Influence of the ballast on the dynamic properties of a truss railway bridge

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Preface

This Master Thesis was carried out at the division of Structural Engineering and Bridges, at the Royal Institute of Technology, KTH, in Stockholm. I would like to express my sincerest gratitude to my supervisors Associate Professor Jean-Marc Battini, and Ph.D researcher Andreas Andersson, at the Department of Civil and Architectural Engineering at KTH for their continuous assistance during this project. Thank you for devoting me valuable time and providing me wise and constructive advice to fulfil this thesis.

This Master Thesis is based on the preliminary work of Jaroslaw Zwolski, researcher at Wroclaw University of Technology, at the Department of Civil Engineering in Poland. The project was initiated in 2010 during the construction of the Malczyce viaduct when the dynamic measurements were performed. All along this thesis, Jaroslaw Zwolski provided me with crucial information about the bridge and the experimental measurements and answered very rapidly to my questions. Thus, I would like to express my sincerest regards to him.

Finally, I would like to thank the research team at the Department of Civil and Architectural Engineering at KTH for their help and advice at key moments.

Stockholm, May 2013

Lucie Bornet
Abstract

To deal with a rapid development of high-speed trains and high-speed railways, constant improvement of the railway infrastructure is necessary and engineers are continuously facing challenges in order to design efficient and optimized structures. Nowadays, more and more railway bridges are built and thus, they require the engineers’ attention both regarding their design and their maintenance. A comprehensive knowledge of the infrastructures and the trains is crucial: their behaviours need to be well known. However, today, the ballast - the granular material disposed on the track and on which the rails lie – is not well known and its effect in dynamic analyses are rarely accounted for. Engineers are still investigating the role played by the ballast in the dynamic behaviour of bridges.

This master thesis aims at quantifying the influence of the ballast on the dynamic properties of a bridge. Is the ballast just an additional mass on the structure or does it introduce any additional stiffness? Thus, this project investigates different alternatives and parameters to propose a realistic and reliable model for the ballast superstructure and the track. For the purpose of this study, a simply supported steel truss bridge located in Poland is studied. The bridge was excited by a harmonic force and the interesting point regarding the experiments is that acceleration measurements were collected before and after the ballasted track setting up on the bridge deck. Then, these data are processed through MATLAB in order to obtain the natural frequencies of the bridge at two different times during its construction. The determined natural frequencies for the un-ballasted case are then compared with analytical values obtained with a 3D finite element model implemented in the software LUSAS. This step aims at calibrating the un-ballasted finite element model so that the bridge is represented as realistically as possible.

Once it has been done, a model both for the ballast and the track is proposed using solid elements for the ballast superstructure and beam elements for the rails, the guard rails and the sleepers. Different parameters influencing the natural frequencies and modes shapes of the bridge are testing and it appears that the ballast introduces an additional stiffness through a bending stiffness in the ballast and a change in the support conditions. Finally, the contribution of these parameters is assessed and discussed: the stiffness of the ballast increases the stiffness of the bridge by more than 20% for the 2\textsuperscript{nd} vertical bending vibration mode and the support conditions increase the bridge’s stiffness by more than 15% and 30% respectively for the 1\textsuperscript{st} vertical bending the 1\textsuperscript{st} torsional vibration modes.

\textbf{Keywords:} Ballast, Railway bridges, Experimental dynamics, Finite element modelling, Natural frequencies, Eigenmodes, Bridge stiffness.
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Chapter 1

1 Introduction

1.1 Introduction

The design phase is a crucial step in the lifecycle of bridges. Swedish codes and Eurocodes provide, to design engineers, calculation methods and safety coefficients which takes into account different parameters: the bridge type (Railway Bridge, Suspension Bridge, Cable Stayed Bridge...), the materials (steel, concrete, pre-stressed concrete...), the geometry of the bridge, the foundations and other constraints. Regarding railway bridges, specific rules and recommendations exist and a dynamic analysis is more and more required in order to adapt the bridge design to the passing train vibrations, especially for high-speed trains. A dynamic analysis is generally required for train speeds over 200 km/h.

Nowadays, high-speed trains and fast railway networks are rapidly developing and a gradual increase of trains speed can be observed. The world speed record is currently held by the TGV (Train à Grande Vitesse, french for "High-Speed Train") and achieved by SNCF, the French national railway in 2007. The maximal reached speed is 574,8 km/h. Thus, this research field is constantly stimulated to develop more efficient technologies, materials and structures. An accurate and comprehensive knowledge of the bridge, the tracks and the ballast behaviour is therefore required to study and model the bridge, track and train interactions.

Railway bridges are complex structures, consisting of several structural components with different mechanical properties and frequently, discrepancies between theoretical results and experimental ones are observed in the dynamic analysis, especially for short railway bridge [1]. The bridge model is often the most accurately defined since the geometrical and material properties are perfectly known by the design office at the end of the design phase. The train model is also, most of
the time, well defined: for a particular type of train (LM71 and SW/2 for statics, HSLM for dynamics),
the design codes provides the axle loads and the geometrical and mechanical characteristics.

However, the track is more complicated to model and there is, so far, no clear
recommendation in design codes about how to take into account the effect of the ballasted
superstructure. Nonetheless, EN 1991-2, Part 6.5.4 provides rules for the combined response of the
track and the bridge [2].

The ballast is a “material such as broken stone, gravel, slag, cinders, burnt clay, etc... (10-60
mm), which is placed on the finished roadbed to form a support for the ties, to provide a means of
draining water away from them, and to make it possible to surface or raise track or make tie
renewals, without disturbing the roadbed”[3]. The ballast layer “provides a firm and even bearing for
the ties by evenly distributing the pressure due to the weight and thrust of trains passing over the
tracks”[3]. Due to its granular character, the properties and the behaviour of the ballast are difficult
to assess. Therefore, modelling the ballast in an accurate way is still nowadays a challenge for
engineers. Although studies show that the ballast has a significant influence on the bridges vibrations
and, possibly on the stiffness of the whole bridge itself, the contribution of the ballast to the bridge
stiffness is still studied. Indeed, the natural frequencies of a structure are proportional to the square
root of the stiffness divided by the mass [4]. The mass and the stiffness are, hence, two antagonist
parameters in relation to the natural frequency and it is crucial to know and understand how the
ballast affects these two parameters.

1.2 Purpose of the study - Aims and scope

The purpose of this project is to estimate the influence of the ballast on the bridge vibrations by
implementing an accurate FEM model of a truss bridge and comparing the analytical natural
frequencies of the bridge with experimental ones. One interesting fact about this project is that
vibrations were measured, first without the ballast and then after the ballast and the track were in
place. The project aims at assessing the influence of the ballast on both the natural frequencies and
on the damping ratio and it aims also at determining if the ballast gives any additional stiffness to the
bridge. Ballasted models will thus be proposed.

1.3 Literature review

Since the behaviour of the ballast is not well known, numerous models of the ballast have been
proposed, considering and analysing different properties of the granular material. Numerous studies
deal with the way of modelling the ballast and railway tracks lying on the ground, and some of them
propose very detailed models for the ballast, using, for instance, the discrete element method [5].
With this method, the grains constituting the ballast layer are modelled as non-deformable polygonal
solids so that it models the interaction between the deformable ground and the track.
However, just few studies are focusing on the ballast behaviour when it comes to railway bridges and to the train-track bridge dynamic interactions. This section aims at presenting and summarizing some of the works published about this particular subject.

Most of the studies about the interaction train-track-bridge [6-8] propose to model the bridge deck and the track as two linear-elastic Bernoulli-Euler beams and the connection between these beams is ensured by a more or less complex springs and dampers system. These models introduce a vertical and horizontal stiffness for the ballast.

In [9], ZACHER et al. implement a 2D model of stiff ballast grains represented as balls with three degrees of freedom. The contact between the grains is then ensured through non-linear springs and viscous dampers. However, the 2D model implemented by ZACHER et al. is not conceivable for the purpose of this thesis with a 3D finite element model.

Other studies show that the stiffness of the ballast is frequency dependent. For that, in [10], HERRON et al. consider a ballast stiffness range from 100 MN/m to 500 MN/m and model the ballast as discrete particles. In contrast, in [11], REBELO et al. model the ballast layer as a plate connected to the bridge deck with springs and take into account only the shear stiffness of the ballast. These two studies result in the observation that the natural frequencies of a structure vary according to the vibration amplitude. Besides, it has been shown that an increase in free vibration’s amplitude result in the decrease of the 1st natural frequency of a bridge.

Then, in [12], LIU et al. implement a 3D finite element model and describe the ballast as solid elements, the sleepers as lumped masses and the rails as linear beams. Appropriate boundary conditions are also applied on the bridge longitudinal direction to simulate the continuity of the rails and the ballast before and after the structure. The connection between the track and the deck is ensured by a spring and damper system. In this study, the influence of the train model is investigated but all the models give a good match with the experiment. Such a model for the ballast seems to provide interesting results and it will be further developed in this thesis.

FINK et al. in [1] and BATTINI et al. [13] study the non-linear effect of the ballast superstructure on the bridge. Both studies introduce a 2D model, consisting of two beams: one modelling the bridge and the other modelling the ballast layer. Then, they study the interaction at the interface between these two beams. In [1] and [12], the effect of ballast is introduced through a non-linear longitudinal stiffness and the slip at the beam interface is taken into consideration into the ballast stiffness matrix. Good agreements between experimental and analytical results are found in both studies. Such a model can also be implemented in a 3D FEM-program.

These different works and conclusions about the train-track bridge dynamic interactions are taken as a starting point of the thesis. No convincing model for the ballast superstructure has been implemented yet and as a result, the thesis will focus exclusively on this purpose and on the different parameters that can have a more or less significant influence on the ballast model and therefore, on the dynamic analysis of a bridge.
1.4 Method and outline of the thesis

The crucial point of this project is to have a finite element model as realistic as possible so that the model does not lead to any source of error or misinterpretation of the experimental data. Therefore, an accurate and comprehensive knowledge of the truss bridge is necessary. It has been possible thanks to the help of Jaroslaw Zwolski who did the first study about the bridge [14]. The engineering analysis software LUSAS is used for all the parts of the project related to finite element modelling and the implementation of the un-ballasted FEM model is detailed in Chapter 2.

The next step consists in processing experimental measurements - first without the ballast and then, after the ballast and tracks were in place - in order to get the lowest natural frequencies and the damping ratios of the bridge. Once again, a deep knowledge of the experimental conditions – weather, exciter properties, sensors and data acquisition systems – is important to extract the natural frequencies of the railway bridge as accurately as possible. The experimental data processing is specified in Chapter 3.

The experimental values for the un-ballasted bridge are then compared with the values from the LUSAS eigenvalue analysis. Thus, the third step of the project aims at optimizing and improving the un-ballasted finite element model of the railway bridge to get it as close as possible to the real behaviour of the structure. Different parameters which might influence the accuracy of the model are then tested: the support conditions, the mesh size or the type of element for instance. The calibration of the un-ballasted model and the influence of these parameters are discussed in Chapter 4.

The final step, which is also the purpose of this project, consists in proposing several alternatives to model the ballast and the track on railway bridges. Previous studies are taken as a starting point and different parameters are studied: the element type, the continuity of the ballast superstructure before and after the bridge, the influence of the mass of the ballast and the support conditions. These different alternatives are presented in Chapter 5.

Finally, the influence of the ballast on the natural frequencies and on the damping of the bridge, and the estimation of its contribution or not to the bridge stiffness is discussed in Chapter 6.
Chapter 2

2 Implementation of the finite element model in LUSAS

An accurate finite element model of the bridge is required to perform a reliable dynamic analysis; the engineering analysis software LUSAS 14.7 has been used to carry out the finite element analysis. A 3D model of the overall bridge was created in order to determine the mode shapes and eigenfrequencies of the bridge.

2.1 The studied bridge

This project is focusing on a simply supported truss bridge. The railway bridge is located in Poland over a main double-track line. It supports a single-track railway line slightly curved with radius 330/290m [14]. The bridge is composed of a truss structure and a bracing system connecting the girders truss arrangement and all its structural components are in steel. The bridge deck is an orthotropic plate where the ballast and the track are placed. The track is composed of main rails and guard rails and due to the track’s curve, the track is inclined with a slope of 2%.

