FINNISH MATHEMATICS TEACHING: A CASE OF UNIQUELY IMPLICIT DIDACTICS

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This paper reports on a qualitative analysis of video-taped mathematics lessons taught by four case study teachers, defined locally as effective, in a provincial university city in Finland. The aim was to examine how teachers conceptualise and present mathematics to their learners and, in so doing, understand the relationship between Finnish mathematics teaching practices, as reflected in case study lessons, and Finnish success on successive PISA assessments. Analysed by means of the process of constant comparison, the data yielded two key characteristics of case study classrooms. Firstly, irrespective of their intended learning outcome, teachers exploited a series of implicit didactic strategies focused on encouraging students to infer meaning. Secondly, three culturally located activities were identified that appeared complementary to this sense of the implicit. These were the systemic encouragement of students to make notes, teachers' exploitation of the confident child and the assumed collaboration of parents.

INTRODUCTION

This paper reports on a qualitative analysis of video-taped mathematics lessons taught by four case study teachers in a provincial university city in Finland. The aim was to examine how these teachers conceptualised and presented mathematics to their learners and, in so doing, understand how they constructed those opportunities for learning contributory to that country's success on successive iterations of the Organisation for Economic Cooperation and Development's (OECD) Programme for International Student Achievement (PISA) (OECD, 2001, 2004, 2007, 2010). A key aspect of the study is that it was undertaken by an outsider looking in; a process necessitating particular methodological and ethical considerations.

Finnish mathematical success

The primary foci of the four cycles of the Programme for International Student Assessment (PISA) have been literacy, mathematical literacy, scientific literacy and literacy respectively. However, each assessment has included, irrespective of the main focus, substantial assessments of students' application of mathematical knowledge and skills to authentic settings, both within mathematics itself and a wider world (Adams 2003). Over these four cycles Finnish students appear not only to have performed consistently higher than those of any other European country but also comparably to those of the highest achieving nations of the Pacific Rim. This is, of course, not quite true as the figures published in the various PISA reports have not distinguished between the autonomous regions of Belgium. Taken as a whole Belgium was among the higher performing European states but, when viewed separately, the Flemish community, which is similar in size to that of Finland, performed at least as well as Finland, and often better, across all four cycles (De Meyer et al., 2002, 2005; De Meyer 2008; De Meyer and Warlop, 2010).

That said, the headline success of Finnish students has been a catalyst for such international interest in its educational processes that over “the past few years, more than 100 delegations from all over the world have beaten their way to the doors of the ministry of education in Helsinki to find out how the Finnish system works” (Crace, 2003). A number of PISA success-related explanations have been proposed by internal commentators. In general, it has been attributed to the quality of the comprehensive school system, high expectations of all participants, a well qualified and committed teaching force that enjoys high status, if not high pay, and the trust of society in general and parents in particular (Välijärvi et al., 2002; Välijärvi, 2004; Tuovinen, 2008). In particular it has been ascribed to the high quality of mathematics teacher education, competition for teacher education places and the esteem in which teaching continues to be held (Malaty, 2007) and high systemic investment in the language acquisition and competence of special educational needs students (Kivirauma and Kari, 2007).

Thus, internal commentators have indicated that Finnish PISA success may be a consequence of various cultural factors, although little evidence has been forthcoming recently with regard to the functioning of Finnish mathematics classrooms in general and the role of participants in particular (Pehkonen et al., 2007). Admittedly, research undertaken a quarter of a century ago highlighted a tradition of teacher-dominated practice that had not only changed little in fifty years but created an intelligence and emotional wasteland (Carlgren et al., 2006). More recently, but preceding Finland’s PISA successes, Norris et al. (1996, p.29) found “rows and rows of children all doing the same thing in the same way whether it be art, mathematics or geography”, adding that they had “moved from school to school and seen almost identical lessons, you could have swapped the teachers over and the children would never have noticed the difference”. Thus, while Finnish classrooms appear to have been historically dominated by traditional approaches to teaching and learning, little is known about the ways in which participants enact their roles in the modern, PISA-driven world. Thus, it would seem timely to suggest that an examination of the practices of Finnish teachers of mathematics and the role they play in the construction of PISA success would be of benefit to policy-makers, educational researchers and curriculum developers alike.

