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Admission Control Based on OFDMA Channel Transformations[*]

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Abstract

It is well known that channel-dependent OFDMA resource assignment algorithms provide a significant performance improvement compared to static (i.e. channel-unaware) approaches. Such dynamic algorithms constantly adapt resource assignments to current channel states according to some objective function. Due to these dynamics, it is difficult to predict the resulting performance for such schemes given a certain scenario (characterized by the number of terminals in the cell and their average channel gains). Hence, previous work on admission control for OFDMA systems neglects the performance improvement from channel-dependent resource assignments and bases analysis on the average channel gains instead. In this paper we provide for the first time an analytical framework for admission control in OFDMA systems applying channel-dependent resource assignments. The framework is based on fundamental transformations of the channel gains caused by the channel-dependent assignment algorithms. We provide closed-form expressions for these transformations and derive from them probability functions for the rate achieved per terminal and frame. These functions can then be used for admission control as demonstrated in this paper for Voice-over-IP streams in IEEE 802.16e systems.

1 Introduction

Over the last decade orthogonal frequency division multiple access (OFDMA) has become one of the major multiple access scheme for broadband wireless systems. Today, OFDMA is already part of the IEEE 802.16e [4] standard for metropolitan area networking while it is also going to be the dominant multiple access technology in 3GPP's Long Term Evolution (LTE) systems for cellular networking. Finally, there is some evidence that OFDMA might

also be implemented in future wireless local area networks i.e. post IEEE 802.11n systems (standardization activity has just started in IEEE 802.11's task group 'ac').

There are several reasons for this success of OFDMA as multiple access scheme. First of all, the proliferation of orthogonal frequency division multiplexing (OFDM) as favorable transmission scheme for broadband wireless links has made OFDMA the "natural" choice for multiple access. Almost all broadband wireless systems, either on the market or under standardization, are based on OFDM due to its resilience to frequency-selective fading paired with a relatively low implementation complexity [10]. OFDMA brings the advantage of allowing fine-grained scheduling of multiple different terminals which is particularly important for packet-oriented wireless networks. In addition, mobile terminals benefit from frequency diversity if uncorrelated parts of the OFDM bandwidth are scheduled for their data transmission.

However, over the last decade one of the strongest arguments made by research for OFDMA lies in exploiting multi-user diversity. OFDM splits the given system bandwidth into small parallel communication channels referred to as subcarriers. Any single link in a broadband OFDM system experiences a varying channel gain for these numerous subcarriers (stemming from frequency-selectivity). In addition, if multiple different receivers (i.e. links) are considered (for example, in the down-link), the different receivers experience very different gains for any given subcarrier (as channel gain is uncorrelated in space over the distances usually separating multiple terminals). So, physical layer efficiency can be improved significantly by allocating "appropriate" subcarriers to terminals in a frequency division multiplexing fashion. Such channel-dependent OFDMA resource assignment is now under discussion for about ten years [16, 1]. Research mainly focuses on assignment algorithms to be applied for the down-link of wireless OFDMA cells. Many different algorithms have been proposed in the past, addressing mainly the trade-off between complexity (i.e. run time of the algorithm) and phys-

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ical layer efficiency. Apart from that, some contributions have also investigated the associated overhead (collecting the channel state information as well as signaling the dynamic assignments). IEEE 802.16e as well as 3GPP's LTE enable the application of channel-dependent OFDMA resource assignments by providing the protocol "hooks" in the system.

While channel-dependent OFDMA resource assignments do provide a significant performance gain, these schemes also cause new problems. One particularly important issue deals with admission control: Assume that a certain algorithm at the base station assigns subcarriers to terminals for each down-link phase in order to improve some metric (for example, proportional fair throughput or overall throughput). Based on stochastic simulations, it is relatively easy to determine the average performance obtained from any such algorithm for some chosen scenario (number of terminals in the cell, distance of the terminals to the base station etc.). However, it is much harder to *predict* the obtained performance of any assignment algorithm for some given scenario due to the channel-dependent nature of these algorithms. Unfortunately, some form of performance prediction is required for admission control. To see this, assume that a certain number of Voice-over-IP (VoIP) calls are currently admitted to a wireless OFDMA cell. Next, five new calls arrive and request admission to the cell. Some method is clearly required to judge if the channel-dependent OFDMA resource assignments can still fulfill the quality-of-service requirements of all the voice streams if the new ones are admitted to the cell. In a more general setting, we need a framework to determine if an OFDMA cell can support a given set of transmission requests (characterized by quality-of-service demands as well as the average channel gains between the base station and the requesting terminal) with a given amount of resources (subcarriers, power, modulation and coding schemes). Notice that due to the stochastic nature of the wireless channel the framework of interest is of stochastic nature.

In this paper we present a novel framework for admission control in OFDMA cells. The framework allows the prediction of achievable rates given a certain amount of resources and a certain set of requests. In contrast to previous work [9, 8, 13] (see also Section 6 for a detailed discussion), our major contribution is that we provide an *analytical* framework for performance prediction of *channel-dependent* OFDMA resource assignments, i.e. a class of algorithms which freely assigns subsets of subcarriers to different terminals for each down-link phase in order to improve some performance metric. Our analytical framework is based on a fundamental insight into such OFDMA resource assignment algorithms regarding the way they modify the fading statistics of *assigned* subcarriers. We provide a closed form expression for these resulting fading statis-

tics (which we refer to as OFDMA channel transformations) and derive from this analytical core a lower performance bound for one example OFDMA subcarrier assignment optimization problem. This lower bound can be used to perform admission control with respect to different policies, as discussed finally in this paper. To the best of our knowledge, this approach is novel.

