Abstract

In some recent economic growth models there can be decreasing pollution along with increasing per capita income, if the rate of improvement in the environmental technology is sufficiently high. A central function describes how gross pollution and environmental technology interact to determine net pollution, which in the previous works has a log-linear form. This note provides an example in which this function is generalized to a CES type. The result is that the environmental technology factor in the long run may be either implausibly potent or almost ineffective in transforming a high gross pollution to a low net pollution if the function deviates from the log-linear case.

Keywords: Pollution and growth, Directed technological change.

JEL classification: O30, Q55.

1 Introduction

In some recent models of economic growth pollution is counteracted by environmental technical development (e.g. Brock and Taylor (2004) and Hart (2004)\textsuperscript{1}). The central result is that there can be sustainable development, here meaning decreasing pollution along with increasing per capita income, if the rate of progress in the development of environmental technology is sufficiently high.

\textsuperscript{*}I gratefully acknowledge comments from Andr\‘e Grimaud and seminar participants at SLU.

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\textsuperscript{1}Technical change is exogenous in the former, while it is endogenous in the latter.

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An important assumption in these papers is that the function describing how gross pollution and environmental technology interact to determine net pollution has a log-linear form. This note provides an example in which this function is generalized to a CES type. The result is that the environmental technology factor in the long run may be either implausibly potent or almost ineffective in transforming a high gross pollution to a low net pollution when the function deviates from the log-linear case.

The model presented in this note thus highlights the importance of making a distinction between the rates of growth of various technology factors, on the one hand, and their power to influence the outcome of the model, on the other. For an interesting, similar discussion of the resource-and-growth literature, see Bretschger (2005).

2 Growth Model

Consider an economy with the constant labor force $L$. Denote by $L_Y$ the quantity of labor used in ordinary production and let $Y = F[K, AL_Y]$ be a production function with the standard neoclassical properties listed in Barro and Sala-i-Martin (2004). In this function, $K$ signifies capital and $A$ is a technology factor. The environmental technology factor is denoted by $B$. The quantities of labor used in research to enhance $A$ and $B$ are $L_A$ and $L_B$, respectively, so the labor constraint is $L = L_Y + L_A + L_B$.

To focus on the feasibility of sustainable development, intertemporal optimization is here replaced by rule-of-thumb allocation. Thus, $s$ is the constant saving rate and capital accumulation is consequently $\dot{K} = sF[K, AL_Y] - \gamma K$, where $\gamma$ is the rate of capital depreciation. Similarly, the use of labor in the two types of research is $L_A = s_A L$ and $L_B = s_B L$ (where $s_A$ and $s_B$ are constant shares), implying that $L_Y = (1 - s_A - s_B) L$.

We assume that the growth rates of the technology factors are

$$g_A = \delta_A s_A L \quad \text{and} \quad g_B = \delta_B s_B L,$$

where we use $g_Z$ to denote the proportional rate of change of variable $Z$, for instance $g_A \equiv \dot{A}/A$. The constants $\delta_A$ and $\delta_B$ capture the productivities in the two types of research.

By standard arguments, the dynamics of the growth model are such that total output in the long run is

$$Y = AL_Y f(\tilde{k}^*) ,$$

where $\tilde{k} \equiv K/(AL_Y)$ and the ‘*’ signifies the steady-state value. It follows that the growth rate of output in the long run will be equal to the rate of change of $A$, i.e. $g_Y = g_A$. 


3 Pollution

The total quantity of net pollution, \( X \), is an increasing function of gross pollution, which we assume to be proportional to \( Y \) (here with the constant of proportion normalized to unity). However, it is also possible to mitigate the polluting consequences of the output by using environmental technology. This can be represented by the function \( X(t) = G(Y(t), B(t)) \), where \( G_Y > 0 \) and \( G_B < 0 \). The first argument can be understood as a scale effect, while the second is a technology effect. A ‘greening’ of technology is thus captured by an increase in \( B \).

To explore the relations between the variables in more detail, it is useful to introduce a specific functional form, and we consider here the CES type

\[
X = \left[a Y^\rho + (1-a) B^{-\rho}\right]^{\mu/\rho},
\]

where \( 0 < a < 1 \), \( -\infty < \rho < \infty \) and \( \mu > 0 \). The long-run change in net pollution is

\[
g_X = \frac{\mu a}{a + (1-a)(BY)^{-\rho}} g_A - \frac{\mu(1-a)}{a(YB)^\rho + (1-a) g_B}.
\]

The time path of pollution will differ significantly, depending on the value of \( \rho \). We first analyze the implications of this model in the Cobb-Douglas case and then come back to the CES version.

