Analysis of vortex movement in an YBCO thin film

Models of the vortex solid-to-liquid transition compared to a sensitive resistive measurement of a strongly pinned YBCO thin film

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Analysis of vortex movement in an YBCO thin film

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Abstract

When a high-temperature superconductor transitions in the presence of a magnetic field, the resistance does not immediately go to zero. Instead, the transition is smoothed by the resistive effects of moving vortices so that perfect conductivity isn’t reached until several K below $T_c$, where the vortices have frozen into a glass state. The properties of this glass depend mainly on the amount and correlation between impurities in the superconductor.

This report describes a study of the glass transition in an YBCO thin film rich in uncorrelated impurities. The aim was to use sensitive resistive measurements to determine which of two models of this transition, the vortex glass model and the vortex molasses model, is more accurate.

After evaluation, both models are found lacking and a third model is put forth. This model, which resembles the normal distribution, excellently describes the YBCO thin film case. When the new model is tested against data from similar measurements, an YBCO single crystal and a TlBaCaCuO thin film, the results are mixed. Further study is suggested to determine the model’s range of validity.

Keywords: High-temperature superconductor, YBCO, YBa$_2$Cu$_3$O$_{7-\delta}$, Tl-BaCaCuO, vortex dynamics, glass transition, vortex glass, vortex molasses
Sammanfattning

En högttemperatur-supraleadare som kyls förbi sin övergångstemperatur i ett magnetfält har en mycket flackare $R(T)$-kurva än andra supraleadare. Detta beror på bildningen av en resistiv vortexvätska, vars resistans avtar med sjunkande temperatur tills dess att vätskan fryser till ett glas, vilket återställer noll-resistansen vi förväntar oss hos en supraleadare. Egenskaperna hos detta vortexglas beror på mängden defekter i supraleadaren samt huruvida dessa har en inbördes ordning eller ej.


Nyckelord: Högttemperatursupraleadare, YBCO, YBa$_2$Cu$_3$O$_{7-\delta}$, TlBaCaCuO, vortexdynamik, glasövergång, vortexglas, vortexmelass
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Chapter 1

Introduction

Superconductors are materials that can conduct electricity without any electric resistance. They have practical applications in magnetic imaging, maglev trains, particle accelerators and prototype quantum computers, and have been known for just over a hundred years. Despite this, our understanding of superconductivity is still very limited. For instance, we have no working theory of how high-temperature superconductivity occurs.

We should bear in mind that the name 'high-temperature' has a slightly different meaning than it does in everyday life: We define high-temperature superconductors as those that become superconducting at temperatures higher than 30 K. That’s a cold day on the surface of Pluto. Compared to that, the nitrogen-cooled operating temperatures of today’s high-temperature superconductors are positively balmy at their 77 K (-196 °C).

As you cool a superconductor, it has a linear resistance like that of a metal until you reach a certain point, the critical temperature \( T_c \), at which the resistance abruptly drops to zero. If you also add an external magnetic field, one of three will things happen: Either superconductivity will stop, the critical temperature will sink, or the drop in resistance will start at the same place as it did without external field but be much less sudden, as in fig. 1.1. It is this last behavior, characteristic of high-temperature superconductors, that will be the focus of this thesis.

This different behavior is due to vortices in the superconductor, as will be explained in section 2.3. These vortices are set in motion by thermal excitation or by interaction with passing current, and as they move they cause resistance. This explains the smoothing of the transition – superconductivity has set in at \( T_c \) but a different mechanism that only exists in the superconducting state destroys the perfect conductivity.

The resistance decreases as the system is cooled down, and vanishes when the vortices freeze into place. Whether they freeze into a neat pattern or an unorganized mess called a glass depends on the microscopic crystal properties of the superconductor, and the glass case is particularly poorly understood despite the fact that the crystal properties that gives glass are also the most useful ones for applications.

The purpose of this thesis is to study the transition and determine which of the two most well-supported models of vortex freezing is correct. These two models are the vortex glass and the vortex molasses models, which were both
Chapter 1. Introduction

Figure 1.1: Schematic illustration of transition of high-temperature superconductor in different fields. The lower the field the sharper the transition.

tested against data from a sample purpose-built to enhance the difference. The somewhat surprising result of this can be found in chapter 5. Before that, I have chosen to include quite a lot of introductory theory geared towards a reader with no in-depth background in superconductivity. The reader familiar with the subject can probably skip ahead to section 3.8 without difficulty and focus on the theory specific to this experiment.
Chapter 2

Superconductors

2.1 Properties

Perfect conductivity

The most defining characteristic of a superconductor is that the electrical resistance drops abruptly to (as far as we can measure) indistinguishably close to zero at a specific temperature known as the \emph{critical temperature} $T_c$. This remarkable property was discovered in 1911 by Dutch physicist and cryogenics pioneer Heike Kamerlingh Onnes, who was investigating the effects of extreme cold on metals in the wake of his own discovery of a way to liquefy helium three years earlier. The perfect conductivity of a sufficiently cool superconductor makes lossless electrical transport possible, a very attractive prospect in a world with ever increasing energy needs. A superconducting wire can also carry about a thousand times more current than one of the same size made from a traditional good conductor (like copper), which makes it possible to build very powerful electromagnets without making them the size of houses. While these powerful magnets have many applications, their most widespread societal use is in various forms of magnetic imaging, which is indispensable to modern medical diagnostics.

The Meissner effect

Superconductors have one other startling and characteristic property in addition to zero resistivity: They completely shield themselves against magnetic fields. This is easily demonstrated by placing a small magnet on a spacer a little bit above a superconductor at room temperature and then cooling the superconductor down to $T_c$. At the transition to superconductivity, you can remove the spacer and the magnet will stay in place, levitating on the magnetic field. This phenomenon is caused by the so called \emph{Meissner effect}, named after one of its discoverers Walther Meissner.

Prior to this discovery in 1933, perfect conductivity had been assumed to be the only defining trait of the superconducting state. The addition of the Meissner effect serves to elevate superconductivity to a true thermodynamic state, meaning that its state is completely determined by its temperature and field immersion, since without magnetic expulsion we would see different be-
Chapter 2. Superconductors

Figure 2.1: Magnetic levitation at KTH’s expo, Dreamhack Winter 2012. Image courtesy of KTH Kista.

behavior upon cooling depending on what the external field was at the time of transition[1].

2.2 Theories of superconductivity

Understanding superconductivity requires reasonable familiarity with several different fields of physics. The most obvious properties, such as perfect conductivity and magnetic expulsion, are electromagnetic effects, but whether or not they occur depends on temperature and magnetic fields which are governed by thermodynamics. Furthermore, all of our theories as to why superconductivity exists describes it as a macroscopic quantum phenomenon which requires quantum mechanics to understand. Because of this, there does not currently exist any one theory of superconductivity which is satisfactory on its own. Instead, we make do with three different main theories, one for each field of physics involved: London theory for electromagnetism, Ginzburg-Landau for thermodynamics and BCS for the quantum approach. Not even these three taken together paint the full picture and many big questions remain unanswered, particularly in the field of high temperature superconductivity.

To be able to discuss superconductivity, a short historical overview of how theories describing it have developed is useful. This will serve to give a basic theoretical understanding as well as to unequivocally define necessary terms and expressions within the scope of this report.

The London model

The first successful theory that explains and predicts some of the facets of superconductivity was proposed by the brothers Fritz and Heinz London in 1935[2]. It is an extension to Maxwell’s equations that describes the elec-
2.2. Theories of superconductivity

tromagnetic response of the superconductor without touching upon what is happening at a microscopic level. The London equations have the benefit of being fully compatible with the models used to describe normal conductors, which makes it easy to include superconducting components in otherwise non-superconducting circuits. Sadly, they also have several shortcomings:

- They do not take temperature dependence into account, which means they are only applicable at $T \ll T_c$ where temperature-dependent effects are next to absent.
- They are only formally proved when the depth to which the magnetic fields penetrate before being neutralized by shielding currents, $\lambda$, is greater than the coherence length $\xi$, which is not true for some type I superconductors.

The concept of coherence length is perhaps best described as the shortest length over which the superconducting carrier concentration can change perceptibly. It wasn’t part of the original London model but was introduced by Brian Pippard in 1965, and reappears in both Ginzburg-Landau and BCS theory with different interpretations.

The London model appends two equations to the ones made familiar by James Clerk Maxwell. Maxwell’s equations themselves, written in differential form as

\[
\begin{align*}
\nabla \cdot D &= \rho \\
\nabla \cdot B &= 0 \\
\n\nabla \times E &= -\frac{\partial B}{\partial t} \\
\n\nabla \times H &= J + \frac{\partial D}{\partial t}
\end{align*}
\]

are normally enough to describe any electromagnetic phenomenon, but since superconductivity is as much a thermodynamic phenomenon and a macroscopic manifestation of quantum mechanics, they are insufficient. What must be added are

1. An expression for zero resistivity, modeled by the London brothers as the scattering time between resistive events diverging to infinity by the use of an equation relating electric field to current density:

\[
E = \mu_0 \lambda_L^2 \frac{\partial}{\partial t} (J)
\]

where $\lambda_L$ is the *London penetration depth* $\lambda_L$, giving how far into the material the magnetic field reaches.