METHODS

The data on which this paper is based derive from an EU-funded study of mathematics teaching in Belgium (Flanders), England, Finland, Hungary and Spain. The main data set comprised video recordings of four sequences of lessons taught in each country on agreed topics by teachers defined locally, in the manner of the learner’s perspective study (Clarke 2006) as effective. Each sequence comprised five lessons. Videographers were instructed to focus on the teachers whenever they were speaking. Teachers wore radio-microphones while a static microphone was placed strategically to capture as much student talk as possible. However, since attention was on how teachers structured opportunities for learning the integrity of data collection was not greatly compromised through lost student talk. After filming, each tape was compressed and transferred to CD for coding, copying and distribution. The first two lessons of each sequence were transcribed and translated into English and subtitled recordings made to facilitate analysis by all project teams. Each of the remaining lessons in each sequence was accompanied by a narrative written by the home team to inform a foreigner’s viewing. The analysis presented in this paper draws predominately on the eight subtitled videos although the remaining twelve were consulted frequently.

Qualitative analysis of such data is not a straightforward process. One approach, particularly in respect of its facilitating our understanding of how “actors respond to changing conditions and the consequences of their actions” (Corbin and Strauss, 1990, p.5), is the constant comparison process exploited by grounded theorists. Firstly, all videos, with and without subtitles, were viewed several times in order to get a *feel* for how participants constructed the playing out of their lessons. Secondly, the first video in the sequence on linear equations, for no other reason than it was the first alphabetically, was viewed again several times to identify categories of teacher activity. With each new category the video was viewed again to determine whether or not the category had been missed in the earlier sections. Once the first video had been completed the second in the sequence was subjected to the same process. However, any new category to emerge from the second video prompted a return to the first to examine whether it, too, had been missed in the earlier viewings. In this manner a set of teacher behaviours emerged on which this paper is based and which, it is suggested, represent a unique Finnish mathematics didactic tradition.

Inevitably, interpreting alien educational systems creates particular ethical issues, not least because cultural insiders, as the taxpaying funders of a system’s policies, have critical rights denied the outsider who is, essentially, a guest in the classrooms of that country. Moreover, when researching contexts culturally different from their own it is important to understand that incomplete understanding of the historical, social and political agendas underpinning participants' beliefs and actions may skew interpretations of events (Liamputtong, 2010). However, a significant methodological strength of an outsider is that he or she exploits a different interpretive lens from the insider. This is because cultures so “shape the classroom processes and teaching practices within countries” (Knipping, 2003, p.282) that teachers' practices are “deep in the background of the schooling process ... so taken-for-granted… as to be beneath mention” (Hufton and Elliott, 2000, p.117). In this paper, therefore, we adopt the position that an outsider will see things hidden to the insider but that data interpretation should show commensurability with Tillman’s (2006) conception of culturally sensitive research in its subordination of dominant theoretical perspectives to the voice of the cultural group under scrutiny.

RESULTS

The four Finnish teachers, here given pseudonyms, taught in different comprehensive schools in a provincial university city with a large teacher education department. All four, three male and one female, were involved in teacher education activities as part of the University's programme and, therefore, located in the norms and values of mathematics education within their respective community. The males, Sami, Antti and Janne were all in their early thirties, while the female, Katja, was in her mid-fifties. Sami taught his sequence of lessons on percentages to a grade 6 class, Antti taught linear equations to a grade 8 class, Janne taught polygons to a grade 6 class while Katja taught geometry to a grade 8 class.

The analysis yielded a variety of categories of teacher actions or behaviours that were, essentially, located within the teaching of the concepts of mathematics and the procedures of mathematics. Therefore, the results are presented against these two broad categorisations that tend to typify mathematics teaching internationally. Interestingly, little evidence was seen of teachers presenting non-routine or authentic problems other than as the vehicles for teacher-managed whole class introductions to new procedures or concepts.

Conceptual knowledge

The development of students' conceptual knowledge was typically manifested in episodes of exposition, various forms of questioning and whole-class reflections on conceptually-focused tasks. In the following, examples from each of the case study teachers are presented before being discussed briefly. By way of explanation, any reference to either a teacher or a student writing should be taken as meaning that they were writing on the chalk board.

