Degree project

Performance of MLSE over Fading Channels

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To my parents, family, siblings, friends and teachers
Research is what I’m doing when I don’t know what I’m doing\textsuperscript{1}

\textit{Wernher Von Braun}

Abstract

This work examines the performance of a wireless transceiver system. The environment is indoor channel simulated by Rayleigh and Rician fading channels. The modulation scheme implemented is GMSK in the transmitter. In the receiver the Viterbi MLSE is implemented to cancel noise and interference due to the filtering and the channel. The BER against the SNR is analyzed in this thesis. The waterfall curves are compared for two data rates of 1 M bps and 2 Mbps over both the Rayleigh and Rician fading channels.
Acknowledgments

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The radio spectrum or the electromagnetic spectrum includes frequencies up until 300 GHz. It is divided into different bands in every country for different purposes. The waves are classified under different bands of frequencies. The waves are referred to as radio waves, microwaves, infrared waves, light waves, ultra violet rays, x-rays, and gamma rays depending on which band they are in. Each band is further divided into smaller bands. The radio band is of our interest, the range of frequencies of which can be seen in the figure 1.1 [1].

The industrial, scientific, and medical (ISM) band as it is called is the band of frequencies which are employed in applications in the aforementioned fields. The federal communications committee (FCC) allowed 3 ISM bands for unlicensed users in the field of communications. The bands can be seen in the table 1.1 [2].

In the ISM band are the waves of frequencies falling in the radio band range that carry energy radiated by applications like microwave ovens and diathermy machines to name some. This band allows alongside applications in the three fields mentioned earlier unlicensed applications in the field of communications. Some communications applications are wireless local area networks LAN s, Blue tooth protocol, Zigbee, Personal area networks, Wifi, IEEE 802.15.4, IEEE 802.11, radio frequency identification applications (RFID) like biometric passports readers and near field communication applications. These are all
Table 1.1: Frequency bands allotted by FCC

<table>
<thead>
<tr>
<th>Name of band</th>
<th>frequency bandwidth for each band</th>
</tr>
</thead>
<tbody>
<tr>
<td>part 15.247</td>
<td>ISM band</td>
</tr>
<tr>
<td></td>
<td>902-928 MHz (26 MHz)</td>
</tr>
<tr>
<td></td>
<td>2400-2483.5 MHz (83.5 MHz)</td>
</tr>
<tr>
<td></td>
<td>5725-2850 MHz (125 MHz)</td>
</tr>
<tr>
<td>part 15 subpart E</td>
<td>U-NII band</td>
</tr>
<tr>
<td></td>
<td>5150-5250 MHz (100 MHz)</td>
</tr>
<tr>
<td></td>
<td>5250-5350 MHz (100 MHz)</td>
</tr>
<tr>
<td></td>
<td>5725-5825 MHz (100 MHz)</td>
</tr>
<tr>
<td>part 15 subpart D</td>
<td>U-PCS band</td>
</tr>
<tr>
<td></td>
<td>1910-1920 MHz (10 MHz)</td>
</tr>
<tr>
<td></td>
<td>2390-2400 MHz (10 MHz)</td>
</tr>
<tr>
<td></td>
<td>1920-1930 MHz (10 MHz)</td>
</tr>
<tr>
<td>part 15.255</td>
<td>59-64 GHz</td>
</tr>
</tbody>
</table>

unlicensed applications working at 915 MHz, 2.450 GHz, or 5.800 GHz. For being unlicensed they must be able to cope with electromagnetic interference from other non-communication devices but in return can not cause any interference to other ISM users. The devices operating according to any of the protocols mentioned are referred to as users. This constraint is set by the FCC. As one can predict this leads them to be very low power devices i.e., maximum 1 Watt (30 dB m). Along with power there is also the frequency domain constraint to satisfy in presence of nearby frequency band users.

1.1 Thesis Contribution

The work in this thesis is a study and performance of a wireless transceiver system operating on 2.4 GHz. Referring to table 1.1 we see 2.4 GHz is in the ISM band. A transceiver is a communication device that houses both the transmitter and a receiver and mostly the circuitry is common to some extent for both parts. This frequency is in the ISM band hence the transceiver is to operate at low power and share with other unlicensed users like wifi, blue tooth etcetera. The objective is to see the performance of this transceiver via the bit error rate (BER) against signal to noise ratio (SNR) curves. The idea is to simulate an indoor channel so as to simulate the situation of a consumer electronics device like a television interacting with a hearing aid device. The room environment is simulated with an indoor channel. The digital data bits transmit through this channel and through all the filters in the transmitter and receiver are bound to be different due to noise in digitization, modulation, channel, and filters. In the receiver we implement a mitigating technique to reduce the errors that the channel and the filters will have induced into the transmitted data sequence wise. The technique is called maximum likelihood sequence estimation (MLSE) which is implemented by the Viterbi decoding method.

1.2 Thesis Layout

The division of this thesis is given as follows. Chapter 1 is an introduction to this thesis and gives an overview of the whole thesis. Chapter 2 discusses the background needed to gradually realize the GMSK scheme which is central to the transmitter of this thesis. Chapter 3 discusses the major blocks that simulate the transmitter in matlab. Chapter 4 discusses the channel that the data encounters in the air. Chapter 5 discusses the receiver simulated in matlab. Chapter 6 shows the simulation results and chapter 7 gives concluding remarks and suggestions for future work.
2.1 Introduction to GMSK

The frequency of operation of the transceiver is the 2.4 GHz frequency range which is very crowded as the spectrum is shared with other communication protocols such as blue tooth. So this transceiver must operate in this crowded environment while not allowing noise, i.e., inter symbol and inter channel interference to undermine its performance. This fact and low power constraint will dictate the choice of the modulation scheme that is implemented. Normally the power radiated out side the band should be 60-80 dB fewer than the power radiated in the adjacent band [3]. To fulfill this the power spectrum has to be engineered but not at the RF stage as there the frequency can vary. The spectrum is manipulated either in the baseband or in intermediate frequency band and then up sampled and modulated and passed through power amplifiers.

In light of the above discussion a narrow-band digital modulation scheme with constant envelope called the Gaussian Minimum Shift Keying GMSK is used in the transceiver. A constant envelope is desired. The reason is that if raw data bits with some data encoding like non return to zero (NRZ) are directly frequency modulated they would be spread over an infinite band in the frequency domain. If some low pass filtering is applied the result would be that in time domain the sharp transitions of data bits will be smoothed and a compact spectrum would be under utilization [4]. In the next section a stepwise development of simpler modulation schemes is presented in arriving at the description of GMSK.

2.2 GMSK Development: A Preface to GMSK

2.2.1 Binary Phase Shift Keying: BPSK

In BPSK the binary information is in the two phase shifts of the carrier corresponding to a binary 0 or a 1, basically the carrier signal takes two amplitude levels. The symbols for both the amplitude levels can be generalized as

\[
S_i(t) = \sqrt{2E_b/T_b}\cos(2\pi f_c t + (i - 1)\pi), \quad \begin{cases} 0 \leq t \leq T_b \\ i = 1, 2 \\ f_c = 1/T_b, \end{cases}
\]  

(2.1)

where \(E_b\) is the bit energy, \(T_b\) is the bit duration and \(f_c\) is the carrier frequency. If \(f_c\) is an integer multiple of \(1/T_b\) phase transitions will occur at the same points of the carrier signal. In equation 2.1 the individual signals of the BPSK scheme are \(S_1(t) = -S_2(t)\). The normalized basis function for BPSK signals is given by

\[
\phi(t) = \sqrt{2/T_b}\cos(2\pi f_c t).
\]  

(2.2)
The vector coefficients for the BPSK signal are given by

\[ S_{11} = \int_0^{T_b} S_1(t)\phi(t) \, dt = \sqrt{E_b} \]

and

\[ S_{21} = \int_0^{T_b} S_2(t)\phi(t) \, dt = -\sqrt{E_b}. \]

\( S_1(t) \) is represented by the amplitude \( \sqrt{E_b} \) while \( S_2(t) \) is represented by the amplitude \( -\sqrt{E_b} \). The symbols that represent \( S_1(t) \) and \( S_2(t) \) can be visualized on a constellation diagram in the figure 2.1 [5, 6]. A 0 can be represented by \( -\sqrt{E_b} \) and a 1 can be represented by \( \sqrt{E_b} \).

![Figure 2.1: constellation diagram for BPSK](image)

The two symbols are farthest away from each other on the constellation diagram. This has the advantage that it would take a lot of noise to corrupt one symbol to be interpreted as the other at the demodulator. The disadvantage is that only one bit is encoded onto one symbol.

**BPSK Modulator**

As already stated A 0 can be represented by \( -\sqrt{E_b} \) and a 1 can be represented by \( \sqrt{E_b} \). This is NRZ encoding of the data bits as shown in figure 2.3. These bits when multiplied with the basis function for BPSK generate the BPSK signal as expressed mathematically in 2.1. A BPSK modulator can be visualized as in figure 2.2 [7]

![Figure 2.2: BPSK Modulator](image)

**Power Spectral Density (PSD) of BPSK signal**

The BPSK signal can be referred to as a bipolar non return to zero (NRZ) random signal that consist of rectangular pulses. Both 0 and 1 are represented by essentially the same pulse of opposite polarity, any of the two bits is equally likely in the stream of bits. The rectangular pulse is let’s say \( g(t) \) with duration \( T_b \) and magnitude \( \pm \sqrt{2E_b/T_b} \). It is shown in the figure 2.3.
The PSD of the rectangular pulse is given as in figure 2.3 and mathematically as:

\[ S_B(f) = (2E_b/T_b)(T_b^2 \text{sinc}^2(fT_b)/T_b) = 2E_b \text{sinc}^2(fT_b). \] (2.3)

The PSD of the BPSK signal is

\[ S(f) = (1/4)[S_B(f - f_c) + S_B(f + f_c)], \] (2.4)

or

\[ S(f) = (E_b/2)\text{sinc}^2[(f - f_c)T_b] + (E_b/2)\text{sinc}^2[(f + f_c)T_b]. \] (2.5)

The PSD of the BPSK signal is shown in the figure 2.4.

In BPSK modulation case minimum occupied bandwidth \( B \) equals the bit rate \( R_b \). The spectral efficiency for BPSK case is \( 1 \text{bit/s/Hz} \). The spectral efficiency measures how much data a modulation scheme can pack which is by definition the ratio of the transmitted bit rate and the occupied bandwidth and given in equation 2.8 as
\[ \rho = \frac{R_b}{B}. \]  
(2.6)

The bit rate and the bit period are reciprocally related, so

\[ R_b = \frac{1}{T_b}. \]  
(2.7)

Hence

\[ \rho = \frac{R_b}{(1/T_b)} = \frac{1}{Hz}. \]  
(2.8)

The bit error probability of BPSK modulated signal is given by

\[ P_{e,BPSK} = Q\left(\frac{\sqrt{2}E_b}{N_o}\right). \]  
[13]  
(2.9)

\subsection*{2.2.2 Quadrature Phase Shift Keying: QPSK}

QPSK is a higher order modulation scheme. It uses four symbols for encoding; two bits encoded into one symbol. So more data can now be encoded. The four QPSK symbols can be generally mathematically represented as

\[ S_i(t) = \begin{cases} \sqrt{2(E_s/T_s)} \cos(2\pi f_c t + (2i - 1)\pi/4), & 0 \leq t \leq T_s \\ 0, & \text{otherwise} \end{cases}, \]  
(2.10)

where \( i = 1, 2, 3, 4 \). \( E_s \) is energy of transmitted symbol, symbol duration is denoted by \( T_s \) and \( f_c \) is the carrier frequency.

