Theoretical Physics

Neutrinos from Dark Matter Annihilation in the Sun

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Abstract

Dark Matter (DM) is believed to consist of Weakly Interactive Massive Particles (WIMPs) which interact only through gravity and the weak nuclear force. These particles can become trapped in gravitational wells such as the Sun and a theoretical value of the capture rate can be calculated. At high particle density the WIMPs annihilating spontaneously into Standard Model (SM) particles. Due to particle equilibrium the total annihilation rate can be related to the capture rate by a simple expression. This report will focus on calculating the capture rate and the related annihilation rate as well as calculating the neutrino flux of the Sun. At first we will give a brief introduction to cosmology and a theoretical argument for the WIMPs as the prime DM candidate. Then we will look at the theoretical background and the mechanism through which WIMPs become trapped and evaporate or annihilate. Finally we will perform a numerical analysis of the WIMP cycle within the Sun and calculate the capture rate for a variety of theoretical WIMP masses. We will look at the capture rate due to scattering both by hydrogen nucleii and by more massive elements. The Scattering by hydrogen will be the prime contributor to the total capture rate and is the only spin dependent contribution.
Keywords: Dark Matter, WIMP annihilation, neutrino, the Sun.

Sammanfattning

Mörk materia (DM) tros bestå av svagt interagerande massiva partiklar (WIMPs) som endast interagerar genom gravitationen och den svaga kärnkraften. Precis som vanlig materia kan dessa partiklar fångas i massiva himmlakroppar såsom solen där ett teoretiskt värde för infångningshastigheten kan beräknas. Vid höga partikeldensiteter kan WIMPs spontant sönderfalla till standardmodell-partiklar (SM). På grund av partikeljämvikt kan den totala sönderfallshastigheten relateras till infångningshastigheten med ett enkelt uttryck. Denna rapport kommer att fokusera på att beräkna infångningshastigheten, den tillhörande sönderfallstakten och neutrinosignalen från solen. Först kommer vi att ge en kort introduktion till kosmologi och ett teoretiskt argument för WIMPs som den främsta DM kandidaten. Sedan kommer vi att titta på den teoretiska bakgrunden och den mekanism genom vilken WIMPs kan fastna i solen, där de sedan avdunstar eller sönderfaller. Slutligen kommer vi att utföra en numerisk analys av WIMP-cykeln i solen och beräkna infångningshastigheten för en rad olika teoretiska WIMP massor. Vi kommer att titta på fångsthastigheten på grund av spridning av både väte och mer massiva grundämnen. Spridningen av väte kommer att vara den främsta bidragsgivaren till den totala fångsthastigheten och är det enda spinnberoende bidrag.
Nyckelord: Mörk Materia, WIMP sönderfall, neutrino, solen.

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Chapter 1

Introduction

The first observation of the 'missing mass problem' was made in the early 1930s when Swiss astronomer Fritz Zwicky found that the rotation of galaxies did not correlate with their light emitting masses. In [1] he proposed that most mass was dark and emitted no radiation, he referred to it as dark matter (DM). Many different candidates for dark matter have been proposed and the most popular being Weakly Interacting Massive Particles (WIMPs) [2] and the MAssive Compact Halo Objects (MACHOs). According to [3] the main difference between the two is that WIMPs are non-baryonic particles while MACHOs are purely baryonic. WIMPs only interact via the weak nuclear force and gravitation and can thus be trapped in strong gravitational wells such as that of the Sun [4]. In the Sun the DM particles annihilate into standard model (SM) particles such as neutrinos, these can then be detected by neutrino detectors on Earth [5].

Chapter 2

Background Material

2.1 Evidence for the existence of dark matter

DM has been a long standing theory to account for the discrepancy between approximation of the mass of the Universe based on observations and the theoretically necessary mass to maintain the observed kinematic movement. The idea was first presented by Zwicky in 1933 [1]. Data collected by Wilkinson Microwave Anisotropy Probe (WMAP) suggests that the universe is flat and contains about 25 % non-baryonic DM and 70 % dark energy, with the remaining 5 % being regular baryonic matter. [2, 6]

An indirect method of searching for dark matter is generally favored due to the weak interactions of DM particles. The annihilation of DM may give rise to a multitude of SM particles such as electrons, protons, photons and neutrinos. By analysing fluxes of charged cosmic rays, measure photon signals or excess of neutrino fluxes from the Sun or other regions with high DM density an indirect mapping of DM is possible. There is furthermore some potential of developing a method of detection based on the observation of antideuterons [2].

Alternative ways of direct detection by examining weak gravitational lensing have been proposed but have yet to be proven viable [7].

2.2 WIMPs explained (WIMPs vs MACHOs)

The most accepted model of DM is the WIMP model. This model is based on the belief that DM particles were freely constructed and destroyed in pairs during the early stages of the Universe before the temperature of the Universe dropped below the mass of the DM particles. The drop in temperature happened in conjunction with the expansion of the Universe and resulted in almost complete suspension of both creation and annihilation of DM particles. The DM particles are believed to have masses in the range of 10 GeV - 1 TeV and only interacts by the weak nuclear force and gravitation. DM particles annihilate in pairs into pairs of SM particles [2].

Another DM candidate is the MACHO model which is a theory for explaining the missing mass of low mass stars that are too light to initiate nuclear burning. A star of this low mass would not emit any light or radiation of their own. Matter in black holes, neutron stars and fainter large stars are sometimes included as MACHOs. MACHOs existing mainly in light stars are thought to be purely baryonic [3].

The big bang nucleosynthesis puts limits on the amount of baryonic matter in the Universe and it can be derived that the vast majority if the DM must be non-baryonic in nature [3]. This makes WIMPs the more probable DM candidate, or in the case that DM consists of both WIMPs and MACHOs, ascertains that the vast majority of the DM mass consist of WIMPs [5]. MACHOs will not be further considered in this examination of DM. This can be justified by noting that the signal on Earth will be proportional to r^{-2}. The Sun is thus the only significant source of DM annihilation signals.