Antti and linear equations

Early in his first lesson on linear equations Antti wrote, *Definition: An equation is two expressions denoted as being of equal magnitude*. He wrote in capitals, taking almost two minutes to complete the definition while his students copied in silence. The definition was given with no comment. Next, six sentences were written on the board and each examined publicly to determine whether or not it was an equation. The third, x = 3, typically, went as follows.

Antti:	What about x equals three? Olli?
Olli:	Yes
Antti:	Yes, there's a sign of equality and expressions exist on both sides.

Following the operationalisation exercise Antti wrote, *Equation solving means finding out when the equation is true or false*". He said,

Antti:	The solution means that we think about when an equation is true... The other possibility is that it might never be true. So then it would be false... So it is settling between those two possibilities, which one it is.

Later, during another operationalisation exercise, also undertaken publicly, the following transpired.

Antti:	What should x be to make x squared equals eight true; x to the second equals eight? What would fit in x's place? Does anyone have any good suggestions? Olli, can you give us something? What would fit in x's place? When you square it, it equals eight.
Antti:	Joonas?
Joonas:	The square root of eight.
Antti:	Yes. If x is the square root of eight, this is true. So, this equation is a conditional equation. This equation is true, when x is the square root of 8.

The above exchanges seemed focused on two key concepts - the nature of an equation and a categorisation of equations as always true, conditionally true or false – introduced by means of two strategies. The first involved definitions written on the board and the second involved opportunities for students to operationalise those definitions. The definitions, which were written in capitals, were presented slowly and without comment. In this way students had time not only to copy but also to read and interpret the text. These were then operationalised through publicly discussed exercises that allowed Antti's desired categorisations to emerge.

With respect to interpretation, the written definitions, particularly in their drawing on mathematically appropriate forms of words, could be construed as explicit. However, Antti neither sought nor offered clarification, seeming to expect his students to infer meaning. Such practices may allude to something implicit to which we return below. The operationalisation exercises could also be viewed as attempts to make explicit his definitions and yet the manner in which they were managed also alluded to a form of implicit presentation. For example, when Olli correctly asserted that x=3 was an equation, Antti's reaction was to repeat the definition of an equation independently of its relation to the equation itself. He neither discussed the nature of the expressions involved nor elicited his students' understanding as to both x and 3 constituting mathematical expressions. In similar vein, when discussing Joonas' solution of $x^2 = 8$, the relationship between the square root of eight and the value of x that would make the equation true, and the role of that relationship in defining a conditional equation, was left for students to infer. Thus, despite several superficially explicit elements, Antti's presentation and operationalisation of his definitions were implicitly managed with students being left to infer meaning.

Sami and percentages

In starting his first lesson on percentages, Sami asked, *what do we mean by the word percent?* Aku suggested that it meant one hundredth, which Sami accepted. Sami informed his students that *there are three ways to denote one hundredth* and invited them to write them in their notebooks. This they did in silence with Sami then suggesting that it should be in equation form. Salla, invited to the board, wrote without hesitation, 1/100 = 1% = 0,01. Later, Sami set an exercise whereby students constructed equations of the same form from 100 squares with various proportions shaded. Several were discussed collectively, as in this typical example:

Sami:	What about the green shading? Anna?
Anna:	Thirty hundredths.
Sami:	Yes. As a percentage, Aku?
Aku:	Thirty.
Sami:	As a decimal number, Jussi?
Jussi:	Zero point thirty.

Both the above exchanges, typical of longer sequences of interaction, appeared conceptually focused on the equivalence of different percentage-related representations. It could be argued, also, by means of Aku's assertion that one percent meant one hundredth, that Sami had provided his students with an implicit definition. However, this implicit definition was operationalised by means of instrumentally presented procedures linking the various percentage-related representations. Thus, particularly as Sami had neither sought nor offered clarification, students appeared to have been left to infer meaning, through individual interpretation of the repeated tripartite equation, as to the meaning of these linkages. Thus, as with Antti and the teaching of equations, Sami's presentation and operationalisation of his definitions were implicitly managed.