The remaining paper is structured as follows. In Section 2 we present the basic system model and state the problem of interest. Then, in Section 3 we discuss the OFDMA channel transformations, i.e. the basic effect that channel-dependent OFDMA resource assignment has on the channel gains of assigned subcarriers. We also present some illustrating numerical examples in the same section. Section 4 contains then derivations of the lower bound for a specific resource assignment problem considered in this paper. This lower bound is investigated numerically in Section 5. The same section contains also a discussion on the application of the framework to admission control. Section 6 discusses relations of this work to previous one. Finally, we conclude the paper in Section 7.

2 System Model

A single cell of a wireless system consists of a base station serving J terminals. We consider a centrally organized system, all data transmissions within the cell are controlled by the base station. Time is split into frames each featuring a down-link and an up-link phase. In the following we focus on the down-link phase and denote its duration by T_{dl}. Via a backbone the base station receives data destined for the terminals in the cell. Upon transmission, this data is queued separately for each terminal. Prior to each down-link phase, the base station schedules (some or all of the) currently queued data for transmission during the next down-link phase. Denote these scheduled data amounts for terminal j by σ_j. Afterwards a resource assignment unit tries to match system resources with scheduled transmission requests as good as possible. In the following, we first describe these system resources in Section 2.1 and focus afterwards on the resource assignment. Finally, we state the problem addressed by this paper in Section 2.3.

2.1 Physical Layer

Data is transmitted via an OFDM system of total bandwidth B [Hz] at a center frequency f_{c} [Hz]. A maximum transmit power of P_{max} can be utilized in this frequency band. The bandwidth is split into N subcarriers on which information is transmitted in parallel by digital symbols of length $T_{\mathrm{s}} = 1/\Delta f = B/N$. We refer to all N subcarrier symbols transmitted simultaneously as an OFDM symbol. In order to mitigate intersymbol interference, a guard

period of length T_{g} is added to each OFDM symbol. Per down-link phase, the system features $S = \lfloor T_{\mathrm{dl}}/\left(T_{\mathrm{s}} + T_{\mathrm{g}}\right)\rfloor$ OFDM symbols for data transmission.

For each subcarrier n and terminal j the gain $g_{j,n}$ varies over time and frequency, i.e. each subcarrier/terminal gain depends on a constant component (path loss, denoted by ρ_j) and a random, time- and frequency-variant fading component. We assume this gain to be exponentially distributed (with mean ρ_j). Matrix $\mathbf{G}$ groups all subcarrier/terminal gains. $\mathbf{G}$ is assumed to stay constant for the duration of one down-link phase but varies over longer time spans. Based on the transmit power p_n per subcarrier and the variance σ^2 of the white Gaussian noise process per subcarrier, we obtain the signal-to-noise ratio (SNR) of subcarrier n and terminal j per down-link phase by:

$$\gamma_{j,n} = \frac{p_n \cdot g_{j,n}}{\sigma^2} \ . \tag{1}$$

We assume M different modulation types to be featured by the transmission system. Modulation type m represents b_m bits per symbol (for example $b_m = 2$ for QPSK). The base station can adapt the modulation type for each subcarrier separately (referred to as adaptive modulation). However, any down-link transmission is constrained by a terminal specific target bit error rate β_j. Hence, a modulation type m is applied if the current SNR for this terminal is within the range $\Gamma_{j,m} \leq \gamma < \Gamma_{j,m+1}$. The SNR range delimiters are determined by modulation specific bit error probability functions, as for example presented in [2]. Denote by $F_j\left(\gamma\right)$ the function returning the amount of bits that can be transmitted per OFDM symbol to terminal j at an SNR of γ. Notice that for any SNR below $\Gamma_{j,1}$ no modulation is applied, i.e. $F_j\left(\gamma < \Gamma_{j,1}\right) = 0$. Also, for modulation M the range has no upper limit.

2.2 Medium Access Control Layer and Resource Assignment

During each down-link phase frequency division multiplexing is applied. Thus, during the resource assignment step disjoint subsets of subcarriers are assigned to terminals based on perfect knowledge of subcarrier gains. Subcarrier assignments are valid throughout the entire down-link phase. The assignment of a subcarrier/terminal pair is denoted by the binary variable $x_{j,n}$. In addition to the subcarrier/terminal assignments, resource assignment might also contain a variable transmit power p_n per subcarrier.

As the scheduler requests the resource assignment unit to transmit the data amount σ_j per terminal during the next down-link phase, the resource assignment unit first identifies the minimum requested data amount $\sigma_{j^\star}$ and computes then for all terminals the proportion factor:

$$\alpha_j = \sigma_j/\sigma_{j^\star} \ . \tag{2}$$

Based on these quantities the resource assignment unit solves the following rate-adaptive optimization problem [14, 3]:

$$\begin{aligned}
\max \quad & \epsilon \\
\text{s. t.} \quad & \sum_j x_{j,n} \leq 1 \quad \forall\, n \\
& S \cdot \sum_n F_j\left(\frac{p_n \cdot g_{j,n}}{\sigma^2}\right) \cdot x_{j,n} \geq \alpha_j \cdot \epsilon \quad \forall\, j \\
& \sum_n p_n \leq P_{\max}
\end{aligned} \tag{3}$$

In this maximization of the minimum rate the factor α_j assures a proportional scaling of the achieved minimum rate in case that the scheduler requests different data amounts per terminals. Note that problem (3) is an NP-hard optimization problem [1]. Different approaches have been suggested how to solve the problem by faster heuristics [3, 6]. We do not consider the specific algorithm further and simply assume that problem (3) can be solved prior to each down-link phase optimally or with negligible performance degradation. Finally, some control channel is needed to inform the terminals of their assignments (subcarriers and modulation types). We assume the existence of such a separate, error-free control channel conveying this information to the terminals.

