Optimal Power Masking in Soft Frequency Reuse based OFDMA Networks

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***Abstract*—Soft frequency reuse is a strong tool for co-channel interference mitigation in cellular OFDMA/LTE networks. The performance of such networks significantly depends on the configuration of the power masks that implement the soft frequency reuse patterns. In this paper, we investigate the performance of different power mask configurations against the optimal case, in which a central entity optimally distributes power and resource blocks among the users of the network. It is shown that large differences exist between the performance of different mask types and the optimal case in both, the overall cell throughput, as well as the cell-edge user performance.**[1]

***Index Terms*—LTE, OFDMA, soft frequency reuse, power masks, optimization**

I. Introduction

The Long Term Evolution (LTE) mobile broadband system [1] uses *Orthogonal Frequency Division Multiple Access* (OFDMA) as combined transmission and multiple access technique in the downlink. With OFDMA, the system bandwidth is split into a number of sub-carriers, each featuring a bandwidth smaller than the system's coherence bandwidth, on which data of different users is transmitted in parallel. While the sub-carrier thinness and the resulting large OFDM symbol time reduces the effect of inter-symbol interference (ISI), the orthogonality among them mitigates inter-carrier interference (ICI). By using appropriate cyclic prefixes, ICI and ISI can almost completely be avoided. OFDMA therefore is a promising technique for use in various systems and scenarios.

When applied to mobile cellular systems, a key issue with OFDMA is co-channel interference (CCI): Especially terminals located at the cell border largely suffer from the power radiated by the base station of neighboring cells in their communication band. There are three major alternatives for mitigating CCI in cellular OFDMA systems: *hard frequency reuse* (HFR), *fractional frequency reuse* (FFR), and *soft frequency reuse* (SFR).

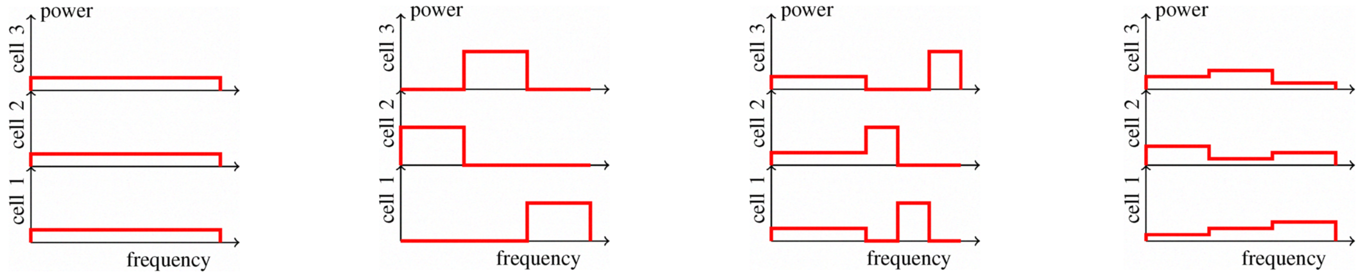
[1]This work has been supported by the German Ministry of Education and Science (BMBF) and Ericsson Research, Germany, in the context of the project ScaleNet.

Hard frequency reuse splits the system bandwidth into a number of distinct sub-bands according to a chosen reuse factor and lets neighboring cells transmit on different sub-bands. Fractional frequency reuse and soft frequency reuse both apply a frequency reuse factor of one to terminals located in the cell's center. For terminals closer to the cell edge, however, a frequency reuse factor greater than one applies. Fractional frequency reuse [2] splits the given bandwidth into an inner and an outer part. The inner part is completely reused by all base stations; the outer part is divided among the base stations with a frequency reuse factor greater than one. With soft frequency reuse [3], the overall bandwidth is shared by all base stations (*i. e.,* a reuse factor of one is applied), but for the transmission on each sub-carrier the base stations are restricted to a certain power bound.

All these approaches to mitigating CCI can be described in terms of cell-specific *power masks* over the system bandwidth. A power mask prescribes the fraction of the maximum transmit power that the base station may use depending at the part of the spectrum. In Fig. 1 we assume a scenario of three neighboring cells. In the case of hard frequency reuse with a reuse factor of three (Fig. 1b), the power masks block all but one third of the spectrum. In our example for fractional frequency reuse (Fig. 1c), the power mask for the first half of the spectrum is uniform, and the second half corresponds to a condensed version of the hard reuse case. Fig. 1d illustrates the soft frequency reuse case.