2. An expression giving a phenomenological description for the magnetic expulsion behavior of the Meissner effect, given as the curl of the current density

\[
\nabla \times J = -\frac{1}{\mu_0 \lambda_L^2} B.
\]
Chapter 2. Superconductors

Ginzburg-Landau theory

The second important theory of superconductivity was developed by Vitaly Ginzburg and Lev Landau and presented in 1950. It starts out as a theory of the superconducting phase transition, unsurprising as Landau had previously engineered a well-received theory of phase transitions. Ginzburg-Landau theory (G-L theory) measures the ‘degree’ of superconductivity as a varying density of superconducting electrons by describing the supercarrier density by the wavefunction \( \Psi(\mathbf{r}) = \sqrt{n_s(\mathbf{r})} e^{i\phi(\mathbf{r})} \), where \( n_s \) is the number of superconducting electrons and \( \phi(\mathbf{r}) \) is the phase. Within Landau’s earlier theory on continuous phase-transitions the free energy for this state is then expressed in terms of the \( \psi(\mathbf{r}) \) and minimized to an equilibrium state[3]. This results in two equations:

\[
\alpha \Psi + \beta |\Psi|^2 \Psi + \frac{1}{2m} \left( \frac{\hbar}{i} \nabla - q \mathbf{A} \right)^2 \Psi = 0 \tag{2.1}
\]

and

\[
J = \frac{\mu}{m} \text{Re} \left\{ \Psi^* \left( \frac{\hbar}{i} \nabla - q \mathbf{A} \right) \Psi \right\}, \tag{2.2}
\]

also known as the supercurrent equation.

More abstractly put, the supercarrier density in the material is the phase coherence of the quantum-mechanical wave function on macroscopic length scales[4]. It serves as what G-L theory calls the order parameter of the superconducting state and can be considered zero above \( T_c \) and increasing with decreasing temperature below it[5].

BCS theory

Neither London nor Ginzburg-Landau theory gives a good reason as to why superconductivity occurs. No such microscopic explanation was available until 1957, when Bardeen, Cooper and Schrieffer (BCS for short) published their joint paper Microscopic Theory of Superconductivity[6]. BCS theory introduces Cooper pairs, electrons bound in pairs by lattice interactions, as the carriers of current in the superconducting state.

Cooper was inspired by the 1950 discovery of the so-called isotope effect – the transition temperature of the same element varies with what isotope is used. This must mean that superconductivity is not solely an electronic phenomenon, since electrons are unaffected by the number of neutrons in their nuclei. Cooper reasoned that the only way atomic mass could affect \( T_c \) was if superconductivity involved not only electron movement but also lattice vibrations, which is subject to the mass of the lattice atoms.

When an electron passes through a lattice of positive ions, the ions are attracted to the electron and bends the lattice slightly to get closer to it. Bear in mind that compared to the small, light and quickly moving electrons, lattice ions are great ponderous behemoths who will take considerable time to regain their equilibrium positions. This slowly developing lattice distortion gives rise to a high local positive charge density even long after the first electron has left, taking its Coulomb repulsive effects with it. It is now energetically favorable for a second electron to enter the scene, in what is effectively electron-electron interaction mediated by the lattice.
2.3. Type I and type II superconductors

The two electrons can now be considered bound to each other, and will stay correlated while traversing the lattice. This correlation means that they can for many interactions be considered as one particle – one with integer spin. Integer spin means the pairs will act similar to bosons, which can occupy the same quantum states and form condensates [8, pp. 70-72].

The distance between the electrons in the pair is given by the correlation length $\xi$, which can be as much as a few $\mu$m, a very large distance for a quantum mechanical phenomena. This means that many Cooper pairs overlap, and when one collides with the lattice, knocking it out of sync with its Cooper partner, it will immediately pair up with another electron. This preserves the overall correlation, and zero resistivity with it [7, pp. 8-9]. Since the Cooper pairs are constantly exchanging partners, their wave functions start to sync up to each other, in a way that approximates all the pairs sharing one phase-coherent macroscopic wave function [1, pp. 54-55]. This makes the superconducting state stable as long as lattice vibrations – the strength of which are directly proportional to temperature – are not strong enough to break up all the Cooper pairs at once.

It is interesting to note that the phonon-interaction that gives rise to superconductivity is the same mechanism that causes resistivity in pure metals. This is why the the good elemental conductors that become superconducting only do so extremely close to $T=0$, while much worse conductors like mercury transitions at a few K. Good room-temperature conductivity is a direct consequence of lattice vibrations being very limited, and some of the best conductors like copper never becomes superconducting at all.

2.3 Type I and type II superconductors

It has previously been mentioned that a central property of a superconductor is the complete expulsion of magnetic fields from its interior. This is not the whole truth. Instead, there exist two kinds of superconductors referred to as type I and type II. They are distinguished by how they respond to a magnetic field:
• Type I will completely expel the magnetic field, and the superconducting state will succumb when the flux density gets too high. The critical magnetic field $H_c$ is typically on the order of a few to a few hundred mT – somewhere between the strength of a fridge magnet and a bar magnet.

• Type II has two critical magnetic fields labeled $H_{c1}$ and $H_{c2}$. Below $H_{c1}$ the superconductor will be in the Meissner phase just like the type I, but between $H_{c1}$ and $H_{c2}$ the superconductor will allow flux lines to pass through it in narrow normal-conducting tubes. Each of these tubes is surrounded by a vortex of shielding current and the state is called the vortex state.

These vortices all carry the exact same flux, a very small amount known as the flux quantum $\Phi_0$,

$$\Phi_0 = \frac{\hbar}{2e} = 2.0678 \cdot 10^{-15} \text{ Tm}^2$$

and they all spin the same way around\(^1\) and thus repulse each other. Because of this repulsion they enact a force upon each other and will in a perfect crystal arrange themselves in a minimum energy triangular lattice known as the Abrikosov lattice. This lattice can be displayed very elegantly by scattering finely ground magnetic powder over a superconductor and observing it in a microscope, a technique known as Bitter decoration, or by more general methods such as the scanning tunneling microscope image found below.

![STM-image of regular Abrikosov lattice in a 1 T field at 1.8 K. Hess et al. [9]](image)

The amount of vortices depend on the external field – at $H_{c1}$ there will be one vortex and at $H_{c2}$ there will be so many vortices that there is no room left inbetween for the superconductor. $H_{c1}$ is typically in the same range as $H_c$ for a type I material, while $H_{c2}$ can be greater than 100 T for some high-temperature superconductors\(^2\).[1]

The discovery of type II superconductors

The first superconductors studied all exist in the Meissner phase, as do the majority of all elemental superconductors. However, these superconductors are

\(^1\)Not quite – vortices spinning the other way are called anti-vortices and make life complicated for physicists. They can exist in pairs with vortices even at zero field. The higher the field the less relative impact they have.

\(^2\)Developing intuition for the magnitude of magnetic fields can be tricky. 100 T is about the highest man-made field ever measured without blowing up the equipment in the process.
2.3. Type I and type II superconductors

Figure 2.4: Schematic illustration of phase diagram of type I and type II superconductor. Espinosa Arronte [7].

unable to retain zero resistivity in the presence of fields higher than about 0.1 T, which means they cannot be used for one of the most desired superconductor applications: Powerful magnets, capable of producing fields unattainable with traditional electromagnets.

During the 1920s, when Kamerlingh-Onnes had finally shared the secrets of liquid helium with the world, attention turned to superconducting alloys. It was soon discovered that the loss of perfect diamagnetism – the absolute refusal of the superconductor to become magnetized by an external field – occurred at much lower field than the return of resistivity. This would later be described as the superconductor leaving the Meissner state and entering the vortex state. Oxford's Kurt Mendelssohn put forth a theory in 1935 that the superconductor was like a sponge of different phases – a network of thin superconducting filaments stretching through an otherwise not superconducting bulk, and that magnetic flux could pass through this normal bulk. This model was disproved a year later by Shubnikov et al[10], who showed the same two critical fields-phenomena in well-annealed single crystals of lead-thallium and lead-indium alloy, and also demonstrated that changing the ratio of the alloy’s composition could control whether the result was what we today would call type I or type II[11].

Superconductivity received attention from physicists for its many interesting properties, but it remained a niche interest until 1986 when Bednorz and Müller[12] discovered a $T_c$ of 30 K in lanthanum-barium-copper-oxide, significantly higher than anything else then known and approaching the limit for the highest possible $T_c$ predicted by the BCS theory[13]. This sparked a flurry of research in related copper perovskites, and within a year superconductivity was recognized in the similar yttrium-barium-copper-oxide with a $T_c$ of 92 K: Higher than the boiling point of liquid nitrogen, which is a much cheaper coolant than the previously required helium.
Chapter 2. Superconductors

High-temperature superconductors

The theoretical framework from which we today understand superconductivity falls short when faced with the high $T_c$ materials. BCS theory’s model of electron-phonon interaction cannot explain superconductivity at temperatures in excess of about 30 K and at the moment there is no general agreement upon a theory.

The first high-temperature superconductors (HTS) were all of a closely related crystal family known as cuprate superconductors. They are highly anisotropic materials with a plane of CuO$_2$ spreading out in conducting layers, tied together by poorly conducting atomic chains. The cuprate superconductors are all derived from a class of materials known as Mott insulators – materials that band theory predict to be metallic but where electron-electron interactions prevent conduction. To create HTSs, these materials are hole-doped: In effect, a nuclei with one additional proton is introduced into a lattice site, which causes it to absorb one conduction electron. What happens then is a matter of intense debate, but as neither of the two main theories – one that posits that the atomic spin pattern in the doped cuprates distort to mediate electron interaction in a way analogous to how lattice atoms vibrate in BCS theory, and one that has chemical bonds between copper atoms acting as Cooper pairs due to the unique properties of the cuprate crystal structure – has much effect on this thesis, we will not delve deeper into them.

