Optimization of Fuel Consumption in Hybrid Electric Vehicles

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Abstract

There are various technologies used for reducing fuel consumption of automobiles. Hybrid electric vehicles is one approach that has been used, which can reduce fuel consumption by 10-30% compared to conventional vehicles.

In this master thesis the minimization of fuel consumption of a power-split type HEV along a given route is considered, where the vehicle speed has been assumed to be known a priori. This minimization was made by first deriving a model of the HEV powertrain, followed by creating a Dynamical programming based program for finding the optimal distribution of torques.

The performance was evaluated through the commercial software GT-Suite. The resulting control from the Dynamic program could follow the reference speed in many situations. However the battery state-of-charge calculated in the Dynamic program did not update properly, resulting in a depleted battery in some cases.

The model derived could follow the dynamics of the vehicle, but there are some parts which could be improved. One of them is the dynamical model of the rotational speed for the engine, \( \omega_e \).

The Dynamic program works for finding the controller, and can be modified to work with improved state-equations.

**Keywords:** Fuel optimization, Hybrid electric vehicle, Dynamic programming, modeling
Sammanfattning

Det finns olika sätt att minska bränsleförbrukningen hos bilar, men ett sätt som använts är el-hybrider. Dessa kan minska bränsleförbrukningen med 10-30% jämfört med konventionella bilar.

I det här examensarbetet undersöks optimering av bränsleförbrukning för en el-hybrid, där hastigheten antas vara känd i förväg. Optimeringen skedde genom att först härleda en modell för drivlinan, och därefter skapades ett Dynamisk programerings baserat program för att hitta den optimal kombinationen av moment.


Modellen som härleddes visade i många fall liknande respons som GT-Suite, men viss förbättring kan ske. En utav dessa förbättringar är rotationsekvationen för bränslemotorn, $\omega_e$.

Den Dynamiska programmeringen som skapades fungerade, och kan modifieras för förbättrade tillståndsekvationer.
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Fredrik Bäberg
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Chapter 1

Introduction

In order to avoid global warming and to resolve energy resources issues it is necessary to improve the fuel consumption of automobiles. This has led to various different technologies for internal combustions engines and hybrid electric vehicles (HEV). Many of these technologies incorporate very advanced algorithms in order to resolve the problem. Unfortunately, although Europe and the US have already strengthened their cooperation with the academic world in order to implement and find new and better control algorithms [1], Japan lags behind [2]. This is a major drawback, considering that Japan is one of the major car exporters in the world with household names such as Toyota, Mazda, Mitsubishi, Honda and Subaru just to mention a few. Therefore it is encouraged that the Japanese industry and Japanese Universities start a joint research for a sustainable future. This is the reason why JSAE (Society of Automotive Engineers of Japan), SICE (The Society of Instrument and Control Engineers) and Japanese automobile industry created ”The Technical Committee on Vehicle Control and Modeling” which came about in the year 2009. The committee posts so called benchmark problems, which are good research themes in the academics as well as having useful applications in the industry. The benchmark problem investigated in this report will be the ”Fuel Consumption Optimization of Commuter Vehicle Using Hybrid Powertrain”.

What is so special with a hybrid powertrain is that it may combine driving force from two power sources, for instance an internal combustion engine and an electrical motor, and can use the energy accumulated by the internal combustion engine freely as either a driving force or as to charge a battery. The internal combustion engine can therefore work under optimal conditions where it can achieve high thermal efficiency on the hybrid powertrain, but not without advanced algorithms. These algorithms are what will be researched in this report such that the fuel consumption of a hybrid powertrain is minimized.
Chapter 2

Background Material

An interest of the hybrid vehicles has grown considerably in the last 15-20 years, although they have existed since before the 1900 [3]. One of the reasons is that different control systems can be used to improve fuel consumption within ranges of 10 – 30% compared with conventional vehicles. The main objective of the energy-management strategy for HEV are all the same, to minimize fuel consumption along some given route. In their article [3], Sciarretta and Guzella brings forth the different methods used for fuel consumption optimization and classifies some research groups active in HEV energy control with their respective method used.

The energy-management approaches are typically divided into two groups, they are heuristic strategies and optimal strategies [4].

The heuristic approaches are usually implemented in real time and are rule based. An example is the Fuzzy logic approach which is used by Schouten et al. [5] and Baumann et al. [6].

The second approach, optimal strategies, uses tools from the optimal control theory and appropriate dynamical models to find the optimal controllers. There are two different kinds of methods [7],[8], global optimal methods that solves the entire problem as a whole, and local optimization methods which divides the global problem into smaller local problems. Under the global methods we find for example dynamic programming (DP) and Pontryagins minimum principle (PMP). Under the local methods, which may use some information about future or past driving conditions although not all of it, we find for example stochastic dynamic programming (SDP), model predictive control (MPC) and equivalent energy consumption minimization (ECMS).

In order to apply theories from optimal control one first need to find some kind of mathematical model describing the dynamics of the HEV. Such attempts have thoroughly been done by Syed et al. [9], Liu et al. [10] and Mansour et al. [11] among others.

After arriving at a successful model, the optimal control strategy chosen depends strongly on what kind of prior information one has. If for instance all knowledge about the future route is known, then DP is a good alternative. This approach is
often used as an benchmark [1,14]. For example, Liu et al. uses DP to compare the performance of an SDP and an ECMS approach in [12]. Although there are existing articles that try to simplify the model enough in order to be able to use DP in real time implementation [13].

There are also approaches that use DP to extract specific rules [7], thus a mixture between an optimal strategy and a heuristic method.

The other global method mentioned above is PMP. The problem though with PMP is that the solution might not be the global optimal solution, however it is faster than DP, which is one of the reasons for Kim et al. to use this method to try to find a real-time PMP algorithm in their article [14]. Also here we find an intermediate approach which this time bases their control rules on PMP [15].

However, although the global optimal solutions are the most attractive, they do demand full knowledge of the given route, which might not always be available. Local methods are therefore well suited for the challenge of finding satisfying energy management controllers with less prior knowledge.

One of the problem faced without knowledge of the routes speed and acceleration is that of the demanded power. This problem is directed in Lin et al. [16], where the power demand is modeled as random Markov process. The optimal control strategy is then obtained using SDP.

Future driving conditions can also be predicted with the help of Intelligent Transportation Systems, this has for example been done in [17], where they use this information in order to establish a prediction based real-time controller structure using MPC. This approach is showed to be comparable to ECMS when the velocity and load prediction is noise free.

Borhan et. al [18] considers two approaches to MPC, and compares them to a controller available in the commercial software PSAT. The first is linear time-varying MPC, which results in an improvement of fuel consumption compared to the PSAT controller, except for one case where the result is similar. For a second approach nonlinear MPC is used, where an improvement of fuel consumption is achieved compared to both LTV-MPC and the PSAT controller.

The last local method mentioned above, ECMS, was first introduced by Paganelli in the year 1999. The strategy is based on the idea that the stored electric energy only functions as a energy buffer and that in the end the energy always comes from the fuel, even the energy from the battery. This is because the energy coming from the battery always has to be replenished later by the fuel from the engine, either directly from the engine or indirectly trough regenerative braking. So in both charge and discharge phase, a virtual fuel consumption may be added to the actual fuel consumption to obtain the instantaneous equivalent fuel consumption which is to be minimized at each moment [1]. The ECMS approach has proven very effective, for example in Sciaretta et al. paper [19], they show how the ECMS almost achieves the same fuel consumption results as that coming from the benchmark DP approach. This might be the reason for Sivertsson to develop an adaptive control strategy based on a map-ECMS approach for the PHEV benchmark problem that was organized by IFP Energies nouvelles in 2012 in [20].
Chapter 3

Problem Description and Background Information

There are at least two kinds of HEVs: Charge-sustaining and plug-in HEV (PHEV). The former aims to keep the state of charge of the battery at a given level, while the latter does not have this requirement. In the benchmark paper [2] it is not specified which kind of HEV is used. However the problem formulation is equivalent in the two different cases with the exception of the end state for the battery, and therefore doesn’t need to be included in the model derivation.

3.1 Hybrid electrical vehicle

The HEV in the considered benchmark problem is equipped with a split type hybrid powertrain. Two electrical motors, EM1 and EM2, plus an engine which are linked together by a planetary gear set, see Fig. 3.1.

![Figure 3.1. Schematic figure of the planetary gear.](image)

EM1 is linked to the sun gear, the engine is linked to the carrier gear and EM2 is linked to the ring gear in the planetary gear set. Both EM1 and EM2 are able to drive the vehicle by using energy from a battery and they can both generate electric power to charge this same battery. EM1 can also be used to start the engine.
CHAPTER 3. PROBLEM DESCRIPTION AND BACKGROUND INFORMATION

However in this study we will only consider the following 5 modes to be possible, and use them in order to construct an energy management strategy.

1. The engine drives the vehicle.
2. EM2 drives the vehicle.
3. Both the engine and EM2 drives the vehicle.
4. The engine generates electrical power using EM1 and charges the battery.
5. EM2 generates electric power when the vehicle is decelerated.

3.1.1 Traffic conditions

For series production HEV, the control algorithm managing the hybrid powertrain is usually designed so that it will achieve moderate fuel consumption for driving situations of all users [2]. This means that it is not optimized for the best fuel consumption for a specific user and driving condition, so an optimization taking regards to the specific driving condition can potentially lower the fuel consumption in comparison to the general case, although more knowledge about the specific route is needed. For a general user, the following can describe common situations,

- Going to the office.
- Going back home.
- Driving on weekends.

When driving to the office on weekdays, there may be traffic jams due to a huge number of commuter vehicles, which results in lower and more varying speeds. While when going home, the traffic might be more disperse, resulting in less traffic jams and higher speeds. The third situation, during weekends, might show a very different driving pattern from that of the weekdays, usually depending on the agenda of the day, see [2] for samples of the three situations.

3.1.2 Problem formulation

The problem considered is to minimize the fuel consumption of an HEV that reconciles the drivers demanded vehicle velocity, $v_d$, by designing a control algorithm for the powertrain. The control algorithm should decide on the power distribution between the combustion engine and the electric motors in order to let the vehicle have a desired speed. At the same time, other constraints should be considered, such that keeping the battery charge (State of charge, $SOC$) within a given level. Also, it is stated in [2] that it is generally known that the fuel economy of a vehicle is improved if the acceleration performance is restricted. However, there must be a trade-off between the drivers demand to accelerate fast and the benefit of restricted
3.1. HYBRID ELECTRICAL VEHICLE

acceleration. Therefore, one of the constraints is also to keep a so called driver satisfaction parameter, $S_d$, at 90% or higher. This parameter is defined as

$$S_d(k) = S_d(k-1) + \Delta S_d(k),$$

where

$$\Delta S_d(k) = \begin{cases} 
0, & |v_d(k) - v_a(k)| \leq \delta_{Vs} \\
0.1P_n, & \delta_{Vs} < |v_d(k) - v_a(k)| \leq \delta_{VL} \\
P_n, & \delta_{VL} < |v_d(k) - v_a(k)|.
\end{cases}$$

We have $\delta_{Vs} = 7.5 \text{ km/h}$, $\delta_{VL} = 15 \text{ km/h}$, $P_n = -1$ and $S_d(0) = 100$. $v_d(k)$ denotes the demanded reference speed and $v_a(k)$ denotes the actual speed at timestep $k$. The number of steps of each route varies.
Chapter 4

Mathematical Model of the HEV

In order to derive an optimal control algorithm for the HEV, system equations are needed in state space form describing the vehicle. A mathematical model based on some simplified physics is therefore derived below. Notation of variables and values of parameters are given in Table A.1 in Appendix A.