Equation 2.10 can be written as

\[ S_{QPSK}(t) = (2E_s/T_s)\cos(2\pi f_c t)\cos[(2i - 1)\pi/4] - (2E_s/T_s)\sin(2\pi f_c t)\sin[(2i - 1)\pi/2], \]  
(2.11)

the basis functions for QPSK are \( \phi_1(t) = \sqrt{2/T_s}\cos(2\pi f_c t) \) and \( \phi_2(t) = \sqrt{2/T_s}\sin(2\pi f_c t) \). In terms of the basis functions equation 2.12 can be written as

\[ S_{QPSK}(t) = E_s\cos[(2i - 1)\pi/4]\phi_1(t) - E_s\cos[(2i - 1)\pi/4]\phi_2(t), i = 1, 2, 3, 4. \]  
(2.12)

The symbols that represent the four possible phases can be visualized on the constellation diagram.

\[ \text{Figure 2.5: constellation diagram for QPSK} \]

\textbf{QPSK Modulator}

The QPSK modulator was mathematically shown in equation equation 2.12 and a block diagram is shown in figure 2.6 [7, 8, 9].
2.2. GMSK DEVELOPMENT: A PREFACE TO GMSK

CHAPTER 2. DIGITAL MODULATION SCHEMES

The input bit stream is converted into an even and an odd bit stream by the serial to parallel converter. The bits are then converted according to the NRZ scheme. Just like the BPSK case we assign a $-\sqrt{E_b}$ to 0 and a $\sqrt{E_b}$ to 1. Just like the BPSK case the NRZ symbols are multiplied with the basis functions but afterwards are added to form the QPSK symbols. The QPSK waveform as an addition of two BPSK waveforms (from the even and odd bit stream) is shown in figure 2.7.

Each symbol comprising of two bits dictates that symbol energy $E_s = 2E_b$ and on the constellation diagram each adjacent symbol is $\sqrt{2E_s}$. In terms of bit energy the adjacent symbols are $2\sqrt{E_b}$ [7]. Depending on if one or both bits change after a particular symbol the phase can change by 90° degrees or 180° degrees respectively for the next symbol. We discuss one case to illustrate the abrupt phase transitions. If we concentrate on the first pair of bits in the original incoming bit stream they are separated and become the first bits of both the even and odd bit streams. The second pair of bits in the original bit stream are separated and become the second bits of the even and odd bit streams. Each bit in the original bit stream are separated and become the second bits of the even and odd bit streams. Each bit in the odd and even bit stream has a bit duration twice as that in the original bit stream. The first odd bit and the first even bit add to give the first QPSK symbol and this continues. Since there are two bits employed in forming a QPSK symbol the symbol rate is half that of the bit rate. In QPSK the carrier phase changes sign after every two bit periods due to the symbol spread over two bits. The first pair of (odd, even) bits is (0, 1). The second is (1, 0). So in transition from one QPSK symbol to the next both the constituent BPSK symbols have changed. This causes a phase transition of 180° degrees and is shown as an abrupt change in the QPSK waveform phase. If only one BPSK symbol had changed in the transition the phase transition of 90° degree would occur. This explanation of phase transitions of 90° degree and 180° degree is evident in the constellation diagram 2.11. The line through the origin indicates a 180° degree phase shift while the remaining lines indicate a 90° degree phase shift between successive
symbols. These sudden shifts of phase will have the effect that the spectrum will expand. To limit the spectrum we can filter the modulated data. That will have the undesired effect that we may not have the constant envelope. At the symbol transition where both bits change the envelope may go to zero for an instant.

Power Spectral Density PSD and Probability of bit error \( P_e \) of QPSK signal

The PSD of QPSK is given as

\[
P_{QPSK} = \frac{E_s}{2}[(\sin\pi(f-f_c)T_s/\pi(f-f_c)T_s)^2 + (\sin\pi(-f-f_c)T_s/\pi(-f-f_c)T_s)^2],
\]

where \( T_s \) is the symbol period. In terms of the bit period \( T_b \) equation 2.13 can be written as

\[
P_{QPSK} = E_b[(\sin 2\pi(f-f_c)T_b/2\pi(f-f_c)T_b)^2 + (\sin 2\pi(-f-f_c)T_b/2\pi(-f-f_c)T_b)^2].
\]  

The PSD of QPSK is shown in the figure 2.8 [7]

![PSD of QPSK](image)

Figure 2.8: PSD of QPSK

and the bit error probability of QPSK in an additive white Gaussian noise (AWGN) channel is

\[
P_{e,QPSK} = Q(\sqrt{2}E_b/N_0).
\]  

The bit error probability for QPSK is the same as BPSK. But twice as much data is sent using QPSK as compared to BPSK according to the relation 2.6 and 2.8 but here the bandwidth of the signal is half the data rate (one bit is spread over two periods). So equation 2.8 becomes

\[
\rho = \frac{R_b}{1/2T_b} = 2\text{bit/s/Hz}.
\]

2.2.3 Offset Quadrature Phase Shift Keying: OQPSK

OQPSK is similar to QPSK. In QPSK there were two rectangular bit streams amplitude modulated on a cosine and a sine wave that both changed after two bit periods. In OQPSK one bit stream is simply delayed by one bit interval initially 2.9.
2.2 GMSK DEVELOPMENT: A PREFACE TO GMSK

CHAPTER 2. DIGITAL MODULATION SCHEMES

Figure 2.9: constellation diagram for QPSK

The bit period is still two. Due to the initial one bit shift of the quadrature bit stream the bits will never change value or the phase will never change phase at the same time. Due to this innovation we have eliminated the phase changes between successive symbols to only 90° degrees and they can occur at every bit i.e., one bit in one bit stream has completely passed while a bit in the other stream is in the process. OQPSK was designed so that after filtering the spectrum to make it compact the envelope may stoop towards 0 at the 90° degrees phase change but will not completely go to zero as in the QPSK case. The constellation diagram for OQPSK is shown. We can compare the difference with QPSK.

Figure 2.10: constellation diagram for QPSK

The PSD of OQPSK is identical to that of QPSK [7]. The PSD of OQPSK and MSK is shown in figure 2.12 [11]. MSK is discussed in the next coming section. By looking at 2.12 one notices the width of the main lobe of the MSK spectrum is 1.5 times wider than that of OQPSK and its sidebands decay to below -60 dB. In our case the wide main band of MSK is not attractive. To cope with this we will discuss GMSK in later sections.

Figure 2.11: constellation diagram for OQPSK
2.2.4 Minimum Shift Keying: MSK

QPSK has abrupt changes of 90° degree and 180° degree in its waveform as seen in 2.7. OQPSK has 90° degree abrupt jumps in its waveforms. These abrupt changes in the time domain have disastrous effects in the frequency domain. The baseband spectrum becomes wide and high frequency components arise [8, 9, 10, 12]. These high frequency components have to be filtered by low pass filters. These filters along with suppressing side lobes cause inter symbol interference. QPSK and OQPSK modulated signals have constant amplitudes but QPSK exhibits phase changes at the symbol rate while OQPSK exhibits phase changes the bit rate. The phase changes occur because the data or the information is in these phase changes. On filtering these signals they undergo amplitude variations at the phase change instants violating the constant amplitude characteristic of these modulated signals [7].

To cope with these problems is a modulation scheme called minimum shift keying (MSK) which has properties distinguishing it from QPSK. The MSK waveform is smoother than either QPSK and QPSK, there are no abrupt jumps in the phase of MSK such as those in QPSK seen in 2.7. The main lobe of the PSD of MSK is 1.5 times wider than that of QPSK as proved in 2.12.

MSK is a Continuous Phase Modulation (CPM) scheme, a special form of binary continuous phase frequency shift keying (CPFSK)[13, 14]. Consider a continuous phase frequency shift keying (CPFSK) signal, which is defined for the interval \(0 \leq t \leq T_b\)

\[
S(t) = \begin{cases} 
\sqrt{2E_b/T_b}\cos[2\pi f_1 t + \theta(0)], & \text{for symbol 1} \\
\sqrt{2E_b/T_b}\cos[2\pi f_2 t + \theta(0)], & \text{for symbol 0},
\end{cases} \tag{2.17}
\]

where \(E_b\) is the transmitted signal energy per bit, and \(E_b\) is the bit duration. The phase \(\theta(0)\), denoting the value of the phase at time \(t=0\), sum up the past history of the modulation process up to time \(t=0\). The frequencies \(f_1\) and \(f_2\) are sent in response to binary symbols 1 and 0 appearing at the modulator input, respectively. Another useful way of representing the CPFSK signal \(S(t)\) is to express it in the conventional form of an angle modulated signal as [13]

\[
S(t) = \sqrt{2E_b/T_b}\cos[2\pi f_c t + \theta(t)], \tag{2.18}
\]

where \(\theta(t)\) is the phase of \(S(t)\). Where the phase \(\theta(t)\) is a continuous function of time, we find that the modulated signal \(S(t)\) itself is also continuous at all times, including the inter bit switching times. The phase \(\theta(t)\) of a CPFSK signal increases or decreases linearly with time during each bit duration of \(T_b\) seconds, as shown by

\[
\theta(t) = \theta(0) \pm (\pi h/T_b), \quad 0 \leq t \leq T_b, \tag{2.19}
\]
where the plus sign corresponds to sending symbol 1, and the minus sign corresponds to sending symbol 0; the parameter $h$ is the deviation ratio. Substituting value of $\theta(t)$ from equation 2.28 in $S(t)$ in equation 2.18 and comparing it with CPFSK signal in 2.17, we get equations 2.20 and 2.21

$$f_c + h/2T_b = f_1$$

$$f_c - h/2T_b = f_2.$$  

(2.20)

(2.21)

The information is frequency modulated on the carrier in the form of these discrete frequencies. We can solve for $f_c$ and $h$ and get the relations

$$f_c = 1/2(f_1 + f_2)$$

and

$$h = T_b(f_1 - f_2).$$

(2.22)

(2.23)

To clarify the phase continuity of the MSK signal a term **phase trellis** is introduced.

**Phase Trellis**

CPFSK is a special case of CPM. As the name suggests this is a digital modulation scheme with the condition that the phase of the signal is continuous. This continuity dictates memory in the frequency modulator. The digital bits transmission via frequency shift keying (FSK) is done so by discrete shifts in the carrier of the signal. If we look at equation 2.28 we see that at time $t = T_b$

$$\theta(T_b) - \theta(0) = \begin{cases} \pi h, & \text{for symbol 1} \\ -\pi h, & \text{for symbol 0}. \end{cases}$$

(2.24)

So the phase of the CPFSK signal 2.17 increases or decreases by $\pm \pi h$ straight lines; the slope of these straight lines are the frequency changes. Figure 2.13 which is a phase tree clarifies the phase trajectory across boundary intervals of the incoming data bits.