The power masks have a significant impact on the system's performance. Previous work [4] shows that soft frequency reuse has a capacity advantage over the plain hard reuse. Furthermore, adapting the power mask used for soft frequency reuse to the current traffic situation has recently been shown to be a great lever on capacity [5]. However, the question remains, how close this and similar adaptive schemes get to optimality.

In this paper, we present a means to evaluate power mask performance in cellular OFDMA systems. Our central contribution is the formulation of a global knowledge exploiting



(a) Uniform (b) Hard reuse 3 (c) Fractional reuse 3 (d) Soft reuse 3

Fig. 1. Different power masks

non-linear optimization problem and the solving of several according problem instances in a basic reference scenario. We compare some existing power mask configurations to the ideal results, in order to show how our model serves as a basis for the performance evaluation of more sophisticated static as well as adaptive power mask configurations. To the best of our knowledge, this is the first work to present results of the non-linear resource scheduling optimization problem related to cellular OFDMA networks.

The remainder of this paper is organized as follows. In the following section, we describe resource scheduling in LTE systems and introduce our scheduling goal. Then, in Sec. III we introduce our scheduling optimization model for the local (Sec. III-A), as well as the global optimization case (Sec. III-B). In Sec. IV, we present our reference scenario and the according reference results. We conclude our work and identify topics for further study in Sec. V.

II. System Model

In LTE, time is slotted into transmission time intervals (TTI) [1] of duration T_{TTI} (in the order of milliseconds). During a single TTI, down-link user data multiplexing is done in frequency division multiplexing (FDM) fashion, where the smallest addressable bandwidth-unit is a *resource block*. Following the localized mapping scheme, a resource block consists of adjacent sub-carriers in the frequency domain. In the time domain, a resource block spans all OFDM symbols available for user data transmission of the respective TTI. Each resource block is expected to experience mostly flat fading throughout a single TTI.

A. The scheduler

For each TTI and in each cell, a base station scheduler assigns the resource blocks to the served terminals. LTE uses adaptive coding and modulation (ACM) per resource block, so the scheduler determines also the modulation type and coding, based on available channel state information (CSI).

In this work, however, we use the theoretical Shannon capacity of a channel [6] instead of referring to the coding and modulation type combinations actually considered for LTE. Our method is nonetheless also applicable to realistic coding schemes. Shannon's theorem states that for a channel with bandwidth B and a given signal-to-interference-and-noise ratio (SINR), there exists a code that achieves a throughput of

$$\text{THR} = B \cdot \log_2(1 + \text{SINR})\,. \tag{1}$$

We assume that the transmit power is prescribed for each resource block by a power mask. For cell i, we will denote the power mask by $p_{i,r}^{\text{mask}} \in [0, 1]$. This value denotes the fraction of the total available output power $p^{(\text{MAX})}$. On resource block r, cell i thus transmits with a power of $p^{(\text{MAX})} \cdot p_{i,r}^{\text{mask}}$.

We denote the channel gain in TTI t by $\gamma_{i,m,r}^{(t)}$ for user m, base station i, and resource block r, and calculate the current SINR as

$$\text{SINR}_{i,m,r}^{(t)} = \frac{p^{(\text{MAX})} \cdot p_{i,r}^{\text{mask}} \cdot \gamma_{i,m,r}^{(t)}}{\sum_{j \neq i} p^{(\text{MAX})} \cdot p_{j,r}^{\text{mask}} \cdot \gamma_{j,m,r}^{(t)} + \eta_r}\,. \tag{2}$$

The above denominator sums up the co-channel interference from concurrently transmitting base stations $j \neq i$ and the noise power η_r. Note that in a real system, the TTIs would not be synchronized among cells, and the terminal would simply measure the current SINR.

III. Scheduler Optimization

Scheduling commonly aims at maximizing system throughput, but fairness has to be taken into account, too. Solely maximizing the raw system throughput can lead to starvation of users at the cell edge and oversupply of bandwidth to users that are easy to serve. Different kinds of fairness constraints circumvent this: guaranteeing each user a certain minimum rate [7], multiplying each user's throughput by an individual proportional fair factor [8], or utility-based per-user throughput optimization [9]. Utility-based optimization is fairest, but it is highly complex.