YBCO

YBa$_2$Cu$_3$O$_{7-\delta}$, the superconductor studied in this work, is one of the best documented high-temperature superconductors, being the first one found that is superconducting when cooled by liquid nitrogen. It is sometimes referred to as the 123-superconductor, as that is the elemental proportions of the metals in the unit cell – one Y, two Ba and three Cu. Around these metal atoms, oxygen atoms stretch out in planes, forming two conducting planes around the Ba atoms with the Y-plane acting as a spacer between. The CuO links above and below the conducting planes serve as charge reservoirs, helping to control the electron density in the conduction planes. YBCO is highly anisotropic, though not so much so as most of its HTS siblings, and conduction along the vertical axis of the primitive cell is quite limited.

The $-\delta$ term in the chemical formula means that YBa$_2$Cu$_3$O$_{7-\delta}$ is a so-called non-stoichiometric compound: The ratio of elements cannot quite be represented by natural numbers, as some atoms that should be present are either absent or overrepresented due to crystal defects. The oxygen content varies over the crystal, and has a pronounced effect on its electronic properties. ‘Optimally doped’ is an oft-heard term, and it means $\delta \approx 0.07$, where almost all oxygen sites are occupied, which gives the highest $T_c$. For $\delta > 0.65$, YBCO has a completely different crystal structure, tetragonal as opposed to orthorhombic, and doesn’t superconduct at any temperature.

YBCO is also prone to twinning, which means that the crystal growth switches orientation partway through. In YBCO the switch is between the a- and b-directions of crystal growth, while the much-different c-axis stays constant. The boundaries between these twin crystals will behave like large, semi-ordered crystal defects and will profoundly impact the crystal’s electronic
2.3. Type I and type II superconductors

Figure 2.5: Crystal structure of the $\text{YBa}_2\text{Cu}_3\text{O}_7$ primitive cell. Espinosa Arronte[7].

Figure 2.6: Twinned $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ single crystal viewed in polarized light. Lundqvist[3]

properties. In practical superconductor applications, twinning is often relied upon to provide strong pinning sites to inhibit vortex motion, which will be explained in the next chapter.

In this case we are using YBCO grown as a thin film, which has effects on crystal structure that should help isolate the effects under scrutiny. A film will have several nucleation sites from which the crystal expands, so that several different near-2 dimensional crystal grains grow into each other. At the borders between these grains, it is probable that the lattice won’t fit together perfectly, and the chance of cylindrical defects stretching all the way through the height of the film is high. These cylindrical defects provide excellent pinning sites, which should counteract the problems Berglund[14] had in a similar study of an YBCO crystal.
Chapter 3

The Vortex State

What is really fascinating about the study of vortex motion and how it interacts with current is how it turns familiar concepts upside down. In a normal conductor like most metals, we deal with a continuous magnetic field and small discrete points of charge, electrons, moving in fairly predictable trajectories.

In a superconductor we have the direct opposite. The macroscopic wave function of the Cooper pairs means that considering a single hole or electron is meaningless. Current is no longer made up of discrete particles, it is a continuous wave that is intrinsically different from its constituents. Magnetic flux, on the other hand, is quantized. Every vortex carries exactly one flux quanta, which is a line in bulk matter and can be considered a point in a sufficiently thin film. This continuous current wave will move the vortices in the same way that magnetic fields move electrons in normal metals, and so turn our normal perception of electromagnetics upside down.

3.1 Vortex formation

A traditional superconductor, like most elemental ones, is superconducting in the Meissner state at low temperature and magnetic field. When temperature and/or field strength is increased, superconductivity breaks down and the material undergoes a first order transition to normal conducting. A type II superconductor, on the other hand, has one critical temperature and two critical magnetic fields: $H_{c1}$ and $H_{c2}$.

Below $H_{c1}$, the superconductor is in the Meissner state and acts like its type I sibling. As the external magnetic field increases, however, normal-conducting tubes start to appear in the material, letting magnetic flux through. These tubes are surrounded by a circling current that screens the superconductor from the magnetic flux. Together the core and the screening current is known as a vortex, and are for our intents and purposes considered as a unit.

Principles of induction give that the direction of vortex spin follows from the applied magnetic field, so all vortices will spin in the same direction. This means they will repulse each other, and will settle in a places that minimize the overall energy of the system. The shape of the vortex lattice depends on the purity of the superconductor – in a perfect crystal it will be triangular, giving equal distance between all vortices, but in the presence of crystal defects it
3.2. The critical current

is favorable for the normal vortex cores to intersect the non-superconducting defects, which will distort the lattice.

$H_{c1}$ is defined as when it is energetically favorable for the first vortex to enter the superconductor. With the external magnetic field increasing, the number of vortices passing into the superconductor will also increase. Each vortex has a radius of $\xi$ – logical, since $\xi$ is the smallest distance over which the density of Cooper pairs can change, and it would waste energy to have the core larger than necessary – so it is then easy to see where the second critical field $H_{c2}$ must be: $H_{c2}$ is where the magnetic field is high enough that the vortices pushed into the material start intersecting so that no superconducting matter is left.

The type II superconductor was predicted by later Nobel laureate Alexei A. Abrikosov [15] in 1957. He was considering Ginzburg-Landau theory for materials with properties different from any then known and realized that if the Ginzburg-Landau parameter $\kappa$, which is the ratio $\frac{\lambda}{\xi}$ – usually less than one in traditional superconductors – were large, completely different magnetic behavior could be expected. Working from G-L theory, he thus predicted the two critical magnetic fields to be

$$H_{c1} = \frac{\Phi_0}{4\pi\mu_0\lambda^2} \ln\frac{\lambda}{\xi}$$

and

$$H_{c2} = \frac{\Phi_0}{2\pi\mu_0\xi^2}$$

Looking at these equations, we can see that a large $\lambda$ gives a low $H_{c1}$ and a large $\xi$ gives a low $H_{c2}$. Abrikosov found that in the case where $\kappa > \frac{1}{\sqrt{2}}$ partial flux penetration would rule rather than the complete magnetic field expulsion familiar from type I superconductors.

For a HTS like YBCO, which was used in the experiments to be described later in this thesis, $\lambda$ is 120 nm and $\xi$ is minuscule 2 nm, which means virtually the entire phase diagram is in the vortex phase.

### 3.2 The critical current

Vortices themselves pose no threat to superconductivity, since the perfect conductivity of the superconducting matter around them makes somewhat narrower current channels not a problem: If resistance $R = \frac{\rho}{A}$ then $R$ will remain zero as long as $\rho$ does, no matter the length and width of the superconductor.

What can and will destroy superconductivity if one isn’t careful is current, which will enact a Lorenz-like force upon the vortices. For a superconducting slab in the xy-plane in a z-directional magnetic field a force

$$\mathbf{f}_L = j \times \Phi_0\hat{z}$$

will push at the vortices and move them in a direction perpendicular to the current and the field. Since a vortex is essentially a tiny bar magnet, moving it through the current will induce an electric field in the direction of the current. An electric field in the current direction means resistivity, and so perfect conductivity is lost.
Since vortex velocity cannot become infinitely large, there must also be some other force, proportional to $f_L$ and counteracting it. We call this the *drag force* 

$$f_\eta = -\eta v$$

where $\eta$ is the vortex motion viscosity[7]. Since type II superconductors do exist, we surmise that there must be one additional force influencing vortex motion. This is the *pinning force*, which creates an energy barrier the vortex must overcome before it can start moving.

### 3.3 Vortex Pinning

Since we cannot prevent the formation of vortices, we must instead make sure that the current cannot move them. In a pure, perfectly crystalline superconductor, only the balance of the Abrikosov lattice keeps the vortices in place. While the vortices remain reluctant to move in close to each other, there is little to keep the entire vortex lattice from moving as a whole across the superconductor.

Better then to pin some of them in place, and hope that the repulsive forces between vortices will keep the free vortices in place. Vortex pinning is achieved by introducing crystal defects in the superconductor. These defects will result in small normal conducting volumes, and since the formation of a vortex core requires energy to break up the resident Cooper pairs, vortices will prefer forming around these defects instead of creating normal cores of their own. Utilizing Ginzburg-Landau theory, the maximum possible pinning force can be calculated as[1]

$$f_p = -\frac{\Phi_0^2}{32\pi\mu_0\lambda^2\xi}.$$ 

As long as vortices stay pinned, dissipation-free current flow is possible. As the Lorenz-like force upon the vortices is a direct consequence of the current density while pinning force depends on disorder in the material and as such does not scale, we can calculate the maximum *critical current density* 

$$j_c = \frac{\Phi_0}{32\pi\mu_0\lambda^2\xi}$$

which is the current density at which a type II superconductor becomes dissipative. It should not be confused with the *depinning current*, which is the current that induces a magnetic field strong enough to break up the Cooper pairs themselves.

