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Global and clustered approaches for stress constrained topology optimization and deactivation of design variables

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1. Abstract
We present a global (one constraint) version of the clustered approach previously developed for stress constraints, and also applied to fatigue constraints, in topology optimization. The global approach gives designs without large stress concentrations or geometric shapes that would cause stress singularities. For example, we solve the well known L-beam problem and obtain a radius at the internal corner.

The main reason for using a global stress constraint in topology optimization is to reduce the computational cost that a high number of constraints impose. In this paper we compare the computational cost and the results obtained using a global stress constraint versus using a number of clustered stress constraints.

We also present a method for deactivating those design variables that are not expected to change in the current iteration. The deactivation of design variables provides a considerable decrease of the computational cost and it is made in such a way that approximately the same final design is obtained as if all design variables are active.
2. Keywords: Topology optimization, Stress constraints, Global, Clustered, Deactivated variables

3. Introduction

In many industrial applications, the aim of structural optimization is to find the lightest design that meets the structural requirements. However, most commercial optimization software in industrial use are based on the traditional formulation of finding the stiffest structure for a prescribed amount of material: a formulation which does not necessarily yield a design that is feasible with respect to actual structural requirements, such as stress and fatigue. Furthermore, in many industrial applications, the stiffness does not necessarily have to be maximized and by allowing a slightly lower stiffness we might find a lighter and more mature design. The reason for using this stiffness based formulation is mostly computational efficiency: the optimization is driven by a global measure (compliance) and no extra (adjoint) system of equations needs to be solved in the sensitivity analysis. Stress constraints on the other hand give a much more expensive problem as stress is a local measure; i.e., a local (every stress evaluation point) quantity has to be constrained rather than a global, which increases the computational cost. However, minimizing the mass subjected to stress constraints has the potential of yielding a more mature and useful final design.

Stress constraints have been discussed since the very first papers on topology optimization: Bendsøe and Kikuchi [3] and Bendsøe [2] mention stress limitations, but do not use stress constraints in the optimization. Duysinx and Bendsøe [7] used local stress constraints, one in each element, but found that the optimization was computationally expensive. In order to decrease the computational cost, the local stresses may be grouped into one global stress constraint, as suggested by Duysinx and Sigmund [8], or several constraints as suggested by Le et al. [12], Paris et al. [13] and Holmberg et al. [11].

The clustered approach developed by Holmberg et al. [11] means that stress constraints are applied to stress clusters, where each cluster contains the stresses from several stress evaluation points. This gives a decent control of the local stresses even though a small number of constraints are used. The global approach is obtained as a special case if only one cluster is used and it allows for creating, at low computational cost, light weight conceptual designs that are free from large stress concentrations and dimensioned with respect to the stress limit.

The main question when using the clustered or global approach is how to create a clustered or global stress measure that approximates the local stresses in an adequate way. We find that different stress measures should be used depending on if a global constraint or several clustered constraints are used.

We use the qp-approach suggested by Bruggi [4], and we penalize the stresses by the square root of the design variables in order to increase the stress for intermediate design variable values. Together with SIMP [2], the stress penalization forces the solution to a black-and-white design and it also avoids the singularity problem which otherwise arises as the design variables approach zero, see Holmberg et al. [11].

The global and the clustered approaches are in this paper shown and compared for stress constrained problems, where the objective is to minimize the mass. Each stress constraint is based on von Mises stresses, but we note that using principal stresses allows for high-cycle fatigue constraints according to the methodology by Holmberg et al. [10].

In order to decrease the computational cost without decreasing the number of degrees of freedom, we have also implemented an idea originally discussed by Holmberg [9], where we allow design variables to be *deactivated*, which means that they are not updated in the current iteration if they are not expected to change. The expensive sensitivity analysis is only calculated with respect to the active design variables, which decreases the computational cost. The element which is related to a deactivated design variable is preserved and we still solve the full Finite Element (FE) problem. If necessary, a deactivated design variable may become active again and as the corresponding element has not been removed, this can be made without any problems. We note that a somewhat similar approach was proposed by Bruns and Tortorelli [6], where elements whose respective design variables approached the lower bound were removed from the structural analysis and the sensitivity analysis if the design variables within its filter radius also had reached the lower bound.

The designs shown in this paper are obtained using a Matlab code which we have developed from the code in [1]. The code has been vectorized to a wide extend so that the efficient matrix operations in Matlab are used.

In the following parts of the paper, we discuss the optimization problem in Section 4, the stress measures used for global or clustered constraints in Section 5 and the deactivation of design variables in Section 6. The examples are shown and discussed in Section 7 and conclusions are drawn in Section 8.