Janne and polygons

During his first lesson on polygons, by means of an over-head projector, Janne progressively revealed rows of quadrilaterals with each row a categorical refinement of the previous. When he reached rectangles, having passed through rows of generic quadrilaterals, trapezia and parallelograms, he asked, *what properties do they have?* Katja replied, *all the angles are right angles*. Janne repeated Katja's words before asking, *how many angles have to be right angles in order to get such a figure?* Having received several different responses Janne invited his students to find out by drawing a rectangle. Eventually, after initiating a vote on the matter, he concluded that three right-angles would be sufficient. Next, having discussed other rectangular properties like equal and parallel sides, he uncovered a row of squares. A question revealed that *all the sides have the same length*, to which he commented, *yes, added to all the things we had earlier*.

In the above, Janne appeared conceptually focused on an increasingly refined categorisation of quadrilaterals. However, his exploitation of the popular vote and decision not to discuss the sufficiency of three left much for students to infer. Thus, although Janne presented a genuine problem for students to solve, a rare occurrence in case study lessons, the management of its solution alluded to an implicitly-derived concept. With respect to the categorisation of quadrilaterals, Janne's linking of the current level of quadrilateral to the previous, despite a superficial explicitness, was always implicitly managed. For example, when discussing the square, and despite making explicit the common length of all sides, Janne made no comment with respect to other properties other than to comment, *added to all the things we had earlier*.

Katja and geometry

During her second transcribed lesson, having invited her class to construct the mid-normal to a segment of their choice, Katja engaged her class in a discussion as to why such constructions yield a perpendicular to the segment. It went:

Katja:	How do we know that this is a right-angle? What is the reason behind our knowing? (Tracing the line of the mid-normal with her finger) What did we draw here? Simo?
Simo:	A mid-normal.
Katja:	A mid-normal. What kinds of properties does a mid-normal have? Leena?
Leena:	It is perpendicular to the segment at the midpoint.
Katja:	Yes. It is perpendicular to the segment. What other property? Can you tell it us again?
Leena:	Which one? Do you mean the one that it goes through the midpoint?
Katja:	Yes. So, it goes through...?
Leena:	The midpoint.
Katja:	Through the midpoint of this segment. Yes. We could get more properties to this segment, which is... perpendicular to the base and goes through the midpoint of the base.

Unlike the previous episodes, Katja's efforts seemed focused not only on revising the properties of a mid-normal but also engaging her students in mathematical reasoning. In this respect her questioning of Leena elicited the desired properties, which Katja repeated, wrote and students copied. Interestingly, despite its comprising more explicit conceptually-focused activity than almost any other observed episode, Katja neither sought nor offered any clarification as to the meaning of any of the terms used. Thus, even in such an explicit exchange, elements were left for implicit students to infer.

Procedural knowledge

The four Finnish case study teachers placed substantial emphasis on their students' procedural knowledge, which was also frequently developed by means of exposition, questioning or reflections on tasks.

Antti and linear equations

Early in his first lesson on equations Antti rehearsed answers to test questions, intended to assess students' calculator competence, posed the previous lesson. One example, written here as $\sqrt{(7+\sqrt{3}.\sqrt{12})}$, was managed as follows:

Antti: You can use the product rule here, 3 times 12 is 36 and the square root of 36 is 6... So, 7 plus 6 is 13 and the square root of 13 is 3.61

During his introduction to the main part of the second lesson, Antti wrote on the board $x/8 + 1 = 4$ and said,

Antti: Tell me, what x should be? We have an equation. There are two expressions that are denoted as being of equal magnitude. What should x be? (He waits) Mika?

Mika: Twenty-four.

Antti: Twenty-four. Let's try it. Twenty-four divided by eight is three. And add one to it. Yes.

During the test answer exchange Antti sought no input from his students. Despite the reference to the product rule, nothing was said about the rule itself or its application to the particular context. In general, it appeared that Antti's intention was on alerting students to the procedures they should employ. With regard to the second example, having reminded students of the definition introduced the previous lesson, Antti accepted Mika's response, but did not ask how the solution was obtained, before presenting a checking procedure. By way of interpretation, Antti's approach to the test question, in its alerting students to the product rule, could have been construed as an explicit instruction. In similar vein, in responding to Mika's equation answer, while electing not to explore Mika's solution process, Antti presented a checking procedure. However, in both cases, procedures were introduced with no explanatory narrative; Antti neither sought nor offered any clarification of any operation, leaving students to infer both the meaning and the manner of their implementation.