### Collective pinning

In order to use the pinned vortices to keep the unpinned ones in place, some thought must be put into where the defects are placed. This problem, referred to as *collective pinning*, was studied in depth by Larkin and Ovchinnikov[16] in 1979. They found that while the Lorenz force on the vortices scales with volume $V$, pinning force on them only scales with $\sqrt{V}$. As a consequence of this, collective pinning will be effective in small bundles, so that a limited number of vortices will stay correlated and locally ordered around each pinning site[7].
Pinning is most likely the main concern for technological superconductor applications. The stronger the pinning, the larger the current that can be passed through without destroying superconductivity.

3.4 Thermal effects

Sometimes, type II superconductors show resistive behavior even when the current density and external fields are low enough that we would ordinarily predict perfect conductivity. This is due to two related effects: Flux creep and thermally assisted flux flow (TAFF). These effects along with a third effect called flux flow affect the transport properties in different regions of the I-V curve, as can be seen in fig 3.1.

Flux flow

For a large current density $j > j_c$, the pinning barriers will not be enough to keep the vortices in place, and the entire vortex lattice will be torn loose and set sliding across the superconductor with reasonable order between vortices sustained. The vortex velocity will be limited only by the drag force, and will reach an equilibrium. Resistivity in this state is still very low compared to conventional conductors, but it is no longer zero. With vortex velocity $v = (j - j_c) \times \Phi_0/\eta$, assuming that $j$ and $\Phi_0$ are perpendicular, and electrical field $E = Bv$, $E = \frac{B\Phi_0}{\eta}(j - j_c)$

we get the flux flow resistivity

$$\rho_{ff} = \frac{B\Phi_0}{\eta}$$

The theory of flux flow, presented by Bardeen and Stephen, [17] considers a vortex core with radius $\xi$ moving through a superconductor. The core is normal and can support an electric field, which induces a resistive current in the vortex core. These currents cause resistivity. Since the number of vortices is proportional to the field, the total fraction of the material that is normal conducting is then a result of the factor $B/B_{c2}$. Flux flow resistivity is then this fraction multiplied with the normal state resistivity.

$$\rho_{ff} = \rho_n \frac{B}{B_{c2}}$$

As can be seen, $B \to B_{c2}$ will pull the material out of the superconducting regime, while $B \to 0$ will restore zero resistivity as there are no vortices left to move.

Flux creep

Flux creep occurs at fairly high temperatures, where thermal energy is so large that it can entirely ignore the energy boundaries set up by the pinning sites. Vortices still repel each other, so they will stay in mostly correlated groups
around the previously pinned vortices, and jump in groups from one pinning site to the next.

The jump frequency is proportional to the frequency of atomic vibrations\[18\] $f_0$ and depends upon temperature and the pinning energy $U_0[19]$:

$$f = f_0 e^{-U_0/k_B T} \tag{3.1}$$

If there is no current flowing through the system, the flux bundles will jump around randomly, which is very hard to observe. In the presence of a current, the current will induce a flux gradient perpendicular to the current that will tug the flux bundles towards areas of lower flux density\[7, p. 18\].

The electric field caused by flux creep can be calculated as\[19\]

$$E(j) \propto e^{-U_0/k_B T} \sinh \left( \frac{jU_0}{j_c k_B T} \right). \tag{3.2}$$

**TAFF**

Sometimes, the superconductor will display resistivity even when current, field and temperature are well within the bounds where we would expect perfect conductivity. This is an effect unique to high temperature superconductors, and will be discussed in more detail in the next section. What happens is that energy from thermal fluctuations and energy added to the system by the current conspire to lift the vortices out of their pinning site potential wells.

A theory for the physics of this mechanism was proposed by Kes et al [20] in 1989, which gives the resistivity as

$$\rho = \rho_0 e^{-U_0/k_B T} \tag{3.3}$$

where $\rho_0$ and $U_0$ are field and temperature dependent, with activation energy on the form $U_0 \propto (1 - T/T_c)^q B^{-\beta}$. The exponent $q$ is suggested to be around 1.5, but a value of 1 has also been suggested\[7\][pp.18-19].
3.5 Vortex matter

We have previously talked about a vortex lattice, reminiscent of a crystal lattice in that it is ordered and regular. Let us now expand upon that model and describe vortex matter in the same way we describe atomic matter, and allow for many different states of vortex matter with different properties.

We have already encountered the ordered Abrikosov lattice, which can be considered the vortex matter equivalent of a crystalline solid. At high temperatures, energy is added to the solid and it melts into a vortex liquid. In this state vortices are free to move about and twine around each other, much like the microscopic constituents of a liquid.

The vortex liquid is unique to high-temperature superconductors: Due to both the high anisotropy and the HTS being in the high-κ limit ($\lambda \gg \xi$), thermal effects are observable in a way they are not in conventional superconductors. Comparing for example a typical material for low-temperature applications like NbTi with high-temperature superconductor YBCO, the thermal effects that are apparent within a few K of $T_c$ for YBCO are only theoretically predicted to occur within about a tenth of a $\mu$K from the critical temperature for NbTi[1, p. 109].

Our final state will be the glassy states, which are the solid forms the vortex matter takes in the presence of disorder. Depending on the density of defects and on how regularly placed they are, the vortex liquid will solidify into one of several kinds of glass with different properties. The physics of these glassy states is still hotly debated, and it is hoped that the experiments to be presented in this report will shed some light on one aspect thereof.

Figure 3.2: Magneto-optical image of distorted vortex lattice in 4 K NbSe$_2$ at low fields, 300 $\mu$T to the left and 600 $\mu$T to the right. The scale bar is 10 $\mu$m. Image from Baziljevich et al [21].

3.6 Vortex liquid phenomena

The existence of a vortex liquid was suspected early on after the discovery of high-temperature superconductivity in 1986. The new materials, which exhibited by-the-book transitions to the superconductive phase in zero field, had a markedly wider phase transition in the presence of a magnetic field[22][23].
Since the effect is absent in zero field, vortices were the natural suspects. What happens is that when a material drops into the superconducting state, it lands in the vortex liquid phase. Here the vortices are energetic enough to be moving about and twining about each other, which causes a low but measurable resistance. This resistance decreases with temperature as the vortex liquid moves from the flux flow regime close to \( T_c \) through the TAFF region at lower temperatures until it solidifies into a glass. This mechanism extends the measurable transition to superconductivity well into the superconducting phase, so that we actually measure the onset of zero resistivity at the glass temperature \( T_g \).

In the liquid phase, vortices have enough energy that the repulsive forces between them can be neglected, so that they move independently of each other without staying in correlated groups.

### 3.7 Glassy states

Glass is the general name given to amorphous solids, in reference to the disordered molecular structure of common household glass as compared to the ordered crystal of the chemically similar quartz. In the case of superconductors, however, it is not molecular structure that is unpredictable but the mass of vortices that solidifies without an orderly long-range pattern.

The properties of the superconductor close to and below the glass transition is governed by two opposing forces: The pinning energy that keeps vortices in place, which must be large for a glass region to exist, and the elastic energy between the disordered vortices, which strive to order the vortices in an Abrikosov lattice.

In the following sections, the two models that make up the principal focus of this report will be introduced: The vortex glass and vortex molasses models. Somewhat confusingly, the molasses model, to be discussed in section 3.9, has its origin in comparison with the physics of household glass, while the perhaps more descriptive name vortex glass was already taken by a earlier model, see section 3.8. The vortex glass was named in analogue to the spin glass, a material wherein the direction of atomic spins is disordered so that the magnetic state of the material becomes non-trivial. And coming back full circle, the spin glass was once named in reference to the disordered molecular state of common glass.

### 3.8 The Vortex Glass Model

The vortex glass model was proposed by Matthew Fisher in 1989[24], in reaction to the startling properties observed in the high \( T_c \) cuprate superconductors. It proposes a way in which a vortex-laden type II superconductor can keep its resistivity zero despite the lack of an ordered vortex lattice. This contradicts the 1964 predictions of Anderson and Kim[25] that a disordered lattice must be in the flux flow regime even at low \( T \)[24]. Fisher instead describes a new thermodynamic phase at lower energies than the vortex liquid and separated by a sharp transition, and calls it the vortex glass.

This model was expanded upon by Fisher, Fisher and Huse[26], who developed a scaling theory that can be used to study the transition between glass
and liquid vortex phases. To understand it, we must first discuss a concept known in statistical mechanics as critical exponents.

**Critical exponents**

Continuous phase transitions, also known as second order, often come with strange behavior at critical points. This was discovered by French scientist and engineer Charles Cagniard de la Tour (who also invented the siren!) in 1822, when he found that superheated liquids – he experimented on water, alcohol, ether and carbon disulfide – all had a distinct point in the phase diagram where liquid and gas became indistinguishable from one another. Interestingly enough, this discovery eventually led Heike Kammerlingh Onnes to the way to liquefy helium, which in turn enabled the discovery of superconductivity.

Today we describe this point in the phase diagram as the point where a continuous transition between liquid and gas takes over from the familiar first order boiling we observe in daily life. Peculiar effects at the transition temperature are common in continuous transitions: Cagniard de la Tour himself observed the cloudiness we today know as critical opalescence, where fluctuations in the supercritical fluid density have length scales comparable to the wavelength of visible light. These peculiarities are caused by singularities in the thermodynamic functions that describe the observable physical properties, which diverge close to $T_c$. In general, close to the singularity physical quantities follow from the difference between the parameter in question (often temperature) and the critical point as a power law – if $\tau = \frac{T - T_c}{T_c}$ then a quantity such as the specific heat $C$ can be described as $C \propto |\tau|^{-\alpha}$, where $\alpha$ is the critical exponent for specific heat. This in itself would not be very useful if not for one brilliant property of critical exponents: They are independent of the system details, and only depend on how many degrees of freedom are involved and the symmetry of the system. This is known as universality, and its practical consequence is that we can do all our calculations on the simplest system for that universality class – all systems with the same symmetry and as many degrees of freedom.