![Figure 2.13: the phase trajectory taken by binary CPFSK](image)
The phase of this CPFSK signal is an odd multiple of \( \pm \pi h \) radians at odd multiples of bit duration \( T_b \) and likewise is an even multiple of \( \pm \pi h \) radians at even multiples of bit duration. If the deviation ratio \( h = 1/2 \) the phase can take on values of only \( \pm \pi/2 \) at odd multiples of \( T_b \) and 0 and \( \pi \) at even multiples of \( T_b \). This info can be verified for the generalized \( h \) and an \( h=1/2 \) in the figures 2.13 and 2.14 respectively. Figure 2.14 is shown for a specified bit sequence of 1101000 with \( \theta(0)=0 \) [9].

![Figure 2.14: the phase trellis, boldfaced path shows the bit sequence 1101000](image)

By virtue of setting the modulation index \( h = 1/2 \) in 2.23 MSK is attained. The modulated carrier signal is [13]

\[
S(t) = \sqrt{2E_b/T_b}\cos[2\pi(f_c + I_n/4T)t - n\pi I_n/2 + \theta(n)], \quad nT \leq t \leq (n+1)T,
\]

where \( I_n \) is a sequence of \( M \)-ary information symbols from the alphabet \( \pm 1, \pm 3, \ldots, \pm (M-1) \). Basing on the value of \( I_n \) in the interval \( nT \leq t \leq (n+1)T \) which can take on values \( \pm 1 \) for the binary CPFSK the equation 2.25 can be expressed as a sinusoid having two possible frequencies defined as \( f_1 = f_c - 1/4T \) \( f_2 = f_c + 1/4T \). Equation 2.25 can be generalized as

\[
S_i(t) = \sqrt{2E_b/T_b}\cos[2\pi f_i t + \theta(n)] + 1/2\pi n(-1)^i - 1], \quad i = 1, 2,
\]

As iterated earlier information is frequency modulated on the carrier in the form of discrete frequency shifts. \( f_2 - f_1 = 1/2T \) is the minimum necessary frequency separation for the orthogonality of the signals over a signal interval \( T \). Hence CPFSK with \( h = 1/2 \) is called MSK. The phase in the \( nth \) signaling interval is a result of a phase continuing from previous adjacent symbols.

**Signal Space Diagram of MSK**

Equation 2.18 can express CPFSK signal in terms of [13]

\[
S(t) = \sqrt{2E_b/T_b}\cos[\theta(t)]\cos(2\pi f_c t) - \sqrt{2E_b/T_b}\sin[\theta(t)]\sin(2\pi f_c t).
\]

We focus on the inphase component \( \sqrt{2E_b/T_b}\cos[\theta(t)] \) with \( h = 1/2 \) we have from equation 2.28

\[
\theta(t) = \theta(0) \pm (\pi/2T_b), \quad 0 \leq t \leq T_b,
\]

where the plus sign corresponds to symbol 1 and the minus sign corresponds to symbol 0. A similar result holds for \( \theta(t) \) in the interval \( -T_b \leq t \leq 0 \), except that the algebraic sign is not necessarily the same in both intervals. Since the phase \( \theta(0) \) is 0 or \( \pi \), depending on the past history of the modulation process, we find that, in the interval \( -T_b \leq t \leq T_b \), the polarity of \( \cos[\theta(t)] \) depends only on \( \theta(0) \), regardless of the sequence of 1’s and 0’s transmitted before or after \( t=0 \). Thus, for this time interval, the inphase component \( S_I(t) \) consists of a half cycle cosine pulse defined as

\[
S_I(t) = \sqrt{2E_b/T_b}\cos[\theta(t)]
\]

\[
= \sqrt{2E_b/T_b}\cos[\theta(0)]\cos(\pi t/2T_b)
\]

\[
= \pm\sqrt{2E_b/T_b}\cos(\pi t/2T_b), \quad -T_b \leq t \leq T_b,
\]

where the plus sign corresponds to \( \theta(0) = 0 \) and the minus sign corresponds to \( \theta(0) = \pi \). Similarly in the interval \( 0 \leq t \leq 2T_b \) the quadrature component of equation 2.27 consists of half cycle sine pulse.
2.2. GMSK DEVELOPMENT: A PREFACE TO GMSK

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it’s polarity depending upon \( \theta(T_b) \) as shown in equation 2.30 as

\[
S_Q(t) = \sqrt{2E_b/T_b}\sin[\theta(t)] = \sqrt{2E_b/T_b}\sin[(\theta(T_b))]\sin(\pi t/2T_b) = \pm \sqrt{2E_b/T_b}\sin(\pi t/2T_b), \quad 0 \leq t \leq T_b,
\]  

(2.30)

where the plus sign corresponds to \( \theta(T_b) = \pi/2 \) and the minus sign corresponds to \( \theta(T_b) = -\pi/2 \). From the previous discussion phase states \( \theta(0) \) and \( \theta(T_b) \) can each assume one of two possible values, any one of the four possibilities can arise, as described in [9].

1. The phase \( \theta(0) = 0 \) and \( \theta(T_b) = \pi/2 \) correspond symbol 1 transmitted.
2. The phase \( \theta(0) = \pi \) and \( \theta(T_b) = \pi/2 \) correspond symbol 0 transmitted.
3. The phase \( \theta(0) = \pi \) and \( \theta(T_b) = -\pi/2 \) correspond symbol 1 transmitted.
4. The phase \( \theta(0) = 0 \) and \( \theta(T_b) = -\pi/2 \) correspond symbol 0 transmitted.

This implies that the MSK signal will assume any of four possible forms, depending on the values of \( \theta(0) \) and \( \theta(T_b) \). From equation 2.27 we deduce the basis functions \( \phi_1(t) \) and \( \phi_2(t) \) for MSK as:

\[
\phi_1(t) = \sqrt{2/E_b}\cos(2\pi f_c t)\cos(\pi t/2T_b), \quad 0 \leq t \leq T_b, 
\]

(2.31)

\[
\phi_2(t) = \sqrt{2/E_b}\sin(2\pi f_c t)\sin(\pi t/2T_b), \quad 0 \leq t \leq T_b. 
\]

(2.32)

MSK signal can now be expressed in the form

\[
S(t) = s_1\phi_1(t) + s_2\phi_2(t), \quad -T_b \leq t \leq T_b, 
\]

(2.33)

where the coefficients \( s_1 \) and \( s_2 \) are related to the phase states \( \theta(0) \) and \( \theta(T_b) \), respectively. To evaluate \( s_1 \), we integrate the product \( S(t)\phi_1(t) \) between the limits \(-T_b \) and \( T_b \) as

\[
s_1 = \int_{-T_b}^{T_b} S(t)\phi_1(t) \, dx = \sqrt{E_b}\cos[\theta(0)], \quad -T_b \leq t \leq T_b. 
\]

(2.34)

To evaluate \( s_2 \), we integrate the product \( S(t)\phi_2(t) \) between the limits \(-T_b \) and \( T_b \) as

\[
s_2 = \int_{0}^{T_b} S(t)\phi_2(t) \, dx = -\sqrt{E_b}\cos[\theta(T_b)], \quad 0 \leq t \leq T_b. 
\]

(2.35)

In equations 2.34 and 2.35

1. Both the integrals are evaluated for two bit periods
2. Upper and lower limits of the integration for \( s_1 \) are shifted a bit period with respect to those for \( s_2 \).
3. \( 0 \leq t \leq T_b \) is the time interval in which both the phase states \( \theta(0) \) and \( \theta(T_b) \) are defined.

The signal constellation for an MSK signal is two dimensional, with four possible message points, as shown in the Figure 2.15. The coordinates of the message points are (\( \sqrt{E_b}, \sqrt{E_b} \)), (\( -\sqrt{E_b}, \sqrt{E_b} \)), (\( -\sqrt{E_b}, -\sqrt{E_b} \)), and (\( \sqrt{E_b}, -\sqrt{E_b} \)). The possible values of the phases of the inphase and quadrature components \( \theta(0) \) and \( \theta(T_b) \) are also included in the figure 2.15. The MSK and QPSK constellation diagrams are similar in that both of them have four message points but there is a subtle difference. In QPSK each message point represents a symbol while in MSK two message points can represent symbol 0 and two message points.
2.2. GMSK DEVELOPMENT: A PREFACE TO GMSK

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<table>
<thead>
<tr>
<th>Transmitted Binary Symbols</th>
<th>Phase States (radians)</th>
<th>Coordinates of Message Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq t \leq T_b$</td>
<td>$\theta(0)$</td>
<td>$\theta(T_b)$</td>
</tr>
<tr>
<td></td>
<td>$0$</td>
<td>$-\pi/2$</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>$-\pi/2$</td>
</tr>
<tr>
<td></td>
<td>$\pi$</td>
<td>$\pi/2$</td>
</tr>
<tr>
<td></td>
<td>$0$</td>
<td>$\pi/2$</td>
</tr>
</tbody>
</table>

$s_1 \quad s_2$

$\sqrt{E_b} \quad \sqrt{E_b}$

$-\sqrt{E_b} \quad \sqrt{E_b}$

$-\sqrt{E_b} \quad -\sqrt{E_b}$

$\sqrt{E_b} \quad -\sqrt{E_b}$

Table 2.1: MSK signal space characteristics

can represent 1 at any particular time. Table 2.1 summarizes the value of the phases $\theta(0)$ and $\theta(T_b)$ as well as corresponding values of $s_1$ and $s_2$ calculated for time intervals $-T_b \leq t \leq T_b$ and $0 \leq t \leq 2T_b$ respectively. The first column of the table indicates whether symbol 1 or symbol 0 was sent in the interval $0 \leq t \leq 2T_b$. The coordinates of the message points, $s_1$ and $s_2$, have opposite signs when symbol 1 is sent in this interval, but the same sign when the symbol 0 is sent. For a given sequence of bits we can use the entries in the table 2.1 to derive the sequence of coefficients on a bit by bit basis to scale $\phi_1(t)$ and $\phi_2(t)$ and hence determine the MSK signal $S(t)$.

Figure 2.15: MSK constellation diagram

$P_e$ and PSD:

The $P_e$ for MSK is the same as that for BPSK and QPSK, so

$$P_e = Q(\sqrt{2E_b}/N_0).$$  \hspace{1cm} (2.36)

The PSD of MSK is obtained from taking the square of the magnitude of the Fourier transform of baseband pulse shaping function is given as in

$$P(t) = \begin{cases} \cos(\pi t/2T_b), & t \leq |T_b| \\ 0, & Elsewhere. \end{cases}$$  \hspace{1cm} (2.37)
The PSD of MSK is given as
\[
P_{\text{MSK}} = \frac{16}{\pi^2}(\cos^2(\pi(f + f_c)T/1.16f^2T^2))^2 + \frac{16}{\pi^2}(\cos^2(\pi(f - f_c)T/1.16f^2T^2))^2
\]
in [15]. The PSD of QPSK and MSK is simulated in figure 2.12.