4. Optimization problem

The optimization problem ($\mathbb{P}$) which we solve in this paper aims at minimizing the structural mass, i.e. the sum of the element masses m_e scaled by the filtered variables $\rho_e(\boldsymbol{x})$, which are obtained as a weighted average of the neighboring design variables x_k. This is done using the design variable filter by Bruns and Tortorelli [5], which reads

$$\rho_e(\boldsymbol{x}) = \frac{\sum\limits_{k\in\Omega_e} w_k x_k}{\sum\limits_{k\in\Omega_e} w_k}, \tag{1}$$

where the set Ω_e includes all the design variables within the filter radius r_0, measured from element e. The weights w_k are given by

$$w_k = \frac{r_0 - r_k}{r_0}, \tag{2}$$

where r_k is the distance from the centroid of element e to the centroid of element k.

The problem ($\mathbb{P}$) below is subjected to a limit on the von Mises stress and the local stresses are clustered into several (n_c) clustered stress constraints or one global constraint ($n_c = 1$). The stress limit $\overline{\sigma}$ is in this paper chosen equal to the yield limit of the material but, as shown in [10], a constraint on high-cycle fatigue can be obtained if the stress limit is related to a critical fatigue stress and the local stresses are represented by the principal stresses instead of the von Mises stresses. The clustered stress for constraint i is denoted $\widetilde{\sigma}_i(\boldsymbol{x})$ and it is normalized with respect to the stress limit $\overline{\sigma}$, as will be discussed in Section 5.

Problem formulation ($\mathbb{P}$) reads

$$(\mathbb{P}) \quad \begin{cases} \min\limits_{\boldsymbol{x}} \sum\limits_{e=1}^{n_e} m_e \rho_e(\boldsymbol{x}) \\ \text{s.t.} \begin{cases} \widetilde{\sigma}_i(\boldsymbol{x}) \leq 1, \ i = 1, .., n_c \\ \epsilon \leq x_e \leq 1, \ e = 1, .., n_e, \end{cases} \end{cases}$$

where n_e is the number of design variables, $\epsilon = 0.001$ and $\widetilde{\sigma}_i(\boldsymbol{x})$ is changed for the global normalized stress measure $\widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x})$ if $n_c = 1$, as is discussed in the following section.

5. Stress measures for global and clustered stress constraints

The choice of global and clustered stress measure has a major influence on the final design. For example, if a constraint would be applied to the mean stress of the structure, there might still be a number of local stresses far above $\overline{\sigma}$ in the final design. The stress measure must also be a continuously differentiable function and it has to provide some smoothness, so that the single highest stress does not have too much influence on the clustered stress measure: the highest stress usually fluctuates and as it may occur in different points in successive iterations it might cause convergence problems.

The modified P-norm stress measure used by Holmberg et al. in [11] and [10] was proven to give a good stress distribution in the final design when using clusters which are created and updated with the *stress level* technique, [11]. The modified P-norm stress measure reads

$$\widetilde{\sigma}_i(\boldsymbol{x}) = \left(\frac{1}{N_i} \sum_{e \in \Omega_i} \left(\frac{\sigma_e^{\mathrm{vM}}(\boldsymbol{x})}{\overline{\sigma}} \right)^p \right)^{\frac{1}{p}}, \tag{3}$$

where $\sigma_e^{\mathrm{vM}}(\boldsymbol{x})$ is the local penalized von Mises stress in the centroid of element e, which belongs to the set of elements Ω_i in cluster i. The P-norm exponent is denoted p and the modification is the division by N_i, which is the number of elements in cluster i. The local stresses are normalized by the stress limit $\overline{\sigma}$.

In the *stress level* technique we sort elements with the n_e/n_c highest stresses into the first cluster, the elements with the next n_e/n_c highest stresses in the second cluster etc. The local stresses in each cluster will then have a fairly similar stress level and the clustered stress measure will become a good approximation to the local stresses. For the special case when all the local stresses in a cluster are the same, i.e. $\sigma_e^{\mathrm{vM}}(\boldsymbol{x}) = \sigma$ for all e in the set Ω_i, we get the desired clustered normalized stress measure $\widetilde{\sigma}_i(\boldsymbol{x}) = \sigma/\overline{\sigma}$. However, for all other cases (and unless $p \to \infty$) $\widetilde{\sigma}_i(\boldsymbol{x})$ will be lower than the highest local normalized stress in Ω_i; it can be shown that, see [8],

$$\widetilde{\sigma}_i(\boldsymbol{x}) \leq \max_{e \in \Omega_i} \frac{\sigma_e^{\mathrm{vM}}(\boldsymbol{x})}{\overline{\sigma}}. \tag{4}$$

Using one global stress measure, the local stresses which it consists of will range from approximately zero up to the highest stresses, and Eq.(3) will for that case not be a good approximation to the local stresses. It will underestimate the highest local normalized stresses much more than when a number of clusters are used, resulting in a final design with too high local stresses. Therefore, for the global normalized stress measure $\widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x})$, we remove the $1/N_i$-term and use a regular P-norm, which reads

$$\widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x}) = \left(\sum_{e=1}^{n_e} \left(\frac{\sigma_e^{\mathrm{vM}}(\boldsymbol{x})}{\overline{\sigma}} \right)^p \right)^{\frac{1}{p}}. \tag{5}$$