The critical exponents that will be involved in the transition discussed in this report are $\nu$, which measures the divergence in correlation length $\xi$ – the distance over which everything is in the same phase – as well as the dynamic critical exponent $z$, which describes how the dynamics of the system reacts to changing temperature.

**FFH scaling**

The transition between vortex liquid and vortex glass is predicted to be second order as opposed to the first order transition between vortex liquid and vortex lattice in clean superconductors. Furthermore, there is expected to be a singularity at the glass temperature $T_g$ where the correlation length $\xi$ diverges. As a function of this $\xi$-divergence the relaxation time $\tau$ diverges as well:

$$\xi \sim |T - T_g|^{-\nu} \text{ and } \tau \sim \xi^z$$

where $\nu$ and $z$ are the static and dynamic critical exponents, which are expected to be constants (see section 3.8). Through scaling, we arrive at an expression
for the ohmic resistivity close to the transition\cite{26}\cite{7}:

$$\rho(T) = \rho_{VG} |T - T_g|^\nu(z+2-d),$$  \hspace{1cm} (3.4)

where $d$ is the number of dimensions.

This can be rewritten, using the combined exponent $s = \nu(z+2-d)$, as

$$\left( \frac{\partial \ln \rho}{\partial T} \right)^{-1} = \frac{1}{s} (T - T_g)$$ \hspace{1cm} (3.5)

which gives us a way to measure $T_g$ and $s$ in experiment.

**Scaling the glass state**

To further improve the model, Rydh et al.\cite{28} suggested replacing the $T - T_g$ relationship with $k_B T - U_0(B,T)$, to incorporate how pinning and condensation energies vary with magnetic field. With this taken into account, equation 3.4 becomes

$$\rho = \rho_{VG} |k_B T - U_0(B,T)|^s,$$ \hspace{1cm} (3.6)

where the glass transition occurs at $k_B T_g = U_0(B,T_g)$. For $U_0$ we use the expression

$$U_0 = k_B T_c \left( \frac{1-T/T_c}{(B/B_0)^\beta} \right) = U_B(1 - \frac{T}{T_c})$$ \hspace{1cm} (3.7)

where $B_0$ and $\beta$ are field independent constants\cite{28}.

![Figure 3.3: Scaled resistivity versus scaled temperature for single crystal YBCO. Note how the different field curves fit into one when scaled. Espinosa Arronte et al\cite{7}.](image)

By inverting 3.6 it is possible to extract $U_0$, as was done by Andersson et al.\cite{29} on optimally doped single crystal YBCO. By introducing the expression
for \( U_0 \) in equation 3.7 into equation 3.6, another useful expression can be obtained, giving the resistivity as

\[
\rho = \rho_{VG} \left| \frac{T(T_c - T_g)}{T_g(T_c - T)} - 1 \right|^s,
\]

(3.8)

where the magnetic field dependence is implicit in \( T_g \). Scaling by this and normalizing the resistivity as \( \rho/\rho_n \) should result in the resistance curves for all fields scaling into one, with the exponent \( s \) given by the slope in the linear region as in fig. 3.3.

3.9 The Vortex Molasses Scenario

An emerging rival to the vortex glass model is the vortex molasses model, proposed in 2000 by Reichhardt et al[30]. Its proponents argue that, due to some peculiar results found when testing the vortex glass model, the supposed vortex glass is in fact a Bose glass, another glass state that appears when the disorder in the material is not randomly distributed. Since a large part of defects in YBCO are long-range twin-boundaries, this may well be the case.

The Bose glass has very similar properties to the vortex glass, with a resistivity that should scale as \( \rho \propto |T - T_{BG}|^{\nu'(z'-2)} \) as compared to the vortex glass’ \( \rho \propto |T - T_g|^{\nu(z+2-d)} \) and they are difficult to tell apart.

In the vortex molasses model, the vortices freeze into place in the same way glasses or polymers solidify. Exactly how this freezing occurs is not yet agreed upon in the scientific community, but all theories have some things in common: Freezing occurs very rapidly, so quickly that the length scale doesn’t have time to diverge. Instead only a diverging time scale can be observed, in accordance with the empirical Vogel-Fulcher-Tamman (VFT) law of glass transitions \( \tau \propto \exp \frac{C}{T - T_g} \), where \( C \) is a material parameter and \( \tau \) is the relaxation time[31].

If we consider resistivity solely a function of vortex movement, then the larger the relaxation time (that is, the higher the viscosity) for moving vortices, the lower the resistivity. We thus get a VFT-derived equation of resistivity,

\[
\rho = \rho_{VM} \exp \frac{-C}{T - T_g},
\]

(3.9)

and by differentiating the logarithm of 3.9 we arrive at the expression

\[
\left( \frac{\partial \ln \rho}{\partial T} \right)^{-1} = \frac{1}{C}(T - T_g)^2,
\]

(3.10)

which is of similar shape to 3.5 but squared. In both these equations, the glass transition temperature can be found by extrapolating the measured curves so that the left hand sides of the expressions equal zero.
Chapter 4

Experiments

The aim of this experiment was to try to distinguish between the vortex glass and the vortex molasses models. To do this, the resistive transition was measured for several fields, namely 0 T, 0.5 T, 1 T, 2 T, 4 T, 6 T, 8 T, 10 T and 12 T, and then analyzed in MATLAB. To optimize our chances, we strove to create as strong a pinning effect as possible, and used the highest current possible through the sample without edging into flux flow territory. In all measurements, the sample was cooled in the field to be measured and the transition was measured with increasing temperature, which should minimize noise.

4.1 The sample

The sample used was a thin film of optimally doped YBCO grown by pulsed laser deposition in a Neocera Pioneer 180 system at pressure below $5 \times 10^9$ Torr, with a deposition temperature of 815 °C. The films were grown on SrTiO$_3$ substrates in the [100] direction to a thickness of 230 nm ± 5 %, which we estimate to be thick enough to exhibit 3D behavior, meaning that the vortices will have room to twine into a glass state. After deposition the film was cooled down to 450 °C at a rate of 10 °C per minute at an oxygen pressure of 300 Torr, kept there for 15 minutes, and then allowed to cool down to room temperature at 20 °C/minute. The film was grown so that the c-axis of the crystal rises perpendicular to the substrate. The sample was characterized in a Tencor P-15 profiler and had a root mean square surface roughness of less than 2 nm. In the cryostat, it was positioned so that the magnetic field was parallel to the c-axis for all measurements.

To improve the measurement resolution, a comparatively high current of 500 µA was used. As a consequence of this, the picovoltmeter used to amplify the signal maxed out at about 0.3 Ω, roughly a third of the normal state resistance just above the transition. This gives the measurement curves a slightly odd appearance, but is irrelevant to analyzing the melting of the solid vortex phase as these equations are only valid for an even smaller fraction of the normal state resistance.
4.2 Equipment

All measurements were performed in an Oxford Instruments cryostat with a NbTi/Nb$_3$Sn helium-cooled superconducting magnet. Sample temperature was controlled by a Lakeshore 340 temperature controller that was plugged into a heating element, and temperature was measured with a carbon glass thermometer in the sample holder. Voltage over the sample was amplified by an EM model P13 picovoltmeter and then measured with a Schlumberger SI7081 voltmeter. The sample was placed in a sample holder[32] with an adjustable sample platform and rotated so that it was parallel to the ground. The sample holder was then sealed with perforated cryogenic tape to minimize turbulence and fixed in the cryostat.

![Figure 4.1: Schematic of the cryostat used in the experiment. Lundquist [3].](image)

4.3 Contacting

Contacts in the form of 0.3 µm thick silver pads were deposited on the YBCO film in the pattern shown in figure 4.2. Copper wire was attached to the film and to the contacts on the sample holder by silver paint. Once the paint had dried contact resistances were on the order of a few Ω.

To make the potential difference measurement as accurate as possible the potential contacts were made smaller than the current contacts and placed
between them, in order to make the current passing under the measurement region as uniform as possible.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{sample_contact_configuration.png}
\caption{Contact configuration on sample.}
\end{figure}
Chapter 5

Analysis and results

To be able to determine which of the vortex glass and the vortex molasses models is accurate, we want to compare plots of the measured data with the predictions of both models. It is possible to simply plot the measured data – stretching from the noise-level of the equipment to about $1/3$ of the normal state resistance, making our usable region a few $\mu\Omega$ to just short of $0.3\ \Omega$ – but the models both match the data well and it is difficult to distinguish the best fit with any certainty.

![Measured data and model predictions, B = 12T](image)

*Figure 5.1: Model predictions and measurement of the beginning of the resistive transition. To increase visual clarity only every fifteenth data point is displayed.*

5.1 Preliminary data treatment

Differentiation and smoothing

In previous work by Berglund[14], most of the analysis was performed on the inverse derivative of the natural logarithm of the data. This expression reduces the near indistinguishable power law behavior of the vortex glass model and the
exponential of the vortex molasses to one linear and one quadratic expression, as derived in sections 3.8 and 3.9. However, this treatment requires taking the derivative of noisy data which tends to amplify the noise level enormously, in this case to the point where it is unworkable without some sort of smoothing algorithm. Smoothing will invariably cause some loss of precision and can if used erroneously indicate trends that never existed in the original data. In this case, a combined smoothing and derivation-algorithm of Savitzky-Golay-type is used, smoothed over five consecutive points. The algorithm is described in detail by Pavel Holoborodko on his website[33], and is suitable with the presumably Gaussian distribution of noise.

