Investigation of the use of Laser Scanning for Deformation Monitoring

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Abstract

The ability of fast and accurate acquiring of large 3D spatial data is the main benefit for consideration of a terrestrial laser scanner in deformation monitoring. The objective of this paper is to discuss this technique with support of practical experiments performed inside a laboratory. It also includes measuring changes from millimetre to sub millimetre level and a comparison of measurements from a terrestrial laser scanner with measurements of other instruments. Various areas of applications are reviewed. The report discusses a surface modeling method to estimate deformation parameters of objects, such as planar, spherical and cylindrical surface representations. Illustrative numerical examples are performed by simulating randomly generated sample point coordinates for estimation of changes of modeled planar and cylindrical surfaces. The practical experiments were performed using a scan of a carton box, a ball and a rounded paper holder, which correspond to the planar, spherical and cylindrical surfaces, respectively. Independent measurements were performed using a total station and a measuring tape to make a comparison with the scanner measurements. A statistical test was performed independently for the changes obtained from each type of modeled surface in order to check whether the movement is real or due to measurement noises. A significant change of the normal of a plane was detected between epochs, and similar results were obtained from both scanner and total station measurements. The normal of the plane was rotated by 19.94′ between scan epochs. A translation of 3.2 and 3.7 millimetres were detected between scan epochs for the center of the sphere and axis of the cylinder, respectively. Only the scanner data was used in this case. From the scanner measurement changes in radii of the sphere and the cylinder were obtained as 1.6 and 3.1 millimetres, respectively between scan epochs. The measurement of the scanner was verified by performing independent measurements using measuring tape. And hence the change in radii of the sphere and the cylinder were obtained as 2.5 and 4 millimetres, respectively.

Keywords Terrestrial laser scanner, Total station, Surface parameters, Surface modeling, Estimation of deformation
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1- Introduction

Deformation monitoring of structures or any other objects is the essence in many engineering field of studies. Analyzing and estimating the size of deformation is very helpful to predict its significance in relation to damage or failure (in case of structure).

In previous works different types of devices and methods were used to monitor and estimate deformation of objects as well as structures. Most of them, excluding the conventional surveying work, have direct contact to the object or the structure to monitor and measure the magnitude of deformation. Those applications might acquire either one or two-dimensional data with limited number of observations.

Laser scanning is one of recently emerging technologies, which is applied in different areas of measuring and monitoring the status of structures and objects. Nowadays it has become a widely applicable engineering technique in deformation monitoring and measurement analysis. Laser scanning has a special feature, which makes it different from the others, no need of direct contact with the object or the structure to be scanned. Hence this makes it most preferable to do scanning in hazard areas, fast and accurate in acquiring of 3D spatial data and ability to result in up to mm level of precision.

The aim of this research is to investigate the potential of laser scanning in monitoring of objects and structures and measuring their deformations. Deformation will estimate from different types of modeled surfaces. Geometrical shapes, which represent real objects in nature, such as planar, spherical and cylindrical surface representations are selected to use for the detail analysis. All those described geometrical surfaces can be obtained by fitting of the scanned laser data points corresponding to each scan epoch. In the planar surface case the normal of the plane of each scan epoch can be estimated from the fitting process and the norm of the differenced normal vectors will be the expected change between scan epochs. Whereas in the case of spherical surface the center coordinate and the radius of the sphere of each scan epoch can be estimated from the fitting process and the norm of the differenced center coordinates will give us the magnitude of the expected change in position of the sphere. The change in size of the sphere can be obtained from the change in radius between scan epochs. Similarly to the spherical case the plane coordinates of the axis of a cylinder and its radius of each scan epoch can be estimated from the fitting process and the norm of the differenced coordinates of the axis of the cylinder will give us the magnitude of the change in position of the cylinder between scan epochs. The change in size of the cylinder can be obtained from the change in radius between scan epochs. From this analysis it is expected to detect changes between 1 mm and 1 cm from the surfaces.

The research started with reviewing previous works related with laser scanning application on deformation monitoring. Hence in chapter 2 different experimental case studies from laboratory and non-laboratory conditions are presented to support the main objective of the paper. Chapter 3 discusses the methodology and approaches to estimate deformations from the measured data, and
it also presents the effectiveness of deriving best fitting surface models from which deformation of smaller magnitude can be recovered than the single point precision. Chapter 4 presents the procedures for the practical test and data analysis, of the experiment performed in particular for this research. Selected sample objects for three basic geometrical surface shapes were scanned to monitor and quantify deformation by using Terrestrial laser scanner (TLS). The deformation measured in this experiment is artificially created by applying a load on the object or by changing the volume/size of the object, which is performed inside the laboratory. Finally, precision analysis on the measured data is presented in chapter 5 by using the precision of the instrument, which is referred from the manufacture technical specification of HDS 2500 laser scanner.

2- Related Works

Over the past decade different authors performed various researches related with deformation monitoring using TLS. Depending on the objective of the research different methods were employed in each research and different objects or structures were used in the practical test of each research experiment. Tunnel, sea harbor door lock, ancient cultural heritage building, lock of hydro power dam and bridge beam are a few of structures which were monitored using laser scanner. In this section selected research works, which are related with deformation monitoring and measurement analysis are reviewed and presented.

In Tsakiri et al. (2006) a case was studied regarding deformation monitoring using TLS. The main concern of the study was to discuss the issue influencing the feasibility of laser scanning which is instrument calibration. In order to determine the instrument performance instrument calibration has to be done before starting the main process. Since millimetre to sub millimetre accuracies are required in such application testing and setting a calibration procedure is a very important task to be performed at the initial stage of the study. These tasks can be performed either indoor or outdoor.

For indoor facilities calibration of instrument can fall into two general categories. The first category is a hardware calibration whereas the second one is hardware plus software calibrations. In the first category, performance evaluation involves the assessment of the instrument in hardware calibration, distance accuracy and precision, pointing accuracy and precision, laser beam divergence, range and angular resolution, minimum distance between objects that is detectable by the sensor, minimum object size that is detectable by the sensor and stability/degradation of the sensor over time. In the second category, performance evaluation involves the assessment of the combination of hardware and software to produce a desired end product. This process involves software to register multiple scans and to mesh the point cloud, as well as methods to clean/filter and to sub-sample the data. In general, the calibrations obtained from category one above could be used for propagating the instrument errors to the end product and to determine how to improve the scanner, whereas the calibration of the second category would be used to measure and explicitly note the performance of the scanner.
A number of case studies were presented in Tsakiri et al. (2006) to highlight the potential application of the laser scanners to deformation monitoring given that their accuracy can be greatly improved by exploiting the 3D point clouds with simple modeling techniques. The purpose of these studies is to assess the sensitivity of laser scanners for measurement of deformations of loaded structures and to investigate their potential for metrological tasks. Through those studies it were discovered that deformation monitoring with conventional surveying is superior in accuracy compared to laser scanning, where individual sample points have low precision. Although it is proved that the use of modeled surfaces is a key for deformation monitoring of laser scanning rather than single points.

The case studied in Tournas & Tsakiri (2008) discusses about a new method to monitor deformation of structures using laser scanner. In this method the terrestrial point clouds from a deformed object were acquired without the use of targets. Instead of targets a number of stationary control points located in the surrounding area of the object were used, which were identified through an automatic registration process. The main objective of the methodology used in this study is to investigate the deformations between two or more discrete epochs and to describe its application in laboratory experiment.

A combination of Cyrax 2500 laser scanner and Nikon 4700 CCD camera were used for this specific experiment. Identification of the control points was carried out on the external images, which were captured by the CCD camera mounted on top of the laser scanner. The identification process is automatic on the photographs taken during the acquisition of the laser scanner from a high resolution CCD camera. The important thing required during image capturing is finding a stable feature on the object that can be identifiable from different views. Several candidate control points might be found from the overlapping images but it is mandatory to filter out the points until sufficient stable set of control points is obtained. For the application of the method a laboratory experiment was performed on a model of 1:3 scale replica of an ancient temple exposed to controlled earthquake vibrations. The maximum displacement obtained from the earthquake test was ±0.1 m in each direction and a maximum acceleration of 2g and 4g in each horizontal and vertical direction, respectively. Two over lapping point clouds were captured before and after the earthquake stress. The registration of the two cloud points were resulted an accuracy of 2mm in the X, Y and 3mm in the Z directions. After having the registration of the point clouds 11 control points were identified in order to calculate the displacement before and after the excitations of the earthquake. The coordinates of the control points were calculated twice to compare them, and a significant movement at the longitudinal direction and less significant movement at the lateral direction were identified.

The other technique used for structural deformation monitoring is surface modeling. In other geodetic monitoring methods small targets are used in order to minimize pointing errors but large structured flat targets or 3D shapes are used in TLS. Single points cannot be used for deformation estimation due to the difficulty of identifying the same points on multiple scans. Since laser scanning is a system that acquires dense 3D spatial information of the surface of an object, hence modeling surface deformation is more favorable than estimating deformation of a
single point in order to get advantage by using the dense information. Although the difficulty to find a fixed benchmark on the surface of the deformed area might be one of the disadvantages of this method Zhou et al. (2011) a case study related with the above-mentioned method is discussed in the next section.

2.1- Deformation of a Harbor Lock

The objective of the experiment described in Lindenbergh & Pfeifer (2005) is to estimate the deformation caused by the change in water level of a small harbor sea lock shown in Figure 2.1, which connects the main shipping channel from Amsterdam to Open North Sea. Using HDS 2500 Laser scanner at fixed position two scan epochs were obtained within short time interval. A statistical deformation analysis was used to detect and measure the magnitude of the movement of the lock. Different types of methods were employed in this experiment. Segmentation of cloud points was the initial process that performed in order to get group of points with similar properties; in here a region growing approach was used for segmentation of a planar patch.

The other method used in this experiment is direct comparison of the observed cloud points, since the scanner position was fixed between the two scans. The emittance directions of the points are parameterized by the horizontal direction and vertical angle (β, ζ) and are the same for the two scans. Those two angles (β, ζ) can be constructed from the Cartesian coordinate xyz and the remaining polar coordinate R is the range directly obtained from the measurements. The deformation can be find by comparing the two ranges R_p(1) and R_p(2) at scan configuration p = (β, ζ), as R_p(1) − R_p(2) = n(p) + d(p) + s(p) where n(p) is the measurement noise, d(p) is the deformation and s(p) is the systematic errors.

Analyzing the movement with unit normal vector of the planar surface is another approach used to detect the presence of movement between epoch scans. A manual interpretation was used to check the movement between the two scans. For each cloud point p the normal vector of the plane is computed by using an adjusting plane through some neighboring points of point p in scan 1 and taking its normal. For a point b in scan 2 the closest point in scan 1 say point a is searched, the projection of the difference vector b-a on the normal n_a can be built. This vector can indicate the movement and its magnitude of each point.

Iterative closest point (ICP) is mentioned as an alternative method to detect deformation if direct comparison method is not possible. To perform ICP method the cloud points of the second scan must be transformed into the coordinate system of the first scan. The main task here is to find the corresponding or closest points in the second scan for every point in the first scan as well as the distance between pairs of points used for sorting of the correspondences. Then the correspondence with the shortest distance is used to determine the six transformation parameters (three translations and three rotations) for minimizing the distance between corresponding points.

Surface modeling is the main method that is used for monitoring and estimation of deformation of the sea entrance lock. A planar surface representation was selected to analyze the deformation.
A least square adjustment mathematical model was used to find the parameters of the planar surface.

The modeling process yields residuals and the distribution of the resulting error indicates the fitting of the model, if the model is perfectly fit the residuals are normally distributed while there is no spatial correlation between them. A variogram/covariance analysis was used to check the correlation between the residuals. If there is no correlation, the variogram should only yield a pure nugget effect, that is, the variogram is approximately a straight line, and its offset corresponds to the noise level. In similar way the modeling process was performed for the planar patch of each scan and to determine the parameters that describes the best fitting plane. For this particular case a 3 by 3 covariance matrix is assumed to describe the quality of the fit for each planar patch. An adjusting plane is found by using individual plane parameters and covariance matrices for the corresponding planes of the two epochs. And finally new parameters of the adjusting plane will be computed in the same way as described in the above. The resulting residuals obtained from the modeling of the adjusting plane are used for statistical test of the fitness of the plane, a brief mathematical expression is found in the paper. The main objective of the test statistics is to check the fitness of the model to the observations, and also to test that whether the two planes in a single epoch shows a deformation or they are stable. The data is analyzed through the method described in the paper and the results are obtained accordingly.

From direct comparison of the observed points Figure 2.2 the range difference showed a 0.3 mm upward systematic movement which although were within the accuracy specifications for single measurements they were not expected in specific parts of the lock. Possible explanations were attributed to strong wind or instability of the scanner during the operation. The sidewall of the lock showed a sideward movement of 1.4 mm between the two scan epochs. On the other method planes fitted to point clouds of an entire segment were compared between the two scan epochs. It was shown that from the four segments in Figure 2.3, the lower test value indicating stability, are those of segment 1 and 3, whereas the higher test values indicating movements are those of segments 2 and 4.

![Figure 2.1. Points from the first scan, representing the lock, photographed by the scanner internal camera](image_url)
While this is the first indication of deformations, further study showed that the plane model is not appropriate for the entire segment and hence analysis on smaller region of the plane is preferred. Therefore the area of one segment was split up into raster cell of equal size. The edge length of the raster was chosen to be 5 cm, which leads to contain roughly 10 points per cell.

One test statistics was provided for each cell and was checked against the critical value using the $\chi^2$-distribution with 3 degrees of freedom. It was found that 1234 cells were accepted that is to be stable, whereas 444 cells were rejected, that is a movement was detected with a significance level of 5%. The difference from the reference plane of the first epoch varied from 0 to 20 mm for the stable cells and 9 to 21 mm for the moving cells.

### 2.2- Deformation of a Tunnel

An experiment was performed by Lindenbergh et al. (2009) for deformation monitoring of a tunnel on Rotterdam CS metro station using terrestrial laser scanning. The notion of the paper is to present an approach to obtain sub noise level accuracies in surveying applications using terrestrial laser scan data. For the detection of deformations between epochs two choices were mentioned. The first choice is reconstruction of the position of apparent reference points from a registered point cloud, which requires targets during scanning process, can allow to asses deformation in all three xyz coordinate if registration is performed very well. The second one is identification of corresponding objects in a time series of point clouds and parameterizes their motion through time. The second one was used for this case study.
In the detail analysis of the experiment two major works were done; the first one is obtaining the optimal quality of cloud points for each scan epoch and secondly detecting deformation by getting benefit from the cloud point redundancy and quality of individual point through adjustment and testing procedure.

The major task in this tunnel-monitoring project was designing the measurement set up. Scanning geometry is the main step, which is needed in order to get the proper intensity and density of point clouds. The change in incidence angle is the factor for the variation of the intensity as well the density of a point clouds. In elongated objects like tunnel the incidence angle increases as the distance from the scanner to the object increases Lindenbergh et al. (2005). For fixed range and surface properties the increase in incidence angle implies that the pulse returning from the object is widened. This indicates that a decrease in intensity and an increase in the noise level. On the other hand with fixed incidence angle the increase in range results a nosier point clouds. And hence incidence angle greater than 60 degree and long scan ranges (depending on the scanner specification) should be avoided in order to get an optimal quality of cloud points Soudarissanane et al. (2008). Before performing design of automatic measurement set up it is required fully characterize the variance of individual scan points depending on the scanner calibration, scanning geometry, surface properties and environmental conditions.

The next major step done in this monitoring project work is registration. Targets are used to serve as a control points to allow additional validation or connection to a global reference frame. But using a limited number of control points will have a problem that small bias of positioning of the control points with scanner will propagate into all consecutive processing steps Alba et al. (2008). Registration must be performed in order to transform the point clouds into the global reference system. Any point cloud registration will result some closing error. Software that is used for registration reports errors found on either the control point or the point representing the stable objects used for the registration.

Object and surface changes were analyzed to obtain optimal quality of the point cloud. For this purpose automatic change detection was processed before the analysis of the actual deformation. The change detection should identify both object and surface. The presence or absence of object on the scanned surface will have different results during segmentation process. So segmentation is applied to identify object parts and to establish correspondences between the same object parts in different epochs.

A point wise deformation analysis is the second part of this research work. Delft adjusting and testing methods were used for the analysis Teunissen (2000a) & (2000b); and only deformation perpendicular to the object surface is considered. An approximately cylindrical tunnel was scanned two times using HDS4500 phase scanner. Artificial deformation were applied during the time of the second scan epoch using small lids, some plates and some boards, that all are less than 2 cm shown in Figure 2.4.
Theoretically the quality of each individual point cloud $p = (p_x, p_y, p_z)$ of different epoch was described by using the variance covariance matrix $\mathbf{Q}_p$ which explains the realistic variance of the three coordinate points and their covariance. But this information is not always available, so for this particular project a global standard deviation $\sigma_R = 7.6$ mm is adapted in the case of Figure 2.4 which approximately describing the average standard deviation of the scan points in the deformation direction. To estimate the deformation from the surface the object surface is parameterized locally and for Figure 4 the parameterization is achieved by fitting a mathematical cylindrical model to the complete point cloud of each epoch.

The point cloud points of each epoch were described in the form of $(p_x, p_y, p_z)$, where $(p_x, p_y)$ express the location of the point on the surface and $p_z$ is the remaining coordinate in the direction of the deformation assumed to perpendicular to the object surface. To hold back the effect of noise to optimally profit from data redundancy, the surface of the object is divided into grid cells of suited size. Its aim is to present possible local deviation and deformation on the surface. For Figure 2.4 a grid size of 15 cm was chosen. And then all points in corresponding grid cell are used to determine range coordinates $(r_1, \sigma_1)$ and $(r_{11}, \sigma_{11})$ for epoch one and two for the middle of the grid cell.