2.3 Problem Statement

This paper addresses the problem of admission control and capacity estimation for OFDMA down-link. When determining the requested data amount per terminal σ_j the scheduler requires some estimate of ϵ. Intuitively, this estimate should depend on the number of terminals J, the two sets of their respective average channel gains $\{\rho_j\}$ and their target bit error rates $\{\beta_j\}$ as well as the resources available for transmission (i.e. the total transmit power $P_{\max}$, the total bandwidth B, the amount of subcarriers N, the length of the down-link phase T_{dl} as well as the set of available modulation types). Notice that such an estimate resembles on a larger time scale the key component to admission control for (the down-link of) OFDMA systems. Flows destined for terminals within the cell request admission and quality-of-service requirements are specified, given for example by their average rate requirement, delay tolerance, packet loss rates etc. In order to decide about admission (with respect to the already admitted flows and their respective quality-of-service requirements) the scheduler needs to determine the impact of admitting the new flow on ϵ.

Any framework enabling the scheduler to estimate ϵ is of probabilistic nature. That is, given a characterization of the load and the available resources, we can only hope to derive probabilities that the solution to problem (3) results in some (required or estimated) ϵ – unless we restrict

ourselves to an instance of matrix $\mathbf{G}$ (in which case we are rather trying to design a suitable algorithm for assigning resources than dealing with analytical performance estimates). Hence, when scheduling data portions σ_j for each terminal, these decisions have to be based on some probability that the resource assignment unit will not be able to transmit the requested capacity. In the following we refer to such "scheduling misses" simply as outages.

3 OFDMA Channel Transformations

Ideally, we could obtain an analytical framework directly derived from problem (3). However, as this problem is NP-hard the general (optimal) solution to it depends on exhaustive search and is therefore not analytically tractable. Still, we are interested in deriving some analytical framework and take therefore the following approach. We pick a suboptimal algorithm which can be captured by analysis. This yields a lower bound on the estimate for ϵ, in fact we can even derive rate probability mass functions for each terminal individually. However, instead of focusing directly on the above described system, we first present the selected algorithm and derive its impact on the PDF of subcarrier gains. The substantial research body on channel-dependent OFDMA resource assignment is based on the fact that any such assignment algorithm modifies the stochastic nature of the respective subcarrier gains per terminal. Thus, for our framework we first derive how our chosen algorithm transforms exponentially distributed channel gains. Note that this analytical core can be applied to further questions and approaches regarding OFDMA resource assignments apart from the ones we consider in the context of assignment problem (3).

3.1 Assignment Algorithm

The underlying algorithm used for analysis takes the channel state information matrix $\mathbf{G}$ as input and works as the following. Initially, each terminal considered for the next down-link phase is allocated a certain number of subcarriers l_j. Note that this only fixes the amount of subcarriers obtained by each terminal, the assignments are still open. Given the allocation of subcarriers, the algorithm starts with some "privileged" terminal j^* and assigns it the corresponding l_{j^*} best subcarriers out of the set of N total subcarriers. A subcarrier is said to be better if its gain is higher. Next, the assigned subcarriers are removed from the list of available ones. Afterwards the algorithm switches to some next terminal and assigns it the l_j best subcarriers from the remaining set. The algorithm continues until all terminals are assigned their share of l_j subcarriers. A more formal description of the algorithm is presented below as Algorithm 1.

Given: Set of gains $\{g_{j,n}\}$ and allocations $\{l_j\}$
Initialize: $\forall j \in J : X_j = \emptyset; \mathcal{N} = \{1, \cdots, N\}$
for $(j \in J)$ **do**
   while $(l_j > |X_j|)$ **do**
      $\tilde{n} = \mathrm{argmax}_{n \in \mathcal{N}}\{g_{j,n}\}$
      $X_j = X_j \cup \tilde{n}$
      $\mathcal{N} = \mathcal{N} \setminus \tilde{n}$
   end
   return X_j
end

Algorithm 1: Scheme of the approximation algorithm.

3.2 Basic Analysis

Two sets of variables determine the resulting performance obtained per terminal: the set of allocated subcarriers, i.e. $\{l_j\}$ and the order with which the terminals are served. In the following let us assume that both are fixed. Let us consider terminal j. Recall that we assume the subcarrier gain of an arbitrarily selected subcarrier for terminal j to be exponentially distributed with mean ρ_j. Denote the random channel gain of an arbitrarily selected subcarrier for terminal j by g_j, then the density function for this random variable is given by:

$$f_{g_j}(x) = \frac{1}{\rho_j} \cdot e^{\frac{-x}{\rho_j}} \qquad (4)$$

with the corresponding distribution function:

$$P[g_j \le x] = F_{g_j}(x) = 1 - e^{\frac{-x}{\rho_j}} . \qquad (5)$$

However, the l_j subcarriers assigned by the algorithm mentioned above are not arbitrarily chosen. Instead, they are the l_j best subcarriers (with respect to their channel gains) out of a (most likely) larger set of subcarriers. Denote by θ_j the set of terminals which get their subcarriers assigned prior to terminal j. Then, at the time the algorithm serves terminal j there are

$$A_j = N - \sum_{i \in \theta_j} l_i \qquad (6)$$

subcarriers left. Denote by $\tilde{g}_{j,(1)}, \ldots, \tilde{g}_{j,(l_j)}$ the random channel gains of the l_j best subcarriers chosen out of the remaining set of subcarriers. We can characterize their density function and distribution function by applying results from order statistics [15].