2.2 Geometry of the bridge

2.2.1 Global geometry

The overall geometry of the bridge is shown in Figures 2.1, 2.2 and 2.4. All dimensions are in meters. The total length of the span is 38,4m and the total width is 6,7m; the height is 6,2m (Figure 2.2). The
The main structural system consists of longitudinal beams connected by angled cross-members forming equilateral triangular units: this truss structure is composed of 16 diagonal elements. The diagonal elements are subject alternatively to tension and compression. The orthotropic deck is composed of two longitudinal beams, 13 cross beams and 14 stringer beams. A thin steel plate covers the deck framework (Figure 2.3 and 2.4). The upper part of the bridge consists of a reinforcement system composed of bracing elements. The bridge is slightly unsymmetrical in the transverse direction due to drainage slopes of the deck plate and to a non-symmetrical arrangement of the stringer beams (Figure 2.4).

Figure 2.1: Photo of a longitudinal view of the bridge.

Figure 2.2: Overall geometry and dimensions.
The beam elements are modelled and meshed as 3D Thick (Timoshenko) Beam elements and the steel plate is modelled as Thin (Kirchhoff) shell elements. The influence of the element length or the number of the elements division will be further investigated.

Figure 2.3: Photo of the truss structure, the steel plate and the load exciter used for the field measurements.

Figure 2.4: Cross section of the bridge.
2.2.2 Cross sectional properties

All dimensions in Table 2.1 are in metres. Table 2.1 shows the cross sections of the different elements of the bridge. The shape of the sections was determined based on drawings provided by the design office of the bridge in Poland.

The longitudinal beams of the deck are I-beams with unequal flanges and their characteristic dimensions and properties are shown in Table 2.1. Their location is shown in Figure 2.4.

According to the drawings provided by the design office, the cross section of the cross beams is varying along the transverse axis in order to create a drainage slope. For the purpose of the study, it has been assumed an average constant cross section for these beams and it has been checked that this assumption does not influence the eigenvalue analysis of the bridge. The 13 cross beams are reversed T beam section and their characteristic dimensions and properties are shown in Table 2.1. Their location is shown in Figure 2.4.

The 14 stringer beams are divided in three geometric categories: two L cross sections and one reversed T cross section. There are seven L-beams S1, one L-section S2 which is a shorter version of S1 aiming at leaving space for pipes to cross the bridge deck and there are six reversed T-beams S3. Their characteristic dimensions and properties are shown in Table 2.1. Figure 2.5 shows the stringer beams arrangement and Figure 2.6 shows an overall view of the bridge deck without the steel plate.

![Figure 2.5: Stiffener beams arrangement.](image)

![Figure 2.6: Arrangement of the beam elements of the deck without the steel plate.](image)
Table 2.1: Cross section properties.

### Cross section definition

#### Type of section: I beam

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Element Name</th>
<th>Number of elements</th>
<th>D</th>
<th>Bt=Bb</th>
<th>T1</th>
<th>T2</th>
<th>t</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal beams bottom</td>
<td>L</td>
<td>2</td>
<td>1.25</td>
<td>0.46</td>
<td>0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>0.065</td>
</tr>
<tr>
<td>Diagonal elements</td>
<td>D1</td>
<td>8</td>
<td>0.52</td>
<td>0.46</td>
<td>0.024</td>
<td>0.024</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>D2</td>
<td>8</td>
<td>0.4</td>
<td>0.44</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Longitudinal beam top</td>
<td>LT1</td>
<td>4</td>
<td>0.508</td>
<td>0.46</td>
<td>0.024</td>
<td>0.024</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>LT2</td>
<td>2</td>
<td>0.52</td>
<td>0.46</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>R</td>
<td>26</td>
<td>0.12</td>
<td>0.013</td>
<td>0.01</td>
<td>0.01</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>Cross beams top</td>
<td>CBT</td>
<td>2</td>
<td>0.34</td>
<td>0.46</td>
<td>0.02</td>
<td>0.02</td>
<td>0.016</td>
<td>0.01</td>
</tr>
</tbody>
</table>

#### Type of section: Reversed T beam

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Element Name</th>
<th>Number of elements</th>
<th>D</th>
<th>B</th>
<th>T</th>
<th>t</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross beams bottom</td>
<td>CBB</td>
<td>13</td>
<td>0.754</td>
<td>0.46</td>
<td>0.03</td>
<td>0.016</td>
<td>0.015</td>
</tr>
<tr>
<td>Stiffening beams</td>
<td>S3</td>
<td>6</td>
<td>0.26</td>
<td>0.1</td>
<td>0.01</td>
<td>0.014</td>
<td>0.015</td>
</tr>
</tbody>
</table>

#### Type of section: L beam

<table>
<thead>
<tr>
<th>Element Type</th>
<th>Element Name</th>
<th>Number of elements</th>
<th>A</th>
<th>B</th>
<th>T</th>
<th>t</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffening beams</td>
<td>S1</td>
<td>7</td>
<td>0.25</td>
<td>0.114</td>
<td>0.01</td>
<td>0.014</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>1</td>
<td>0.16</td>
<td>0.114</td>
<td>0.01</td>
<td>0.014</td>
<td>0.015</td>
</tr>
</tbody>
</table>
The 16 diagonal elements are I-beams with equal flanges and their characteristic dimensions and properties are shown in Table 2.1. These elements are named D1 and D2 and their location is shown in Figure 2.7.

The longitudinal beams of the upper part of the bridge are I-beams with equal flanges and their characteristic dimensions and properties are shown in Table 2.1. These elements are named LT1 and LT2 and their location is shown in Figure 2.7 and Figure 2.8.

![Figure 2.7: Diagonal elements D1,2 location and upper longitudinal beam LT1,2 location.](image)

The two cross beams of the upper part of the bridge are I-beams with equal flanges and their characteristic dimensions and properties are shown in Table 2.1. These elements are named CBT and their location is shown in Figure 2.8.

The 16 bracing beams of the upper part of the bridge are I-beams with equal flanges and their characteristic dimensions and properties are shown in Table 2.1. These elements are named R and their location is shown in Figure 2.8.

![Figure 2.8: View of the bracing system from above.](image)

### 2.2.3 Orthotropic deck

In order to have the most realistic model of the deck, all the geometrical points of the model belong to the mid-surface of the plate (Figure 2.2) and then, offsets are applied (Table 2.2) to have the correct locations for each element. The steel plate is directly connected to the cross beams, the longitudinal beams and the stiffening beams and it rests above the top of the reversed T-shaped cross beams. The inclined edge parts of the steel plate (Figure 2.4) are taken into account by increasing the thickness of the edge parts of the steel plate from 0.015 m to 0.025 m but in conserving the same mass of steel (Figures 2.9 and 2.10). Two models had been compared: one with
inclined edge parts and one with thicker horizontal edge parts. The difference between the two models was negligible and the simplest model was kept for the rest of the analysis. Therefore, the following thickness has been used:

\[
t_{\text{steel plate central part}} = 0.015 \text{ m}
\]

\[
t_{\text{steel plate edge parts}} = 0.025 \text{ m}
\]

The values of the offsets are the following (Table 2.2):

<table>
<thead>
<tr>
<th>Element</th>
<th>Offset value (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBB</td>
<td>-0.387</td>
</tr>
<tr>
<td>CBT</td>
<td>+0.09</td>
</tr>
<tr>
<td>R</td>
<td>+0.2</td>
</tr>
<tr>
<td>S1</td>
<td>-0.028</td>
</tr>
<tr>
<td>S2</td>
<td>-0.077</td>
</tr>
<tr>
<td>S3</td>
<td>-0.026</td>
</tr>
<tr>
<td>Main steel plate part</td>
<td>+0.185</td>
</tr>
<tr>
<td>Edges steel plate parts</td>
<td>+0.19</td>
</tr>
</tbody>
</table>

Figure 2.9: Transverse view of the steel deck.

Figure 2.10: Overall view of the steel deck.
2.2.4 Foundations

The bridge foundations are not studied in the project and therefore, they are not included in the FE-model. Nonetheless, realistic support conditions must be specified to perform the calculation. These boundary conditions have been applied at the bottom of the longitudinal beams to model the real support conditions as realistically as possible (Figure 2.11 and Figure 2.12).

Figure 2.11: Support conditions.

In order to apply the boundary conditions at the bottom of the longitudinal beams, a stiff beam has been added between the centre of gravity of the longitudinal beam cross section and the bottom of the longitudinal beam (Figure 2.12). This stiff beam is modelled as one 3D Thick Beam element, a Timoshenko beam, where the following properties have been adopted for the un-ballasted model (Table 2.3). The dimensions of the stiff beam are five times higher than the thickness of the web of the longitudinal beams.

![Image of Rigid Beam](image)

Figure 2.12: Stiff beam.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>L = 0.57 m</td>
</tr>
<tr>
<td>Rectangular cross sections</td>
<td>A = 0.1 x 0.1 m</td>
</tr>
<tr>
<td>Mass density</td>
<td>ρ = 100 kg/m³</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>E = E_steel = 205 GPa</td>
</tr>
</tbody>
</table>

Table 2.3: Stiff beam properties.

The influence of the stiffness, i.e. the Young’s Modulus of this stiff beam, on the bridge natural frequencies is studied in Chapter 4.2 and it will be further investigated in Chapter 5.3.3 for the ballasted model. The values presented here are the ones which fit the best with the real behaviour of the bridge without the ballast.

The boundary conditions are then applied according to the drawings from the design office. They are represented on Figure 2.13. Figure 2.14 shows a view of the final model without the ballast.
Figure 2.13: Boundary conditions in the xy plane.

Figure 2.14: 3D model of the bridge.
2.3 Materials and loads

2.3.1 Material

The bridge consists entirely of steel components. Table 2.4 shows the steel properties that have been used for the 3D FEM model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>E = 205 GPa</td>
</tr>
<tr>
<td>Mass density</td>
<td>ρ = 7 849 kg/m³</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>ν = 0.3</td>
</tr>
</tbody>
</table>

Table 2.4: Steel properties.

2.3.2 Permanent loads

The weight of the structure is considered based on the geometry obtained from the finite element model and taking the density of steel as 7 849 kN/m³. In LUSAS, the self-weight is applied as a body force with linear acceleration of 9.81 m/s² in the vertical direction.

The total mass of the exciter is about 1050kg. The exciter’s dead weight is modelled by a lumped mass applied in the vertical direction at two different positions (Figure 2.15): one corresponding to the 1st test without the ballast and one corresponding to the 2nd test with the ballast.

![Exciter’s dead weight position in LUSAS](image)

Figure 2.15: Exciter’s dead weight position in LUSAS,
a) test without the ballast, b) test with the ballast and the track in place.
2.4 Quality assurance

Throughout the different stages of the modeling work, it is crucial to perform a quality control of the model. In fact, wrong inputs in LUSAS gives wrong outputs and the FEM results are wrong. This step of the design process is a necessity for all bridge designers and must be a natural and common part of their work.

2.4.1 Mass checking

The total mass of the model need to be compared with hand calculations to ensure the model accuracy. Table 2.11 summarizes the different steps done to perform the hand calculation for the mass checking. Then, the hand calculated value is compared with the sum of the four vertical reaction forces obtained from LUSAS when the self-weight of the model is applied (Figure 2.15).

Figure 2.16: Self-weight vertical reactions (N).

According to Table 2.11, the difference between the result obtained from LUSAS and the hand calculation is very low: only 0.046 %. This value is more than acceptable.
Table 2.5: Mass model checking.