2.3 Distribution of dark matter in the galaxy

DM is believed to gather in stellar clusters such as Sagittarius, Draco and the Milky Way galactic center. There is furthermore some evidence of higher concentrations of DM in the galactic halo. On a lower scale DM will gather at strong gravitational fields such as in the Sun or the Earth much like the case with regular matter [2]. Only neutrino signals originating in the Sun will be examined.

2.4 Solar neutrino problem

Another important problem which has emerged as more accurate measure techniques have been developed is the 'solar neutrino problem'. This problem is based on the observation that the Sun produces less neutrinos than expected [4, 8]. The explanation of this discrepancy is explained by considering neutrino oscillations. As neutrino telescopes on Earth can only detect muon neutrinos and a significant portion will have oscillated into electron neutrinos at the Sun-Earth distance. Solar neutrinos have energies ≤ 15 MeG, while those from DM annihilations have energies in the range of 10 to 1000 GeV [5].

Chapter 3

Investigation

This investigation will focus on the theoretical derivation of the total annihilation rate. The analytical derivation will be followed by a brief numerical investigation of the system model. Finally the neutrino flux by annihilation of DM will be investigated.

3.0.1 Annihilation channels

DM particles are supposed to annihilate in pairs into pairs of SM fermions:

$$\chi\chi \to b\bar{b}, c\bar{c}, \tau\bar{\tau}, \nu\bar{\nu}, \mu\bar{\mu} \tag{3.1}$$

where it is supposed that all neutrino flavours are created as $\nu = (\nu_\tau + \nu_\mu + \nu_e)/3$. Other annihilations would cause weak neutrino signals for low DM masses and are thus insignificant for WIMP masses < 100 GeV [9, 10].

Another possible annihilation channel is:

$$\chi\chi \to 4\tau, \tag{3.2}$$

where the energy is distributed equally between the taus. Neutrino telescopes are sensitive to this annihilation channel [9].

For higher WIMP masses > 200 GeV a few other annihilation channels become contributing factors:

$$\chi\chi \to W^+W^-, ZZ, t\bar{t} \tag{3.3}$$

where W and Z are the weak bosons that mediate the weak nuclear force [10].

Photons are also believed to be emitted by DM annihilations. These photons are emitted by three different mechanics; gamma-rays emitted directly from the DM annihilation, gamma-rays from the inverse compton scattering of electrons and positrons produced as part of the annihilation chain with energies between a fraction of to almost the full DM mass and lastly synchrotron emission emitted in the magnetic field of the Galaxy by electrons and positrons [2].

The photons can be used for indirect detection in regions of high DM density or where the astrophysical background radiation is low. While there are some regions fulfilling these criterias no experiment have so far produced any DM signal via photon detection [2].

Antideuterons are deuterons consisting of an antiproton and an antineutron and seem to be a promising tool for detection of DM. These particles have however not been detected as of yet. Antideuterons are produced when an antiproton and antineutron with spatially aligned and comparable in magnitude momenta coalesce. The particles will have a flux peak around a fraction of a GeV, where very little background antideuterons are present. If any antideuterons were found within this energy range it would be strong evidence for the existence of DM [2]. There are currently two main experiments looking for these antideuterons; GAPS [11] and AMS-02 [12].

3.1 Problem

3.1.1 Setting up the mass equilibrium differential equation in the Sun

As DM particles collide with the particles in the Sun they will scatter and lose energy. If the energy loss is substantial enough their velocity will drop below the escape velocity and they will become gravitationally bound to the Sun. This will trap the DM particles and the increase in local DM density will cause annihilation and evaporation [9]. By setting up a particle conservation equation in the Sun it is possible to calculate the actual neutrino creation rate.

3.1.2 Cross sections for annihilations

The cross section of interaction between a proton and a WIMP is extremely important for the capture rate and governs both particle equilibrium conditions and the total capture rate. The spin dependent WIMP-proton cross section can be approximated by analysing data from neutrino telescopes on earth. The value used in the calculations was derived from the Baksan Underground Scintillator Telescope as $\sigma_p \approx 10^{38}$ cm^2 with an experimental time of 24.12 years. The fact that the cross section is slightly dependent on WIMP mass is not considered as it will be negligible compared to the other quantities [13]. The WIMP nucleus cross section for nucleus i would be given by,

$$\sigma_i = \sigma_p A_i^2 \frac{\mu_i^2}{\mu_p^2} \tag{3.4}$$

where A_i is the mass number. μ_i is the reduced mass of the nucleus and μ_p is the reduced mass of a proton.

3.2 Model

3.2.1 Composition and physical qualities of the Sun

The composition of the Sun is important as any approximations or uncertainties can potentially cause significant deviations in the capture rate. Most notably the density and various element abundances are of great importance as they enter the equations as a differential contribution to the capture rate and need to be integrated over the total solar volume. The quantities used in the following calculations are taken from the AGSS09 solar model [14].