Sami and percentages

Midway through his first lesson on percentages, several text-based questions were solved individually before students were invited to share responses. Aku, having been invited to share his solution to the first, wrote, over three lines, 100% - (15% + 30%) = 100% - 45% = 55%, V: 55%. Aku's contribution went undiscussed. In similar vein, Salla wrote up the solution to the second. It took her a minute and a half while the class looked on in silence. She wrote, over several lines, 100% - (20%+12%+25%) = 100% - 57% = 43%, V: 43%. This process was repeated for all the questions in the exercise with no commentary from teacher or student.

During his second lesson on percentages Sami asked his students how they would write one sixth as a decimal. The following conversation transpired:

Sami:	Well, who can start the division? Sanni?
Sanni:	Six goes into one zero times.
Sami:	Then we put the point there (he inserts the decimal point into the calculation). Pirkka, you continue.
Pirkka:	For the sake of the point we have to add zero. Six goes into ten once (Sami calculates the remainder and writes the four into the calculation and brings down the zero to create forty)
Sami:	Then, Markus continues.
Markus:	Six goes into forty six times. Again the remainder is four (Sami adds six to the growing number on top and adds in the remainder four and so on).
Sami:	I asked for an accuracy of one hundredth. What is the answer? Aku?
Aku:	0,17

One construal of the first exchange is that the repeated pattern of correct solutions provided a model for those students who were, as yet, unsure as to the procedure they were expected to learn. The fact that Sami neither offered nor sought any additional comment supports this conjecture. It is also interesting to note that all the questions in this episode were word problems but no reference was made such contexts during the reporting of their solutions. Throughout the second exchange Sami's emphasis appeared to be on a revision of a particular procedure. Even those students who contributed did so in ways that reinforced the mechanical nature of the task; at no point did Sami offer or invite either commentary or justification. Thus, procedures, which incorporated veneers of explicitness, offered only implicit opportunities for those who did not follow to learn.

Janne and polygons

During Janne's five lessons on polygons, few episodes were observed in which the primary focus was on procedures. However, in the following can be seen two similar but typical examples. During his first transcribed lesson he invited his students to determine how many triangles could be found in a particular figure. After two or three minutes of independent seatwork Janne, having observed some disagreement as to the answer, asserted there were eight before asking, *how can we check if there are fewer or more than eight triangles?* He received little response and so asserted that naming them all would suffice. However, he appended his instruction with, *let's not check and just move on*. Later students spent some time on a similar problem before Janne asserted that there were seven triangles. In this case

Janne suggested that they should *not check them all. It would take too long. By naming them we could get them all*.

In both episodes Janne asserted an answer and alluded to the procedure by which students should be able to derive it. However, in both cases, he also asserted that implementing the procedure was a waste of time. Thus, students were left to infer what they could with regard to their being confident in the answer they obtained. In both cases, Janne offered procedures, derived implicitly, focused on the systematic identification of triangles that, for reasons unknown, he chose not to exploit in any observed lesson.

Katja and geometry

As with Janne, much of Katja's teaching was focused on the development of her students' conceptual rather than procedural knowledge. However, the following highlights well how her few procedurally focused episodes were managed. During her second transcribed lesson, having spent the first lesson constructing and discussing the properties of twin circles, Katja invited her class to draw an arbitrary segment AB and then construct a mid-normal. Katja, having constructed an arbitrary segment on the board quietly constructed her arcs with no comment. Unbidden, students did the same, some seemingly independently and others taking their lead from Katja's actions. On completion of her arcs Katja told her students to mark the upper intersection of the two arcs with a letter C. The episode continued:

Katja	Connect points A and C, and B and C... What do you get? Elisa?
Elisa	A triangle.
Katja	Yes, you get a triangle... What can you say about that triangle? What can you say about the lengths of triangle ABC's sides? Ilja?
Ilja	They have the same length.
Katja	All of them? Have you drawn all the sides of equal length? Simo?
Simo	A to C and B to C are the same. A to B is not.
Katja	OK. So AC's length is equal in magnitude to BC's length. Why? (Katja writes, $\lvert AC \rvert = \lvert BC \rvert$) Leena?
Leena	The distance to the centres is the same.
Katja	So, how has it been drawn? How did you get the point C? I don't want go on about it, but I want to have clear reasons. So I am still asking why? How did we get that point? Elisa?
Elisa	It is on the mid-normal
Katja	Yes, the mid-normal. How was it drawn?
Katja	Lisa-Maria?
Lisa-Maria	By drawing twin circles.
Katja	Yes, twin circles. And this proves that they have the same length, because they have the same radius.