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>Number of element</th>
<th>Area (m²)</th>
<th>Length (m)</th>
<th>Density</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal beams bottom</td>
<td>2</td>
<td>0.0506268</td>
<td>38.4</td>
<td>7849</td>
<td>30518.00</td>
</tr>
<tr>
<td>Cross beams bottom</td>
<td>13</td>
<td>0.0254806</td>
<td>6.7</td>
<td>7849</td>
<td>17419.76</td>
</tr>
<tr>
<td>D1</td>
<td>8</td>
<td>0.0368858</td>
<td>7.841</td>
<td>7849</td>
<td>18160.80</td>
</tr>
<tr>
<td>D2</td>
<td>8</td>
<td>0.0248858</td>
<td>7.841</td>
<td>7849</td>
<td>12252.58</td>
</tr>
<tr>
<td>Cross beams top</td>
<td>2</td>
<td>0.0232858</td>
<td>6.7</td>
<td>7849</td>
<td>2449.12</td>
</tr>
<tr>
<td>LT1</td>
<td>4</td>
<td>0.0313658</td>
<td>9.6</td>
<td>7849</td>
<td>9453.70</td>
</tr>
<tr>
<td>LT2</td>
<td>2</td>
<td>0.0368858</td>
<td>9.6</td>
<td>7849</td>
<td>5558.72</td>
</tr>
<tr>
<td>Reinforcements 1</td>
<td>16</td>
<td>3.32E-03</td>
<td>4.633</td>
<td>7849</td>
<td>1932.53</td>
</tr>
<tr>
<td>Reinforcements 2</td>
<td>8</td>
<td>3.32E-03</td>
<td>6.7</td>
<td>7849</td>
<td>1397.36</td>
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<tr>
<td>Reinforcements 3</td>
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<td>3.713</td>
<td>7849</td>
<td>387.19</td>
</tr>
<tr>
<td>S1</td>
<td>7</td>
<td>4.55E-03</td>
<td>38.4</td>
<td>7849</td>
<td>9596.03</td>
</tr>
<tr>
<td>S2</td>
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<td>3.29E-03</td>
<td>38.4</td>
<td>7849</td>
<td>991.10</td>
</tr>
<tr>
<td>S3</td>
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<td>4.54E-03</td>
<td>38.4</td>
<td>7849</td>
<td>8215.46</td>
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<tr>
<td>Steel plate 1</td>
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<td>0.1005</td>
<td>38.4</td>
<td>7849</td>
<td>30290.86</td>
</tr>
<tr>
<td>Steel plate 2</td>
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<td>0.00166</td>
<td>38.4</td>
<td>7849</td>
<td>1000.65</td>
</tr>
<tr>
<td>Steel plate 3</td>
<td>1</td>
<td>0.010663</td>
<td>38.4</td>
<td>7849</td>
<td>3213.85</td>
</tr>
<tr>
<td>Exciter</td>
<td>1</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>1050</td>
</tr>
<tr>
<td>Hand calculated total mass (kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>153888</td>
</tr>
<tr>
<td>Total mass from LUSAS (kg)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>153958</td>
</tr>
<tr>
<td>Percentage of difference (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.046</td>
</tr>
</tbody>
</table>

2.4.2 Influence of the elements’ number

Then, a convergence analysis needs to be performed to evaluate the influence of the mesh size and the number of divisions for each element constituting the bridge. This analysis is crucial to test the accuracy of our finite element model and to find the most efficient mesh size to have an acceptable CPU time and accurate results.

The following numbers of divisions are used:

\[ N_{\text{cross beams bottom}} = 1 \]
\[ N_{\text{longitudinal beams bottom}} = 5 \]
\[ N_{\text{stiffening beams}} = 5 \]
\[ N_{\text{cross beams top}} = 5 \]
\[ N_{\text{longitudinal beams top}} = 5 \]
\[ N_{\text{bracing elements}} = 5 \]

The difference in frequency between the mesh described above and a finer mesh is not higher than 0.1%. The current mesh enables to have both a good accuracy and an efficient CPU time.
Chapter 3

3 Experimental data processing

Experimental dynamic aims at determining the real behavior of a structure and its dynamic properties. Usually, experimental dynamic is used to verify and calibrate finite element models. These types of field measurements are very costly due to the high accuracy and quality of the instrumentation material (sensors and data acquisition systems). The two mains steps of experimental dynamic are of crucial importance: the data acquisition and then, the data processing, using signal analysis particularly based on Jean-Baptiste Fourier and Harry Nyquist theories [4]. The first step has already been carried out and therefore, this thesis is going to focus on the experimental data processing only. The method and the results are described is the following chapter.

3.1 Procedure of the vibration tests

The vibration tests were carried on by Dr. Jaroslaw Zwolski from Wroclaw University of Technology, Department of Civil Engineering in Poland. Two sessions of tests were performed; the first one took place on the 06/10/2010, before the ballast and tracks’ installation; the second one took place on the 09/04/2011 after the tacks’ installation. The source of vibration is a Rotational Eccentric Mass (REM) exciter (Figures 3.1 and 3.2) which generates a sinus excitation with continuously variable frequency and its positions for the first and the second testing sessions are shown in Figures 3.3 and 3.4. The mass of the exciter is 1050 kg.
For the first test, the exciter was fixed to a short section of the track so as to prevent the exciter from bouncing while the REM excited the bridge at high frequencies i.e. when the excitation force exceeded the exciter’s weight. For the second testing session, the exciter was placed on the track.

The exciter generated a sweep signal in the predefined frequency range from 3 to 30 Hz. However, it became rather unstable over 15 Hz (Figure 3.5) and after 270 s, the force does not increase as a parabola. The duration of both tests was 6 min and 47 sec and a sample frequency of 400 Hz was used.

The response of the bridge has been measured by twelve accelerometers for the first test without the ballast and ten for the second test with it. They were used in pair: one accelerometer measuring the vertical acceleration and the other one measuring the transverse acceleration. The 1st test session consisted of 12 tests corresponding to 12 arrangements of accelerometers and the 2nd test session of 14 arrangements of accelerometers (Appendix 1). Two pairs of accelerometers were fixed at the same position as the REM exciter as the reference for test 1 and one pair for test 2.
3.2 Analysis of the experimental data

The raw experimental data for each accelerometer and each setup has been provided by Dr. Jaroslaw Zwolski. For each setup, the interesting accelerometers or combination of accelerometers are studied. The experimental data processing aims at extracting the eigenfrequencies of the bridge, the corresponding mode shapes and damping ratios for both tests i.e. before and after the setting up of the track.

3.2.1 Extract the frequency value

For each frequency, the same process has been adopted to extract the value. Figure 3.6 displays the process to extract one eigenfrequency of the bridge. The corresponding code in MATLAB is shown in Appendix B.

Step 1:
Plot the FFT of one signal recorded by an accelerometer and identify a peak on the FFT spectrum plot of the whole signal.

↓

Step 2:
Apply a band pass filter “Butterworth” to isolate the part of the signal which gives the frequency corresponding to the peak.

↓

Step 3:
Apply a window function “Hamming” and a zero-padding on both sides of the signal. The interesting part of the signal is thus isolated and optimized to get a peak as accurate as possible.

↓

Step 4:
Plot the FFT of the improved isolated signal and extract the frequency value by peak-picking.

Figure 3.6: Eigenfrequency extraction method.
The zero-padding procedure consists in completing a signal with $n$ zeros. It aims at improving the accuracy of the signal analysis by increasing the number of points. The peak value is thus more precisely pinpointed.

3.2.2 Mode shape identification

Figure 3.7 displays the process to identify a vibration’s mode shape. The corresponding code in MATLAB is shown in Appendix B.

**Step 1:**
Plot on the same graph the improved isolated signal used to determine the frequency value of two interesting accelerometers.

**Step 2:**
Compare the phase of the two signals: Are they in phase or not?

**Step 3:**
Compare the phase of the corresponding displacements (Numerical displacement method) to confirm the previous observation.

**Step 4:**
Do the same for different pairs of accelerometers and identify the mode shape. Also compare the intensity of the signal for transversal and vertical accelerometers to determine if it is a horizontal or vertical mode. Try to see if a peak is missing on a FFT plot.

Figure 3.7: Mode shape identification method.

3.2.3 Damping ratio determination

Figure 3.8 displays the process to determine the damping ratio corresponding to one natural frequency of the bridge. The corresponding code in MATLAB is shown in Appendix B.
Step 1: Plot the FFT of one signal recorded by an accelerometer and identify a peak corresponding to an eigenfrequency on the FFT spectrum plot of the whole signal.

Step 2: Apply a band pass filter “Butterworth” to identify the part of the signal which gives the frequency corresponding to the peak.

Step 3: Plot the interesting part of the raw signal and its corresponding FFT.

Step 4: Use the “Half power Bandwidth method” to determine the damping ratio.

Figure 3.8: Damping ratio determination method.

Method: Half power bandwidth method [3,14]:

This method aims at determining the damping ratio $\xi$ for an eigenfrequency corresponding to a resonance of the bridge. It uses the frequency spectrum plot to determine the damping ratio. First, the maximal amplitude $A_1$ is determined and then, the amplitude $A_2$ is calculated:

$$A_2 = \frac{A_1}{\sqrt{2}} \quad (3.1)$$

The two frequencies corresponding to the value of the amplitude $A_2$ are determined on the frequency plot. They are located on both side of the frequency peak as shown in Figure 3.9 [4,15].

![Frequency plot](image)

Figure 3.9: Half Power Bandwidth method [4].

Then, the damping ratio is obtained using the following relation:

$$\xi = \frac{f_b - f_a}{f_b + f_a} \quad (3.2)$$
3.3 Experimental results

Table 3.1 shows the results extracted from the experimental data. The following chapter presents the different graphs enabling to extract these values and to conclude on the type of mode shape. All the values presented in Table 3.1 are an average of 4 values determined from relevant accelerometers.

Table 3.1: Experimental results.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Without the track – Test 1</th>
<th>With the track – Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency value (Hz)</td>
<td>Damping ratio estimation (%)</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>8,05</td>
<td>2,5</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>9,68</td>
<td>/</td>
</tr>
<tr>
<td>1st torsional</td>
<td>10,1</td>
<td>0,4</td>
</tr>
<tr>
<td>1st horizontal bending</td>
<td>14,0</td>
<td>/</td>
</tr>
<tr>
<td>1st horizontal bending</td>
<td>14,4</td>
<td>/</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>18,5</td>
<td>/</td>
</tr>
<tr>
<td>3rd vertical bending</td>
<td>21,6</td>
<td>/</td>
</tr>
</tbody>
</table>

3.3.1 Test 1: before the installation of the ballast and the track

Examples of a vertical and transverse acceleration’s plots and their corresponding FFT are shown in Figure 3.11 and 3.12. Figure 3.10 shows where the studied accelerometers were located.

![Figure 3.10: Test 1 – Accelerometers’ arrangement, Setup 04.](image)
Figure 3.11: Signal Z9 processing – Vertical acceleration – Setup 04 – Test 1.

Figure 3.12: Signal Y6 processing – Transverse acceleration Setup 04.
1st frequency

As the Fast Fourier Transform plot on Figure 3.11 shows, a peak is clearly visible between 7,5 Hz and 8,5 Hz. The band pass filter is then applied between these two limits. Figure 3.13 shows the part of the time signal we are looking at and Figure 3.14 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding: Figure 3.15 shows the result of this step. The FFT of the isolated filtered signal is then plotted in Figure 3.16.

Figure 3.13: Part of the time signal - Z9, Setup 04.

Figure 3.14: Filtered signal Z9 – Setup 04.

Figure 3.15: Isolated filtered signal Z9 – Setup 04.

Figure 3.16: Frequency spectrum of the isolated filtered signal Z9 – Setup 04.

An average between four values obtained from relevant accelerometers has been done and it has been observed that the value of the first frequency oscillates around the following value:

\[ f_{1, \text{vertical}} = 8,05 \pm 0,01 \text{ Hz} \]

Then, two isolated filtered signals and the corresponding displacements are plotted in order to compare the phase between them. As Figures 3.17 and 3.18 show, accelerometers Z5 and Z9 are in
phase for Setup 04; it is the same for Z1-Z3 or Z3-Z11. In addition, the studied accelerations have the same amplitude. Therefore, it can be concluded that the 1st frequency extracted corresponds to a first vertical bending mode.

The corresponding damping ratio is then calculated and is equal to: \( \xi = 2.5\% \). This value is an average of 4 values with an uncertainty of \( \pm 0.05\% \).