3.2.2 Capture by the Sun

As WIMPs pass through the Sun they might scatter with nuclei near the core. If they lose enough of their velocity to drop below the local escape velocity they become gravitationally bound and the local DM density will increase. The differential expression of the capture rate is given as [9]

$$\frac{dC_{\odot,i}}{dV} = \frac{\rho_\chi \rho_{\odot,i}(r)}{2m_\chi \mu_i^2}\sigma_i \int_0^\infty du \frac{f(u)}{u} \int_{E_{R,min}}^{E_{R,max}} dE_R |F(E_R)|^2. \tag{3.5}$$

In which $\rho_{\odot,i}(r)$ is the mass density of element i at radius r in the Sun. m_χ is the WIMP mass the capture rate is calculated for. Notice that this parameter is altered as the independent variable. ρ_χ is the WIMP density and can through measuring the kinematics of stars be approximated to $\simeq 0.3$ GeV cm^{-3} [15]. μ_i is the reduced mass and σ_i is the WIMP nucleus cross section. $f(u)$ is a Maxwellian velocity distribution of WIMPs due to the gravitational potential of the Sun and is shifted by the velocity of the Sun. It can be described as, [9]

$$\frac{f(u)}{u} = \frac{1}{\sqrt{\pi} v_\odot^2}\left(\exp\left[-\frac{(u-v_\odot)^2}{v_\odot^2}\right] - \exp\left[-\frac{(u+v_\odot)^2}{v_\odot^2}\right]\right) \tag{3.6}$$

where $v_\odot = 220$ km/s is the velocity of the Sun. $|F(E_R)|$ is the form factor describing the decoherence effects in the scattering. If supposed to be Gaussian in order to simplify the calculations it is given as, [16]

$$|F(E_R)|^2 = \exp\left[-\frac{E_R}{E_i}\right]. \tag{3.7}$$

E_R is the recoil energy of the nucleus and E_i can be expressed as,

$$E_i = \frac{3}{2m_i R_i^2} \tag{3.8}$$

in which m_i is the mass of element i and R_i is its RMS charge radius. The form factor is integrated between the minimal energy loss for the WIMP in order to trap it $E_{R,min}$ and the maximum energy loss allowed by kinematic limitations $E_{R,max}$. The limits are given as [9]

$$E_{R,min} = \frac{1}{2} m_\chi u^2, \ E_{R,max} = \frac{2\mu_i^2}{m_i}(u^2 + v_{esc}^2(r)) \tag{3.9}$$

where $v_{esc}(r)$ is the internal escape velocity of the Sun. The total solar capture rate can then be expressed as,

$$C_\odot = 4\pi \sum_i \int_0^{R_\odot} dr r^2 \frac{dC_{\odot,i}}{dV} \tag{3.10}$$

where the sum is over all elements in the Sun.

There is a slight difference between spin-dependent and spin-independent scattering; resulting in spin-dependent capture only having relevant capture rate at hydrogen. This reduces the sum to one term and results in a lower capture rate [9].

3.2.3 Annihilation in the Sun

As the WIMPs become trapped within the Sun the increase in local DM density will cause the WIMPs to annihilate into SM particles. Trapped WIMPs are distributed according to a Maxwellian velocity distribution at the temperature T_χ. The distribution is expressed as, [9]

$$f(v) = \sqrt{\frac{2m_\chi^3}{\pi T_\chi^3}} v^2 \exp\left[-\frac{m_\chi v^2}{2T_\chi}\right]. \tag{3.11}$$

The WIMPs will gather close to the center of the Sun so that solar temperature $T_\odot(r)$ and density $\rho_\odot(r)$ are roughly constant. This means that they can be approximated as $T_\chi \approx T_\odot(\bar{r})$ and $\rho_\odot(r) \approx \rho_\odot(\bar{r})$ where $\bar{r}$ is the mean WIMP orbit radius. Thus the WIMP number density can be expressed as, [9]

$$n_\chi(r) = n_0 \exp\left[-\frac{m_\chi \phi(r)}{T_\chi}\right] \tag{3.12}$$

where $\phi(r)$ is the gravitational potential. The potential is given as,

$$\phi(r) = \frac{2\pi}{3}\rho_\odot(\bar{r}) r^2 G. \tag{3.13}$$

The average of r can be calculated as,

$$\bar{r} = \sqrt{\frac{6T_\odot(\bar{r})}{\pi^2 G \rho_\odot(\bar{r}) m_\chi}}. \tag{3.14}$$

The total WIMP number annihilation rate can be expressed as, [9]

$$\Gamma_\odot = \frac{1}{2} A_\odot N^2. \tag{3.15}$$

The annihilation rate $A_\odot$ can then be expressed as,

$$A_\odot = \frac{1}{N^2} \int dr 4\pi r^2 n_\chi^2(r) \left\langle \sigma v_{rel} \right\rangle_\odot = \left(\frac{\sqrt{2}}{\pi \bar{r}}\right)^3 \left\langle \sigma v_{rel} \right\rangle_\odot . \tag{3.16}$$

For WIMP masses $m_\chi > 1$ GeV, the annihilation rate can be approximated by,

$$A_\odot \approx 4.5 \cdot 10^{-30} \text{cm}^{-3} \left(\frac{m_\chi - 0.6 \text{ GeV}}{10 \text{ GeV}}\right)^{\frac{3}{2}} \left\langle \sigma v_{rel} \right\rangle_\odot . \tag{3.17}$$

The thermal average is thus calculated by, [9]

$$\left\langle \sigma v_{rel} \right\rangle_\odot = a + \frac{6T_\odot(\bar{r})}{m_\chi} b \tag{3.18}$$

3.2.4 Evaporation in the Sun

The captured WIMPs will continue scattering with other nuclei in the Sun and may regain kinetic energy. If the WIMPs can gain enough energy they can escape the gravitational pull of the Sun and become unbound once again [9, 17, 18]. This process is very dependent on the WIMP mass,

$$E_\odot \propto \frac{1}{t_\odot} \exp\left[-30\frac{m_\chi - m_{evap}}{m_{evap}}\right], \tag{3.19}$$

where $t_\odot$ is the age of the Sun. The annihilation rate can be approximated by, [17]

$$E_\odot \approx \frac{8}{\pi}\frac{\sigma_{evap}}{\bar{r}^3}\bar{v}\frac{E_{esc}}{T(\bar{r})} \exp\left[-\frac{E_{esc}}{2T(\bar{r})}\right] \tag{3.20}$$

where E_{esc} is the energy needed to escape from the center of the Sun and σ_{evap} is the evaporation cross section. The mean WIMP speed $\bar{v}$ of the distribution expressed in (3.11) can be calculated as,

$$\bar{v} = \sqrt{\frac{8T_\odot(\bar{r})}{\pi m_\chi}}. \tag{3.21}$$

In this case an approximation of the evaporation mass is given by, [9]

$$m_{evap} = m_0 + 0.32 \text{ GeV } \log\left(\frac{\sigma_p}{10^{-40}cm^2}\right) \tag{3.22}$$

and $m_0 = 3.5$ GeV for spin-independent collisions, otherwise $m_0 = 3.02$ GeV .