Let $X_1, X_2, \ldots, X_N$ be N independent and identically distributed random variables. Denote by $\tilde{X}_{(k/N)}$ the k-th smallest of these N random variables. Then from order statistics it is well known that the density function of

$\tilde{X}_{(k/N)}$ is given by:

$$f_{\tilde{X}_{(k/N)}}(x) = k \cdot \binom{N}{k} P(X \leq x)^{k-1}$$
$$\cdot P(X > x)^{N-k} \cdot f_X(x) \qquad (7)$$

and correspondingly we have for the distribution function:

$$P\left(\tilde{X}_{(k/N)} \leq x\right) = \sum_{i=k}^{N} \binom{N}{i} P(X \leq x)^i \cdot P(X > x)^{N-i} . \qquad (8)$$

Based on Equations (4), (5) and (7), we obtain the density function of the k-th best subcarrier[1] selected for terminal j out of the set of A_j remaining ones by:

$$f_{\tilde{g}_{j,(k)}}(x) = \frac{A_j - k + 1}{\rho_j} \cdot \binom{A_j}{A_j - k + 1}$$
$$\cdot \left(1 - e^{\frac{-x}{\rho_j}}\right)^{A_j - k} \cdot \left(e^{\frac{-x}{\rho_j}}\right)^k . \qquad (9)$$

Correspondingly, we obtain for the distribution function from Equations 4, 5 and 8:

$$F_{\tilde{g}_{j,(k)}}(x) = \sum_{i=A_j-k+1}^{A_j} \cdot \binom{A_j}{i} \cdot \left(1 - e^{\frac{-x}{\rho_j}}\right)^i \cdot \left(e^{\frac{-x}{\rho_j}}\right)^{A_j-i} . \qquad (10)$$

Note that these equations are based on the assumption of independence and identical distribution of the subcarrier gains. While the property of identical distribution is likely to exist, independence strongly depends on the spacing between subcarriers and the coherence frequency of the environment. We restrict our analysis to cases where independence can be assumed due to a coherence bandwidth which equals roughly the bandwidth of a subcarrier.

Equations (9) and (10) can be used for any probabilistic bound, for example on the rate or the power consumption. It is clear that regarding any such objective function, there exist more and less suitable choices for the set of allocations $\{l_j\}$ and the selection order (which determines for any terminal the remaining amount of A_j subcarriers to choose from). However, the fact that we have analytical expressions for the stochastic behavior of the channel gains allows us to study some insights of channel-dependent OFDMA resource assignments before continuing the specific analysis with respect to optimization problem (3).

3.3 Example Numerical Results

Let us consider the following scenario: We deal with a total of $N = 48$ subcarriers and $J = 6$ terminals. Subcarrier allocation is kept simple, each terminal is allocated

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[1]Notice that Equation (7) and Equation (8) are stated for the k-th worst random variable. In contrast, we derive here the PDF and distribution function for the k-th best subcarrier which is obtained by substituting k by $K - k + 1$ in Equation (7) and Equation (8).
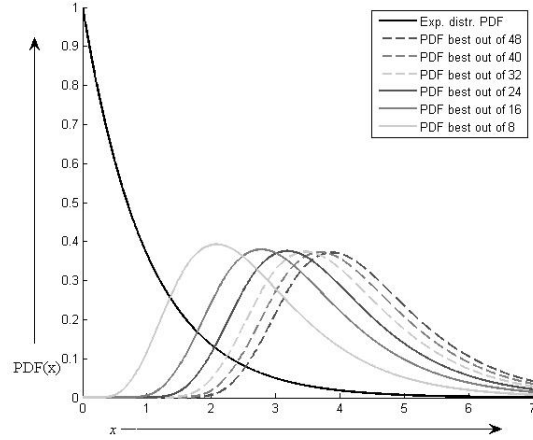

Figure 1. Density functions of the first (i.e. best) subcarrier selected for each terminal (48 subcarriers, 6 terminals considered).

$l_j = 8$ subcarriers. All terminals have an average channel gain of $\rho_j = 1$ (no path loss as well as assuming them to be positioned along a circle around the base station). For each down-link phase subcarriers are assigned according to the approximation algorithm. From Equation (9) we derive the density functions for the best and worst selected subcarrier for each terminal and plot them in Figure 1 and 2 (also showing for each plot the exponential PDF of the underlying subcarrier gains). Clearly, for the first four terminals the resulting PDF for the best and worst subcarriers of their selected ones provide a better stochastic characteristic (meaning that more often the resulting gain is above 1) than the exponential PDF. Even the best subcarrier of the last terminal features such a better behavior (while the worst subcarrier of the last terminal clearly does not).

4 Probabilistic Rate Guarantees

Given the analytical framework for the subcarrier gains by Equation (9) and (10) we develop in this section probabilistic expressions for the rate per terminal. First, we develop these expressions for an arbitrary setting of the allocations $\{l_j\}$ and a respective selection order $\{A_j\}$.

4.1 Derivation

In the following we assume that the transmission power is equally distributed over all subcarriers, i.e. $p_n = P_{\max}/N = P_{tx}$. At the end of this section, we comment on more general cases. Recall from Section 2.1 that there
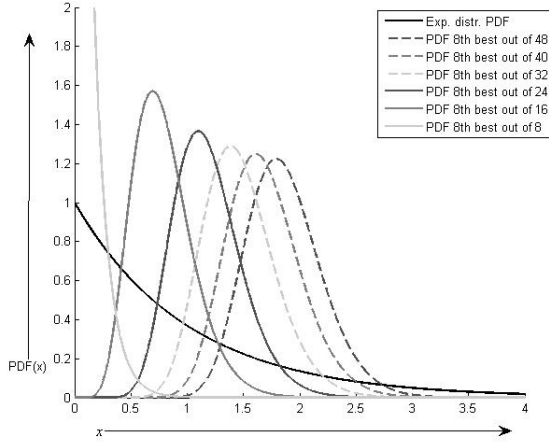
Figure 2. Density functions of the last (i.e. worst) subcarrier selected for terminal (each out of a set of 48 down to 8 subcarriers (48 subcarriers, 6 terminals considered).

exist M different modulation types in the system and for each terminal j there exists a target bit error rate β_j, resulting in SNR thresholds $\Gamma_{j,m}$. Based on the continuous distribution function for the subcarrier gains, we can derive (approximately) discrete probability mass functions for the (random) amount of data that can be transmitted on an assigned subcarrier per down-link phase. Denote by $z_{j,(k)}$ the random amount of bits that can be transmitted during the down-link phase on the k-th best subcarrier to terminal j. We can characterize its probability mass function by:

$$
P\left[z_{j,(k)} = S \cdot b_m\right] = F_{\tilde{g}_{j,(k)}}\left(\frac{\Gamma_{j,m+1} \cdot \sigma^2}{P_{\text{tx}}}\right)
$$
$$
- F_{\tilde{g}_{j,(k)}}\left(\frac{\Gamma_{j,m} \cdot \sigma^2}{P_{\text{tx}}}\right) \quad (11)
$$