Question can be posed if the difference between $r_1$ and $r_{11}$ is significant given their variances. It is described that the deformation can be detected if this difference is significant otherwise the difference is more likely to be explainable by random deviation on the scan measurements. A statistical test was performed to answer this question, which is evaluated against a critical value.

In the top left image of Figure 2.4, the test statistics for each grid cell is shown. Most are gray, indicating low-test statistic values but in the bottom right considerable larger values can be observed at the location where the plates were placed. The stability test also performed to evaluate the test statistic, using a chi-square distribution with one degree of freedom by setting zero hypothesis which states stability. A reliability of 5 % was used for Figure 2.4 top right, to obtain the critical value and evaluated against the test statistic. In Figure 2.4 of the top right image the red color grid points are with a test statistics larger than the critical value and are therefore tested unstable, whereas the blue colored are stable. It is also explained that artificial deformations were found back by the testing method even though some other points were tested instable. It is discussed that these locations are places where coincide with gripper holes in the concrete tunnel plates. A possible explanation given for the artificial differences is small errors in registration between epochs. And improved method of segmentation was suggested as a solution to remove scan points in the hole that causes the error. At the end the result of the experiment was described that differences above 15 mm were always tested unstable and most differences below 15 mm were stable.

As described above the first point cloud of each epoch was obtained with a high individual point quality and then realistic variance values of each point also obtained through statistical analysis of adjustment and testing theory. Finally it is concluded that prior characterization of the quality of the point clouds is under investigation but not yet available.
A methodology for measuring structural deformation, relying on theoretical aspects of beam mechanics and implemented by constrained least squares curve fitting, has been developed by Gordon et al. (2004). It is shown in two structural deformations monitoring experiments, involving a concrete beam being loaded in a load testing frame, and a filed case involving a span of a timber bridge beam, were used to test the analytical modeling strategy. The analytical model was derived from the first principle of beam deflection by integration, which results the low order polynomial Eq.s. This modeling avoids the arbitrary nature inherent in some other methods, such as gridding. All experiments were controlled with convergent digital photogrammetry (close range photogrammetry that is used to establish fast and adequate 3D object model in real time measurement) Gordon et al. (2004). The study also aimed to evaluate the sensitivity of the laser scanner in regard with deformation monitoring of a structural element as well.

In the first experiment a controlled loading of a timber beam on an indoor test frame, which had a dimension of 5.0m x 0.2m x 0.1m was involved. A total of eight load increments were applied on each occasion by inducing a nominal 5 mm vertical displacement at the center of the beam. The loading was applied by a hydraulic jack that was positioned at the center of the beam to assist the determination of each 5 mm increments of the displacement. The total downward deflection measured at the center of the beam was approximately 40 mm.

A Cyra Cyrax 2500 (Leica Geosystems, 2004) and a RiegL LMS-Z210 (RiegL, 2004) terrestrial laser scanners (TLSs) were used in these experiments. A Kodak DC420 with a CCD array of 1524 by 1012 pixels fitted with 14 mm lens was used for the photogrammetric range measurement analysis.

The positions of the two laser scanners were fixed until the testing was completed. Scanning was performed during the loading time and a high resolution scans were collected at each epoch by the two scanners. Since the LMS-Z210 scanner offers a coarse coordinate precision compared to the Cyrax 2500, hence three repeated scans were taken and averaged in order to get a single
mean scan, 25 photogrammetric targets were affixed on the face of the beam to use as control points. After the completion of pre processing of the scan data the vertical deflection of the beam was derived using the analytical beam model. This derivation was performed using the estimated model for each eight load epochs. All TLS data for a single epoch were processed in one adjustment, thus simultaneously solving for the parameters of the analytical models of beam deflection. The mean number of points used for each solution was 7364 for the Cyrax 2500 and 1099 for the LMS-Z210. The overall RMS of residuals from the least squares adjustments was ±0.6 mm and ±5.4 mm for Cyrax 2500 and LMS-Z210, respectively, the difference in the size of residuals largely reflects the observational precision of each scanner.

The x and y coordinates of each 13 photogrammetric targets were passed through the estimated model to compute the z coordinate which were then used to determine the vertical deflections between the epochs. The estimated models using Cyrax 2500 data, compared to the benchmark photogrammetry, give an overall RMS of differences of ±0.29 mm and the overall RMS of differences for the LMS-Z210 is ±3.6 mm as it is shown in Table 2.1. The maximum RMS is ±5.0 mm for the 25 mm deflection case. The overall RMS values represent a factor of improvement (in precision) of 21 times for Cyrax 2500 and 7 times for the LMS-Z210 over the coordinate precision of each laser scanner.

Table 2.1 RMS of differences between TLS-derived and photogrammetry-derived vertical deflections using 13 targets per deflection case, Gordon et al. (2004).

<table>
<thead>
<tr>
<th>Nominal Vertical Deflection (mm)</th>
<th>RMS of Differences (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cyrax 2500</td>
</tr>
<tr>
<td>5</td>
<td>±0.12</td>
</tr>
<tr>
<td>20</td>
<td>±0.14</td>
</tr>
<tr>
<td>15</td>
<td>±0.47</td>
</tr>
<tr>
<td>20</td>
<td>±0.26</td>
</tr>
<tr>
<td>25</td>
<td>±0.24</td>
</tr>
<tr>
<td>30</td>
<td>±0.27</td>
</tr>
<tr>
<td>35</td>
<td>±0.30</td>
</tr>
<tr>
<td>40</td>
<td>±0.34</td>
</tr>
<tr>
<td>Total RMS</td>
<td>±0.29</td>
</tr>
</tbody>
</table>

The second experiment was also performed in similar way with the first one, except the difference in the type of beam, its size and the loading condition. An L-shaped concrete beam with size 7.0 m x 0.5 m x 0.5 m was used for this experiment, shown in Figure 2.5. Like the previous case the plan metric coordinates of 12 photogrammetric targets were passed through the estimated model in order to compute the z coordinate, the vertical deflection were computed by differencing the z coordinate. Similar results were obtained from the experiment, which was loaded in increments up to 240 KN (approximately 13 mm of vertical deflection). The Rieg
LMS-Z210 laser data achieved the estimation of the model parameters at an accuracy of ±2.4 mm level (1σ).

**Figure 2.5** Concrete beam and the Riegl LSM-Z210, Image taken from Gordon et al. 2004.

A third experiment involved placing larger weights over a span of an ageing timber bridge and measuring the deformation at critical sites on the various structural members. A Reigl LMS-Z210 laser scanner was used to acquire a scan per each load epoch. The derived vertical deflection from the scanner data was computed using the estimated parameters of the analytical model. A comparison of the laser derived deflections and photogrammetric derived deflections gave RMS differences ranging from ±2.6 mm for the stringer (longitudinal timber member underneath the bridge deck) closest to the scanner to ±13.0 mm for the stringer furthest from the scanner, which are at the same level of accuracy as results of the laboratory-experiments. At the end the paper concluded that the sub-millimetre result from Cyrax 2500 is with the same accuracy level with the close range photogrammetry (for non metric cameras).

### 3- Methodology

As it is already explained in section 2.1 that the surface modeling technique is used to monitor and measure artificially created deformation of sample objects. Under the surface modeling method we can have different types of surface representations depending on the nature of the surface of the objects used for the experiment and the objective of the study as well. In this practical experiment three basic types of parametric surface representations were used to estimate deformation from the surface Figure 3.1. In the context of this research the main aim of using those different parametric surfaces is to compare and identify suitable surface modeling methods for deformation monitoring of an object or structure with respect to precision and accuracy.

A carton with size (53x48x48) cm is used for the estimation and analysis of change on the planar surface. In the first scan epoch the object is placed at a distance of 5.4 metre and scanned without creating any deformation on the object. In the second epoch the position of the scanner was kept on the same place and a 10 kN load is applied on the object to create an artificial deformation and scanning is performed in similar fashion.
For the estimation of a spherical surface parameters a ball with initial diameter of 0.2 metre is used; in the first epoch the ball is scanned as it is without any change on its size next in the second epoch scan some air was released from the ball in order to change its size. The position of the scanner is unchanged in both scan epochs.

In the case of the cylindrical object a rounded and semi flexible paper holder is selected to do the experiment. In similar way to the spherical case the first scan epoch is performed before any change has been made on the shape or the size of the cylinder and then the second scan epoch is performed after creating an artificial deformation on the object.

![Image of sample objects](image)

**(a)** For planar (b) For Spherical and (c) For Cylindrical Surfaces

**Figure 3.1:** Images of sample objects used in the experiment (a) For planar (b) For Spherical and (c) For Cylindrical Surfaces

### 3.1- Surface Reconstruction and Modeling

The main target of surface reconstruction is in accordance with the procedure described below. In a given set of sample points \( P \), assumed to lie on or near an unknown surface \( (s) \), create a surface model \( (s') \) that approximates \( (s) \). A surface reconstruction procedure cannot guarantee the recovering of \( (s) \) exactly, since we have information about \( (s) \) only through a finite set of sample points. Sometimes additional information of the surface (e.g. break line) can be available and, in general, as the sampling density increases the output result \( (s') \) is more likely topologically correct and converges to the original surface \( (s) \) – see Fabio (2003).

In Fabio (2003) different classifications of surface reconstruction methods were discussed in detail. However, all these methods require the following four basic steps: a) Pre-processing, which deals with eliminating erroneous data or sampling points in order to reduce computation time. b) Determination of global properties of the object’s surface, which includes derivation of neighborhood relations between adjacent parts of the surface. This needs steps to consider possible ‘constraints’ (e.g. break lines) to preserve special features (like edges). c) Generation of polygonal surfaces such as triangular or tetrahedral meshes but also parametric surfaces (e.g. low order polynomials over a user-defined reference plane or more general free form surfaces) and implicit surface representations (e.g. for plane, spheres, cylinders and tori) are used, and finally d) Post-processing, when the model is created, editing operations are commonly applied to refine and perfect the polygonal surface.

In this work Cyclone 3D modeling software was used for the analysis of the experiments and also Matlab codes were developed for determination of the best fitting parameters of each
surface type. The parameters obtained from Cyclone were compared with the one obtained from Matlab code as it is described in the following sections.

3.1.1- Segmentation of Point Clouds

Segmentation is a process of partitioning a point cloud into meaningful regions or estimating important features from point data. Most point data segmentation methods can be categorized into three groups: edge detection, region-growing and hybrid methods. The edge detection method is used to detect discontinuities of surface that makes close boundaries of components in the point data whereas region-growing method perform segmentation of point data by detecting continuous surfaces that have homogeneity or similar geometric properties. The hybrid method is a combination of the two; a brief explanation of the methods is found in Fabio (2003). In the case of region-growing method nearby points which have a common properties under a given homogeneity criterion (like the direction of a locally estimated normal vector) can be grouped to estimate smooth surfaces. But in this particular experiment “fit to cloud” option is used in Cyclone to get a best-fitted surface. In cyclone the fit to cloud segmentation method uses a least square algorithm and the Matlab code also developed in similar fashion to make a comparison between the results. Least squares fitting (LSF) is an iterative algorithm of finding the best fit surface with the least squares constraints of distances of the scanned points from the surface.

3.1.2- Fitting Surface Models to Measured Data

Depending on the type of algorithm to be used in the process we can have two approaches for fitting of geometrical surfaces of objects; the first one is iterative whereas the other is non-iterative. Geometrical shapes like plane, cylinder and sphere can describe most man-made objects. These shapes are described by a few parameters, which allow fitting of such shapes with robust non-iterative methods that detect clusters in a parameter space (Vosselman et al. 2004). But in our experiment an iterative approach is selected to perform the fitting. Hence as it is already explained in sub section 3.1.1 a least square-fitting algorithm is used for the estimation of parametric surfaces from point clouds.

Planar Model

A plane is the most frequent surface shape found in many man-made objects. Segmentation of a planar patch will be performed according to the procedure discussed in sub section 3.1.1. Once we carried out segmentation of point clouds the next step is to estimate the parameters of the plane. In 3D Euclidian space a plane can be expressed by the equation

\[ ax + by + cz + d = 0 \]  \hspace{1cm} (3.1)

where \((a, b, c)\) are the coordinates of the normal vector to the plane, \((x, y, z)\) are coordinates in the plane and \(\frac{d}{\|a b c\|}\) is the perpendicular distance from the origin to the plane.

In the process of segmentation of sub section 3.1.1 or for the later stage of deformation analysis it is necessary to find a mathematical parameterization for each selected surface model. The planar model parameter \(X\) of the planar model Eq. (3.1) and the residual vector of the model can be obtained as follows. First, Eq. (3.1) can be rewritten in the following form
\[
\begin{pmatrix} a \\ b \\ c \end{pmatrix} x + \begin{pmatrix} a \\ b \\ c \end{pmatrix} y + \begin{pmatrix} a \\ b \\ c \end{pmatrix} z = -1
\]  

By considering multiple data points on the plane, Eq. (3.2) can be further rearranged and written as a system of observation equations

\[
\begin{align*}
Ax_1 + By_1 + Cz_1 &= -1 - \varepsilon_1 \\
Ax_2 + By_2 + Cz_2 &= -1 - \varepsilon_2 \\
&\vdots \\
Ax_n + By_n + Cz_n &= -1 - \varepsilon_n
\end{align*}
\]

where \( A = \frac{a}{d} \); \( B = \frac{b}{d} \); and \( C = \frac{c}{d} \) will be determined by least squares solution and \((\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)\) are the residuals corresponding to each observation equation. Geometrically the residuals are distances between points and the plane. Eq. (3.3) can be written in matrix form as

\[
AX = L - \varepsilon
\]

where \( X = [A \ B \ C]^T \) is the vector which contains the parameters of the plane to be determined,

\[
A_{(nx3)} = \begin{bmatrix}
{x_1} & {y_1} & {z_1} \\
{x_2} & {y_2} & {z_2} \\
& \vdots & \\
{x_n} & {y_n} & {z_n}
\end{bmatrix}
\]

is the matrix containing the co-ordinates of points in the segmented point cloud for \( n \) number of sample points,

\[
L_{(nx1)} = \begin{bmatrix}
-1 \\
-1 \\
& \vdots \\
-1
\end{bmatrix}
\]

is the column matrix and \( \varepsilon \) is the residual vector.

From least squares solution the model parameter can be obtained as

\[
\hat{X} = (A^T A)^{-1} A^T L
\]

and the model residual can be computed as

\[
\hat{\varepsilon} = L - A\hat{X}
\]

In order to show the process of the change detection from the planar surface an illustrative numerical example is presented below. For this illustration purpose two sets of 6 points belonging to the same plane were generated. Random errors with zero mean and standard deviation of 4 mm were added to each coordinate in both sets. Numerical values of the simulated
coordinates are given in Appendix 1 and Appendix 2 denoted as \( A_1 \) and \( A_2 \), respectively. The simulated values of the parameters (A B C) are \((0, 0, 1)\) for both sets of point coordinates.

Using Eq. (3.5a) the parameters are estimated from the first and second sets of points as

\[
\hat{\mathbf{X}}_1 = [0.0014 \quad 0.0009 \quad 0.4028]^T \quad \text{and} \quad \hat{\mathbf{X}}_2 = [0.0003 \quad -0.0009 \quad 0.3985]^T,
\]

for the first set of points and

\[
\hat{\mathbf{X}}_2 = [0.0036 \quad 0.0022 \quad 1.0000]^T \quad \text{and} \quad \hat{\mathbf{X}}_2 = [0.0006 \quad -0.0023 \quad 1.0000]^T.
\]

The Aposterior variance factor is computed as

\[
\hat{\sigma}_{01}^2 = \frac{\hat{e}_1^T \hat{e}_1}{n-k} = 3.09 \text{ mm}^2
\]

for the first set of points and

\[
\hat{\sigma}_{02}^2 = \frac{\hat{e}_2^T \hat{e}_2}{n-k} = 9.54 \text{ mm}^2
\]

for the second set of points, where \( n \) is the number of points and \( k \) is the number of parameters.

The propagated variance-covariance matrix of the parameters for the first set of points

\[
C_{1xx} = \hat{\sigma}_{01}^2 (A_1^T A_1)^{-1} = 10^{-5} \begin{bmatrix} 0.0920 & -0.0364 & 0.0704 \\ -0.0364 & 0.0680 & 0.0222 \\ 0.0704 & 0.0222 & 0.1089 \end{bmatrix}
\]

and for the second set of points

\[
C_{2xx} = \hat{\sigma}_{02}^2 (A_2^T A_2)^{-1} = 10^{-5} \begin{bmatrix} 0.1268 & -0.0212 & 0.0922 \\ -0.0212 & 0.1566 & 0.2037 \\ 0.0922 & 0.2037 & 0.4064 \end{bmatrix}
\]

Once the parameters are estimated from each fitted planar surface, the next step is to detect the change between them. In the following computation \( \hat{\mathbf{n}}_1 \) and \( \hat{\mathbf{n}}_2 \) are the estimated unit normal vectors of the plane in epoch 1 and epoch 2, respectively:

\[
\hat{\mathbf{n}}_1 = \frac{\hat{\mathbf{x}}_1}{\|\hat{\mathbf{x}}_1\|} = [0.0036 \quad 0.0022 \quad 1.0000]^T \quad \text{and} \quad \hat{\mathbf{n}}_2 = \frac{\hat{\mathbf{x}}_2}{\|\hat{\mathbf{x}}_2\|} = [0.0006 \quad -0.0023 \quad 1.0000]^T.
\]

The difference between the normal vectors becomes

\[
d_{\hat{\mathbf{n}}} = \hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2 = [0.0030 \quad 0.0045 \quad 0.0000]^T
\]

is a measure of the rotation of the plane. Its norm is computed as

\[
\|d_{\hat{\mathbf{n}}}\| = \sqrt{d_{\hat{\mathbf{n}x}}^2 + d_{\hat{\mathbf{n}y}}^2 + d_{\hat{\mathbf{n}z}}^2} = 0.0054
\]

is the angle between the two normal vectors \( \hat{\mathbf{n}}_1 \) and \( \hat{\mathbf{n}}_2 \) since
\[
\sin \left( \frac{\alpha}{2} \right) = \frac{0.0054}{2} \quad \text{and} \quad \alpha \approx 0^\circ.31 = 18.6' \quad (3.14)
\]

For small angles sine of the angle is approximately the same as the angle itself and hence \(\alpha \approx \|d_\mathbf{n}\| = 0^\circ.31 = 18.6'\), which is the rotational angle of the normal of the plane.