If the WIMP mass slightly exceeds the evaporation mass the evaporation effect will be insignificant compared to the annihilation rate. As m_{evap} is fairly low compared to normal WIMP masses the evaporation rate will be exponentially decreasing with m_χ. Therefore the evaporation can be neglected for the masses under consideration.

3.2.5 Finding the total annihilation rate

The total number of WIMPs can be described by the following particle conservation [9],

$$\frac{dN}{dt} = C_\odot - A_\odot N^2 - E_\odot N \tag{3.23}$$

where N is the total number of WIMPs in the Sun. As discussed earlier the evaporation is exponentially suppressed for most WIMP masses and can thus be neglected. The simpler equation is solved by,

$$N(t) = \sqrt{\frac{C_\odot}{A_\odot}} \tanh\left(\sqrt{C_\odot A_\odot} t\right). \tag{3.24}$$

The total annihilation rate as stated earlier can then be expressed in terms of the capture rate as [9]

$$\Gamma_\odot = \frac{1}{2}A_\odot N^2 = \frac{1}{2}C_\odot \tanh^2\left(\sqrt{C_\odot A_\odot} t_\odot\right). \tag{3.25}$$

Considering that $\sqrt{C_\odot A_\odot} t_\odot \gg 1$ for all relevant t it can be seen that particle equilibrium between capture and annihilation is reached and; [9]

$$\Gamma_\odot = \frac{1}{2}C_\odot. \tag{3.26}$$

3.2.6 Neutrino interactions and oscillations

The three flavours of neutrinos are electron, muon and tau. A neutrino can oscillate between the different flavours due to the eigenstates of mass and flavour being linear combinations of each other. Thus a neutrino created as a certain flavour in the Sun does not necessarily have to be detected as the same flavour when it has propagated to a neutrino telescope on earth. A large portion of muon neutrinos will at the time of passing the Earth be electron neutrinos [5].

The dominating factor in neutrino interactions is different for different neutrino energies. For energies above 10 GeV the dominating factor is deep inelastic scattering. As most neutrinos from DM annihilations have > 10 GeV this is the most important factor. For energies below 3 GeV the dominating factor is quasi-elastic scattering and single pion production [9].

3.2.7 Neutrino flux on Earth

In order to reconstruct the WIMP properties the theoretical neutrino flux on Earth has to be calculated and compared to the observations of a neutrino telescope. Neutrinos will propagate through empty space almost unobstructed once they leave the Sun. This leaves the neutrino oscillations as the main mechanism that has to be taken into account. The neutrino telescopes on Earth can today only detect muon neutrinos. Assuming the neutrinos spread equally in all directions the average flux at Earth will be given as,

$$\phi_\nu = \frac{C_\odot}{4\pi R_{AU}^2} \tag{3.27}$$

where R_{AU} is the distance between the Sun and the Earth. One third of these neutrinos will be muon neutrinos which can be detected by present technology. IceCube is a neutrino telescope on the south pole with the dimensions of roughly a cylinder with a height of 2800 m and a radius of 750 m. This means that an area of roughly 4.2 km^2 will be exposed to the neutrino flux. The average muon neutrino flux from WIMP annihilations through the IceCube neutrino telescope can then be expressed as,

$$A_{IC}\phi_{\nu_\mu} = 4.2\frac{C_\odot}{12\pi R_{AU}^2}. \tag{3.28}$$

It must however be noted that there will be major fluctuations in the signal as the distans between the Sun and Earth varies greatly. Also the season will have some effect on the signal as the effective area of IceCube will vary depending on the incident angle.

3.3 Analytical calculations

In this section we will look at the two integrals in the differential capture rate contribution over dE_R and du which can be integrated analytically for all types of nuclei. Furthermore the escape velocity will be derived.

3.3.1 Escape velocity from inside the Sun

$\frac{m_\chi v_{esc}^2}{2} + U(r) = 0$ where $U(r)$ is the gravitational potential energy of the DM particle. We use this expression to find the escape velocity inside the Sun. The general expression for calculating the gravitational potential thus can be found by

$$U(r) = \int_{\infty}^{r} F(r)dr = \int_{\infty}^{r} \frac{Gm_\chi M(r)}{r^2}dr = -\frac{Gm_\chi M_\odot}{R_\odot} - Gm_\chi \int_{r}^{R_\odot} \frac{M(r)}{r^2}dr. \tag{3.29}$$

where $M(r)$ is the total mass inside a sphere of radius r centered on the Sun. This results in the escape velocity being defined by,

$$v_{esc}(r) = \sqrt{2G\left(\frac{M_\odot}{R_\odot} + \int_{r}^{R_\odot} \frac{M(r)}{r^2}dr\right)}. \tag{3.30}$$

