This is a submitted version of a paper published in *Series of Physical-Technical and Mathematical sciences*.

Citation for the published paper:
"Stability of the fiber surrounded by two subsequental shells in the elastic matrix"
*Series of Physical-Technical and Mathematical sciences*, XVIII(3-4): 162-166

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TRANSACTIONS
OF ACADEMY OF SCIENCES OF AZERBAIJAN

FIZİKA-TEXNIKA VƏ RİYAZİYYAT EMLƏRLİ SERİYASI
SERIES OF PHYSICAL-TECHNICAL AND MATHEMATICAL SCIENCES

CİLD XVIII VOLUME

RİYAZİYYAT VƏ MEXANİKA
MATHEMATICS AND MECHANICS

№ 3-4

1998
"ELM" NƏŞRİYYATI - "ELM" PUBLISHING HOUSE

BAKI -- BAKU
MECHANICS

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STABILITY OF THE FIBER, SURROUNDED BY TWO SUBSEQUENTIAL SHELLS, IN THE ELASTIC MATRIX

Abstract

In the case of the piecewise - homogeneous body model with the use of the three-dimensional linearized theory of elastic stability under the second variant of small precritical deformations the problem of stability of the fiber, surrounded by two subsequential shells in the elastic matrix, is investigated. In this case it is assumed, that the external compressive forces act at infinity in the direction along the fiber.

By the present time the investigations of stability of fibers in the elastic matrix have been carried out in many works ([1-4] and other) and on that the inhomogeneity of the fiber material in the radial direction and existence of undersurface zone between the maretials of the fiber and the matrix have not been considered. The mentioned properties are characteristic for fiber composites and influence of those properties to loss of stability of the fiber in the elastic matrix is of considerable interest.

In the present work it is offered the model for considering of above mentioned properties at investigation of loss of stability of the fiber in the elastic matrix. The offered model consists of that existence of inhomogeneity of the fiber material and existence of the zone, which covers the surface of the fiber, represent as existence of two sequential shells, surrounding the fiber. So it is considered the stability problem of the fiber, surrounded by two sequential shells, in the elastic matrix. On that, the investigations are carried out on the base of model of piecewise homogeneous body under the three-dimensional linearized theory of elastic stability.

The precritical state is determined in a view:

\[
\begin{align*}
\chi^{0f} & = \epsilon^{f0} = \epsilon^{s0} = \epsilon^{0s} = \epsilon^0 \\
\chi^{01} & = \epsilon^{0s} = \epsilon^{0s} = \epsilon^{0s} = \epsilon^{0s} = \epsilon^{0s} = \epsilon^{0s}
\end{align*}
\]  

(1)

The conditions of complete bond of vectors of forces and displacements are fulfilled on the surface of separation of the fiber, the first and the second shells, and the matrix

\[
\begin{align*}
P^f_{r=0} & = P^s_{r=0} & P^f_{r=0} & = P^s_{r=0} & P^f_{r=0} & = P^s_{r=0} \\
u^f_{r=0} & = u^s_{r=0} & u^f_{r=0} & = u^s_{r=0} & u^f_{r=0} & = u^s_{r=0} \\
P^s_{r=0} & = P^s_{r=0} & P^s_{r=0} & = P^s_{r=0} & P^s_{r=0} & = P^s_{r=0} \\
u^s_{r=0} & = u^s_{r=0} & u^s_{r=0} & = u^s_{r=0} & u^s_{r=0} & = u^s_{r=0}
\end{align*}
\]

(2)

We take the forms of loss of stability of the considered body are periodic in direction of axis 0\(x_1\) and introduce the conditions of damping for \(r \to \infty\), that is
\[ p'_{i}^{r} |_{r \to \infty} \to 0; \quad p'_{j}^{u} |_{r \to \infty} \to 0; \quad p'_{s}^{u} |_{r \to \infty} \to 0 \]

Besides that we take
\[ \alpha_{j}^{f} |_{r \to \infty} < \alpha; \quad u_{i}^{f} |_{r \to \infty} < \infty \]