The stress measure in Eq.(5) will instead be higher than the highest local normalized stress. For the case when $i = 1$ and $\Omega_1 = \{e | 1 \leq e \leq n_e\}$, we conclude that

$$\widetilde{\sigma}_1(\boldsymbol{x}) \leq \max_{1 \leq e \leq n_e} \frac{\sigma_e^{\mathrm{vM}}(\boldsymbol{x})}{\overline{\sigma}} \leq \widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x}). \tag{6}$$

5.1. P-norm exponent

Increasing the P-norm exponent brings both $\widetilde{\sigma}_i(\boldsymbol{x})$ and $\widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x})$ closer to the maximum local normalized stress, i.e. it will make the clustered measure in Eq.(3) higher and the clustered stress measure in Eq.(5) lower. This implies that the highest stresses in the final design will be closer to the stress limit and the final design will thus not be unnecessarily heavy or have too thin structural parts. We therefore suggest a higher P-norm exponent for the global stress measure in Eq.(5) compared to what we previously have used for the clustered measure in [11] and [10]. However, choosing p too high implies that the single highest stress has too much influence on the clustered value. In this paper we use $p = 24$, which is twice that of what we used in [10]. We note that the choice of p does influence the design, but $p = 24$, which we use for all designs in this paper, appears to be an appropriate choice. Of course, different scale factors,

replacing $1/N_i$ in Eq.(3), could be suggested, which perform well in specific problems; or, an adaptive scale factor as suggested in [12] could be employed. However, the aim in this paper is to find a simple stress measure, which gives a design without large stress concentrations and with local stresses close to the stress limit. The topology optimization should not replace the final sizing of the design, but it should give a good starting point for further analysis.

5.2. Difference between Eq.(3) and Eq.(5)

To visualize the difference between the stress measure in Eq.(3), for only one cluster, and the stress measure in Eq.(5), we compare stress values obtained for a static analysis ($\overline{\sigma} = 1$) using a simple tensile load case as shown in Fig. 1. The element size is 1×1 mm and a 100 N load is distributed over three nodes. The width is first 4 mm (4 elements), which gives a case with an even stress distribution. This is to resemble the case when clusters are used and the stresses creating the cluster are very similar, as in the *stress level* approach. The width is then increased to 12 mm, so that some elements will have a low stress, which resembles the distribution of high and low stresses which are clustered together in the global approach. The elements close to the applied load and close to the fixed support are not contributing to the stress measure, only the elements inside the dashed lines in Fig. 1. Table 1 summarizes the stresses for the different cases and the deviation between the global stress and the highest local stress. It is seen that Eq.(3) is a much better approximation to the highest local stresses when there are similar stresses and that Eq.(5) is preferable when the global measure is created by a mix of high and low stresses.

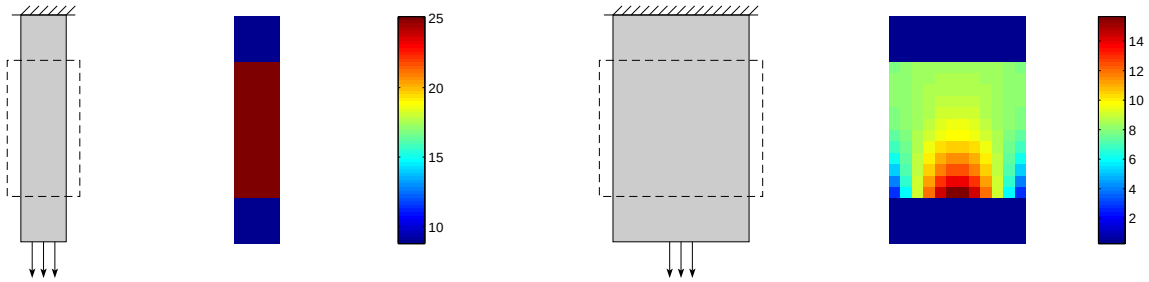


Figure 1: Domains with 4 mm and 12 mm width, respectively, and plots of the corresponding stresses, which should be viewed in color

Table 1: Global stress measure using Eq.(3) or Eq.(5) for the domains in Fig. 1

Equation	Width	Highest stress [MPa]	Mean stress [MPa]	Global value [MPa]	Deviation
(3)	4	25.11	25.01	25.01	-0.5%
(5)	4	25.11	25.01	29.39	17.0%
(3)	12	15.65	8.84	13.19	-16.0%
(5)	12	15.65	8.84	16.22	3.5%