The covariance matrix of \(d_\mathbf{n}\) can be obtained as

\[
C_{d_\mathbf{n}} = C_{\mathbf{f}_1} + C_{\mathbf{f}_2} = 10^{-4} \begin{bmatrix}
0.0547 & -0.0144 & 0.0406 \\
-0.0144 & 0.0562 & 0.0566 \\
0.0406 & 0.0566 & 0.1290
\end{bmatrix} \quad (3.15)
\]

where

\[
C_{\mathbf{f}_1} = \left( \frac{1}{\|X_1\|^2} \right) C_{X_1} = 10^{-5} \begin{bmatrix}
0.2284 & -0.0904 & 0.1748 \\
-0.0904 & 0.1688 & 0.0551 \\
0.1748 & 0.0551 & 0.2704
\end{bmatrix} \quad (3.16)
\]

\[
C_{\mathbf{f}_2} = \left( \frac{1}{\|X_2\|^2} \right) C_{X_2} = 10^{-4} \begin{bmatrix}
0.0318 & -0.0053 & 0.0231 \\
-0.0053 & 0.0393 & 0.0511 \\
0.0231 & 0.0511 & 0.1020
\end{bmatrix} \quad (3.17)
\]

\(C_{X_1}\) and \(C_{X_2}\) are the variance covariance matrices of estimated parameters of the first and the second sets of points and \(C_{\mathbf{f}_1}\) and \(C_{\mathbf{f}_2}\) are the matrices of unit normal vectors \(\mathbf{f}_1\) and \(\mathbf{f}_2\), respectively.

The standard error of \(\|d_\mathbf{n}\|\), \(\sigma_{\|d_\mathbf{n}\|}\) is a nonlinear function of \(d_{nx}, d_{ny}\) and \(d_{nz}\), i.e. \(\sigma_{\|d_\mathbf{n}\|} = f(d_{nx}, d_{ny}, d_{nz})\), as expressed in Eq. (3.11). The law of error propagation for a non-linear function is used to compute \(\sigma_{\|d_\mathbf{n}\|}\) as described below. By using the computed value of the standard error \(\sigma_{\|d_\mathbf{n}\|}\) we can determine the limit of the uncertainty that will tell us whether the detected change is significant or not. According to Fan (2010) the law of error propagation for a non-linear function can be expressed as

\[
\epsilon_{\|d_\mathbf{n}\|} \approx \frac{\partial \|d_\mathbf{n}\|}{\partial d_{nx}} \epsilon_{d_{nx}} + \frac{\partial \|d_\mathbf{n}\|}{\partial d_{ny}} \epsilon_{d_{ny}} + \frac{\partial \|d_\mathbf{n}\|}{\partial d_{nz}} \epsilon_{d_{nz}} = a_{xx} \epsilon_{d_{nx}} + a_{yy} \epsilon_{d_{ny}} + a_{zz} \epsilon_{d_{nz}} = A \epsilon \quad (3.18)
\]

where \(\epsilon = \begin{bmatrix} \epsilon_{d_{nx}} & \epsilon_{d_{ny}} & \epsilon_{d_{nz}} \end{bmatrix}^T\) is the vector of residuals containing the residuals which correspond to the changes of the normal vector along x, y and z axis and \(A = [a_{xx} \quad a_{yy} \quad a_{zz}]\) is the matrix that contains the partial derivatives of the non-linear function 

\[
\|d_\mathbf{n}\| = \sqrt{d_{nx}^2 + d_{ny}^2 + d_{nz}^2} \text{ computed as}
\]

\[
a_{xx} = \frac{\partial \|d_\mathbf{n}\|}{\partial d_{nx}} \quad a_{yy} = \frac{\partial \|d_\mathbf{n}\|}{\partial d_{ny}} \quad a_{zz} = \frac{\partial \|d_\mathbf{n}\|}{\partial d_{nz}} \quad (3.19)
\]

Then the variance of \(\|d_\mathbf{n}\|\) can be obtained from

\[
\sigma_{\|d_\mathbf{n}\|}^2 = E\{[\|d_\mathbf{n}\| - E(\|d_\mathbf{n}\|)]^2\} = AC_{d_\mathbf{n}}A^T \quad (3.20)
\]

16
where $C_{d\hat{n}}$ is the variance-covariance matrix of the difference vector.

Therefore the value of $\sigma_{\|d\hat{n}\|}$ is obtained as

$$\sigma_{\|d\hat{n}\|}^2 = AC_{d\hat{n}}A^T = 0.00000425 \text{ rad}^2$$

where $A = [0.5547 \ 0.8321 \ 0.0000]$

and $\sigma_{\|d\hat{n}\|} = 0.0021 \text{ rad}$

Since the errors are normally distributed over the fitted model, a coverage factor 3 is chosen from the normal distribution table for the determination of the confidence interval of the uncertainty that corresponds to a confidence level of 99.7% with a risk level of 0.3%. The uncertainty limit become $3\sigma_{\|d\hat{n}\|} = 0.0062 \text{ rad}$. As $\|d\hat{n}\|$ is only 0.0054 rad, the result of the statistical test shows that no significant change is detected between epochs 1 and 2.

**Spherical Model**

Four parameters are needed to describe a sphere. These parameters include three coordinates $(x_c, y_c, z_c)$ of the center of the sphere and its radius $r$ to determine its size. So any point $(x_i, y_i, z_i)$ on the sphere satisfies the equation

$$(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 = r^2$$

(3.21)

where $(x_i, y_i, z_i)$ are the coordinates of any point on the sphere.

A minimization function has to be identified to obtain the estimate of the center coordinates and the radius of the sphere from Eq. (3.22), and the function could be $f = \sum_{i=1}^{n}(r_i^2 - r^2) = \min (x_c, y_c, z_c, r^2)$, where $r_i$ is the radial distance from the center of the sphere to each point obtained from the measured data.

The spherical model parameter $X = (x, y, z, r)^T$ of the model of Eq. (3.21) and the residuals can be found as per following procedure. Linearizing the left hand part of the model equation around the initial estimate of the center of the sphere $(x_{co}, y_{co}, z_{co})$ results in the following equations, which are rearranged and written for multiple data points

$$
\begin{align*}
2(x_1 - x_{co})dx + 2(y_1 - y_{co})dy + 2(z_1 - z_{co})dz + r^2 &= (x_1 - x_{co})^2 + (y_1 - y_{co})^2 + (z_1 - z_{co})^2 + \varepsilon_1 \\
2(x_2 - x_{co})dx + 2(y_2 - y_{co})dy + 2(z_2 - z_{co})dz + r^2 &= (x_2 - x_{co})^2 + (y_2 - y_{co})^2 + (z_2 - z_{co})^2 + \varepsilon_2 \\
2(x_n - x_{co})dx + 2(y_n - y_{co})dy + 2(z_n - z_{co})dz + r^2 &= (x_n - x_{co})^2 + (y_n - y_{co})^2 + (z_n - z_{co})^2 + \varepsilon_n
\end{align*}
$$

(3.22)

where $\varepsilon_i = r_i - r$ and $(dx, dy, dz)$ are the increments of the center coordinates to be determined by least squares solution.

Eq. (3.22) can be written in matrix form as

$$AdX = L + \varepsilon$$

(3.23a)
where
\[ A_{(nx4)} = \begin{bmatrix}
2(x_1 - x_o) & 2(y_1 - y_o) & 2(z_1 - z_o) & 1 \\
2(x_2 - x_o) & 2(y_2 - y_o) & 2(z_2 - z_o) & 1 \\
\vdots \\
2(x_n - x_o) & 2(y_n - y_o) & 2(z_n - z_o) & 1
\end{bmatrix} \]

is the matrix containing coefficients of the left hand parts of Eq. (3.22),
\[ dX_{(4x1)} = [dx \ dy \ dz \ r^2]^T \]

is the vector which contains increments in center coordinates to be determine by the least squares solution,
\[ L_{(nx1)} = \begin{bmatrix}
(x_1 - x_o)^2 + (y_1 - y_o)^2 + (z_1 - z_o)^2 \\
(x_2 - x_o)^2 + (y_2 - y_o)^2 + (z_2 - z_o)^2 \\
\vdots \\
(x_n - x_o)^2 + (y_n - y_o)^2 + (z_n - z_o)^2
\end{bmatrix} \]

is the vector containing the right hand parts of Eq. (3.22).

The least squares solution can be solved as
\[ d\bar{X} = (A^TA)^{-1}A^TL \]

the estimated residual can be computed as
\[ \varepsilon = L - Ad\bar{X} \]

and the final estimate of the parameters of the sphere can be obtained as
\[ \bar{X} = [x_o + dx, \ y_o + dy, \ z_o + dz, \ r]^T \]

The covariance matrix of the parameters and the residuals can be determined in similar fashion as in the planar model case above.

**Cylindrical Model**

A Gauss–Newton algorithm as described in NSS (2012) was used to find the parameters of a cylinder surface model. To start with the basic mathematical derivation, any line can be specified by giving a point on the line and direction cosines (a, b, c). So 6 numbers are required to describe a line: 3 directional cosines (a, b, c) and 3 coordinates (x, y, z). Its constraint is \( a^2 + b^2 + c^2 = 1 \). Given two of the components, the third can be determined. Consider a constraint on a line where c=1, which is a vertical line. Then it is enough to specify two direction cosines a and b, and the coordinates x and y, z can be determined from the relationship
Equation (3.26) states:

\[ ax + by + cz = 0 \]  \hspace{1cm} (3.24)

since \( c = 1, z = -ax - by \).

The advantage of this constraint is to reduce the number of parameters from 6 to 4, which are \( a, b, x \) and \( y \). It also reduces the complication of solving the Jacobean matrix and time to evaluate, because the derivatives of the distance are computed when using the Gauss-Newton method. Due to this, the basic Gauss-Newton algorithm is modified here in our case in order to get the advantage described. The first modification is, to translate the coordinate system at the beginning of each iteration, so that the point on the axis is the origin of the coordinate system. It means \( x = y = 0 \). Second, rotate the coordinate system, so that the direction of the axis is along the \( z \)–axis, yielding \( a = b = 0 \) and \( c = 1 \). In general, based on the principle discussed above, a cylinder can be described by five parameters: the direction cosines \( a \) and \( b \), the center coordinates of the circle on a plane \((x_o, y_o)\) and the radius of the cylinder \( r \).

Estimating the parameters of a cylindrical surface cannot be performed within short steps like the planar or the spherical surfaces; rather it needs a multiple stage process. The parameters for a cylindrical surface are those already mentioned above and the estimation can be performed in the following procedure according to NSS (2012). To start with, the equation of a cylinder can be written as

\[ A^2 + B^2 + C^2 = R^2 \]  \hspace{1cm} (3.25a)

where

\[ A = c(y_1 - y_o) - b(z_1 - z_o) \]  \hspace{1cm} (3.25b)
\[ B = a(z_1 - z_o) - c(x_1 - x_o) \]  \hspace{1cm} (3.25c)
\[ C = b(x_1 - x_o) - a(y_1 - y_o) \]  \hspace{1cm} (3.25d)
\[ R = \text{Radius of cylinder} \]

In the above equations, \( a, b, c \) are the direction cosines of the axis, \( x_o, y_o \) and \( z_o \) are the coordinates of a point on the axis of the cylinder and \( x_1, y_1 \) and \( z_1 \) are the coordinates of any point on the cylinder.

Then by substituting Eq. (3.25b), (3.25c) and (3.25d) in Eq. (3.25a) results in

\[ [c(y_1 - y_o) - b(z_1 - z_o)]^2 + [a(z_1 - z_o) - c(x_1 - x_o)]^2 + [b(x_1 - x_o) - a(y_1 - y_o)]^2 = R^2 \]  \hspace{1cm} (3.26)

Eq. (3.26) can be simplified to

\[ Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ex_iz_i + Fy_iz_i + Gx_i + Hy_i + Iz_i + J = 0 \]  \hspace{1cm} (3.27)

where

\[ A = (b^2 + c^2), B = (a^2 + c^2), C = (a^2 + b^2), D = -2ab, E = -2ac, F = -2bc \]
\[ G = -2(b^2 + c^2)x_o + 2aby_o + 2acz_o, H = 2abx_o - 2(a^2 + c^2)x_o + 2bcz_o \]
\[ I = 2acx_o + 2bcy_o - 2(a^2 + b^2)z_o \]
\[ J = (b^2 + c^2)x_0^2 + (a^2 + c^2)y_0^2 + (a^2 + b^2)z_0^2 - 2bcy_0z_0 - 2acz_0x_0 - 2abx_0y_0 - R^2 \]

Dividing both sides of Eq. (3.27) by A and rearranging will result in the following linear equation:

\[ \frac{B}{A}y_1^2 + \frac{C}{A}z_1^2 + \frac{D}{A}x_1y_1 + \frac{E}{A}x_1z_1 + \frac{F}{A}y_1z_1 + \frac{G}{A}x_1 + \frac{H}{A}y_1 + \frac{I}{A}z_1 + \frac{J}{A} = -x^2 - \varepsilon \quad (3.28) \]

In order to solve the coefficients, Eq. (3.28) can be written in matrix form as

\[ A_1X_1 = L_1 - \varepsilon \quad (3.29) \]

where

\[ A_1 = \begin{bmatrix} y_1^2 & z_1^2 & x_1y_1 & x_1z_1 & x_1 & y_1 & z_1 & 1 \\ y_2^2 & z_2^2 & x_2y_2 & x_2z_2 & x_2 & y_2 & z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_n^2 & z_n^2 & x_ny_n & x_nz_n & x_n & y_n & z_n & 1 \end{bmatrix} \]

is the matrix containing the measured coordinate values,

\[ X_1 = \begin{bmatrix} B/A \\ C/A \\ D/A \\ E/A \\ F/A \\ G/A \\ H/A \\ I/A \\ J/A \end{bmatrix}^T \]

is the vector containing coefficients of Eq. (3.28) to be solved by least squares solution

\[ L_1 = [-x_1^2 \ -x_2^2 \ \ldots \ -x_n^2]^T \]

is a column matrix and \( \varepsilon \) is the residual vector.

Hence the least squares solution can be solved as

\[ X_1 = (A_1^T A_1)^{-1} A_1^T L_1 \quad (3.30) \]

But to find the direction cosines \((a, b, c)\) it is necessary to rewrite the equations in another form. Let the redefine coefficients of Eq. (3.28) be

\[ C' = \frac{B}{A}, \quad D' = \frac{D}{A}, \quad E' = \frac{E}{A}, \quad F' = \frac{F}{A}, \quad G' = \frac{G}{A}, \quad H' = \frac{H}{A}, \quad I' = \frac{I}{A}, \quad J' = \frac{J}{A} \quad (3.31) \]

The values of \(C', D', E', F', G', H', I'\) and \(J'\) are already solved in \(X_1\) and, if \(|D'|, |E'|, |F'|\) are close to 0 then \(B'\) close to 1 implies \((a, b, c) = (0, 0, 1)\), and \(C'\) close 1 implies \((a, b, c) = (0, 1, 0)\), otherwise we need to use another formula to find the direction cosines.
Let us take \( k \) a factor, which is computed as

\[
k = \frac{2}{1+B'+C'}
\]  

(3.32a)

and then new numerical values can be computed as

\[
\begin{align*}
A &= k, & B &= kB', & C &= kC', & D &= kD', & E &= kE', \\
F &= kF', & G &= kG', & H &= kH', & I &= kI', & J &= kJ'
\end{align*}
\]  

(3.32b)

If \( A \) and \( B \) are close to 1,

\[
\begin{align*}
&c' = (1 - C)^{1/2} \quad a' = \frac{E}{-2c'} \quad \text{and} \quad b' = \frac{F}{-2c'}
\end{align*}
\]  

(3.32c)

If \( A \) is close to 1, \( B \) is not close to 1,

\[
\begin{align*}
&b' = (1 - B)^{1/2} \quad a' = \frac{D}{-2b'} \quad \text{and} \quad c' = \frac{F}{-2b'}
\end{align*}
\]  

(3.32d)

If \( A \) is not close to 1,

\[
\begin{align*}
&a' = (1 - A)^{1/2} \quad b' = \frac{D}{-2a'} \quad \text{and} \quad c' = \frac{E}{-2a'}
\end{align*}
\]  

(3.32e)

And the direction cosines \((a',b',c')\) should be normalized in order to get the direction cosines \((a,b,c)\). Then the point on the axis of the cylinder \((x_o,y_o,z_o)\) can be solved by using the direction cosines \((a,b,c)\), the values of \(G\), \(H\) and \(I\) from Eq. (3.27) and Eq. (3.24). Due to the noise in the measured points, these equations will not be fulfilled exactly. Considering normally distributed error terms \( \epsilon \), we can form linear system of equations as

\[
A_2X_2 = L_2 - \epsilon
\]  

(3.33)

where

\[
A_2 = \begin{bmatrix}
-2(b^2 + c^2) & 2ab & 2ac \\
2ab & -2(a^2 + c^2) & 2bc \\
2ac & 2bc & -2(a^2 + b^2)
\end{bmatrix}
\]

is the matrix containing coefficients of Eq. (3.24) and \( G, H, I \) of Eq. (3.27)

\[
X_2 = \begin{bmatrix}
x_o \\
y_o \\
z_o
\end{bmatrix}^T
\]

is the vector containing coordinates of a point on the axis of the cylinder and
\[ \mathbf{L}_2 = [G \ H \ I \ 0]^T \]

is a column matrix. \( \mathbf{X}_2 \) is the vector containing the coordinates of a point on the axis of the cylinder and can be obtained from least squares solution as

\[
\mathbf{X}_2 = (\mathbf{A}_2^T \mathbf{A}_2)^{-1} \mathbf{A}_2^T \mathbf{L}_2
\]

and finally the radius of the cylinder can be computed from the definition of \( J \) in Eq. (3.27).