In this equation essentially the probability is calculated that the k-th best subcarrier regarding terminal j can be employed by modulation type m (where b_m is the amount of bits that can be transmitted per symbol by this modulation type). Due to the required bit error probability, this modulation type can only be applied for an SNR larger $\Gamma_{j,m}$ and lower than $\Gamma_{j,m}$. As the transmit power is fixed and noise power is assumed to be on average of strength σ^2, the corresponding channel gains for these lower and upper bounds are equal to the expressions in the brackets of the distribution function in Equation 11. From Equation (11) we obtain a PMF for $z_{j,(k)}$:

$$
p\left(z_{j,(k)}\right) = \left[P\left(z_{j,(k)} = S \cdot b_0\right), \ldots, P\left(z_{j,(k)} = S \cdot b_M\right)\right]
$$
$$
\quad (12)
$$

As terminal j receives a total of l_j (best) subcarriers, we proceed with considering the random amount of data that can be transmitted over all these subcarriers, denoted by Z_j. It is given by the sum of the single subcarrier rate random variables $z_{j,(i)}$.

$$
Z_j = \sum_{i=1}^{l_j} z_{j,(i)} \quad (13)
$$

Under the assumption of independent random variables $z_{j,(i)}$ we obtain the PMF for Z_j by the convolution of the PMFs for all $z_{j,(i)}$ - denoting the convolution operator by $\odot$. Note that in fact the random variables $z_{j,(i)}$ are not independent as they are obtained from a selection order which imposes a dependency. However, we still treat the random variables $z_{j,(i)}$ to be independent at the cost of obtaining only an approximation for the probability mass function of the overall amount of data transmitted in one down-link phase per terminal:

$$
p\left(Z_j\right) = \bigodot_{i=1}^{l_j} p\left(z_{j,(i)}\right) \quad (14)
$$

Recall the assumption of a fixed transmit power for all subcarriers. In order to obtain a result for Equation (11) we require some transmit power setting. Clearly, it does not have to be the same for all subcarriers, but for the analysis it needs to be fixed for the k-th best subcarrier of terminal j. As the channel gain distribution function is at hand from Equation (10), we could apply a statistical power loading to improve the PMF in Equation (12). We consider these extensions to be out of scope of this paper.

Finally, we remark that the same framework (i.e. Equations (13) and (14)) can also be used to determine the rate PMF in case that no time-varying subcarrier assignments are performed (i.e. a static or channel-independent subcarrier assignment where we still adapt the modulation type). To see this, let us again assume a certain set of subcarrier allocations $\{l_j\}$. As channel-dependent subcarrier assignment is not performed, subcarrier allocations are either done contiguously or interleaved. In either case, we can obtain a PMF for the data amount that can be transmitted on a single subcarrier from the fact that the channel gains are exponentially distributed (ignoring in the contiguous case the correlation in frequency). Thus, we obtain the rate PMF per subcarrier from Equation (11) but using the exponential distribution function F_{g_j} from Equation (5).

4.2 Allocation Setting

Let us come back to the estimation of the minimum amount of data ϵ as obtained from the optimal solution to problem (3). Given a certain allocation set $\{l_j\}$ we yield by the above framework a set $\{E[Z_j]\}$ of expected data

amounts that we can transmit during a down-link phase to each terminal. The smallest rate $E[Z_{j^*}]$ among this set is the estimate for ϵ. Clearly, there is some relationship between the allocation set and the resulting estimate for ϵ.

It is not the goal of this paper to develop an optimal allocation. Instead, we simply discuss here an iterative allocation strategy which we will use in the numerical part to evaluate the framework with respect to estimating ϵ. The iterative approach works as follows: Given a set of requested transmission data amounts σ_j, a certain outage probability $\hat{\pi}$ is set for all terminals. Then, we start allocating subcarriers to some initially chosen terminal j^* until we have:

$$P[Z_{j^*} \leq \sigma_{j^*}] = \sum_{i=0}^{\sigma_{j^*}-1} P[Z_{j^*} = i] \leq \hat{\pi} \qquad (15)$$

Assume that an amount of $\hat{a}_j$ subcarriers provides a sufficient outage probability. We then switch to the next terminal and redo the initial step until the outage probability is sufficiently low. As we do this for all terminals, we either run out of subcarriers (without having allocated for every terminals a sufficient amount of subcarriers) or some subcarriers are left over. In the first case the system actually can not handle the requested load and we therefore have to increase the outage probability π by $\Delta^+\pi$ in order to redo the allocation. In the second case, we either keep the allocation (if only very few subcarriers are left over) or we decrease the outage probability π by $\Delta^-\pi$ and redo the allocation if a significant amount of subcarriers ($\geq J$) is left over. Hence, the algorithm terminates[2] once we have determined an allocation set which can support the requested data amounts with a homogeneous outage probability π. A more formal representation of the algorithm is given below as Algorithm 2.

5 Numerical Evaluation

In this section we initially investigate the quality of our lower bound in comparison to the optimal solution of problem (3). After performing this evaluation (including a validation of the framework by simulations), we draw our attention to the problem of admission control in the down-link of an OFDMA (WiMAX-like) system. We consider here the specific case of Voice-over-IP streams for which we are interested in the maximum number of streams that can be served due to several different quality-of-service constraints.

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[2]There is some issue regarding oscillations in this algorithm depending on the setting of $\Delta^-\pi$ and $\Delta^+\pi$. In practice this is controlled by tracking the outcome and aborting the algorithm as soon as it starts repeating allocations.