As the original resistance over time data makes a smooth curve, but is difficult to derive reliably because of unevenly spaced temperature data points (as it was time, not temperature, that was the constant interval in the measurement) it was deemed safe to use splines to interpolate new, equidistant points for the derivative.

![Measured values and equidistant interpolated points](image)

**Figure 5.2:** Original data and interpolated equidistant points for all fields. Higher fields cause transition temperatures to sink, so from left to right we have 12 T, 10 T and so on up to near-vertical 0 T at the far right.

**Measurement artefacts**

Despite these precautions for a nice, smooth derivative, there was a problem with it: A strange bump appearing every five K. Some digging revealed that the bumps coincide with the calibration points for the carbon glass thermometer – it appears that the algorithm that transforms the thermometer resistance into the temperature that the user receives as output is not continuously differentiable. To fix this it was assumed that the temperature controller used linear interpolation between its calibration points, and a spline was made of how a continuously differentiable resistance-to-temperature curve could have looked. The recorded temperature-values were then reverted to thermometer-resistance, and what temperature that should correspond to was calculated with the spline. This mapping introduced some extra noise, but a slightly
5.1. Preliminary data treatment

bumpier curve around the true values was deemed more valuable than one with obvious systematic error.

**Constants**

It is often useful to compare the resistivity in any one point with the normal state resistivity for the same temperature. This temperature is simple to find by plotting a resistivity curve that reaches far into the metallic region above the transition, and extrapolating the straight line one finds there down to sub-$T_c$ temperatures.

![Sample resistance and extrapolated normal resistance](image)

*Figure 5.4: Zero field resistance of the sample and its extrapolated normal resistance. The low temperature values have been padded with expected zeros; the lowest temperature measured was 85 K.*

In this case the sample was measured from room temperature and down while cooled in a LN$_2$ dewar without further temperature control. The unevenness of the line at high temperature in fig. 5.4 is due to turbulence as the
sample was lowered into the dewar. This measurement was taken with a lower current over the sample, which is why the cutoff at 0.3 Ω is absent.

5.2 Model fitting

Vortex glass

If the VG model is true, \((\frac{\partial \ln R}{\partial T})^{-1}\) should form a straight line from the upper right corner down to zero just outside the bottom left. Extrapolating this line to where it crosses the x-axis should give \(T_g\), while the inverse slope of the curve gives \(s\). The expected value of \(s\) is about 5 to 7. Since it is a sum of critical exponents, \(s\) should be approximately the same for all fields.

The equation used for the vortex glass was

\[
\left(\frac{\partial \ln R}{\partial T}\right)^{-1} = \frac{1}{C_1} \times (T - C_2),
\]

(5.1)

where \(C_1\) and \(C_2\), representing \(s\) and \(T_g\), are being fitted to the data by MATLAB’s lsqcurvefit.

![Predicted model curves](image)

*Figure 5.5: Predicted results for VG and VM models at typical temperatures and parameters. The data is expected to conform to one of these curves.*

Vortex molasses

If the vortex molasses model is correct, \((\frac{\partial \ln R}{\partial T})^{-1}\) should curve like a quadratic function centered on \(T_g\), which will be somewhere to the left, at a lower temperature than the measured region. The parameter \(C\) moderates the slope, and while it doesn’t have the same rigorous physical interpretation as \(s\) it must reasonably be quite a bit larger, since it suppresses the rise of a quadratic curve that is expected to not stray very far from the VG-model predictions. It should also make the exponent \(-\frac{C}{T - T_g}\) unitless, so it is likely something \(\times T\).
The equation used for vortex molasses is
\[
\left( \frac{\partial \ln R}{\partial T} \right)^{-1} = \frac{1}{C_1} \times (T - C_2)^2,
\]
where \(C_1\) is the parameter \(C\) and \(C_2\) is \(T_g\).

5.3 Fitting and residuals

When fitting a model to data, one of the simplest ways to tell whether the model is good or not is to look at the residuals, the differences between each individual data point and the corresponding value for the fitted curve. For a good model, these residuals should seem to oscillate around zero without noticeable patterns. Patterns appearing in the residuals are a powerful indicator that the model is flawed.

The results of fitting the vortex glass and molasses models to data from the 12 T measurement can be seen in fig. 5.6, along with the resulting residuals: Note how the data points are consistently higher than either model curve at both edges, and lower in the middle. This is echoed in the residual plot, which has a large uneven shape on it where it is supposed to be a straight if noisy line.

![Graph showing fit and residuals](image)

Figure 5.6: Fit of vortex glass and molasses to data (top) and the residuals of that fit (bottom). The fit is poor, the data curving away from both models, and the residuals are not centered on zero.

The residual pattern repeats itself over all measurements, though only one is displayed here. More examples can be seen in section 5.5, where the pattern
shown in the topmost half of fig. 5.6 is apparent for all fields. This leads us to conclude that neither one of the vortex glass and vortex molasses models is in fact a good model for how the vortices are moving in the sample, and by extension for the transition between the liquid and solid states of vortex matter. A different model is needed.

5.4 A possible Gaussian model

It is always intriguing when something unexpected happens in physics. In this case, neither vortex glass nor vortex molasses achieved a convincing fit to the data. What did fit the data much better was a set of equations based on the fact that \( \frac{\partial \ln R}{\partial T} \) fits very neatly with a straight line, while both the VG and VM models predict the line will bend to accommodate a discontinuity at \( T = T_g \).

![Figure 5.7](image)

*Figure 5.7: \( \frac{\partial \ln R}{\partial T} \) of data fitted with VG, VM and a linear function for \( B = 10 \, T \).*

Fitting this data with a straight line offers a far better fit without obvious systematic errors, and it also explains the unexpected manner in which \( \frac{\partial \ln R}{\partial T}^{-1} \) doesn’t appear to be heading towards zero in the low temperature limit, but instead appears to even out toward some non-zero value. Unfortunately, the upper \( 2/3 \) of the transition of this sample was never measured due to the limited range of the picovoltmeter, but low-current measurements from the calibration phase of the experiment suggests that the extrapolated straight line hits zero at \( T_c \).

This suggests that \( T_c \) is the important parameter, filling the mathematical role \( T_g \) does in the VG and VM models. An important consequence of this should be the lack of a glass temperature – for a sample where this holds true, there will be a non-zero (albeit very low) resistance even at \( T = 0 \, K \). The vortex glass paper explicitly states that the zero-resistive glass state will not occur in a 2D material [24], so perhaps our sample wasn’t as 3-dimensional as we thought and this mechanism takes over instead.

A suitable format for the equation should be \( \frac{\partial \ln R}{\partial T}^{-1} = \alpha \frac{1}{T - T_c} \), where \( \alpha \) is some negative constant and the valid interval is \( T < T_c \). Fitting the curve for all fields (see fig. 5.8) suggests an inverse relationship to the magnetic field, so that \( \alpha = \frac{\beta}{B} \).
5.4. A possible Gaussian model

Figure 5.8: The fitted parameter $\alpha$ plotted against the magnetic field and compared to $-4B + 0.04$. $\alpha$ appears to be some unexplained constant divided by the field. The added 0.04 in the denominator likely corresponds to the background magnetic field.

Integrating $\frac{\partial \ln R}{\partial T}$ and de-logarithming it gives an expression reminiscent of the Gaussian distribution $f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$:

$$R = R_0 \exp \left( -k \frac{(T_c - T)^2}{B} \right),$$

(5.3)

where $R_0$ must be proportional to $R_n$ or to $R_{ff}$, $k$ must be in the units $\frac{T}{K^2}$ to make the units add up and the magnetic field $B$ takes the place of the standard deviation to control the slope of the transition curve.

Figure 5.9: All three models show a good fit to measured resistance data. When extrapolated to higher temperatures, VG and VM will diverge to very large values while the Gaussian model curves down again in a bell curve with its peak at about $1/2$ of $R_n$.

Could it be that what we’re seeing in the transition is the summed effect of many discrete events, such as vortices moving in leaps with a temperature-
dependent probability for jumping, so that the central limit theorem makes all these events form a normal distribution? This scenario is similar to the flux creep phenomenon, though in flux creep vortices jump in correlated groups.

Of course, this new equation raises more questions than it answers. What does it mean that $\frac{\partial \ln R}{\partial T}$ appears to go to zero at $T_c$? The logarithmic derivative can be considered a measure of $\frac{R'}{R}$, so this would either indicate that the change in $R$ goes to zero or that $R$ goes to infinity, neither of which sits well with observation. Granted, this problem can be avoided by stating that this equation is only valid at $T < T_c$.

To test this Gaussian model, the equation was tried on some other data from the archives. For a single crystal of YBCO, measured in 1998 by Lundqvist et al. [34], the Gaussian model was a poor fit at low fields and a rather good fit at high fields. It is presented in section 5.6. The data was also compared to another thin film, this one of Tl$_2$Ba$_2$CaCu$_2$O$_8$. TlBaCaCuO is highly anisotropic, so if the Gaussian behavior is a thin film effect it should be even more pronounced here. The TlBaCaCuO film is previously described in reference [35], also by Lundqvist et al, and will be compared to the Gaussian model (and the others) in section 5.7.