3.1 Cobb-Douglas

When \( \rho = 0 \), the expression for the change in pollution in (3) boils down to a weighted average of growth rates where the weights are constant:

\[
g_X = \mu a g_A - \mu(1-a) g_B.
\]

It is obvious that the rate of green technology factor growth, compared to \( A \), is not the only factor that matters. The change in pollution also depends on the ability of \( B \) in reducing pollution, which in this case is captured by the constants \( \mu \) and \( 1-a \). Recalling (1), the condition for sustainable development is \( s_B > \frac{a}{1-a} \frac{\delta_A}{\delta_B} s_A \), which occurs if there are sufficiently good incentives to conduct environmental research. The higher \( a \) is, the larger \( s_B \) must be.

There is however a special case in which only the relation between the growth rates matters for the direction of pollution change, namely when \( a = 1/2 \). Then we have that \( g_X = \mu/2 [g_A - g_B] \). Obviously, \( g_X < 0 \) is then feasible if and only if \( g_A < g_B \).²

²Essentially, Brock and Taylor (2004) and Hart (2004) study the case when \( a = 1/2 \) and \( \mu = 2 \), implying that \( X = Y/B \).
3.2 CES

Returning to the CES case, we first note that the product $YB$ approaches infinity with time, because both these variables are growing. It is therefore important whether $\rho$ is positive or negative, because this greatly influences the values of the weights in front of the growth rates in (3) in the long run.

When $\rho > 0$ we have the very gloomy implication that the factor in front of $g_B$ in (3) falls toward zero over time, while the other weight tends to $\mu$. Consequently, we have that

$$g_X \to \mu g_A \text{ as } t \to \infty.$$ 

Irrespective of how fast the green technology develops, it will not be potent enough to prevent the growth of the scale from dominating the determination of long-run pollution. An increasing net pollution is then unavoidable as $Y$ grows.

On the other hand, the prospects for the economy are very bright if $\rho < 0$. The factor in front of $g_A$ in (3) falls toward zero over time, while the weight of the green technology tends to $-\mu$. That is,

$$g_X \to -\mu g_B \text{ as } t \to \infty.$$ 

The scale effect is now easily counteracted by the technique effect. No matter how fast gross pollution grows, a tiny improvement in environmental technology will guarantee declining net pollution.

To understand these results, we use (2) to examine the iso-$X$ curve in $B - Y$ space. This curve is positively sloping and the second-order derivative, in elasticity terms, is\(^3\)

$$\frac{d^2 Y}{dB^2} \cdot B \cdot \left(\frac{dY}{dB}\right)^{-1} = (1 - \rho) \frac{1}{(YB)\rho} \frac{1 - a}{a} - (\rho + 1).$$

For $\rho > 0$ the elasticity approaches $-(\rho + 1)$ in finite time; a constant $X$ requires that the slope decreases more in percent than $B$ increases. When $Y$ increases, relatively more of $B$ is needed to keep $X$ constant; as the scale of the economy grows, it becomes more and more difficult to counteract pollution.

In contrast, the slope of the isocline is rapidly increasing and tending to infinity when $\rho < 0$, implying that the ratio of $B/Y$, necessary to keep $X$ constant, decreases as the scale grows.\(^4\)

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\(^3\)The function in (2) is not homothetic in $Y$ and $B$. Therefore, it is not straightforward to apply a concept like the elasticity of substitution.

\(^4\)If $\rho = 0$, the elasticity boils down to $(1 - 2a)/a$. The iso-$X$ curve is a straight line if $a = 1/2$; it is neither more nor less difficult to counteract pollution as the scale increases. If $a > 1/2$ ($a < 1/2$) the slope decreases (increases) as the scale grows.
4 Conclusion

The central message from the literature that assumes a log-linear net pollution function is that a sustainable development will emerge if investments in new environmental technology are sufficiently extensive, compared to investments in ordinary technical progress. This is hopefully true, but the $G$ function is not sufficiently explored empirically, and in particular the matter of the value of $\rho$ is not resolved. Therefore this paper examines the consequence of the slightest deviation from the underlying assumption, in the previous works that, $\rho = 0$.

The example with $\rho < 0$ is rather implausible; it is not realistic that the environmental problem should rapidly become easier to solve when the scale of the economic activity expands.

While the case with $\rho > 0$ is hopefully unnecessarily pessimistic, it cannot be ruled out entirely. The example thus suggests some caution in assuming that the environmental technology effect always can be made strong enough to guarantee sustainable development. It would thus be wise to keep the door open for other functions, for which the log-linear case may be an asymptote or envelope.\textsuperscript{5} Alternatively, the relation could be CES in the short run, but Cobb-Douglas in the long run. This could imply a Cobb-Douglas relation if $Y$ and $B$ grow moderately fast, but a CES relation if they grow very fast.

References


\textsuperscript{5}Confer for example Jones (2003), where the ordinary production function converges only asymptotically to a Cobb-Douglas.