A. Local optimization model

Discussing different scheduling and fairness policies is beyond the scope of this paper. We use a simple fairness model to compare different soft frequency reuse scenarios, but our approach easily adapts to other fairness notions. We assume that for each user there is a maximum throughput

$\text{THR}^{(\text{MAX})}$, and that any throughput beyond $\text{THR}^{(\text{MAX})}$ is useless. In other words, we try to maximize the system throughput while assuring that none of the users gets more than a certain maximum rate. This approach corresponds to a simple piecewise linear utility function.

Formally, we can write our scheduling goal as an optimization model by introducing the binary user/resource block assignment variable $x^{(t)}_{m,r}$, which is 1 if user m obtains resource block r, and 0 otherwise [10]. The sets of all active users and of all resource blocks are denoted by $\mathcal{M}$ and $\mathcal{R}$, respectively. The task of the scheduler then is described by the following integer linear program:

$$\max \quad \sum_m \sum_r x^{(t)}_{m,r} \cdot \hat{\text{THR}}^{(t)}_{m,r} \tag{3a}$$

$$\text{s.t.} \quad \sum_m x^{(t)}_{m,r} \leq 1 \qquad \forall\, r \in \mathcal{R} \tag{3b}$$

$$\sum_r x^{(t)}_{m,r} \cdot \hat{\text{THR}}^{(t)}_{m,r} \leq \text{THR}^{(\text{MAX})} \qquad \forall\, m \in \mathcal{M} \tag{3c}$$

The scheduling objective (3a) is to maximize the total expected throughput $\hat{\text{THR}}^{(t)}_{m,r}$ of all users m on all resource blocks r. Constraint (3b) ensures that each resource block is assigned to at most one user at a time (*i. e.,* m exclusively uses r at TTI time t). Constraint (3c) is the utility constraint that guarantees that user m does not get more than the maximum required throughput $\text{THR}^{(\text{MAX})}$.

The expected throughput $\hat{\text{THR}}^{(t)}_{m,r}$ of user m on block r depends on the expected SINR, which we will denote by $\hat{\text{SINR}}^{(t)}_{i,m,r}$. The expected SINR is derived from the latest SINR measurement. According to Eq. (1), the expected throughput is

$$\hat{\text{THR}}^{(t)}_{m,r} = \Delta f \log_2(1 + \hat{\text{SINR}}^{(t)}_{i,m,r}), \tag{4}$$

if m is located in cell j out of the set of active cells $\mathcal{J}$. Note that Δf is the resource block bandwidth.

B. Global optimization model

The global optimization model aims at scheduling the available system resources such that the system wide throughput is maximized. We assume that there is a central entity equipped with all information necessary to achieve this goal. The central entity not only is in charge of determining the optimal user/resource block assignments, but also of finding the optimal power levels for each resource block in each cell j out of the set of active cells $\mathcal{J}$. Note that in this case, the optimization problem is not subject to power masking, but to an overall maximum power $p^{(\text{MAX})}$ constraint that is allowed to be radiated by each base station (expressed in Constraint 5c). As a consequence, we modify our local resource scheduling optimization problem 3 to include the three-dimensional cell/user/resource block optimization variable $x^{(t)}_{j,m,r}$:

$$\max \quad \sum_j \sum_m \sum_r x^{(t)}_{j,m,r} \cdot \hat{\text{THR}}^{(t)}_{j,m,r} \tag{5a}$$

$$\text{s.t.} \quad \sum_m x^{(t)}_{j,m,r} \leq 1 \qquad \forall j, r \tag{5b}$$

$$\sum_r y^{(t)}_{j,r} \leq p^{(\text{MAX})} \qquad \forall j \tag{5c}$$

$$\sum_r x^{(t)}_{j,m,r} \cdot \hat{\text{THR}}^{(t)}_{j,m,r} \leq \text{THR}^{(\text{MAX})}_m \qquad \forall j, m \tag{5d}$$

Optimization variable $y^{(t)}_{j,r}$ is the power assignment variable. The expected user throughput is computed following Eq. 4. The expected SINR, however, is now computed as follows:

$$\hat{\text{SINR}}^{(t)}_{i,m,r} = \frac{y^{(t)}_{i,r} \cdot \gamma^{(t)}_{i,m,r}}{\sum_{j \neq i} y^{(t)}_{j,r} \cdot \gamma^{(t)}_{j,m,r} + \eta_r}. \tag{6}$$

Combining Eqs. 5 and 6, it can easily be seen, that the global resource scheduling problem is non-linear in nature. Moreover, since the user/resource block assignment variable is binary, whereas the resource block/power assignment is a real variable, the global resource optimization problem is a non-linear mixed integer problem, which is known to be extremely hard to solve.