4.1 Model derivation

4.1.1 Vehicle dynamics

The HEV uses a planetary gear which enables the vehicle to use different control modes, as described in the previous chapter. EM1 is connected to the sun gear, the engine is connected to the carrier gear and EM2 is connected to the ring gear. Fig. 4.1 is an illustration on how the different parts are linked together and how they affect each other. The torques direction is positive to the left as indicated by $\omega_+$. The dotted box indicates the planetary gear set. It is assumed that the planetary gears share a collective loss which does not depend on which part the power is coming from, i.e if it is from the engine, EM1 or EM2 in order to simplify the derivation. Since the primary function of EM1 is as a generator, and of EM2 as a motor, they will be called generator and motor, abbreviated \(g\) and \(m\).

By applying the rigid body equation for a fixed axis [21], the following equations may be derived for the planetary gears seen in Fig. 4.1

\begin{equation}
\dot{\omega}_s I_s = F \cdot R_s - T_s, \tag{4.1}
\end{equation}

\begin{equation}
\dot{\omega}_c I_c = T_c - F(R_r + R_s), \tag{4.2}
\end{equation}

\begin{equation}
\dot{\omega}_r I_r = F \cdot R_r - T_r, \tag{4.3}
\end{equation}

where \(T_s\), \(T_c\) and \(T_r\) are the torques on the sun gear, carrier gear and the ring gear. Their respective inertia is denoted \(I_s\), \(I_c\) and \(I_r\). \(R_r\) and \(R_s\) represent the ring
CHAPTER 4. MATHEMATICAL MODEL OF THE HEV

Figure 4.1. Torques working on the planetary gear set.

gear and the sun gears radius from the center, \( F \) denotes the internal force working between the gears.

The equations for the generator, engine and motor may be derived in similar fashion,

\[
\dot{\omega}_g I_g = T_g + T_s, \quad (4.4)
\]

\[
\dot{\omega}_e I_e = T_e - T_c, \quad (4.5)
\]

\[
\dot{\omega}_m I_m = T_m + T_r - T_d, \quad (4.6)
\]

where \( T_d \) is shown in Fig 4.2. The definition of \( T_g \) is such that when \( T_g \) and \( \omega_g \) has opposite signs, i.e. \( P_g = T_g \omega_g < 0 \), the generator will charge the battery. This will be more clear when the model of the battery is developed.

Now Eq. (4.4) and (4.1) together with \( \dot{\omega}_g = \dot{\omega}_g \) gives

\[
\dot{\omega}_g (I_g + I_s) = F \cdot R_s + T_g. \quad (4.7)
\]

Eq. (4.5) and (4.2) together with \( \dot{\omega}_e = \dot{\omega}_e \) gives

\[
\dot{\omega}_e (I_e + I_c) = T_e - F \cdot (R_r + R_s). \quad (4.8)
\]

And Eq. (4.6) and (4.3) together with \( \dot{\omega}_m = \dot{\omega}_m \) gives

\[
\dot{\omega}_m (I_r + I_m) = F \cdot R_r + T_m - T_d. \quad (4.9)
\]

From Fig 4.2 we may derive the following relationships, which will eventually lead to an expression for \( \dot{\omega}_m \). Note that for the inertia of the vehicle, we consider the
4.1. MODEL DERIVATION

![Diagram of power balance between the ring gear and the power acting externally on the vehicle.]

Figure 4.2. Power balance between the ring gear and the power acting externally on the vehicle.

The vehicle as a point mass, so that $I_v = MR_{tire}^2$. For the driveshaft, differential-gear, 4 axles and finally the vehicle we get

$$\dot{\omega}_d I_d = \eta_m G_f T_d - T_{dg}, \quad (4.10)$$

$$\dot{\omega}_{dg} I_{dg} = T_{dg} - T_a, \quad (4.11)$$

$$\dot{\omega}_a I_a = \eta_{dg} T_a - T_v, \quad (4.12)$$

$$\dot{\omega}_v I_v = T_v - T_f. \quad (4.13)$$

$\eta_m$ represents the energy losses that are present in the planetary-gear. $G_f$ is the differential gear ratio, i.e. how fast the rotational speed of the driveshaft is compared to the rotational speed of the ring gear. $\eta_{dg}$ represents the transmission efficiency of the differential gear. Eq. (4.10), (4.11), (4.12) and (4.13) with $\dot{\omega}_v = \dot{\omega}_a = \dot{\omega}_{dg} = \dot{\omega}_d$, eliminating $T_{dg}, T_a$ and $T_v$, gives

$$\dot{\omega}_d (I_v + 4I_a + \eta_{dg} I_{dg} + \eta_{dg} I_d) = \eta_{dg} \eta_m G_f T_d - T_f. \quad (4.14)$$

Eq. (4.9) and (4.14) with $\omega_d = \omega_m/G_f$ gives

$$\dot{\omega}_m (\frac{I_v + 4I_a + \eta_{dg} I_{dg} + \eta_{dg} I_d}{G_f} + \eta_{dg} \eta_m G_f (I_r + I_m)) = \eta_{dg} \eta_m G_f (F \cdot R_r + T_m) - T_f. \quad (4.15)$$

In Eq. (4.15) $T_f$ is an expression involving the friction brake and the opposing forces [22]. The expression for $T_f$ is

$$T_f = T_{fb} + R_{tire} (\mu_r M g \cos(\theta) + \frac{1}{2} \rho AC_d v^2 + M g \sin(\theta)). \quad (4.16)$$
The first term on the right hand side of Eq. (4.16) is $T_{fb}$ which is the torque caused from friction braking. The next one $\mu F R \cos(\theta)$ is caused by the friction forces between the wheels and the road. $\frac{1}{2} \rho AC_d v^2$ is the friction forces caused by the air resistance and last $M g \sin(\theta)$ is the forces caused by gravity.

The planetary gear functions as a speed-summing unit [22], with the angular velocities related as

$$\omega_m R_r + \omega_g R_s = \omega_e (R_r + R_s)$$  \hfill (4.17)

There are now four unknown variables, $\omega_g$, $\omega_e$, $\omega_m$, and $F$ and four equations, (4.7), (4.8), (4.15), and (4.17). With these equations we may solve for $\omega_e$ and $\omega_m$, which will constitute two out of three state variables that will be considered. The last one will be the state of charge (SOC). The results are,

$$\dot{\omega}_e = \frac{(I'_g R^2 + I'_g R^2 \eta) T_e + (R_r + R_s) I'_g R_r \eta T_m}{I'_g I'_e (R_r + R_s)^2 + I'_e I'_e R^2 + I'_g I'_g R^2 \eta} \hfill \tag{4.18}$$

and

$$\dot{\omega}_m = \frac{(R_r + R_s) R_s I'_g \eta T_e + (I'_g \eta (R_r + R_s)^2 + I_e \eta R^2 \eta) T_m}{I'_g I'_e (R_r + R_s)^2 + I'_e I'_e R^2 + I'_g I'_g R^2 \eta} - \frac{I'_e R_r R_s \eta T_g - (I'_e R^2 + I'_g (R_r + R_s)^2) T_f}{I'_g I'_e (R_r + R_s)^2 + I'_e I'_e R^2 + I'_g I'_g R^2 \eta}, \hfill (4.19)$$

where

$$I'_g = I_g + I_s, \hfill (4.20)$$

$$I'_e = I_e + I_c, \hfill (4.21)$$

$$I'_b = \frac{I_b + 4 I_a + \eta_{dg} I_a + \eta_{dg} I_d}{G_f} + \eta_{dg} \eta_m G_f (I_r + I_m) \hfill (4.22)$$

and

$$\eta = \eta_{dg} \eta_m G_f \hfill (4.23)$$

### 4.1.2 Battery

The third state is the state of charge, or $SOC$, given in % where 100% represents a fully charged battery and 0% a depleted battery. The $SOC$ will be represented by a number between 0 and 1. The dynamics is given by

$$SOC = - \frac{I_b}{Q_b} \hfill (4.24)$$

12
where $I_b$ is the battery current and $Q_b$ is the maximum capacity of the battery. For modeling the battery an internal resistance model is used, as shown in Fig. 4.3, where $U$ is the battery voltage, $U_{oc}$ the open circuit voltage of the battery and $R_b$ denotes the internal resistance of the battery. The model is simplified in that it does not describe all the dynamics in detail. This is an approach that has been used earlier, for instance in [10] and [11].

From basic electric circuit analysis, we have Ohm’s law

$$U = R \cdot I,$$

$$P = U \cdot I.$$  \hspace{1cm} (4.25)

From this it follows that the power output from the battery can be written as

$$P_b = UI_b.$$  \hspace{1cm} (4.26)

Using Kirchoff’s voltage law to find $U = U_{oc} - U_b$, where $U_b = R_bI_b$, Eq. (4.26) becomes

$$P_b = U_{oc}I_b - I_b^2R_b.$$  \hspace{1cm} (4.27)

By solving Eq. (4.27) for $I_b$, we get

$$I_b = \frac{U_{oc} \pm \sqrt{U_{oc}^2 - 4P_bR_b}}{2R_b}.$$  \hspace{1cm} (4.28)

From this the state equation for $SOC$, which is the final state equation needed, is given by

$$\dot{SOC} = - \left( \frac{U_{oc} - \sqrt{U_{oc}^2 - 4P_bR_b}}{2R_bQ_b} \right).$$  \hspace{1cm} (4.29)
with the minus sign in Eq. (4.28) since with $P_b = 0$ the SOC should be constant. It can be noted that when $P_b < 0$ the battery is charging, since $SOC > 0$ and when $P_b > 0$ the battery is discharging. Furthermore, since the battery power flow is either from or to the motor or the generator we have the following relationship,

$$P_b = \eta_{me} P_m + \eta_{ge} P_g,$$

(4.30)

where

$$\eta_{me} = \begin{cases} \eta_{me, discharge}, & P_m > 0 \\ \eta_{me, charge}, & P_m < 0. \end{cases}$$

while $\eta_{ge}$ only takes on one value since it is considered to only be able to charge the battery.

**Voltage**

Since the voltage $U_{oc}$ varies with the SOC, a mathematical model is needed. GT-Suite\(^1\), which will be considered the reference model, includes a map of the battery voltage at different levels of the SOC. From the map it can be seen that the voltage takes values between 202 and 237.29 V.

By performing a polynomial fit to this data, the expression obtained was

$$U_{oc}(SOC) = a_{u0} + a_{u1} SOC + a_{u2} SOC^2 + a_{u3} SOC^3 + a_{u4} SOC^4 + a_{u5} SOC^5.$$  (4.31)

The values of the coefficients are

$$a_{u0} = 202.0361 \quad a_{u1} = 95.3121 \quad a_{u2} = -284.6488 \quad a_{u3} = 604.1528 \quad a_{u4} = -696.0737 \quad a_{u5} = 316.5064.$$  

A plot of $U_{oc}(SOC)$ compared to the data from the map is seen in Fig. 4.4.

---

\(^{1}\)GT-Suite HEV model is provided by Yuji Yasui, Honda R&D Co.
4.1. MODEL DERIVATION

Figure 4.4. A fifth degree polynomial approximation of the voltage as a function of $SOC$.