For functions \( \psi \) and \( \chi \) we find the following expressions:

\[ \psi_{f} = A_{f} \frac{\pi}{l} I_{1} \left[ \frac{\pi}{l} \xi_{3} r \right] \sin \theta \sin \left( \frac{\pi}{l} x_{3} \right); \]

\[ \chi_{f} = \left[ B_{f} I_{1} \left( \frac{\pi}{l} \xi_{3} r \right) + C_{f} I_{1} \left( \frac{\pi}{l} \xi_{3} r \right) \right] \cos \theta \cos \left( \frac{\pi}{l} x_{3} \right); \]

\[ \psi_{a} = \frac{\pi}{l} \left[ \frac{\pi}{l} \xi_{3} r \right] + A_{a} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) \sin \theta \sin \left( \frac{\pi}{l} x_{3} \right); \]

\[ \chi_{a} = \left[ B_{a} I_{1} \left( \frac{\pi}{l} \xi_{3} r \right) + B_{a} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) + C_{a} I_{1} \left( \frac{\pi}{l} \xi_{3} r \right) + \right. \]

\[ \left. + C_{a} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) \right] \cos \theta \cos \left( \frac{\pi}{l} x_{3} \right), \quad k = 1, 2; \]

\[ \psi_{n} = A_{n} \frac{\pi}{l} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) \sin \theta \sin \left( \frac{\pi}{l} x_{3} \right); \]

\[ \chi_{n} = \left[ B_{n} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) + C_{n} K_{1} \left( \frac{\pi}{l} \xi_{3} r \right) \right] \cos \theta \cos \left( \frac{\pi}{l} x_{3} \right); \]

\[ l_{1}(\chi) \] is Bessel function of pure imaginary argument;
\[ K_{1}(\chi) \] is Macdonald's function.

\[ A_{f}, B_{f}, C_{f}; A_{a}, A_{a}; B_{a}, B_{a}; C_{a}, C_{a} \quad \text{\( k = 1, 2 \)}; \quad A_{n}, B_{n}, C_{n} \]

are unknown constants.

\[ u_{r} = \frac{1}{r} \frac{\partial}{\partial \theta} \psi - \frac{1}{r} \frac{\partial^{2}}{\partial \theta \partial x_{3}} \psi; \]

\[ u_{\theta} = - \frac{\partial}{\partial r} \psi - \frac{1}{r} \frac{\partial^{2}}{\partial r \partial x_{3}} \psi; \]

\[ u_{z} = \left( \omega_{101} + \omega_{111} \right) \Delta \left( \omega_{101} \Delta + \omega_{111} \frac{\partial^{2}}{\partial x_{3}^{2}} \right) \chi \]

\[ \Delta = \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^{2}} \frac{\partial^{2}}{\partial \theta^{2}} \]

For compressible bodies for small precritical deformations:
\begin{align*}
P_r &= \omega_{1111} \frac{\partial u_1}{\partial r} + \omega_{1112} \left( \frac{1}{r} \frac{\partial u_1}{\partial \theta} + \frac{1}{r} u_1 \right) + \omega_{1133} \frac{\partial u_3}{\partial \xi_3} \\
P_\theta &= \omega_{1211} \frac{\partial u_2}{\partial r} + \omega_{1212} \left( \frac{1}{r} \frac{\partial u_2}{\partial \theta} - \frac{1}{r} u_2 \right) \\
P_3 &= \omega_{1311} \frac{\partial u_3}{\partial r} + \omega_{1312} \frac{\partial u_3}{\partial \xi_3} \\
\det \beta_{i,j} &= 0 \quad i, j = 1, 2, 3, 18 \\
N &= \frac{\pi R}{l}, \quad N_1 = \frac{\pi R_2}{l}, \quad N_2 = \frac{\pi R_3}{l}
\end{align*}

The pointed characteristic equation is obtained in the general view for the cases, when the materials of the fiber, shells (which surround the fiber) and the matrix are transversal isotropic, which axis of elastic symmetry coincides with the axis of the fiber (the axis \( \alpha_x \)).

The concrete numerical investigations were obtained in the case, when the materials of the fiber, the shells and the matrix are homogeneous and isotropic with elastic characteristics \( E^f, \nu^f, E^h, \nu^h, E^m, \nu^m \) (\( E \) is Young's modulus, \( \nu \) is Poisson coefficient). At the all considered cases it was taken, that \( \nu^f = \nu^h = \nu^m = \nu \) and by that the homogeneity of the precritical state was provided, which is determined in a view

\begin{align*}
\sigma_{33} &= \frac{E^f}{E^m} P; \quad \varepsilon = \varepsilon_{33} = \varepsilon_{33} = \varepsilon_{33} = \frac{P}{E^m}; \\
\sigma_{33} &= \frac{E^h}{E^m} P; \quad \sigma_{33} = \frac{E^h}{E^m} P; \\
\varepsilon_{11} &= \varepsilon_{22} = \frac{E^h}{E^m} P; \quad \varepsilon_{11} = \varepsilon_{22} = \frac{E^h}{E^m} P;
\end{align*}