3.3.2 Spin dependent capture rate

Analytical calculations for spin dependent capture are particularly easy as the form factor is given by $|F(E_R)| = 1$ and the integral solves to $E_{R,max} - E_{R,min}$. This makes the du integral a simple sum of polynomials and exponentials as

$$\begin{aligned} &\frac{\frac{2\mu_H^2}{m_H} - m_\chi}{\sqrt{\pi}v_\odot^2} \int_{0}^{\infty} u^2 \left(\exp\left[-\frac{(u-v_\odot)^2}{v_\odot^2}\right] - \exp\left[-\frac{(u+v_\odot)^2}{v_\odot^2}\right]\right) du + \\ &+\frac{2\mu_H^2}{\sqrt{\pi}m_H v_\odot^2} v_{esc}^2(r) \int_{0}^{\infty} \left(\exp\left[-\frac{(u-v_\odot)^2}{v_\odot^2}\right] - \exp\left[-\frac{(u+v_\odot)^2}{v_\odot^2}\right]\right) du \end{aligned} \tag{3.31}$$

which solves into

$$A \cdot \frac{v_\odot}{\sqrt{\pi}} \left(\frac{2\mu_H^2}{m_H} - m_\chi\right) + B \cdot \frac{2\mu_H^2}{\sqrt{\pi}m_H v_\odot} v_{esc}^2(r). \tag{3.32}$$

A and B are real valued, dimensionless constants derived by numerically solving the du integrals. This is done by using the change of variables $\frac{u-v_\odot}{v_\odot} = x$ as well as $\frac{u+v_\odot}{v_\odot} = x$ to make the integrations dimensionless. This then gives the differential contribution to the capture rate as

$$\frac{dC_{\odot,H}}{dV} = \frac{\rho_\chi \rho_{\odot,H}(r)}{2m_\chi \mu_H^2}\sigma_H \left(A \cdot \frac{v_\odot}{\sqrt{\pi}} \left(\frac{2\mu_H^2}{m_H} - m_\chi\right) + B \cdot \frac{2\mu_H^2}{\sqrt{\pi}m_H v_\odot} v_{esc}^2(r)\right). \tag{3.33}$$

This expression is then integrated numerically over the entire Sun to get the total spin dependent capture rate $C_{\odot,H}$.

3.3.3 Spin independent capture rate

Generally the dE_R integration takes a more complicated form as the form factor can not be approximated by 1. In the spin independent capture rate the first integration yields

$$E_i \left(\exp\left[-\frac{E_{R,min}}{E_i}\right] - \exp\left[-\frac{E_{R,max}}{E_i}\right]\right) \tag{3.34}$$

for nuclei i. This makes the du integral a sum of exponential functions, which we solve numerically.

3.4 Numerical analysis

By using the theoretical expression the capture rate in the Sun can be calculated for various m_χ. We then relate this to the annihilation rate and derive any desired physical quantities.

3.4.1 Spin dependent capture rate

The first integrations which have to be done numerically are the du integrations

$$\int_0^\infty u^2 \left(\exp\left[-\frac{(u-v_\odot)^2}{v_\odot^2} \right] - \exp\left[-\frac{(u+v_\odot)^2}{v_\odot^2} \right] \right) du \tag{3.35}$$

and

$$v_{esc}^2(r) \int_0^\infty \left(\exp\left[-\frac{(u-v_\odot)^2}{v_\odot^2} \right] - \exp\left[-\frac{(u+v_\odot)^2}{v_\odot^2} \right] \right) du. \tag{3.36}$$

These can be reduced to dimensionless integrals with the two substitutions $\frac{u-v_\odot}{v_\odot} = x$ and $\frac{u+v_\odot}{v_\odot} = x$. The first integral has the solution $2.6083818 \cdot v_\odot^3$ and the second $1.4936482 \cdot v_\odot \cdot v_{esc}^2(r)$. This can then be used in conjunction with the derived theoretical differential contribution to the capture rate to numerically integrate it over the total volume of the Sun.

In order to make an accurate integral of the velocity distribution of the WIMPs it can be noted that at high velocities the capture rate contribution will become very limited. As can be expected the capture rate contribution is basically 0 past the exterior escape velocity of the Sun $v_{esc}(R_\odot) \approx 600$ km/s. This is shown in Figure 3.1. It should furthermore be noted that the fact that the graph below only shows the contribution distribution for $m_\chi = 50$ GeV is no real limitation as the velocity distribution will not be very sensitive of the WIMP mass.

3.5 Results

Using Matlab we can calculate the capture rate and any derived physical quantities from the theoretical estimations in earlier sections. By inserting the appropriate parameters the capture rate becomes a function of the WIMP mass. This correlation is then plotted in Figure 3.2.

3.5.1 Capture rate

The capture rate is calculated numerically as a function of m_χ. Elements up to Ni have been summed over as beyond this the number density of the various elements is extremely low and will not significantly change the appearance of the graph. As can be seen in Figure 3.2 hydrogen is major contributor but however far from the only. It must be noted that as more elements are included the capture rate strictly increases; this means that Figure 3.2 presents a lower limit of the capture rate.