2.3 Gaussian Minimum Shift Keying:GMSK

MSK was superior to QPSK and OQPSK in the time domain because it was a true continuous phase modulation scheme. The continuous phase and constant envelope are the features desirable because the idea was to keep low power consumption and narrow bandwidth utilization. The time domain property due to which MSK was arrived at is clear. The reason we did not employ MSK as our choice of the modulation scheme was that its spectrum is not compact enough. Its main lobe is wider than that of OQPSK by 50% as seen in figure 2.12 even though its side lobes are smaller. This is where the need for GMSK arose. MSK itself can be implemented by directly giving as input to a frequency modulator with a modulation index of 0.5. To make the spectrum slimmer or to reduce the energy in the upper side lobes of MSK all that is needed is to low pass filter the data stream before presenting it as input to the frequency modulator. This filter is a premodulation low pass filter referred to as a baseband pulse-shaping filter. The impulse response must have the following properties

1. Narrow bandwidth frequency response and sharp cutoff characteristics
2. Low overshoot impulse response
3. As in MSK the phase of the carrier of the modulated signal takes values ±\(\pi/2\) at odd multiples of \(T_b\) and at even multiples of \(T_b\) it takes on values of 0 and \(\pi\).

The first property makes the wide MSK spectrum compact suppressing the high frequency components. Care is taken so as not to disturb the constant envelope of the waveform. The second property ensures that frequency deviations of the frequency modulated signal do not exceed their their limit. The third property ensures detection of the modulated frequency modulated signal. NRZ binary data stream when passed through this baseband pulse shaping filter whose impulse response is a Gaussian function. The frequency response of a Gaussian function is also Gaussian. The data is Gaussian filtered MSK modulated or GMSK modulated. The GMSK impulse response is given
\[
h(t) = \sqrt{\frac{2\pi}{\log 2}} W \exp((-2\pi^2 / \log 2)W^2 t^2)
\]
as in [9]. The response of this Gaussian filter to a rectangular pulse of unit amplitude and bit duration \(T_b\) is given by
\[
f(t) = \int_{-T_b/2}^{T_b/2} h(t - \tau) d\tau
= \sqrt{2\pi/\log 2} W \int_{-T_b/2}^{T_b/2} \exp((-2\pi^2 / \log 2)W^2 T_b(t - \tau)^2) d\tau,
\]
where \(W\) is the 3 dB bandwidth of the Gaussian filter. Equation 2.40 can be expressed as the difference of two complementary error functions as
\[
f(t) = \frac{1}{2}\left[\text{erfc}(\pi \sqrt{(2/\log 2)WT_b(t/T_b - 1/2)}) - \text{erfc}(\pi \sqrt{(2/\log 2)WT_b(t/T_b + 1/2)})\right].
\]
The impulse response of the Gaussian filter for BT values of 0.25 and 0.5 are shown in figure 2.16.
In equation 2.41 the time bandwidth product $WT_b$ is a design parameter. We note that as the $WT_b$ product decreases the pulse is spread in time and vice versa. The PSD of GMSK is compared with that of MSK and then with QPSK and BPSK in 2.17 and 2.18.

Figure 2.16: Gaussian impulse response of filter for BT values of 0.25 and 0.5

Figure 2.17: PSD of GMSK and MSK
2.3. GAUSSIAN MINIMUM SHIFT KEYING: GMSK

CHAPTER 2. DIGITAL MODULATION SCHEMES

Due to its narrow bandwidth it is superior to MSK and the other modulation schemes in our application. It also has drawbacks. When a stream of NRZ data bits is passed through the GMSK filter the data is spread over onto the adjacent bits, inter symbol interference (ISI) is generated. The lower the $WT_b$ product the farther out the tails of the Gaussian impulse response of the filter causes the modulating signal to spread out. The choice of $WT_b$ offers a tradeoff between compact spectrum and ISI\[9\].

Figure 2.18: PSD of GMSK and MSK

![PSD of GMSK and MSK](image)

Figure 2.19: response of filter to individual pulses

![Response of filter to individual pulses](image)
Transmitter Implemented in Matlab

We proceed to elaborate on the whole transmitter that fulfills our requirements of low power consumption via constant envelope modulation technique and narrow spectrum occupation. The major blocks implemented in Matlab are first listed and explained.

1. Incoming data:
   The data from the mac layer enters the transmitter at two data rates of 1 and 2 M bps depending on the mode.

3.1 Over sampler:

Up sampling is the first and easiest choice when reconstructing a signal from its samples or the opposite. We up sample the incoming data at rates of 1 and 2 M bps by an oversampling rate (OSR) of 8. Up sampling a signal by a frequency at least twice as much as the highest frequency in the signal according to Nyquist criterion is absolutely necessary for reconstructing a signal from its samples but in practical applications not sufficient. In practice it is up sampled by a higher factor. Oversampling is useful in coping with aliasing as will be shown. Mathematically up sampling operation can be represented as

\[ h(k) = \begin{cases} 
  g(k/L), & \text{if } (k/L) \text{ is an integer,} \\
  0, & \text{otherwise.} 
\end{cases} \quad (3.1) \]

In actuality the up sampler inserts \( L - 1 \) zeros in between the sample values entering into it, where \( L \) is the up sampling factor. Higher up sampling is used so as to leave room for the transition gap between the spectral replicas so that a low pass filter can recover the continuous signal from its samples. To illustrate, equation 3.1 assumes that a signal is sampled according to the Nyquist rate so the spectral replicas won’t have any gap in between them. To recover the continuous time signal \( g(t) \) from the samples \( \tilde{g}(t) \) we would need an ideal low pass filter.
This isn’t possible. An infinite time delay in the response of filter could approximate the spectrum $G(\omega)$ of the samples $g(t)$ but that is impractical. The transition band is non existent and to solve this exact problem is why we oversampled by a factor of 8 which is evident in the figure 3.1. This provides ample space between spectral replicates of $G(\omega)$ which has the spectrum of $G(\omega)$ repeated at integral multiples of the sampling frequency. To recover $G(\omega)$ is easy now by low pass filtering as there is space for the transition band in which the spectrum sample can decay considerably before the next replica starts. Choosing a high enough data rate relaxes the strict requirement on the cutoff frequency of the filter. Our signals, without any shape filtering, are finite time bits with bit time period $T_b = 1/R$ where $R$ is the rate of the data which is 1 and 2 M bps. Time limited signals have infinite band spectra. The spectrum $G(\omega)$ of samples $g(t)$ will overlap no matter how high an OSR we oversample with. $G(\omega)$ due to overlapping tails of the copies of spectrum of $G(\omega)$ won’t contain a pure $G(\omega)$ and no amount of filtering won’t be able to retract $G(\omega)$ or $g(t)$ but only a distorted version which will resemble $G(\omega)$ closely if the up sampling factor is chosen high enough. The reason for this aliasing is the spectral folding of tail of $G(\omega)$ beyond $|f| > fs/2$ [16]. In practice we use a cutoff filter which attenuates the spectrum beyond the $fs/2$ frequency. The zero order hold does this partially discussed next.

### 3.2 Zero Order Hold (ZOH)

The zero order hold block is essentially a reconstruction filter. The ultimate objective on the transmitter side is to shape digital pulses into continuous time and band limited train of symbols and then to frequency modulate them on a high frequency carrier to transmit them through a wireless channel. We give a brief description of the ZOH. The ZOH has a rectangular shaped impulse response $h(t) = rect(t/T_s)$ of amplitude 1 that we will refer to as a gate pulse as shown in figure 3.2.

The digital data entering the zero order hold is essentially samples which we denote by $f'(t)$. When these gate pulses pass through the reconstruction filter $h(t)$ a continuous signal should be produced but as we will see the result will be only an approximation of a continuous signal. Let’s say the $j$th impulse of $f'(t)$ has magnitude $f(jT_s)$ at $t = jT_s$ in time. It can be written as $f(jT_s)delta(t - jT_s).$ This $j$th impulse will produce a gate pulse $f(jT_s)rect(t/T_s)$ of strength $f(jT_s)$ centered at

![Figure 3.1: oversampling rates: Nyquist rate and by a factor of 8.](image1)

![Figure 3.2: Impulse response of the ZOH](image2)
$t = jT_s$ when it passes through the ZOH. Every sample will generate a gate pulse proportional to its magnitude and give an output. The result is mathematically expressed as

$$x(t) = \sum_{j=1} f(jT_s) \text{rect}(t/T_s)$$  \hspace{1cm} (3.2)$$

and is shown in the figure 3.3 [16]

![Figure 3.3: Impulse response of the ZOH](image)

We see it to be a staircase approximation of $x(t)$. It isn’t smooth. The Fourier transform of the impulse response of the ZOH is denoted by $F(\Omega)$. We use the Nyquist rate $T_s = 1/2B$,

$$f(t) = \text{rect}(t/T_s) = \text{rect}(2Bt),$$  \hspace{1cm} (3.3)$$

$$F(\Omega) = Ts\text{sinc}(\omega Ts/2) = (1/2B)\text{sinc}(\omega/4B)$$  \hspace{1cm} (3.4)$$

The absolute value of the Fourier transform of the impulse response of the ZOH $|F(\Omega)|$ is shown in figure 3.4.

![Figure 3.4: absolute value of the Fourier transform of the impulse response of the ZOH](image)

The effect that this ZOH has in time domain is a crude form of continuity even though it is only staircase and in the frequency domain the attenuation of the higher frequency components.

### 3.3 Gaussian filter

GMSK was proposed in this system study. It is the central point of discussion in the transmitter side of this transceiver. It is implemented by this Gaussian filter. This Gaussian impulse response filter provides all the desired properties of waveform and spectrum of GMSK namely a constant envelope waveform as well a narrower main lobe of the spectrum compared to MSK. We discussed at length in chapter 2 that GMSK is superior to MSK according to the above two properties. The constant waveform is realized after the filter output passes through the voltage controlled oscillator which is doing the frequency modulation with a modulation index of 0.5.
We notice that for BT values of 0.25 the filter is spreading each pulse over it’s adjacent pulses on both sides. This is inter symbol interference (ISI), an inherent problem with this filtering. So for a narrower bandwidth ISI in the data is the compromise. We will have to equalize this at the receiver. When we add these individual responses we get 3.5

![Figure 3.5: sum of the responses of individual pulses](image)

### 3.4 Analog front end

2. Digital to Analog converter (DAC) In the code the DAC output voltage is between 0 and 1 volt and therefore we center the DAC input around 0.5V and we clip on 1V.

3. An analog frequency modulator: A function was implemented to simulate the voltage controlled oscillator (VCO). VCO does direct frequency modulation generation. According to the VCO the oscillation frequency is a linear function of the control voltage. The message signal or the modulating signal is used as the control signal. The relationship between them can be expressed according to [15]

\[
\omega_i(t) = \omega_c + kf_m(t),
\]

(3.5)

where \(\omega_i(t)\) is the instantaneous frequency at any instant, \(\omega_c\) is the carrier frequency, \(k\) is a constant and \(m(t)\) is the message signal that is doing frequency modulation. There are numerous ways to construct a VCO with the characteristic functionality of equation 3.5. To name two methods it can be made with a Schmidt trigger circuit, an oscillator circuit (the capacitance and/or the inductance of the resonant circuit is varied to achieve equation 3.5 characteristics of the VCO). A reversed biased semiconductor can give the behavior characteristics of a capacitor the capacitance of which varies with the bias voltage. It is in approximately linear dependence of the bias voltage from the message signal acting as the modulating signal. The frequency of oscillation \(\omega_0\) of some oscillators is given by [15]

\[
\omega_0 = 1/\sqrt{LC},
\]

(3.6)

where \(C\) is the capacitance and \(L\) is the inductance. If as mentioned the variation in capacitance is control signal \(m(t)\) dependent then

\[
C = C_0 - km(t),
\]

(3.7)
where $C$ is instantaneous capacitance and $k$ is a constant.

$$
\omega_0 = \frac{1}{\sqrt{L/C_0[1 - km(t)/C_0]}} \\
= \frac{1}{(\sqrt{L/C_0[1 - km(t)/C_0]})^{1/2}} \\
\approx (1/\sqrt{LC_0})[1 + km(t)/2C_0] \\
= \omega_c[1 + km(t)/2C_0] \\
= \omega_c + k_f m(t).
$$

where $\omega_c = 1/\sqrt{LC_0}$ and $k_f = k \omega_c/2C_0$. Hence from the variation of capacitance as a function of the modulating signal equation 3.5 is arrived at again. The oscillator frequency varies depending on the modulating signal. The frequency deviation needed in the FM generation is produced by this method referred to as the direct method and no frequency multiplication is required. It is however not stable with the running frequency. This can be dealt with by comparing output frequency with the running frequency generated by a crystal oscillator. The difference error signal is fed back to the oscillator for error elimination.