Let introduce the designations:

\[ h_1 = R_1 - R; \quad h_2 = R_2 - R; \]

\[ N = \frac{\pi R}{l}, \quad N_1 = \pi(1 + h_1), \quad N_2 = \pi(1 + h_1 + h_2) \]

As \( h_1 \) and \( h_2 \) the thicknesses of the shells were denoted with external radiuses \( R_1 \) and \( R_2 \), correspondingly. Also we take, that \( \nu^f = \nu^h = \nu^m = \nu = 0.3 \)

Let investigate dependence between \( \varepsilon \) and \( N \) for different values of the parameters of the problem. On that, we introduce also the designation

\[ \varepsilon_{\alpha} = \min \{ \varepsilon(\alpha) \} \text{ (*)} \]

and we denote value \( N \) as \( \varepsilon_{\alpha} \) (\( N \) is parameter of wave formation), which corresponds to \( \varepsilon_{\alpha} \). Condition

\[ E^f > E^h > E^m \text{ (***)} \]

supports fulfilling of (*) for \( \varepsilon_{\alpha} = 0 \), or (***) supports appearance of internal loss of stability in the body with structure, represented in fig.1.
From analyses of the numerical results it follows, that for all chosen \( h_i / R \) and \( h_i / R \) the growth of \( E^i / E \), and also the growth of \( E^f / E \) reduces to monotone decrease of \( \varepsilon_\alpha \) values.

The comparison of the values, obtained for different \( h_i / R \) and \( h_i / R \), shows, that for fixed \( h_i / R \) the increase of \( h_i / R \), and also for fixed \( h_i / R \) the increase of \( h_i / R \) reduces to monotone increase of \( \varepsilon_\alpha \). On that the values of \( \varepsilon_\alpha \), obtained in the cases \( h_i / R = 0 \) (we denote these values as \( \varepsilon_{\alpha,2.0} \)) and \( h_i / R = \infty \) (we denote these values as \( \varepsilon_{\alpha,2.\infty} \)), coincide with the corresponding values of \( \varepsilon_\alpha \), obtained for the fiber with one shell. Besides that, the values of \( \varepsilon_\alpha \), obtained for different \( h_i / R \), fulfill the inequality

\[
\varepsilon_{\alpha,2.0} < \varepsilon_\alpha < \varepsilon_{\alpha,2.\infty}.
\]

So, existence of the second shell, which directly contacts with the matrix, for insignificant difference of the moduluses of elasticity of this shell's material and of the matrix material (e.g. for \( E^i / E = 2 \)) reduces to significant increase of the external compressible fail load.

Besides that, with growth of \( E^i / E \) the values of \( \varepsilon_\alpha \), growth, too. The results certify on that, for fixed \( h_i / R, h_i / R, E^i / E \) the growth of \( E^i / E \) and \( E^f / E \) reduces to decrease of \( \varepsilon_\alpha \). For comparative small values of \( E^i / E \), \( h_i / R, E^i / E \) with increase of \( h_i / R \) the values of \( \varepsilon_\alpha \) monotonely increase. However, the abovesaid is failed for comparatively big values of \( E^i / E \) and \( h_i / R \). The pointed fault is observed also at analysis of dependences \( \varepsilon = \varepsilon(x) \), which are obtained for comparatively big values of \( h_i / R \). Graphics of these dependences for \( h_i / R = 0.8; E^i / E = 10; E^i / E = 20 \) have been given in fig. 2 in the case, when \( E^f / E = 300 \). Graphics, denoted by figures 1-1, were constructed for \( h_i / R = 1.0; 1.5; 2.5; 3.5; 4.5; 5.5; 6.5; 7.5; 8.5; 10; \infty \).

From these graphics it follows, that for comparatively big values of \( h_i / R \) the dependence \( \varepsilon = \varepsilon(x) \) has a complicated character. The pointed complication in the
character of dependences between $\varepsilon$ and $\chi$ is observed also for comparatively big values of $E'/E''$. On that, with growth of $E'/E''$ and $E'/E''$ the explained complication gradually disappears.

![Graph showing dependences between $\varepsilon$ and $\chi$.]

**Fig. 2.**

**References**


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Received January 28, 1998; Revised April 10, 1998.
Translated by authors.