- **2nd frequency**

As the Fast Fourier Transform plot on Figure 3.11 shows, a peak is clearly visible between 9.4 Hz and 9.8 Hz. A band pass filter is then applied between these two limits. Figure 3.19 shows the part of the time signal we are looking at and Figure 3.20 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding: Figure 3.21 shows the results of this step. The isolated filtered signal’s FFT is then plotted (Figure 3.22).
An average between four values obtained from relevant accelerometers has been done and it has been observed that the value of the second frequency oscillates around the following value:

\[ f_{1\, \text{vertical}} = 9.68 \pm 0.03 \, \text{Hz} \]

Then, two isolated filtered signals and their corresponding displacements are plotted in order to compare the phase between them. As Figures 3.23 shows, accelerometers Z5 and Z9 are in phase for Setup 04; it is the same for Z1-Z3 or Z3-Z11. It can be concluded that the 2\textsuperscript{nd} frequency extracted corresponds to another first vertical bending mode as well. The behaviour of the bridge’s upper part would enable to differentiate these two modes.
- 3rd frequency

As the Fast Fourier Transform plot on Figure 3.11 shows, a peak is clearly visible between 10 Hz and 10.2 Hz. A band pass filter is then applied between these two bounds. Figure 3.24 shows the part of the time signal we are looking at and Figure 3.25 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.26 shows the results of this step. The isolated filtered signal’s FFT is plotted in Figure 3.27.

![Graphs showing time and frequency signals](image)

An average between four values obtained from relevant accelerometers has been done and it has been observed that the value of the peak oscillates around the following value:

$$f_{1\text{ torsional}} = 10.10 \pm 0.04 \text{ Hz}$$

As Figures 3.28 and 3.29 show, accelerometers Z5 and Z9 are in phase opposition for Setup 04; it is the same for Z1-Z3 or Z3-Z11. The displacements of these accelerometers are also out of phase. The
opposition is not perfect and this might be due to the bridge deck’s slight asymmetry in the transverse direction. Nonetheless, it can be concluded that the 3rd extracted frequency corresponds a first torsional mode. Moreover, the amplitudes of two opposite accelerometers are not equal; that means that the torsional deformations are combined with vertical ones.

- 4th frequency

As the Fast Fourier Transform plot on Figure 3.11 shows, a peak is observed between 13,8 Hz and 14,1 Hz. The band pass filter is then applied between these two limits. Figure 3.30 shows the part of the time signal we are looking at and Figure 3.31 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.32 shows the results of this step. The isolated filtered signal’s FFT is then plotted in Figure 3.33.
An average between four values obtained from relevant accelerometers has been done and it has been observed that the value of the peak oscillates around the following value:

\[ f_{1 \text{ transverse}} = 13.95 \pm 0.04 \text{ Hz} \]

As Figures 3.34 and 3.35 show, accelerometers Y6 and Y10 are in phase for Setup 04; it is the same for Y2-Y4. It can be concluded that the 4th extracted frequency corresponds to a first horizontal bending mode.
- 5th frequency

As the Fast Fourier Transform plot on Figure 3.12 shows, a peak is noticeable between 14.2 Hz and 14.5 Hz. A band pass filter is then applied between these two limits. Figure 3.36 shows the part of the time signal we are looking at and Figure 3.37 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.38 shows the results of this step. The isolated filtered signal’s FFT is then plotted in Figure 3.39.

An average between four values obtained from relevant accelerometers has been done and it has been observed that the frequency value oscillates around the following value:

\[ f_{1 \text{ transverse}} = 14.39 \pm 0.007 \, \text{Hz} \]
As Figures 3.40 and 3.41 show, accelerometers Y2-Y4 are in phase for Setup 04; it is the same for Y6-Y10 and Y8-Y12. It can be concluded that the 5th frequency extracted corresponds also to a first horizontal bending mode.

![Figure 3.40: Phase comparison Y2-Y4 Setup 04.](image)

![Figure 3.41: Phase comparison Y8-Y12 Setup 04.](image)

- **6th frequency**

As the Fast Fourier Transform plot on Figure 3.42 shows, a peak is visible between 18.4Hz and 18.6Hz. This peak has been identified by plotting the FFT of the signal between a large frequency interval (18Hz-19Hz) and the peak around 18.5Hz was clearly dominating among the others.

The band pass filter is then applied between 18.4Hz and 18.6Hz. Figure 3.43 shows the part of the time signal we are looking at and Figure 3.44 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.45 shows the results of this step. The isolated filtered signal’s FFT is then plotted in Figure 3.46.

![Figure 3.42: FFT Frequency spectrum of the signal Z9 – Setup 03.](image)
An average between four values obtained from relevant accelerometers has been done and it has been observed that the peak value oscillates around the following value:

\[ f_{2\text{vertical}} = 18.48 \pm 0.04 \text{ Hz} \]

As Figures 3.47 shows, accelerometers Z1-Z9 are out of phase for Setup 05; it is the same for Z1 and Z7 (Figure 3.48). However, Z5 and Z9, for example, are in phase (Figure 3.49). It can be concluded that the 6th frequency extracted corresponds to a second vertical bending mode.
As the Fast Fourier Transform plot on Figure 3.50 shows, a peak is visible between 21 Hz and 22 Hz. The band pass filter is then applied between these two limits. Figure 3.51 shows the part of the time signal we are looking at and Figure 3.52 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.53 shows the results of this step. The isolated filtered signal is then plotted in Figure 3.54.
Figure 3.50: FFT Frequency spectrum of the signal Z7 – Setup 03.

Figure 3.51: Part of the time signal – Z7, Setup 03.

Figure 3.52: Filtered signal Z7 - Setup 03.

Figure 3.53: Isolated filtered signal Z7 - Setup 03.

Figure 3.54: Frequency spectrum of the isolated filtered signal Z7 – Setup 03.
An average between four values obtained from relevant accelerometers has been done and it has been observed that the frequency value oscillates around the following value:

\[ f_{3^{rd}/4} \text{ vertical} = 21.60 \pm 0.05 \text{ Hz} \]

As Figures 3.55 shows, accelerometers Z1-Z9 are out of phase for Setup 05; it is the same for Z3-Z5 (Figure 3.56). However, accelerometers Z5-Z9, Z1-Z3 and Z3-Z5 are in phase (Figures 3.57, 3.58 and 3.59). It can be concluded that the 6th frequency extracted corresponds also to a combination between a **third vertical bending mode** and a **fourth vertical mode**. Figure 3.60 summarizes all these observations. The points which have the same colour are in phase.

![Signals Comparison](image1)

**Figure 3.55:** Phase comparison Z1 (red) - Z9 (blue) Setup 04.

![Signals Comparison](image2)

**Figure 3.56:** Phase comparison Z3 (red) - Z5 (blue) Setup 04.

![Signals Comparison](image3)

**Figure 3.57:** Phase comparison Z1 (blue) - Z3 (red) Setup 04.

![Signals Comparison](image4)

**Figure 3.58:** Phase comparison Z5 (red) - Z9 (blue) Setup 03.
Figure 3.59: Phase comparison Z3 (blue) - Z5 (red), Setup 05.

Figure 3.60: Bridge deck’s point’s phase.

This combination of 3\textsuperscript{rd} and 4\textsuperscript{th} vertical bending mode will be confirmed in Chapter 4 with the dynamic analysis of the FEM model in LUSAS and Figure 3.61 extracted from LUSAS shows the corresponding mode shape.

Figure 3.61: Mode shape for the 3\textsuperscript{rd}/4\textsuperscript{th} vertical bending frequency, from LUSAS.
3.3.2 Test 2: after the installation of the ballast and the track

Examples of a vertical and transverse acceleration plots and their corresponding FFT are shown in Figure 3.63 and 3.64. Figure 3.62 shows where the studied accelerometers are located on the bridge’s deck.

![Figure 3.62: Test 2 – Accelerometers’ arrangement Setup 02.](image)

![Figure 3.63: Signal Z5 processing – Vertical acceleration – Setup 02 – Test 2.](image)
• 1st frequency

As the Fast Fourier Transform plot on Figure 3.63 shows, a peak is observed between 5.1 Hz and 5.8 Hz. The band pass filter is then applied between these two bounds. Figure 3.65 shows the part of the time signal we are looking at and Figure 3.66 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.67 shows the results of this step. The isolated filtered signal’s FFT is then plotted in Figure 3.68.
An average between four values obtained from relevant accelerometers has been done and it has been observed that the value of the first frequency oscillates around the following value:

\[ f_{1\text{ vertical}} = 5.387 \pm 0.015 \text{ Hz} \]

Then two filtered isolated signals and their corresponding displacements are plotted in order to compare the phase angle between them. As Figures 3.69 shows, accelerometers Z5 and Z7 are in phase for Setup 02; it is the same for Z5-Z7, Setup 03 or Z1-Z5, Setup 03 (Figure 3.70). In addition, the studied accelerations have the same amplitude when the accelerometers have the same longitudinal position. Therefore, it can be concluded that the 1st frequency extracted corresponds to a first vertical bending mode. It is confirmed by the displacements.
The corresponding damping ratio is then calculated: \( \xi = 2.21\% \). This value is an average of 4 values with an uncertainty of ±0.01%.

- 2\textsuperscript{nd} frequency

As the Fast Fourier Transform plot on Figure 3.63 shows, a peak is clearly noticeable between 9Hz and 9.4Hz. The band pass filter is then applied between these two limits. Figure 3.71 shows the part of the time signal we are looking at and Figure 3.72 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.73 shows the results of this step. The FFT of the isolated filtered signal is then plotted in Figure 3.74.

An average between four values obtained from relevant accelerometers has been done and it has been observed that the peak value oscillates around the following value:

\[
f_{1\text{ torsional}} = 9.262 \pm 0.007 \text{ Hz}
\]
Then, two filtered isolated signals and their corresponding displacements are plotted in order to compare the phase angle between them. As Figures 3.75 and 3.76 show, accelerometers Z5-Z7 and Z1-Z5 are in phase opposition for Setup 03; it is the same for Z5-Z7, Setup 02 or Z1-Z5, Setup 03. The same observations can be drawn by studying the displacements. In addition, the studied accelerations have the same amplitude when the accelerometers have the same position along the longitudinal direction. However, in Figure 3.77, the signals are in phase. Therefore, it can be concluded that the 2nd frequency extracted corresponds to a first torsional mode.

![Figure 3.75: Phase comparison Z5 (red) – Z7 (blue) Setup 03.](image1)

![Figure 3.76: Phase comparison Z5 (red) – Z1 (blue) Setup 03.](image2)

![Figure 3.77: Phase comparison Z1 (blue) – Z7 (red), Setup 03.](image3)

- 3rd frequency

As the Fast Fourier Transform plot on Figure 3.63 shows, a peak is clearly visible between 10 Hz and 10.5 Hz. The band pass filter is then applied between these two limits. Figure 3.78 shows the part of the time signal we are looking at and Figure 3.79 shows the acceleration plot obtained after filtering.
the signal. The next step consists in applying the window function and the zero-padding and Figure 3.80 shows the results of this step. The isolated filtered signal’s FFT is then plotted in Figure 3.81.

Figure 3.78: Part of the time signal – Z7, Setup 03.

Figure 3.79: Filtered signal Z7 – Setup 03.

Figure 3.80: Isolated filtered signal Z7 – Setup 03.

Figure 3.81: Frequency spectrum of the isolated filtered signal Z7 – Setup 03.

An average between four values obtained from relevant accelerometers has been done and it has been observed that the frequency value oscillates around the following value:

$$f_{2\text{ vertical}} = 10.25 \pm 0.025 \text{ Hz}$$

Then, two filtered isolated signals and their corresponding displacements are plotted in order to compare the phase angle between them. As Figure 3.82 show, accelerometers Z5 and Z7 are in phase for Setup 03; it is the same for Z5-Z7, Setup 02 or Z5-Z7, Setup 04. In addition, the studied accelerations have the same amplitude when the accelerometers have the same abscise. The same observations can be drawn by studying the displacements. However, Z1-Z5 and Z1-Z9 for Setup 03 are in phase opposition (Figures 3.83 and 3.84). Besides, one can observe on Figure 3.85 that the
peak is not visible for an accelerometer located in the middle of the bridge deck. Therefore, it can be concluded that the 3rd frequency extracted corresponds to a second vertical bending mode.

Figure 3.82: Phase comparison Z5 (blue) – Z7 (red) Setup 03.

Figure 3.83: Phase comparison Z1 (blue) – Z9 (red) Setup 03.

Figure 3.84: Phase comparison Z1 (red) – Z5 (red) Setup 03.

Figure 3.85: FFT plot – Signal Z3 – Setup04.
- 4\textsuperscript{th} frequency

As the Fast Fourier Transform plot on Figure 3.63 shows, a peak is observed between 12 Hz and 12.5 Hz. The band pass filter is then applied between these two bounds. Figure 3.86 shows the part of the time signal we are looking at and Figure 3.87 shows the acceleration plot obtained after filtering the signal. The next step consists in applying the window function and the zero-padding and Figure 3.88 shows the results of this step. The FFT of the part of the signal is then plotted in Figure 3.89.