The VCO is doing the analog frequency modulation with a frequency deviation. The modulation index is kept at $h = 0.5$. The high frequency carrier is 2.4 GHz. The frequency modulation is resilient to fluctuations in amplitude so it helps in realization of our constant amplitude of GMSK. The GMSK frequency modulated signal that is to pass through a power amplifier and an antenna is shown in figure 3.6.

![Figure 3.6: the final gmsk modulated signal to be transmitted wirelessly](image)

In the receiver we will have to extract these digital pulses from an analog carrier modulated signal. The transmitter is shown in the figure 3.7. The analog front end at both sides is not the main center of discussion. The digital part is focused upon.

![Figure 3.7: GMSK modulation implemented in the transmitter](image)
4.1 Introduction

In wireless communications the main hindrance is the wireless medium or the channel along with the noise due to filtering and processing of the channel. The radio waves have to travel any sort of terrain such as ground, buildings, trees, hills, atmosphere, within buildings, and atmosphere. During travel through different terrains the wave accumulates noise as well as is absorbed, reflected, diffracted, refracted and scattered. The job of the receiver is to undo the effects of all the phenomena that the channel induces and to reconstruct and detect the original data.

There are several models to model the kind of fading experienced by the signal. In our case the application is an indoor one in a room. We will consider models that model an indoor room environment. Despite the presence of different mathematical models to model any sort of channel there is always noise added into a signal.

4.2 AWGN Channel

The Additive White Gaussian Noise (AWGN) channel is a channel model that only adds wide band or white noise process with a constant spectral density and an amplitude of Gaussian distribution to the transmitted signal. It is a simplistic model and does not account for the effects of fading, interference, dispersion, or frequency selectivity. It is mathematically expressed as

\[ y(t) = s_m(t) + n(t), \]  

where \( s_m(t) \) is one of the M possible signal points and \( n(t) \) is the zero mean white Gaussian noise process with power spectral density of \( N_0/2 \) and \( y(t) \) is the received waveform. The channel model is shown in figure 4.1.

![Diagram](#)

**Figure 4.1:** model for received signal through AWGN channel
4.3 Fading Channels

Disturbance experienced by radio waves during propagation is fading. Fading is the fluctuation in the amplitude, phase, frequency, delay in arrival time of the many waves arriving at the receiver. The transmitter transmits one wave but due to the surroundings like ground, buildings, building edges, trees, bridges, lampposts, atmosphere, hills etcetera the waves are reflected, diffracted, scattered, absorbed. Due to these hindrances at the receiver we get replicas of the original transmitted wave with different amplitudes, and phases all superimposing. This superposition of different replicas is destructive addition and hence the term fading is coined. There are two types of fading: large scale and small scale. These fading models the propagation mechanisms of the radio waves in different scenarios. The propagation models focus on the mean signal strength at any distance from the transmitter. Large scale propagation models predict the signal strength for any transmitter receiver distance. Small scale propagation models give the signal strength or the signal fluctuations over short travel distances (few wavelengths) or short time durations (few seconds).

4.3.1 Large Scale Fading

Different possible propagation mechanisms are modeled by different large scale fading models depending upon the particular geographical scenario.

Free Space model

The simplest channel model between the transmitter and the receiver have a clear unobstructed line of sight (LOS) with no objects in the surroundings. This models the situation of satellite communications. There is no obvious obstruction to attenuate the signal energy. The waves spread spherically from the transmitter antenna. The free space model shows decay in power usually as a function of 2\textsuperscript{nd}, 3\textsuperscript{rd}, or 4\textsuperscript{th} power of the distance between the transmitter and receiver. The received power at the receiving antenna at a distance \(d\) is given as in [8]

\[
P_r(d) = P_t G_t G_r \lambda^2 / ((4\pi d)^2 L),
\]

(4.2)

where \(P_t\) is the transmitted power, \(P_r(d)\) is the received power as a function of the distance \(d\) between the transmitter and the receiver, \(G_t\) is the transmitter antenna gain, \(G_r\) is the receiver antenna gain, \(L\) is the system loss factor \((L \geq 1)\) is not related to propagation and \(\lambda\) is the wavelength in meters. The gain of an antenna is related to it’s effective aperture, \(A_e\), given by

\[
G = 4\pi A_e / \lambda^2.
\]

(4.3)

The effective aperture is related to the physical size of the antenna and \(\lambda\) is related to the carrier frequency by

\[
\lambda = c / f = 2\pi c / \omega_c,
\]

(4.4)

where \(f\) is carrier frequency in Hertz, \(\omega_c\) is the carrier frequency in radians per second and \(c\) is the speed of light in meters/seconds. \(P_t\) and \(P_r\) must be expressed in same units and \(G_t\) and \(G_r\) are dimensionless quantities.

Two Ray Model

In another case the presence of the ground can be taken into consideration to model a base station propagation radio waves for example. The ground will reflect waves before they reach the receiver. The reflected waves will have different phase shifts resulting in reducing the net received power. A two-ray approximation for path loss is as in [21]

\[
P_r = P_t G_t G_r h_t^2 h_r^2 / d^4,
\]

(4.5)

where \(h_t\) and \(h_r\) are antenna heights of the transmitter and the receiver. A major difference is observed from the expression for the free space model. Antenna heights are in this expression, the wavelength is absent, the distance has power 4. An empirical formula for the path loss is
\[ P_r = P_t P_0 (d_0/d)^\alpha \]

where \( P_0 \) is the power at a distance \( d_0 \) and \( \alpha \) is the path loss exponent. The path loss is given by:

\[ PL(d)dB = PL(d_0) + 10\alpha \log(d_0/d) \]

Where \( PL(d_0) \) is the mean path loss in dB at a distance \( d_0 \). The thick dotted line in figure 4.2 shows the received power as a function of the distance from the transmitter.

**Figure 4.2:** path loss, shadowing and multi path effects on received power

**Shadowing**

A much more realistic model is when objects such as buildings, hills and trees are considered to encumber the path of the propagating radio wave resulting in signal loss via reflection, scattering, diffraction, and absorption. This propagation model is called shadowing. Instead of radio waves if the transmitting antenna was a light source then due to the middle building blocking the LOS path there would be a shadow cast on the receiving antenna. This is illustrated in the figure 4.4

**Figure 4.3:** Shadowing

The net path loss due to shadowing is

\[ PL(d)dB = PL(d_0) + 10\alpha \log(d_0/d) + \chi \]

\( \chi \) is a Gaussian distributed random variable with variance \( \sigma \) and it represents shadowing effects. The effect of shadowing is shown in the figure 4.2 as the thin dashed line. Due to shadowing power received at the same points away from the transmitter is different and has log normal distribution. This is called log normal shadowing.

**4.3.2 Small Scale Fading/Multi path fading**

When reflection, diffraction, and scattering occur due to indoor structures the transmitted radio waves reach the receiver via more than one path. The signals through direct and indirect paths superimpose to give a distorted version of the original signal. This phenomenon is referred to as multi path fading.
be seen in figure 4.2. It has different effects on narrow band and wide band transmission. In narrow band transmission signals through the multi path channel add in amplitude and phase to give a fluctuating received signal. In wide band transmission for each transmitted pulse there are a series of delayed and attenuated pulses.

### 4.3.3 Impulse Response Characterization

Multi path fading degrades the performance of communication systems inside buildings and it can’t be avoided. The disturbing effects of the mobile radio channel can be fully characterized as a linear time varying impulse response filter. Due to motion of the receiver in space the impulse response is time varying. The impulse response is a wide band channel characterization and contains all the information necessary to analyze and understand any sort of radio transmission through the channel. It is useful in the sense that narrow band impulse response can be derived from the wide band model as well by making some adjustments in the variables.

#### Wide band Impulse Response model

A signal in a multi path channel arrives at the receiver through multi paths, the many replicas of the original signal being amplitude attenuated, delayed differently in time, and phase shifted. Hence the multi path mobile radio propagation channel model impulse response can be modeled as a linear time varying filter in 3-dimensional space as [8]

\[
h(t, \tau) = \sum_{i=0}^{N-1} a_i(t, \tau) \delta[\tau - \tau_i(t)] e^{j(2\pi f_c \tau_i(t)) + \phi_i(t, \tau)},
\]

where \( t \) represents the time variations due to motion, \( \tau \) is the channel multi path delay for a particular \( t \), \( N \) is the total number of multi path components, \( \delta[\tau - \tau_i(t)] \) is the delayed impulse that determines the specific multi path bins having components at time \( t \) and delays \( \tau_i \), \( a_i(t, \tau) \) and \( \tau_i(t) \) are the real amplitudes and excess delays respectively of the \( i \)th multi path component at time \( t \). The phase term \((2\pi f_c \tau_i(t)) + \phi_i(t, \tau)\) in equation 4.9 is the phase shift due to free space propagation of the \( i \)th multi path component, plus any additional phase shifts encountered in the channel. Generally the phase term is simply \( \theta_i(t, \tau) \) and it encapsulates all the mechanisms for the phase delays of a single multi path component within the \( i \)th time interval of the multi path delay axis. These path variables in this mathematical model completely describe the channel. The mathematical model is shown in the figure 4.4.

\[
x(t) \xrightarrow{h(t, \tau)} y(t) = \int_{-\infty}^{\infty} x(\tau) h(t, \tau) d\tau + n(t)
\]

**Figure 4.4:** Impulse Response model of the channel

The above impulse response is elaborated upon further and graphically shown in figure 4.5.
In multi path fading in indoor channel model the may copies of the signal that arrive at the receiver are amplitude attenuated, arrive from different directions with different phase shifts are encapsulated in the mathematical expression 4.9. A graphical view is presented by fist assuming a discretization of the multi paths channel impulse response. Snapshots of the impulse response at different times $t_i$, $i = 0, 1, 2$, at different points in space can be considered in figure 4.5. Focusing attention on a point in space and time the delay in arrival of different replicas is w.r.t the first component called multi path excess delay. This axis can be discretized into equal time intervals called excess delay bins. Each excess delay bin width is $\tau_{i+1} - \tau_i = \Delta \tau$. Any delay bin is $i\Delta \tau$ with $N\Delta \tau$ being the maximum excess delay where $i = 0, 1, \ldots, N - 1$. $N$ is the total number of arriving multi path components. If more than one signal arrives in the $i_{th}$ bin $i\Delta \tau$ they are represented by single resolvable multi path component. The width of the excess delay bins $\Delta \tau$ determines the resolution of the model and signals of bandwidth $1/2\Delta \tau$ are analyzable. In this discretization the impulse response at every time with excess delay bins can be expressed by 1’s and 0’s. 1 indicates presence of path in a bin and 0 indicates no path. An amplitude and phase is associated with every 1 meaning with a path [22].