**Battery Resistance**

The battery resistance $R_b$ also needs to be modelled. In a similar way as for the voltage, a map from GT-Suite is used. From this it can be seen that the resistance depends on both the $SOC$ and if the battery is charging or discharging. The approximation is given as

$$R_b(SOC) = \begin{cases} 
  a_{d0} + a_{d1}SOC + a_{d2}SOC^2 + a_{d3}SOC^3 + \cdots \\
  a_{d4}SOC^4 + a_{d5}SOC^5 + a_{d6}SOC^6, & \text{discharging} \\
  a_{c0} + a_{c1}SOC + a_{c2}SOC^2 + a_{c3}SOC^3 + \cdots \\
  a_{c4}SOC^4 + a_{c5}SOC^5 + a_{c6}SOC^6, & \text{charging} 
\end{cases}$$

(4.32)

with coefficients

- $a_{d0} = 0.7022, a_{d1} = 0.1398, a_{d2} = -17.1832, a_{d3} = 76.0844$
- $a_{d4} = -138.7320, a_{d5} = 115.9729, a_{d6} = -36.5850$
- $a_{c0} = 0.7012, a_{c1} = 0.1422, a_{c2} = -15.1353, a_{c3} = 64.0950$
- $a_{c4} = -112.5153, a_{c5} = 91.2406, a_{c6} = -28.1699$

See Fig. 4.5 and 4.6 for an illustration of Eq. (4.32). From the map it is also found that the resistance is between 0.357 and 0.7 $\Omega$. 

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Electric efficiency of the motor and generator

In the problem description the electric efficiency of the generator and motor, \( \eta_{ge} \) and \( \eta_{me} \), are not given. Therefore they are estimated by using data from GT-Suite. From the equation for \( P_b \) (4.30),

\[
P_b = \eta_{me} P_m + \eta_{ge} P_g
\]

and the state equation for the \( SOC \) (4.29),

\[
\dot{SOC} = - \frac{U_{oc} - \sqrt{U_{oc}^2 - 4P_b R_b}}{2R_b Q_b}
\]

we find

\[
\dot{SOC} = - \frac{U_{oc} - \sqrt{U_{oc}^2 - 4R_b(\eta_{me} P_m + \eta_{ge} P_g)}}{2R_b Q_b}.
\] (4.33)

By obtaining the state of charge, \( SOC \) from GT-Suite in specific intervals were the motor and generator power is either only positive or negative, we can estimate \( U_{oc} \) from Eq. (4.31), \( R_b \) from Eq. (4.32) and approximate \( \dot{SOC} \) from Euler forward. With these values together with the power of the motor, \( P_m \) and the generator, \( P_g \),
4.1. MODEL DERIVATION

also received from GT-Suite we can find an approximate value of the efficiencies through an iterative least squares estimation. After this had been done we plugged these values into our model and compared it to the GT-Suite response, we found that by manually adjusting these values we could come closer to the GT-Suite response. The estimates obtained were found to be \( \eta_{ge} = 0.8425 \),

\[
\eta_{me} = \begin{cases} 
\eta_{me,\text{disscharge}} = 1.258, & P_m > 0 \\
\eta_{me,\text{charge}} = 0.818, & P_m < 0
\end{cases}
\]

4.1.3 Fuel Consumption

For modeling the fuel consumption \( m_f \), the equation

\[
BSFC = \frac{\dot{m}_f}{P_e},
\]

relating the fuel flow with the power and the BSFC (Brake Specific Fuel Consumption) [23], will be used. The BSFC is measured from the engine, and is provided as a map in GT-Suite. By approximating the map as a function of the engine torque and the
engine rotational speed expressed in [rpm], the following function can be found

$$BSFC(T_e, N_e) = b_0 + b_1 N_e + b_2 T_e + b_3 N_e^2 + b_4 N_e T_e + b_5 T_e^2 + b_6 N_e^3 + b_7 N_e^2 T_e + b_8 N_e T_e^2 + b_9 T_e^3$$

with coefficients

- \(b_0 = 6.9933 \cdot 10^2\)
- \(b_1 = -6.5760 \cdot 10^{-3}\)
- \(b_2 = -1.8639 \cdot 10^1\)
- \(b_3 = 3.1841 \cdot 10^{-6}\)
- \(b_4 = -9.7399 \cdot 10^{-5}\)
- \(b_5 = 2.27 \cdot 10^{-1}\)
- \(b_6 = -2.6412 \cdot 10^{-10}\)
- \(b_7 = 2.1534 \cdot 10^{-8}\)
- \(b_8 = -8.4366 \cdot 10^{-7}\)
- \(b_9 = -8.5085 \cdot 10^{}\)

The unit of BSFC is \(\frac{g}{kW\cdot h}\). Since \(P = T \omega\) the fuel consumption per second can be calculated as

$$m_f = \frac{T \omega_e BSFC(T_e, N_e)}{3.6 \cdot 10^6}, \quad \left[\frac{g}{s}\right]$$

A plot of BSFC as a function of \(T_e\) and \(N_e\) can be seen in Fig. 4.7. The BSFC takes values between 201 and 720 \(\frac{g}{kW\cdot h}\) according to the GT-Suite map.

![Figure 4.7. Approximation of BSFC as a function of \(T_e\) and \(N_e\), together with map data from GT-Suite](image-url)
4.2. MODEL VERIFICATION

4.2 Model verification

4.2.1 Verification of $\omega_e$ and $\omega_m$

In order to verify the derived model, Eq. (4.18) and (4.19) was implemented in Simulink and compared with the GT-Suite response. It showed considerable dynamical similarities with the GT-Suite output. However, Jiangyan Zhang, at Sophia University 2013, had also found an empirically based model which showed even greater similarities with the GT-Suite response. In Fig. 4.8 and 4.9 the two models for $\omega_e$ are compared with the GT-Suite response, and in Fig. 4.10 and 4.11 a comparison is made for $\omega_m$. For $\omega_e$ and $\omega_m$ we also found that the two derived dynamical equations would follow better with some constraints. For $\omega_e$ we set if $\omega_e(k-1) \leq 0$ and $\dot{\omega}_e(k) < 0$ or $\omega_e(k) < 0$ then $\omega_e(k) = 0$. If $\omega_e(k-1) \geq \omega_{e,max}$ and $\dot{\omega}_e(k) > 0$ or $\omega_e(k) > \omega_{e,max}$ then $\omega_e(k) = \omega_{e,max}$. We found similar constraints for $\omega_m$, if $\omega_m(k-1) \leq 0$ and $\dot{\omega}_m(k) < 0$ or $\omega_m(k) < 0$ then $\omega_m(k) = 0$. These constraints are set so that $\omega_e$ and $\omega_m$ does not go outside the feasible range.

![Figure 4.8](image.png)

**Figure 4.8.** Comparison of $\omega_e$ between the derived model, the empirical model, and the GT-Suite output, for a time period of 160 s.
Figure 4.9. Comparison of $\omega_e$ between the derived model, the empirical model, and the GT-Suite output, for a time period of 2600 s.
4.2. MODEL VERIFICATION

![Graph comparing angular velocity](image1)

**Figure 4.10.** Comparison of $\omega_m$ between the derived model, the empirical model, and the GT-Suite output, for a time period of 160 s.

![Graph comparing angular velocity](image2)

**Figure 4.11.** Comparison of $\omega_m$ between the derived model, the empirical model, and the GT-Suite output, for a time period of 2600 s.

The coefficients from Eq. (4.18) and (4.19), with values from data given in Table A.1 in Appendix A, is presented together with the coefficients from the empirically derived model in Table 4.4.

**Table 4.4:** Comparison of coefficients from the derived model and the GT-Suite empirically derived model.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$T_e$</th>
<th>$T_m$</th>
<th>$T_g$</th>
<th>$T_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_e$, Derived</td>
<td>2.0086</td>
<td>0.0567</td>
<td>7.0838</td>
<td>-0.0148</td>
</tr>
<tr>
<td>$\omega_e$, Empirical</td>
<td>2.6841</td>
<td>0.0876</td>
<td>10.2663</td>
<td>-0.0153</td>
</tr>
<tr>
<td>$\omega_m$, Derived</td>
<td>0.0567</td>
<td>0.0785</td>
<td>-0.0947</td>
<td>-0.0300</td>
</tr>
<tr>
<td>$\omega_m$, Empirical</td>
<td>0.0616</td>
<td>0.1249</td>
<td>-0.1030</td>
<td>-0.0293</td>
</tr>
</tbody>
</table>
4.2.2 Verification of SOC

The mathematical model of the SOC was also compared with the SOC from the GT-Suite when the same power was applied. In Fig. 4.12 the SOC model with the derived efficiencies is compared to the actual SOC output from GT-Suite.

Since the SOC becomes complex when \( U_{oc}^2 - 4P_b R_b \) is negative, we set a restriction so that the square root term in Eq. (4.29) is excluded whenever it does not take on real numbers.

![Graph comparing SOC between GT-Suite and the derived model.](image)

**Figure 4.12.** Comparison of SOC between GT-Suite and the derived model.

4.3 State dynamical equations used

Although the derived dynamical models for \( \omega_e \) and \( \omega_m \), Eq. (4.18) and (4.19) show great dynamical similarities with the GT-Suite response, we decided to continue our research with the more accurate empirically derived model in order to eventually approach a lower fuel consumption. The dynamical models that are to be used are,

\[
\dot{\omega}_e = 2.6841T_e + 0.0876T_m + 10.2663T_g - 0.0153T_f \quad (4.37)
\]

\[
\dot{\omega}_m = 0.0616T_e + 0.1249T_m - 0.1030T_g - 0.0293T_f \quad (4.38)
\]

and the unchanged

\[
SOC = -\left( \frac{U_{oc} - \sqrt{U_{oc}^2 - 4P_b R_b}}{2R_b Q_b} \right). \quad (4.39)
\]
Chapter 5

Control and Optimization

We will now derive and state the optimization problem.

5.1 General form of the optimization problem

The general mathematical form of an optimal control problem usually contains similar features [24]. They are seen in Eq.(5.1)-(5.5),

\[
\begin{align*}
\text{minimize} & 
J(t_0, x_0, u(\cdot)) = \Phi(x(t_f)) + \int_{t_0}^{t_f} f_0(t, x(t), u(t)) dt \\
\text{subject to} & 
\dot{x}(t) = f(t, x(t), u(t)) \quad (5.2) \\
& x(0) = x_0 \quad (5.3) \\
& x(t) \in X(t) \quad (5.4) \\
& u(t) \in U(t), \quad (5.5)
\end{align*}
\]

where \(x(t)\) is the state vector and \(u(\cdot)\) an admissible control vector on \([t_0, t_f]\).

5.2 Dynamic Programming

5.2.1 General concepts

Dynamic programming is a method that solves an optimal control problem by dividing the problem into several sub-problems which are to be solved and eventually added into a final solution [25]. The method is based on Bellman’s principle of optimality, which can be stated as follows,

Let \(u^* : [t_0, t_f] \rightarrow \mathbb{R}^m\) be an optimal control for \(\min_{u(\cdot)} J(t_0, x_0, u(\cdot))\) that generates the optimal trajectory \(x^* : [t_0, t_f] \rightarrow \mathbb{R}^n\). Then, for any \(t' \in (t_0, t_f]\), the restriction of the optimal control to \([t', t_f]\), \(u^*|_{[t', t_f]}\), is...
optimal for \( \min_{u(t')} J(t', x^*(t'), u(\cdot)) \) and the corresponding optimal trajectory is \( x^*|_{[t', t_f]} \) [24].

This means that the optimal path from any of the intermediate steps to the end corresponds to the optimal solution at this point to the end.