During the episode, which was structured by six questions eliciting responses from five students, Katja's apparent objective was the procedure for constructing a mid-normal or perpendicular bisector to a segment and why it yielded the result it did. Indeed, her comment, *I don't want go on about it, but I want to have clear reasons*, seems to confirm that objective.

However, the means by which Katja managed her students’ responses alludes to something different. For example, Ilja's and Leena's contributions were received in ways that left observers uncertain as to whether they were acceptable. Even those responses that received a *yes* were followed by a question intended to refocus thinking in some way. Thus, despite an apparent explicitness, the episode comprised several implicit elements, not least of which being that no comment was offered with respect to justifying why the invocation of twin-circles *proves that they have the same length, because they have the same radius*. Furthermore, although the initial task could have been presented as an opportunity for students to engage with a non-routine problem, Katja’s modelling of the procedure offered students only limited opportunity to solve the problem for themselves. Katja's behaviour appeared to reflect an implicit differentiation according to students’ independence.

DISCUSSION

This study was undertaken in order to understand how Finnish teachers of mathematics facilitate their students' learning. It exploited a constant comparison analysis that precluded predetermined categorisations of mathematics didactics and facilitated an emergent understanding of culturally located patterns of behaviour. However, as the analysis proceeded, the question posed by Clarke and Xu (2008, p.965) became increasingly pertinent; “who is responsible for the public generation of mathematical knowledge in the classroom and how is this responsibility distributed between the teacher and the students?”

The following considers this question. First, however, the analysis indicated that all four teachers emphasised the development of their students' conceptual knowledge and procedural knowledge. In this regard there seemed little evidence of culturally-informed uniqueness. This is not surprising as recent studies have shown that when broad and inclusive categories of analysis have been exploited, mathematics teachers, irrespective of their location, appear to act similarly (Andrews, 2011; LeTendre et al., 2001; Van de Grift, 2007). Indeed, with respect to teachers' development of their students' conceptual knowledge and procedural knowledge, both of which could be described as broad variables, the data showed case study teachers behaving in similar ways to those of Flanders, England, Hungary and Spain (Andrews, 2009). Similarly, there was little surprise in the ways in which these outcomes were realised, with all four teachers employing exposition, various forms of whole class discussion and whole-class reflections on generative tasks. However, in this regard, it is interesting to note that Finnish teachers appear to continue, as they have for more than fifty years, to talk for the majority of the time, with pupils giving generally short responses (Carlgren et al., 2006). Thus, at very general and possibly superficial levels, not only do Finnish teachers appear to privilege outcomes similar to those of their colleagues elsewhere but they behave in ways similar to those who have gone before them.

Of course, case study research seeks to go beyond the identification and reporting of similarities and attempts to identify those characteristic and culturally located practices typical of the system under scrutiny. In this respect, the analyses indicate that few teacher-initiated public exchanges did not involve one or more act of implicit teaching and it is this sense of the implicit that seems to characterise the unique nature of Finnish mathematics teaching.