Still, in the next section we present a basic reference scenario for which solving the global resource allocation problem is possible under common optimization problem solving soft- and hardware conditions.

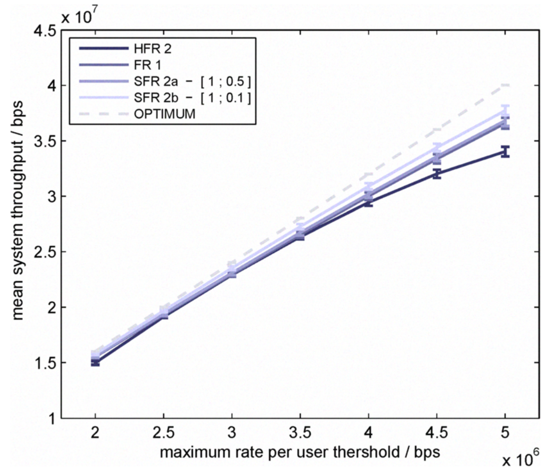
IV. Reference Scenario

A. Setup

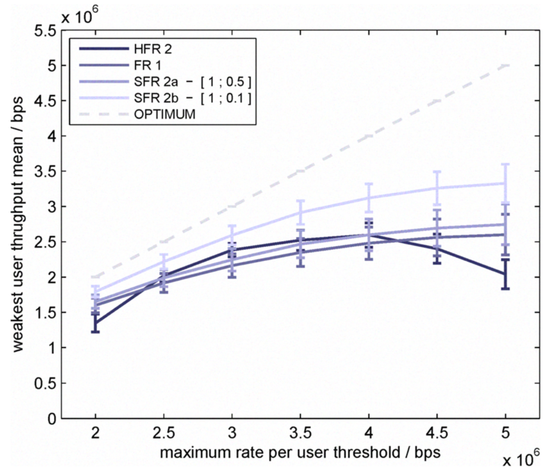
Our reference system and channel model parameterization (path loss, shadowing, and fading model) largely follows the parameters for the UTRA and EUTRA simulation case 1 as presented in Tables A.2.1.1-1 and A.2.1.1-2 of [4] with an inter-site distance of 500 m and users dropped uniformly in each hexagonal cell. Differing parameters are shown in Table I. We have considered two hexagonal cells. The reason for this is that for significantly larger scenarios, the global optimization problem is not solvable within reasonable time constraints using regular hard- and software equipment. Our LTE system-level simulator is based on the free timed discrete

Parameter	Symbol	Value
Cells in the system	$\|\mathcal{J}\|$	2
Users	$\|\mathcal{M}\|$	8
Resource blocks	$\|\mathcal{R}\|$	24
Resource block frequency spacing	Δf	180 kHz
Maximal transmission power per cell	$p_{\max}$	43 dBm

TABLE I
Simulation Parameters.



(a) Mean system throughput (bps), 99 % confidence intervals



(b) Mean throughput (bps) of weakest user, 99 % confidence intervals

Fig. 2. All users achieve their maximum required rate if the resources are distributed optimally. In the case of local knowledge based resource assignement optimization, case SFR02b, which features a power level difference between the one of SFR02a and the hard frequency reuse HFR2 scheme achieves the best performance, especially in the weakest user performance (b).

event simulation library OMNeT++ [11]. The instances of non-linear global optimization problem (5) have been solved using LINDO's LINGO non-linear optimization problem solver [12]. The local scheduling optimization problem instances (3) have been solved using ILOG's CPLEX linear problem solver [13].

We have simulated 100 different user distributions. In each run, the instances of the respective global and local scheduling optimization problem were solved for each cell. Thereby, we have compared the performance of the global optimal scheduling decision to the performance of the locally optimal scheduler featuring the following power masks:

Hard Frequency Reuse 2 (HFR2). Each cell may only use half of the total system bandwidth. An according power mask for hard frequency reuse 3 is depicted in Fig. 1b.

Uniform/Frequency Reuse 1 (FR1). The available power is distributed evenly across all resource blocks in all cells, *cf.* Fig. 1a.

Soft Reuse 2 (SFR2). There are two different power levels: high, and low. Each cell uses half of the spectrum with each power level, see Fig. 1d for the according soft reuse 3 mask. There are two versions of soft frequency masks: (a) SFR2a $[1; 0.5]$ and (b) SFR2b $[1; 0.1]$, which means that in the first case the the low power level equals half, and in the latter one tenth of the high power level.