Given: Set of requested rates $\{\sigma_j | \forall j \in J\}$
Initialize: $\pi = \hat{\pi}$
repeat
    Initialize Allocations: $\forall j \in J : l_j = 0$
    for $(j^* \in J)$ **do**
        while $P[Z_{j^*} \leq \sigma_{j^*}] \geq \pi$ **do**
            if $N == 0$ **then**
                $\pi = \pi + \Delta^+\pi$;
                break (the for loop);
            else
                $l_{j^*} = l_{j^*} + 1$;
                $N = N - 1$;
                Recompute $P[Z_{j^*}]$;
            end
        end
    end
    if $N \geq J$ **then**
        $\pi = \pi - \Delta^-\pi$;
    end
until $(\forall j \in J : P[Z_j \leq \sigma_j] \leq \pi) \,\&\, (N < J)$;
return l_j;

Algorithm 2: Schema of the allocation algorithm.

5.1 Basic Evaluation Scenario

We pick data transmission in 802.16e-2005 [4] systems as basic scenario. This standard defines a set of OFDMA systems with multiple "allocation modes". We consider here a total system bandwidth of $B = 10$ MHz. Each subcarrier has a bandwidth of 11.16 kHz. Hence, the system bandwidth is split into $N = 865$ net subcarriers (the total number equals 1024 but 159 of them are used as guard bands) out of which 96 are used as pilots. The symbol time results to $T_s = 89.6$ μs while the considered setting for the guard time equals $T_g = 11.2$ μs, yielding a total length of an OFDM symbol of 100.8 μs. We focus explicitly on the AMC (adaptive modulation and coding) transmission mode, therefore the net subcarriers are divided into bands of 36 subcarriers each which are again subdivided into bins of 9 sub-carriers (one pilot and 8 data subcarriers). There are 24 bands and 96 bins. A bin is the smallest share of bandwidth that can be allocated separately. In the following we assume that a bin is assigned to a certain terminal throughout the whole down-link phase (in fact, the standard allows for time sharing of bins, where the bin can be reassigned after 6 OFDM symbols). Furthermore, we assume a frame length of 5 ms out of which $T_{dl} = 2.5$ ms are available for the down-link transmission (assuming TDD). This results to $S = 24$ symbols available per bin per down-link phase. From these 24 symbols we assume 20 to be available for data transmission (the remaining ones reserved for control

signaling). The adaptive modulation system of 802.16e features BPSK, QPSK, 16-QAM and 64-QAM.

Terminals in the cell are located with an equal distance to the base station. Hence, for all considered scenarios we have the same basic channel gain ρ_j for all terminals (for different scenarios we vary this basic channel gain which equals different distances between base station and terminals). Fading components are generated due to an exponential distribution. We do not consider correlation in time or in frequency of the channel gains. Also, terminals are assumed to be stationary.

As traffic we assume the transmission of Voice-over-IP flows encoded according to the G.711 standard. Such streams generate a packet every $d = 20$ ms of size 80 Bytes (resulting in an application layer rate of 32 kBit/s). Adding to this RTP, UDP and IP packet overheads yields a final size of $\sigma = 120$ Bytes. We assume them to be encoded according to a rate $3/4$ convolutional encoder. Finally, 802.16e adds a MAC overhead of $O = 10$ Byte (6 Byte header plus a CRC32), which yields a total MAC packet size of 170 Byte (which equals a required physical layer rate of 68 kBit/s). Packet transmissions are subject to a packet error rate of $p_j^{\mathrm{Err}} = 0.01$. Due to the usage of a $3/4$ convolutional code this yields a target bit error rate of $\beta_j = 0.0052$ (following the derivations in [12] and assuming hard-decision Viterbi decoding with independent errors).

5.2 Methodology

Initially, we are interested how our lower bound performs in comparison to the optimal solution of problem (3). This is also interesting because of the approximation step taken in order to derive Equation 14. As metric we consider the average minimum rate per down-link phase of all terminals (i.e. we consider ϵ). Apart from that we also consider the outage probability of a VoIP packet (of size 170 Byte) as metric. The following schemes are compared with each other regarding these two metrics.

- The optimal solution to problem (3), setting $\alpha_j = 1$ for all terminals j. In order to determine it, we generate channel gains for a large number of down-link phases, calculate from them the SNR values and the respective adaptive modulation states, and essentially formulate the corresponding integer programming problem. This is then passed to the IP solver CPLEX [5]. The resulting optimal assignments are then further processed for statistical purposes by a script. Hence, the optimal solution to problem (3) is determined by stochastic simulations. From these simulations we obtain the average down-link rate as well as the outage probability per terminal. We refer to this in the following as the "IP Optimum". Notice that very close performance to the IP

optimum can be achieved for example by applying linear relaxation techniques to simplify the problem [3].

- The analytical lower bound for channel-dependent OFDMA assignment as derived in Section 4.1. The evaluation is done by Matlab. Notice that we consider the allocations according to the algorithm discussed in Section 4.2. Hence, for the bound we obtain average down-link rate and outage probability by analysis. We refer to this as the "Lower Bound" in the following.

- In order to verify the analysis leading to the lower bound, we also provide simulated performance results. We have programmed the allocation and assignment algorithm presented in Section 3.1 and 4.2 and applied it to the same set of channel gains used for the stochastic simulation of the IP optimum. We refer to this scheme as "Simulated Bound". Notice that the difference between this simulated bound and the analytical lower bound reveals the error stemming from the assumption of independent rate PMFs in the derivation of Equation 14.

- Finally, the above framework can also be used to predict the performance if no channel-dependent subcarrier assignments are performed. Notice that we still assume the usage of adaptive modulation as channel-dependent means. But terminals receive a certain block of subcarriers and reuse them throughout all down-link phases. Hence, the performance metrics of this scheme are evaluated by analysis. We refer to this scheme as "Static Bound". The static approach essentially resembles what has been suggested for admission control in previous work [9, 8, 13].