That is, irrespective of the focus of attention, whether students' conceptual knowledge or procedural knowledge, teachers' actions and utterances were consistently implicit in their facilitation of student learning. A particular manifestation is evidenced in teachers' management of publicly-posed questions. Even when an offering was accepted as correct, typically indicated by a *yes* on the part of the teacher, observers of the exchange were rarely offered insight into why the response had been accepted. Similarly, when an offering was deemed incorrect or irrelevant, teachers tended to ignore what was said. For example, during his first lesson on percentages, Sami asked students to give examples of one hundredth. For reasons known only to her, Liisa suggested *the circumference of the earth compared with the circumference of the sun*, a response that Sami ignored before turning to another student for his response, which turned out to be related to the proportion of fat in foods. In other words, the data suggest that case study teachers appeared to play no part in alerting students, whether active or passive in the exchange, to either the strengths or the weaknesses of students' offerings. In similar vein, teacher exposition was managed with few explicit concessions to student understanding. Whether it was Janne's management of the number of triangles problems, Sami's dealings with the percentages word problems or Antti's dealing with the answers to the earlier test, students were offered procedures with no explicit justification as to the warrant for either the procedure itself or the manner of its implementation. Indeed, an important component of this emergent sense of implicit didactics, as noted repeatedly above, lay in the frequently observed finding that case study teachers neither sought nor offered clarification with respect to public utterances, whether teacher or student.

However, the emergent sense of implicit didactics was complemented by three issues highlighted by the analyses. The first was that students were encouraged in both implicit and explicit ways, to make extensive notes. Implicitly, teachers wrote extensive notes on the board, as in Antti's definitions and categorisations of equations or Katja's construction of the mid-normal to a segment. In such circumstances, students were observed to spend time copying and annotating what teachers had written. Explicitly, though these were rarer, there were several occasions when teachers commented or instructed in ways that implied an instruction to make notes. For example, when asked a procedural question in Katja's second lesson on geometry, Sauli commented, *I don't know how to do it*. In response, Katja commented that, *that is why we write it down; so that it would not stay unclear to anyone*. More explicitly, Sami indicated to his students, during the first lesson on percentages, that they should *draw a picture of it in your notebook*. Interestingly, three of the four case study teachers always wrote in capitals, which not only seemed to provide more legible text for copying but also slowed the writer in ways that facilitated student note-taking.

The second complementary feature was that case study teachers regularly appeared to exploit those students perceived as likely to make appropriately meaningful contributions. For example, all four teachers invited high proportions of responses from a small number of confident and competent students, as in Sami's frequent exploitation of Aku or the fact that Janne asked Taneli, a confident boy in his class, an average of nine questions per lesson. On one occasion, lasting more than five minutes, Janne's class observed passively as Janne and Taneli conducted a conversation as though in private but clearly intended for public consumption. Significantly, Such practices frequently resulted in students, particularly those like Taneli, spending long periods of their lessons waiting, having completed a task, for their peers to catch up. This latter characteristic of Finnish lessons was highlighted by Gameran, (2008), a journalist working for the Wall Street Journal, when she interviewed Fanny, a straight A student. Fanny admitted that *she sometimes doodles in her journal while waiting for others to catch up*.

Thirdly, there were several occasions during the four sequences where teachers alluded to the involvement of parents in their students' work. For example, Antti, having gone through the test results that started his first lesson, instructed his students to *return your paper tomorrow with a signature showing your parents know at home how the test went*. A more telling exchange emerged during Janne's first transcribed lesson. He had instructed his students to mark on their paper and connect three points to construct an arbitrary triangle. Joona had placed his three points in a straight line with the consequence that he was unable to identify any shape. This following ensued:

Janne:	This is a task for you. Tell me at 10 o'clock tomorrow what sort of figure you have.
Joona:	You should know because you're the teacher.
Janne:	No, I want you to find out for me. Tomorrow at 10 o'clock you tell me what kind of a figure you made. You have some kind of a figure there, a geometrical figure. Find out what it is.
Joona:	I can't make it out.
Janne:	Then you'll just have to think about it at home with mum and dad.

In this episode can be seen evidence of two distinctive elements of case study teachers' didactic tradition. Firstly, in informing Joona that he would have to solve the problem for himself, Janne further confirmed the extent to which Finnish mathematics teaching draws on implicit didactics. Secondly, in telling Joona to discuss the problem with his parents, Janne was highlighting the explicit role that Finnish parents play in the education of their children. This latter observation accords with both external and internal analyses of Finnish classrooms. In its recent review of Finnish mathematics education, the English education inspectorate highlighted the multi-faceted role of parents in the education of their children (Office for Standards in Education 2010). From the perspective of internal commentators, Välijärvi et al. (2002, p.46) concluded that Finnish educational success draws on a

> network of interrelated factors, in which students' own areas of interest and leisure activities, the learning opportunities provided by schools, parental support and involvement as well as the social and cultural context of learning and of the entire education system combine with each other.