An average between four values obtained from relevant accelerometers has been done and it has been observed that the peak value oscillates around the following value:

\[ f_{3\text{rd vertical}}^{\text{rd frequency}} = 12.26 \pm 0.08 \text{ Hz} \]
Then, two filtered isolated signals and their corresponding displacements are plotted in order to compare the phase angle between them. As Figures 3.91 shows, accelerometers Z1-Z7 are in phase opposition for Setup 02; it is the same for Z1-Z5 and Z1-Z9 (Figure 3.92). However, accelerometers Z1-Z7, Setup 04 and Z5-Z7, Setup 02 are in phase (Figures 3.90 and 3.93). It can be concluded that the 6th extracted frequency corresponds once again to a combination between a third vertical bending mode and a fourth vertical bending mode. This observation has been confirmed by using the process OMA (Operational Modal Analysis) with the software Artemis (Figure 3.94).
3.3.3 Discussion

The experimental data processing through MATLAB has enabled to extract seven frequencies for the un-ballasted case and four for the ballasted case (Table 3.1). However, the transverse frequencies determined for the un-ballasted case are not as accurately known as the vertical and torsional ones, since the bridge had only been excited vertically. For the rest of this project, the vertical and torsional frequencies will be used and compared with frequencies extracted from eigenvalue analyses with the finite element software LUSAS. One can observe from Table 3.1 that influence of the ballast is more important for vertical bending modes: a decrease in frequency of 33% to 44% is noticed; for the torsional mode, the decrease is only of 8%. This diminution is due to the additional mass brought by the ballast superstructure but might also be due to an increase of the bridge stiffness, induced by the ballast stiffness itself. This is what Chapter 5 will focus on.

Furthermore, from this chapter, it can also be noticed that a value for the damping ratio was determined only for the 1st vertical bending mode for both cases (Table 3.1). Indeed, an accurate and reliable estimation of the damping ratio could not be obtained for other frequencies: large differences had been observed. The raw signal was directly used to determine to the damping ratio and the corresponding FFT plot for the other frequencies did not have enough points to enable to determine an accurate value of the damping ratio.

Nonetheless, it can be concluded that, for the 1st vertical bending mode, the ballast seems to have a slight influence on the damping but it is not possible to draw a general conclusion on that point. One would expect that the ballast increases the damping ratio, due to the energy dissipation. However, the damping ratio is lower after the ballast is in place and it can be explained by a decrease of the amplitude of the vibrations due to the additional mass. This conclusion needs to be confirmed for higher amplitudes. Moreover, it has been observed from OMA that the damping ratio for the 1st torsional mode in both cases is equal to 0.4%. Hence, it seems that the ballast does not influence the damping ratio for torsional modes.
Chapter 4

4 Eigenvalue analysis and calibration of the un-ballasted FEM model

The un-ballasted finite element model has been implemented according to information, drawings and indications from Dr Jaroslaw Zwolski. The main remarkable fact here is that there are few unknown parameters on which it is possible to play: the support conditions, the mesh size or the type of element for instance. The influence of these crucial parameters has been tested in order to get a model as realistic and accurate as possible and thus, the support conditions are going to be thoroughly studied. For that, an eigenvalue analysis of the finite element model is performed and the frequency values are compared with the experimental ones. The mode shapes will also be analysed.

4.1 Eigenvalue analysis results

An eigenvalue analysis is performed for the un-ballasted 3D finite element model established in Chapter 2. The results displayed in this chapter are obtained for the most realistic model, presented in Chapter 2. This analysis aims at extracting the natural frequencies of the bridge corresponding to several eigenmodes (Table 4.1). Numerous modes are extracted and most of them are local bending or local torsional modes, due mainly to the behaviour of the upper part of the bridge and its diagonal elements. To extract only the global modes, two tools are used.
The mass participation factors enable to determine which modes require the most important contribution of the mass of the bridge (Table 4.1). The local modes are easy to identify because they require less than 0,001% of the mass of the bridge in the three directions. Height global modes are identified.

Table 4.1: Frequency values and mass participation factors.

<table>
<thead>
<tr>
<th>Mode number</th>
<th>Mode type</th>
<th>Frequency values (Hz)</th>
<th>Mass participation factors (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>5</td>
<td>1st vertical bending</td>
<td>8,14</td>
<td>0,05</td>
</tr>
<tr>
<td>7</td>
<td>1st vertical bending</td>
<td>8,82</td>
<td>0,004</td>
</tr>
<tr>
<td>8</td>
<td>1st vertical bending</td>
<td>9,89</td>
<td>0,005</td>
</tr>
<tr>
<td>9</td>
<td>1st torsional</td>
<td>10,2</td>
<td>0</td>
</tr>
<tr>
<td>36</td>
<td>2nd vertical bending</td>
<td>18,3</td>
<td>0,004</td>
</tr>
<tr>
<td>41</td>
<td>3rd/4th vertical bending</td>
<td>21,2</td>
<td>0</td>
</tr>
<tr>
<td>44</td>
<td>3rd/4th vertical bending</td>
<td>21,6</td>
<td>0</td>
</tr>
<tr>
<td>46</td>
<td>3rd/4th vertical bending</td>
<td>22,1</td>
<td>0,004</td>
</tr>
</tbody>
</table>

Then, another confirmation of the global modes determination is done by generating a steady state analysis of the bridge. This analysis aims at solving the following frequency response spectrum [4,15]:

\[-\omega^2 M + i\omega C + K]\cdot x(\omega) = F \quad (4.1)

Where:
- \( M \) is the mass matrix,
- \( C \) is the damping matrix,
- \( K \) is the stiffness matrix,
- \( F \) is the force vector,
- \( x \) is the steady state response,
- \( \omega \) is the natural frequency.

This analysis uses a harmonic loading of 1kN applied in one node, recreating the experimental conditions. The response is generated at the point on the steel plate shown in Figure 4.1. No damping has been introduced in this steady state analysis. As Figure 4.2 shows, eight peaks can be observed and they correspond to global modes. The intensity of the peaks at 18,3 Hz and 21,6 Hz is too high to allow seeing clearly the other peaks; hence, to visualize correctly the frequency spectrum, they have been truncated.

One can observe from Figure 4.2 and Table 4.1 that three frequencies are extracted for both the first vertical and third vertical bending modes. The modes which required the most mass and have the maximal intensity on the frequency spectrum are used for the rest of the project. The difference between these three frequencies for both modes is induced by the behaviour of the upper part of the bridge. Finally, five eigenmodes are extracted (Table 4.2).
As Table 4.2 shows, a good agreement is obtained between the implemented un-ballasted finite element model and the experimental results, both for the natural frequencies values and the modes shapes, especially regarding the combination of the 3rd and 4th vertical bending mode which is a type of mode shape specific to this particular truss bridge. This is encouraging for the ballasted model implementation. The global and main mode shapes are then presented in Table 4.3 and on Figure 4.3 are plotted of the displacements of a longitudinal beam belonging to the deck along the x axis. As one can notice from Figure 4.3 and Table 4.3, the 3rd/4th vertical bending mode is unsymmetrical, as it has already been highlighted in Chapter 3.3.1. This asymmetry is confirmed through the FEM analysis and can be explained by the asymmetry of the bridge itself in the transverse direction.
Indeed, the stringer beams’ arrangement is not symmetric and this can lead to a global asymmetrical behaviour of the bridge when it comes to high vibration’s amplitudes.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>LUSAS values (Hz)</th>
<th>Experimental values (Hz)</th>
<th>% of difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} vertical bending</td>
<td>8,14</td>
<td>8,05</td>
<td>1,11</td>
</tr>
<tr>
<td>1\textsuperscript{st} vertical bending</td>
<td>9,89</td>
<td>9,68</td>
<td>2,21</td>
</tr>
<tr>
<td>1\textsuperscript{st} torsional</td>
<td>10,2</td>
<td>10,1</td>
<td>0,70</td>
</tr>
<tr>
<td>2\textsuperscript{nd} vertical bending</td>
<td>18,3</td>
<td>18,5</td>
<td>-0,98</td>
</tr>
<tr>
<td>3\textsuperscript{rd}/4\textsuperscript{th} vertical bending</td>
<td>21,6</td>
<td>21,6</td>
<td>0,19</td>
</tr>
</tbody>
</table>

However, this good correlation results from a comprehensive study of the influence of different parameters which is detailed thoroughly in the next Chapter 4.2.
Table 4.3: Eigenmodes shapes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1st vertical bending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane XZ</td>
<td><img src="image1" alt="Mode 5 Isometric View" /></td>
</tr>
<tr>
<td>$f_{1,\text{vertical}}$ = 8.14 Hz</td>
<td><img src="image2" alt="Mode 5 Plane XZ" /></td>
</tr>
<tr>
<td>Plane XZ</td>
<td><img src="image3" alt="Mode 8 Isometric View" /></td>
</tr>
<tr>
<td>$f_{1,\text{vertical}}$ = 9.68 Hz</td>
<td><img src="image4" alt="Mode 8 Plane XZ" /></td>
</tr>
<tr>
<td>Mode 9: 1st torsional</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td>Plane YZ</td>
<td></td>
</tr>
<tr>
<td>$f_{1 \text{ torsional}} = 10.2 \text{ Hz}$</td>
<td></td>
</tr>
<tr>
<td>Plane XZ</td>
<td></td>
</tr>
<tr>
<td>Isometric view</td>
<td></td>
</tr>
<tr>
<td>Mode 36 : 2\textsuperscript{nd} vertical bending</td>
<td></td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Plane XZ</td>
<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Mode 36: 2\textsuperscript{nd} vertical bending image" /></td>
<td></td>
</tr>
<tr>
<td>$f_{z \text{ vertical}} = 18.3 \text{ Hz}$</td>
<td></td>
</tr>
<tr>
<td>Isometric view</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Mode 36: Isometric view image" /></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode 42 : 3\textsuperscript{rd}/4\textsuperscript{th} vertical bending mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane XZ</td>
</tr>
<tr>
<td><img src="image3.png" alt="Mode 42: 3\textsuperscript{rd}/4\textsuperscript{th} vertical bending image" /></td>
</tr>
<tr>
<td>$f_{z \text{ vertical}} = 21.6 \text{ Hz}$</td>
</tr>
<tr>
<td>Isometric view</td>
</tr>
<tr>
<td><img src="image4.png" alt="Mode 42: Isometric view image" /></td>
</tr>
</tbody>
</table>
4.2 Calibration of the un-ballasted FEM model

The influence of different parameters on the frequencies has been thoroughly investigated in this chapter. First, the influence of the mesh size has been studied and as a result, an optimal mesh size for each element has been adopted to perform all the analyses. However, the mesh size does not have a significant influence on the results as long as realistic element sizes are chosen. Then, the influence of the type of element has been investigated. For instance, the transverse beams, which are slightly unsymmetrical due to the drainage slope, have been modelled in a more realistic way than the one presented in Chapter 2. The difference observed from the eigenvalue analyses between symmetrical and unsymmetrical cross beams was insignificant: around 0.3% at most. Therefore, symmetrical cross beams are used so that the model is simplified.

The parameter which appears to influence most the natural frequencies, and consequently the vibrations of the bridge, is the type of support conditions. Thus, this parameter is thoroughly studied and both the influence of the stiff beam characteristics and the boundary conditions type are investigated (Table 4.4). First, the boundary conditions were applied at the node at the intersection between the centre of gravity of the longitudinal beams and the cross beams. From this analysis, it appears that the stiffness of the bridge is too high in torsion: the difference for the 1st torsional frequency between the experiments and the model is about 6% whereas the agreement for the other frequencies remains acceptable (Table 4.4). Then, to model as realistically as possible the support conditions, a stiff beam between the centre of gravity of the beam and the bottom flange of the beam is defined. However, it appears from a comprehensive analysis that the Young’s Modulus of the stiff beam has a significant influence on the natural frequencies and as a result, different values of the Young’s Modulus have been tested.

