evaluate the amount of splitting one can use the staggering function, $S(I)$, to visualize the differences more clearly by means of comparing the energy of a given level, $E(I)$, with the average of the energies of the signature partner levels with one unit of spin higher or lower. A more negative value of the staggering function indicates that its relevant signature is favored. The deduced staggering function is compared for three even-$A$ and three odd-$A$ tantalum neutron deficient isotopes as shown in Fig. 5.4.

For $^{162}$Ta (paper II) the signature splitting of the band with the $\pi h_{11/2} \otimes \nu i_{13/2}$ configuration is generated by splitting of the proton orbital, that is, the coupling of $\alpha = +1/2$ signature of the $\nu i_{13/2}$ orbital to both signature partners of the $\pi h_{11/2}$ orbital. The size of the signature splitting increases at high spins for $^{162}$Ta which can be interpreted as being due to the core-polarizing effect of the second and the third aligned $i_{13/2}$ valence neutrons. Another striking effect, observed in a number of rotational bands in odd-odd nuclei, is the signature inversion where the unfavored signature is lower in energy than the favored signature. Different theoretical calculations have been developed to explain this effect. It is shown by Hamamoto that signature inversion at low spins can occur as the result of angular momentum couplings [103]. However, in a few odd-odd nuclei the signature inversion extends even to high spins. This was observed in two mass regions: $A \approx 130$ for bands having a $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration and for $A \approx 160$ for bands with a $\pi h_{11/2} \otimes \nu i_{13/2}$ configuration. Bengtsson et al. has discussed that the positive triaxial deformations, $\gamma > 0$, in $^{152}$Eu and $^{154}$Tb may give rise to signature inversion [104]. The inclusion of the residual neutron-proton interaction in the particle-rotor model has also been studied extensively and it is shown that the signature inversion can be reproduced with such modifications in the Hamiltonian [105]. In another attempt it is shown by Xu et al. that even for axially symmetric nuclei the appearance of signature inversion is expected due to the contribution of the quadrupole-pairing interaction to the mean field potential [106]. In $^{162}$Ta and $^{164}$Ta the splitting is inverted which implies that the assumed energetically favored signature ($\alpha = 0$), which is expected to have lower staggering values, becomes the unfavored one which has higher values in the staggering plot. The earlier systematic study of energy staggering of $\pi h_{11/2} \nu i_{13/2}$ bands indicates that the amount of signature inversion is the largest at $N = 89$ and decreases as $N$ increases [107]. This trend has also been observed in the systematic study of energy staggering in $N = 89$ nuclei $^{158}$Tm, $^{160}$Lu and $^{162}$Ta shown in figure 6 of paper II. The experimentally observed signature splittings in the odd-$A$ tantalum $^{161,163,165}$Ta isotopes are large and lie in the range of about $\pm 200$ keV. As expected the favored signature ($\alpha = -1/2$) has lower values of staggering in the plot.
Figure 5.4: Energy staggering as a function of spin $I$ for the yrast band in 6 neutron-deficient tantalum isotopes. The filled symbols correspond to $\alpha = -1/2$ in the top plot and to $\alpha = 1$ in the bottom plot.
Chapter 6

Summary of Papers

In this chapter a brief summary of the experimental results and the author’s contribution to papers I to IV is presented. The aim of the experiment presented in paper I was to search for evidence for a new neutron-proton coupling scheme in the $^{92}$Pd nucleus. The $^{58}\text{Ni}(^{36}\text{Ar},2n)^{92}\text{Pd}$ heavy-ion fusion evaporation reaction was used to produce the $^{92}\text{Pd}$ nuclei and the prompt $\gamma$-rays together with the emitted neutrons were detected using the EXOGAM and Neutron Wall, respectively. The charged particle detector DIAMANT was used for detection of any charged particles and to veto them in all $2n$-gated spectra. The experiment was performed at the GANIL accelerator facility (Grand Accélérateur National d’Ions Lourds) in Caen, France. The second experiment, see paper II, was designed principally to study the $^{163}$Re nucleus but the high statistics of the $3pn$-exit channel allowed spectroscopy of excited states in $^{162}\text{Ta}$. The $^{106}\text{Cd}(^{60}\text{Ni},3pn)^{162}\text{Ta}$ fusion evaporation reaction was used to populate excited states in $^{162}\text{Ta}$. The experiment was performed at the Accelerator Laboratory of the University of Jyväskylä, Finland. The products of the fusion evaporation reactions were separated from the beam particles by the gas-filled RITU separator which was coupled to the GREAT spectrometer for the subsequent detection of products. Prompt $\gamma$ rays at the target position were detected with the JUROGAM Ge-detector array. The excited states in $^{162}\text{Ta}$ were identified from a $\gamma\gamma\gamma$ coincidence cube. The third experiment resulting in paper III and paper IV was originally aimed at identifying excited states in the $N = Z$ nucleus $^{96}\text{Cd}$. This experiment used a similar experimental set-up to the first experiment with a different target beam system. The EXOGAM detector array was used as a Compton polarimeter. Due to the technical interruption during the experiment the production yield of $2n$ reaction channel leading to $^{96}\text{Cd}$ was not enough to be observed. However the high production yield of other reaction channels such as $3p$ and $4p$ allowed for construction of directional $\gamma-\gamma$ angular correlation and $\gamma-\gamma$ direction polarization correlation matrices for spin-parity assignments of excited states in $^{94}\text{Ru}$ and $^{95}\text{Rh}$ nuclei.
6.1 Paper I