6. Deactivating design variables

When using the design variable filter in Eq.(1), the sensitivity analysis becomes more involved and more computationally expensive than if no filter, or e.g. a sensitivity filter [14], is used. As each filtered variable is a function of the neighboring design variables, the sensitivity with respect to one design variable is not only a function of the stress and stiffness in the corresponding element, but also of those in neighboring elements. The sensitivity analysis of the clustered stress measure in Eq.(3) is given by Holmberg et al. in [11] and, with trivial modifications, it is also valid for Eq.(5). The final expression is repeated here and adapted to the nomenclature used in this paper; it reads

$$\frac{\partial \widetilde{\sigma}_i(\boldsymbol{x})}{\partial x_b} = \sum_{e\in\Omega_i} \frac{\partial \widetilde{\sigma}_i(\boldsymbol{x})}{\partial \sigma_e^{\mathrm{vM}}} \left(\frac{\partial \sigma_e^{\mathrm{vM}}(\boldsymbol{x})}{\partial \boldsymbol{\sigma}_e}\right)^T \frac{\partial \eta_S(\rho_e(\boldsymbol{x}))}{\partial \rho_e} \frac{\partial \rho_e(\boldsymbol{x})}{\partial x_b} \boldsymbol{E}\boldsymbol{B}_e \boldsymbol{u}(\boldsymbol{x}) - \eta_S(\rho_e(\boldsymbol{x})) \boldsymbol{\lambda}_i^T \left[\sum_{r=1}^{n_e} \frac{\partial \boldsymbol{K}(\boldsymbol{\rho}(\boldsymbol{x}))}{\partial \rho_r} \frac{\partial \rho_r(\boldsymbol{x})}{\partial x_b} \boldsymbol{u}(\boldsymbol{x})\right], \quad (7)$$

where $\boldsymbol{\sigma}_e(\boldsymbol{x})$ are the stress components in element e, $\eta_S(\rho_e(\boldsymbol{x}))$ is the stress penalization function, $\boldsymbol{E}$ is the constitutive matrix, $\boldsymbol{B}_e$ is the strain-displacement matrix corresponding to the stress evaluation point in element e and $\boldsymbol{u}$ is the vector of nodal displacements. The included derivatives are found in [11].

The sensitivity analysis is the most expensive calculation in each iteration of the stress constrained topology optimization. The sums in Eq.(7) are due to the filter and the number of nonzero terms depends on the filter size. Also, since Eq.(7) needs to be calculated for each x_b, the computational cost is a function of the number of design variables. For this reason, it is desirable to keep the number of design variables to a minimum, but without decreasing the number of degrees of freedom. Therefore, if a design variable is not expected to change in the current iteration, it may be deactivated, meaning that it is excluded from the sensitivity analysis and not updated in the current iteration. When design variables are deactivated, the number of calculations needed in the sensitivity analysis of the stress constraints, Eq.(7), is reduced, even though the calculation of the adjoint variable vectors $\boldsymbol{\lambda}_i$ is equally costly as when all design variables are active.

We still solve the full FE-problem, which, compared to the sensitivity analysis, is relatively cheap. Previously deactivated design variables can therefore be made active again without any problems.

Design variables are considered unlikely to change if they are at the lower bound, $x_e = \epsilon$, and totally surrounded by void elements, or, if they are at the upper bound, $x_e = 1$, and totally surrounded by elements representing solid material. Such design variables are now identified in a simple, but efficient, way by using the design variable filter Eq.(1), where this information, i.e. when $\rho_e(\boldsymbol{x}) = x_e$, already exists. If all design variables inside design variable x_e's filter radius are equal to x_e, we have

$$\rho_e(\boldsymbol{x}) = \frac{\sum\limits_{k\in\Omega_e} w_k x_k}{\sum\limits_{k\in\Omega_e} w_k} = x_e \frac{\sum\limits_{k\in\Omega_e} w_k}{\sum\limits_{k\in\Omega_e} w_k} = x_e. \quad (8)$$

Thus, the design variables may be deactivated when $x_e = 1$ or $x_e = \epsilon$ and $|\rho_e(\boldsymbol{x}) - x_e| < \xi$, where ξ is a small positive value that allows for some numerical tolerance. As the topology changes during iterations, a deactivated design variable may be active again if $|\rho_e(\boldsymbol{x}) - x_e| \geq \xi$. That is, the sets of active and deactivated design variables are updated every iteration.

Reduction of the computational cost will be obtained first when solid or void areas are created and the computational cost will decrease as the solution converges towards a black-and-white design. We note that the decrease in computational time is dependent on the implementation: in the current implementation where the code has been vectorized and the efficient matrix multiplications in Matlab are used to a wide extend, the gain is not as substantial as when the code is based exclusively on loops.