It can be observed from Table 4.4 that the Young’s Modulus of the stiff beam need to be equal to the Young’s Modulus of steel in order to get the best agreement between the experiments and the finite element model. Indeed, the agreement remains good for the vertical bending modes no matter the stiffness of the stiff beam is. However, the agreement for the torsional mode is not acceptable at all when the stiffness of the stiff beam is 100 times higher than the Young’s Modulus of steel.

It is quite hard to explain what phenomenon happens here but one hypothesis can be that the web of the longitudinal beam is bending in the transverse direction when the structure is in resonance for the first torsional mode. The web of the longitudinal beams would then contribute to the transverse flexibility of the bridge. It would have been interesting to model the longitudinal beams’ web belonging to the deck as shell elements but by lack of time, it has not been done. This shell modelling may have been the appropriate solution to model the behaviour of the web.
Table 4.4: Influence of the stiff beam definition.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental frequencies (Hz)</th>
<th>Reference model $E_{\text{stiff beam}} = E_{\text{steel}}$</th>
<th>At the longitudinal beam’s centre of gravity</th>
<th>At the bottom of the longitudinal beam and $E_{\text{stiff beam}} = 100E_{\text{steel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical bending</td>
<td>8,05</td>
<td>8,14</td>
<td>8,15</td>
<td>8,08</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>9,68</td>
<td>9,89</td>
<td>9,89</td>
<td>9,86</td>
</tr>
<tr>
<td>1st torsional</td>
<td>10,1</td>
<td>10,2</td>
<td>10,8</td>
<td>11,2</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>18,5</td>
<td>18,3</td>
<td>18,3</td>
<td>19,1</td>
</tr>
<tr>
<td>3rd/4th vertical bending</td>
<td>21,6</td>
<td>21,6</td>
<td>21,3</td>
<td>21,8</td>
</tr>
</tbody>
</table>

Reference model’s support conditions applied:
Nonetheless, the web of the longitudinal beam seems to deform in two different ways both in the longitudinal and transverse directions with different stiffness for different modes. These deformations influence mainly the torsional mode.

Another conclusion can be drawn: the type of boundary conditions has no significant influence for the un-ballasted model. Indeed, there is no difference in the eigenvalue analysis when all the supports are fixed in all directions or if they are not (see boundary conditions detailed in Chapter 2.1.4). The stiffness of the stiff beam is a much more important parameter to study when the ballast is not in place yet. Nevertheless, the boundary conditions detailed in Chapter 2.1.4 and specified by the design office are adopted for the un-ballasted case.
Chapter 5

5 Ballast and tracks models

5.1 Ballast superstructure and properties of the track

The information about the ballast and the track has been extracted from CAD drawings, from directions given by Dr Jaroslaw Zwolski and from [14]. The track is composed of wooden sleepers, rails UIC60 and guard rails UIC49. The shape and the mass of the ballast profile (Figure 5.4) have been determined according to Figures 5.1, 5.2 and 5.3. However, it is important to mention that there is an uncertainty on the ballast profile: an accurate mass of the ballast has not been determined after the ballast has been placed on the bridge bed. Table 5.1 summarizes all the adopted properties for the track and ballast elements.

![Load exciter](image)

Figure 5.1: A view along the bridge with the track in place.

Table 5.1: Track element properties.
<table>
<thead>
<tr>
<th>Wooden sleepers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
</tr>
<tr>
<td>Space between two sleepers</td>
</tr>
<tr>
<td>Number of elements</td>
</tr>
<tr>
<td>Young’s modulus</td>
</tr>
<tr>
<td>Mass density</td>
</tr>
<tr>
<td>Total mass</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rails UIC60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear mass</td>
</tr>
<tr>
<td>Number of elements</td>
</tr>
<tr>
<td>Cross section area</td>
</tr>
<tr>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>Mass density</td>
</tr>
<tr>
<td>Total mass</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guard rails UIC49</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear mass</td>
</tr>
<tr>
<td>Number of elements</td>
</tr>
<tr>
<td>Cross section area</td>
</tr>
<tr>
<td>Young’s Modulus</td>
</tr>
<tr>
<td>Mass density</td>
</tr>
<tr>
<td>Total mass</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ballast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density</td>
</tr>
<tr>
<td>Volume</td>
</tr>
<tr>
<td>Cross section area</td>
</tr>
<tr>
<td>Mass</td>
</tr>
</tbody>
</table>

The total mass of the track is: \( m_{track} = 265\,600 \) kg and the total mass of the bridge without the ballast is: \( m_{steel\ bridge} = 154\,000 \) kg. The mass of the track is then 1,7 times larger than the mass of the steel bridge.
Figure 5.2: Ballast cross section in the middle of the bridge.

Figure 5.3: Ballast cross section at the ends of the bridge.

Figure 5.4: Ballast cross sections.

Area on the drawing: 3,6 m²
Area used for the calculations: 3,42 m²
5.2 Modelling the track

Several alternatives to model the ballast have been tested in the literature and some throughout this project. The most relevant results are shown in this chapter. For the purpose of the project, the ballast is modelled as 3D solid elements and the rails and the sleepers as 3D thick beams.

To model the ballast layer, solid elements are created and arranged on the bridge deck and 16 m before and after the bridge in order to recreate the continuity of the ballast (Figure 5.8). The solid elements are meshed as a volume with tetrahedral elements. The slope of the track is taken from the drawings and an equivalent cross section is determined (Table 5.1 and Figure 5.5). The points and lines of the steel plate and of the ballast bottom surface are merged.

Then, the rails and the guard rails are created and meshed as 3D thick beams. The cross sections are determined through reference tables [16]. An example of the rails cross section is shown in Figure 5.6. The steel rails are separated from the guard rail by 0.3 m. Regarding the wooden sleepers, a rectangular cross section of 0.26 x 0.15 m is adopted. In order to simplify the model, 48 sleepers are created instead of the 44 originally present on the bridge. In reality, the space between the sleepers is equal to 0.6 m but here, it is equal to 0.56 m. An equivalent density for the wood is then calculated and it is equal to 688 kg/m$^3$. An offset of -0.075 m is applied to the sleepers and of +0.08 m and +0.085 m respectively for the guard rails and the rails. Figure 5.7 shows a cross section of the track and Figure 5.8 shows an overall view of the ballasted model.
The ballast, which is not on the bridge deck, is vertically fixed at the bottom surface and the ballast layer ends are fixed in the longitudinal direction and in the transverse direction (Figures 5.9 and 5.10). Besides, the rails and the guard rails are blocked in the longitudinal direction. The boundary conditions are shown by green arrows on Figure 5.9 and Figure 5.10.
5.3 Different alternatives to model the ballast

5.3.1 Influence of the ballast stiffness

The first step of the ballasted model implementation consists in evaluating the ballast stiffness and the contribution of the ballast stiffness to the bridge stiffness.

For the first analysis, the model described in the previous section is used and the support conditions described in Chapter 2.2.4 are considered. A Young’s modulus of 10 MPa is chosen for the ballast: the ballast is thus considered just as an additional mass and does not bring any stiffness to the bridge.

The following results are obtained (Table 5.2):

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental frequencies (Hz)</th>
<th>LUSAS values for $E_{ballast} = 10$ MPa (Hz)</th>
<th>% of difference from the experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical bending</td>
<td>5,39</td>
<td>4,83</td>
<td>-11,6</td>
</tr>
<tr>
<td>1st torsional</td>
<td>9,26</td>
<td>8,10</td>
<td>-14,4</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>10,3</td>
<td>8,97</td>
<td>-14,2</td>
</tr>
<tr>
<td>3rd/4th vertical bending</td>
<td>12,3</td>
<td>9,89</td>
<td>-24,0</td>
</tr>
</tbody>
</table>

As one can notice, the agreement between experimental values and analytical ones is not good: a difference higher than 10% is observed for all the frequencies. This difference may either come from the stiffness of the ballast itself or from other parameters of the model of the bridge and this is what will be investigated in the rest of this chapter. Therefore, it can be concluded that the ballast appears to give an additional stiffness to the bridge: from 25% to 50% according to the mode.
Thereafter, the influence of the ballast stiffness on the bending stiffness of the bridge is tested. A Young's Modulus equal to 300 MPa is taken for this analysis. This value has been determined according to several studies on the subject, notably [8, 9, 10] and according to discussions with researchers from the department. Except the value of the ballast Young's Modulus, the exact same bridge model parameters are kept.

The following results are obtained (Table 5.3):

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental frequencies (Hz)</th>
<th>LUSAS values $E_{\text{ballast}} = 300$ MPa (Hz)</th>
<th>% of difference from the experiment</th>
<th>% of difference between $E_{\text{ballast}} = 10$ MPa or 300 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical bending</td>
<td>5,39</td>
<td>4,94</td>
<td>-8,94</td>
<td>2,38</td>
</tr>
<tr>
<td>1st torsional</td>
<td>9,26</td>
<td>8,20</td>
<td>-13,0</td>
<td>1,21</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>10,3</td>
<td>9,91</td>
<td>-3,41</td>
<td>9,48</td>
</tr>
<tr>
<td>3rd/4th vertical bending</td>
<td>12,3</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

One can notice, once again, that the agreement is not acceptable, especially for the 1st vertical bending and the 1st torsional modes. The 3rd/4th vertical bending is not observed. However, the value for the 2nd vertical bending frequency is closer to the experiments than the one from the previous analysis. Table 5.3 also shows that for the two first modes, the difference between the frequencies of the model using a low Young's Modulus value (10 MPa) and the one using a higher value (300 MPa) is very low. It can be concluded that for the two first eigenmodes of the studied bridge, the ballast does not bring any stiffness to the structure in a significant way and that the end moments do not play a significant role. Nonetheless, one can assume that the contribution of the ballast stiffness to the bridge stiffness may be non-negligible for the second vertical bending: around 9,5% according to Table 5.3.

The contribution of the ballast stiffness does not enable to match the experimental results for all the eigenfrequencies. Other parameters need to be further investigated so that a good correlation between the experiment and the FEM model is found.

### 5.3.2 Influence of the ballast mass

As it has been mentioned in Chapter 5.1, there is a relative uncertainty regarding the mass of ballast which has been placed on the bridge. This uncertain parameter can be a source of error and therefore, it must be further investigated.

A minimal mass density of the ballast is calculated in order to see the influence of the mass of the ballast on the natural frequencies of the bridge. This value is calibrated so that the 1st numerical vertical bending frequency matches the corresponding experimental frequency. This mass density, unrealistically low, is equal to:

$$\rho_{\text{ballast}} = 1 355 \, \text{kg/m}^3$$
In order to compare similar models, the ballast shape previously used is kept and an equivalent minimal mass corresponding to the minimal mass density is thus calculated. This minimal mass corresponds to a value of:

\[ m_{\text{minimal}} = 191\,000\,\text{kg} \]

This minimal mass corresponds to a reduction of 24.7% from the mass previously used. Figure 5.11 shows this reduction in terms of area.

Figure 5.11: Minimal mass cross section.

Once again, the model described in Chapter 5.2 is used, using the support conditions described in Chapter 2.1.4 and a Young’s Modulus for the ballast equal to 300 MPa.

Table 5.4: Eigenvalue analysis for \( m_{\text{minimal}} \).

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental frequencies (Hz)</th>
<th>LUSAS values for ( m_{\text{minimal}} )</th>
<th>% of difference from the experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^{st}) vertical bending</td>
<td>5,39</td>
<td>5,39</td>
<td>-0,02</td>
</tr>
<tr>
<td>1(^{st}) torsional</td>
<td>9,26</td>
<td>8,68</td>
<td>-6,65</td>
</tr>
<tr>
<td>2(^{nd}) vertical bending</td>
<td>10,3</td>
<td>10,9</td>
<td>6,16</td>
</tr>
<tr>
<td>3(^{rd})/4(^{th}) vertical bending</td>
<td>12,3</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

As one can notice from Table 5.4, the difference for the 1\(^{st}\) torsional and the 2\(^{nd}\) vertical bending modes is around 6.5% which is not good enough, although the agreement is good for the 1\(^{st}\) vertical bending mode.
The uncertainty of the mass of the ballast does not enable to explain the differences between the experimental values and those obtained from the FEM model. Other parameters regarding the model itself need to be further studied, since the uncertain parameters of the ballast have already been tested: both its mass and its stiffness. The only not well known parameters are the characteristics of the bearings.