**Narrow band Impulse Response model**

The time invariant impulse response is used to describe the multi path fading channel in a small time interval as in [22]

$$h(\tau) = \sum_{i=0}^{N-1} a_i \delta[\tau - \tau_i] e^{-j\theta_i}. \quad (4.10)$$

### 4.4 One number Parameters for Multi paths

The square of the amplitude of the impulse response is called the power delay profile $|h(t; \tau)|^2$ and is the source from where some quantifying parameters of any small scale fading channels can be derived. Two time dispersive parameters that quantify the multi path channels are mean excess delay: $\overline{\tau}$ and rms delay spread: $\sigma_\tau$. $\overline{\tau}$ is the first moment of the power delay profile as

$$\overline{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2}, \quad (4.11)$$

where $a_k$ are the amplitude coefficients of the multi paths and $\tau_k$ are the respective multi path delays.

**Delay Spread:** $\sigma_\tau$

$\sigma_\tau$ is the square root of the second moment of the power delay profile defined as in [8]
\[ \sigma_\tau = \sqrt{\tau^2 - \tau^2}. \]  

(4.12)

where

\[ \tau = \frac{\sum_k a_k^2 r_k^2}{\sum_k a_k^2}. \]  

(4.13)

The mean excess delay and the rms delay spread are measured w.r.t the first arriving signal at \( \tau_0 \) in the figure 4.5. The power delay profile from which these one number quantities are derived is an average of many impulse responses measured over a local region. \( \sigma_\tau \) is in microseconds in the outdoor environment and is in nanoseconds in indoor channels. The maximum excess delay \( (\chi dB) \) of the power delay profile is the excess delay in arrival of a multi path during which energy falls \( (\chi dB) \) below the maximum level.

If the Fourier transform of the power delay profile is taken frequency response of the channel is obtained. From the power delay profile the rms delay spread \( \sigma_\tau \) is derived to parametrize the channel. From the frequency response of the channel a parameter called coherence bandwidth \( B_c \) is derived and which quantizes the behavior of the channel in frequency domain. The coherence bandwidth and the delay spread are inversely related to each other.

**Coherence Bandwidth: \( B_c \)**

\( B_c \) is a statistical measure of range of frequencies over which the channel is flat; It passes all the frequencies in the range without any attenuation or any nonlinear phase. In other words \( B_c \) is the bandwidth in which the frequency components are amplitude correlated. If the frequency correlation between components is taken as 0.9 then the relation between \( B_c \) and \( \sigma_\tau \) is

\[ B_c \approx 1/50\sigma_\tau \]  

(4.14)

as in [8]. If the frequency correlation is 0.5 then

\[ B_c \approx 1/5\sigma_\tau. \]  

(4.15)

**Doppler Spread: \( B_D \)**

\( B_D \) measures the spectral widening of the signal due to time rate of change of the channel. Doppler shift \( f_d \) is the shift in frequency of the carrier wave propagating due to motion of the transmitter and or the receiver. The shift in frequency centers around the frequency of wave propagation by \( f_c \pm f_d \). The spectral widening depends on \( f_d \) which is a function of relative velocity \( V \) of mobile and angle \( \theta \) between direction of motion of mobile and direction of travel of wave. \( B_D \) is the range of frequencies over which the receiver Doppler spectrum is nonzero. If the signal bandwidth \( B_s > B_D \) the Doppler spread has negligible effects at the receiver. This is called slow fading.

**Coherence Time: \( T_c \)**

As in the case of delay spread and coherence bandwidth Doppler spread has a time dual called coherence time \( T_c \). It is related to the maximum Doppler shift as

\[ T_c \approx 1/f_m \]  

(4.16)

in [8], where \( f_m \) is the maximum Doppler shift. The \( T_c \) statistically measures the time duration during which the channel impulse response is invariant. \( T_c \) quantifies how similar the channel impulse response is at different times. During the duration of \( T_c \) the amplitude of the received signal can be correlated. If the baseband signal symbol period \( T_s \) is greater than the coherence time \( T_s > T_c \) of the channel, the channel will have changed before the symbol period or before the message is transmitted. This will lead to distortion in the signal at the receiver. If the \( T_c \) is defined as that time during which amplitudes of two received signals are correlated by more than 0.5 then the coherence time is approximately

\[ T_c \approx 9/16\pi f_m \]  

(4.17)
where \( f_m = V/\lambda \) and \( \lambda \) is the wavelength of the signal of propagation. \( B_D \) and \( T_c \) characterize frequency dispersiveness or time selectivity fading of the channel.

### 4.5 Small Scale Fading Types

Depending on the properties of the signal \((T_s, B)\) and those of the channel \((\sigma_\tau, B_D)\) the multi path signals will experience four different types of fading.

#### 4.5.1 Multi path fading due to Delay Spread

**Flat Fading**

The conditions for flat fading are

\[
B_s < B_c \quad (4.18)
\]

\[
T_s > \sigma_\tau \quad (4.19)
\]

as in [8]. If the range of frequencies that the channel passes are unattenuated and not phase distorted is much greater than the bandwidth of the signal \( B_s < B_c \) the signal will experience this very common flat fading. The amplitudes of all the spectral components will have the same gain and the spectrum is preserved. The gain of the received signal will change with time when the multi path channel fluctuates in gain.

The symbol period of the transmitted signal is much greater than the delay spread of the multi path channel in a flat fading channel. This implies that there won’t be any excess delay components in the impulse response \( h(t; \tau) \), only a delta at \( \tau = 0 \).

Flat fading channels are known by names such as amplitude varying channels or narrow band channels since \( B_s < B_c \). Flat fading channels can cause severe fading and transmitter may need 20 dB to 30 dB more power for low BER during these deep fades as compared to communication systems having to experience non flat fading channels. The instantaneous gain of the flat fading channels is required in the designing radio links. The instantaneous gains assume Rayleigh distribution for amplitudes in flat fading channels.

**Frequency selective fading**

The conditions for frequency selective fading are

\[
B_s > B_c \quad (4.20)
\]

\[
T_s < \sigma_\tau \quad (4.21)
\]

as in [8]. Now if the range of frequencies that the channel passes are unattenuated and not phase distorted or the coherence bandwidth of the channel is less than the signal bandwidth then different frequency components of the signal will undergo different fading. This is referred to as frequency selective fading. Looking at this case in the time domain the symbol period of the transmitted signal is less than the delay spread of the channel. This distorts the transmitted signal and multiple attenuated and delayed versions are received at the receiver. \( \sigma_\tau \) greater than \( T_s \) time disperses the symbol in the channel introducing ISI. With \( B_c < B_s \) different components experience different gains. Frequency selective fading and time dispersive channels are known also as wide band channels since the bandwidth of the signal is greater than \( B_c \) or channel impulse response. With time the received signals are distorted since the channel amplitude and phase response vary with time. If \( T_s < 10 \sigma_\tau \) the channel is frequency selective.

#### 4.5.2 Multi path fading due to Doppler Spread: \( B_D \)

**Fast Fading**

The conditions for fast fading are summarized in the relations

\[
B_s < B_D \quad (4.22)
\]
\[ T_s > T_c \]  \hspace{1cm} (4.23)

as in [8]. If the channel impulse response varies faster than the symbol period the channel is a fast fading one. A smaller coherence time as compared to symbol time period dictates that the Doppler spread be greater than the transmitted signal bandwidth. This Doppler spread causes distortion in the signal. The greater the Doppler spread the greater the fast fading. So a signal undergoes fast fading if equation 4.22 and 4.23 hold.

Fast and slow fading is different from flat and frequency selective fading. Fast fading is due to the time rate of change of the channel due to motion and a flat fading channel Impulse response can be approximated by a single \( \delta \) function with no multi path. Very low data rates experience fast fading. A flat fast fading channel is one in which the \( \delta \) function amplitude changes before the transmitted signal. A frequency selective fast fading channel is one in which multi path component amplitudes, phases, and delays change faster than the transmitted signal.

**Slow Fading**

The conditions for slow fading are summarized in the relations

\[ B_s > B_D \]  \hspace{1cm} (4.24)

\[ T_s < T_c \]  \hspace{1cm} (4.25)

as in [8]. If the time duration during which the channel impulse response varies is large as compared to the symbol period the channel is termed as slow fading. Frequency domain implication of this definition of slow fading channel is that the signal bandwidth is much larger than Doppler spread so the signal experiences slow fading according to equation 4.24 and 4.25. The Doppler spread depends on the motion of the mobile and hence the velocity indicates whether the fading occurs in the channel is fast or slow. All types of fading are given in chart 4.6.

**Figure 4.6:** Fading channels categorized

### 4.6 Rayleigh Fading Distribution

Flat fading channels were also named amplitude varying channels. Multi path amplitudes in a small scale flat fading channel follow distributions depending on factors like measurement area, presence or absence of a line of sight (LOS) component between transmitter and receiver. In absence of strong LOS the amplitudes of multi path fluctuate according to the Rayleigh distribution. The probability distribution function (Pdf) of the Rayleigh distributed amplitude fluctuation is given by [22]

\[ P(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}, \ r \geq 0 \]  \hspace{1cm} (4.26)

where \( \sigma \) is the Rayleigh parameter and \( r \) is the amplitude of a random multi path. Mean of this distribution is \( \sqrt{\pi/2}\sigma \) and variance is given by \( (2 - \pi/2)\sigma^2 \). In multi path fading amplitudes of multi
paths according to Rayleigh distribution is theoretically explained by Clarke’s channel model. According to this the transmitted signal reaches the receiver as complex replicas via $N$ different directions where each path is $r_ie^{j\theta_i}$. All these signals are added at the receiver vectorially giving

$$re^{j\theta} = \sum_i r_ie^{j\theta_i}.$$  \hspace{1cm} (4.27)

In small areas $r_i = r'$ for $i = 1, 2, \ldots N$ and in the absence of LOS path

$$re^{j\theta} = r' \sum_i r_ie^{j\theta_i}. \hspace{1cm} (4.28)$$

The path phase $\theta_i$ varies in the range of $[0, 2\pi)$ when the path length changes by a wavelength which is a fraction of a meter and decreases further with increasing frequency. The phase is uniform distributed. The resultant received signal amplitude and phase is a random variable. The resultant signal quadrature components are independent and Gaussian distributed random variables according to the central limit theorem. Lord Rayleigh found from the joint distribution of amplitude $r = \sqrt{T^2 + Q^2}$ and phase $\theta = \arctan(Q/I)$ that $r$ and $\theta$ are independent with the phase being uniform distributed and the amplitude $r$ being Rayleigh distributed. The assumption that Clarke’s model assumed of all multi paths being equal is erroneous and implies some attenuation for each path. The Rayleigh distribution is still valid for amplitude fluctuation if any one path amplitude is not a major contributor to the received power (i.e., if $r_i^2 < \sum_i r_i^2$, $i = 1, 2, \ldots N$). Rayleigh characteristics were observed in measurements in indoor communications (both antennas indoor). If there was strong component meaning no human body obstructing the path the distribution became Rician.