In order to show the change detection process from the cylindrical surface a numerical example is presented below. For this illustration purpose two sets of 15 points belonging to the same cylinder were generated. Random errors with zero mean and standard deviation of 4 mm were added to each coordinate in both sets. Numerical values of the simulated coordinates are given in Appendix 3 and Appendix 5 denoted as \( \text{Coord}_1 \) and \( \text{Coord}_2 \), respectively. The simulated radius \( r \) and directional cosines \((a \ b \ c)\) are 1.2 m and \((0 \ 0 \ 1)\), respectively, for both sets of points.

The matrix \( \mathbf{A}_1 \) in Eq. (3.29) containing coordinates of points given in Appendix 4 and Appendix 6 denoted as \( \mathbf{A}_1(1) \) and \( \mathbf{A}_1(2) \) for the first and second sets of points.

Using Eq. (3.30) \( \mathbf{X}_1 \) is solved as

\[
\begin{bmatrix}
0.9770 & 0.0626 & 0.0103 & 0.4466 & -0.8581 & 0.0019 & -0.0057 & 0.6908 & -1.4241
\end{bmatrix}^T
\]

and

\[
\begin{bmatrix}
1.0023 & 3.8932 & 0.0133 & 1.7681 & -2.2186 & 0.0027 & -0.0050 & 0.5421 & -1.4364
\end{bmatrix}^T
\]

for the first and second set of points, respectively. According to Eq. (3.31) the values of \( \mathbf{X}_1 \) are

\[
\begin{align*}
C' &= \frac{C}{A} = 0.0626 & D' &= \frac{D}{A} = 0.0103 & E' &= \frac{E}{A} = 0.4466 & F' &= \frac{F}{A} = -0.8581 \\
G' &= \frac{G}{A} = 0.0019 & H' &= \frac{H}{A} = -0.0057 & I' &= \frac{I}{A} = 0.6908 & J' &= \frac{J}{A} = -1.4241
\end{align*}
\]

for the first set of points and

\[
\begin{align*}
C' &= \frac{C}{A} = 3.8932 & D' &= \frac{D}{A} = 0.0133 & E' &= \frac{E}{A} = 1.7681 & F' &= \frac{F}{A} = -2.2186 \\
G' &= \frac{G}{A} = 0.0027 & H' &= \frac{H}{A} = -0.0050 & I' &= \frac{I}{A} = 0.5421 & J' &= \frac{J}{A} = -1.4364
\end{align*}
\]

for the second set of points.
Using Eq. (3.32a) the value of k is computed as 0.8666 and 0.3392 for the first and the second sets of points, respectively. And then according Eq. (3.32b) and using the value of k the new numerical values are computed as

\[
\begin{align*}
A &= k = 0.9806, & B &= kB' = 0.9580, & C &= kC' = 0.0614, & D &= kD' = 0.0101 \\
E &= kE' = 0.4379, & F &= kF' = -0.8415, & G &= kG' = 0.0019, & H &= kH' = -0.0027 \\
I &= kl' = -0.0056, & J &= kJ' = -1.3964
\end{align*}
\]

for the first set of points and

\[
\begin{align*}
A &= k = 0.3392, & B &= kB' = 0.3400, & C &= kC' = 1.3207, & D &= kD' = 0.0045 \\
E &= kE' = 0.5998, & F &= kF' = -0.7526, & G &= kG' = 0.0009, & H &= kH' = -0.0009 \\
I &= kl' = 0.1839, & J &= kJ' = -0.4873
\end{align*}
\]

for the second set of points.

The above numerical results are checked based on the criteria given in Eq. (3.32c), Eq. (3.32d) and Eq. (3.32e). Both Eq. (3.32c) and Eq. (3.32d) were not satisfied the condition, so the next step is to use Eq. (3.32c) to find the direction cosines. The direction cosines \((a', b', c')\) for the first and second sets of points are computed as \((-3.4950 \ 0.4343 \ 0.9688)\) and \((0.8129 - 0.0028 - 0.3689)\), respectively. These values are normalized in order to get the direction cosines \((a, b, c)\) and obtained as \((-0.1983 \ 0.2843 \ 1.9550)\) and \((0.0561 \ 0.0090 \ 0.6208)\) for the first and second sets of points, respectively.

The next step is computing the coordinates of a point on the axis of the cylinder. Using Eq. (3.34) \(\hat{\mathbf{x}}_2\) is computed as \([0.0979 \ 0.0408 - 0.3334]^{\mathrm{T}}\) and \([0.1196 \ 0.0006 - 0.0855]^{\mathrm{T}}\) m for the first and second sets of points, respectively.

Finally by using the definition of J in Eq. (3.27) the value of r is computed as 1.1092 m and 1.1282 m for the first and second sets of points, respectively.

Once the parameters are estimated from the fitted cylindrical surface the next stage is to detect the change between them. From least squares solution of \(\hat{\mathbf{x}}_2\) only x and y coordinates are taken for the change detection analysis since they belong to parameters of the cylinder. Accordingly the covariance matrix of the parameters are split into two, the first part containing a covariance matrix of the plane coordinates of the axis of the cylinder whereas the second part containing the variance of the radius of the cylinder. So the variance covariance matrix of the parameters can be expressed as
where \( \mathbf{C}_{\text{co}} \) is the variance covariance matrix of the plane coordinates of the axis of the cylinder and \( \sigma_r^2 \) is the variance of the radius of the cylinder.

Hence \( \mathbf{X}_{2(1)} \) and \( \mathbf{X}_{2(2)} \) are the estimated parameters of the cylinder containing plane coordinates of the axis of the cylinder for the first and second sets points, respectively, obtained as

\[
\mathbf{X}_{2(1)} = [0.0979, 0.0408]^T \\
\text{and} \\
\mathbf{X}_{2(2)} = [0.1196, 0.0006]^T
\]

the difference between the parameters

\[
\mathbf{d}_\mathbf{X} = \mathbf{X}_{2(1)} - \mathbf{X}_{2(2)} = [0.0217, -0.0402]^T
\]

is the measure of the translation of the axis of the cylinder. Its norm computed as

\[
\|\mathbf{d}_\mathbf{X}\| = \sqrt{\mathbf{d}_{XX}^2 + \mathbf{d}_{XY}^2} = 45.7 \text{ mm}
\]

The change in radii of the cylinder between the first and second sets of points were computed as

\[
d_r = r_2 - r_1 = 19 \text{ mm}
\]

where \( r_1 = 1.1092 \) m and \( r_2 = 1.1282 \) m.

The covariance matrix of \( \mathbf{d}_\mathbf{X} \) can be obtained as

\[
\mathbf{C}_{\mathbf{d}_\mathbf{X}} = \mathbf{C}_{\mathbf{X}_{2(1)}} + \mathbf{C}_{\mathbf{X}_{2(2)}} = 10^{-3} \begin{bmatrix}
0.4405 & -0.1996 \\
-0.1996 & 0.1148
\end{bmatrix}
\]

where

\[
\mathbf{C}_{\mathbf{X}_{2(1)}} = 10^{-3} \begin{bmatrix}
0.4000 & -0.2000 \\
-0.2000 & 0.1000
\end{bmatrix}
\text{ and }
\mathbf{C}_{\mathbf{X}_{2(2)}} = 10^{-4} \begin{bmatrix}
0.4050 & 0.0040 \\
0.0040 & 0.1480
\end{bmatrix}
\]

\( \mathbf{C}_{\mathbf{X}_{2(1)}} \) and \( \mathbf{C}_{\mathbf{X}_{2(2)}} \) are the propagated variance covariance matrices of the plane coordinates of the axis of the cylinder for the first and second sets of points, respectively, which are similar to \( \mathbf{C}_{\text{co}} \) in Eq. (3.35).

The standard error of \( \|\mathbf{d}_\mathbf{X}\|, \sigma_{\|\mathbf{d}_\mathbf{X}\|} \) is a function of \( \mathbf{d}_{XX} \) and \( \mathbf{d}_{XY} \), and similar principle like the planar surface case in the above is applied to compute \( \sigma_{\|\mathbf{d}_\mathbf{X}\|} \). The law of error propagation for the non-linear function in Eq. (3.37) expressed as
\[ \varepsilon \|d_X\| \approx \frac{\partial \|d_X\|}{\partial d_{xx}} \varepsilon_{dxx} + \frac{\partial \|d_X\|}{\partial d_{xy}} \varepsilon_{dxy} = A \varepsilon \] (3.40)

where \( \varepsilon = \begin{bmatrix} \varepsilon_{dxx} & \varepsilon_{dxy} \end{bmatrix}^T \) is the vector of residuals containing residuals corresponding to each parameter and \( A = \begin{bmatrix} a_{xx} & a_{xy} \end{bmatrix} \) is the matrix containing the partial derivatives of the non-linear function \( \|d_X\| = \sqrt{d_{xx}^2 + d_{xy}^2} \), computed as

\[ a_{xx} = \frac{\partial \|d_X\|}{\partial d_{xx}} \quad a_{xy} = \frac{\partial \|d_X\|}{\partial d_{xy}} \] (3.41)

The variance of \( \|d_X\| \) can be obtained from

\[ \sigma^2_{\|d_X\|} = E\left\{\left[\|d_X\| - E(\|d_X\|)\right]^2\right\} = AC_{d_X}A^T \] (3.42)

where \( C_{d_X} \) is the variance covariance matrix of the difference of the parameters, and \( \sigma^2_{\|d_X\|} \) is the variance of \( \|d_X\| \) and the value of \( \sigma_{\|d_X\|} \) is obtained as

\[ \sigma^2_{\|d_X\|} = AC_{d_X}A^T = 355 \text{ mm}^2 \quad \text{where} \quad A = \begin{bmatrix} 0.4750 & -0.8800 \end{bmatrix} \]

\[ \text{and} \quad \sigma_{\|d_X\|} = 18.85 \text{ mm} \]

The significance of the change in radius is also checked as follows. The variance of \( d_r \) can be computed as

\[ \sigma^2_{d_r} = \sigma^2_{r_1} + \sigma^2_{r_2} = 1.21 \times 10^{-4} \] (3.43)

where

\[ \sigma^2_{r_1} = 10^{-4} \quad \text{and} \quad \sigma^2_{r_2} = 2.13 \times 10^{-5} \]

and therefore

\[ \sigma_r = 11.01 \text{ mm} \quad \text{and} \quad 3\sigma_r = 33.03 \text{ mm} \]

\( \sigma^2_{r_1} \) and \( \sigma^2_{r_2} \) are the variances of the radius of the cylinder for the first and second sets of points, respectively and \( \sigma_r \) is the standard error of \( d_r \).

The standard error \( \sigma_{\|d_X\|} \) is obtained as 18.85 mm and a coverage factor 3 is chosen from the normal distribution table since we assumed that the errors are normally distributed. Then the uncertainty limit become \( 3\sigma_{\|d_X\|} = 56.55 \text{ mm} \). So based on this statistical test no significant changes are detected neither in translation of the axis of the cylinder nor in change in radius between sets of points 1 and 2.
4- Practical Test and Data analysis

4.1- Direct Comparison of Observed points

The direct comparison of observed points is used in the case of planar surface experiment. For detail analysis of a planar surface the left and right segments of the carton box were taken as shown in Figure 4.1, these parts of the box are more visible from the position of the scanner and it is expected that change can be easily detected from these surfaces of different scan epochs.

![Segmented point clouds of a carton for planar surface analysis.](image)

**Figure 4.1** a) Image of the carton box of the first scan as photographed by the internal camera of the laser scanner b) Segmented point clouds of a carton for planar surface analysis.

Since the position of the scanner was kept on the same place in both scan epochs, direct comparison of points was performed by differencing the ranges between scan epochs of points which have the same $\beta$ (horizontal direction) and $\zeta$ (vertical angle). For this analysis 20 points were marked on the surface of the carton in order to easily identify them in both scan epochs. The range difference indicates the movements of each marked point between scan epochs. And the computed range difference of the points is presented in Table 4.1. Those points also used for independent measurement comparison purpose in section 4.3. The significance of the range differences of points between scan epochs is checked as presented below.

The range difference of each point can be computed as

$$d_r = r_2 - r_1$$  \hspace{1cm} (4.1)

where $r_1$ and $r_2$ are ranges of points in epochs 1 and 2, respectively and $d_r$ is the range difference between epochs.

The standard uncertainty of the range difference can be obtained as

$$\sigma_{dr}^2 = \sigma_{r_1}^2 + \sigma_{r_2}^2$$  \hspace{1cm} (4.2)

where $\sigma_{r_1} = \sigma_{r_2} = 4$ mm, which are standard uncertainties corresponding to measured ranges in epochs 1 and 2, which is taken from the technical specification of the instrument.

Then $\sigma_{dr}$ is computed as
\[ \sigma_{dr} = \sqrt{32} = 5.7 \text{ mm} \]

Since \(3\sigma_{dr} = 17.1 \text{ mm} \) the range differences between points which were computed and presented in Table 4.1 are not significant in this case. So a movement is not detected using direct comparison of observed points. But there is a deformation in the actual case, which is detected by using other method using the same experimental data as presented in section 4.3.

Table 4.1 Computed range differences of marked points on the surface of the carton of segment 1

<table>
<thead>
<tr>
<th>Point ID</th>
<th>Range ( (r_1) ) m</th>
<th>Range ( (r_2) ) m</th>
<th>( dr = r_2 - r_1 ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A101</td>
<td>5.414</td>
<td>5.409</td>
<td>5.300</td>
</tr>
<tr>
<td>A102</td>
<td>5.368</td>
<td>5.364</td>
<td>3.300</td>
</tr>
<tr>
<td>A103</td>
<td>5.319</td>
<td>5.320</td>
<td>-0.900</td>
</tr>
<tr>
<td>A104</td>
<td>5.261</td>
<td>5.267</td>
<td>-5.300</td>
</tr>
<tr>
<td>A105</td>
<td>5.392</td>
<td>5.386</td>
<td>6.100</td>
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</tr>
<tr>
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<td>-3.300</td>
</tr>
<tr>
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<td>5.243</td>
<td>0.600</td>
</tr>
<tr>
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<td>5.370</td>
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</tr>
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</tr>
<tr>
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<td>5.273</td>
<td>3.200</td>
</tr>
<tr>
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<td>5.229</td>
<td>5.226</td>
<td>3.100</td>
</tr>
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<td>5.367</td>
<td>0.200</td>
</tr>
<tr>
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</tr>
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<td>5.255</td>
<td>0.300</td>
</tr>
<tr>
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<td>5.209</td>
<td>2.500</td>
</tr>
<tr>
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<td>5.357</td>
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</tr>
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<td>5.289</td>
<td>-2.600</td>
</tr>
<tr>
<td>A119</td>
<td>5.248</td>
<td>5.244</td>
<td>4.000</td>
</tr>
<tr>
<td>A120</td>
<td>5.200</td>
<td>5.194</td>
<td>6.100</td>
</tr>
<tr>
<td>Average Value</td>
<td></td>
<td></td>
<td>0.295</td>
</tr>
</tbody>
</table>

4.2- Detection of Movement with Normal Vector

The normal vector movement detection is mainly used for deformation analysis of planar surfaces but not for other surface types. The visualization of the movements of a normal of each triangular surface (face normal) is shown in Figure 4.2 for illustration purpose, but this doesn’t represent the normal of the whole planar surface. A Matlab code was developed using delaunay triangulation function to generate face normal of each triangular surfaces. The normal vectors of the selected segments of the two scan epochs were obtained from patching of their point clouds. And the angle between the two unit normal vectors also computed in order to check the movements between scan epochs. If a change in direction is obtained between the two vectors (angle between normal vectors) that shows the presence of movement between scan epochs.
Figure 4.2 Visualization of the movement of the normal vectors of planar surfaces on both scan epochs (Face normal of each triangular surface), (a) segment 1 of 1st scan epoch and, (b) segment 1 of second scan epoch (c) plot of (a) and (b)

In the graphical visualization of Figure 4.2 above, (a) shows the normal of each triangular face of the plane of scan epoch 1 and similarly Figure (b) shows the normal of each triangular face of a plane of scan epoch 2. Figure (c) shows the plot of (a) and (b) together. As we can see from Figure (c) the blue and red arrows are not in the same position as well as orientation and this can used as an indicator for the presence of movement between scan epochs.