With regard to the particularities of mathematics, Hannula et al. (2007), writing of the impact of the home on teacher education students' learning of school mathematics, found that few students had not received help with their mathematics at home and that the family not only supported their learning but also provided positive attitudes towards the subject. Thus, the role of parents in the education of their children appears a long established characteristic of Finnish schooling that is well understood by all participants in accordance with research from around the world showing that “parents who are involved in their children's education contribute not only to higher academic achievement, but also to positive behaviours and emotional development” (Cai 2003, p.87).

In returning to Clarke’s and Xu’s (2008) question, one conclusion is that the responsibility for the public generation of mathematical learning lies with teachers and students, with both having clearly defined, if not publicly articulated, and distinctive roles. However, these roles offer only a partial picture with, it seems, parents playing complementary roles facilitating students’ making sense of the plethora of tasks and activities on which their teachers invite them to work. Thus, in attempting to understand the ways in which mathematics teaching plays out in Finnish classrooms, researchers need to understand the role of participants outside those classrooms in the generation of mathematical knowledge.

CONCLUSIONS

In sum, the Finnish mathematics education tradition, and its hitherto unacknowledged drawing on implicit didactics, seems to exploit a partnership between teachers, students and parents that reflective of a characteristic collective mentality of the Finnish people (Simola, 2005). A particular manifestation of this collective mentality was seen in the ways that the class, as a unit, appeared to learn together, with none of the four teachers ever being seen to differentiate the tasks they presented. Indeed, as Välijärvi (2004, p.47) observed, an effective “school works as a community, and the results depend on its ability to employ the students' individual and special skills to benefit the common good”. In this manner, case study teachers’ reliance on the confident student and parental collaboration highlight this systemic manifestation of the collective in ways that may go further towards explaining Finnish PISA success than an analysis of classrooms on their own would ever do.

That said, the notion of the collective seems to beyond merely describing the prosaics of mathematics teaching and learning. Simola (2005, p.458) has commented that “there is something archaic, something authoritarian, possibly even something eastern, in the Finnish culture and mentality” and, in some ways, this accords with Välijärvi et al.'s (2002) attribution of Finnish literacy success to positive student attitudes, an engagement and interest in reading and, in particular, the tradition of reading at home during the long winter evenings. Indeed, in this latter characteristic of Finnish culture may be found a more powerful explanation of not only the Finnish mindset but also PISA success. According to Linnakylä (2002), for more than four hundred years, reading competence was a prerequisite for receiving Holy Communion and, therefore, permission to marry. Failure in the public examination, or kinkerit, meant a denial of such permission with the consequence that Finns have, for centuries, been raised in a culture of high expectations with respect to learning. Such a tradition would help explain how the mathematics didactic tradition described above came into being and continues to dominate classroom discourse. Thus, as Simola (2005, p.465) notes,

> it is still possible to teach in the traditional way in Finland because teachers believe in their traditional role and pupils accept their traditional position. Teachers' beliefs are supported by social trust and their professional academic status, while pupils' approval is supported by the authoritarian culture and mentality of obedience. The Finnish 'secret'

of top-ranking may therefore be seen as the curious contingency of traditional and post-traditional tendencies in the context of the modern welfare state and its comprehensive schooling.

The analysis presented in this paper confirms not only that Finnish mathematics teaching remains much as it was but also that traditional, in the Finnish sense, means something unique in the mathematics education literature. Traditional Finnish didactics seem to fall outside the descriptive frameworks used to describe mathematics teaching and learning in other countries. Indeed, the evidence of this paper cautions those looking to high achieving countries for solutions to domestic educational problems not to assume that educational achievement and didactic excellence are synonymous. As our case study teachers have shown, understanding the role of culture in the construction of student achievement is crucial. Finally, this paper reminds us that when comparing or analysing classrooms from a system other than one's own, broad categorisations of variables, as in the study of LeTendre et al (2001), are unlikely to be helpful in identifying those practices likely to discriminate one system's didactic traditions from another's. Researchers need to find ways of burrowing beneath the layers of superficially similar practice to expose those characteristic behaviours typically hidden from view, especially from the insider.

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