### 5.2.2 Discrete Dynamic Programming

If the continuous optimal control problem in Eq. (5.1)-(5.5) instead is discretised, we find the multistage decision problem

\[
\text{minimize} \quad J(0, x_0) = \Phi(x_N) + \sum_{k=0}^{N-1} f_0(k, x_k, u_k) \\
\text{subject to} \quad \begin{align*}
x_{k+1} &= f(k, x_k, u_k) \\
x_0 &\text{ given} \\
x_k &\in X_k \\
u_k &\in U(k, x_k).
\end{align*}
\] (5.6)

The principle of optimality can in the discrete state be formulated as,

If \( \{u_k^*\}_{k=0}^{N-1} \) is an optimal control for the problem (5.6)-(5.10), then \( \{u_k^*\}_{k=n}^{N-1} \) is optimal for the subproblem obtained by considering an optimization on the problem (5.6)-(5.10) but with initial condition \((n, x^*(n))\) i.e., we restart the optimization from somewhere along the optimal path [24].

With

\[
J^*(n, x_n) = \min_{u \in U(n, x_n)} \left\{ f_0(n, x_n, u) + J(n+1, f(n, x_n, u)) \right\}, \quad n = N - 1, N - 2, \ldots, 0
\] (5.11)

and the principle of optimality we can find the following theorem.

**Theorem 1** Suppose there exist a finite solution to the backwards dynamic programming recursion

\[
J(N, x_N) = \begin{cases} 
\Phi(x_N), & x_N \in X_N \\
\infty, & x_N \notin X_N
\end{cases}
\]

\[
J(n, x_n) = \min_{u \in U(n, x_n)} \{ f_0(n, x_n, u) + J(n + 1, f(n, x_n, u)) \}, \quad n = N - 1, N - 2, \ldots, 0
\]

where the optimization over \( U(n, x_n) \) is restricted to those control variables for which \( f(n, x_n, u) \in X_{n+1} \). Then there exists an optimal solution to problem (5.6)-(5.10) and
5.3. HEV FUEL CONSUMPTION MINIMIZATION PROBLEM

a) \( J^*(n, x_n) = J(n, x_n) \) for all \( n = 0, \ldots, N, \ x_n \in X_n \).

b) The optimal feedback control in each stage is obtained as

\[ u^*_n = \mu(n, x_n) = \arg\min_{u \in U(n, x_n)} \{ f_0(n, x_n, u) + J(n + 1, f(n, x_n, u)) \}. \]


5.3 HEV fuel consumption minimization problem

In our specific problem we want to minimize the fuel consumption while still keeping the driver satisfaction parameter \( S_d \) within prescribed limits. However, since the difference in velocity \( |v_d(k) - v_a(k)| \) can only be above \( \delta_{Vs} = 7.5 \text{ km/h} \) 10 times before it is considered an infeasible solution, we decided to have a constraint saying that our solution gives a very large cost if \( |v_d(k) - v_a(k)| > \delta_{Vs} \), this in order for the solution to stay feasible also when implemented in GT-Suite.

In the discrete case the cost function in Eq.(5.6) has been decided to look like

\[ J(0, x_0) = \Phi(x_N) + \sum_{k=0}^{N-1} (\hat{m}_f(k, x_k, u_k) \Delta t + \Delta S(k)) \]  

(5.12)

where \( \Delta t = t_f/N, \ N \) being the number of time-steps considered and \( t_f \) is the final time. The function corresponding to \( f_0(k, x_k, u_k) \) consists of two terms, where the first term \( \hat{m}_f(k, x_k, u_k) \Delta t \) is the fuel consumed for every cycle, with the fuel consumption rate in \([g/s]\) given as

\[ \hat{m}_f(k, x_k, u_k) = \frac{P_e \cdot BSFC(T_e, N_e)}{3.6 \cdot 10^6} \]  

(5.13)

\( N_e \) denotes the engine rotational speed expressed in \([\text{rpm}]\) and \( T_e \) is the engine torque, see Section 4.1.3 The term \( \Delta S(k) \) in (5.12) is defined as

\[ \Delta S(k) = \begin{cases} 0, & |v_d(k) - v_a(k)| \leq \delta_{Vs} \\ P, & \delta_{Vs} < |v_d(k) - v_a(k)| \end{cases} \]

where \( P \) is set to be a sufficiently large real number to punish the cost of the solution if the velocity difference is too big. However this term will not be needed in the DP approach since the desired velocity will be assumed to be the actual velocity. \( \Phi(x_N) \) will be included to assure that \( x_N \in X_N \), this through letting

\[ \Phi(x_N) = \begin{cases} 0, & x_N \in X_N \\ \infty, & x_N \notin X_N \end{cases} \]

The control variables we can access are the torques of the engine, \( T_e \), the electric motor, \( T_m \), the generator, \( T_g \), and the friction brake, \( T_{fb} \), so we let
$u = [T_e, T_m, T_g, T_{fb}]^T$. The states we decided to have are the rotational speed of the engine, $\omega_e$, the rotational speed of the motor, $\omega_m$, and the state of charge, $SOC$, thus $x = [\omega_m, \omega_e, SOC]^T$, with their respective dynamics governed by Eq. (4.37), (4.38) and (4.39).

5.3.1 DP approach to HEV fuel consumption minimization problem

By using Dynamic Programming (DP) the optimal fuel consumption, and corresponding torques, can be determined for a given route. Since data for the route is needed beforehand, such as speed, acceleration and road slope, it is unsuitable for on-line implementation. However, it is well suited for obtaining benchmark values to compare other methods with.

Since we receive the data about desired velocity, desired acceleration and road slope in the form of discrete data the problem is well suited for the discrete DP. Moreover, in this case it is preferable to reduce the number of states and controllers in order to simplify the problem so that it can be implemented and eventually solved with a computer. This is because as the number of state variables increases the computational time increase exponentially [26]. For the DP we will assume that the desired velocity and acceleration is the actual velocity and acceleration. We can therefore remove one of the states $\omega_m$ using $\omega_m = v_s G_f / R_{tire}$ and reduce the controllers from three to one by the derivation below.

From Ch. 4 we have the two dynamical equations, Eq. (4.37) and (4.38), which can be stated as

\[
\begin{align*}
\dot{\omega}_e &= a_1 T_e + a_2 T_m + a_3 T_g + a_4 T_f \\
\dot{\omega}_m &= b_1 T_e + b_2 T_m + b_3 T_g + b_4 T_f.
\end{align*}
\]

We also found Eq. (4.30)

\[P_b = \eta_{me} P_m + \eta_{ge} P_g,\]

where $\eta_{me}$ and $\eta_{ge}$ denotes the efficiency of the motor and generator respectively. For $\eta_{me}$ we have

\[
\eta_{me} = \begin{cases} 
\eta_{me, \text{disscharge}}, & P_m > 0 \\
\eta_{me, \text{charge}}, & P_m < 0.
\end{cases}
\]

For the angular velocities we found

\[\omega_m R_e + \omega_g R_s = \omega_e (R_e + R_s).\]  

The demanded power, $P_d$, comes from either the engine, $P_e$, or the battery, $P_b$. Since we assume that there will be some energy losses from the engine and the battery the expression

\[P_d = \eta_e P_e + \eta_b P_b\]

will be used, where $\eta_e$ is the efficiency of the engine and $\eta_b$ is the efficiency of the battery power which depends on the sign of $P_b$. 

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\[ \eta_b = \begin{cases} \eta_{b,\text{discharge}}, & P_b > 0 \\ \eta_{b,\text{charge}}, & P_b < 0 \end{cases} \]

see Appendix [15]. \( P_b \) positive corresponds to the case of power going from the battery through the motor to the wheels. Negative values of \( P_b \) corresponds to power coming from the generator or motor to the battery.

We find \( P_d \), the power needed for a given speed, from

\[ P_d = \left( \sum F_R + M \dot{v}_a \right) v_a, \tag{5.19} \]

where \( \sum F_R \) is the external forces acting on the vehicle and \( v_a \) is the speed of the specific time period. By replacing \( \sum F_R \) with \( T_f \), where \( T_f \) is given by (4.16),

\[ T_f = T_{fb} + R_{tire}(\mu_r Mg \cos(\theta) + \frac{1}{2} \rho AC_d v_a^2 + Mg \sin(\theta)), \]

as a function of speed and slope, the value of \( P_d \) can be determined from knowing the velocity and acceleration.

Now using

\[ P = T \omega \tag{5.20} \]

with the above relationships, the number of controllers can be reduced from three to one.

There are three possible cases of \( P_d \) which will now be considered. In all the cases it is assumed that \( \omega_e > 0 \), and except for the case \( P_d = 0 \) it is assumed that \( \omega_m \neq 0 \) by the definition of \( P_d \).

\( P_d > 0 \) When the demanded power is positive the torque of the engine, \( T_e \), can be expressed with the help of Eq. (5.18) and (5.20) as a function of \( \omega_e \), \( P_d \) from (5.19), and \( P_b \). We find that

\[ P_d = \eta_e P_e + \eta_b P_b \implies P_e = \frac{P_d - \eta_b P_b}{\eta_e} \implies T_e = \frac{P_d - \eta_b P_b}{\eta_e \omega_e}. \tag{5.21} \]

Eq. (5.16) can be used to find an expression for \( T_m \). Using (5.16), (5.20) and the fact that the motor uses energy from the battery to drive the vehicle, i.e. \( P_m > 0 \),

\[ P_b = \eta_{m,\text{discharge}} P_m + \eta_{ge} P_g \implies P_m = \frac{P_b - \eta_{ge} P_g}{\eta_{m,\text{discharge}}} \]

\[ \implies T_m = \frac{P_b - \eta_{ge} T_g \omega_g}{\omega_m \eta_{m,\text{discharge}}}. \tag{5.22} \]

If now (5.14) and (5.15) is used to eliminate \( T_m \), an expression for \( T_g \) can be obtained on the form

\[ T_g = \beta_1 \dot{\omega}_m + \beta_2 \dot{\omega}_e + \beta_3 T_e + \beta_4 T_f. \tag{5.23} \]
By inserting Eq. (5.21), (5.22) and (5.23) into (5.14) an expression for $\omega_e$ which does not depend on the torques, but rather on $P_b$, can be obtained on the form

$$\gamma_1 \omega_e = \gamma_2 \omega_m + \gamma_3 P_d + \gamma_4 P_b + \gamma_5 T_f,$$

(5.24)

where $\gamma_i, i = 1, \ldots, 5$ are coefficients depending on $\omega_e$ and $\omega_m$. More details about the coefficients are given in appendix C. In the expression for $T_f$ seen in Eq. (5.24), $T_{fb} = 0$. Also in Eq. (5.24), the battery power can be either positive or negative. If it is positive, i.e. discharging the battery, it would not be effective to provide more power than necessary. Also, the power cannot be more than what the motor can provide, therefore the upper bound on $P_b$. If the battery power is negative it will be assumed to be limited by the generator, the range for the battery power will therefore be

$$P_b \in \left[ \max \left( -\eta_{ge} P_{g,\text{max}}, \min \left( 0, \frac{\eta_{ge}}{\omega_m} \left( \frac{P_d}{\eta_e} - P_{e,\text{max}} \right) \right) \right), \min \left( P_{m,\text{max}} \frac{\eta_{me,\text{disscharge}}}{\omega_m}, \frac{P_d}{\eta_b} \right) \right].$$

(5.25)