5.5 YBCO thin film

As can be seen in figures 5.10 and 5.11, the Gaussian model is a superior fit for all but the zero-field curve, where it performs about equal to the VG
Figure 5.11: $\left(\frac{\partial \ln R}{\partial T}\right)^{-1}$ for YBCO thin film compared with the three models, high fields.
and VM curves. That the curve for $B = 0 \, \text{T}$ looks somewhat different is not unexpected: For a true zero-field environment we wouldn’t expect to see any vortices at all, though there can exist vortex/anti-vortex pairs with properties that are outside the scope of this thesis.

The goodness of fit for the different curves was evaluated by computing the $R^2$-value,

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}},$$

where $SS_{res}$ is the sum of squares of the residuals and $SS_{tot}$ is the total sum of squares, as well as graphically studying the residuals. A $R^2$-value typically stretches between 0 and 1, where 1 is a perfect fit. Using $R^2$-values with non-linear fits can sometimes return negative values when the fitting didn’t work at all, as it does for single crystal YBCO at zero field.

The interested reader can find the complete Matlab code for the YBCO thin film in Appendix A. Due to the varying data formats the code must be modified slightly for the YBCO single crystal and the TlBaCaCuO film, which will be explained in more detail in their respective sections.

<table>
<thead>
<tr>
<th>$B$ (T)</th>
<th>$VG$</th>
<th>$VM$</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.95</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>0.5</td>
<td>0.86</td>
<td>0.89</td>
<td>0.95</td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>0.92</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>0.89</td>
<td>0.91</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>0.87</td>
<td>0.90</td>
<td>0.96</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>0.89</td>
<td>0.97</td>
</tr>
<tr>
<td>8</td>
<td>0.84</td>
<td>0.88</td>
<td>0.96</td>
</tr>
<tr>
<td>10</td>
<td>0.85</td>
<td>0.90</td>
<td>0.98</td>
</tr>
<tr>
<td>12</td>
<td>0.86</td>
<td>0.91</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 5.2: $R^2$-values of the three models fitted to YBCO single crystal.

5.6 YBCO single crystal

For the YBCO crystal displayed in figures 5.12 and 5.13, the Gaussian model was a poor fit at low field, particularly $0 \, \text{T}$ where it fails completely, but a quite good fit for higher field. The correspondence to the Gaussian model seems to peak at around 6 to 9 T and then decreases when $B = 12 \, \text{T}$ where the data curve straightens out again.

Since the single crystal lacks grain boundaries, pinning is much weaker than in the thin film. Therefore we expect some difficulties in isolating exactly what transition mechanism has the upper hand at any given moment, and the results may well be a blend of several different models.

This measurement was very detailed and among other things recorded the fraction of resistivity and normal state resistivity for every point. Data with $\frac{\rho}{\rho_n} < 0.15$ was used for the fit.

<table>
<thead>
<tr>
<th>$B$ (T)</th>
<th>$VG$</th>
<th>$VM$</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.98</td>
<td>0.99</td>
<td>-14.6</td>
</tr>
<tr>
<td>0.5</td>
<td>0.98</td>
<td>0.99</td>
<td>0.73</td>
</tr>
<tr>
<td>1</td>
<td>0.99</td>
<td>0.99</td>
<td>0.60</td>
</tr>
<tr>
<td>2</td>
<td>0.99</td>
<td>0.99</td>
<td>0.86</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>0.99</td>
<td>0.96</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>0.98</td>
<td>0.96</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>9</td>
<td>0.93</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>12</td>
<td>0.94</td>
<td>0.98</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 5.2: $R^2$-values of the three models fitted to YBCO single crystal.
Figure 5.12: $\left( \frac{\partial \ln R}{\partial T} \right)^{-1}$ for YBCO single crystal compared with the three models, low fields. Data from Lundqvist et al [34]. Figure continues over next page.
Figure 5.13: $\left( \frac{\partial \ln R}{\partial T} \right)^{-1}$ for YBCO single crystal compared with the three models, high fields. Data from Lundqvist et al [34].
5.7 TlBaCaCuO film

TlBaCaCuO is a much more anisotropic material than YBCO, which may lend some support to the apparent way the Gaussian model fits well for low fields and poorly for higher ones. The data presented in figures 5.15 and 5.16 is just a fraction of the curves measured by Lundqvist et al.: There were many more high-field curves that all displayed the same behavior as 4 and 8 T. We can see the Gaussian model having an excellent fit for very low fields and pulling away from the data at somewhere around 0.2 T.

![Transition of TlBaCaCuO at 1 mT](image)

*Figure 5.14: Transition of TlBaCaCuO at 1 mT, the lowest field measured.*

The high anisotropy of TlBaCaCuO makes the resistive transition wide even at very low fields. This makes it difficult to determine $R_n$, as the sudden downturn we easily find on low field curves for YBCO is smoothed out, see fig. 5.14. By extrapolating a straight line from the uppermost ten data points, it looks like the transition begins at about 105 K where $R_n$ is roughly 5 Ω, which puts 15% of $R_n$ at 0.75 Ω.

<table>
<thead>
<tr>
<th>Field (T)</th>
<th>VG</th>
<th>VM</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.90</td>
<td>0.93</td>
<td>0.99</td>
</tr>
<tr>
<td>0.05</td>
<td>0.97</td>
<td>0.98</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.92</td>
<td>0.95</td>
<td>0.99</td>
</tr>
<tr>
<td>0.2</td>
<td>0.94</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>0.5</td>
<td>0.98</td>
<td>0.99</td>
<td>0.97</td>
</tr>
<tr>
<td>1</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0.88</td>
</tr>
<tr>
<td>8</td>
<td>0.98</td>
<td>0.88</td>
<td>0.64</td>
</tr>
</tbody>
</table>

*Table 5.3: $R^2$-values of the three models fitted to TlBaCaCuO thin film. Data for several higher fields is available but the Gaussian model is a poor fit to all of them.*
Chapter 5. Analysis and results

Figure 5.15: \((\frac{\partial \ln R}{\partial T})^{-1}\) for TiBaCaCuO thin film compared with the tree models, low fields. Data from Lundqvist et al [35].
Figure 5.16: $(\frac{\partial R}{\partial T})^{-1}$ for TlBaCaCuO thin film compared with the tree models, high fields. Data from Lundqvist et al [35].
5.8 Scaling behavior

The scaling proposed by Rydh et al. [28] was developed to improve the vortex glass model, but its central tenets is in no way exclusive to it: It makes sense to consider the real struggle between pinning forces and thermal energy rather than the abstraction of temperature differences. Applying this to the glass and molasses models was straightforward by simply letting \( \frac{T_c - T_g}{T_g(T_c - T)} - 1 \) replace \( T - T_g \).

![Figure 5.17: Scaling behavior of vortex glass model, with lighter color for higher field. The solitary tail sticking out to the left is the zero-field curve. Compare fig. 3.3 for expected result.](image1)

![Figure 5.18: Scaling behavior of vortex molasses model, with lighter color for higher field. The tail is the zero-field curve. Compare fig. 3.3 for expected result.](image2)
Chapter 6

Conclusion

The original purpose of this experiment was to determine which of the vortex glass and vortex molasses models better described the resistive transition in a YBCO thin film. As the analysis progressed, it became more and more clear that neither model displayed the same tendencies as the data, and plots of the residuals after fitting were riddled with systematic errors. An unexpected but clear inverse linear relationship was then spotted between temperature and $\frac{\partial \ln R}{\partial T}$.

This relationship gave an equation for the often studied $\left(\frac{\partial \ln R}{\partial T}\right)^{-1}$ which offered a much better fit. Integrating this to an expression for $R(T, B)$ gives an expression reminiscent of the normal distribution, with the magnetic field playing the part of standard deviation. This could suggest that vortices are jumping in discrete steps between pinning sites, with a probability of jumping determined by temperature.

This was discovered with only a few weeks left of the project, and there wasn’t time to explore the new model beyond testing it on previously available data from similar measurements. The results were mixed: For a larger sample of bulk YBCO, the model fit well for high fields but not for low, and for a thin film of TlBaCaCuO there was an excellent match between the new model and data at the lowest fields, and none whatsoever at even modest fields. It would be interesting to study this further, by testing the model on other materials and devising experiments to test its predictions. Since it has already been tested on YBCO in two and three dimensions and TlBaCaCuO in two, measuring a bulk sample of TlBaCaCuO seems like a logical first step. This would give a starting position to determine what effects depend on anisotropy and what depend on the dimensionality of the system.
Acknowledgements

Thank you to:

My advisor Magnus Andersson for a very interesting project idea and support and encouragement along the way, and for putting up with incessant interruptions when the tedium of hours of Matlab grew too much for me.

Sergiy Khartsev was an invaluable resource, without whom none of this would have gotten past the planning stage. He grew the samples and figured out how to make contacts on them, without which I would still be sitting in the lab swearing at the soldering iron.

Sana Ullah, who showed me around the lab and helped me keep the helium flowing. He also very conscientiously refrained from laughing at me when I was having a terrible day in the lab, and Mattias Ekström, for frequent sanity-checks, company and excellent nit-picking of this report.