Gamma-ray transitions have been identified for the first time in the extremely neutron-deficient, $N = Z$ nucleus $^{92}_{\text{Pd}}$ and the energies of the lowest excited states have been deduced. The results have revealed evidence for a transition from normal superfluidity and seniority coupling scheme, to an isoscalar spin-aligned coupling scheme in the ground states and low-lying excited states of the heaviest $N = Z$ nuclei. This new neutron-proton paired phase is different from the earlier predictions of a neutron-proton BCS type of pairing condensate and is predicted to have a considerable impact on the level structures and ground state properties of $^{92}_{\text{Pd}}$. In paper I the experimental observation of lowest-lying excited states of $^{92}_{\text{Pd}}$ are presented and compared with the shell model predictions. The details of the experiment are described in supplementary information attached to the paper.

6.2 Paper II

In this paper the rotational yrast band structure of the odd-odd neutron deficient nucleus $^{162}_{\text{Ta}}$ was identified up to a tentative spin and parity of $I^\pi = 30^-$. The experimental $B(E2)/B(M1)$ branching ratios together with the energy splitting between the two signatures of the band were used to establish the quasiparticle configuration. The band was assigned to the configuration $\pi h_{11/2} \otimes \nu i_{13/2}$ before the paired band crossing rotational frequency after comparison with total Routhian surface calculations and the Dönau-Fraundorf semi-classical model. The striking feature of the energy splitting between the signature partners being inverted throughout the yrast band in $^{162}_{\text{Ta}}$ nucleus was compared with the energy staggering and signature inversion behavior of some odd-odd neutron deficient isotopes in the neighborhood of $^{162}_{\text{Ta}}$.

6.3 Paper III

The experiment resulted in the first direct measurement of spins and parities of excited states in the semi-magic neutron deficient nucleus $^{94}_{\text{Ru}}$ by means of Direction-Polarization Correlation (DPC) and Directional Correlation from Oriented states (DCO) techniques. The placement of various states in the level scheme and associated spin-parities were confirmed, or in some cases reassigned. Various possibilities of new coincident transitions were also examined. The majority of deduced multipoarities of strong transitions in the yrast structure were found to be of stretched M1, E1 and E2, type and mostly in agreement with previous tentative assignments. Notably, the state at 6358 keV excitation energy has been reassigned to spin-parity $12_1^-$ rather than $12_2^+$ as proposed previously. This reassignment is based on the deduced multipolarity and linear polarization of 257, 1641 and 1869 keV transitions. Within the framework of large-scale shell model calculations, the presence of the $12_1^-$ state is interpreted as a pure proton-hole state dominated by the $\pi(p_{1/2}^\uparrow \otimes g_{9/2}^-)$
and $\pi(p^{-1}_{3/2} \otimes g^{-5}_{9/2})$ configurations. A new positive-parity state is observed at 6103 keV which is tentatively assigned as $12^+_2$. A further reassignment is the spin change of the 7970 keV level from the earlier proposed $14^+_1$ to $13^+_1$. This state is interpreted as dominated by neutron particle-hole core excitations. Additionally, a signature of core-excited configurations is found from the measurement of the strengths of several $E1$ transitions.