But in here the movement of the normal vectors between scan epochs for the whole planar surface is computed. As the result presented in Table 4.2 the normal vectors of segment 1 for the 1\textsuperscript{st} and 2\textsuperscript{nd} scan epochs are obtained as $\mathbf{n}_1 = [0.0256 - 0.2202 - 0.0005]$ and is $\mathbf{n}_2 = [0.0253 - 0.2207 - 0.0007]$. And the angle between them can be computed by using vectors dot product as

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = ||\mathbf{n}_1|| ||\mathbf{n}_2|| \cos \theta$$  \hspace{1cm} (4.3)
where \( \mathbf{n}_1 \) and \( \mathbf{n}_2 \) are normal vectors of the plane in epochs 1 and 2 and \( \| \mathbf{n}_1 \| \) and \( \| \mathbf{n}_2 \| \) are their respective norms.

Then the angle between the normal vectors can be computed as

\[
\theta = \cos^{-1} \left( \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\| \mathbf{n}_1 \| \| \mathbf{n}_2 \|} \right)
\]

(4.4)

The computed angle between the two normal vectors obtained as \( \theta = 0^\circ.32 \), which shows the change in orientation of the normal of the plane between scan epochs but a statistical test is required in order to check the significance of this change.

From the covariance matrix of plane fitting process \( \sigma_{d\theta} \) obtained as \( 0^\circ.023 \). If we select a coverage factor of 3 from the normal distribution table which corresponds to a confidence level of 99.7 % the uncertainty limit become \( 3\sigma_{d\theta} = 0^\circ.07 \). So the change in orientation of the normal of the plane is significant and it is rotated by \( 0^\circ.32 \) (19.38').

4.3- Estimation of Deformation from Surfaces

Planar Surface

A modeled planar surface shown in Figure 4.3 is used to estimate the surface parameters of both scan epochs. The parameters of these surfaces were computed through plane fitting using fit to cloud method, which is a function in Cyclone software and also a Matlab code was developed for this purpose. Both fitting methods are working based on the concept of least squares fitting. The residuals and covariance of the parameters are obtained from the Matlab results. The differences of the parameters of both surfaces from the two scan epochs also computed to find the magnitude of the change between epochs. Since the same results are obtained from both Cyclone and Matlab only Matlab results are presented in Table 4.2 and 4.3 for scanned points and Table 4.4 and 4.5 for surveyed points.

**Figure 4.3** Fitted planar surface of segment 1  (a) First scan epoch  (b) Second scan epoch
Table 4.2 Estimated parameters and their standard errors of a planar surface of scanned points.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>First scan epoch (segment 1)</th>
<th>Second scan epoch (segment 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Values of Parameters</td>
<td>Standard error of Parameters</td>
</tr>
<tr>
<td>A</td>
<td>0.0256</td>
<td>0.00014</td>
</tr>
<tr>
<td>B</td>
<td>-0.2202</td>
<td>0.00011</td>
</tr>
<tr>
<td>C</td>
<td>-0.0005</td>
<td>0.00013</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>First scan epoch (segment 2)</th>
<th>Second scan epoch (segment 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Values of Parameters</td>
<td>Standard error of Parameters</td>
</tr>
<tr>
<td>A</td>
<td>0.4095</td>
<td>0.00058</td>
</tr>
<tr>
<td>B</td>
<td>0.0439</td>
<td>0.00037</td>
</tr>
<tr>
<td>C</td>
<td>-0.0015</td>
<td>0.00030</td>
</tr>
</tbody>
</table>

Table 4.3 Fit quality of the fitted planar surface and the point location of the normal of scanned points.

<table>
<thead>
<tr>
<th>Point of the normal</th>
<th>First scan epoch (segment 1) (-2.909, 4.200, 0.970) m</th>
<th>Second scan epoch (segment 1) (-2.910, 4.201, 1.106) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit quality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error mean (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Error Std Deviation (m)</td>
<td>0.002</td>
<td>0.003</td>
</tr>
<tr>
<td>Abs Error (m)</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>Max Abs error (m)</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>Number of points</td>
<td>864</td>
<td>864</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point of the normal</th>
<th>First scan epoch (segment 2) (-2.888, 4.201, 1.183) m</th>
<th>Second scan epoch (segment 2) (-2.890, 4.206, 1.252) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit quality</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error mean (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Error Std Deviation (m)</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Abs Error (m)</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Max Abs error (m)</td>
<td>0.013</td>
<td>0.012</td>
</tr>
<tr>
<td>Number of points</td>
<td>864</td>
<td>864</td>
</tr>
</tbody>
</table>

The descriptions in the first column of Table 4.3 and other similar tables are fit quality parameters. The definitions of these parameters were not found in Cyclone user manual and the following definition is given based on the meaning of statistical terminologies.

Error Mean – the mean value of the residuals, which can be computed as

\[ \varepsilon_m = \frac{\sum_{i=1}^{n} \varepsilon_i}{n} \quad \text{where: } i = 1, 2 \ldots n \]

Error Standard Deviation – is the standard deviation can be obtained as

\[ \sigma = \sqrt{\frac{\sum_{i=1}^{n} (r_i - r)^2}{n - 1}} \]
where \( i = 1, 2 \ldots n, r_i \) is the range of each point and \( r \) is the mean of ranges of all points

Mean Absolute Error – the mean of the absolute values of the residuals

Maximum Absolute Error – maximum of the absolute values of the residuals

Table 4.4 Estimated parameters and their standard errors of a planar surface of surveyed points

<table>
<thead>
<tr>
<th>Parameters</th>
<th>First scan epoch (segment 1)</th>
<th>Second scan epoch (segment 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.2209 0.00038</td>
<td>-0.2206 0.00038</td>
</tr>
<tr>
<td>b</td>
<td>0.0245 0.00029</td>
<td>0.0251 0.00010</td>
</tr>
<tr>
<td>c</td>
<td>-0.0015 0.00032</td>
<td>-0.0003 0.00042</td>
</tr>
</tbody>
</table>

Table 4.5 Fit quality of the fitted planar surface and the point location of the normal of surveyed points

<table>
<thead>
<tr>
<th>Point of the normal</th>
<th>First scan epoch (segment 1)</th>
<th>Second scan epoch (segment 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3.067, 4.179, 1.158) m</td>
<td>(-3.067, 4.183, 1.155) m</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fit quality</th>
<th>First scan epoch (segment 1)</th>
<th>Second scan epoch (segment 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error mean (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Error Std (m)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Abs Error (m)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Max Abs error (m)</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Number of point clouds</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point of the normal</th>
<th>First scan epoch (segment 2)</th>
<th>Second scan epoch (segment 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2.917, 4.478, 1.159) m</td>
<td>(-2.897, 4.285, 1.155) m</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fit quality</th>
<th>First scan epoch (segment 2)</th>
<th>Second scan epoch (segment 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error mean (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Error Std (m)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Abs Error (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Max Abs error (m)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Number of point clouds</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

In order to make a comparison with the laser measurements an independent measurement of marked points on the surface of a carton was performed using a total station. In this measurement the two faces of the carton were gridded with a size of 10 x 10 cm. Twenty points were marked on both faces of the carton and three fixed target points also assigned externally to use for registration process of the scanned points later. The target points are included in each scan epoch and at the same time their coordinates also measured by total station. Finally using these fixed targets as a control point registration of cloud points has been performed in order to transform scanned and surveyed point coordinates to the same local coordinate system. From the total station measurements coordinate differences between scan epochs of all marked points of segment 1 were computed and presented in Table 4.6, 4.7 and 4.8. The data in Table 4.8 shows
the differences of coordinate differences of scanned and surveyed points and a maximum of 8 mm coordinate difference is obtained between the scanner and the total station measurement. Additionally the ranges of the marked points from scanner and total station measurements also computed and compared as shown in Table 4.9. Range differences of marked points between scan epochs are computed for both the scanned and the surveyed points. The last column of Table 4.9 shows a maximum of 8.7 mm difference of range difference.

The significance of the coordinate differences of the scanner and total station measurements is checked as follows. The covariance of the coordinate differences can be obtained as

\[
\mathbf{C} = \mathbf{C}_{\text{SC}} + \mathbf{C}_{\text{TS}} = \begin{bmatrix}
8.3800 & -6.9112 & 0.3282 \\
-6.9112 & 9.6150 & -3.0690 \\
0.3282 & -3.0690 & 2.3178
\end{bmatrix} \text{mm}^2
\]  

(4.5)

where

\[
\mathbf{C}_{\text{SC}} = \begin{bmatrix}
5.7535 & -7.2932 & 2.1670 \\
-7.2932 & 9.5189 & -2.7974 \\
2.1670 & -2.7974 & 0.9411
\end{bmatrix} \text{mm}^2
\]

\[
\mathbf{C}_{\text{TS}} = \begin{bmatrix}
2.6265 & 0.3821 & -1.8388 \\
0.3821 & 0.0961 & -0.2716 \\
-1.8388 & -0.2716 & 1.3767
\end{bmatrix} \text{mm}^2
\]

\(\mathbf{C}_{\text{SC}}\) and \(\mathbf{C}_{\text{TS}}\) are the precision of a 3D point from laser scanner and total station measurements, respectively, computed by using Eq. (5.2) and Eq. (5.3). The square root of the diagonal elements of the covariance matrix \(\mathbf{C}\) gives the precision of the coordinate differences and obtained as

\[
\sigma_x = 2.9 \text{ mm} \quad \sigma_y = 3.1 \text{ mm} \quad \sigma_z = 1.5 \text{ mm}
\]

A coverage factor 3 is chosen from the normal distribution table and the uncertainty limit become \(3\sigma_x = 8.7 \text{ mm}, \quad \sigma_y = 9.3 \text{ mm}\) and \(\sigma_z = 4.5 \text{ mm}\).

So as the results are presented in Table 4.8 significant differences were obtained only in \(z\) coordinate differences between the laser scanner and total station measurements. A maximum of 8 mm coordinate difference was obtained for the point ID A113.

<p>| Table 4.6 coordinate differences of scanned marked points (Segment 1). |
|--------------------------|--------------------------|--------------------------|--------------------------|
| First Scan Epoch | Second Scan Epoch | Difference |
| Point ID | x | y | z | Point ID | x | y | z | dx | dy | dz |
| A101 | 4.162 | -3.265 | 1.154 | C101 | 4.159 | -3.259 | 1.157 | 0.003 | -0.006 | -0.003 |
| A102 | 4.173 | -3.171 | 1.158 | C102 | 4.171 | -3.167 | 1.161 | 0.002 | -0.004 | -0.003 |
| A103 | 4.181 | -3.076 | 1.162 | C103 | 4.182 | -3.075 | 1.165 | -0.001 | -0.001 | -0.003 |
| A104 | 4.191 | -2.964 | 1.154 | C104 | 4.195 | -2.967 | 1.156 | -0.004 | 0.003 | -0.002 |
| A105 | 4.161 | -3.262 | 1.057 | C105 | 4.157 | -3.256 | 1.060 | 0.004 | -0.006 | -0.003 |
| A106 | 4.171 | -3.167 | 1.062 | C106 | 4.172 | -3.167 | 1.065 | -0.001 | 0.000 | -0.003 |</p>
<table>
<thead>
<tr>
<th>Point ID</th>
<th>First Scan Epoch</th>
<th>Second Scan Epoch</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>A101</td>
<td>4.158</td>
<td>-3.267</td>
<td>1.160</td>
</tr>
<tr>
<td>A102</td>
<td>4.168</td>
<td>-3.166</td>
<td>1.157</td>
</tr>
<tr>
<td>A103</td>
<td>4.179</td>
<td>-3.067</td>
<td>1.158</td>
</tr>
<tr>
<td>A104</td>
<td>4.192</td>
<td>-2.969</td>
<td>1.157</td>
</tr>
<tr>
<td>A105</td>
<td>4.158</td>
<td>-3.267</td>
<td>1.060</td>
</tr>
<tr>
<td>A106</td>
<td>4.168</td>
<td>-3.166</td>
<td>1.059</td>
</tr>
<tr>
<td>A107</td>
<td>4.179</td>
<td>-3.067</td>
<td>1.059</td>
</tr>
<tr>
<td>A108</td>
<td>4.191</td>
<td>-2.970</td>
<td>1.060</td>
</tr>
<tr>
<td>A109</td>
<td>4.158</td>
<td>-3.267</td>
<td>0.960</td>
</tr>
<tr>
<td>A110</td>
<td>4.169</td>
<td>-3.165</td>
<td>0.860</td>
</tr>
<tr>
<td>A111</td>
<td>4.180</td>
<td>-3.069</td>
<td>0.860</td>
</tr>
<tr>
<td>A112</td>
<td>4.192</td>
<td>-2.971</td>
<td>0.860</td>
</tr>
<tr>
<td>A113</td>
<td>4.161</td>
<td>-3.270</td>
<td>0.758</td>
</tr>
<tr>
<td>A114</td>
<td>4.171</td>
<td>-3.167</td>
<td>0.759</td>
</tr>
<tr>
<td>A115</td>
<td>4.182</td>
<td>-3.071</td>
<td>0.759</td>
</tr>
<tr>
<td>A116</td>
<td>4.193</td>
<td>-2.971</td>
<td>0.759</td>
</tr>
</tbody>
</table>

Table 4.7 Coordinate difference of marked points surveyed by Total station (Segment 1).

<table>
<thead>
<tr>
<th>Point ID</th>
<th>Difference of dx</th>
<th>Difference of dy</th>
<th>Difference of dz</th>
</tr>
</thead>
<tbody>
<tr>
<td>A101</td>
<td>0.007</td>
<td>-0.006</td>
<td>-0.006</td>
</tr>
<tr>
<td>A102</td>
<td>0.007</td>
<td>-0.005</td>
<td>-0.005</td>
</tr>
<tr>
<td>A103</td>
<td>0.004</td>
<td>-0.001</td>
<td>-0.006</td>
</tr>
<tr>
<td>A104</td>
<td>-0.001</td>
<td>0.004</td>
<td>-0.005</td>
</tr>
<tr>
<td>A105</td>
<td>0.006</td>
<td>-0.004</td>
<td>-0.006</td>
</tr>
<tr>
<td>A106</td>
<td>0.002</td>
<td>0.001</td>
<td>-0.006</td>
</tr>
<tr>
<td>A107</td>
<td>0.001</td>
<td>0.003</td>
<td>-0.005</td>
</tr>
<tr>
<td>A108</td>
<td>0.003</td>
<td>-0.001</td>
<td>-0.006</td>
</tr>
<tr>
<td>A109</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.005</td>
</tr>
<tr>
<td>A110</td>
<td>0.000</td>
<td>0.001</td>
<td>-0.005</td>
</tr>
<tr>
<td>A111</td>
<td>0.004</td>
<td>-0.004</td>
<td>-0.007</td>
</tr>
</tbody>
</table>

Table 4.8 Difference between coordinate differences measured by total station and laser scanner.
<table>
<thead>
<tr>
<th>Point ID</th>
<th>Scanned Points</th>
<th>Surveyed Points</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Range $r_1$ mm</td>
<td>Range $r_2$ mm</td>
<td>$dr_1=r_1-r_2$ mm</td>
</tr>
<tr>
<td>A101</td>
<td>5414.3</td>
<td>5409.0</td>
<td>5.3</td>
</tr>
<tr>
<td>A102</td>
<td>5367.5</td>
<td>5364.2</td>
<td>3.3</td>
</tr>
<tr>
<td>A103</td>
<td>5319.1</td>
<td>5320.0</td>
<td>-0.9</td>
</tr>
<tr>
<td>A104</td>
<td>5261.3</td>
<td>5266.6</td>
<td>-5.3</td>
</tr>
<tr>
<td>A105</td>
<td>5391.8</td>
<td>5385.7</td>
<td>6.1</td>
</tr>
<tr>
<td>A106</td>
<td>5343.7</td>
<td>5345.1</td>
<td>-1.4</td>
</tr>
<tr>
<td>A107</td>
<td>5291.5</td>
<td>5294.8</td>
<td>-3.3</td>
</tr>
<tr>
<td>A108</td>
<td>5243.7</td>
<td>5243.1</td>
<td>0.6</td>
</tr>
<tr>
<td>A109</td>
<td>5369.5</td>
<td>5370.2</td>
<td>-0.7</td>
</tr>
<tr>
<td>A110</td>
<td>5320.9</td>
<td>5323.0</td>
<td>-2.1</td>
</tr>
<tr>
<td>A111</td>
<td>5276.0</td>
<td>5272.8</td>
<td>3.2</td>
</tr>
<tr>
<td>A112</td>
<td>5228.8</td>
<td>5225.7</td>
<td>3.1</td>
</tr>
<tr>
<td>A113</td>
<td>5366.7</td>
<td>5366.5</td>
<td>0.2</td>
</tr>
<tr>
<td>A114</td>
<td>5300.8</td>
<td>5306.7</td>
<td>-5.9</td>
</tr>
<tr>
<td>A115</td>
<td>5255.2</td>
<td>5254.9</td>
<td>0.3</td>
</tr>
<tr>
<td>A116</td>
<td>5211.0</td>
<td>5208.5</td>
<td>2.5</td>
</tr>
<tr>
<td>A117</td>
<td>5350.7</td>
<td>5357.3</td>
<td>-6.6</td>
</tr>
<tr>
<td>A118</td>
<td>5286.7</td>
<td>5289.3</td>
<td>-2.6</td>
</tr>
<tr>
<td>A119</td>
<td>5247.5</td>
<td>5243.5</td>
<td>4.0</td>
</tr>
<tr>
<td>A120</td>
<td>5199.7</td>
<td>5193.6</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 4.9 Range differences of marked points measured by Laser scanner and Total station.