$P_d = 0$ With $P_d = 0$ the vehicle should not provide any translational power, although the engine can still charge the battery with the help of the generator. For the engine power the expression

$$P_e = -\frac{P_b}{\eta_{ge}} \Rightarrow T_e = -\frac{P_b}{\eta_{ge} \omega_e}.$$  

(5.26)

is used. Since there is no reason to provide any further power, $P_m = 0 \Rightarrow T_m = 0$. So using this fact with Eq. (5.16)

$$P_g = \frac{P_b}{\eta_{ge}} \Rightarrow T_g = \frac{P_b}{\eta_{ge} \omega_g},$$

(5.27)

assuming $\omega_g \neq 0$. Using (5.26), (5.27) and $T_m = 0$ it follows that

$$\dot{\omega}_e = \left( \frac{a_3}{\eta_{ge} \omega_g} - \frac{a_1}{\eta_{ge} \omega_e} \right) P_b + a_4 T_f.$$  

(5.28)

In case $\omega_g = 0$ we let $T_g = T_{g^*}$, constant, so that

$$\dot{\omega}_e = -\frac{a_1}{\eta_{ge} \omega_e} P_b + a_3 T_{g^*} + a_4 T_f.$$  

(5.29)

We summarize these equations as

$$\dot{\omega}_e = \epsilon_1 P_b + \epsilon_2 T_f + \epsilon_3,$$  

(5.30)

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where $\epsilon_3$ is a constant containing $T_g^*$. Also in this equation $T_{fb} = 0$, which is a term included in the expression for the opposing torques $T_f$. In Eq. (5.30), the motor is assumed to not be used, but the generator can charge the battery. The battery power $P_b$ will therefore be limited to

$$P_b \in [-\eta_{ge} P_{g,max}, 0]. \quad (5.31)$$

$P_d < 0$ When the demanded power is negative the engine should not have to provide power. Therefore $P_e = 0$ is assumed, which implies that $T_e = 0$. This also gives $T_g = 0$ since the generator is assumed to only work when charging the battery with energy coming from the engine. Since the motor has a maximum power limit, $P_b$ will be limited. It is further assumed that it is best to generate as much power as possible, therefore we find,

$$P_b = \max\left( P_{b_{min}}, P_d \eta_d \right),$$

where $\eta_d$ is an efficiency from demanded power to the battery and $P_{b_{min}} = -\eta_{me,\text{charge}} P_{m,max}$. The remaining power needed will be provided by the friction brakes, $T_{fb}$, when $P_b = P_{b_{min}}$, given by

$$T_{fb} = \begin{cases} \left( \frac{P_e + P_{b_{min}}/\eta_d}{\omega_m} \right) G_f, & P_b = P_{b_{min}} \\ 0, & \text{else.} \end{cases}$$

From Eq. (5.16) and $P_m < 0$, the expression for $T_m$ becomes

$$T_m = \frac{P_b}{\eta_{me,\text{charge}} \omega_m}. \quad (5.32)$$

Using Eq. (5.32), $T_g = 0$ and $T_e = 0$, the expression for $\dot{\omega}_e$ becomes

$$\dot{\omega}_e = \frac{a_2}{\eta_{me,\text{charge}} \omega_m} P_b + a_4 T_f. \quad (5.33)$$

We summarize the equation as

$$\dot{\omega}_e = \kappa_1 P_b + \kappa_2 T_f. \quad (5.34)$$

Where

$$P_b = \max\left( -\eta_{me,\text{charge}} P_{m,max}, P_d \eta_d \right). \quad (5.35)$$

The only states that are needed now are $\omega_e$ and SOC, with the single controller $P_b$. By using Euler forward, the problem can be discretized and stated in the form required for discrete DP. The full problem can now be stated as follows.
CHAPTER 5. CONTROL AND OPTIMIZATION

\[
\minimize_{P_b(k) \in U(P_d)} \Phi(x_N) + \sum_{k=0}^{N-1} \frac{P_e \cdot BSFC(T_e, N_e) \cdot \Delta t}{3.6 \cdot 10^6}
\]

subject to

\[
\begin{align*}
\omega_e(k + 1) &= \omega_e(k) + f_e(\omega_e, \omega_m, \dot{\omega}_m, P_d, P_b) \cdot \Delta t \\
SOC(k + 1) &= SOC(k) - \left( \frac{U_{oc} - \sqrt{U_{oc}^2 - 4P_bR}}{2R_bQ_b} \right) \cdot \Delta t \\
\omega_e(0) &= \omega_{e0} \\
SOC(0) &= SOC_0 \\
\omega_e &\in [\omega_{emin}, \omega_{emax}] \\
SOC &\in [SOC_{min}, SOC_{max}] \\
P_b &\in U(P_d).
\end{align*}
\]

Where

\[
f_e(\omega_e, \omega_m, \dot{\omega}_m, P_d, P_b) = \begin{cases} 
(\gamma_2\omega_m + \gamma_3P_d + \gamma_4P_b + \gamma_5T_f) / \gamma_1, & P_d > 0 \\
\epsilon_1P_b + \epsilon_2T_f + \epsilon_3, & P_d = 0 \\
\kappa_1P_b + \kappa_2T_f, & P_d < 0.
\end{cases}
\]

and \(U(P_d)\) equals

\[
P_d > 0 \\
P_{b,low} = \max \left( -\eta_{ge}P_{g,\text{max}}, \min \left( 0, \eta_{ge} \left( \frac{P_d}{\eta_e} - P_{c,\text{max}} \right) \right) \right) \\
P_{b,high} = \min \left( P_{m,\text{max}}\eta_{me,\text{discharge}}, \frac{P_d}{\eta_p} \right) \\
P_b \in \{P_{b,low}, P_{b,high}\}.
\]

\[
P_d = 0 \\
P_b \in \{-\eta_{ge,\text{charge}}P_{m,\text{max}}, P_d\eta_d\}.
\]

\[
P_d < 0 \\
P_b = \max \left( -\eta_{me,\text{charge}}P_{m,\text{max}}, P_d\eta_d \right).
\]
Chapter 6

Results and Analysis

By using the results from the dynamic programming in GT-Suite, a comparison with the GT-Suite reference controller could be made. There were several weeks of data available divided into separate days and if traveling away or back home. However, we will only present the result from two of these days which were selected because of shorter computational time, a Sunday and a Tuesday.

6.1 Settings for the dynamic programming

When using the Dynamic program that was written in MATLAB, there were some parameters which had to be set. In the Dynamic program, the time step was set to 0.5 s, but since GT-Suite calculates its updates every 0.02 s the result from the dynamic program was decided to be interpolated when used in GT-Suite. The grid of $\omega_e$ was set to 21, the grid for $SOC$ to 30, and the grid of $P_b$ was set to 1000. For the conditions of the $SOC$, it was set with 60% as initial and final value, i.e. $SOC(0) = 0.6$ and $SOC(N) = 0.6$, with an allowed range between 0.58 − 0.62. This means that we want the vehicle to be charge-sustaining. The initial value of $\omega_e$ was set to 100 rad/s, i.e. $\omega_e(0) = 100$ which was the minimum allowed speed of the engine when it was operating. The final value of the engine speed was allowed to be any value between the minimum and maximum engine speed, i.e. 100 − 480 rad/s.

6.2 Results for Sunday

6.2.1 Leaving home

The trip when driving away from home lasted 539 seconds. As can be seen in Fig. 6.1, the speed obtained by Dynamic Programming was similar to the GT-Suite reference controller, and the driver satisfaction parameter was kept at 100% during the entire trip. The state of charge and the fuel economy for the same trip is presented in Fig. 6.2, where it can be seen that the $SOC$ was much lower at the end of the dynamic programming compared to the reference controller, although it
is still feasible. The use of the engine, motor and generator is shown in Fig. 6.3 - 6.5 and as can be seen in the figures the dynamic programming uses the engine less than the reference controller and the generator power is both positive and negative, although we assumed it to only be positive when deriving our problem formulation. Finally, a comparison of the total fuel used for the same trip is presented in Fig. 6.6.

Figure 6.1. Speed and driver satisfaction results for Sunday, driving away from home.
6.2. RESULTS FOR SUNDAY

![SOC and fuel economy results for Sunday, driving away from home.](image1.png)

![Engine power used during Sunday, driving away from home.](image2.png)

Figure 6.2. SOC and fuel economy results for Sunday, driving away from home.

Figure 6.3. Engine power used during Sunday, driving away from home.
CHAPTER 6. RESULTS AND ANALYSIS

Figure 6.4. Motor power used during Sunday, driving away from home.

Figure 6.5. Generator power used during Sunday, driving away from home.
6.2. RESULTS FOR SUNDAY

![Fuel consumed over time graph](image)

Figure 6.6. Fuel used during Sunday, driving away from home.

In order to evaluate the effect of the $SOC$ grid, one simulation was made with a $SOC$ grid of 101, see Fig. [6.7] However, this gave a very similar result to the $SOC$ in Fig. [6.2] which had a grid of 30.
Figure 6.7. Fuel used during Sunday, driving away from home. Using higher SOC grid.
6.2. RESULTS FOR SUNDAY

6.2.2 Coming home

For the return trip during the same Sunday, the result is presented in Fig. 6.8. As can be seen in the results for going back during the Sunday, the driver satisfaction parameter decreases and so does the SOC although the solution is still feasible.

Figure 6.8. Speed and driver satisfaction results for Sunday, driving back home.
CHAPTER 6. RESULTS AND ANALYSIS

Figure 6.9. SOC and fuel economy results for Sunday, driving back home.

Figure 6.10. Engine power used during Sunday, driving back home.
6.2. RESULTS FOR SUNDAY

Figure 6.11. Motor power used during Sunday, driving back home.

Figure 6.12. Generator power used during Sunday, driving back home.
Figure 6.13. Fuel used during Sunday, driving back home.
6.3 Results for Tuesday

6.3.1 Leaving home

The results for going away are presented in Fig. 6.14-6.19. The driver satisfaction parameter seen in Fig. 6.14 was less than 90% at the end of the trip, thus making the solution infeasible. Also the end state of the SOC in Fig. 6.15 is infeasible.

![Graph showing speed and driver satisfaction results for Tuesday, driving to work.](image)
CHAPTER 6. RESULTS AND ANALYSIS

Figure 6.15. SOC and fuel economy results for Tuesday, driving to work.

Figure 6.16. Engine power used during Tuesday, driving to work.
6.3. RESULTS FOR TUESDAY

![Motor power](image1.png)

**Figure 6.17.** Motor power used during Tuesday, driving to work.

![Generator power](image2.png)

**Figure 6.18.** Generator power used during Tuesday, driving to work.
Figure 6.19. Fuel used during Tuesday, driving to work.
6.3. RESULTS FOR TUESDAY

6.3.2 Coming home

In Fig. 6.20 - 6.25 the results for driving back home the same Tuesday is presented, however the DP solution is infeasible because it does not satisfy the driver satisfaction parameter and the final state of the $SO_{C}$ is not feasible.

![Graph showing speed and driver satisfaction results for Tuesday, driving back home.](image)

**Figure 6.20.** Speed and driver satisfaction results for Tuesday, driving back home.
Figure 6.21. SOC and fuel economy results for Tuesday, driving back home.

Figure 6.22. Engine power used during Tuesday, driving back home.
6.3. RESULTS FOR TUESDAY

Figure 6.23. Motor power used during Tuesday, driving back home.