5.3 Bound Evaluation

Regarding the above metrics we have evaluated the four different schemes in a setting where the number of terminals in the cell increases from initially 10 terminals up to 32 terminals. In Figure 3 we present the resulting minimum average rate ϵ per down-link phase while Figure 4 presents the corresponding outage probability. From Figure 3 we see that the lower bound approximates the performance of the IP optimum with a gap equal to about 15% of the optimal performance. While from a mathematical point of view this bound is not tight, we have to keep in mind that the IP optimum is based on exhaustive search. Hence, it often applies assignments which cannot be captured by any analytical framework. Also, the only alternative to using the bound from the channel-dependent OFDMA resource assignment is to base the analysis on average channel gains which leads to the estimate of ϵ represented by the static approach. Taking these two arguments into account, the proposed lower
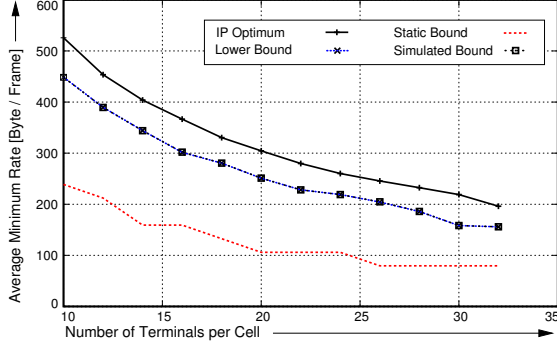
Figure 3. Minimum average rate per frame per terminal for an increasing number of terminals (average SNR of 10 dB).

Figure 4. Maximum outage probability per frame for a packet of size 170 Byte over an increasing number of terminals (average SNR of 10 dB).

bound for channel-dependent OFDMA resource assignment is a significant improvement. Finally, from Figure 3 we also observe that the lower bound is valid, as the curve of the simulated bound almost exactly matches the derived curve (notice that confidence intervals are below 1% of the total performance for each point and are not shown in Figure 3). Hence, regarding the *average minimum rate per terminal* in the cell the approximation applied for Equation 14 does not have a strong impact. Correspondingly, in Figure 4 we observe the outage probability of the four schemes. Essentially, the same statements can be made about this graph. The lower bound provides a much better estimate of the outage probability than what is provided by the static approach (not taking adaptive subcarrier assignments into account). However, there remains some gap between the lower bound and the IP optimum. Finally, note from Figure 4 that there is some difference between the simulated and analytical results for the lower bound (around 22 terminals in the cell) which stems from the (essentially wrong) assumption of statistical independent of the variables in deriving Equation 14. So, regarding the outage statistics this assumption leads to some better result than what the scheme provides in reality. However, the bound is still significantly below the IP optimum and that is our main goal in this work.

5.4 Application to Admission Control

We now come to the application of the analytical framework to admission control. We illustrate this by considering the down-link transmission of VoIP streams (parameters as described in Section 5.1). Each VoIP stream has a (constant) bit rate requirement on the physical layer (including all packet overheads) of 68 kBit/s. However, the stream is divided into packets with an (average) interarrival time of 20 ms. We assume in the following that packets should be
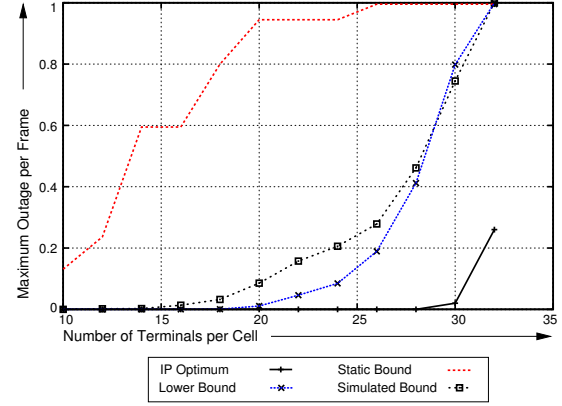
transmitted in one piece during a single down-link phase. Hence, the question comes up how many such packets can be transmitted (each one to a different terminal) in a single down-link frame. Once we have determined this number, the total VoIP capacity of the system is simply this number times four, as we consider a frame time of 5 ms (and packet interarrival time is 20 ms). Notice that there exist other "scheduling" variants where for example the transmission of a packet might be split over several down-link phases (causing a higher control overhead). We do not investigate this any further.

As quality-of-service requirement we consider two different regimes. In the first case we simply assume that every stream requires its packets to be transmitted successfully *on average*. That is, we can admit as many VoIP streams to a single down-link phase as we have for all admitted flows:

$$E\left[Z_j\right] \geq 170 \text{ Byte} \qquad (16)$$

We refer to this admission control scheme as "average rate QoS criteria". In this case we do not care about outage probabilities. In contrast, the second admission control scheme requires for each VoIP packet transmission a maximum outage probability of 0.05, i.e. we admit as many streams to each down-link phase of the system as for each of them we have:

$$P\left[Z_j \leq 170 \text{ Byte}\right] \leq 0.05 \qquad (17)$$

In Figure 5 we show the corresponding VoIP capacity when considering the average rate QoS criteria. The figure shows the maximum number of VoIP streams that can be admitted for a single down-link phase versus an increasing average SNR of all terminals. We show the resulting admission ca-
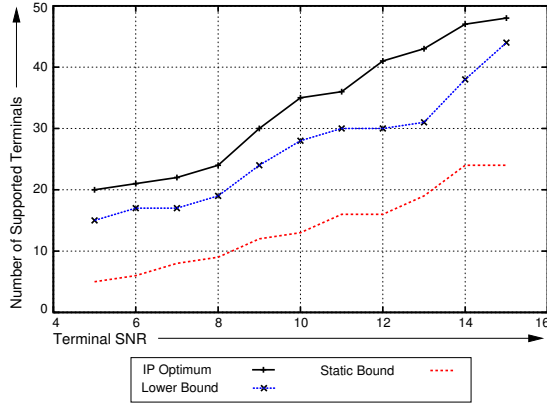
Figure 5. Maximum number of VoIP calls that can be supported in a single WiMAX frame according to an average rate QoS criteria over an increasing average SNR per terminal.
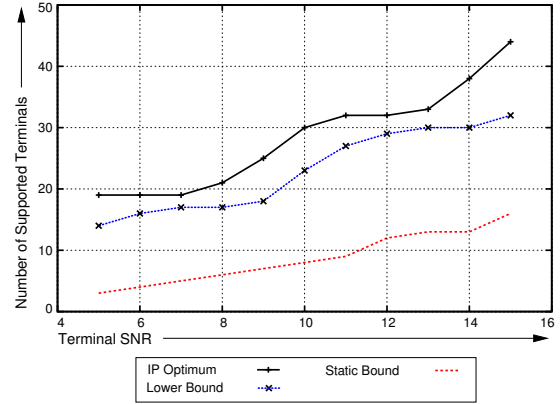

Figure 6. Maximum number of VoIP calls that can be supported by a single WiMAX frame according to an outage-based QoS criteria over an increasing average SNR per terminal (maximum outage probability of 0.05 is assumed).

pacity of the static approach, the lower bound and the (simulated) optimal capacity of the system. Clearly, the static approach is limited by the fact that it "ignores" multi-user diversity. Compared to the IP optimum, admission control by such a scheme misses capacity by about 50 to 75%. In contrast, the lower bound achieves a much better capacity estimate still with some gap in comparison to the optimum (which is basically the same gap as observed in Figure 3.) Finally, consider the resulting VoIP capacity if the outage admission regime is considered. Here the full strength of the analytical framework can be observed as outage behavior has rarely been investigated in the literature. But due to channel transformations core it can be easily analyzed. Figure 6 shows the corresponding VoIP capacity in this case again for an increasing average SNR per terminal. In contrast to Figure 5 we observe that the lower bound provides now a better estimate of the IP optimum than in case of the average QoS criteria. Notice that this is due to the approximation error, which is substantial in case of considering outage probabilities (as also observed in Figure 4). Still, the analytical bound is strictly below the IP optimum.

6 Related Work

The issue of channel-dependent OFDMA resource assignment (in the down-link) is well investigated where typical studies [3, 7, 16] focus on suboptimal algorithms for either minimizing the transmit power or maximizing the rate per terminal. Numerical evaluation, based on the simulation of numerous down-link phases, show that on average channel-dependent schemes outperform static ones significantly. Neither do these contributions address the prob-

lem of admission control nor do they provide an analytical framework for expected performance of the proposed algorithms.

Admission control for OFDMA networks is addressed by [9, 8, 13]. In [9] the authors investigate Poisson arrivals to a base station with finite buffers. Based on a queuing model, blocking probabilities are derived which an admission control scheme can base its operation on. However, the authors lack an analytical framework for the expected rate obtained from channel-dependent OFDMA resource assignments. Instead, they base their proposed admission control scheme on the average channel gain, i.e. average subcarrier rate. This clearly underestimates the true system capacity as shown above. This shortcoming applies also to [8, 13] where different admission control strategies are studied for two distinct systems and scenarios, still the achievable rate per subcarrier/subchannel is always derived from the average channel gain, underestimating the system's capacity significantly. In [13] the authors investigate admission control schemes for OFDMA cells under the power minimization regime. Per call, the required resources are planned according to the average channel gain, not taking multi-user diversity into account. Instead, the authors focus on the effect of hand-off multimedia traffic streams for which some system resources have to be reserved. In contrast to these three investigations, [11] studies a queuing analysis framework for static and channel-dependent OFDMA resource assignments based on weighted fair queuing. While the provided queuing-theoretic framework is quite deep in general, the

authors do not determine analytical expressions for the instantaneous terminal rate obtained from channel-dependent OFDMA resource assignments. In order to still perform a queuing-theoretic analysis, they obtain average terminal rates (under channel-dependent OFDMA resource assignments) from simulations. While for the specific considered case this serves very well, it is certainly not practical to simulate each and every possible situation in a wireless OFDMA cell that might have impact on the average terminal rates. Furthermore, no outage based admission control can be performed from this approach which is another significant disadvantage.

Finally, we comment on some application constraints for this approach. Primarily, the question arrises how complex the approach is to apply it in real time. Note that the real time constraints are related to admission control, which is less time critical (than for example packet delivery of admitted flows). We have implemented the scheme in C and execution time is in the range of milliseconds (depending on the parameterization of the wireless cell) which is acceptable for admission control. One further crucial assumption in the framework is the model of the fading channel. The exponentially distributed fading gain model (i.e. Rayleigh fading) is one of the most "'pessimistic'" models matching measured data. Often, real channels behave better which essentially makes the system capacity better and does not harm the lower bound. Extending the approach to other models can result in severe analytical difficulties in the calculcation of the distribution functions of the "'transformed'" channel gains in Equation 10. If these derivations can be done, the framework can also be applied to a mixture of different fading models. However, in practise it is quite difficult to identify appropriate fading models of the currently active terminals.

7   Conclusions

In this paper we have derived an analytical framework for performance prediction of channel-dependent OFDMA resource assignment algorithms. The framework is based on closed-form expressions for the impact of such algorithms on the stochastic characteristics of subcarrier gains. This core is then applied to find a lower performance bound for the well-known rate-adaptive optimization problem in OFDMA systems. We demonstrate the utility of the bound in the context of admission control for Voice-over IP streams where the analytical approach allows to predict down-link OFDMA performance with respect to different quality-of-service constraints. While the bound is not a precise prediction of the optimal performance (which is very difficult due to the NP-hard nature of the optimization problem), it is a significant improvement of state-of-the-art admission control schemes (which neglect the perfor-

mance improvement due to exploiting multi-user diversity in OFDMA systems).

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