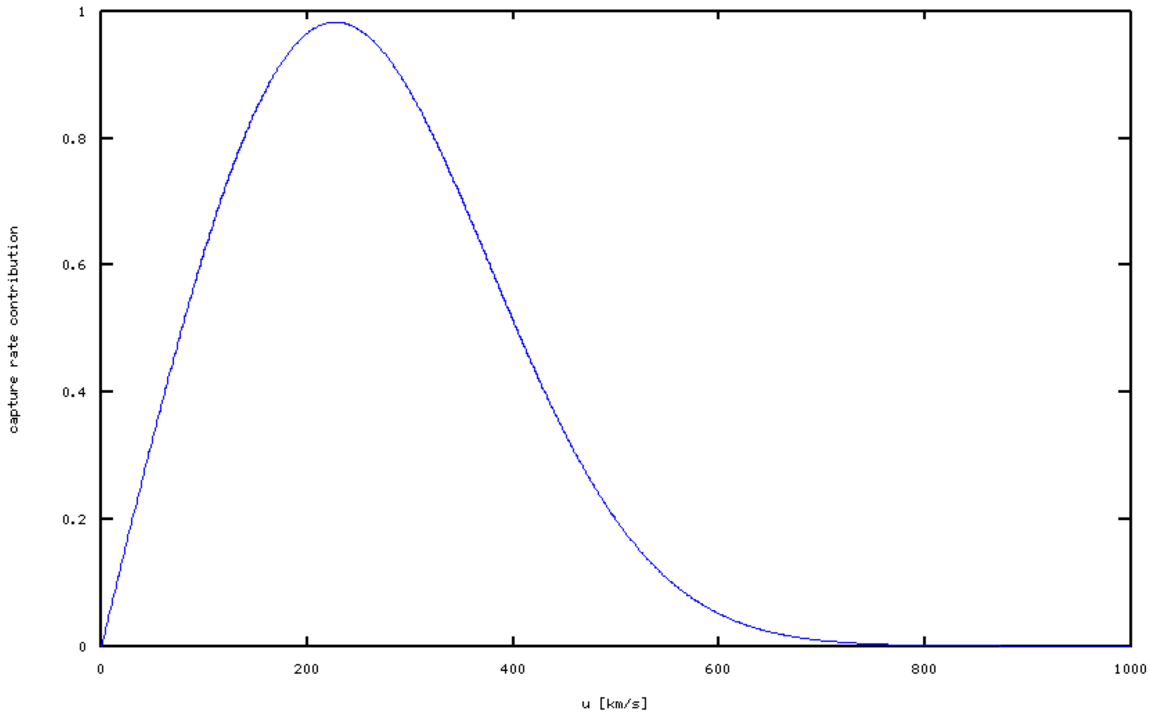

Figure 3.1: Differential contribution to the capture rate for various WIMP velocities at $m_\chi = 50$ GeV. The graph presents the function $\sqrt{\pi} v_\odot^2 \frac{f(u)}{u}$.

This capture rate can then be correlated to the incident signal on IceCube by (3.28) and be plotted in Figure 3.4. In order to appreciate the mass of the DM particles this function can be compared to the observed signal from IceCube. Notice however that very few of the neutrinos passing through IceCube actually will be detected. As the detected signal is weak the statistical probability of a neutrino being detected will cause major deviations from the average value. By using a Monte Carlo method of simulating this the uncertainty can be minimised.

3.6 Discussion

3.6.1 The choice of Solar model

The AGSS09 Solar model quantities were determined through spectroscopic measurements where the emitted radiation was studied. The model is slightly conflict with helioseismological measurements where pressure waves are studied. The newer solar models, which have slightly higher metallicity than older models, agree better with helio seismological values than other low metallicity models but not as good as high metallicity models [14].

As the mass of the particles increases the actual number density decreases and subsequently the differential capture rate decreases. The slightly older model BPS08(AGS) and AGSS09 have very similar mass fractions and distribution of hydrogen. AGSS09 was used as it was newer than BPS08(AGS) and had addressed some issues with its predecessor.

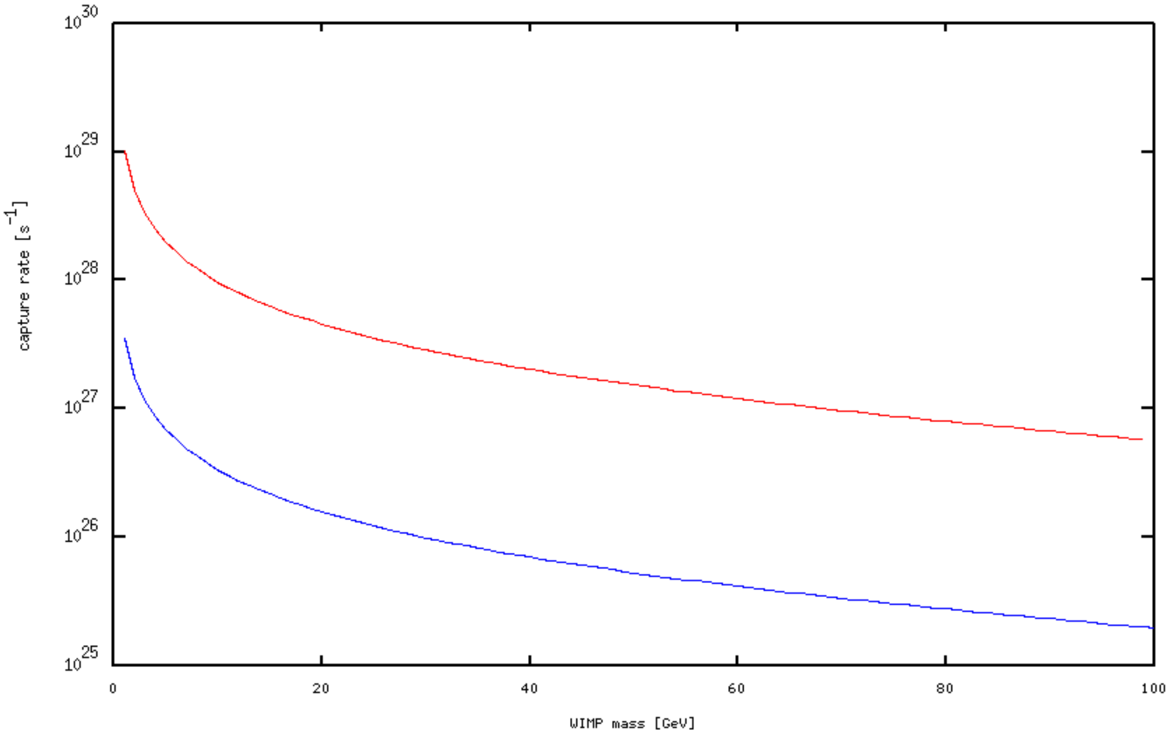

Figure 3.2: Spin-dependent and spin-independent total capture rate as a function of WIMP mass. The blue line represents spin-dependent capture and the red represents spin-independent capture. For the spin-independent capture elements up to Ni were considered.