5.3.3 Influence of the support conditions

5.3.3.1 Reference mass of ballast

In this chapter, the support conditions are studied; both the influence of the stiffness of the stiff beam and the type of boundary conditions. It appears that the bridge is not stiff enough if the boundary conditions are kept in the same way as the un-ballasted model, according to the previous analyses. For this analysis, the ballast stiffness is assumed equal to 300 MPa and the ballast mass originally determined in Chapter 5.1 is used.

Table 5.5 summarizes the influence on the natural frequencies of the two parameters mentioned above. It can be noticed from Table 5.5 that the stiff beam definition, i.e. its stiffness, plays a significant role on the natural frequencies of the bridge.

Indeed, if the Young’s Modulus of the stiff beam is equal to the Young’s Modulus of steel, no matter what type of boundary conditions is applied (translations fully fixed or not), it is not possible to match the experimental frequencies. Furthermore, if the initial boundary conditions are used and the stiffness of the stiff beam is increased to be 100 times higher than the steel stiffness (\(E_{\text{stiff beam}} = 2.05 \times 10^4 \text{ GPa}\)), one can notice from Table 5.5 that the agreement for the three vertical bending modes is still not acceptable, especially for the 3rd/4th vertical bending. Nevertheless, it can also be observed that the agreement with the experimental value is very good for the 1st torsional mode.

Then, the translations of the four bearings are fully fixed in the three directions and it can be noticed from the last column of Table 5.5 that the agreement is acceptable for all the modes, both the vertical bending ones and the torsional one.

The type of boundary conditions influences significantly the results in that case and the increase of the stiffness of the stiff beam enables to improve the agreement for the torsional mode.
Table 5.5: Influence of the support conditions on the natural frequencies.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental values (Hz)</th>
<th>Reference boundary conditions $E_{\text{stiff beam}} = E_{\text{steel}}$</th>
<th>All supports fixed $E_{\text{stiff beam}} = E_{\text{steel}}$</th>
<th>Reference boundary conditions $E_{\text{stiff beam}} = 100E_{\text{steel}}$</th>
<th>All supports fixed $E_{\text{stiff beam}} = 100E_{\text{steel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LUSAS values (Hz)</td>
<td>% of difference</td>
<td>LUSAS values (Hz)</td>
<td>% of difference</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>5.39</td>
<td>4.94</td>
<td>-8.94</td>
<td>4.95</td>
<td>-8.74</td>
</tr>
<tr>
<td>1st torsional</td>
<td>9.26</td>
<td>8.20</td>
<td>-13.0</td>
<td>8.28</td>
<td>-11.9</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>10.3</td>
<td>9.91</td>
<td>-3.41</td>
<td>9.91</td>
<td>-3.37</td>
</tr>
<tr>
<td>3/4th vertical bending</td>
<td>12.3</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>
A steady state analysis, done in the same way as it is described in Chapter 4.1, confirms that four global mode can be extracted from this analysis and that a good correlation with the experiments is achieved (Figure 5.12).

As a result, one can conclude that the stiff beam stiffness plays a significant role especially on the torsional mode; and the boundary conditions, in contrast, have an important influence on the vertical bending modes. It would have been interesting to model the longitudinal beams belonging to the deck using shell elements, as it has already been mentioned in Chapter 4.2. What can be concluded from the above analysis is that the additional mass brought by the ballast changes the support conditions types. First, it appears to increase the stiffness of the stiff beam and therefore the stiffness of the longitudinal beam web in the transverse direction. In addition, it can be assumed that the mass of the ballast and the track, which represent almost twice the mass of the bridge, makes all the supports unable to move.

It is also interesting to compare the mode shapes for the un-ballasted model and the best ballasted model in order to confirm the agreement through the mode shapes (Figure 5.13 to 5.16). The vectors of the displacements have been normalized to have a norm equal to one. One can observe that the global shape of the four modes remains the same from the un-ballasted model to the ballasted one. Nonetheless, small difference can be noticed. For the 1st vertical bending mode and the 1st torsional mode (Figures 5.13 and 5.14), the effect of the end moment is clearly visible for the ballasted case. Besides, differences can also be noticed for the 2nd vertical bending mode and the 3rd/4th vertical bending mode (Figures 5.15 and 5.16). It appears that the ballast modifies slightly the vibration’s amplitude and shapes.
Figure 5.13: Mode shapes comparison between the un-ballasted case and the ballasted case – 1\textsuperscript{st} vertical bending mode.

Figure 5.14: Mode shapes comparison between the un-ballasted case and the ballasted case – 1\textsuperscript{st} torsional mode.
Figure 5.15: Mode shapes comparison between the un-ballasted case and the ballasted case – 2\textsuperscript{nd} vertical bending mode.

Figure 5.16: Mode shapes comparison between the un-ballasted case and the ballasted case – 3\textsuperscript{rd}/4\textsuperscript{th} vertical bending mode.
Therefore, one can conclude from these observations that solid and beam elements, associated to realistic boundary conditions for both the bridge and the ballast layer, seem to be a good alternative to model the ballast and the track.

However, this model results from a comprehensive and thorough analysis of different parameters influencing the natural frequencies of the truss bridge. Thus, it is also interesting to test the influence of the support conditions on the previous models implemented in Chapters 5.3.1 and 5.3.2.

5.3.3.2 Minimal ballast mass

The minimal ballast mass density determined in Chapter 5.3.2 is used here to study the influence of the support conditions on the natural frequencies if the ballast mass is minimal. A similar study of the support conditions is done with the minimal mass and Table 5.6 summarizes the results from the analysis.

From Table 5.6, it cannot be found an acceptable model which matches the natural frequencies for all the modes with this minimal ballast mass. Even though the mass of the ballast is not accurately known, it can be concluded from this analysis that the uncertainty on the ballast mass is not higher than 20% and also that the one provided by the design office seems correct according to Table 5.5. Indeed, the results from the last column of Table 5.5 are good, using the model described in Chapter 5.3.3.1.
Table 5.6: Influence of the ballast mass (minimal mass) and the support conditions on the natural frequencies.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental values (Hz)</th>
<th>Reference boundary conditions $E_{\text{stiff beam}} = E_{\text{steel}}$</th>
<th>All supports fixed $E_{\text{stiff beam}} = E_{\text{steel}}$</th>
<th>Reference boundary conditions $E_{\text{stiff beam}} = 100 \cdot E_{\text{steel}}$</th>
<th>All supports fixed $E_{\text{stiff beam}} = 100 \cdot E_{\text{steel}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>/</td>
<td>Reference boundary conditions</td>
<td>All supports fixed</td>
<td>Reference boundary conditions</td>
<td>All supports fixed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$E_{\text{stiff beam}} = E_{\text{steel}}$</td>
<td>$E_{\text{stiff beam}} = E_{\text{steel}}$</td>
<td>$E_{\text{stiff beam}} = 100 \cdot E_{\text{steel}}$</td>
<td>$E_{\text{stiff beam}} = 100 \cdot E_{\text{steel}}$</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>5,39</td>
<td>5,39 -0,02</td>
<td>5,39 0,14</td>
<td>5,45 1,14</td>
<td>5,85 8,64</td>
</tr>
<tr>
<td>1st torsional</td>
<td>9,26</td>
<td>8,68 -6,23</td>
<td>8,77 -5,34</td>
<td>9,62 3,87</td>
<td>9,82 6,01</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>10,3</td>
<td>10,9 6,59</td>
<td>10,9 6,59</td>
<td>10,6 3,68</td>
<td>10,8 5,74</td>
</tr>
<tr>
<td>3/4th vertical bending</td>
<td>12,3</td>
<td>/ /</td>
<td>/ /</td>
<td>11,9 -3,10</td>
<td>13,7 11,9</td>
</tr>
</tbody>
</table>
5.3.3.3 Ballast stiffness

Chapter 5.3.1 has highlighted the contribution of the stiffness of the ballast, mainly for the 2nd vertical bending mode. The influence of the support conditions has been confirmed in this chapter and thus, it is interesting to study the influence of this parameter if the ballast stiffness is decreased so that the ballast is considered just as an additional mass. The ballast stiffness is therefore taken equal to 10 MPa and a study, similar to the previous ones, about the influence of the support conditions is done. Table 5.7 presents the results when all the supports are fixed in the three directions and a stiff beam’s Young’s modulus is equal to 100 times the steel one.

Table 5.7: Results from the analysis studying the influence of the support conditions when the ballast considered as an additional mass.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>Experimental frequencies (Hz)</th>
<th>LUSAS values for $E_{ballast} = 300$ MPa (Hz)</th>
<th>LUSAS values for $E_{ballast} = 10$ MPa (Hz)</th>
<th>% of difference between the two models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st vertical bending</td>
<td>5,39</td>
<td>5,37</td>
<td>5,22</td>
<td>2,72</td>
</tr>
<tr>
<td>1st torsional</td>
<td>9,26</td>
<td>9,42</td>
<td>9,22</td>
<td>2,04</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>10,3</td>
<td>9,84</td>
<td>8,93</td>
<td>9,25</td>
</tr>
<tr>
<td>3rd/4th vertical bending</td>
<td>12,3</td>
<td>12,32</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

As one can notice from Table 5.7, the contribution of ballast stiffness to the bridge stiffness is approximately the same, no matter what boundary conditions are used (Tables 5.3 and 5.7). Besides, this analysis confirms the higher contribution of the ballast stiffness to the bridge stiffness for the second vertical bending mode. In contrast, the ballast stiffness does not play any significant role when it comes to the 1st vertical bending and 1st torsional modes.
Chapter 6

6 Discussion and conclusion

6.1 Comparison of the experimental and analytical results

6.1.1 Reference un-ballasted and ballasted models

This study results in the implementation of two optimized finite element models: one corresponding to the un-ballasted case and one corresponding to the ballasted case. Differences between the two bridge models have been highlighted in Chapter 5. They are mainly related to the support conditions. The difference between the support conditions of the two models are summarised in Table 6.1.

Table 6.2 summarises the results obtained from the eigenvalue analysis for both models. For both cases, a good agreement between the experimental eigenfrequencies and mode shapes, determined in Chapter 3, and the analytical results is achieved.
Table 6.1: Support conditions for both final models.

<table>
<thead>
<tr>
<th>Support conditions</th>
<th>UN-BALLASTED CASE</th>
<th>BALLASTED CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiff beam</td>
<td>Young’s Modulus</td>
<td>Young’s Modulus</td>
</tr>
<tr>
<td></td>
<td>( E_{\text{stiff beam}} = 205 \text{ GPa} )</td>
<td>( E_{\text{stiff beam}} = 100 \cdot E_{\text{steel}} = 205 \cdot 10^4 \text{ GPa} )</td>
</tr>
<tr>
<td>Boundary conditions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Translations</td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>Support 1</td>
<td>free</td>
<td>free</td>
</tr>
<tr>
<td>Support 2</td>
<td>fixed</td>
<td>free</td>
</tr>
<tr>
<td>Support 3</td>
<td>free</td>
<td>fixed</td>
</tr>
<tr>
<td>Support 4</td>
<td>fixed</td>
<td>fixed</td>
</tr>
</tbody>
</table>

Table 6.2: Results from the eigenvalue analysis for un-ballasted and ballasted final finite element models.

<table>
<thead>
<tr>
<th>Mode type</th>
<th>UN-BALLASTED CASE</th>
<th>BALLASTED CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Experimental</td>
<td>LUSAS</td>
</tr>
<tr>
<td></td>
<td>values (Hz)</td>
<td>values (Hz)</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>8,05</td>
<td>8,14</td>
</tr>
<tr>
<td>1st vertical bending</td>
<td>9,68</td>
<td>9,89</td>
</tr>
<tr>
<td>1st torsional</td>
<td>10,1</td>
<td>10,2</td>
</tr>
<tr>
<td>2nd vertical bending</td>
<td>18,5</td>
<td>18,3</td>
</tr>
<tr>
<td>3rd/4th vertical bending</td>
<td>21,6</td>
<td>21,6</td>
</tr>
</tbody>
</table>

6.1.2 Modelling the ballast

Table 6.3 summarizes the contribution to the stiffness of the structure for the ballasted model of the different parameters studied above. The contribution of the stiffness of the stiff beam is determined for the initial boundary conditions and \( E_{\text{ballast}} = 300 \text{ MPa} \) and therefore, the contribution of the boundary conditions is assessed once all the translations at the supports are fixed.