New structural members most often develop from areas with intermediate design variable values. When a larger area has become void, it is very unlikely for a structural member to arise within that area, as it is an area with a weak structural response. However, with this method, new structural members can arise in a void and solid members can be removed, but it will require several iterations more than usual as the design variables have to become active before they can change.

7. Examples

The optimization problem ($\mathbb{P}$) is solved using 25 clustered stress constraints or one global stress constraint, using the formulations in Eq.(3) and Eq.(5), respectively. The computational cost and the results are compared for global versus clustered constraints and when solving the full problem or allowing design variables to be deactivated. We use the well known L-shaped beam and the MBB-beam to verify the theoretical discussions on the expected results. The L-shaped beam is seen to the left in Fig. 2, where $F = 500$ N and $L = 100$ mm. The MBB-beam is seen to the right in Fig. 2 and the same load magnitude is used, but the length is $L = 40$ mm. All elements have the same size, 1×1 mm, and the thickness is 1 mm. The filter radius is 1.5 mm.

An aircraft aluminium with Young's modulus 71000 MPa, Poisson's ratio 0.33 and with the density $\rho = 2.8 \times 10^{-9}$ ton/mm^3 is used as design material. The stress limit is 350 MPa, which corresponds to the yield limit of the material.

All designs are started with $\rho_e(\boldsymbol{x}) = 0.5$ and the elements close to the applied loads and close to the vertical support in the MBB-beam are not part of the design space. The convergence criteria is that the objective function value should not change more than 0.01 % in three successive iterations and that the constraint values are no more than 1 % away from the constraint limit. The solution may also converge if the change in objective function value is less than 5×10^{-4} %, for which no feasible design is considered to be possible, or too expensive. This can occur if the problem is ill-posed or if there are some high stresses close to the applied loads or supports. The optimization problem is solved using the Method of Moving Asymptotes [15], which has been modified to be more conservative.

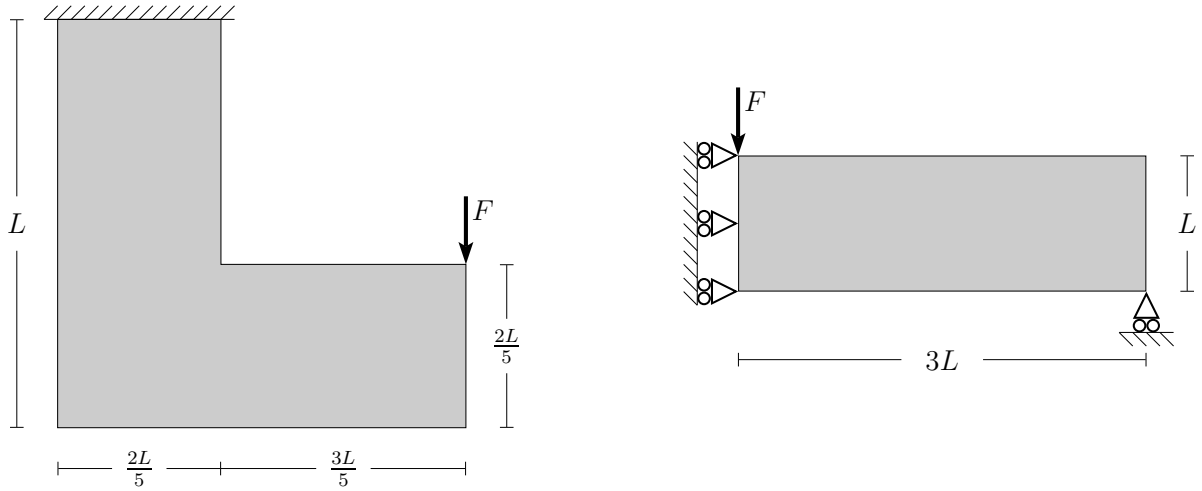


Figure 2: Geometry and boundary conditions for the L-shaped beam and the MBB-beam