3.6.2 Limit on WIMP masses by evaporation

The simplifying assumption that the evaporation rate can be neglected is only valid for $m_\chi > m_{evap}$. m_{evap} is dependent on the quotient $\frac{\sigma_p}{10^{-40}\text{cm}^2}$ which with our assumed σ_p will be 10^2. This leads to $m_{evap} = 4.14$ GeV for spin independent interactions, or 3.66 GeV for spin dependent interactions.

This becomes our equilibrium absolutely lowest limit, and we find that already at $m_\chi = 5$ GeV, $E_\odot = \frac{1}{t_\odot} \cdot 2 \cdot 10^{-3}$ or $\frac{1}{t_\odot} \cdot 1.7 \cdot 10^{-5}$ for spin dependent interactions. This is negligible when compared to the capture rate or annihilation rate and can thus safely be neglected for any $m_\chi > 5$ GeV.

3.6.3 Upper WIMP mass limit

The upper limit is found when $\Delta|F(E_R)|^2 = 0$, that is when the incoming WIMPs cannot lose enough energy by recoiling against nuclei in the Sun to become gravitationally bound. We find this limit through inspection by increasing m_χ until we see a drop in our calculated capture rate to 0. This can be seen in Figure 3.3.

We numerically found this value to be around $m_\chi \approx 500$ GeV, as the model is mainly valid for low WIMP masses [9]. This should perhaps be viewed as a limit on the theoretical model rather than a limit on capturable WIMP masses themselves as several simplifications have been performed.

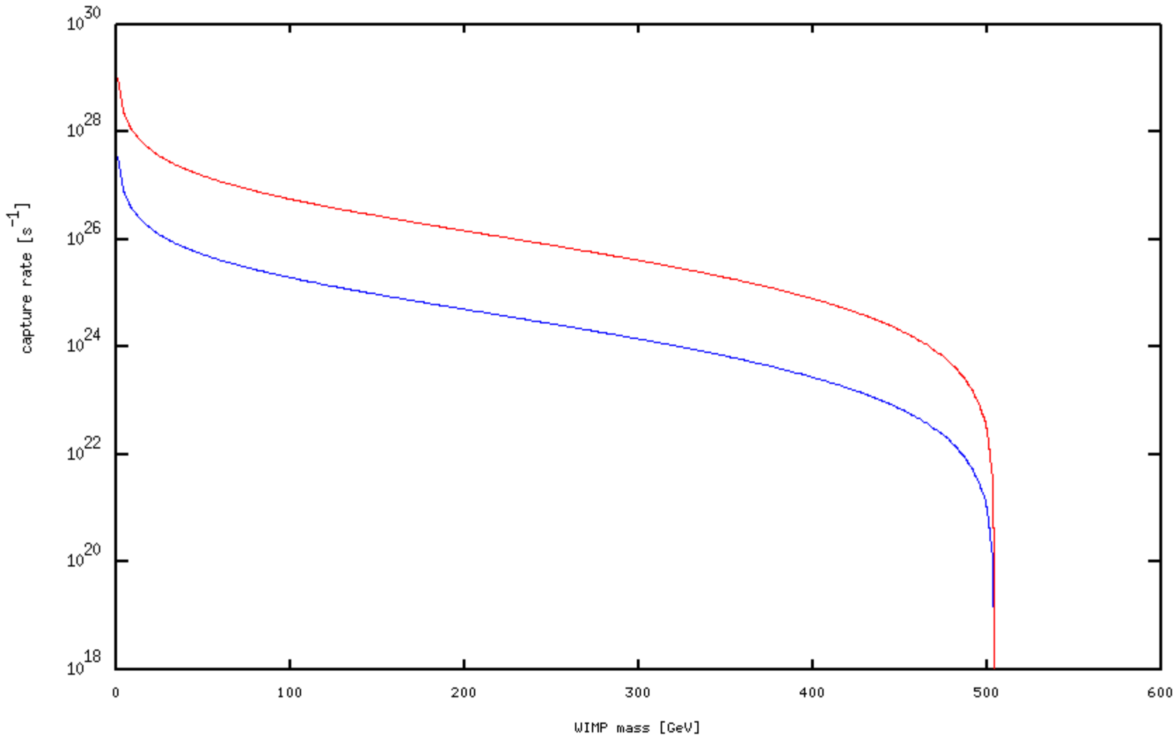

Figure 3.3: Extended range of WIMP masses showing the upper mass limit of the WIMP model.

3.6.4 Equilibrium conditions

The Sun will only be in capture-annihilation equilibrium if $\sqrt{C_\odot A_\odot} t_\odot \gg 1$, where $A_\odot$ needs to be larger than or roughly the same magnitude as the reciprocal of $C_\odot$. In turn this gives us a limit on $\langle \sigma v_{rel} \rangle_\odot$ as according to our approximation $A_\odot \approx 10^{-30} \cdot \langle \sigma v_{rel} \rangle_\odot$. For a typical $C_\odot$ of $10^{25} \frac{1}{\mathrm{s}}$ this means $\langle \sigma v_{rel} \rangle_\odot \gg 10^{-30} \frac{\mathrm{cm}^3}{\mathrm{s}}$. According to our thermal averaging we know $\langle \sigma v_{rel} \rangle_\odot = a + \frac{6T_\odot(\bar{r})}{m_\chi} b$, where a is the velocity independent contribution and b the velocity dependent contribution.