It has been shown that the ballast gives an additional stiffness to the bridge: from 25% to 50% according to the mode. Moreover, from Table 6.3, different conclusions can be drawn. First, it appears that the ballast stiffness has a non-negligible influence on the 2nd vertical bending mode of the studied truss bridge. Other studies [8, 9] have shown that the stiffness of the ballast may be frequency-dependent. In the presented case, when the stiffness of the ballast is multiplied by 30, the stiffness of the model is increased by 22%. However, for the two first modes, its contribution represents less than 5%.
Furthermore, the ballast seems to alter the support conditions: the mass of the ballast changes the behaviour of the bearings of the bridge but also it modifies the behaviour of the web of the longitudinal beams.

Table 6.3 highlights the significant influence of the definition of the stiff beam on the 1\textsuperscript{st} torsional frequency for the ballasted case. Besides, the influence had also been noticed for the un-ballasted case (see Chapter 4.2). For the 1\textsuperscript{st} torsional mode, if the stiffness of the stiff beam is multiplied by 100, the bridge stiffness is increased by 27%, whereas for the two other modes, the stiff beam stiffness has a minor influence (less than 5%).

Finally, the crucial role of the boundary conditions is emphasized in the above table where it can be observed that the additional mass brought by the ballast appears to fix all the bearings of the bridge: they are thus unable to move in all directions.

To conclude, the influence of the support conditions can be questioned and it is surely specific to each bridge but it can partly explain the difference between the experimental and the numerical analysis. It is also important to mention that the uncertainty of the ballast mass is clearly a limitation in this project. A better and more accurate estimation of the ballast mass is necessary to strengthen the conclusions drawn in this thesis.

### 6.2 Further research

Regarding the bridge studied in this thesis, an interesting suggestion to broaden the analysis would be to model the two longitudinal beams of the deck by shell elements. It might enable to model more accurately the behaviour of the webs because they seem to bend in the transverse direction when the bridge is in resonance for the 1\textsuperscript{st} torsional mode.

The next step of this project would consist in using the implemented models for high-speed train applications. It would be interesting to compare experimental measurements of high-speed trains passing with a numerical dynamic response to study the influence of a passing train on the ballast model. However, the ballasted model proposed in this project might result in very time consuming analyses for high-speed train applications. First, a 2D truss model can be implemented, in first...
assumption, only including the mass of the ballast. Indeed, a mismatch in frequency would result in a change in resonance speed.

All the conclusions drawn in this thesis need, of course, to be strengthened and confirmed by additional studies about other bridges of different types. It would be of interest to implement a similar ballasted model than the one used in this project and to test the influence of the highlighted parameters: the stiffness of the ballast and the support conditions.

Finally, the interesting fact regarding the Polish truss bridge is the experimental measurements of the acceleration under a dynamic loading, before and after the track installation: that is what makes this study unique. It is rare to be allowed to perform experimental dynamic measurements before the track is placed; railway bridges are indeed not designed to be excited before their completion. This could be one of the main potential hindrances to pursue of the study.
References

[1] FINK, J., MÄHR, T., (2009), Influence of the ballast superstructure on the dynamics of slender steel railway bridges, Institute of Structural Engineering, Department of Steel Structure, Vienna University of Technology, Vienna, Austria.


[3] BUELL, D. C., (1914), Ballast: a reprint of The Railway Educational Bureau’s Track course Lesson, University of Wisconsin.


Appendix A

Accelerometers arrangements for test 1 and test 2
A.1 Setups for the un-ballasted bridge

Figure A.1: Accelerometers' arrangement Setup 01.

Figure A.2: Accelerometers' arrangement Setup 02.

Figure A.3: Accelerometers' arrangement Setup 03.
Figure A.4: Accelerometers’ arrangement Setup 04.

Figure A.5: Accelerometers’ arrangement Setup 05.

Figure A.6: Accelerometers’ arrangement Setup 06.
Figure A.7: Accelerometers' arrangement Setup 07.

Figure A.8: Accelerometers' arrangement Setup 08.
Figure A.9: Accelerometers’ arrangement Setup 09.

Figure A.10: Accelerometers’ arrangement Setup 10.
Figure A.11: Accelerometers’ arrangement Setup 11.

Figure A.12: Accelerometers’ arrangement Setup 12.
A.2 Setups for the ballasted bridge

Figure A.13: Accelerometers’ arrangement Setup 00.

Figure A.14: Accelerometers’ arrangement Setup 01.

Figure A.15: Accelerometers’ arrangement Setup 02.
Figure A.16: Accelerometers’ arrangement Setup 03.

Figure A.17: Accelerometers’ arrangement Setup 04.

Figure A.18: Accelerometers’ arrangement Setup 05.
Figure A.19: Accelerometers’ arrangement Setup 06.

Figure A.0.10: Accelerometers’ arrangement Setup 07.
Figure A.21: Accelerometers’ arrangement Setup 08.

Figure A.22: Accelerometers’ arrangement Setup 09.
Figure A.23: Accelerometers’ arrangement Setup 10.

Figure A.24: Accelerometers’ arrangement Setup 11.
Figure A.25: Accelerometers’ arrangement Setup 12.

Figure A.0.26: Accelerometers’ arrangement Setup 13.
Appendix B

MATLAB codes to process the experimental data
B.1 Natural frequency extraction

%% Setup & Acceleration
load('setup04');
acceleration = x13;

%% Whole signal processing
% Plot of the whole signal
figure(1); subplot (2,1,1), plot(time,acceleration1,'b'); grid;
xlabel('Time (s)','FontSize', 11)
ylabel('Acceleration amplitude','FontSize',11)
title('Whole Signal');

% Plot of the whole signal FFT
fft_acceleration1 = (abs(fft(acceleration1))/N);
figure(1); subplot(2,1,2), plot(f(f<20),fft_acceleration1 (f<20),'r');grid;
axis ([0, 20, 0, 0.002]);
xlabel('Frequency (Hz)','FontSize', 11)
ylabel('FFT signal','FontSize',11)
title('FFT Whole Signal');

%% Frequency extraction
%% 1st Frequency
%% Variables
filter_order=4;
freq_inf = 7.8;
freq_sup = 8.3;
start_window= 52500;
end_window=70000;
N=end_window-start_window;

%% Filter
% butter_filter (filter_order , freq_inf , freq_sup, signal)
filtered_signal=butter_filter (filter_order , freq_inf , freq_sup,
acceleration1);

%% Isolate signal
% [isolated_signal,isolated_signal_freq]=isolate_signal (start_window,
end_window, signal)
[isolated_signal1,isolated_signal_freq]=isolate_signal (start_window,
end_window, filtered_signal);

%% Eigenfrequency value
% [peak_position] = extract_eigenfrequency(signal)
[peak_position] = extract_eigenfrequency(isolated_signal_freq)
% The same process is adopted for the other eigenfrequencie
Functions used:

- Butterworth function:

```matlab
function filtered_signal = butter_filter (filter_order , freq_inf , freq_sup, signal)
load('variables_global')
[B,A]=butter( filter_order , [freq_inf freq_sup]/(0.5*Fs) , 'bandpass');
filtered_signal = filter(B,A,signal);
%
% Plot the filtered signal
figure;
plot(time, filtered_signal, 'g'); grid;
xlabel('Time (s)');ylabel('Acceleration amplitude (m/s2)');
title('Filtered Signal');
end
```

- Signal isolation function:

```matlab
function [isolated_signal,isolated_signal_freq] = isolate_signal
(start_window, end_window, signal)
load('variables_global')
N= end_window - start_window;
%
% zero padding
% Put 0 before the chosen window
for i = 1:start_window
    signal(i) = 0;
end
% Put 0 after the chosen window
for i = end_window:length(signal)
    signal(i) = 0;
end
%
% compute the Hamming window
win_hamming = zeros(length(signal),1);
win_hamming(start_window:end_window-1, 1) = hamming(end_window-
start_window);
%
% compute the isolated signal (signal * windows)
isolated_signal = signal.*win_hamming;
figure, plot(time,isolated_signal, 'b'); grid;
xlabel('Time (s)');ylabel('Aceeleration amplitude (m/s2)');
title('Isolated filtered signal');
```
% compute fft
isolated_signal_freq = (abs(fft(isolated_signal))/N);

% plot fft
figure,plot(f(f<20), isolated_signal_freq (f<20), 'r'); grid;
xlabel('Frequency (Hz)','FontSize', 11)
title('FFT of isolated filtered signal');
end

Eigenfrequency extraction:

function [peak_position] = extract_eigenfrequency(signal)
load('variables_global.mat');
[peak_value, peak_position]=max((signal));

% give the peak frequency value
peak_position = f(peak_position);
end

B.2 Mode shape identification

acceleration1=x13;
acceleration2=x7;

%% Mode shape
%% Phase acceleration comparison

% Signal 1
filtered_signal=butter_filter (filter_order , freq_inf , freq_sup, acceleration2);
[isolated_signal2,isolated_signal_freq]=isolate_signal (start_window, end_window, filtered_signal);

% Signal 2
filtered_signal=butter_filter (filter_order , freq_inf , freq_sup, acceleration2);
[isolated_signal2,isolated_signal_freq]=isolate_signal (start_window, end_window, filtered_signal);

% Plot the time signals on the same graph
figure; hold on
plot(time,isolated_signal1, 'b');
plot(time,isolated_signal2, 'r'); grid;
xlabel('Time (s)','FontSize', 11)
ylabel('Acceleration amplitude (m/s2)','FontSize',11)
title('Signals Comparison');
%% Displacement comparison: Newmark method
[a,v,d,t]=Newmark(beta, gamma, x_bis, rate, fmin, fmax)
fmin=7.5;
fmax=8.5;

[a1,v1,d1,t1]=Newmark(1/4,1/2,acceleration1,Fs,fmin,fmax);
[a2,v2,d2,t2]=Newmark(1/4,1/2,acceleration2,Fs,fmin,fmax);

figure(2);
hold on
plot(time,d1, 'b');
plot(time,d2, 'r');
xlabel('Time (s)','FontSize', 11);
ylabel('Displacements ','FontSize',11); grid;
title('Displacement comparison');

B.3 Damping estimation

%% Damping ratio of the 1st Frequency
%% Variables
filter_order=2;
freq_inf = 5.28;
freq_sup = 5.4;

%% Filter
% butter_filter (filter_order , freq_inf , freq_sup, signal)
filtered_signal=butter_filter (filter_order , freq_inf , freq_sup, acceleration);

%% Isolated signal for damping calculation

t_inf=78;
delta_t=18;
acceleration_2=acceleration(time>t_inf);
t=time(time>t_inf)-t_inf;
acceleration_isolated=acceleration_2(t<delta_t);
t=t(t<delta_t);
figure(3);
plot(t,acceleration_isolated,'b');grid;
T_2=max(t);
N=length(t);
f_2 = 0:1/T_2:Fs;
isolated_fft = (abs(fft(acceleration_isolated))/N);
figure(4);
plot(f_2(f_2<20),isolated_fft(f_2<20), 'r');
axis ([0, 20, 0, 0.00125]);grid;
%% Peak frequency & Damping ratio

%(peak_position, damping_ratio) = damping_calculation(signal)
[peak_position, damping_ratio] = damping_calculation(isolated_fft)

Functions used:

- Damping calculation:

```matlab
function [peak_position, damping_ratio] = damping_calculation(signal)

load('variables_global.mat');
[peak_value, peak_position] = max((signal));

% compute the threshold
A = peak_value/sqrt(2);

% compute the lowest bandwidth frequency
freq_inf = peak_position;
while signal(freq_inf) > A
    freq_inf = freq_inf - 1;
end

% choose the most accurate point
if abs(A - signal(freq_inf)) > abs(A - (signal(freq_inf) - 1))
    freq_inf = freq_inf - 1;
end

% compute the lowest bandwidth frequency
freq_sup = peak_position;
while signal(freq_sup) > A
    freq_sup = freq_sup + 1;
end

% choose the most accurate point
if abs(A - signal(freq_sup)) > abs(A - (signal(freq_sup) - 1))
    freq_sup = freq_sup - 1;
end

% Compute the damping ratio
damping_ratio = 100 * (freq_sup - freq_inf) / (freq_sup + freq_inf);

% Give the peak frequency value
peak_position = f(peak_position);
end
```