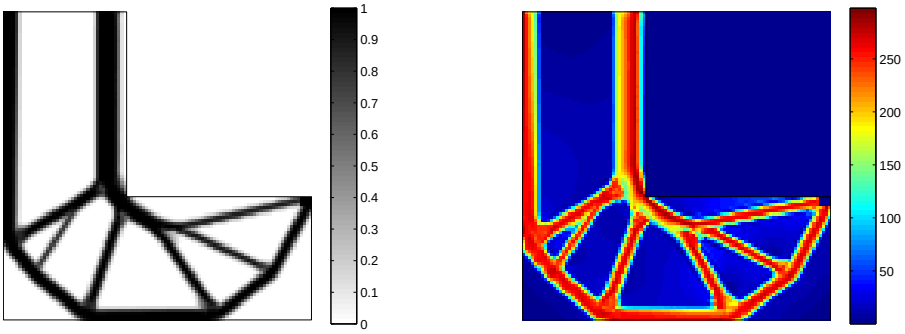
7.1. L-shaped beam

The optimized designs for the L-shaped beam are seen in Fig. 3 and data is presented in Table 2. By comparing Fig. 3a with Fig. 3b and Fig. 3c with Fig. 3d we find that the deactivation of design variables has not affected the final design extensively; approximately the same topology is obtained, but with some smaller local differences. We also find that approximately the same topology, but with different sizes of the structural members, is obtained with the global and the clustered approaches. However, this is more of a coincidence rather than a rule; we note that different number of clusters may result in somewhat different topologies, as the highest local stress in the final design depends on the clustering. As shown by Eq.(4) and Eq.(6), and seen in Fig. 3, the global approach gives designs where all stresses are below $\overline{\sigma}$ and the clustered approach gives designs where some local stresses are above $\overline{\sigma}$.

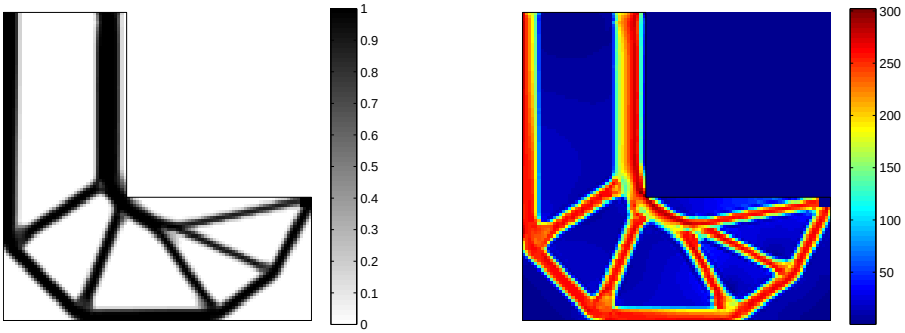
Table 2: Results corresponding to the plots in Fig. 3

Subplot in Fig. 3	Mass $[kg \times 10^{-3}]$	Design variables in last iteration	Total time [s]	Number of iterations	$\max \widetilde{\sigma}_i(\boldsymbol{x})$ or $\widetilde{\sigma}_{\mathrm{G}}(\boldsymbol{x})$	$\max \sigma_e^{\mathrm{vM}}(\boldsymbol{x})$ [MPa]
(a)	5.43	6388	5331	681	1.017	298
(b)	5.41	2253	2905	657	1.006	302
(c)	4.27	6388	12592	536	1.023	377
(d)	4.23	2387	9181	558	1.023	378

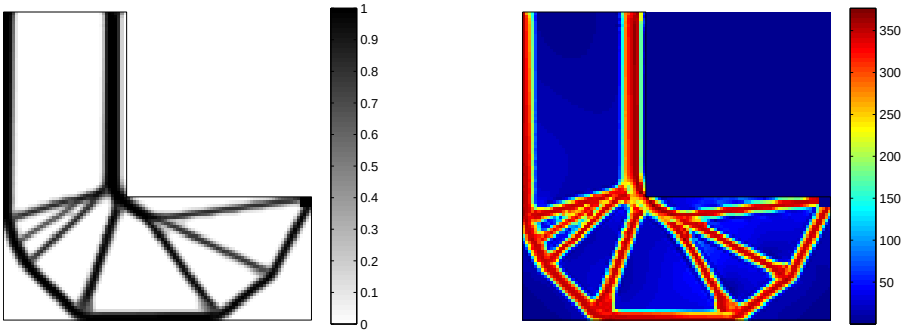
The computational time for the four different optimizations is compared in Fig. 4, where the number of iterations is plotted against the total time. A steeper incline means that the computational time per iteration is shorter, and when design variables are deactivated, it can be seen how the incline increases. As the total time depends on the convergence rate, the incline (or time per iteration) is a better measure of time for this comparison. As expected, the time per iteration using the global constraint is shorter than the time for 25 clustered constraints.