Figure 6.24. Generator power used during Tuesday, driving back home.
Figure 6.25. Fuel used during Tuesday, driving back home.
6.4 Analysis of the Results

Table 6.1: Results for different days. $SOC_0$ is the initial value of the $SOC$ and $SOC_f$ the final value at the end of the trip.

<table>
<thead>
<tr>
<th>Day</th>
<th>Direction</th>
<th>Algorithm</th>
<th>$SOC_0$</th>
<th>$SOC_f$</th>
<th>fuel used (L)</th>
<th>$S_d$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>Away</td>
<td>Reference</td>
<td>0.6</td>
<td>0.5807</td>
<td>0.1407</td>
<td>100</td>
</tr>
<tr>
<td>Sunday</td>
<td>Away</td>
<td>Dynamic P</td>
<td>0.6</td>
<td>0.2629</td>
<td>0.0989</td>
<td>100</td>
</tr>
<tr>
<td>Sunday</td>
<td>Back</td>
<td>Reference</td>
<td>0.6</td>
<td>0.5863</td>
<td>0.1531</td>
<td>100</td>
</tr>
<tr>
<td>Sunday</td>
<td>Back</td>
<td>Dynamic P</td>
<td>0.6</td>
<td>0.2572</td>
<td>0.1099</td>
<td>99.6</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Away</td>
<td>Reference</td>
<td>0.6</td>
<td>0.5849</td>
<td>0.6394</td>
<td>100</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Away</td>
<td>Dynamic P</td>
<td>0.6</td>
<td>−0.9066</td>
<td>0.3657</td>
<td>74.4</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Back</td>
<td>Reference</td>
<td>0.6</td>
<td>0.5895</td>
<td>0.6132</td>
<td>100</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Back</td>
<td>Dynamic P</td>
<td>0.6</td>
<td>−0.9840</td>
<td>0.3446</td>
<td>64.2</td>
</tr>
</tbody>
</table>

6.4 Analysis of the Results

In the figures for the generator power, Fig. 6.5, 6.12, 6.18 and 6.24, we can see how the power is both positive and negative. Although in our derivation for the generator we have only assumed it to be negative, thus charging the battery. The reason for this deviation is unclear, since the controllers follow the speed so well. However we do have some likely ideas, either we have misunderstood the way to extract the resulting data from GT-Suite or our model for $\omega_e$ is not sufficiently correct, since $\omega_g = f(\omega_e)$, see Eq. (5.17) and $P_g = T_g\omega_g$.

Also as can be seen by the results for both Sunday and Tuesday, the $SOC$ at the end of the trip is lower for the dynamic programming than for the reference controller, although on Tuesday the solution is infeasible.

A plausible reason for the deviation is that the model for the $\omega_e$ does not follow the dynamics well enough. We have used Simulink to verify our model, and when doing this we found that if we plug in the torques to see how our model for $\omega_e$, Eq. (4.37) and $\omega_m$, Eq. (4.38) behaves, $\omega_e$ does not behave as good as $\omega_m$. Now if we then use these equations together with Eq. (4.17) to find the power produced or received from the motor, $P_m = T_m\omega_m$, see Fig. 6.26 and generator, $P_g = T_g\omega_g$, see Fig. 6.27, we can observe that the absolute value for the power of the generator is to large in a lot of places. With these equations for $P_m$ and $P_g$ plugged into $P_b$, Eq. (4.30), we may update the $SOC$ using Eq. (4.39), the result is seen in Fig. 6.28. As can be observed in the figure, our $SOC$ increases much more than what the GT-Suite reference $SOC$ does. If this is also what happens in the DP-solver then the program will think that it has more battery power left than what is actually the case, thus resulting in the low $SOC$ seen in the results.

Another reason might be that the $SOC$ does not update properly during the dynamic programming. This could be due to the grid used, since if the grid is not sufficiently large the $SOC$ will not be able to update properly. However, if we use a bigger grid we instead run out of memory.

The necessary grid for updating the $SOC$ can be estimated. The $SOC$ is dis-
cretized into \( N_{SOC} \) parts, the grid, from \( SOC_{min} \) to \( SOC_{max} \). To have an increase or decrease of the \( SOC \) at the next time step, the difference in \( SOC \) must be at least half the distance of two adjacent points in the \( SOC \) grid. With the time step \( \Delta t \) we get the requirement that

\[
S\dot{O}C \Delta t \geq \frac{SOC_{max} - SOC_{min}}{2N_{SOC}}, \tag{6.1}
\]

or expressed for the grid

\[
N_{SOC} \geq \left| \frac{SOC_{max} - SOC_{min}}{2S\dot{O}C \Delta t} \right|. \tag{6.2}
\]

If we now estimate the value of \( S\dot{O}C \), using Eq. (4.29) with \( P_b \) calculated from Eq. (5.16), some guidelines of the \( SOC \) grid can be obtained. Different values of \( P_m \) and \( P_g \) were chosen to show the effect on the required grid if the motor and generator are used separately. The values chosen are for the motor at full power, \( 2/5 \), and \( 1/5 \) of the maximum continuous power. For the generator the values presented are full power, \( 2/3 \) and \( 1/3 \) of the maximum continuous power. The results are presented in the table below. \( \Delta t = 0.5 \) s in the data presented.

<table>
<thead>
<tr>
<th>( P_m (kW) )</th>
<th>( P_a (kW) )</th>
<th>( P_b (kW) )</th>
<th>( SOC_{min} )</th>
<th>( SOC_{max} )</th>
<th>( N_{SOC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0</td>
<td>20</td>
<td>0.58</td>
<td>0.62</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>8</td>
<td>0.58</td>
<td>0.62</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>4</td>
<td>0.58</td>
<td>0.62</td>
<td>51</td>
</tr>
<tr>
<td>0</td>
<td>-15</td>
<td>-13</td>
<td>0.58</td>
<td>0.62</td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
<td>-8</td>
<td>0.58</td>
<td>0.62</td>
<td>27</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>-4</td>
<td>0.58</td>
<td>0.62</td>
<td>51</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>20</td>
<td>0.55</td>
<td>0.65</td>
<td>22</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>8</td>
<td>0.55</td>
<td>0.65</td>
<td>61</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>4</td>
<td>0.55</td>
<td>0.65</td>
<td>127</td>
</tr>
<tr>
<td>0</td>
<td>-15</td>
<td>-13</td>
<td>0.55</td>
<td>0.65</td>
<td>45</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
<td>-8</td>
<td>0.55</td>
<td>0.65</td>
<td>66</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
<td>-4</td>
<td>0.55</td>
<td>0.65</td>
<td>128</td>
</tr>
</tbody>
</table>

By looking at the results, such as Fig. 6.4 and 6.5 it can be seen that the motor power is less than \( 10 \) kW and the generator is bigger than \(-5 \) kW most of the time. The table 6.2 gives that if they work alone they might need a bigger \( SOC \) grid than 30, however one run with \( SOC \) grid of 101 was made for Sunday driving away from home resulting in almost the exact same results, see Fig. 6.7. This was the reason the grid was not set to be bigger.

Further more, since the \( SOC \) is lower for the DP it is hard to compare if its
6.4. ANALYSIS OF THE RESULTS

Figure 6.26. Calculating the power given from $P_m = T_m \omega_m$.

Figure 6.27. Calculating the power given from $P_g = T_g \omega_g$. 
solution is better than the reference solution in terms of energy savings. In order to evaluate these two different solutions with one another we need to estimate how much the difference in $SOC$ is worth in fuel. We had two ideas for this, which both had as the main idea to charge the battery to equal $SOC$ at the end of the simulation, so that the cost in fuel could be compared. The first approach was to let the battery get charged during $t_f$ seconds with a constant increase in $SOC$, $S\hat{O}C = (SOC(t_f) - SOC(0))/t_f$. Then we calculated what generator power, $P_g$, this corresponded to and since we have assumed $P_e = -P_g$ we could find the engine power needed. Next we plugged this into a function that gives us the lowest BSFC for this requested power, $P_e$, and from Eq. 4.36 we can find the fuel consumption per second. By summing these we found the total fuel consumed to charge the battery to the same $SOC$ level as the reference $SOC$ during a time period of $t_f$ s. The other idea we had was to see how much it would cost if we let the generator run on full capacity $P_g = -P_{g,max}$, thus having $P_e = P_{g,max}$ until we had charged the battery to the same level as the reference end state. We calculated how much this would cost in terms of fuel. Since only the solution for the Sunday is feasible, and also because we can not consider negative $SOC$ values in our models for the battery, we only present the analysis for this day. After we found how much fuel had been consumed in grams we used the density of fuel, 750 gram/litres [27] to calculate how many litres this corresponded to, see Tab. 6.3.

Table 6.3: Amount of fuel needed to charge the battery to the same $SOC$ level as the reference solution.

<table>
<thead>
<tr>
<th>Day</th>
<th>Direction</th>
<th>Fuel needed (L), constant $S\hat{O}C$</th>
<th>Fuel needed (L), $P_e = -P_{g,max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>Away</td>
<td>0.1506</td>
<td>0.1500</td>
</tr>
<tr>
<td>Sunday</td>
<td>Back</td>
<td>0.1560</td>
<td>0.1538</td>
</tr>
</tbody>
</table>

Now if we add the extra fuel it takes to charge the battery we find that Sunday going away uses approximately 0.2492 L instead of the 0.0989 L seen in the Tab. 6.1. Sunday going back uses approximately 0.2648 L instead of the former 0.1099 L, so both of these solutions uses more fuel than the reference solutions.

---

1Called eLine in some literature, relating the minimum BSFC with each combination of $T_e$ and $N_e$. 

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6.4. ANALYSIS OF THE RESULTS

Figure 6.28. SOC as a function of $\omega_c$ and $\omega_m$. 
Chapter 7

Summary and Discussion

We have in this thesis tried to find specific controller combinations in the form of torques for the engine, $T_e$, the motor, $T_m$ and the generator, $T_g$, so that the HEV will reconcile the speed of different routes and at the same time use as little fuel as possible. The data used for the control was the velocity for the entire trip.

First we develop three dynamical equations, one for the rotational speed of the engine, $\omega_e$, Eq (4.18), one for the rotational speed of the motor, $\omega_m$, Eq (4.19) and one for the battery, $SOC$, Eq. (4.29). After verifying these we found that the physical reasoning behind these equations were good. Although the empirically derived equations for $\dot{\omega}_e$ and $\dot{\omega}_m$, Eq. (4.37) and (4.38), fitted the GT-Suite response better and for that reason we decided to make these our state equations instead. However, when inspecting Table 4.4 one sees that the coefficients are approximately the same size. The reason they are not closer is probably because we have missed, overestimated or underestimated some inertia in the derivation of Eq. (4.18) and (4.19). Although we did not look further into this since we at the time thought we had found somewhat satisfactory state equations, Eq. (4.37) and (4.38).

Now with these state equations we tried to simplify our problem by reducing the number of states and controllers, see Chapter 5. Finally we reached our problem formulation Eq. (5.36)-(5.43). This problem was eventually solved with DP through a program written in MATLAB, the program has been verified to work well in Appendix E. There were several days that could have been solved for, where each day was divided into going away and going back to home. However some of the days were to large in the amount of data that had to be considered and could therefore not be solved using our present computers and program. While others, were small enough to be solved, as has been presented in the results.