My mother, for being unfailingly enthusiastic about my abilities even when I was not, and poking and prodding me when I’d rather give up than try again.

My father, for always answering my calls and messages when I just wanted to vent or chat, and for teaching me to write.

David Ersson, for love and support and an unending supply of hugs, and our cats, Edison and Tesla, for being cuddly little balls of gluttony and smelliness. Keep supporting your human and there will be much tuna in your future.
Bibliography


Appendix A

MATLAB code

This appendix features straight MATLAB code that can be copied into MATLAB directly or modified slightly to run in Octave. Parts that have to be changed depending on the user’s operating system or file hierarchy are in curly brackets.

A.1 Main program

% Plot raw data and fitted model curves
clear
clc

% Fetch all data
{Move to data-folder}
file0='0T.txt';
filehalf='halfT.txt';
file1='1T.txt';
file2='2T.txt';
file4='4T.txt';
file6='6T.txt';
file8='8T.txt';
file10='10T.txt';
file12='12T.txt';

res0=importdata(file0);
reshalf=importdata(filehalf);
res1=importdata(file1);
res2=importdata(file2);
res4=importdata(file4);
res6=importdata(file6);
res8=importdata(file8);
res10=importdata(file10);
res12=importdata(file12);

p=1; % smoothing parameter for splines. 1 = no smoothing -> linear interpolation
A.1. Main program

\[
t\_values = \text{struct('zero',res0.data(:,2),'half',reshalf.data(:,2),'one', res1.data(:,2),'two',res2.data(:,2), 'four',res4.data(:,2), 'six',res6.data(:,2), 'eight',res8.data(:,2), 'ten',res10.data(:,2), 'twelve',res12.data(:,2));}
\]

\[
r\_values = \text{struct('zero',res0.data(:,3),'half',reshalf.data(:,3),'one', res1.data(:,3),'two',res2.data(:,3), 'four',res4.data(:,3), 'six',res6.data(:,3), 'eight',res8.data(:,3), 'ten',res10.data(:,3), 'twelve',res12.data(:,3));}
\]

% extract names from structs
names=fieldnames(t_values);

B = [0 0.5 1 2 4 6 8 10 12];

for i=1:numel(names)
  clf
  t\_measured=t\_values.(names{i}); % measured t has errors due to temperature control
  r\_orig=r\_values.(names{i}); % measured r is okay

  % Remove bumps. r\_thermo is the calculated resistance of the
t\_points = [65.1308 70.1293 75.1255 80.1181 85.1122 90.1046 95.1013]; % calibration points
  thermo\_spline = csaps(t\_points, r\_points,p); % recreate calibration curve
  spline\_values = ppval(thermo\_spline,t\_measured); % calibration curve values for all measured T
  r\_thermo = interp1(t\_points,r\_points,t\_measured,'linear'); % actual thermometer resistance at all measured T

  % Match actual thermometer resistance to closest point on calibration
  % curve, get index to that point. The same index in temperature vector
  % should be approximately the true temperature
  for n=1:length(t\_measured)
    [pos,pos]=GetClosestValue(spline\_values,r\_thermo(n));
    t\_true(n) = t\_measured(pos);
  end
  t\_true = smooth(t\_true);

  % VG and VM models valid to ~ 15% of Rn, Rn\textasciitilde 1
  r\_max = 0.15;
  r=r\_orig(r\_orig<r\_max);
  t\_true = t\_true(r\_orig<r\_max);

  % Minimum value: The smallest value that doesn't give too much noise in
  % the result, determined by trial and error
  r\_min = 3e-5;
  index=find((r<r\_min),1,'last');
  r(1:index)=[];
  t\_true(1:index)=[];

  % Make x-axis equidistant. T is approximately linear --> evenly spaced
  % vector between min and max is good approximation
  t\_lin=linspace(min(t\_true),max(t\_true),length(t\_true));

\]
% Smooth r and make it equidistant using cubic splines
r_spline = csaps(t_true,r,p);
r_lin = ppval(r_spline,t_lin);

lnr=log(r_lin);
% Use savitzky-golay filter to differentiate/smooth derivative
t_red,dlnr=lanczos_gradient(t_lin,lnr,5);
% Invert dlnr
inv_dlnr=1./dlnr;

% Define equations for three models and give initial values
% Initial values are typical results known from running similar code
% before
F_VG = @(cVG,t_red)1/cVG(1).*(t_red - cVG(2));
guessVG = [5 12];
F_VM = @(cVM,t_red)1/cVM(1).*(t_red-cVM(2)).^2;
guessVM = [50 min(t_red)];

% Assume that dlnr = 0 corresponds to Tc
TcX(i) = calculateTcX(t_red',dlnr');
F_X = @(cX,t_red)1./(cX(1).*(t_red-TcX(i)));
guessX = [-3];

% Least square fit of function curves to data
[cVM] = lsqcurvefit(F_VM,guessVM,t_red,inv_dlnr);
[cVG] = lsqcurvefit(F_VG,guessVG,t_red,inv_dlnr);
[cX] = lsqcurvefit(F_X,guessX,t_red,inv_dlnr);

% Plot prettily and export as pdf
hold on
plot(t_red,F_VG(cVG,t_red),'b--','LineWidth',2)
plot(t_red,F_VM(cVM,t_red),'r-','LineWidth',2)
plot(t_red,F_X(cX,t_red),'g-^','MarkerSize',5,'LineWidth',2,'MarkerFaceColor','g')
plot(t_red,inv_dlnr,'kd-','MarkerSize',5,'LineWidth',1,'MarkerFaceColor','k')
title(strcat(num2str(B(i)), ' T'))
set(gca,'YTick',[]);
goodplot
axis([min(t_red) max(t_red) 0 max(inv_dlnr)*1.1])
hold off
A.2 Helper functions

if B(i)==0.5
    imageName = strcat('inv_dlnr_X_',num2str(0),num2str(10*B(i)),'.pdf');
else
    imageName = strcat('inv_dlnr_X_',num2str(B(i)),'.pdf');
end
{Move to directory you want images in}
print(gcf, '-dpdf', '-r150', imageName);
{Move to Matlab directory}

% Compute residuals. Matlab's documentation is confusing --> do it
% manually so I know what's what
residVG=inv_dlnr - F_VG(cVG,t_red);
residVM=inv_dlnr - F_VM(cVM,t_red);
residX=inv_dlnr - F_X(cX,t_red);

% Compute coefficient of determination according to
% http://www.mathworks.se/help/matlab/data_analysis/linear-regression.html

% Sum of squares of residuals
SSresidVG = sum(residVG.^2);
SSresidVM = sum(residVM.^2);
SSresidX = sum(residX.^2);

% Total sum of squares = variance * number of observations - 1
SStotal = (length(inv_dlnr)-1) * var(inv_dlnr);

% R^2 = 1-SSres/SStot, 1 is best. Choosing not to use adjusted R^2 (see
% Mathworks link) since X-model not even close to polynomial
R2_VG(i) = 1 - SSresidVG/SStotal;
R2_VM(i) = 1 - SSresidVM/SStotal;
R2_X(i) = 1 - SSresidX/SStotal;
end

A.2 Helper functions

Function to match values

% Code from
% http://programming-tips-and-tricks.blogspot.se/2013_04_01_archive.html,
% read 2013-07-22

function [x_ii, ii]=GetClosestValue(x, x_T)

% x= 1D array
% x_T= target value
% x_ii = final value
% ii = index corresponding to the final value
% Find the closest value (x_ii) and index (ii) in the given array with respect to the
% entered target value (x_T)

[mm, nn] = size(x);

if nn>mm
    x=x';
end

matrix = [abs(x-x_T), x];

% sorting with respect to the first column,
% i.e. to the absolute differences among x and x_T

matrix = sortrows(matrix);

x_ii = matrix(1,2);
ii = find(x==x_ii);

Function to derive and smooth derivative

% Code from http://cresspahl.blogspot.se/2013_01_01_archive.html, 
% based on Holoborodko algorithm, read 2013-07-22,

function [x2,y2] = lanczos_gradient(x1,y1,DOrder)

% -----------------------------------------------
% usage :
% [x2,y2] = lanczos_gradient(x1,y1,o);

% parameters :
%
% x1 : x values (x2 will be cropped by 2 times o)
% y1 : function values to be list to differenciated
% o : order of Lanczos-Differentiation (odd number)

% IMPORTANT : x1-values have to be aequidistant !
% -----------------------------------------------

NumPoints = length(y1);
h = x1(2)-x1(1);
y2 = []; m = (DOrder-1)/2;
for i=m+1:NumPoints-m
    NewValue = 0;
    for k = 1:m
        printf("\n NumPoints %d und %d ",i+k,i-k)
A.2. Helper functions

\[
\text{NewValue} = \text{NewValue} + \frac{3}{h} * k * \frac{(y_1(i+k) - y_1(i-k))}{m * (m+1) * (2m+1)};
\]
end
\[
y_2 = [y_2, \text{NewValue}];
\]
end
\[
x_2 = x_1(m+1: \text{NumPoints}-m);
\]
end

Function to calculate $T_c$ in Gaussian model

function [Tc] = calculateTcX(dt,dlnr)

% Calculate intersection of dlnr and x-axis, suppress outliers

\[
f = \text{fit}(dt,dlnr,'poly1','Robust','Bisquare');
\]
\[
Tc = - f.p2/f.p1;
\]
end