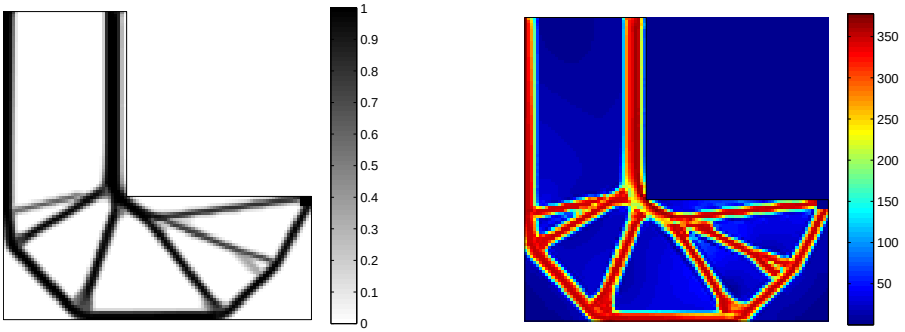
(a) Global stress constraint



(b) Global stress constraint with deactivation of design variables



(c) 25 clustered stress constraints



(d) 25 clustered stress constraints with deactivation of design variables

Figure 3: Topology and stress results for the L-shaped beam

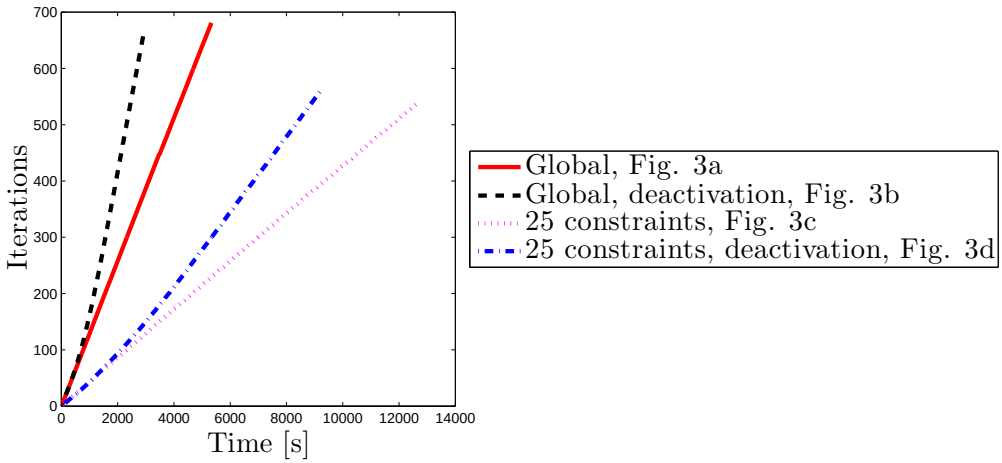


Figure 4: Iterations as a function of time for the optimizations in Fig. 3

7.2. MBB-beam

The optimized designs for the MBB-beam are seen in Fig. 6 and data is presented in Table 3. As for the L-shaped beam, we obtain approximately the same design when design variables are deactivated as when we solve the full problem. However, due to the difference between the allowable local stress in the global and clustered approaches, the obtained topologies are quite different. The computational time is compared in Fig. 5 and again we find that the time per iteration decreases as more design variables are deactivated.

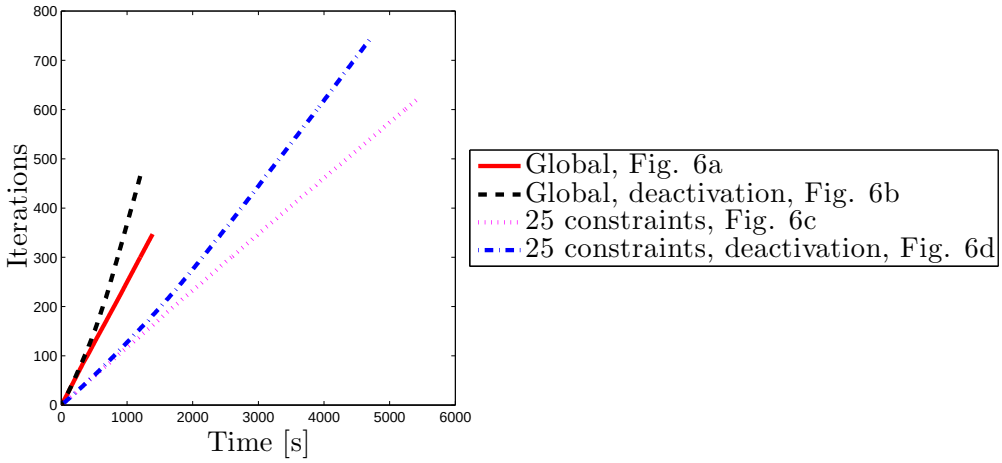


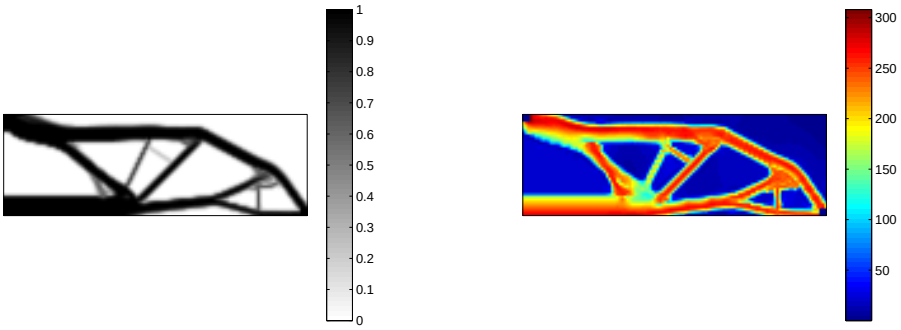
Figure 5: Iterations as a function of time for the optimizations in Fig. 6