Change detection using the laser data:

The estimated parameters from each fitted planar surface were used to detect the change between scan epochs. In the following computation $\hat{n}_1$ and $\hat{n}_2$ are the estimated unit normal vectors of the plane in epochs 1 and 2, respectively

$$
\hat{n}_1 = \frac{\hat{X}_1}{\|\hat{X}_1\|} = [0.1156 \ -0.9933 \ -0.0022]^T
$$

and

$$
\hat{n}_2 = \frac{\hat{X}_2}{\|\hat{X}_2\|} = [0.1139 \ -0.9935 \ 0.0031]^T
$$

34
and their respective standard errors are obtained as \( \sigma_1 = 0.00056 \) rad and \( \sigma_2 = 0.00064 \) rad.

The difference between normal vectors

\[
d_{\vec{n}} = \vec{n}_1 - \vec{n}_2 = [0.0017 \ 0.0002 \ -0.0053]^T
\]  

(4.6)

is a measure of the rotation of the plane. Its norm computed as

\[
\|d_{\vec{n}}\| = \sqrt{d_{n_x}^2 + d_{n_y}^2 + d_{n_z}^2} = 0.0056
\]  

(4.7)

is the angle between the two normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \) and according to Eq. 3.14 \( \alpha \approx \|d_{\vec{n}}\| = 0.0056 \) rad which is the rotational angle of the normal of the plane.

The covariance matrix of \( d_{\vec{n}} \) can be obtained as

\[
\mathbf{C}_{d_{\vec{n}}} = \mathbf{C}_{\vec{n}_1} + \mathbf{C}_{\vec{n}_2} = 10^{-6}
\begin{bmatrix}
0.1987 & 0.1491 & -0.0028 \\
0.1491 & 0.1230 & -0.0475 \\
-0.0028 & -0.0475 & 0.1887
\end{bmatrix}
\]  

(4.8)

where

\[
\mathbf{C}_{\vec{n}_1} = \left( \frac{1}{\|\vec{x}_1\|^2} \right) \mathbf{C}_{\vec{x}_1} = 10^{-7}
\begin{bmatrix}
0.8634 & 0.6482 & -0.0135 \\
0.6482 & 0.5350 & -0.2066 \\
-0.0135 & -0.2066 & 0.8178
\end{bmatrix}
\]  

(4.9)

\[
\mathbf{C}_{\vec{n}_2} = \left( \frac{1}{\|\vec{x}_2\|^2} \right) \mathbf{C}_{\vec{x}_2} = 10^{-6}
\begin{bmatrix}
0.1124 & 0.0843 & -0.0015 \\
0.0843 & 0.0695 & -0.0269 \\
-0.0015 & -0.0269 & 0.1070
\end{bmatrix}
\]  

(4.10)

\( \mathbf{C}_{\vec{x}_1} \) and \( \mathbf{C}_{\vec{x}_2} \) are the variance covariance matrices of estimated parameters of the first and the second scan epochs and \( \mathbf{C}_{\vec{n}_1} \) and \( \mathbf{C}_{\vec{n}_2} \) are matrices of unit normal vectors \( \vec{n}_1 \) and \( \vec{n}_2 \), respectively.

The standard error of \( \|d_{\vec{n}}\| \), \( \sigma_{\|d_{\vec{n}}\|} \) is computed based on the procedure presented in section 3.1.2 of the planar model part. So the variance of \( \|d_{\vec{n}}\| \) can be obtained from

\[
\sigma_{\|d_{\vec{n}}\|}^2 = E\{\|d_{\vec{n}}\| - E(\|d_{\vec{n}}\|))^2\} = \mathbf{A}\mathbf{C}_{d_{\vec{n}}}\mathbf{A}^T
\]  

(4.11)

where \( \mathbf{C}_{d_{\vec{n}}} \) is variance-covariance matrix of the difference vector.

Therefore the value of \( \sigma_{\|d_{\vec{n}}\|} \) obtained as

\[
\sigma_{\|d_{\vec{n}}\|}^2 = \mathbf{A}\mathbf{C}_{d_{\vec{n}}}\mathbf{A}^T = 0.0000002 \text{ rad}^2
\]  

where \( \mathbf{A} = [0.3052 \ 0.0359 \ -0.9516] \)

and \( \sigma_{\|d_{\vec{n}}\|} = 0.0004 \) rad
A coverage factor 3 is chosen from the normal distribution table in order to determine the confidence interval. This correspond to a confidence level of 99.7 % with a risk level of 0.3 %. The standard error of ∥d∥ is obtained as σ∥d∥ = 0.0004 rad and the limit of the uncertainty become 3σ∥d∥ = 0.0012 rad, so a significant change is detected between scan epochs, which is ∥d∥ = 0.0056 rad. Hence the normal of the plane is rotated by 19.25′ between the scan epochs.

Change detection using total station data:

Similar procedure with the above is used to detect the change between epochs using the surveyed data of the plane. The unit normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ of epochs 1 and 2 obtained as

$$\mathbf{n}_1 = [-0.9939 \ 0.1101 \ -0.0066]^T \quad \text{and} \quad \mathbf{n}_2 = [-0.9936 \ 0.1132 \ -0.0015]^T$$

and their respective standard errors are computed as $\sigma_1 = 0.00021$ rad and $\sigma_2 = 0.00026$ rad.

The difference between normal vectors computed as

$$d_\mathbf{n} = [-0.0003 \ -0.0031 \ -0.0051]^T$$

and its norm computed as

$$\|d_\mathbf{n}\| = \sqrt{d_{nx}^2 + d_{ny}^2 + d_{nz}^2} = 0.0058$$

is the angle between the two normal vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ and according to Eq. 3.14 $\alpha \approx \|d_\mathbf{n}\| = 0.0058 \text{ rad}(19.94')$ which is the rotational angle of the normal of the plane.

Based on Eq. (4.9), Eq. (4.10) and Eq. (4.11) the covariance matrix of $d_\mathbf{n}$ is obtained as

$$\text{C}_{d_\mathbf{n}} = 10^{-5} \begin{bmatrix} 0.1026 & 0.1290 & -0.0268 \\ 0.1290 & 0.1737 & 0.0026 \\ -0.0268 & 0.0026 & 0.1255 \end{bmatrix}$$

Using Eq. (4.14) the variance $\sigma^2_{\|d_\mathbf{n}\|}$ obtained as 0.0000015 rad$^2$ and then $\sigma_{\|d_\mathbf{n}\|}$ computed as 0.0012 rad.

The standard error of $\|d_\mathbf{n}\|$ is obtained as $\sigma_{\|d_\mathbf{n}\|} = 0.0012$ rad and the limit of the uncertainty become $3\sigma_{\|d_\mathbf{n}\|} = 0.0036$ rad, so a significant change is detected between scan epochs, which is $\|d_\mathbf{n}\| = 0.0056 \text{ rad}(19.94')$. Hence the normal of the plane is rotated by 19.94′ between scan epochs. As we have seen both results almost the same change in rotation has been detected from laser and surveyed data.

**Spherical Surface**

For spherical surface analysis a scan of a ball was performed as shown in Figure 4.4. An independent measurement for the size of the ball also taken before and after the change had been
made on it. For an independent measurement of the sphere a measuring tape and a string were used in conjunction. First the string is rounded on the ball on the approximate center to measure the circumference of the ball and then a measuring tape is used to measure the string, the result is recorded as the first measurement. This procedure is repeated for ten times and from each measurement diameter of the ball is calculated corresponding to each measured circumference. Finally the average diameters were obtained as 207 and 202 milimeters for the first and second scan epochs, respectively. Comparison was made with the radius measured by laser scanner as presented in Table 4.9. In spherical surface case it is not possible to find a unit normal vector for the whole surface but in order to visualize the movement between epoch scans the face normal of each triangular surfaces were computed as shown in Figure 4.5. A Matlab code was used for the generation of face normals of the sphere and the scanned data of the sphere used for this purpose. But from this doesn’t give us the magnitude of the change; so the better way is to compute the difference between the parameters of the two scan epochs.

**Figure 4.4** Image of a ball of the first scan as photographed by the internal camera of the laser scanner
Figure 4.5 Visualization of the movement of normal vectors of a spherical surface (face normal of each triangular surface) (a) for first scan epoch and (b) for second scan epoch (c) plot of (a) and (b).

In graphical visualization above Figure 4.5 (a) shows the normal of each triangular face of the sphere of scan epoch 1 and similarly Figure (b) shows the normal of each triangular face of a sphere of scan epoch 2. Figure (c) shows the plot of (a) and (b) together. As we can see from Figure (c) the blue and red arrows are not in the same position as well as orientation. So this can be used as an indicator for the presence of movement between scan epochs.

Segmentation of point clouds was made from both scan epochs of a ball to fit a sphere and estimate the parameters of the surface Figure 4.6. Like the planar case in the above the fitting of the sphere points is done both using Cyclone software function (fit to cloud) and Matlab code. The results obtained from Matlab are described in Table 4.10.

Figure 4.6 (a) Segmented point clouds, (b) Fitted sphere points of first scan epoch
Table 4.10 Comparison of manually and laser measured radius of the sphere.

<table>
<thead>
<tr>
<th></th>
<th>Radius From Manual Measurement (mm)</th>
<th>Radius from Laser Scanner Measurement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Scan Epoch</td>
<td>103.5</td>
<td>101.4</td>
</tr>
<tr>
<td>2nd Scan Epoch</td>
<td>101.0</td>
<td>99.8</td>
</tr>
<tr>
<td>(d_r) (m)</td>
<td>2.5</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 4.11 Estimated parameters, their standard errors and the fitting quality of a sphere.

<table>
<thead>
<tr>
<th></th>
<th>Value of parameter (m)</th>
<th>Standard Error of Parameter</th>
<th>Fit Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Error mean (m)</td>
</tr>
<tr>
<td>(X_c)</td>
<td>0.0486</td>
<td>0.000099</td>
<td>Error Std (m)</td>
</tr>
<tr>
<td>(Y_c)</td>
<td>-0.2387</td>
<td>0.000103</td>
<td>Abs Error (m)</td>
</tr>
<tr>
<td>(Z_c)</td>
<td>-2.9730</td>
<td>0.000220</td>
<td>Max Abs error (m)</td>
</tr>
<tr>
<td>(R)</td>
<td>0.1014</td>
<td>0.000033</td>
<td>Number of points</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Value of parameter (m)</th>
<th>Standard Error of Parameter</th>
<th>Fit Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Error mean (m)</td>
</tr>
<tr>
<td>(X_c)</td>
<td>0.0486</td>
<td>0.000098</td>
<td>Error Std (m)</td>
</tr>
<tr>
<td>(Y_c)</td>
<td>-0.2418</td>
<td>0.000099</td>
<td>Abs Error (m)</td>
</tr>
<tr>
<td>(Z_c)</td>
<td>-2.9724</td>
<td>0.000209</td>
<td>Max Abs error (m)</td>
</tr>
<tr>
<td>(R)</td>
<td>0.0998</td>
<td>0.000030</td>
<td>Number of points</td>
</tr>
</tbody>
</table>

The estimated parameters from the fitted spherical surface were used to detect the change between epochs 1 and 2. According to Eq. (3.23d) the parameters of the sphere are 4 but for the purpose of the change detection these parameters were divided into two parts, the first part containing the center coordinates of the sphere whereas the second part containing the radius of the sphere. In accordance to this the covariance matrix of the parameters can be expressed as

\[
C_x = \begin{bmatrix} C_{\text{coord}} & 0 \\ 0 & \sigma_r^2 \end{bmatrix}
\]

(4.12)

where \(C_{\text{coord}}\) is the variance covariance matrix of the center coordinates of the sphere and \(\sigma_r^2\) is the variance of radius of the sphere.

So \(\hat{X}_1\) and \(\hat{X}_2\) are the estimated parameters containing center coordinates of the sphere in epochs 1 and 2, respectively, obtained as

\[
\hat{X}_1 = [0.0486 - 0.2387 - 2.9300]^T
\]

and

\[
\hat{X}_2 = [0.0486 - 0.2418 - 2.9724]^T
\]
and their respective standard errors are computed as \( \sigma_1 = 0.24 \text{ mm} \) and \( \sigma_2 = 0.23 \text{ mm} \).

The difference between the parameters

\[
\mathbf{d}_{\mathbf{x}} = \mathbf{\bar{x}}_1 - \mathbf{\bar{x}}_2 = [0 \ 0.0031 \ -0.0006]^T \tag{4.13}
\]

is the measure of the translation of the center of the sphere. Its norm computed as

\[
\|\mathbf{d}_{\mathbf{x}}\| = \sqrt{\mathbf{d}_{\mathbf{x}x}^2 + \mathbf{d}_{\mathbf{x}y}^2 + \mathbf{d}_{\mathbf{x}z}^2} = 3.2 \text{ mm} \tag{4.14}
\]

The change in radius of the sphere between epochs 1 and 2 computed as

\[
d_r = r_2 - r_1 = 1.6 \text{ mm} \tag{4.15}
\]

where \( r_1 = 0.1014 \text{ m} \) and \( r_2 = 0.0998 \text{ m} \).

The covariance matrix of \( \mathbf{d}_{\mathbf{x}} \) can be obtained as

\[
\mathbf{C}_{\mathbf{d}_{\mathbf{x}}} = \mathbf{C}_{\mathbf{x}_1} + \mathbf{C}_{\mathbf{x}_2} = 10^{-7}\begin{bmatrix} 0.1954 & -0.0031 & -0.0205 \\ -0.0031 & 0.2051 & 0.0800 \\ -0.0205 & 0.0800 & 0.9206 \end{bmatrix} \tag{4.16}
\]

where

\[
\mathbf{C}_{\mathbf{x}_1} = 10^{-7}\begin{bmatrix} 0.0987 & -0.0022 & -0.0126 \\ -0.0022 & 0.1064 & 0.0474 \\ -0.0126 & 0.0474 & 0.4851 \end{bmatrix}
\]

and

\[
\mathbf{C}_{\mathbf{x}_2} = 10^{-7}\begin{bmatrix} 0.0967 & -0.0009 & -0.0079 \\ -0.0009 & 0.0987 & 0.0326 \\ -0.0079 & 0.0326 & 0.4355 \end{bmatrix}
\]

\( \mathbf{C}_{\mathbf{x}_1} \) and \( \mathbf{C}_{\mathbf{x}_2} \) are the propagated variance covariance matrices of the parameters of epochs 1 and 2, respectively, obtained from \( \mathbf{C}_{\text{coord}} \) of Eq. (4.14).

The standard error of \( \|\mathbf{d}_{\mathbf{x}}\| \), \( \sigma_{\|\mathbf{d}_{\mathbf{x}}\|} \) is computed according to the procedure presented in the planar surface case above.

Then the variance of \( \|\mathbf{d}_{\mathbf{x}}\| \) can be obtained from

\[
\sigma^2_{\|\mathbf{d}_{\mathbf{x}}\|} = \text{E}\{[\|\mathbf{d}_{\mathbf{x}}\| - \text{E}(\|\mathbf{d}_{\mathbf{x}}\|)]^2\} = \mathbf{A}\mathbf{C}_{\mathbf{d}_{\mathbf{x}}}\mathbf{A}^T \tag{4.17}
\]

where \( \mathbf{C}_{\mathbf{d}_{\mathbf{x}}} \) is the variance covariance matrix of the difference of the parameters, and \( \sigma^2_{\|\mathbf{d}_{\mathbf{x}}\|} \) is the variance of \( \|\mathbf{d}_{\mathbf{x}}\| \) and the value of \( \sigma_{\|\mathbf{d}_{\mathbf{x}}\|} \) is obtained as

\[
\sigma^2_{\|\mathbf{d}_{\mathbf{x}}\|} = \mathbf{A}\mathbf{C}_{\mathbf{d}_{\mathbf{x}}}\mathbf{A}^T = 0.02 \text{ mm}^2
\]
where \( A = [0.000 \ 0.9818 - 0.1900] \)
and \( \sigma_{\|d_X\|} = 0.14 \text{ mm} \)

The significance of the change in radius is also checked as follows. The variance of \( d_r \) can be obtained as
\[
\sigma_{d_r}^2 = \sigma_{r_1}^2 + \sigma_{r_2}^2 = 1.95 \times 10^{-9}
\]
where
\[
\sigma_{r_1}^2 = 1.06 \times 10^{-7} \quad \text{and} \quad \sigma_{r_2}^2 = 8.9 \times 10^{-10}
\]
and therefore
\[
\sigma_r = 0.04 \text{ mm} \quad \text{and} \quad 3\sigma_r = 0.12 \text{ mm}
\]

\( \sigma_{r_1}^2 \) and \( \sigma_{r_2}^2 \) are variances of the radius of the sphere in epochs 1 and 2, respectively, obtained from Eq. (4.12) and \( \sigma_r \) is the standard error of \( d_r \).

And hence the standard error of \( \|d_X\| \) is obtained as \( \sigma_{\|d_X\|} = 0.14 \text{ mm} \), a coverage factor 3 is chosen from the normal distribution table and the uncertainty limit become \( 3\sigma_{\|d_X\|} = 0.42 \text{ mm} \). So a significant change is detected between scan epochs with a confidence level of 99.7%. The magnitude \( \|d_X\| = 3.2 \text{ mm} \) shows translation of the center of the sphere. Therefore the origin of the sphere is moved by 3.2 mm and the size of the sphere is reduced by 1.6 mm between scan epochs.