B. Results

The results of our simulations are shown in Fig. 2. Moreover, in order to increase the readability of the results, the mean values are also presented in Tables II and III. Fig. 2a shows the mean system throughput averaged over all simulation runs for the different power mask schemes detailed above. Fig. 2b shows the average throughput obtained by the weakest user, *i. e.,* the individual user that received the least throughput in each of the 100 different user distribution scenarios. In most cases, this user is closest to the cell-edge (does occasionally not apply under certain fading conditions). In addition, the error bars display confidence intervals with a confidence level of 99 %.

	Maximum Rate per User Threshold / Mbps						
Approach	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
HFR2	14.97	19.12	22.89	26.32	29.43	32.00	34.01
FR1	15.50	19.29	22.95	26.52	30.00	33.35	36.57
SFR2a	15.56	19.37	23.07	26.71	30.19	33.57	36.81
SFR2b	15.76	19.66	23.50	27.25	30.89	34.37	37.71
Optimum	16.00	20.00	24.00	28.00	32.00	36.00	40.00

TABLE II
SIMULATION RESULTS: MEAN SYSTEM THROUGHPUT/MBPS.

	Maximum Rate per User Threshold / Mbps						
Approach	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
HFR2	1.346	2.018	2.382	2.521	2.595	2.401	2.039
FR1	1.597	1.916	2.162	2.349	2.479	2.560	2.600
SFR2a	1.651	1.988	2.245	2.466	2.597	2.693	2.745
SFR2b	1.792	2.217	2.591	2.913	3.120	3.259	3.327
Optimum	2.000	2.500	3.000	3.500	4.000	4.500	5.000

TABLE III
SIMULATION RESULTS: WEAKEST USER THROUGHPUT/MBPS.

Addressing first the mean system throughput in Fig. 2a, the results show that all power masks show close to optimal performance, if the maximum rate per user is around 2Mbps. With increasing maximum rate threshold, however, the differences in performance become clear. As expected, the hard frequency reuse scheme achieves the worst performance values. This is mainly due to the fact that its bandwidth is limited and its interference advantage does not pay off in the higher max required rate cases, which favor the users closer to the base station (that are less susceptible against interference). Interestingly, none of the soft frequency reuse cases SFR2a and SFR2b perform significantly better than the equal power level frequency reuse 1 (FR1) case, when it comes to the mean system throughput.

In terms of weakest user performance, however, significant differences become visible when looking at Fig. 2b. Here, SFR2a achieves an increase of app. 5 % compared to FR1, whereas SFR2b even has an app. 25-30 % gain over FR1. This is an immense gain, considering the fact that the gain solely stems from masking the resource block power levels.

Another interesting effect shows the weakest user throughput curve of the hard frequency reuse case HFR2. Up to a maximum required rate of 4Mbps, the performance of the cell edge users is better than in the frequency reuse 1 and soft frequency reuse SFR2a case. This advantage traces back to the fact that there is zero interference from the neighbor cell, and, thus, the channel states of the cell edge users are generally better than in the frequency reuse 1 or the soft frequency reuse case. Due to the limited resources in the hard frequency reuse HFR2 case, however, the weakest users cannot take advantage of the increasing maximum rate above that 4Mbps threshold. This is mainly because the cell edge users hardly get any resources at all, if the stronger users are allowed to consume resources for such high rates. Accordingly, their mean throughput decreases with the increasing max rate after that turning point. This is very likely to happen to the other schemes as well at different points on the max rate threshold axis.

In general, none of the locally optimized schemes gets close to the optimum in the higher maximum required rate range. Note that all users achieve the maximum required rate in the considered range, if the resources are distributed optimally. Even though local optimization strategies are very unlikely to get to a performance similar to the global optimum, there is much space for improvements. Using our reference model, promising candidates can be judged with respect to global optimality.

V. Conclusion

We have presented a means to evaluate power mask performance in cellular OFDMA systems. It is based on solving a global knowledge exploiting non-linear optimization problem.

We have solved several according problem instances in a basic OFDMA/LTE reference scenario. Thereby, we have shown that there is a significant gap in performance between the application of simple static power masks in combination with a locally optimal scheduler and the global optimum. This is true especially for the cell-edge user performance.

According future works include, thus, the development of more sophisticated static power masks, as well as schemes for power mask adaptation, and a comparison of their performance to the optimal case indicated in this paper. Consequently, solving the presented non-linear optimization model in a larger reference model is also of major interest.

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