Table 3: Results corresponding to the plots in Fig. 6

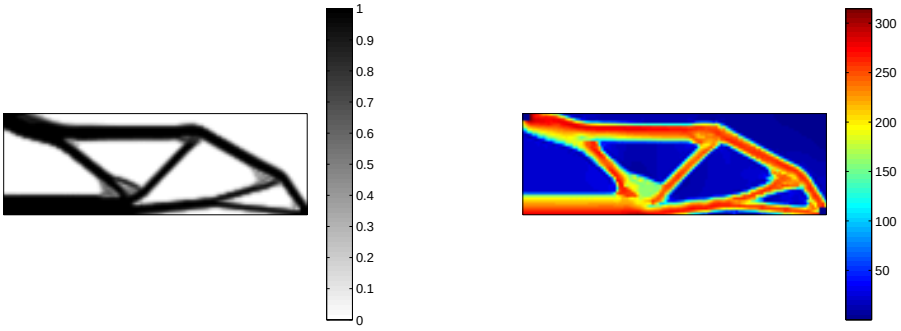
Subplot in Fig. 6	Mass $[kg \times 10^{-3}]$	Design variables in last iteration	Total time [s]	Number of iterations	$\max \overline{\sigma}_i(\boldsymbol{x})$ or $\overline{\sigma}_{\mathrm{G}}(\boldsymbol{x})$	$\max \sigma_e^{\mathrm{vM}}(\boldsymbol{x})$ [MPa]
(a)	4.90	4782	1389	347	1.000	308
(b)	4.81	1824	1201	465	1.009	315
(c)	3.96	4782	5403	619	1.009	387
(d)	3.73	2105	4688	741	1.026	394

8. Conclusions

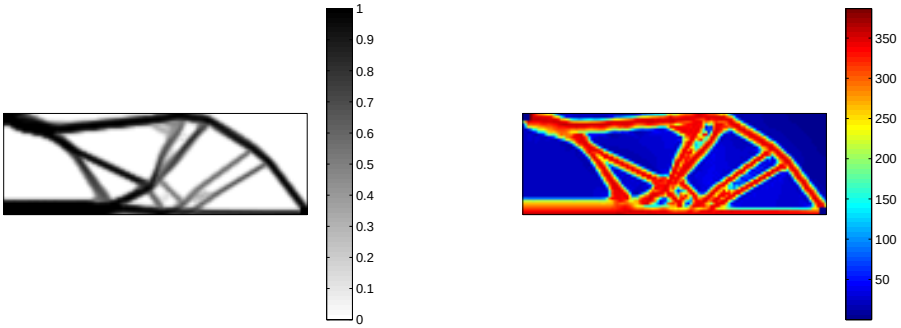
We have shown that a global stress constraint may be used in order to find a design which is free from



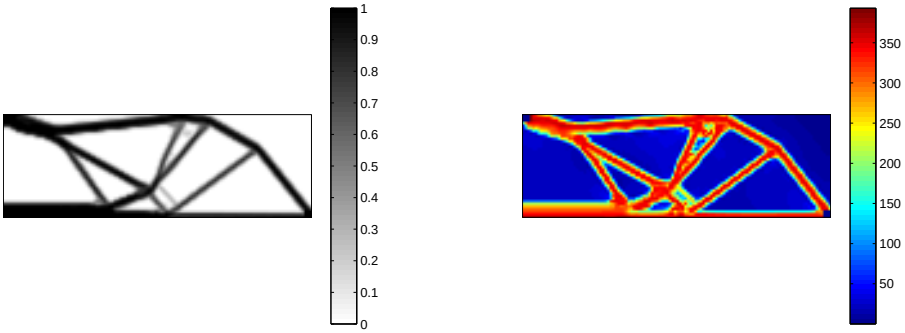
(a) Global stress constraint



(b) Global stress constraint with deactivation of design variables



(c) 25 clustered stress constraints



(d) 25 clustered stress constraints with deactivation of design variables

Figure 6: Topology and stress results for the MBB-beam

large stress concentrations and which is guaranteed to have local stresses below the stress limit. The computational cost for the global approach is much lower than for the clustered approach. The global and clustered approaches give the user a choice to determine if the design obtained with the global approach is sufficient for further design work, or if it is worth the additional computational cost to obtain a better sized design with the clustered approach.

For the two test examples in this paper, deactivation of design variables gives approximately the same topology as solving the full problem and decreases the computational time considerably. The implementation is rather straight-forward and it still allows the topology optimization to be an add-on to an existing FE-program and optimization solver, as these do not have to be changed in any way.

Even though the local stresses are below the stress limit, the global approach has been proven to provide conceptual designs with an even stress distribution, which will be good starting points for further analysis.

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