It is usually assumed that the cross section is mainly velocity independent so that $\langle \sigma v_{rel} \rangle_\odot \approx a$ and a is in the region of $10^{-26} \frac{\mathrm{cm}^3}{s}$ [9]. If this is true equilibrium is easily reached.

However if the main contribution is velocity dependent the annihilation cross section is reduced and equilibrium may not be reached depending on the order of magnitude of a. In which case the capture rate acts as an upper limit to the total annihilation rate so that $\Gamma_\odot \leq \frac{1}{2} C_\odot$. It is very likely that equilibrium is reached and we will assume $\Gamma_\odot = \frac{1}{2} C_\odot$ as deviations will be negligible for all m_χ greater than m_{evap}.

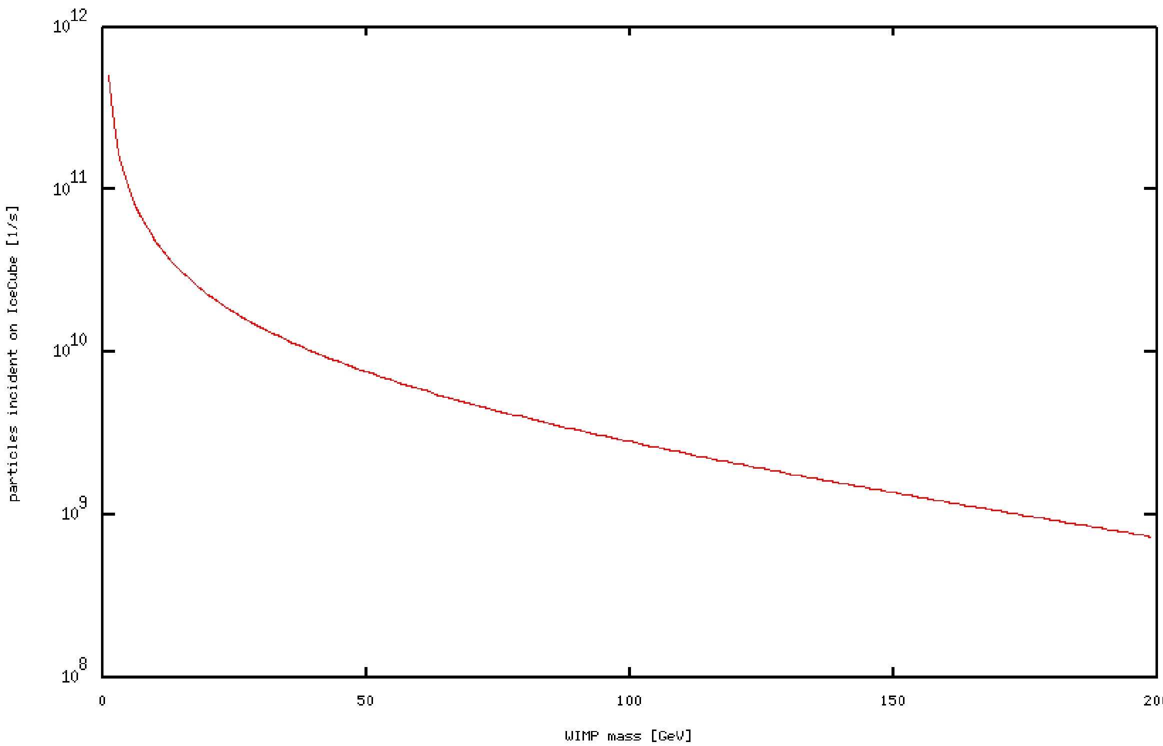

Figure 3.4: Average incident neutrino signal on IceCube over a significant period of time.

Chapter 4

Summary and conclusion

In this thesis we numerically studied the Suns capture rate of WIMPs for various masses between $5 < m_\chi < m_{\chi,max}$ GeV and then derived the total annihilation rate. WIMPs are theorised to annihilate in pairs into pairs of standard model particles. Among these would be neutrinos of all three flavours, most interestingly would be the muon-neutrinos which can then be detected by neutrino telescopes on Earth. To correctly calculate the muon-neutrino yield on Earth one has to take into account neutrino oscillations. This will cause to a lower than expected flux.

We looked at a particle conservation in the Sun taking into account capture rate, annihilation rate and evaporation rate and showed that the evaporation rate can be neglected for WIMP masses higher than m_{evap}. We then analysed this simpler conservation to relate total annihilation rate to the capture rate and then derive any related physical quantities.

The annihilation rate and capture rate will be in equilibrium if the annihilation cross section $\langle\sigma v_{rel}\rangle_\odot$ is big enough. Most approximations of $\langle\sigma v_{rel}\rangle_\odot$ easily puts the Sun into particle equilibrium. If this is not the case the capture rate will then act as an upper limit to the total annihilation rate as $\Gamma_\odot < C_\odot$. In any calculation where the total annihilation rate is used we assumed equilibrium within the Sun is reached.

We mainly focus on the spin dependent capture rate which only has significant capture by hydrogen. This is also the most significant contribution to the overall capture rate. Calculations for elements up to Ni were also performed, more massive nuclei than that are rare in the Sun and do not significantly alter the total capture rate. For more precise results the sum should be extended to all nuclei present within the Sun. The AGSS09 model does however only contain information of element abundances up to Ni, heavier elements should still occur in limited abundances.

The properties of WIMPs can then in theory be reconstructed by comparing the calculated neutrino flux from different WIMP masses to the observed neutrino flux on Earth. In practice this is very hard to do as neutrino telescopes have a very low detection rate and the direct $\nu\bar{\nu}$ channel often is suppressed. The reconstruction of WIMP properties will likely have to wait for much more advanced neutrino telescopes before this will be an accurate method of determining the WIMP mass.

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