### 6.4 Paper IV

The yrast structure of the semi-magic nucleus $^{95}$Rh was also studied by means of $\gamma-\gamma$ coincidence measurement and several new transitions in both the positive and negative parity yrast bands were placed in the level scheme. Firm spin-parity assignments of excited states up to high spins were obtained using the same DPC+DCO techniques as in paper III. The observation of opposite parity bands is described within the framework of large-scale shell model calculations which suggests that the yrast states of this nucleus are dominated by configurations where a $g_{9/2}$ proton is coupled to the yrast structure observed in $^{94}$Ru and the odd valence proton mainly acts as a "spectator". As with $^{94}$Ru, the measured $E1$ transition strengths are used to test large scale shell model calculations. The most hindered $E1$ transition observed corresponds to the decay $17/2^-_1 \rightarrow 17/2^+_1$ indicating that the two states are dominated by the shells $0g_{9/2}$ and $1p_{1/2}$ which do not allow any $E1$ transitions. A few much stronger transitions, in particular the $29/2^+_1 \rightarrow 27/2^-$ transition, are also observed. In this case the two states are dominated by the one-neutron core excitation from $0g_{9/2}$ to $1d_{5/2}$.

### 6.5 Author’s Contributions

A short summary of the author’s contributions to the four articles forming the basis of this thesis is found below. While the articles contain both experiment and theory the main emphasis has been on the experimental issues.

**paper I:**

The author of this thesis contributed in preparation and tests of the experimental set-up, took part in the experiment and performed the online analysis and was the main responsible to perform the offline data analysis, and took part in preparation of the paper.

**paper II:**

The author of this thesis performed the offline data analysis and was the principal author of the paper.

**paper III:**

The author of this thesis took part in the preparation of the experimental set-up, participated in the experiment, performed most of the online and offline data analysis and was the principal author of the paper.
paper IV:
The author of this thesis contributed in the preparation of the experimental set-up, participated in the experiment, and performed most of the online and offline data analysis and was the principal author of the paper.
Svensk populärvetenskaplig sammanfattning

Atomkärnan, atomens “medelpunkt”, innehåller mer än 99.9% av atomens massa och definierar genom sin elektriska laddning grundämnet och dess kemiska egenskaper. Atomkärnor innehåller allt från enstaka till hundratals neutroner och protoner (nukleoner). Nukleonerna har i sin tur en inre struktur bestående av kvarkar och gluoner. Egenskaperna hos atomkärnor med olika nukleonsammansättning avgörs av samspelet mellan tre av de fyra kända naturkrafterna - den starka, svaga och elektromagnetiska kraften. Den starka kraften dominerar i atomkärnan men saknar fortfarande en detaljerad och konsistenter teoretisk beskrivning. Den mångfacetterade växelverkan mellan nukleonerna i kärnan ger upphov till ett brett spektrum av fenomen som kan karakteriseras av allt från rent kvantkaos till regelbundna strukturer som avspeglar enkla rotationer och vibrationer. För att få nya svar på de mest grundläggande frågorna om de grundläggande växelverkningarna i atomkärnor behöver vi lämna de stabila kärnorna och studera mycket exotiska, kortlivade atomkärnor så att nya teoretiska modeller kan testas på ett avgörande sätt. Denna avhandling bygger på experiment där olika typer av kärnkollisioner ger upphov till instabila atomkärnor långt från stabilitet, i vissa fall i högt exciterade tillstånd. Dessa atomkärnors olika kvanttilstånd har studerats genom att med känsliga instrument detektera den strålning de utsänder i form av högenergetiska fotoner (gamma), neutroner, protoner och andra partiklar när de de-exciteras (“kyls ned”) mot grundtillståndet. Experimenten har bl.a. inriktats på studier av mycket instabila kärnor med nära lika antal neutroner och protoner. I kärnor med lika neutron- och protonantal ($N$ respective $Z$) framträder den unika karaktären av atomkärnan som ett system bestående av två olika typer av fermioner särskilt tydligt eftersom utökade korrelationer uppstår mellan neutroner och protoner som befinner sig i orbital (“banor”) med samma kvanttal. Kärnor med vissa proton- och neutronantal är extra stabila (”magiska” tal). Kärnexcitationer nära magiska tal ger information om växelverkansmekanismerna mellan ett fåtal nukleoner i specifika kvanttilstånd. I närheten av den förmodad dubbelmagiska $N = Z$-kärnan $^{100}$Sn har fenomen observerats som ger indikationer om en ny typ av neutron-proton växelverkan vilken skiljer sig från den typ av suprafluiditet som återfinns i kärnor närmare stabilitet och som bygger på korrelationer mellan neutroner och protoner var för sig. Bundna
N = Z-kärnor med masstal \( A > 90 \) kan endast skapas i acceleratorlaboratorier med mycket låga reaktionstvärsnitt. I detta arbete har de experimentella svårigheterna att observera exciterade tillstånd i \( N = Z = 46 \) kärnan \(^{92}\text{Pd}\) övertvunna genom användning av ett mycket känsligt detektorsystem ("EXOGAM - Neutron Wall - DIAMANT") för mätning av gammastrålning, neutroner och elektriskt laddade partiklar som emitteras i samband med dess skapande i fusionsreaktioner. Det observerade excitementsspektrot gav det första experimentella beviset för en ny typ av neutron - proton-korrelerad fas i kärnans inre struktur. Exciterade tillstånd i \(^{94}\text{Ru}\) och \(^{95}\text{Rh}\), nära de dubbla magiska talen \( N = Z = 50 \) har studerats i syfte att reda ut oklarheter i spinn och paritet från tidigare observationer, något som är central för den teoretiska tolkningen av observationerna. De uppmätta energispektra och andra observationer relaterade till polarisation och vinkelkorrelationer för fotoner emitterade från dessa kärnors exciterade tillstånd har jämförts med resultat från storskaliga teoretiska beräkningar. Bl.a. har styrkan i starkt hindrade s.k. E1-övergångar använts som ett känsligt test av de teoretiska modellparametrarna. Exciterade tillstånd i den udda-N, udda-Z-kärnan \(^{162}\text{Ta}\) har observerats med hjälp av ett annat detektorsystem - JUROGAM / RITU. Denna mycket instabila kärna är belägen i ett övergångsområde på "nuclidkartan" som ligger mellan nära sfäriska kärnor och väl deformerede kärnor. Detta erbjuder en möjlighet att studera framväxten av kollektiva fenomen och kärndeformation (särskilt graden av triaxialitet). Resultaten, som till viss del avviker från förutsägelser från teoretiska s.k. "total Routhian surface"-beräkningar tyder på ett rotationsspektrot som bygger på en nästan rent axiellt symmetrisk kärndeformation. I denna kärna tyder de experimentella observationerna även på att s.k. signaturinversion förekommer mellan de två huvudgrenarna i rotationsspektrot, ett fenomen som ännu saknar unik teoretisk förklaring och som också återfinns bl.a. i närbeliggande isotoper.
Acknowledgements