**Cylindrical Surface**

Parameters of the cylinder were estimated from the segmented point clouds of two scan epochs. The same values of parameters were obtained by fitting points to the functional Cylinder surface model using cyclone and Matlab. The standard errors of each parameter are also computed for additional comparison of results. The values obtained from Matlab are presented in Table 4.12 and 4.13. In order to verify the measurements of laser scanner the radius and the height of the cylinder were measured independently using a measuring tape. Hence from independent measurement results, radius and height of the cylinder were measured as 0.054 m and 0.188 m for the first scan epoch and 0.05 m and 0.189 m for the second scan epoch, respectively. A comparison was made between manually and laser measured radii of the cylinder as presented in Table 4.12.

*Figure 4.7* Image of a cylinder of the first scan as photographed by the internal camera of the laser scanner.
Table 4.12 Comparison of manually and laser measured radius of the cylinder.

<table>
<thead>
<tr>
<th></th>
<th>Radius From Manual Measurement (m)</th>
<th>Radius from Laser Scanner Measurement (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Scan Epoch</td>
<td>0.0540</td>
<td>0.0513</td>
</tr>
<tr>
<td>2nd Scan Epoch</td>
<td>0.0500</td>
<td>0.0482</td>
</tr>
<tr>
<td>(d_r) (m)</td>
<td>0.0040</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

Table 4.13 Estimated parameters and their standard errors of a cylinder.

<table>
<thead>
<tr>
<th>Designation</th>
<th>First Epoch Scan</th>
<th>Second Epoch Scan</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_0) (m)</td>
<td>0.0723</td>
<td>0.0686</td>
</tr>
<tr>
<td>(Y_0) (m)</td>
<td>-0.1473</td>
<td>-0.1475</td>
</tr>
<tr>
<td>(Z_0) (m)</td>
<td>-3.0909</td>
<td>-3.0890</td>
</tr>
<tr>
<td>(a)</td>
<td>-1.2891</td>
<td>-1.4648</td>
</tr>
<tr>
<td>(b)</td>
<td>-0.0393</td>
<td>0.0554</td>
</tr>
<tr>
<td>(c)</td>
<td>1.1825</td>
<td>1.2707</td>
</tr>
<tr>
<td>(R) (m)</td>
<td>0.0513</td>
<td>0.0482</td>
</tr>
</tbody>
</table>

Table 4.14 Fitting quality of a cylinder from data points Cyclone results.

<table>
<thead>
<tr>
<th>Fitting Quality</th>
<th>First Epoch Scan</th>
<th>Second Epoch Scan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Mean (m)</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Error Std Deviation (m)</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Absolute Error Mean (m)</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>Maximum Absolute Error (m)</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>Number of points</td>
<td>228.00</td>
<td>228.00</td>
</tr>
</tbody>
</table>

Once the parameters are estimated from the fitted cylindrical surface the next step is to detect the change between scan epochs. Similar procedure was applied in here as it is presented in the numerical example of the cylinder model above.

So \(\hat{X}_{2(1)}\) and \(\hat{X}_{2(2)}\) are the estimated parameters of the cylinder containing plane coordinates of the axis of the cylinder in epochs 1 and 2, respectively, obtained as

\[
\hat{X}_{2(1)} = [0.0723 - 0.1473]^T \text{ m}
\]

and

\[
\hat{X}_{2(2)} = [0.0686 - 0.175]^T \text{ m}
\]

and their respective standard errors are computed as \(\sigma_1 = 2\) mm and \(\sigma_2 = 3.1\) mm.

The difference between the parameters

\[
d_X = \hat{X}_{2(1)} - \hat{X}_{2(2)} = [0.0037 \ 0.0002]^T \text{ m}
\]

(4.19)
is the measure of the translation of the axis of the cylinder. The norm computed as

$$\|d_x\| = \sqrt{d_{xx}^2 + d_{xy}^2} = 3.7 \text{ mm}$$ (4.20)

The change in radius of the cylinder between epochs 1 and 2 computed as

$$d_r = r_2 - r_1 = 3.1 \text{ mm}$$ (4.21)

where $r_1 = 0.0513 \text{ m}$ and $r_2 = 0.0482 \text{ m}$.

The covariance matrix of $d_x$ can be obtained as

$$C_{d_x} = C_{\bar{x}_{2(1)}} + C_{\bar{x}_{2(2)}} = 10^{-3} \begin{bmatrix} 0.0004 & 0.0000 \\ 0.0000 & 0.0003 \end{bmatrix}$$ (4.22)

where

$$C_{\bar{x}_{2(1)}} = 10^{-3} \begin{bmatrix} 0.0001 & 0.0000 \\ 0.0000 & 0.0001 \end{bmatrix}$$

and

$$C_{\bar{x}_{2(2)}} = 10^{-3} \begin{bmatrix} 0.0003 & 0.0000 \\ 0.0000 & 0.0002 \end{bmatrix}$$

$C_{\bar{x}_{2(1)}}$ and $C_{\bar{x}_{2(2)}}$ are the propagated variance covariance matrices of the plane coordinates of the axis of the cylinder in epochs 1 and 2, respectively, which is the same as $C_{c_{a}}$ in Eq. (3.35).

The standard error of $\|d_x\|$, $\sigma_{\|d_x\|}$ is a function of $d_{xx}$ and $d_{xy}$, which is computed as per the procedure discussed in the numerical example of cylinder model above.

Then the variance of $\|d_x\|$ can be obtained from

$$\sigma_{\|d_x\|}^2 = E\{[\|d_x\| - E(\|d_x\|)]^2\} = AC_{d_x}A^T$$ (4.23)

where $C_{d_x}$ is the variance covariance matrix of the difference of the plane coordinates of the axis of the cylinder, and $\sigma_{\|d_x\|}^2$ is the variance of $\|d_x\|$ and the value of $\sigma_{\|d_x\|}$ obtained as

$$\sigma_{\|d_x\|}^2 = AC_{d_x}A^T = 0.4 \text{ mm}^2 \quad \text{where} \quad A = [0.9985 \ 0.0540]$$

and

$$\sigma_{\|d_x\|} = 0.6 \text{ mm}$$

The significance of the change in radius is also checked as follows. The variance of $d_r$ can be obtained as

$$\sigma_{d_r}^2 = \sigma_{r_1}^2 + \sigma_{r_2}^2 = 3 \times 10^{-7}$$ (4.24)

where

$$\sigma_{r_1}^2 = 10^{-7} \quad \text{and} \quad \sigma_{r_2}^2 = 2 \times 10^{-7}$$

and therefore
\[ \sigma_r = 0.55 \text{ mm and } 3\sigma_r = 1.65 \text{ mm} \]

\[ \sigma_{r_1}^2 \text{ and } \sigma_{r_2}^2 \text{ are the variances of the radius of the cylinder in epochs 1 and 2, respectively and } \sigma_r \text{ is the standard error of } d_r. \]

The standard error \( \sigma_{\|d_X\|} \) is obtained as 0.6 mm and a coverage factor 3 is chosen from the normal distribution table and the uncertainty limit become \( 3\sigma_{\|d_X\|} = 1.8 \text{ mm} \). So a significant change is detected between epochs 1 and 2 with a confidence level of 99.7 %. The magnitude \( \|d_X\| = 3.7 \text{ mm} \) shows a translation on the x-y plane that means the axis of the cylinder is moved by 3.7 mm and the size of the cylinder is changed by 3.1 mm.

**4.4- Analysis of the change in orientation**

In this section the change in orientation of the plane was analyzed by using the normal of the surface shown in Figure 4.8. A data from the planar surface experiment was taken to do the analysis. In this analysis we check only the change in orientation of the plane, but not for the change in position (translation). The change in orientation of the plane can be expressed by the change in orientation angles \( \beta \) and \( \alpha \), which are the vertical angle and the horizontal direction of the normal vector of the plane. The normal of the plane was already computed in section 4.3 so it can be possible to compute the orientation angles \( \beta \) and \( \alpha \) using the computed parameters. \( \beta \) and \( \alpha \) are functions of the surface parameters \((a \ b \ c)\) and \((a \ b)\), respectively. They can be obtained by using a formula derived through the trigonometric relationship

\[ \beta = \arcsin\left( \frac{c}{\sqrt{a^2+b^2+c^2}} \right) \]  
and

\[ \alpha = \arcsin\left( \frac{b}{\sqrt{a^2+b^2}} \right) \]

\[ \text{Figure 4.8 Analysis of change in orientation of the normal of a plane between scan epochs.} \]
The propagated variance covariance matrix of the orientation angles $\beta$ and $\alpha$ can be obtained as
\[
C_{\beta\alpha} = JQ_{xx}J^T = \begin{bmatrix}
\frac{\partial \beta}{\partial a} & \frac{\partial \beta}{\partial b} & \frac{\partial \beta}{\partial c} \\
\frac{\partial \alpha}{\partial a} & \frac{\partial \alpha}{\partial b} & \frac{\partial \alpha}{\partial c}
\end{bmatrix}
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \beta}{\partial a} & \frac{\partial \beta}{\partial b} & \frac{\partial \beta}{\partial c} \\
\frac{\partial \alpha}{\partial a} & \frac{\partial \alpha}{\partial b} & \frac{\partial \alpha}{\partial c}
\end{bmatrix}^T = \begin{bmatrix}
\sigma_{\beta}^2 & \sigma_{\beta}\sigma_{\alpha} \\
\sigma_{\alpha}\sigma_{\beta} & \sigma_{\alpha}^2
\end{bmatrix} \tag{4.33}
\]

where $J$ is a matrix containing the derivatives of Eq. (4.31) and Eq. (4.32) and $Q_{xx}$ is the cofactor matrix of the plane parameters $(a, b, c)$.

Using Eq. (4.31), Eq. (4.32) and Eq. (4.33) the values for the two orientation angles and their standard errors are computed and presented in Table 4.14. The change in orientation angle analysis is performed for both segments of the carton but only the result of segment 1 is presented here.

**Table 4.15** Orientation angles and their standard error of the normal of a planar surface of first and second scan epochs of segment 1.

<table>
<thead>
<tr>
<th>Segment 1</th>
<th>1st Epoch Scan</th>
<th>2nd Epoch Scan</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ (Degree)</td>
<td>-0.126</td>
<td>0.178</td>
</tr>
<tr>
<td>$\alpha$ (Degree)</td>
<td>-83.362</td>
<td>-83.460</td>
</tr>
<tr>
<td>$\sigma_{\beta}$ (Degree)</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\sigma_{\alpha}$ (Degree)</td>
<td>2.836</td>
<td>2.882</td>
</tr>
</tbody>
</table>

The difference in orientation angles $\beta$ and $\alpha$ between epochs 1 and 2 computed as
\[
d_{\beta} = \beta_2 - \beta_1 = 0^\circ.304
\]
and
\[
d_{\alpha} = \alpha_2 - \alpha_1 = 0^\circ.098
\]

where $\beta_1, \beta_2, \alpha_1$ and $\alpha_2$ are computed orientation angles in epochs 1 and 2.

The standard error of the orientation angles $\beta$ and $\alpha$ obtained as $\sigma_{\beta} = 0^\circ.001$ and $\sigma_{\alpha} = 2^\circ.836$. Then a coverage factor 3 is chosen from the normal distribution table in order to determine the uncertainty limits. Then the uncertainty limit becomes $3\sigma_{\beta} = 0^\circ.003$ and $3\sigma_{\alpha} = 8^\circ.508$, so a significant change is detected only in the orientation angle $\beta$ with a confidence level of 99.7%, which means the orientation of the normal of the plane is changed only in the vertical direction by $0^\circ.304 (18.24')$.

### 5- Precision analysis

In this chapter precision analysis is performed to check the achievable quality of the planar surface parameters. Cyrax 2500 laser scanner is used for the measurement and a single measurement precision analysis is applied in here. This type of scanner allows only a single
range measurement to a single point. The parameters of the plane and their precision were estimated in chapter 4 through analytical method using the scanned data of the plane. But in here it is necessary to estimate the precision of the parameters theoretically, in order to make a comparison with the precision obtained from the experimental data. The final goal of this precision analysis is to determine the theoretical minimum detectable difference that can be obtained from the change of a normal of a plane determined by LSQ fit to scanned data. So for the detail analysis approximate parameters are considered as mentioned above which are taken from estimated parameters of the plane from experimental data. A laser data from a scan of a carton box of segment 1 only is used for this particular analysis, in which the point cloud is exported as a text file in the form of Cartesian coordinate.

Precision is a parameter that describes the degree of repeatability in measurement. Usually precision is mixed up with accuracy in the system specification. It can be applied to check the quality of raw data; processed data and 3D model as well. As it is discussed in Reshetyuk (2006) there are two kinds of precision, single measurement and averaged measurement precision. The averaged measurement precision is associated with a number of measurements collected at the same point and averaged. Some scanners, e.g. MENTI GS 100, allow a multiple range measurements (shots) to a single point.

The more realistic estimate of precision of 3D point coordinates would be a 3x3 covariance matrix described in Eq. (5.1). This matrix can be obtained by propagating the individual errors associated with the factors influencing the data accuracy. The estimate of precision of a 3D point can be expressed as

$$
\mathbf{C}_{\text{3DPoint}} = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix}
$$

(5.1)

If all significant systematic errors influencing the TLS measurements have been corrected for, and the standard errors in their determination are included in the computation of the standard error matrix $\mathbf{C}_{\text{3DPoint}}$, then Eq. (5.1) will give the expression for the 3D point coordinate accuracy – see Reshetyuk (2006).

The measured coordinates of a point can expressed as

$$
\mathbf{X}_j = \begin{bmatrix}
x_j \\
y_j \\
z_j
\end{bmatrix} = \begin{bmatrix}
r_j \cos \varphi_j \cos \theta_j \\
r_j \sin \varphi_j \cos \theta_j \\
r_j \sin \theta_j
\end{bmatrix}
$$

(5.2)

where $\mathbf{X}_j$ is the coordinate vector of point $j$ in the scanner system and $r_j$, $\varphi_j$ and $\theta_j$ are the measured range, horizontal direction and the vertical angle of the $j^{th}$ point in the point cloud.

The variance covariance matrix of the coordinate vector $\mathbf{X}_j$ can be computed as (Reshetyuk 2006)
\[ C_{x_j} = J C_{\text{inst}} J^T \]

\[
= \begin{bmatrix}
\frac{\partial x_j}{\partial r_j} & \frac{\partial x_j}{\partial \phi_j} & \frac{\partial x_j}{\partial \theta_j} \\
\frac{\partial y_j}{\partial r_j} & \frac{\partial y_j}{\partial \phi_j} & \frac{\partial y_j}{\partial \theta_j} \\
\frac{\partial z_j}{\partial r_j} & \frac{\partial z_j}{\partial \phi_j} & \frac{\partial z_j}{\partial \theta_j}
\end{bmatrix}
\begin{bmatrix}
\sigma_r^2 + \sigma_{\text{beam}}^2 \\
\sigma_{\phi}^2 \\
\sigma_{\theta}^2
\end{bmatrix}
\begin{bmatrix}
\frac{\partial r_j}{\partial x_j} & \frac{\partial \phi_j}{\partial x_j} & \frac{\partial \theta_j}{\partial x_j} \\
\frac{\partial r_j}{\partial y_j} & \frac{\partial \phi_j}{\partial y_j} & \frac{\partial \theta_j}{\partial y_j} \\
\frac{\partial r_j}{\partial z_j} & \frac{\partial \phi_j}{\partial z_j} & \frac{\partial \theta_j}{\partial z_j}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\sigma_{x_j}^2 & \sigma_{x_jy_j} & \sigma_{x_jz_j} \\
\sigma_{y_jx_j} & \sigma_{y_j}^2 & \sigma_{y_jz_j} \\
\sigma_{z_jx_j} & \sigma_{z_jy_j} & \sigma_{z_j}^2
\end{bmatrix}
\]

(5.3)

where \( J \) is a matrix containing the derivatives of Eq. (5.2) and \( \sigma_r^2, \sigma_{\phi}^2 \) and \( \sigma_{\theta}^2 \) are variances of the measured range, horizontal direction and the vertical angle of the \( j \)-th point in the point cloud, respectively. The variances are usually obtained in the manufacturers specification and \( \sigma_{\text{beam}} \) is the uncertainty due to the beam width. Hence in this analysis the value of \( \sigma_r \) is taken as 4 mm and \( \sigma_{\phi} \) and \( \sigma_{\theta} \) are taken as 60 micro rad as stated in the manufacturers specification of laser scanner HDS 2500. The uncertainty due to the beam width is considered as zero in this case. So the precision of each measured point, which is a 3x3 covariance matrix, can be computed using Eq. (5.3). So the variance covariance matrix of the whole measured data points can be computed as

\[
C_{LL(n,n)} = \begin{bmatrix}
C_{11} & 0 & 0 & \ldots & 0 \\
0 & C_{22} & 0 & \ldots & 0 \\
0 & 0 & C_{33} & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & C_{nn}
\end{bmatrix}
\]

(5.4)

where \( C_{11}, C_{12}, C_{13} \) and \( C_{nn} \) are a 3x3 variance covariance matrix of each point.