### 4.7 Rician Fading Distribution

Another distribution that the fluctuating amplitude follows is the Rician distribution. A strong path usually exists in the channel between the transmitter and the receiver in a room for example. In such a case the total received signal is a sum of two components, the strong LOS path and the rest of the multi paths. The Rayleigh distributed paths represented by a vector of an amplitude and phase being a random variable. The vector representing the LOS path which isn’t random. Let $ue^{j\alpha}$ be random where $u$ is Rayleigh distributed and $\alpha$ is uniform distributed. Let $ve^{j\beta}$ be the fixed component with $v$ and $\beta$ both deterministic. The received signal vector $re^{j\theta}$ is the sum of $ue^{j\alpha}$ and $ve^{j\beta}$. The Rician joint pdf of $r$ and $\theta$ is given in [22] as

$$P(r, \theta) = \frac{(r/2\pi\sigma^2)e^{-r^2+v^2-2rv\cos(\theta-\beta)/2\sigma^2}}{I_0(rv/\sigma^2)}, r \geq 0, -\pi \leq (\theta - \beta) \leq \pi.$$  \hspace{1cm} (4.29)

The amplitude of the deterministic component was not rightly assumed to be deterministic. The length and phase of the fixed path can vary with time. $\beta$ is a random variable uniformly distributed. $r$ and $\theta$ become independent with $\theta$ being uniform distributed and $r$ Rician distributed with the randomizing assumption of the strong path component. $r$ being Rician distributed is given by pdf

$$p(r) = (r/\sigma^2)e^{-r^2+v^2/2\sigma^2}I_0(rv/\sigma^2), r \geq 0.$$  \hspace{1cm} (4.30)

where $I_0$ is the zeroth order modified Bessel function of first kind, $v$ is the magnitude of the LOS component and $\sigma^2$ is proportional to the power of the Rayleigh scattered component. If $v = 0$ the amplitude distribution becomes Rayleigh. If $v$ is large $r$ and $\theta$ distributions become Gaussian.
The figure 5.1 is a block diagram of the detection and demodulation part of the receiver implemented in matlab. The Frame synchronization part is discussed afterwards. The main blocks of this receiver are given and then elaborated upon individually. This receiver is a superheterodyne receiver with IQ down mixing.

5.1 Receiver RF Front End

The analog front-end of the receiver consists of the blocks listed below.

1. A matching network to match the antenna to the passive mixer input impedance.

2. A passive mixer. This is a low noise amplifier (LNA)-less receiver architecture so the antenna is directly connected to the input of the mixer. An LNA-less receiver can handle higher interference levels and the absence of an LNA plus passive mixer reduces the power consumption. The penalty is a higher noise figure.

3. A quadrature local oscillator (LO) generation to mix down the incoming RF signal to a low intermediate frequency (IF) of 1 MHz.

4. Baseband anti-aliasing filter stage and programmable gain. The baseband anti-aliasing filters filter the incoming quadrature data with coefficients obtained from 3rd order Chebychev low pass filters. These analogue filters will also suppress interferers and noise. The wanted signal is centered around the IF frequency which for the 1 M bps mode is 500 kHz and for the 2 M bps it is DC. The bandwidth of the low pass filter is 2 MHz. The reason was to allow an IF frequency of up to 1 MHz. However the power consumption of the filter with this high bandwidth is too high. Therefore the IF frequencies were reduced to 500 kHz and DC. The filter passband bandwidth could be reduced but the filter has to cope with incoming signals that have a frequency offset up to 300 kHz. For the stop band attenuation characteristics the most critical regions are the frequency bands that are folding back in band after sampling at 16 MHz. These bands are: 14 MHz - 18 MHz, 30 MHz - 34 MHz... At higher frequencies, the interference suppression will be even more if limited by the anti-alias filter performance.

5. An (ADC) analog to digital converter running at 16 Msps. The 16 MHz bandwidth eases the design of the anti-alias filter. The ADC has a 50 dB dynamic range and a 10 bit resolution. A quantization error in incurred by the ADC. The higher the resolution the less is the quantization error. The ADC samples the continuous input signal and if the sampling rate doesn’t satisfy the Nyquist criterion, then the digital signal won’t be a correct representation of the input signal and aliasing will occur. The sampling rate chosen is 16 Msps and it is a safe oversampling. The frequencies above half the sampling rate that is 8 MHz of the signal entering the ADC are filtered by the anti-aliasing low pass analogue filter before being input to the ADC.
5.2 The Digital Baseband Processing

Crystal Frequency

The system has to work with crystals with 60 parts per million accuracy. This means that there can be a frequency deviation of 150 kHz \((\frac{60}{10^6} \times 2.4 \times 10^9 = 144 kHz \approx 150 kHz)\) at carrier frequency (2.4 GHz) at 1 side, transmitter or receiver. In the worst case the transmitter carrier frequency is 150 kHz off in one direction and the receiver local oscillator frequency is 150 kHz off in the other direction. The baseband processing will see an overall frequency deviation of 300 kHz. The digital receiver baseband has to be able to cope with a frequency offset of \(\pm 300 kHz\). Before the data from the analog front end enters the digital processing stage it is clipped to a maximum and minimum value of \(\pm 512\). This clipping is applied to the real and imaginary part of the signal.

Decimation Filter

In 2 M bps mode the filtering and synchronization is done at 16 MHz which is to be explained in following sections. In 1 M bps mode the filtering and synchronization is done at 8 MHz since the oversampling factor was initially chosen to be 8. In 1 M bps mode the decimation filter is followed by a decimate by 2 which is decimating the incoming stream. Decimation is the exact opposite operation of oversampling. It under samples by the factor specified. The processing is being done in fixed point format. In fixed point format integers are used as the filter coefficients. The coefficients in the table 5.1 each divided by 1024 are the filter coefficients employed in the low pass filter. The decimation filter suppresses noise and interferers in the band between 6 and 8 MHz because these will fold back in band after down sampling with a factor 2. The analogue baseband anti alias filters are attenuating noise between 6 and 8 MHz, so the overall performance doesn’t suffer if the spectrum is folded back after down sampling. However filtering is still needed to prevent interferers between 6 and 8 MHz from folding back. The dynamic range from the ADC is 50 dB, the interferer can be at most 40 dB higher than the wanted signal. To avoid that the interferer impact the desired signal too much after folding back it is suppressed by 50 dB to 60 dB.

Local Oscillator

Before entering into the synchronization mode DC offset and IQ mismatch compensation is conducted. The data from the ADC or the decimation filter is mixed with a Local oscillator at 1 MHz. From this point onwards the receiver has two branches one of the synchronization and the other of the detector and the equalization.

5.2.1 Synchronization

The frame synchronization mechanism is serving different purposes. It detects the start of the frame
and finds out when the data payload starts. Further it measures the initial frequency offset and detect the frame start in the presence of such a frequency offset. Also the initial bit time of the synchronization is found. This is the optimal sampling time for recovering the symbols from the sample stream. For the frame synchronization and detection some changes have to be made to the receiver and transmitter system. These changes are applied during the frame detection and synchronization mode. Once the frame start is detected the receiver switches back to the demodulation mode. This is shown in the receiver diagram with the synchronization switch.

**Synchronization Filter**

The matched filter is a filter different in purpose than the conventional filters. The conventional filters let pass and block some spectral components of the signals. These filters provide a uniform gain and a linear phase against frequency characteristic in the specific band for which they are designed to pass and attenuate the frequencies outside the band. The matched filter is of different purpose. It optimizes the signal to noise ratio at the sampling moment. The conventional filters try to preserve the meaningful signal in a random signal input to them while the matched filters have impulses responses matched to the known signals.

The matched filter is to be discussed in later sections. It is to have an impulse response and hence the frequency response identical to that of the Gaussian pulse shaping filter in the transmitter. The filter discussed in the transmitter has a time bandwidth product of 0.25. This small value ensures the narrowest main lobe in the frequency band as compared to QPSK and MSK but on the contrary smears the pulse in the time domain to the adjacent symbols. The lower this value was the more the pulse was spread.

A matched filter is used as the synchronization filter as well as in the demodulation mode. The filter coefficients are different however. The coefficients are given in the table 5.2 and each one is multiplied with $1/8^8$. Also the matched filter is not wide band enough to cope with the frequency offset requirement of $(\pm 300 kHz)$. Therefore a more wide band filter is used during synchronization. The synchronization filter is a Gaussian filter with a time bandwidth product value of 0.55. Before the Gaussian filter a prefilter which is a copy of the decimation filter in the analog receiver part. The decimation filter was used to suppress noise and interferers in the 6 to 8 MHz band. It is used here for the same purpose.

**Table 5.2: Matched filter Coefficients**

<table>
<thead>
<tr>
<th>1</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>22</th>
<th>26</th>
<th>22</th>
<th>16</th>
<th>8</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
</table>

The correlation with the synchronization word is done in the complex domain and not in the differential domain. The reason for this is again the related to the potential frequency offset. An initial frequency offset will result in a DC offset in the differential phase domain. This makes synchronization peak detection very hard in the presence of a frequency offset. If the correlation is done in the complex domain, the frequency offset results in a phase shift but it is not seen in the magnitude of the correlator output. And as such it does not impact the peak detection that is happening in the magnitude domain. It can be summarized that in the differential phase domain the frequency offset is seen as an additive effect, whereas in the complex domain it is seen as a multiplicative effect what is more favorable since it is purely phase related and not magnitude. As already mentioned before the time bandwidth product equal to 0.25 of the Gaussian filter is introducing a lot of ISI. During payload reception an MLSE is used to take into account this ISI for better BER performance. However this can’t be done during the correlation for the synchronization. During the synchronization the purpose is different, that of matching the received waveform to that of the known transmitted waveform and to optimize the SNR at those sampling moments and so we leave a greater margin of the bandwidth and hence a time bandwidth of 0.5. The time bandwidth product at the transmission side is increased to 0.5. This increases the eye-opening. So that the eye is open enough for synchronization without ISI equalizer. The transmission side Gaussian filter with time bandwidth product of 0.25 and 0.5 is simulated.
Transmit Synchronization Word

The synchronization word has a time bandwidth product of 0.5. This is done by keeping all the same filtering but just sending every symbol from the synchronization word twice after each other. The synchronization word is a Barker-11 sequence. The synchronization word as it is sent is given in the table 5.3.

Table 5.3: Barker-11 Sequence as the Transmission Synchronization-Word

| -1 | -1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 |

The synchronization word as it is shown in the table 5.4 is just sent at the same data rate as the payload, so either 1 M bps or 2 M bps.

Differential Detection

Differential detection has to be done to be insensitive to the phase offset between the receiver and the transmitter. The present symbol is multiplied with the complex conjugate of the previous symbol. There is no information at all in the magnitude of the samples, therefore the samples are normalized. All the samples have the same magnitude and only the phase information remains. This normalization gives better results because the amplitude noise can’t impact the cross correlation anymore. The magnitude of the complex symbols can be approximated by some simple operations on the normalized real and imaginary parts.