First of all I would like to thank my main supervisor Prof. Bo Cederwall for inviting me to work in nuclear physics group of KTH and for his scientific support. A warm thank you goes to my co-supervisor Dr. Torbjörn Bäck: thank you for sharing your knowledge on physics, for providing computer and programming support, for your enormous patience with my questions and your continuous support even in the hard times and for nice explanations about Swedish culture and lifestyle. I would like, in particular, to thank Prof. Arne Johnson for his careful proof reading and criticizing this thesis as I have greatly benefited from the rewarding discussions I have had with him. I would also like to express my gratitude to Prof. Jan Blomqvist, Prof. Ramon Wyss, Prof. Roberto Liotta, Dr. Chong Qi and Prof. Ayse Ataç Nyberg for their nice and fruitful collaborations and Lars-Olov Norlin for discussion on some interesting physics topics and his great help to set-up the experiments in the laboratory. I am also very grateful to my colleagues Sara Asiyeh Changizi, Maria Doncel and Hongjie Li with whom I have had good times and good discussion about physics. Special thanks to my colleagues: Aila Engelbach, Vasily Arzhanov, Pär Olsson, Pia Thörngren Engblom, Zhenxiang Xu, Jitka Zakova, Milan Tesinsky, Erdenechimeg Suvdantsetseg, Youpeng Zhang, Zhongwen Chang, Odd Runevall, Merja Pukari, Luca Messina, Antoine Claise, Kyle Johnson, Ionut Anghel, Diana Caraghiaur and many other great friends that I have not mentioned their names but all of them are included, thank you all for your friendship and accompany. I would also like to thank the Serpoushan family: Asghar, Pari, Arash, Esmeray, Ati and Lili for their great support and kindness to me and my family. I wish to express my sincere thanks to my dear friend Ulrika Engström: you were the first one to support me from the very first moment that I arrived in Sweden until now, thanks for your true friendship and thanks for being my most invaluable friend.

Finally I would like to express my deepest gratitude to my family, my dear sister Behnaz, my dear brother Furzad and in particular to my dear parents: thank you for all you have done. I simply cannot find enough words to say it. I am most grateful to my dearest Carsten: thank you for standing by my side and being there for me, for your support and encouragement, your assistance and inspiration and most of all for the great patience of yours during this time. A warm thank you also goes to Chris, Joke, Jolise and Dieter: I have always had pleasant times with you and thank you for your kind support.
Bibliography