A condition adjustment with unknown method is used as described below in order to find the quality of the parameters. Based on the data used for this analysis we have 864 points i.e. the number of conditional equations also 864:

\[
ax_1 + by_1 + cz_1 + 1 = 0 \\
ax_2 + by_2 + cz_2 + 1 = 0 \\
\vdots \\
ax_n + by_n + cz_n + 1 = 0
\]

(5.6)

In order to find the variance covariance matrix of the estimated parameters of the plane the following mathematical model is introduced (Leick 1995)
\[ F(L_a, X_a) = 0 \quad \text{or} \quad F(L_b + V, X_0 + X) = 0 \] (5.7)

where \( X = X_a - X_0 \), \( X_a \) and \( X_0 \) are the vectors containing adjusted and approximate parameters of the plane, respectively, \( L_b \) is an observation vector and \( V \) is the residual vector.

The approximate parameters of the plane taken as

\[ X_{0(r,1)} = \begin{bmatrix} a_0 & b_0 & c_0 \end{bmatrix}^T = \begin{bmatrix} 0.0256 & -0.2202 & -0.0005 \end{bmatrix}^T \]

and the observation vector \( L_b \) can be

\[ L_b = \begin{bmatrix} x_1 & y_1 & z_1 & x_2 & y_2 & z_2 & \cdots & x_n & y_n & z_n \end{bmatrix}^T \]

Since Eq. (5.7) is a non-linear function, can be linearized as

\[ F(L_a, X_a) = F(L_b, X_0) + \left( \frac{\partial F}{\partial L} \right) V + \left( \frac{\partial F}{\partial X} \right) X \] (5.8)

and Eq. (5.8) can be expanded in a linear form as

\[ ax_i + by_i + cz_i + 1 = a_0x_i + b_0y_i + c_0z_i + a_0v_{x_i} + b_0v_{y_i} + c_0v_{z_i} + x_i\Delta_a + y_i\Delta_b + z_i\Delta_c = 0 \] (5.9)

Finally Eq. (5.9) can be written in matrix form as

\[ B_{(n,u)} V_{(n,1)} + A_{(n,r)} X_{(r,1)} + W_{n,1} = 0 \] (5.10)

where \( V \) denote the vector of the residuals, and \( W \) denotes the value of the mathematical function,

\[ A_{(n,r)} = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ \vdots \\ x_r & y_r & z_r \end{bmatrix} \]

is the matrix containing the data points and

\[ B_{(n,u)} = \begin{bmatrix} a_0 & b_0 & c_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & a_0 & b_0 & c_0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & a_0 & b_0 & c_0 \end{bmatrix} \]

is the matrix containing the approximate parameters of the plane as block diagonal element, \( r \) is the number of unknowns, \( n \) is the number of points and \( u=3n \).

Then from the law of error propagation the cofactor matrix of the parameters can be obtained as

\[ Q_{xx} = (A^T M^{-1} A)^{-1} \] (5.11)
where $\mathbf{M} = \mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T$, $\mathbf{P}$ is the weight matrix of the parameters

since $\sigma_0 = 1$ for this analysis, then $\mathbf{C}_{LL} = \sigma_0^2 \mathbf{Q}_{LL} = \mathbf{Q}_{LL}$ and $\mathbf{Q}_{LL} = \mathbf{P}^{-1} = \mathbf{C}_{LL}$.

Finally the variance covariance matrix of the approximate parameters of the plane can be obtained as

$$
\mathbf{C}_{xx} = \sigma_0^2 \mathbf{Q}_{xx} = (\mathbf{A}^T \mathbf{M}^{-1} \mathbf{A})^{-1} = 10^{-6} \begin{bmatrix} 0.3479 & 0.2603 & -0.0052 \\ 0.2603 & 0.2140 & -0.0827 \\ -0.0052 & -0.0827 & 0.3287 \end{bmatrix}
$$

(5.12)

So the square root of the diagonal elements of $\mathbf{C}_{xx}$ give us the quality of the estimated planar surface parameters as $\sigma_a = 0.00059$, $\sigma_b = 0.00046$ and $\sigma_c = 0.00057$. This result are compared with the one obtained from the actual data as presented in Table 4.16.

**Table 4.16** Comparison of the precision of planar surface parameters computed from experimental data and theoretical analysis.

<table>
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<tr>
<th>Designated Standard Error</th>
<th>From experimental data</th>
<th>From theoretical analysis</th>
</tr>
</thead>
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<tr>
<td>$\sigma_a$</td>
<td>0.00014</td>
<td>0.00059</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.00011</td>
<td>0.00046</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>0.00013</td>
<td>0.00057</td>
</tr>
</tbody>
</table>

Once we have the variance covariance matrix of the parameters then we can compute the minimum difference of $\mathbf{d}_{\bar{R}}$, which indicate the motion of a plane.

The variance covariance matrix of $\mathbf{d}_{\bar{R}}$ is obtained as

$$
\mathbf{C}_{\bar{R}} = 2\mathbf{C}_{\bar{R}_1} = 10^{-3} \begin{bmatrix} 0.1416 & 0.1059 & -0.0021 \\ 0.1059 & 0.0871 & -0.0337 \\ -0.0021 & -0.0337 & 0.1338 \end{bmatrix}
$$

(5.13)

where

$$
\mathbf{C}_{\bar{R}_1} = \left( \frac{1}{||\hat{\mathbf{X}}||^2} \right) \mathbf{C}_{xx} = 10^{-4} \begin{bmatrix} 0.7079 & 0.5297 & -0.0106 \\ 0.5297 & 0.4355 & -0.1683 \\ -0.0106 & -0.1683 & 0.6689 \end{bmatrix}
$$

(5.14)

and $\mathbf{C}_{\bar{R}_1} = \mathbf{C}_{\bar{R}_2}$.

According to Eq. (4.12), Eq. (4.13) and Eq. (4.14) of the planar case analysis in chapter 4 standard error of $||\mathbf{d}_{\bar{R}}||$ is computed as

$$
\sigma_{||\mathbf{d}_{\bar{R}}||}^2 = \mathbf{A} \mathbf{C}_{\bar{R}} \mathbf{A}^T = 0.0000031 \text{ rad}^2
$$

where

$$
\mathbf{A} = \begin{bmatrix} 0.0256 & -0.2202 & -0.0005 \end{bmatrix}
$$

and $\sigma_{||\mathbf{d}_{\bar{R}}||} = 0.0018 \text{ rad}$
Since we assume that the residuals are normally distributed, a coverage factor 3 is chosen from the normal distribution table, then the uncertainty limit become \(3\sigma_{d} = 0.0053 \text{ rad}\). So the minimum difference of \(d_{R}\) that indicates the rotation of the plane determined as 0.0053 rad (18.22').

6- Results and Discussions

In the planar surface experiment the direct range comparison analysis was performed as the result presented in Table 4.1. The range differences between the marked points were computed to find the movements of each individual point between scan epochs. A movement was not detected using direct comparison of observed points but the deformation was detected by other method using a data from the same measurement epoch.

As the result described in section 4.3 the coordinate difference of each marked point was computed for both total station and the scanner measurements. The results from the two measuring instruments were compared and the coordinate difference obtained from total station measurement is slightly less than the one obtained from the laser scanner. A difference of the coordinate differences from the measurements of laser scanner and total station was computed. Significant coordinate differences were obtained only in z coordinate differences according to the statistical test result presented in section 4.1. A maximum of 8 mm difference was obtained for a point ID A113.

The main method used to detect changes between scan epochs is the estimation of parameters from modeled surfaces. This method was used for all type of geometrical shapes of objects. As it is presented in Table 4.2 the parameters of the planar surface were estimated for both scan epochs of a carton. The norm of the change in parameters shows the movement of the normal of the planar surface between scans epochs. As the statistical test result shows that the change detected between epochs is significant. As the result obtained using laser scanner data the normal of a plane is rotated by 19.94' between scan epochs. In order to verify the measurements of the laser scanner the change detection also performed using the data from total station measurements. Similar rotation of the normal of the plane was obtained from the total station measurement.

In similar way surface parameters were estimated from both spherical and cylindrical surfaces in order to determine the change between scan epochs. Similar procedure was used as the planar surface case in order to check movements between scan epochs. Using a data measured by the laser scanner a change detection analysis was performed. And as the statistical test results in section 4.2 shows that the center of the sphere and the axis of the cylinder were moved by 3.2 and 3.7 millimetres between scan epochs, respectively. And the size of the sphere and the cylinder were changed by 1.6 and 3.7 millimetres between scan epochs, respectively. And the scanner measurements were verified by performing independent measurements of the radius of the sphere and the cylinder using a measuring tape. And the changes in radii of 2.5 and 4 millimetres were obtained between scan epochs for sphere and cylinder, respectively.
From the analysis of the change in orientation of the normal of the plane significant change is detected only in the vertical direction of the normal of the plane whereas no significant change has been detected in the horizontal direction of the normal of a plane.

Finally from precision analysis the expected quality of the parameters of the plane was estimated. A comparison between the results from the actual data and results obtained from the theoretical analysis is presented in Table 4.16. Finally the minimum detectable difference of the normal of the plane is determined as 18.22′, which indicate the motion of a plane.

7- Conclusions

The use of laser scanner in deformation monitoring and estimating were reviewed and discussed in this paper. Calibrations of instruments are very vital in order to avoid systematic errors since estimation of deformation requires a significant measurement precision. From the practical experiment performed in the laboratory a precision to the mm level was obtained from the laser scanner measurement. Previous studies made by other people indicate that the terrestrial laser scanner has equivalent capacity as other instruments in estimating deformations.

Similar precisions of measurements were obtained between the scanner measurements and the conventional surveying method, even though the conventional surveying method is superior in measurement precision of a single point. As it is discussed in section 4.1 a movement could not be detected from direct comparison of observed points, but a movement was detected from the modeled surface using data from the same measurement epochs. So this indicates that estimating deformation from a modeled surface is more advantageous than comparing a single point for reliable deformation analysis. Obtaining a set of dense 3D data from the surface of a deforming object within a short period of time makes the laser scanning an advanced technology in monitoring of deformation. The laser scanner also provided better measurement results than measurements performed by a measuring tape as our comparison has shown in section 4.3. Our experimental results show that the scanner used in this study is capable of measuring a movement of ± 3.2 mm when the change is estimated from the surface. Finally the statistical analysis is very important in order to assess the results obtained from the laser data.
Bibliography


Fabio R (2003). From Point Cloud to Surface: The modeling and Visualization Problem. Institute of Geodesy and Photogrammetry, Swiss Federal Institute of Technology, ETH Zurich, Switzerland.


Appendix 1
Simulated input data for numerical example of planar surface of first set of points.

| \(C_1=\) | 1.500 | 1.000 | -2.500 |
| 2.000 | 3.000 | -2.500 |
| 3.000 | 1.500 | -2.500 |
| 3.000 | 2.000 | -2.500 |
| 3.500 | 4.000 | -2.500 |
| 4.000 | 2.500 | -2.500 |

| \(\delta_1=\) | 0.007 | 0.004 | 0.012 |
| -0.001 | 0.000 | 0.003 |
| -0.009 | 0.006 | 0.006 |
| -0.003 | -0.008 | -0.004 |
| 0.005 | -0.001 | -0.002 |
| -0.004 | -0.005 | -0.001 |

| \(A_1=C_1+\delta_1=\) | 1.507 | 1.004 | -2.488 |
| 1.999 | 3.000 | -2.497 |
| 2.991 | 1.506 | -2.494 |
| 2.997 | 1.992 | -2.504 |
| 3.505 | 3.999 | -2.502 |
| 3.996 | 2.495 | -2.501 |

Appendix 2
Simulated input data for numerical example of planar surface of second set of points.

| \(C_2=\) | 1.000 | 3.000 | -2.500 |
| 1.000 | 4.500 | -2.500 |
| 2.500 | 2.000 | -2.500 |
| 2.500 | 4.000 | -2.500 |
| 3.500 | 3.000 | -2.500 |
| 4.000 | 5.000 | -2.500 |

| \(\delta_2=\) | 0.006 | 0.002 | 0.004 |
| 0.006 | 0.004 | -0.005 |
| 0.003 | 0.003 | -0.004 |
| -0.005 | -0.001 | -0.003 |
| 0.003 | 0.001 | -0.012 |
| 0.007 | -0.003 | 0.006 |
Appendix 3
Simulated input data for numerical example of cylindrical surface first set of points.

\[ A_z = C_z + \delta_z = \begin{array}{ccc}
1.006 & 3.002 & -2.496 \\
1.006 & 4.504 & -2.505 \\
2.503 & 2.003 & -2.504 \\
2.495 & 3.999 & -2.503 \\
3.503 & 3.001 & -2.512 \\
4.007 & 4.997 & -2.494 \\
\end{array} \text{ m} \]

\[ C_1 = \begin{array}{ccc}
0.3708 & 1.1413 & 0.0000 \\
-0.9708 & 0.7053 & 0.0000 \\
-0.9708 & -0.7053 & 0.0000 \\
0.3708 & -1.1413 & 0.0000 \\
1.2000 & 0.0000 & 0.0000 \\
0.3708 & 1.1413 & 0.0020 \\
-0.9708 & 0.7053 & 0.0020 \\
-0.9708 & -0.7053 & 0.0020 \\
0.3708 & -1.1413 & 0.0020 \\
1.2000 & 0.0000 & 0.0020 \\
\end{array} \text{ m} \]

\[ \delta_1 = \begin{array}{ccc}
-0.002 & 0.016 & 0.002 \\
0.002 & 0.006 & 0.011 \\
0.008 & -0.004 & -0.011 \\
-0.006 & -0.001 & 0.000 \\
-0.004 & 0.002 & 0.000 \\
0.024 & 0.002 & -0.019 \\
-0.005 & 0.010 & 0.010 \\
0.015 & 0.009 & 0.023 \\
0.008 & -0.003 & -0.002 \\
-0.005 & -0.001 & 0.002 \\
-0.001 & 0.006 & -0.004 \\
-0.002 & 0.001 & 0.003 \\
0.004 & 0.009 & -0.003 \\
0.011 & -0.006 & -0.001 \\
-0.013 & 0.004 & 0.013 \\
\end{array} \text{ m} \]
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Appendix 4
A matrix containing coordinate values of the first set of points in numerical example of cylindrical surface

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Appendix 5
Simulated input data for numerical example of cylindrical surface second set of points.

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\[ C_2 = \begin{array}{ccc} 0.371 & 1.141 & 0.003 \\ -0.971 & 0.705 & 0.003 \\ -0.971 & -0.705 & 0.003 \\ 0.371 & -1.141 & 0.003 \\ 1.200 & 0.000 & 0.003 \\ 0.371 & 1.141 & 0.006 \\ -0.971 & 0.705 & 0.006 \\ -0.971 & -0.705 & 0.006 \\ 0.371 & -1.141 & 0.006 \\ 1.200 & 0.000 & 0.009 \end{array} \]

\[ \delta_2 = \begin{array}{ccc} 0.013 & -0.014 & 0.002 \\ -0.003 & -0.001 & 0.001 \\ 0.001 & 0.003 & -0.009 \\ -0.005 & 0.003 & -0.005 \\ -0.001 & 0.009 & -0.005 \\ 0.006 & 0.003 & 0.008 \\ -0.010 & -0.001 & -0.001 \\ 0.002 & 0.008 & 0.005 \\ -0.006 & 0.017 & 0.005 \\ 0.006 & 0.000 & -0.002 \\ -0.016 & -0.002 & -0.016 \\ -0.016 & 0.008 & 0.003 \\ 0.013 & -0.004 & 0.006 \\ -0.001 & -0.012 & -0.006 \\ -0.016 & -0.001 & 0.000 \end{array} \]

\[ m \]

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Appendix 6
A matrix containing coordinate values of the second set of points in numerical example of cylindrical surface

\[
\begin{array}{ccccccccc}
1.270 & 0.000 & 0.435 & 0.000 & 0.000 & 0.386 & 1.127 & 0.000 & 1.000 \\
0.495 & 0.000 & -0.683 & 0.001 & -0.001 & -0.971 & 0.704 & -0.001 & 1.000 \\
0.496 & 0.000 & 0.681 & 0.011 & 0.008 & -0.968 & -0.704 & -0.011 & 1.000 \\
1.299 & 0.000 & -0.420 & -0.003 & 0.008 & 0.368 & -1.140 & -0.007 & 1.000 \\
0.000 & 0.000 & 0.009 & -0.009 & 0.000 & 1.202 & 0.007 & -0.008 & 1.000 \\
1.306 & 0.000 & 0.433 & 0.003 & 0.010 & 0.379 & 1.143 & 0.009 & 1.000 \\
0.495 & 0.000 & -0.689 & 0.000 & 0.000 & -0.979 & 0.704 & 0.000 & 1.000 \\
0.487 & 0.000 & 0.675 & -0.006 & -0.004 & -0.967 & -0.698 & 0.006 & 1.000 \\
1.268 & 0.000 & -0.413 & 0.002 & -0.007 & 0.367 & -1.126 & 0.006 & 1.000 \\
0.000 & 0.000 & -0.001 & -0.001 & 0.000 & 1.208 & -0.001 & -0.001 & 1.000 \\
1.296 & 0.000 & 0.407 & -0.004 & -0.014 & 0.357 & 1.138 & -0.012 & 1.000 \\
0.508 & 0.000 & -0.702 & -0.007 & 0.005 & -0.984 & 0.713 & 0.007 & 1.000 \\
0.505 & 0.000 & 0.679 & -0.009 & -0.007 & -0.956 & -0.711 & 0.010 & 1.000 \\
1.332 & 0.000 & -0.429 & -0.001 & 0.002 & 0.372 & -1.154 & -0.002 & 1.000 \\
0.000 & 0.000 & -0.002 & 0.005 & 0.000 & 1.186 & -0.002 & 0.004 & 1.000 \\
\end{array}
\]
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