Receiver Synchronization Word

The synchronization word that is used in the receiver for the cross correlation is the following:

Table 5.4: Receiver Synchronization-Word

| -j | 1 | j | 1 | -j | -j | 1 | j | 1 | -j | -j | 1 | j | j | j | j |

Normally a -1 is represented by -j and a +1 by a +j in the complex domain. However at the transition between a -j and a +j a 1 is put to take into account the ISI. the coefficients from the synchronization sequence are kept simple. Just -1 or 1 and either imaginary or real and these simplify the multipliers in the cross correlator. Based on the position of the synchronization peak the data payload start and the initial value of the optimal sampling moment (initial bit time synchronization) is detected and the control goes to the matched filter and the equalization branch of the receiver. Also the initial frequency offset is derived from the phase value of the sample at the synchronization peak.

Initial Frequency Offset

The initial frequency offset estimation is done during the frame synchronization. If there is no frequency offset the phase at the correlation peak is zero since the receiver and the transmitter synchronization words must be in phase. If there is a frequency offset there will be a fixed phase offset between the transmitter and the receiver synchronization sequence. This fixed phase offset is also the phase of the peak value of the cross correlation. The relation between the phase offset after differential modulation and the frequency offset is: \( \Delta f = \Delta \phi s / 2 \pi \). Since the maximum phase difference that can be measured is \( \pm \pi \), the maximum frequency deviation that is detectable in 1 M bps is \( \pm 500 kHz \) and in 2 M bps it is \( \pm 1 MHz \). The frequency offset compensation mechanism is working in the feed forward way with an initial estimation during the frame synchronization. The complete digital receiver diagram with the synchronization part is shown in the figure 5.2.
5.2.2 Demodulation

Before switching to the payload data reception part the frequency offset is compensated for as well as the IF down mixing is again updated.

Matched Filter: Theory

GMSK is a non linear modulation scheme and the matched filter is not applicable. A kind of a matched filter is derived based on the linear representation of GMSK. The linear representation of GMSK is also called the linear decomposition [17, 18]. The idea behind the matched filter based on the Laurent decomposition is summarized. Continuous phase modulation can be represented as a linear superposition of amplitude modulated pulses in the IQ domain. There are only \(2^L - 1\) different pulses (with \(L\) being the length of the pulse shaping filter in symbols). The first pulse \(h_0(t)\) contains the most significant part of the signal energy. \(h_0(t)\) can be used as a simple linear approximation of the GMSK signal and so it can also be used as the impulse response for the matched filter.

![Matched Filter Frequency Response](image)

**Figure 5.3:** Matched Filter Frequency Response

The figure 5.3 shows the frequency response of the matched filter. It is narrow. This is a problem in
case of a big frequency offset, because then the signal is distorted too much by the matched filter. The out of band suppression of this filter is high which is good for suppressing any remaining interferers.

**Matched Filter: In Practice**

The matched filter is already discussed in the synchronization part as it is the same filter as the synchronization filter but with different coefficients. The matched filter optimizes the signal to noise ratio at the sampling moment. However the matched filter as explained already is not ISI free and will create additional ISI on top of the transmission filter and the ISI from the differential demodulation in the receiver. The filter is calculated for the GMSK modulation with time bandwidth product BT = 0.5 and the optimal solution is obtained. Also the additional ISI added by this matched filter is taken into account in a relatively simple Viterbi MLSE with 8 states (1 symbol differential demodulation). In case of GMSK with BT = 0.25 the optimal matched filter is not possible because it causes too much ISI and a simple Viterbi MLSE with 16 states (2 symbol differential demodulation) is not enough to cope with this additional ISI. However the filter coefficients calculated for BT = 0.5 result in a more wide band filter with less ISI so that a 16 states Viterbi MLSE is sufficient.

The coefficients of the matched filter are given in table 5.5 and each one is multiplied with $1/2^8$

| 1 | 2 | 4 | 7 | 10 | 14 | 17 | 20 | 22 | 23 | 24 | 23 | 22 | 20 | 17 | 14 | 10 | 7 | 4 | 2 | 1 |
|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|---|---|---|---|

**5.2.3 Demodulation**

The demodulation process covers the sent data bits from the received and filtered signal stream. The sample stream coming out of the matched filter is down sampled with a factor of 8 to symbol rate. So there is one sample per data bit that is sent to the demodulator. Since there is almost no information in the amplitude of the received signal, the demodulation is done in the frequency domain. This is done by calculating the differential phase. This is a non coherent receiver. The difference between two successive phase values is taken and hence the receiver is independent of the carrier phase offset between the receiver and the transmitter. As already mentioned there is a lot of ISI on the received data. To have a demodulator performing well enough this ISI needs to be taken into account during demodulation. This is done by using maximum likelihood sequence estimation (MLSE) detection mechanism.

**5.2.4 Detection**

**Optimal Sampling Time**

The initial bit time synchronization or the optimal sampling time is found by implementing a bit time tracking loop. The bit time tracking loop continuously updates the sampling time with a feed back loop based on the output of the timing error detector. As timing error detector the Mueller and Muller algorithm is used. This algorithm has the advantage that it is only based on symbols and that no additional sampling points need to be calculated. The timing error detector is based on the formula given in 5.1

$$e_n = x_n \hat{x}_{n-1} - \hat{x}_n x_{n-1}$$

The feedback loop used consists of a proportional and an integral path. Bit time tracking is enabled since it needs to compensate for symbol clock differences between receiver and transmitter.

**5.2.5 Maximum Likelihood Sequence Estimation(MLSE) Viterbi**

In this most important step all the ISI that was introduced during all the filtering and also due to the channels is undone and the best possible approximation of the transmitted bits is done. This MLSE scheme will not just look at one received symbol but also at the adjacent symbols. This information is compared with all possible sequences that can be received and most likely one is chosen. Based on this
information the received symbol is detected to be a 1 or a 0. The most convenient way to implement this MLSE mechanism is by means of a Viterbi algorithm.

This mechanism counts on the a priori knowledge of the introduced ISI. If the ISI by the filtering is taken into account this information is available. The Viterbi algorithm is implemented according to [19]. The basic Viterbi algorithm takes into account the adjacent symbols for the ISI. Therefore 4 or 8 Viterbi states are required. The 4 Viterbi states are due to one symbol differential detection and the 8 Viterbi states are due to two symbol differential detection.

The viterbi algorithm say for 8 states requires 2 adjacent symbols, $2^3 = 8$. All these 8 states are updated with arrival of new bits into the Viterbi decoder. Every state can be arrived at from two states in the previous instant. For both the transitions the distance of each transition from a pre calculated reference metric is calculated. The transition that has the smaller distance from the pre calculated reference metric survives. This continues for a sequence of bits. So the transmitted bit sequence in itself is not retracted from the transmitted and received bit sequence but a near approximation of the bit sequence is guessed basing on comparison and selection. This is illustrated in the figure 5.5. The digital receiver part is shown in figure 5.4.

![Figure 5.4: The Receiver diagram:the Digital Demodulator Part](image)

![Figure 5.5: A trellis diagram with a chosen path by the Viterbi algorithm](image)
Simulation Results

To summarize the chain of the code the simulations are carried out such that the data is generated digitally, Gaussian pulse shape filtered, then analog modulated on a 2.4 GHz carrier via a voltage controlled oscillator. The analog frequency modulated wave is passed through fading channels that simulate indoor channel environments. Rayleigh and Rician fading channel are simulated that distort the original data. At the receiver the signal is demodulated down to baseband and then digitally processed. In this part the frames of data are extracted first by synchronization and frequency offset compensation. After synchronization the data is matched filtered with that of the Gaussian pulse shaping impulse response same as that at the generating side. The sampling time at the receiver is updated and finally the Viterbi MLSE is applied to extract the transmitted bits. The BER against the SNR waterfall curves are calculated.

For both the Rayleigh and the Rician fading four multipaths are chosen with different delays. They are in the order of nano seconds. This indicates that indoor multipaths are simulated. The four multipaths are chosen to simulate the waves reflecting from the floor, side walls, and the ceiling before arriving at the receiver from the transmitter.

6.1 Rayleigh Fading Channel

The four multipaths chosen in figure 6.1 have path delays as [1 ns, 3 ns, 5 ns, 8 ns] with gains [-10 dB m, -30 dB m, -20 dB m, -40 dB m]
6.2 Rician Fading Channel

The four multipaths chosen in figure 6.3 have path delays as [0 ns, 3 ns, 5 ns, 8 ns] with gains [0 dB m, -10 dB m, -20 dB m, -40 dB m]
6.2. RICIAN FADING CHANNEL

CHAPTER 6. SIMULATION RESULTS

Figure 6.3: BER vs. SNR over 4 multipaths through Rician fading channel

The four multipaths chosen in figure 6.4 have path delays as [0, 2 ns, 5 ns, 7 ns] with gains [0 dB m, -15 dB m, -35 dB m, -40 dB m]

Figure 6.4: BER vs. SNR over 4 multipaths through Rician fading channel

In both the Rayleigh and Rician fading channels the BER found is higher in lower SNR region than when only AWGN is present. However in the higher SNR region the BER drops well below that of the AWGN case. Also in the waterfall curve for the 2 Mbps case at any given SNR the BER is smaller. The ability of the Viterbi MLSE is seen when 1 and 2 Mbps cases are compared in every case. The 2 Mbps data rate means a wider Gaussian pulse shaping filter. The time bandwidth product was taken to
be 0.25. This means the spreading of the pulse over two adjacent pulses. For a larger bandwidth pulse there is more ISI. By this MLSE in the receiver path advantage is taken from the memory in the signal for doing better detection.
7.1 conclusion

A transceiver system is studied for wireless communication over indoor channels. The modulation scheme employed in the transmitter is GMSK due to the constant amplitude time domain waveform and the compact power spectral density as compared to QPSK and MSK. The waveforms corresponding to the pulses undergo frequency modulation. It is more noise resilient as compared to Lets say amplitude modulation. The signal modulates a 2.4 frequency signal. The data is transmitted through an indoor simulation of a Rayleigh and Rician fading channel for two data rates of 1 and 2 M bps. The data received at the receiver is downconverted to baseband before any digital processing. The data is down sampled by a factor a 8 as it was up sampled by the same factor in the transmitter. The signal undergoes frame synchronization to detect the start of the data payload. The initial frequency offset is measured to detect the start of a frame in the presence of a frequency offset. The optimal sampling time is found for recovering of the symbols from the sample train. For this the sampling time is updated continuously as well as a matched filter is implemented to remove noise and interferers. A differential phase detector is implemented and then an MLSE is implemented.

7.2 Future Work

The MLSE Viterbi detection mechanism was employed as an equalizer. It is a sequence estimation technique. Alternative methods for equalization can be ones which decide on a bit by bit basis whether a bit is correct or not. Other methods like the zero forcing equalizer, minimum square error (MSE) equalizer method and decision feedback equalizer (DFE) are some methods whose results can be compared with that of MLSE. MLSE is computation extensive and resource expensive. The techniques could be compared with respect to the BER versus SNR results but also with respect to computation time and resource measures. The modulation index can be varied in this MLSE case due to which different reference metrics for the Viterbi demodulator will result in the receiver side. The set of reference metrics were defined a priori. This can be changed and modulation index can be measured before the reception of a packet and then the Viterbi reference metrics can be calculated basing on the estimated transmission modulation index.