In the analysis of the results we discuss the solutions. What we found was that our solution for Sunday, which is the only feasible solution, is probably not as good as the reference solvers, this was concluded through the $SOC$ to fuel discussion we made in the results. Here we found that with the fuel it takes to charge the battery to the same level, DP uses 0.2492 L while the reference only uses 0.1407 L going away on Sunday. Coming home on Sunday, DP uses 0.2648 L vs 0.1531 L for the
CHAPTER 7. SUMMARY AND DISCUSSION

reference solver. One of the probable reasons for this higher fuel consumption is
that the model for \( \dot{\omega}_e \), Eq. (4.37), does not follow the dynamics well enough and
will therefore update the SOC badly.

As a first step in future work we would want to try to find a better dynamical
model for the \( \omega_e \) state. One could also consider finding a better estimates on the
efficiencies \( \eta_{me} \) and \( \eta_{ge} \) since they are probably functions of the power received or
given to the battery. This could also be done for \( \eta_b \) and \( \eta_e \), but here \( \eta_b \) would be
a function of the power from the battery and \( \eta_e \) would be a function of the power
from the engine. In our derivation for \( \eta_b \) and \( \eta_e \), Appendix B we found \( \eta_e = 0.8 \),
\( \eta_{b,\text{discharge}} = 0.84 \) and \( \eta_{b,\text{charge}} = 0.95 \) which are reasonable values according to
Mingxin Kang at Sophia University 2013. When the vehicle is breaking the battery
can recharge the battery with the help of the motor. The amount of kinetic energy
that is converted to potential energy in the battery is 30 \%, this was found to be the
value that best fitted with the GT-Suite response, and is also reasonable according to
[29].

If one eventually finds a sufficiently good model for the two states \( \omega_e \) and \( \text{SOC} \) a
plausible functionality of our research would be as a benchmark solution. One could
solve the problem with the desired method of choice, either a heuristic strategy or a
optimal strategy. Then plug the velocities that one receives in to our DP program
and see how much further one could reduce in fuel consumption.
Appendix A

Physical Parameters

Table A.1: Physical Parameters and Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c$</td>
<td>cylinder volume</td>
<td>$1497 \times 10^{-6}$</td>
<td>$\text{m}^3$</td>
</tr>
<tr>
<td>$M$</td>
<td>vehicle mass</td>
<td>1460</td>
<td>$\text{kg}$</td>
</tr>
<tr>
<td>$R_{tire}$</td>
<td>radius of the tire</td>
<td>$298.2 \times 10^{-3}$</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>air density</td>
<td>1.293</td>
<td>$\text{kg/m}^3$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>drag coefficient</td>
<td>0.33</td>
<td>[-]</td>
</tr>
<tr>
<td>$A$</td>
<td>frontal area of vehicle</td>
<td>3.8</td>
<td>$\text{m}^2$</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>coefficient of rolling resistance</td>
<td>0.015</td>
<td>[-]</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity coefficient</td>
<td>9.82</td>
<td>$\text{m/s}^2$</td>
</tr>
<tr>
<td>$G_f$</td>
<td>final differential gear ratio</td>
<td>4.113</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta_{dg}$</td>
<td>transmission of differential gear</td>
<td>0.97</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>planetary-gear energy loss</td>
<td>0.96</td>
<td>[-]</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>planetary gear ratio</td>
<td>0.3846</td>
<td>[-]</td>
</tr>
<tr>
<td>$R_r$</td>
<td>radius of the ring gear</td>
<td>$57.5 \times 10^{-3}$</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$R_s$</td>
<td>radius of the sun gear</td>
<td>$\epsilon R_r$</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$R_c$</td>
<td>radius of the carrier gear</td>
<td>$0.5(R_r + R_s)$</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$I_r$</td>
<td>Inertia of the ring gear</td>
<td>0.003</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_s$</td>
<td>Inertia of the sun gear</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_c$</td>
<td>Inertia of the carrier gear</td>
<td>$1.0 \times 10^{-9}$</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_m$</td>
<td>Inertia of the motor (EM2)</td>
<td>0.035</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_g$</td>
<td>Inertia of the generator (EM1)</td>
<td>0.0265</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_e$</td>
<td>Inertia of the engine crankshaft</td>
<td>0.16</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_a$</td>
<td>Inertia of the axle</td>
<td>1.4</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_{dg}$</td>
<td>Inertia of the differential gear</td>
<td>0.05</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$I_d$</td>
<td>Inertia of the drive shaft</td>
<td>0.04</td>
<td>$\text{kg}\cdot\text{m}^2$</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>rotational speed of the ring gear</td>
<td></td>
<td>$\text{rad/s}$</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>rotational speed of the sun gear</td>
<td></td>
<td>$\text{rad/s}$</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>rotational speed of the carrier gear</td>
<td></td>
<td>$\text{rad/s}$</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>rotational speed of the motor</td>
<td></td>
<td>$\text{rad/s}$</td>
</tr>
</tbody>
</table>

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## APPENDIX A. PHYSICAL PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_g$</td>
<td>rotational speed of the generator</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_e(N_e)$</td>
<td>rotational speed of the engine</td>
<td>[rad/s] ([rpm])</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>rotational speed of the drive-shaft</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_dg$</td>
<td>rotational speed of the differential gear</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_a$</td>
<td>rotational speed of the axles</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$\omega_v$</td>
<td>rotational speed of the tires</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>$F$</td>
<td>inner forces on the pinion gear</td>
<td>[N]</td>
</tr>
<tr>
<td>$T_r$</td>
<td>torque on the ring gear</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_s$</td>
<td>torque on the sun gear</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_c$</td>
<td>torque on the carrier gear</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_m$</td>
<td>torque from the motor</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_g$</td>
<td>torque from the generator</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_e$</td>
<td>torque from the engine</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_d$</td>
<td>torque on the the drive-shaft</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_dg$</td>
<td>torque on the the differential gear</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_a$</td>
<td>torque on the the axles</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_{af}$</td>
<td>torque on the the wheels</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_f$</td>
<td>torque from the opposing forces</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$T_{fb}$</td>
<td>torque from the friction brakes</td>
<td>[N·m]</td>
</tr>
<tr>
<td>$P_m$</td>
<td>power from motor</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_g$</td>
<td>power from generator</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_e$</td>
<td>power from engine</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_b$</td>
<td>power from battery</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{m,max}$</td>
<td>maximum continuous motor power</td>
<td>25 [kW]</td>
</tr>
<tr>
<td>$P_{g,max}$</td>
<td>maximum continuous generator power</td>
<td>15 [kW]</td>
</tr>
<tr>
<td>$P_{e,max}$</td>
<td>maximum continuous engine power</td>
<td>51 [kW]</td>
</tr>
<tr>
<td>$v_a$</td>
<td>actual speed of vehicle</td>
<td>[km/h]</td>
</tr>
<tr>
<td>$v_d$</td>
<td>desired speed of vehicle</td>
<td>[km/h]</td>
</tr>
<tr>
<td>$I_b$</td>
<td>battery current</td>
<td>[A]</td>
</tr>
<tr>
<td>$R_b$</td>
<td>battery resistance</td>
<td>[Ω]</td>
</tr>
<tr>
<td>$U_{oc}$</td>
<td>battery open-circuit voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$Q_{bmax}$</td>
<td>battery maximum charge capacity</td>
<td>$6.5 \times 3600$ [A·s]</td>
</tr>
<tr>
<td>$\theta$</td>
<td>road angle (grading)</td>
<td>[rad]</td>
</tr>
<tr>
<td>$SOC$</td>
<td>battery State-of-Charge</td>
<td>[-]</td>
</tr>
<tr>
<td>$\dot{m}_f$</td>
<td>fuel mass flow rate</td>
<td>[g/s]</td>
</tr>
<tr>
<td>$\eta_{ge}$</td>
<td>generator electrical efficiency</td>
<td>0.8425 [_-]</td>
</tr>
<tr>
<td>$\eta_e$</td>
<td>engine efficiency</td>
<td>0.80 [_-]</td>
</tr>
<tr>
<td>$\eta_{e,charge}$</td>
<td>battery charge efficiency</td>
<td>$\eta_e/\eta_{ge}$ [_-]</td>
</tr>
<tr>
<td>$\eta_{b,discharge}$</td>
<td>battery discharge efficiency</td>
<td>0.84 [_-]</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>regenerative breaking efficiency</td>
<td>0.3 [_-]</td>
</tr>
<tr>
<td>$\eta_{me,charge}$</td>
<td>motor charge efficiency</td>
<td>0.818 [_-]</td>
</tr>
<tr>
<td>$\eta_{me,discharge}$</td>
<td>motor discharge efficiency</td>
<td>1.258 [_-]</td>
</tr>
</tbody>
</table>
Appendix B

Efficiencies $\eta_b$ and $\eta_e$

In Eq. (5.18) the efficiencies $\eta_e$ and

$$\eta_b = \begin{cases} \eta_{b,\text{discharge}}, & P_b > 0 \\ \eta_{b,\text{charge}}, & P_b < 0, \end{cases}$$

had to be determined. This was done with the help of Eq. (5.19),

$$P_d = (\sum F_R + M\dot{v}_a) v_a,$$

which was plotted vs

$$P_d = \eta_e P_e + \eta_b P_b$$

for different values on $\eta_e$ and $\eta_b$. The efficiencies were then manually adjusted so that the two equations would be as close to one another as possible. The result can be seen in Fig. B.1 and in Fig. B.2. The values obtained were

$\eta_e = 0.80$,

$\eta_{b,\text{discharge}} = 0.84$,

$\eta_{b,\text{charge}} = \eta_e/\eta_ge \approx 0.95$.  

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Figure B.1. Comparison of $\eta_e P_e + \eta_b P_b$ with $(\sum F_R + M \dot{v}_a) v_a$ for a time period of 1000 s.
Figure B.2. Comparison of $\eta_c P_c + \eta_b P_b$ with $(\sum F_R + M \dot{v}_a) v_a$ for a time period of 200 s.
Appendix C

Coefficients from DP

The expression for the coefficients from the Dynamic Programming in the case $P_d > 0$ will now be given in more detail. First, some of the parameters needed are stated. Using Eq. (5.14) and (5.15), written as

$$\dot{\omega}_e = a_1 T_e + a_2 T_m + a_3 T_g + a_4 T_f$$ \hspace{1cm} (C.1)

$$\dot{\omega}_m = b_1 T_e + b_2 T_m + b_3 T_g + b_4 T_f$$ \hspace{1cm} (C.2)

we can find an expression for $T_g$ in the form

$$T_g = \beta_1 \dot{\omega}_m + \beta_2 \dot{\omega}_e + \beta_3 T_e + \beta_4 T_f,$$ \hspace{1cm} (C.3)

where the different coefficients are given in Table C.1.

<table>
<thead>
<tr>
<th>$\dot{\omega}_e$</th>
<th>$\dot{\omega}_m$</th>
<th>$T_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$ 2.6841</td>
<td>$b_1$ 0.0616</td>
<td>$\beta_1$ -0.0678</td>
</tr>
<tr>
<td>$a_2$ 0.0876</td>
<td>$b_2$ 0.1249</td>
<td>$\beta_2$ 0.0967</td>
</tr>
<tr>
<td>$a_3$ 10.2663</td>
<td>$b_3$ -0.1030</td>
<td>$\beta_3$ -0.2554</td>
</tr>
<tr>
<td>$a_4$ -0.0153</td>
<td>$b_4$ -0.0293</td>
<td>$\beta_4$ -0.0005</td>
</tr>
</tbody>
</table>

With

$$T_e = \frac{P_d - \eta_b P_b}{\eta_e \omega_e},$$ \hspace{1cm} (C.4)

$$T_m = \frac{P_b - \eta_g T_g \omega_g}{\omega_m \eta_{me,disscharge}}$$ \hspace{1cm} (C.5)

and Eq. (C.3) plugged into Eq. (C.1) we can get a new expression for $\dot{\omega}_e$ that only depends on the variable $P_b$,

$$\gamma_1 \dot{\omega}_e = \gamma_2 \dot{\omega}_m + \gamma_3 P_d + \gamma_4 P_b + \gamma_5 T_f.$$ \hspace{1cm} (C.6)
The coefficients are given by the following expressions,

\[
\begin{align*}
\gamma_1 &= 1 - \beta_2 \left( a_3 - \frac{a_2 \eta_g \omega_g}{\eta_{me, discharge} \omega_m} \right) \\
\gamma_2 &= \beta_1 \left( a_3 - \frac{a_2 \eta_g \omega_g}{\eta_{me, discharge} \omega_m} \right) \\
\gamma_3 &= \frac{a_1 + \beta_3 \left( a_3 - \frac{a_2 \eta_g \omega_g}{\eta_{me, discharge} \omega_m} \right)}{\eta_e \omega_e} \\
\gamma_4 &= \frac{a_2}{\eta_{me, discharge} \omega_m} \frac{a_1 + \beta_3 \left( a_3 - \frac{a_2 \eta_g \omega_g}{\eta_{me, discharge} \omega_m} \right)}{\eta_e \omega_e} \\
\gamma_5 &= a_4 + \beta_4 \left( a_3 - \frac{a_2 \eta_g \omega_g}{\eta_{me, discharge} \omega_m} \right).
\end{align*}
\]

The expression for \( \omega_g \) is

\[
\omega_g = \frac{\omega_e (R_r + R_s) - \omega_m R_r}{R_s},
\]

and the expression for \( T_f \) is

\[
T_f = T_{fb} + R_{tire} (\mu_r M g \cos(\theta) + \frac{1}{2} \rho A C_d v_a^2 + M g \sin(\theta)),
\]

with \( T_{fb} = 0 \).
Appendix D

Implementation to GT-Suite

Some of the input required by GT-Suite is the generator torque, motor power, engine throttle and engine ignition. Since the Dynamic Programming results in only power and torque, some of the other input to GT-Suite has to be converted before implemented in to the GT-Suite.

D.1 Engine throttle

The engine throttle can be expressed as a function of the BMEP, or Brake Mean Effective Pressure, and the engine speed. From GT-Suite there is data relating the engine speed, BMEP and throttle. By using that data a function can be approximated for calculating the throttle. After fitting a polynomial the throttle can be described as

\[ \theta = p_{00} + p_{10}N_e + p_{01}\text{BMEP} + p_{20}N_e^2 + p_{11}N_e\text{BMEP} + \]
\[ +p_{02}\text{BMEP}^2 + p_{30}N_e^3 + p_{21}N_e^2\text{BMEP} + p_{12}N_e\text{BMEP}^2 \]

with the coefficient values

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_{00})</td>
<td>13.76</td>
</tr>
<tr>
<td>(p_{10})</td>
<td>(-7.195 \cdot 10^{-3})</td>
</tr>
<tr>
<td>(p_{01})</td>
<td>1.524 \cdot 10^{-4}</td>
</tr>
<tr>
<td>(p_{20})</td>
<td>(-3.262 \cdot 10^{-8})</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>(-1.478 \cdot 10^{-12})</td>
</tr>
<tr>
<td>(p_{02})</td>
<td>(-2.278 \cdot 10^{-10})</td>
</tr>
<tr>
<td>(p_{30})</td>
<td>3.898 \cdot 10^{-12}</td>
</tr>
<tr>
<td>(p_{21})</td>
<td>2.729 \cdot 10^{-16}</td>
</tr>
</tbody>
</table>

The BMEP can be calculated from the expression

\[ \text{BMEP} = \frac{4\pi T_e}{V_c \cdot 10^3} \quad [\text{kPa}] \quad \text{(D.1)} \]

where \(V_c\) is the cylinder volume. In Fig. D.1, the estimated function, compared to the data points, is presented. In Fig. D.2 the output of the throttle function is presented, and compared to the actual GT-Suite throttle. Note that the maximum
output is limited to 100\%, and the throttle is set to −1 if the power or speed of the engine is negative.

D.2 Ignition

The ignition has a binary representation by 0 and 1. After comparing GT-Suite output the ignition was set in the following way. At each time step the ignition is set to 1 if the engine torque at the next time step is not zero, otherwise the ignition is set to 0.

D.3 Interpolation of data

Since GT-Suite uses a time step of 0.02s, while the dynamic programming can use any time step from 0.02s, the data has to be interpolated to be used in GT-Suite. This is made by using linearisation between each set of time steps.
D.3. INTERPOLATION OF DATA

Figure D.1. Estimation of throttle as a function of BMEP and $N_e$.

Figure D.2. Comparison of GT-Suite throttle and throttle from the estimated function.
Appendix E

Verification

E.1 Verification of DP code

In order to verify that the dynamic programming code works, a simpler problem is solved. The problem is to follow a reference speed by controlling the acceleration. This problem, which will be called the simplified problem, can be stated as

\[
\text{minimize } u(k) \sum_{k=0}^{N-1} (x_1(k) - v(k)) \quad (E.1)
\]

subject to

\[
x_1(k+1) = x_1(k) + u(k) \cdot \Delta t \quad (E.2)
\]

\[
x_2(k+1) = x_2(k) \quad (E.3)
\]

\[
x_1(0) = x_{10} \quad (E.4)
\]

\[
x_2(0) = x_{20} \quad (E.5)
\]

\[
x_1 \in [x_{1,\text{min}}, x_{1,\text{max}}] \quad (E.6)
\]

\[
x_2 \in [x_{2,\text{min}}, x_{2,\text{max}}] \quad (E.7)
\]

\[
u \in [u_{\text{min}}, u_{\text{max}}]. \quad (E.8)
\]

The velocity \( v \) is generated by random numbers coming from a standard uniform distribution on the interval \([0, 30]\). The state variable \( x_1 \) and the control variable \( u \) are set to be within such intervals that the problem is feasible, meaning that \( x_1 \in [v_{\text{min}}, v_{\text{max}}] \) and \( u \in [a_{\text{min}}, a_{\text{max}}] \), where \( a(k) = \frac{v(k+1)-v(k)}{\Delta t} \). \( x_2 \) does not play a role in this simpler problem but does in the fuel consumption optimization problem. This problem can be solved by the DP program that we made in order to solve the fuel consumption problem. The outline of the program is as follows.

1. Discretize the number of possible states and controllers,

\[
x_1 \in [x_1^1, x_1^2, \ldots, x_1^{N_1-1}, x_1^{N_1}], \quad x_2 \in [x_2^1, x_2^2, \ldots, x_2^{N_2-1}, x_2^{N_2}] \quad \text{and}
\]

\[
u \in [u^1, u^2, \ldots, u^{N_u-1}, u^{N_u}], \quad \text{where } N_1, N_2 \text{ and } N_u \text{ is the grid of } x_1, x_2 \text{ and } u \text{ respectively.}
\]
2. For every state \((x_1(k), x_2(k))\) in the grid, apply every controller 
\(u(k) \in [u^1, u^2, \ldots, u^{N_u-1}, u^{N_u}]\) and calculate the next state. In our *simplified problem* this means that 
\(x_1(k + 1) = x_1(k) + u(k) \cdot \Delta t\) and 
\(x_2(k + 1) = x_2(k)\). For these next states, \((x_1(k + 1), x_2(k + 1))\), find which values it is closest to in the grid. Save the corresponding controllers, next states in the grid, \((x_{1\text{next}}(k + 1), x_{2\text{next}}(k + 1))\) and costs corresponding to this state and controller. Go through the unique set of next states that the set of controllers can take the present state to. Find which controller minimizes the norm

\[
\frac{|x_1(k + 1) - x_{1\text{next}}(k + 1)|}{Sx_1} + \frac{|x_2(k + 1) - x_{2\text{next}}(k + 1)|}{Sx_2},
\]

and make this the controller that takes the current state to the specific state in the grid. Set the cost between these states to be the one calculated earlier when the specific controller was applied. \(Sx_1\) and \(Sx_2\) equals the distance between two adjacent states in the grid of \(x_1\) and \(x_2\). If there is not a controller that can take a state to another, then the cost of this will be set to infinity. A cost map with corresponding controllers should now have been set up.

3. Use Theorem 1 from the Discrete Dynamic Programming section to solve the problem using the cost map received from step 2.

In Fig. E.1, E.2, E.3 and E.4 results from the simplified problem is shown. The grid, or resolution, of \(x_1\) is set to 100 which provides a good result. In Figure E.5 and E.6 the same problem is solved but where the grid of \(x_1\) is 10. To see how the grid affects the solution and the computation time some data is provided in Table E.1. The number of time steps is 100, the grid of \(x_2\) is 2, the grid of \(u\) is set to 1000, and \(x_1 \in [0, 30]\). The cost is the sum over all time steps of the error between the given speed and the speed obtained using the control. As can be seen by changing the grid both the cost and runtime changes. The problem is solved with MATLAB and uses a parallel for-loop which makes it possible to do several calculations in parallel. Data is for a PC running Windows 7 with an 3.0 GHz Intel Core2Duo CPU and 2.0 GB RAM.
Table E.1: Data for different grid of $x_1$ for the DP verification model.

<table>
<thead>
<tr>
<th>$N_1$</th>
<th>cost</th>
<th>runtime [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>78.3</td>
<td>1.5</td>
</tr>
<tr>
<td>20</td>
<td>38.7</td>
<td>3.6</td>
</tr>
<tr>
<td>30</td>
<td>23.7</td>
<td>6.1</td>
</tr>
<tr>
<td>40</td>
<td>20.5</td>
<td>9.8</td>
</tr>
<tr>
<td>50</td>
<td>16.1</td>
<td>14.1</td>
</tr>
<tr>
<td>60</td>
<td>13.1</td>
<td>19.4</td>
</tr>
<tr>
<td>70</td>
<td>10.9</td>
<td>27.9</td>
</tr>
<tr>
<td>80</td>
<td>10.1</td>
<td>36.7</td>
</tr>
<tr>
<td>90</td>
<td>8.0</td>
<td>44.9</td>
</tr>
<tr>
<td>100</td>
<td>7.2</td>
<td>54.0</td>
</tr>
<tr>
<td>200</td>
<td>3.5</td>
<td>202.1</td>
</tr>
<tr>
<td>300</td>
<td>2.6</td>
<td>512.1</td>
</tr>
</tbody>
</table>
Figure E.1. Speed using calculated control, compared to reference speed, from the DP program for the simplified problem. $x_1$ grid is 100, number of time steps is 100.

Figure E.2. Control compared to reference acceleration, from the DP program for the simplified problem. $x_1$ grid is 100, number of time steps is 100.
E.1. VERIFICATION OF DP CODE

Figure E.3. Speed using calculated control, compared to reference speed, from the DP program for the simplified problem. $x_1$ grid is 100, number of time steps is 20.

Figure E.4. Control compared to reference acceleration, from the DP program for the simplified problem. $x_1$ grid is 100, number of time steps is 20.
Figure E.5. Speed using calculated control, compared to reference speed, from the DP program for the simplified problem. $x_1$ grid is 10, number of time steps is 20.

Figure E.6. Control compared to reference acceleration, from the DP program for the simplified problem. $x_1$ grid is 10, number of time steps is 20.
Bibliography


BIBLIOGRAPHY


